Beam Shear Design with Welded Wire Fabric:

ACI 318 vs. AASHTO LRFD

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ABSTRACT

Since 1963, the shear design of reinforced and prestressed concrete beams has become unnecessarily complicated. There are currently two different procedures used to design and detail the steel shear reinforcement in concrete beams. The first method, used by the American Concrete Institute (ACI), is purely empirical. Their current design guide, the Building Code Requirements for Structural Concrete (ACI 318-95) and Commentary (ACI 318R-95)[1] uses the results of beam tests to produce the current code shear reinforcement requirements. The second method, recently adopted by the American Association of State Highway and Transportation Officials (AASHTO) in their Load and Resistant Factor Design (LRFD) code[2] is called the Modified Compression Field Theory (MCFT) approach. The MCFT approach, also referred to as the General method, was developed by Professor Michael P. Collins at the University of Toronto. The MCFT approach models the beam's steel reinforcement as a variable angle truss. According to Collins, the resulting steel shear reinforcement is designed based on a "physically significant" quantity.[3] The method relates the average strain in the longitudinal steel reinforcement with the principal strains in the beam. The AASHTO method greatly simplifies the design approach, since the same equations apply to both prestressed and non-prestressed concrete beams and to any beam geometry.

According to two separate studies conducted by Collins and Mitchell,[3,4] the MCFT approach produces more accurate answers than the empirical ACI method. Collins

concludes that the "General method" is more accurate than the ACI method, especially when used to determine the shear reinforcing required in "large, lightly reinforced members and members subjected to high axial compression where the ACI equations can be seriously unconservative" (Collins, 1996). He further states that the ACI approach is overly conservative for "uniformly loaded members, members with inclined prestressing tendons and members subjected to high axial tension" (Collins, 1996). In his study, Mitchell predicted the shear capacity of four beams based on both the ACI and MCFT approaches. His predictions were then verified by experimental testing. In his conclusions, he states that "the predictions of shear capacity using the modified compression field theory are more accurate than the predictions using ACI 318M-89[5] expressions" (Mitchell, 1994).

The ACI and AASHTO shear design methods seem to be diverging in approach. While both methods provide safe designs, they present a different methodology towards shear reinforcement design. As Robert W. Cannon states in *Concrete International*, the changes in shear design are "due to academic research and not based on past performance of structures, or (their) impact on construction procedures."[6] The MCFT approach adopted by AASHTO is not a result of the failure of the ACI code in design practice. Rather, it is an attempt to provide a rational approach to shear design.

A recent development in thin-webbed precast concrete beams is the use of Welded Wire Fabric (WWF) as shear reinforcement. The use of WWF has been shown to greatly reduce the time, cost and the difficulty of on-site construction.[7]

INTRODUCTION AND HISTORICAL REVIEW

Over the last several years, two methods have primarily been used to design the shear reinforcement in prestressed and non-prestressed concrete beams. The focus of my research was prestressed concrete beams using Welded Wire Fabric (WWF) as shear reinforcement. Specifically, I wanted to compare the shear design approach used by ACI 318 with that used by the AASHTO LRFD design specifications for WWF.

The history of shear design methods can be traced back to about the year 1890 as is shown in Figure 1. About the turn of this century, Morsch[8] investigated the diagonal cracking in concrete railroad beams and developed the 45 degree truss analogy still used as the basis for the ACI code provisions today.

ACI 318 Method

In 1955, a failure of an air force warehouse[9,10] led to the first major changes in the ACI shear provisions. During the 1960's, ACI-ASCE Committee 326 refined the ACI shear provisions based on the results of tests conducted on over 924 beams. Similarly, during the 1970's, ACI-ASCE Committee 426 further refined the shear provisions based on beam tests. As shown in Figure 1, research in the shear behavior of concrete beams has increased dramatically in recent years.

