CHAPTER 6

Investment Decision Rules

Chapter Synopsis

6.1 NPV and Stand-Alone Projects

The net present value (NPV) of a project is the difference between the present value of its benefits and the present value of its costs. Since a project's NPV represents its value in terms of cash today, the **NPV investment rule,** which states that all positive NPV projects should be accepted, is consistent with maximizing the value of the firm.

The internal rate of return (IRR) is the rate of return that makes the net present value of a stream sof cash flows equal to zero. Thus, accepting projects with an IRR above the required return, or cost of capital, is generally equivalent to accepting projects with a positive NPV. The difference between the cost of capital and the IRR can be thought of as the maximum amount of estimation error in the cost of capital estimate that can exist without altering the original decision.

6.2 The Internal Rate of return

The **IRR investment rule** advises taking investment opportunities in which the IRR exceeds the opportunity cost of capital. The IRR rule will give the same answer as the NPV rule in many, but not all, applications. The following are cases in which the IRR rule may fail to reliably provide the correct decision.

■ When the initial cash flow is positive and all later cash flows are negative, the IRR rule will provide the opposite decision provided by the NPV rule.

■ In some cases, such as when there is no required investment for a project, the IRR does not exist.

■ There may be multiple IRRs when the sign of the project's cash flows changes more than once, so the IRR rule cannot be relied upon.

While these are limitations to the usefulness of the IRR rule, the IRR itself remains a useful tool. Not only does the IRR measure the sensitivity of the NPV to estimation error in the cost of capital, but it also measures the average return of the investment.

6.3 The Payback Rule

The **payback investment rule** is based on the idea that an opportunity that pays back its initial investment quickly is a good idea. To apply the payback rule, you first calculate the amount of time it takes to pay back the initial investment, called the **payback period.** If the payback period is less than a pre-specified length of time, you accept the project. The payback rule is not a reliable method of determining if projects will increase the value of the firm since it does not consider the timing of a project's cash flows or cost of capital.

6.4 Choosing Between Projects

Sometimes a firm must choose among **mutually exclusive projects** in which only one of two or more projects being considered can be selected. In this case, the NPV rule advises picking the project with the highest NPV and provides the best answer.

Picking one project over another simply because it has a larger IRR can lead to errors.

- ◼ Because the IRR measures only the return of the investment opportunity, it does not depend on the scale of the investment opportunity. Hence the IRR rule cannot be used to compare projects of different scales because larger scale projects may be more valuable.

- ◼ Investment opportunities with the same NPV can have different IRRs because the IRR depends on the timing of the cash flows even when a change in timing does not affect the NPV. By altering the timing of the cash flows, it is possible to change the ranking of the IRRs of two mutually exclusive projects without changing either project's NPV.

The **incremental IRR investment rule** applies the IRR rule to the difference between the cash flows of the two mutually exclusive alternatives. For example, assume you are comparing two mutually exclusive opportunities, A and B, and the IRRs of both opportunities exceed the cost of capital. If you subtract the cash flows of opportunity B from the cash flows of opportunity A, then you should take opportunity A if the incremental IRR exceeds the cost of capital. Otherwise, you should take opportunity B. Although the incremental IRR rule often provides a reliable method for choosing among mutually exclusive projects, it can be difficult to apply correctly, and it is much simpler to just use the NPV rule.

6.5 Project Selection with Resource Constraints

Sometimes there is a fixed supply of capital, or other resources, so that all possible opportunities cannot be undertaken. The **profitability index** can be used to identify the optimal combination of projects to undertake in such situations, where:

$$\text{Profitability Index } = \frac{\text{NPV}}{\text{Resource Consumed}}$$

Projects should be selected in order of profitability index ranking starting with the project with the highest index and moving down the ranking until the resource is consumed. While this procedure generally leads to the most valuable combination of projects, the only guaranteed way to find the best combination of projects is to search through all of them. Linear programming techniques have been developed to solve this kind of problem.

Selected Concepts and Key Terms

Net Present Value (NPV) Investment Rule

Select all projects that have a positive net present value (NPV), where NPV is the difference between the present value of an investment's benefits and the present value of its costs. A project's NPV represents its value in terms of cash today. Choosing this alternative is equivalent to receiving its NPV in cash today, so positive NPV projects should be accepted. When choosing among mutually exclusive alternatives, the alternative with the highest NPV should be selected.

NPV profile

A graph of a project's NPV over a range of discount rates.

Internal Rate of Return (IRR) Investment Rule

Take investment opportunities in which the IRR exceeds the opportunity cost of capital. The **internal rate of return** (IRR) is the rate of return that makes the net present value of a stream of cash flows equal to zero. The IRR investment rule will give the same answer as the NPV rule in many, but not all, applications.

