

# BC Calculus Diagnostic Test

# **AP®** Calculus BC Exam

SECTION I: Multiple-Choice Questions

# DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

# At a Glance

Total Time 1 hour and 45 minutes Number of Questions 45 Percent of Total Grade 50% Writing Instrument Pencil required

# Instructions

Section I of this examination contains 45 multiple-choice questions. Fill in only the ovals for numbers 1 through 45 on your answer sheet.

CALCULATORS MAY NOT BE USED IN THIS PART OF THE EXAMINATION.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

#### Sample Question



 $A \odot C D E$ 

- Chicago is a
- (A) state
- (B) city
- (C) country
- (D) continent
- (E) village

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all the multiple-choice questions.

# **About Guessing**

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. Multiple choice scores are based on the number of questions answered correctly. Points are not deducted for incorrect answers, and no points are awarded for unanswered questions. Because points are not deducted for incorrect answers, you are encouraged to answer all multiple-choice questions. On any questions you do not know the answer to, you should eliminate as many choices as you can, and then select the best answer among the remaining choices. THIS PAGE INTENTIONALLY LEFT BLANK.

#### CALCULUS BC

#### SECTION I, Part A

#### Time—55 Minutes

Number of questions-28

#### A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

<u>Directions</u>: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

1.  $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 5x + 6} =$ 

(A) 0

(B) 1

(C) 3

(D) 5

(E) The limit does not exist.

2.  $\lim_{x \to 0} \frac{\sec x}{\csc x} =$ 

(A) 0

(B) 1

(C) 2π

 $\infty \quad (D)$ 

(E) The limit does not exist.

3. 
$$\frac{d}{dx} \left( \frac{2x+3}{(x-4)^2} \right) =$$
  
(A) 
$$\frac{x-7}{(x-4)^3}$$
  
(B) 
$$\frac{-2x-14}{(x-4)^3}$$
  
(C) 
$$\frac{2x+14}{(x-4)^3}$$
  
(D) 
$$\frac{-2x-14}{(x-4)^4}$$
  
(E) 
$$\frac{x+14}{(x-4)^4}$$

4.  $\lim_{x \to 0} \left( \frac{2 \sin 3x}{3 \sin 2x} \right) =$ (A) -1 (B) 0 (C) 1

(D) ∞

(E) The limit does not exist.

5. What type(s) of discontinuity/ies does the function,  $f(x) = \frac{x^2 - 7x - 18}{x^2 - 12x + 27}$ , have?

- (A) jump
- (B) point
- (C) essential
- (D) jump and removable
- (E) essential and removable

6. Find the fourth derivative of  $f(x) = \frac{x^4 - 4x^3 + 3x^2 - 4x}{x^3}$ .

(A) 
$$\frac{x^3 - 3x + 8}{x^3}$$
  
(B)  $\frac{6x - 24}{x^4}$   
(C)  $\frac{-18x + 96}{x^5}$   
(D)  $\frac{x^3 - 12x^2 + 3x - 4}{x^2}$   
(E)  $\frac{7x - 480}{x^6}$ 

7. 
$$\frac{d}{dx} \left( \frac{\sec 5x}{5} \right) =$$
(A)  $\tan 5x$ 
(B)  $\sec 5x$ 
(C)  $\csc 5x$ 
(D)  $\sec 5x \tan 5x$ 

(E) 5sec  $5x \tan 5x$ 

8. What is the volume of the solid formed by rotating the region between the curves  $y = 6x^2 - x$  and  $y = x^2 - 6x$  about the *y*-axis?

(A)	$\frac{70\pi}{12}$
(B)	$5\pi$
(C)	$\frac{3\pi}{2}$
(D)	π
(E)	$\frac{5\pi}{6}$

9.  $\int x^3 \ln 2x \, dx =$ 

(A) 
$$\frac{x^4}{16}(4\ln 2x - 1) + C$$
  
(B)  $\frac{x^4}{4}(\ln 2x + 1) + C$   
(C)  $x^4(\ln 2x - 1) + C$   
(D)  $\frac{x^4}{4}(\ln 2x - 1) + C$   
(E)  $\frac{x^4}{16}(4\ln 2x + 1) + C$ 

10. 
$$\int \frac{x+10}{2x^2-5x-3} \, dx =$$

(A) 
$$3\ln|x+3| - \frac{1}{2}\ln|2x-1| + C$$
  
(B)  $\ln|x+3| - 3\ln|2x-1| + C$   
(C)  $\ln|x+3| - \frac{3}{2}\ln|2x-1| + C$   
(D)  $3\ln|x+3| + \frac{3}{2}\ln|2x-1| + C$   
(E)  $-\ln|x+3| + \frac{3}{2}\ln|2x-1| + C$ 

(E) 
$$-\ln|x+3| + \frac{3}{2}\ln|2x-1| + 6$$

11.

x	-2	-1	0	1	2
f''(x)	4	1	0	-2	3

The polynomial function f has selected values of its second derivative, f'', given in the table above. Which of the following could be false?