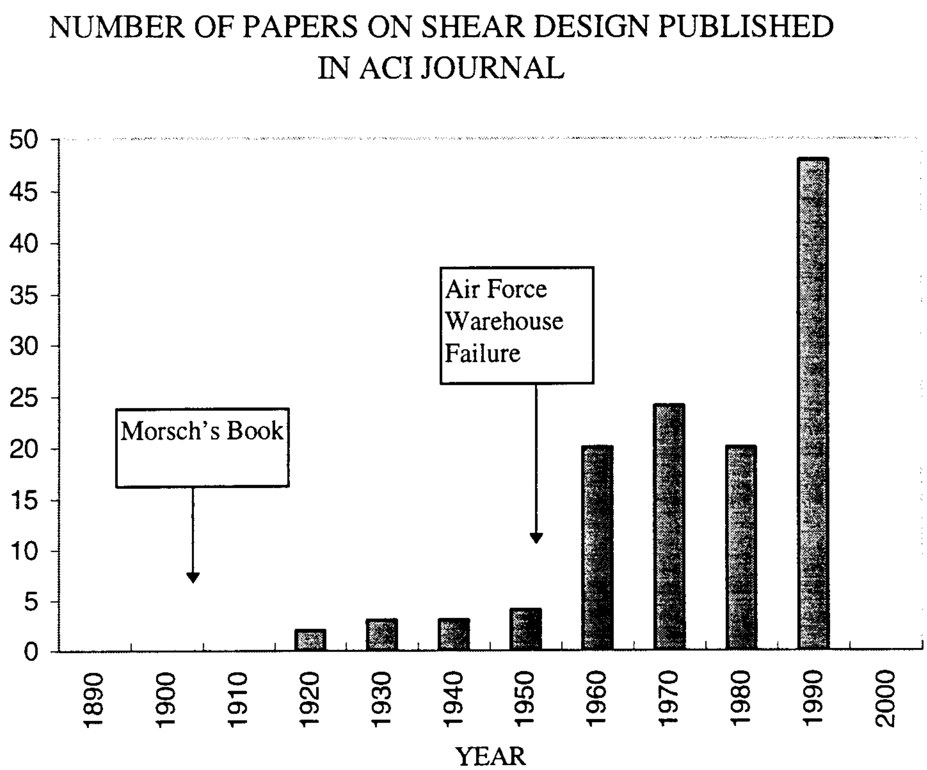

Figure 1. Number of papers on shear design (Collins, 1996)

The history of the ACI empirical approach to shear design has a well-documented history. The ACI 318 specifications have become increasingly complex over the years. As shown in Figure 2, the number of equations required for beam shear design has risen significantly. Up until approximately 1963, the ACI 318 specifications included only 4 equations for shear design and analysis. The ACI 318-95 specification currently includes over 40 equations for shear design and analysis. Even though these equations do not all need to used or checked for each design, the number of complex equations in the ACI code makes it extremely difficult to use without adequate training and experience.

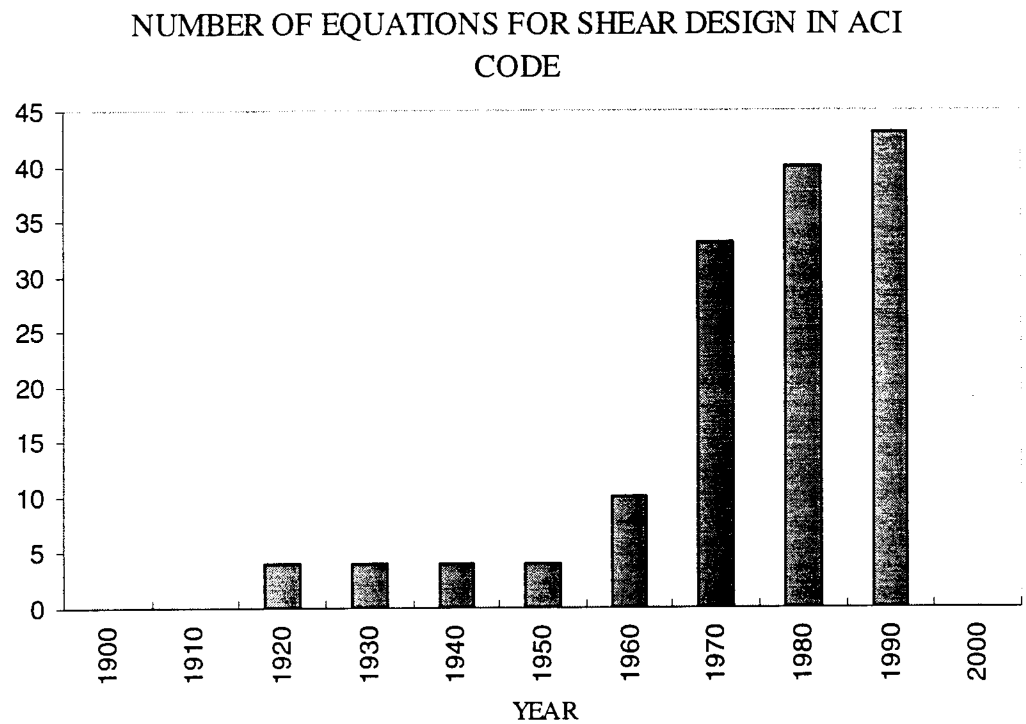

Figure 2. Number of equations for Shear Design in ACI Code (Collins, 1996)