Mutually Exclusive Projects

A situation where only one of two or more projects being considered can be selected. In this case, the NPV rule provides the best answer: Pick the project with the highest NPV. Picking one project over another simply because it has a larger IRR can lead to errors.

Incremental IRR

The **incremental IRR investment rule** applies the IRR rule to the difference between the cash flows of the two mutually exclusive alternatives. Although the incremental IRR rule often provides a reliable method for choosing among mutually exclusive projects, it can be difficult to apply correctly, and it is much simpler to just use the NPV rule.

Profitability Index

The NPV of a project divided by the amount of a resource (such as capital) consumed. When there is a limited resource (such as capital), projects should be selected in order of profitability index ranking starting with the project with the highest index and moving down the ranking until the resource is consumed.

Payback Investment Rule

If the payback period is less than a pre-specified length of time, you accept the project. The **payback period** is the amount of time it takes to pay back the initial investment. The payback rule is not a reliable method of determining if projects will increase the value of the firm since it does not consider the timing of a project's cash flows or cost of capital.

Concept Check Questions and Answers

6.1.1. Explain the NPV rule for stand-alone projects.

The NPV rule for stand-alone projects states that when choosing among alternatives, we should take the project with the highest positive NPV.

6.1.2. What does the difference between the cost of capital and the IRR indicate?

In general, the difference between the cost of capital and the IRR is the maximum amount of estimation error in the cost of capital estimate that can exist without altering the original decision.

6.2.1. Under what conditions do the IRR rule and the NPV rule coincide for a stand-alone project?

The IRR rule is only guaranteed to work for a stand-alone project if all of the project's negative cash flows precede its positive cash flows. If this is not the case, the IRR rule can lead to incorrect decisions.

6.2.2. If the IRR rule and the NPV rule lead to different decisions for a stand-alone project, which should you follow? Why?

When investment rules conflict, you should follow the NPV rule because following the alternative rules means you are not taking a positive NPV project, and thus, you are not maximizing wealth. In these cases, the alternative rules lead to bad decisions.

6.3.1. Can the payback rule reject projects that have positive NPV? Can it accept projects that have negative NPV?

Yes, because the payback rule does not take into consideration the required rate of return and the exact timing of the cash flows.

6.3.2. If the payback rule does not give the same answer as the NPV rule, which rule should you follow? Why?

The NPV rule because it correctly accounts for the required rate of return and the exact timing of the cash flows while the payback rule does not.

6.4.1. For mutually exclusive projects, explain why picking one project over another because it has a larger IRR can lead to mistakes.

For mutually exclusive projects, picking one project over another because it has a larger IRR can lead to mistakes. Problems arise when projects have differences in scale (require different initial investments) and when they have different cash flow patterns.

6.4.2. What is the incremental IRR rule and what are its shortcomings?

The incremental IRR rule applies to the difference between the cash flows of two mutually exclusive projects. Suppose you compare two mutually exclusive projects, A and B, and the IRR of both projects exceeds the cost of capital. If you subtract the cash flows of project B from the cash flows of project A, then you should choose project A if the incremental IRR exceeds the cost of capital. Otherwise, choose project B.

6.5.1. Explain why ranking projects according to their NPV might not be optimal when you evaluate projects with different resource requirements.

When there is a fixed supply of the resource so that you cannot undertake all the mutually exclusive projects, choosing the highest NPV project may not lead to the best decision. The project that has the highest NPV may use up the entire resource. Therefore, it would be a

mistake to take it. A combination of other projects may produce a combined NPV that exceeds the NPV of the best single project.

6.5.2. How can the profitability index be used to identify attractive projects when there are resource constraints?

Practitioners often use the profitability index to identify the optimal combination of projects to undertake because the profitability index measures the value created in terms of NPV per unit of resources consumed. After computing the profitability index, practitioners rank projects from the highest index down until the resource is used up.

Examples with Step-by-Step Solutions

Solving Problems

Problems using the concepts in this chapter generally involve determining the NPV or IRR for a simple project. The ability to evaluate mutually exclusive projects using the NPV rule may be asked as well. Finally, there may be applications involving selecting projects in the presence of a limited amount of capital (or some other resource) using the profitability index. The examples below demonstrate these three types of problems.

Examples

1. Microsoft is considering moving 1,000 employees from a help-desk call center in Seattle to Bombay. The total after-tax cost of a Seattle worker is $50,000 per year and the total after-tax cost of a Bombay worker is $30,000 per year. The move would require paying an upfront severance package worth $40,000 after taxes per former Seattle employee. Assume for this analysis that the cost savings would last forever and that Microsoft's cost of capital is 20%.
   [A]   Should the project be accepted based on the NPV rule?
   [B]   What is the IRR of the project?
   [C]   Can the IRR be relied on in this application?