- A) The graph of f changes concavity in the interval (-2, 1).
- (B) There is a point of inflection on the graph of f at x = 0.
- (C) The graph of *f* has a point of inflection at x = 1.5.
- (D) The graph of *f* is concave down at x = 1.
- (E) The graph of f changes concavity in the interval (1, 2).

12. If 
$$y = \sqrt[3]{\frac{(x+2)^2(x-4)^3}{x^3-1}}$$
,  $\frac{dy}{dx} =$ 

(A) 
$$y\left(\frac{2}{3(x+2)} + \frac{1}{x-4} - \frac{x^2}{x^3-1}\right)$$
  
(B)  $y\left(\frac{2}{x+2} + \frac{3}{x-4} - \frac{3x^2}{x^3-1}\right)$ 

(C) 
$$y\left(\frac{2}{3(x+2)} - \frac{1}{x-4} - \frac{x^2}{x^3-1}\right)$$

(D) 
$$y\left(\frac{2}{3(x+2)} - \frac{1}{x-4} + \frac{x}{x^3-1}\right)$$
  
(E)  $y\left(\frac{2}{x+2} - \frac{3}{x-4} + \frac{3x^2}{x^3-1}\right)$ 

13. If 
$$y = x^3 + 3x^2 + 5$$
, then  $\frac{dy}{dx} =$   
(A)  $3x^3 + 6x^2 + 5$   
(B)  $3x^2 + 6x$   
(C)  $\frac{x^4}{4} + x^3 + 5x$   
(D)  $15x^5$   
(E)  $3x^2 + 6x + 5$ 

14. If 
$$f(x) = \frac{x^3 + 2x - 1}{x^2 + x}$$
, then  $f'(x) =$   
(A)  $\frac{(x^2 + x)(3x^2 + 2) - (x^3 + 2x - 1)(2x + 1)}{(x^2 + x)}$   
(B)  $\frac{(x^2 + x)(3x + 2) - (x^3 + 2x - 1)(2x + 1)}{(x^2 + x)^2}$   
(C)  $\frac{(x^2 + x)(3x^2 + 2) - (x^3 + 2x - 1)(2x + 1)}{(x^2 + x)^2}$   
(D)  $\frac{(x^2 + x)(3x^2 + 2) - 2x(x^3 + 2x - 1)}{(x^2 + x)^2}$   
(E)  $\frac{(x^2 + x)(3x^2 + 2) + (x^3 + 2x - 1)(2x + 1)}{(x^2 + x)^2}$ 

- 15. If  $f(x) = 2 \sin(\cos x)$ , then f'(x) =
  - (A)  $-2\sin x \cdot \cos(\cos x)$
  - (B)  $-2\cos x \cdot \sin(\sin x)$
  - (C)  $2\sin x \cdot \cos x$
  - (D)  $-2\sin x \cdot \cos x$
  - (E)  $2\sin x \cdot \cos(\cos x)$

16. Find 
$$\frac{dy}{dx}$$
 if  $x^3 + 3x^2y + 3xy^2 + y^3 = 27$ .  
(A) 27  
(B)  $\frac{3x^2 + 3y^2}{6x}$   
(C) 1  
(D)  $\frac{-3x^2 - 3y^2}{6x}$   
(E) -1

17. Water is filling a conical cup at a rate of  $\frac{2}{3}\pi$  in<sup>3</sup>/sec. If the cup has a height of 18 in and a radius of 6 in, how fast is the water level rising when the water is 6 in deep?

