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Michael D. Lee and Eric-Jan Wagenmakers
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Bayesian Cognitive Modeling

A Practical Course

Bayesian inference has become a standard method of analysis in many fields of science. Students and researchers in experimental psychology and cognitive science, however, have failed to take full advantage of the new and exciting possibilities that the Bayesian approach affords. Ideal for teaching and self study, this book demonstrates how to do Bayesian modeling. Short, to-the-point chapters offer examples, exercises, and computer code (using WinBUGS or JAGS, and supported by Matlab and R), with additional support available online. No advance knowledge of statistics is required and, from the very start, readers are encouraged to apply and adjust Bayesian analyses by themselves. The book contains a series of chapters on parameter estimation and model selection, followed by detailed case studies from cognitive science. After working through this book, readers should be able to build their own Bayesian models, apply the models to their own data, and draw their own conclusions.

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ERIC-JAN WAGENMAKERS



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For Colleen and David, and Helen and Mitchell — Michael

Preface

This book, together with the code, answers to questions, and other material at www.bayesmodels.com, teaches you how to do Bayesian modeling. Using modern computer software—and, in particular, the WinBUGS program—this turns out to be surprisingly straightforward. After working through the examples provided in this book, you should be able to build your own models, apply them to your own data, and draw your own conclusions.

This book is based on three principles. The first is that of *accessibility*: the book's only prerequisite is that you know how to operate a computer; you do not need any advanced knowledge of statistics or mathematics. The second principle is that of *applicability*: the examples in this book are meant to illustrate how Bayesian modeling can be useful for problems that people in cognitive science care about. The third principle is that of *practicality*: this book offers a hands-on, “just do it” approach that we feel keeps students interested and motivated.

In line with these three principles, this book has little content that is purely theoretical. Hence, you will not learn from this book why the Bayesian philosophy to inference is as compelling as it is; neither will you learn much about the intricate details of modern sampling algorithms such as Markov chain Monte Carlo, even though this book could not exist without them.

The goal of this book is to facilitate and promote the use of Bayesian modeling in cognitive science. As shown by means of examples throughout this book, Bayesian modeling is ideally suited for applications in cognitive science. It is easy to construct a basic model, and then add individual differences, add substantive prior information, add covariates, add a contaminant process, and so on. Bayesian modeling is flexible and respects the complexities that are inherent in the modeling of cognitive phenomena.

We hope that after completing this book, you will have gained not only a new understanding of statistics (yes, it can make sense), but also the technical skills to implement statistical models that professional but non-Bayesian cognitive scientists dare only dream about.

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The material in this book is not independent of our publications in the cognitive science literature. Sometimes, an article was turned into a book chapter; at other times, a book chapter spawned an article. Here we would like to acknowledge our published articles that contain text and figures resembling, to varying degrees, those used in this book. These articles often may be consulted for a more extensive and formal exposition of the material at hand.

Chapter 1: The basics of Bayesian analysis

- Wagenmakers, E.-J., Lodewyckx, T., Kuriyal, H., & Grasman, R. (2010). Bayesian hypothesis testing for psychologists: A tutorial on the Savage–Dickey method. *Cognitive Psychology*, *60*, 158–189.

Chapter 6: Latent-mixture models

- Ortega, A., Wagenmakers, E.-J., Lee, M. D., Markowitsch, H. J., & Piefke, M. (2012). A Bayesian latent group analysis for detecting poor effort in the assessment of malingering. *Archives of Clinical Neuropsychology*, *27*, 453–465.

Chapter 7: Bayesian model comparison

- Scheibehenne, B., Rieskamp, J., & Wagenmakers, E.-J. (2013). Testing adaptive toolbox models: A Bayesian hierarchical approach. *Psychological Review*, *120*, 39–64.

- Wagenmakers, E.-J., Lodewyckx, T., Kuriyal, H., & Grasman, R. (2010). Bayesian hypothesis testing for psychologists: A tutorial on the Savage–Dickey method. *Cognitive Psychology*, *60*, 158–189.

Chapter 8: Comparing Gaussian means

- Wetzels, R., Raaijmakers, J. G. W., Jakab, E., & Wagenmakers, E.-J. (2009). How to quantify support for and against the null hypothesis: A flexible WinBUGS implementation of a default Bayesian *t* test. *Psychonomic Bulletin & Review*, *16*, 752–760.

Chapter 9: Comparing binomial rates

- Wagenmakers, E.-J., Lodewyckx, T., Kuriyal, H., & Grasman, R. (2010). Bayesian hypothesis testing for psychologists: A tutorial on the Savage–Dickey method. *Cognitive Psychology*, *60*, 158–189.

Chapter 10: Memory retention

- Shiffrin, R. M., Lee, M. D., Kim, W., & Wagenmakers, E.-J. (2008). A survey of model evaluation approaches with a tutorial on hierarchical Bayesian methods. *Cognitive Science*, *32*, 1248–1284.

Chapter 11: Signal detection theory

- Lee, M. D. (2008). BayesSDT: Software for Bayesian inference with signal detection theory. *Behavior Research Methods*, *40*, 450–456.
- Lee, M. D. (2008). Three case studies in the Bayesian analysis of cognitive models. *Psychonomic Bulletin & Review*, *15*, 1–15.

Chapter 13: Extrasensory perception

- Wagenmakers, E.-J. (2012). Can people look into the future? Contribution in honor of the University of Amsterdam's 76th lustrum.
- Wagenmakers, E.-J., Wetzels, R., Borsboom, D., van der Maas, H. L. J., & Kievit, R. A. (2012). An agenda for purely confirmatory research. *Perspectives on Psychological Science*, *7*, 627–633.

Chapter 14: Multinomial processing trees

- Matzke, D., Dolan, C. V., Batchelder, W. H., & Wagenmakers, E.-J. (in press). Bayesian estimation of multinomial processing tree models with heterogeneity in participants and items. *Psychometrika*.

Chapter 15: The SIMPLE model of memory

- Shiffrin, R. M., Lee, M. D., Kim, W., & Wagenmakers, E.-J. (2008). A survey of model evaluation approaches with a tutorial on hierarchical Bayesian methods. *Cognitive Science*, *32*, 1248–1284.

Chapter 16: The BART model of risk taking

- van Ravenzwaaij, D., Dutilh, G., & Wagenmakers, E.-J. (2011). Cognitive model decomposition of the BART: Assessment and application. *Journal of Mathematical Psychology*, *55*, 94–105.

Chapter 17: Generalized context model

- Lee, M. D. & Wetzels, R. (2010). Individual differences in attention during category learning. In R. Catrambone & S. Ohlsson (Eds.), *Proceedings of the 32nd Annual Conference of the Cognitive Science Society*, pp. 387–392. Austin, TX: Cognitive Science Society.
- Bartlema, A., Lee, M. D., Wetzels, R., & Vanpaemel, W. (2012). Bayesian hierarchical mixture models of individual differences in selective attention and representation in category learning. Manuscript submitted for publication.

Chapter 18: Heuristic decision-making

- Lee, M. D. & Newell, B. R. (2011). Using hierarchical Bayesian methods to examine the tools of decision-making. *Judgment and Decision Making*, *6*, 832–842.

Chapter 19: Number concept development

- Lee, M. D. & Sarnecka, B. W. (2010). A model of knower-level behavior in number-concept development. *Cognitive Science*, *34*, 51–67.
- Lee, M. D. & Sarnecka, B. W. (2011). Number knower-levels in young children: Insights from a Bayesian model. *Cognition*, *120*, 391–402.