Step 1. Put the cash flows on a time line.

The time 0 cost is $40,000(1,000) = $40 million. The annual savings is $50,000(1,000) = $50 million, and the new annual cost is $30,000(1,000) = $30 million, so the annual net incremental cash flow is $20 million.

Year	0	1	8	9	10
	-$40 million	$20 million	$20 million	$20 million	$20 million

Step 2. Determine the NPV. Since the cash flows after time 0 are a perpetuity:

$$\text{NPV} = \sum_{n=0}^{N} \frac{C_n}{(1+r)^n} = \sum_{n=0}^{\infty} \frac{C_n}{(1.2)^n} = C_0 + \frac{C}{r} = -40,000,000 + \frac{20,000,000}{.2} = \$60 \text{ million}$$

Since the NPV > 0, the project should be accepted.

Step 3. Determine the IRR by setting the NPV equal to zero and solving for the rate.

$$\text{NPV} = 0 \Rightarrow \sum_{n=0}^{\infty} \frac{C_n}{(1+\text{IRR})^n} = -40,000,000 + \frac{20,000,000}{\text{IRR}} = 0 \Rightarrow \text{IRR} = \frac{20,000,000}{40,000,000} = 50\%$$

Since the IRR > 20%, the IRR rule says to accept the project as well.

Step 4. Determine if the IRR rule can be relied on.

The IRR rule can be relied on here because the cash flow at time 0 is negative and all future cash flows are positive. Also, the decision being made involves a stand-alone project, not mutually exclusive projects, in which case the IRR could not be relied in.

2. Pulte Homes purchased 100 acres in suburban Los Angeles. They are considering the following development options:

	NPV in millions	Acres used
Housing development A	$30	100
Housing development B	$24.5	70
Drug store	$3	3
Strip mall	$3.5	7
Golf course	$8	20

Which project(s) should the firm choose?

Step 1. Since the amount of land is a limited resource, calculate the profitability indices for each project relative to how much land they use.

$$\text{Profitabiliy Index} = \frac{\text{NPV}}{\text{Land Used}}$$

	Profitability Index	Acres used
Housing development A	.30	100
Housing development B	.35	70
Drug store	1.00	3
Strip mall	.50	7
Golf course	.40	20

Step 2. Rank the projects based on how much land they use:

	Rank	Acres used
Drug store	1	3
Strip mall	2	7
Golf course	3	20
Housing development B	4	70
Housing development A	5	100

Step 3. Select the projects in descending order of profitability index until all the land is used.

Select the drug store, strip mall, golf course, and housing development A.

3. You are deciding between two mutually exclusive investment opportunities. They both require the same initial investment of $10 million. Project X generates $5 million per year (starting at the end of the first year) in perpetuity. Project Y generates $4 million at the end of the first year and will grow at 5% per year for every year after that. The cost of capital is 10%.
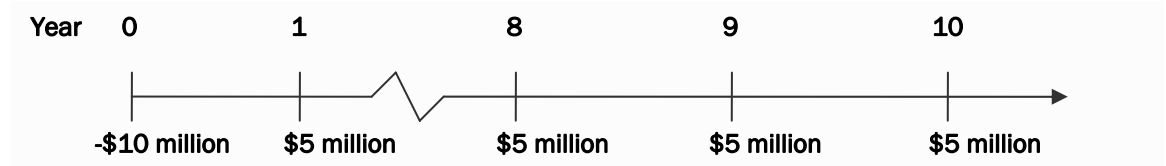   [A]   Which investment has the higher IRR?
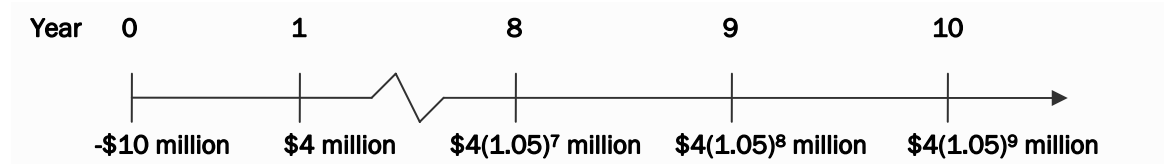   [B]   What project should be chosen?
   [C]   In this case, when does picking the higher IRR give the correct answer as to which investment is the best opportunity?

Step 1. Put the cash flows of each project on a time line.

Project X

Year	0	1	8	9	10
	-$10 million	$5 million	$5 million	$5 million	$5 million

Project Y

Year	0	1	8	9	10
	-$10 million	$4 million	$4(1.05)^7$ million	$4(1.05)^8$ million	$4(1.05)^9$ million

Step 2. Calculate the IRR of each project.