(A) 
$$\frac{1}{6}$$
 in/s

(B) 6 in/s

(C) 
$$\frac{1}{4}$$
 in/s  
(D)  $\frac{8}{3}$  in/s

(E) 
$$\frac{1}{12}$$
 in/s

18. Find the derivative of  $\log_8 (x^2 + 2)^3$ 

(A) 
$$\frac{6x}{(x^2+2)^3 \ln 8}$$
  
(B)  $\frac{6x}{(x^2+2) \ln 8}$   
(C)  $\frac{6x(x^2+2)^3}{\ln 8}$   
(D)  $\frac{6}{(x^2+2) \ln 8}$ 

(E) 
$$\frac{6x\ln 8}{\left(x^2+2\right)}$$

- 19. What curve is represented by  $x = \cos^2 t$  and  $y = \sin^2 t$ ?
  - (A) y = x + 1(B) y = -x + 1(C) y = -x - 1
  - (D) y = x 1
  - (E) y = x
- 20. Given the position function  $x(t) = t^3 18t^2 84t + 11$  for  $t \ge 0$ , for what values of t is speed increasing?
  - (A)  $0 \le t < 14$
  - (B)  $0 \le t < 6 \text{ and } t > 14$
  - (C)  $0 \le t < 6$
  - (D) t > 6
  - (E) 6 < t < 14

- 21. Find the derivative of  $f(x) = e^{\sin^2 x}$ 
  - (A)  $2e^{\sin^2 x} \sin x \cos x$
  - (B)  $e^{\sin^2 x}$
  - (C)  $e^{\sin^2 x} \sin^2 x$
  - (D)  $2e^{\sin^2 x} \sin x$
  - (E)  $e^{\sin^2 x} \sin x \cos x$

#### 22. Given:

x	0	2	6	7	9	12	16
f(x)	1	3	7	5	3	6	9

Use a left-hand Riemann sum with the six subintervals indicated by the data to approximate  $\int_{0}^{16} f(x) dx$ .

(A) 63

(B) 99

(C) 40 (D) 64

- (E) 100
- 23.  $\int \frac{18x^2 + 9}{3x^3 + x} dx =$ 
  - (A)  $2\ln|x^2+1|+C$
  - (B)  $\frac{1}{2}\ln|3x^3 + x| + C$
  - (C)  $\ln |3x^3 + x| + C$
  - (D)  $\ln |x^2 + 1| + C$
  - (E)  $2\ln|3x^3 + x| + C$

24. 
$$\int e^x \sin x \, dx =$$

(A) 
$$\frac{e^x \sin x - e^x \cos x}{2} + C$$
  
(B) 
$$\frac{e^x \sin x + e^x \cos x}{2} + C$$
  
(C) 
$$e^x \sin x - e^x \cos x + C$$
  
(D) 
$$e^x \cos x - e^x \sin x + C$$
  
(E) 
$$\frac{e^x \cos x - e^x \sin x}{2} + C$$

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25. Which of the following integrals converges?

I. 
$$\int_{0}^{\infty} \frac{dx}{1+x^{2}}$$
  
II. 
$$\int_{0}^{\infty} \frac{dx}{\sqrt{1-x^{2}}}$$
  
III. 
$$\int_{1}^{\infty} \frac{dx}{x}$$
  
(A) I  
(B) II  
(C) III  
(D) I & II

(E) I, II, & III

26. Find the area of the region in the plane enclosed by  $r = 5 + 2 \cos \theta$ .

- (A) 23π (B) 24*π*
- (C) 25π
- (D) 26π
- (E) 27π

27. If  $\frac{dy}{dx} = \frac{\cos x}{y^2}$  and y = -1 when x = 0, then the equation for the curve is

(A)  $y^3 = 3\sin x - 1$ (B)  $y^3 = 3\cos x - 1$ (C)  $y = \sin x - 1$ (D)  $y = 3\sin x + 1$ (E)  $y^3 = \sin x$ 

28. Which of the following series converges?

I.  $\sum_{n=1}^{\infty} 2\left(\frac{1}{3}\right)^{n-1}$ II.  $\sum_{n=1}^{\infty} \frac{n}{3^n}$ III.  $\sum_{n=1}^{\infty} \frac{6n^2}{n^3 + 1}$ (A) I (B) II

(C) III

(D) I & II

(E) I & III

# END OF PART A, SECTION I IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

#### CALCULUS BC

#### SECTION I, Part B

#### Time—50 Minutes

#### Number of questions-17

#### A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

<u>Directions</u>: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

- 1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- 2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.
- 29. A spherical balloon is losing air at a rate of  $-24\pi$  in<sup>3</sup>/sec. How fast is the balloon surface area of the balloon shrinking when the radius of the balloon is 2 in?
  - (A)  $-20\pi \text{ in}^2/\text{sec}$
  - (B)  $-24\pi \operatorname{in^2/sec}$
  - (C)  $-48\pi \text{ in}^2/\text{sec}$
  - (D)  $-12\pi \text{ in}^2/\text{sec}$
  - (E)  $-60\pi \text{ in}^2/\text{sec}$