Project X is a perpetuity and project Y is a growing perpetuity, so set the NPV of those valuation equations equal to zero.

$$\text{NPV}_A = 0 = -10,000,000 + \left(\frac{5,000,000}{\text{IRR}}\right) \Rightarrow \text{IRR} = \frac{5,000,000}{10,000,000} = 50\%$$

$$\text{NPV}_B = 0 = -10,000,000 + \left(\frac{4,000,000}{\text{IRR}-.05}\right) \Rightarrow \text{IRR} = \frac{4,000,000}{10,000,000} + .05 = 45\%$$

Step 3. Since the projects are mutually exclusive, the NPV must be calculated to determine which investment is better.

$$\text{NPV}_A = -10,000,000 + \left(\frac{5,000,000}{.10}\right) \Rightarrow \text{NPV} = \$40 \text{ million}$$

$$\text{NPV}_B = -10,000,000 + \left(\frac{4,000,000}{.10-.05}\right) \Rightarrow \text{NPV} = \$70 \text{ million}$$

Since the NPV of Project Y is much higher, it is the best project. In this case relying on the IRR rule would lead to the wrong conclusion since Project X has a higher IRR.

Questions and Problems

1. You own a gold mining company and are considering opening a new mine. The mine is expected to generate $10 million for the next 21 years. After 21 years, the gold is expected to be depleted, but the site can be sold for an expected $20 million. If the cost of capital is 8%, what is the most you should invest to open the mining operation at time 0?

2. You are considering opening a new hotel. The hotel will cost $150 million upfront and will be built immediately. It is expected to produce profits of $20 million every year forever. Calculate the NPV of this investment opportunity if your cost of capital is 10%. Should you make the investment? Calculate the IRR and use it to determine the maximum deviation allowable in the cost of capital estimate to leave the decision unchanged.

3. The Professional Golf Association (PGA) is considering developing a new PGA-branded golf ball. Development will take 3 years at a cost of $250,000 per year. Once in production, the ball is expected to make $250,000 per year for 5 years at which time new technology will make it obsolete. The cost of capital is 10%. Calculate the NPV of this investment opportunity. Should the PGA make the investment?

4. You are considering making a movie. The movie is expected to cost $100 million upfront and take a year to make. After that, it is expected to make $85 million in the first year it is released and $5 million for the following 20 years. What is the payback period of this investment? If you require a payback period of two years, will you make the movie? Does the NPV rule agree with the payback rule if the cost of capital is 10%?

5. Your corporation has $1 million to spend on capital investments this year and is evaluating four investments. The following table summarizes NPV and cost of these investments.

	NPV	Cost
1	$400,000	$400,000
2	$300,000	$200,000
3	$650,000	$400,000
4	$150,000	$600,000

Which project(s) should the firm choose?

Solutions to Questions and Problems

1. Using X as the initial investment:

$$\text{NPV} = X + \frac{10}{.08}\left(1 - \frac{1}{(1.08)^{21}}\right) + \frac{20}{(1.08)^{21}} = X + 100 + 4 = 0 \Rightarrow X = -\$104 \text{ million.}$$

Thus, the most you should invest is $104 million.

2. $\text{NPV} = -150 + \dfrac{20}{.10} = -150 + 200 = \$50 \text{ million, so you should accept the project.}$

The IRR can be found by setting the NPV = 0:

$$\text{NPV} = -150 + \frac{20}{\text{IRR}} = 0 \Rightarrow \text{IRR} = \frac{20}{150} = 13.3\%.$$

So the cost of capital can be underestimated by 3.3% without changing the decision.

3. $\text{NPV} = \dfrac{-250,000}{.10}\left(1 - \dfrac{1}{(1.10)^3}\right) + \left(\dfrac{1}{(1.10)^3}\right)\dfrac{250,000}{.10}\left(1 - \dfrac{1}{(1.10)^5}\right)$

$= -621,713 + 712,019 = 90,036 > 0.$

NPV > 0, so the company should take the project.

4. It will take 4 years to pay back the initial investment, so the payback period is 4 years. You will <u>not</u> make the movie.

$$\text{NPV} = -100 + \frac{85}{(1.10)^2} + \frac{5}{.10}\left(1 - \frac{1}{(1.10)^{20}}\right)\frac{1}{(1+r)^2}$$

$= -100 + 70.2 + 42.6 = \$12.8 \text{ million} > 0.$

So the NPV does not agree with the payback rule in this case.

5.

	Profitability Index	Cost
1	1.000	$400,000
2	1.500	$200,000
3	1.625	$400,000
4	0.250	$600,000

Select the projects in descending order of profitability index until all the money is used. They should select 3, 2, and 1.