30. Find 
$$\lim_{x \to 0} \frac{(3+2x)^{\frac{3}{2}} - 7x}{2x^{2}}$$
.  
(A)  $\frac{\sqrt{3}}{4}$   
(B)  $\frac{4\sqrt{3}}{3}$   
(C)  $\frac{3}{4}$   
(D)  $\frac{1}{4}$   
(E)  $\frac{1}{3}$ 

31. When are the horizontal and vertical components of the velocity of a curve whose motion is given by  $x = \frac{5}{2}t^2 + 6$  and  $y = 2t^3 - t^2 + t$  equal?

- (A) t = -1
- (B) t = 2
- (C)  $t = \frac{1}{6}$
- (D) t = 6
- (E) t = -2

- 32. Find the derivative of  $x^2 + 3x^2y + 3xy^2 + y^3 = 2$  at (3,2).
  - (A) 7 (B)  $\frac{19}{25}$ (C)  $\frac{75}{54}$ (D)  $-\frac{18}{25}$ (E)  $-\frac{25}{18}$
- 33. A kid on a bike is riding home in the woods on a straight path that is 50 m from the nearest point on the road. His home is 1500 m from the nearest point on the road. If the kid rides at 3 m/s in the woods and 5 m/s on the road, how far from his house should the kid cross to the road to get home in the shortest time?
  - (A) 37.5 m
  - (B) 1500 m
  - (C) 300.5 m
  - (D) 1200 m
  - (E) 1462.5 m
- 34. A shoe company determined that its profit equation (in millions of dollars) is given by  $P = 2x^3 105x^2 + 1500x 1200$ , where *x* is the number of thousands of pairs of shoes sold and  $0 \le x \le 50$ . Optimize the manufacturer's profit.
  - (A) \$5.3 billion
  - (B) \$61.3 billion
  - (C) \$1.925 billion
  - (D) 1.2 billion
  - (E) \$65 billion

- 35. If the function  $f(x) = x^4$  has an average value of 5 on the closed interval [0, k], then k =
  - (A) 5
  - (B)  $\sqrt{5}$
  - (C) 1
  - (D)  $\sqrt{3}$
  - (E)  $\sqrt{2}$

36.  $\int 3x(7^{3x^2+2})dx =$ 

(A) 
$$\frac{49 \cdot 343^{x^2}}{\ln 49} + C$$
  
(B)  $\frac{343^{x^2}}{2 \ln 7} + C$   
(C)  $\frac{49^{x^2}}{\ln 49} + C$   
(D)  $\frac{49 \cdot 343^{x^2}}{\ln 7} + C$   
(E)  $\frac{7^{3x^2+2}}{\ln 7} + C$ 

- 37. A rectangle with one side on the *x*-axis has its upper vertices on the graph of  $y = \cos x + 1$ . What is the minimum area between the rectangle and the graph of  $y = \cos x + 1$  on the interval  $(-\pi \le x \le \pi)$ ?
  - (A) 6.283
  - (B) 2.988
  - (C) 1.307 (D) 3.296
  - (E) 5.022

38. 
$$\int \frac{dx}{x^2 + 6x + 10} =$$
  
(A)  $\cot^{-1}(x + 3) + C$   
(B)  $\sin^{-1}(x + 3) + C$   
(C)  $\sec^{-1}(x + 3) + C$   
(D)  $\tan^{-1}(x + 3) + C$   
(E)  $\cos^{-1}(x + 3) + C$ 

39. If the path of the particle is given by  $x(t) = 2t^2 - 3t + 1$ , how far does the particle travel between t = 0 and t = 4?

(A) 20 (B) 21 (C)  $\frac{169}{8}$ (D) 22 (E)  $\frac{89}{4}$ 

40. Given the following values for x and f(x), what is the area under f(x). Use a left-hand Riemann sum to approximate.

:	x	0	1	3	7	8	10	13	15
<i>f</i> (	x)	2	6	3	4	8	9	12	13

(A) 46

(B) 57

(C) 116

(D) 207

(E) 253

- 41. Find the length of the curve given by  $y = \frac{1}{3}(x^2 2)^{\frac{3}{2}}$  from x = 0 to x = 4.
  - (A) 12 (B) 16 (C)  $\frac{64}{3} - 4$ (D)  $\frac{64}{3}$ (E)  $\frac{64}{3} + 4$
- 42. Find  $\frac{d}{dx} \int_{1}^{x} (t t^{4}) dt$ . (A)  $x - x^{4} + 1$ (B)  $x^{4} - x + 1$ (C)  $x^{4} - x$ (D)  $x - x^{4} - 1$ (E)  $x - x^{4}$

43. A cylindrical pool is filling at a rate of  $96\pi \frac{\text{ft}^3}{\text{hr}}$ . If the radius of the pool is 4 feet, how fast is the height changing?

- (A) 3 ft/hr
- (B) 4 ft/hr
- (C) 5 ft/hr
- (D) 6 ft/hr
- (E) 7 ft/hr

44. A child jumps on a trampoline and rises at a rate of 10 ft/min. The child's mother is watching from the patio 40 ft away. How fast, in rad/sec, is the angle of elevation between the trampoline and the mother's line of sight of her child increasing when the child is 30 ft in the air?

(A) 
$$\frac{1}{70}$$
 rad/sec  
(B)  $\frac{1}{375}$  rad/sec  
(C)  $\frac{1}{13}$  rad/sec

(D) 
$$\frac{1}{180}$$
 rad/sec

(E) 
$$\frac{1}{3}$$
 rad/sec

45. Find the derivative of  $y = \frac{\ln(2x^3)}{e^{2x}}$ .

(A) 
$$\frac{3+2\ln(2x^3)}{e^{2x}}$$
  
(B)  $\frac{3-2\ln(2x^3)}{e^{2x}}$   
(C)  $\frac{3}{xe^{2x}} - \frac{2\ln(2x^3)}{e^{2x}}$   
(D)  $\frac{3}{xe^{2x}} + \frac{2\ln(2x^3)}{e^{2x}}$ 

(E) 
$$\frac{3+\ln(4x^2)}{e^{2x}}$$

# STOP

END OF PART B, SECTION I IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY. DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

# SECTION II GENERAL INSTRUCTIONS

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

# A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example,  $\int_{-\infty}^{5} x^2 dx$  may not be written as fnInt (X<sup>2</sup>, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

# SECTION II, PART A Time—30 minutes Number of problems—2

#### A graphing calculator is required for some problems or parts of problems.

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- 1. Let *y* be the function satisfying f'(x) = -x(1 + f(x)) and f(0) = 5.
  - (a) Use Euler's method to approximate f(1) with a step size of 0.25.
  - (b) Find an exact solution for f(x) when x = 1.
  - (c) Evaluate  $\int_0^\infty -x(1+f(x))dx$ .
- 2. Let R be the region enclosed by the graphs of  $y = x^2 x 6$  and  $y = x^3 2x^2 5x + 6$  and the lines x = -2 and x = 2.
  - (a) Find the area of R.
  - (b) The horizontal line y = 0 splits R into two parts. Find the area of the part of R above the horizontal line.
  - (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the *x*-axis is an equilateral triangle. Find the volume of this solid.
  - (d) What is the volume of the solid generated by the region R when it is revolved about the line x = -3.

# SECTION II, PART B Time—1 hour Number of problems—4

#### No calculator is allowed for these problems.

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

- 3. The derivative of a function *f* is  $f'(x) = (2x+6)e^{-x}$  and f(2) = 15.
  - (a) The function has a critical point at x = -3. Is there a relative maximum, minimum, or neither at this point on f? Justify your response.
  - (b) On what interval, if any, is the graph of f both increasing and concave down? Explain your reasoning.
  - (c) Find the value of f(5).

4. Consider the equation  $x^3 - 2x^2y + 3xy^2 - 4y^3 = 10$ .

- (a) Write an equation for the slope of the curve at any point.
- (b) Find the equation of the normal line to the curve at the point x = 1.
- (c) Find  $\frac{d^2y}{dx^2}$  at x = 1.

5. Given that  $f(x) = \sin x$ :

- (a) Find the 6<sup>th</sup> degree Maclaurin series.
- (b) Use the polynomial to estimate  $\sin 0.2$ .
- (c) Estimate the remainder of the approximation.

- 6. Two particles travel in the *xy*-plane for time  $t \ge 0$ . The position of particle A is given by x = 2t 3 and  $y = (2t + 1)^2$  and the position of particle B is given by x = t 1 and y = t + 23.
  - (a) Find the velocity vector for each particle at t = 3.
  - (b) Set up, but do not evaluate, an integral expression for the distance traveled by particle A from t = 3 to t = 5.
  - (c) At what time do the two particles collide? Justify your answer.

**STOP** 

# **END OF EXAM**