AASHTO LRFD Method

The recently adopted 1994 AASHTO LRFD design guide uses the Modified Compression Field Theory (MCFT) approach developed by Professor M.P. Collins at the University of Toronto. The approach is "based on equilibrium, compatibility and the stress-strain characteristics of cracked reinforced concrete" (Collins, 1996). In fact, Collins developed the MCFT approach based on the desire of the 1973 ACI-ASCE Shear Committee[11] that the "design regulations for shear strength can be integrated, simplified, and given a physical significance." Testing done since 1971 has shown that, "in general, the angle of inclination of the concrete compression is not 45 degrees, and that equations based on a variable angle truss provide a more realistic basis for shear design" (Collins, 1996). The MCFT method provides a rational and simplified approach to shear design. As a result, it has been adopted by several design codes in Canada and in the AASHTO LRFD specifications in the United States.

In his text, <u>Reinforced Concrete: Mechanics and Design</u> ,[12] James G. MacGregor disagrees with the MCFT approach used by Collins. He states that one cannot predict flexure-shear cracks based on the principal tensile stresses in a section, unless web shear cracks precede them (which sometimes occurs in prestressed concrete beams but rarely occurs in reinforced concrete beams). MacGregor, a member of the ACI 318 committee, believes an empirical approach (such as ACI 318-95) is the only way to predict beam cracking. He and several other "subject matter experts" advocate the use of arching action in "disturbed" regions of a beam (such as near a support) to explain shear failure.

Welded Wire Fabric

Recently, the use of Welded Wire Fabric (WWF), primarily in prestressed concrete construction, has taken on added importance. For instance, in Germany, WWF constitutes over 40% of the steel reinforcement market and has been used successfully for over 50 years (Mitchell, 1994). The use of prefabricated WWF cages instead of conventional L-shaped or U-shaped stirrups for shear reinforcement has led to time savings of 70-75% during actual construction (Mitchell, 1994). In addition, it has greatly reduced the amount of steel reinforcement required in a given concrete beam, thereby greatly reducing the cost. However, here in North America, the use of WWF for shear reinforcement is just beginning to be an accepted practice. For instance, in 1980, the Precast Concrete Institute (PCI) and the Wire Reinforcing Institute (WRI) formed an ad hoc joint PCI/WRI committee to study the use of WWF for shear reinforcement.[13] This has led to ACI acceptance of WWF for use as shear reinforcement. ACI has currently restricted the allowable yield strengths in the wire material (see Table 1 below).

ACI Code Version	Allowable Yield Strength, Fy (ksi)
ACI 318-77	60 ksi
ACI 318-89 (Revised 1992)	60 ksi
ACI 318-95	60+ ksi (up to stress corresponding to 0.35 percent strain)

Table 1. Allowable Yield Strength for Wire Reinforcement Based on ACI 318-XX.

One of the main advantages of using WWF is that it may be fabricated with whatever size wires or spacings that the designer deems necessary. Instead of the standard WWF typically used for concrete slabs, the designer can now specify exactly what types of steel reinforcement he wishes to use. If crack control is a concern, he can reduce the wire spacing. If strength is a primary concern, the designer can use stronger deformed wire or increase the wire size in the fabric.

Most of the recent testing of wire reinforcing has focused on the wire properties: ductility, yield strength and bonding characteristics (smooth and deformed wire). Other parameters investigated were the requirements for proper anchorage, the quality and strength of welds at wire intersections and the need for proper wire sizes and spacings when WWF is used for shear reinforcement (Mitchell, 1994). In fact, Mansur, Lee and Lee[14,15] concluded that the use of WWF as shear reinforcement results in smaller measured maximum crack widths than those cracks resulting from the use of regular mild steel stirrups. They found that these smaller cracks were a direct result of the smaller steel spacings and the more effective "staple action" across the potential crack. They also concluded that deformed WWF performed better than the smooth WWF in terms of anchorage and overall crack control. This was verified by tests done by Pincheira, Rizkalla, and Attiogbe.[16] Mitchell confirmed that deformed WWF did lead to smaller cracks at service loads than those beams reinforced with conventional stirrups. He also stated that there was sufficient anchorage of the WWF when two horizontal wires were used and that WWF exhibited sufficient ductility for use as shear reinforcement. Mitchell also concluded that the "nominal yield stress of 500 Mpa (about 72.5 ksi) could be used in design calculations" (Mitchell, 1994). This eventually led to ACI Committe 318 adoption of the ultimate yield stress of 60+ ksi (that is, up to the stress corresponding to a 0.35 percent strain) in ACI 318-95.

THEORY

Different types of cracking behavior are shown in Figure 3 below. Overall, there are two critical types of inclined cracking in concrete beams. Flexure-shear cracks start as vertical flexure cracks. The flexure-shear crack then develops "when the combined shear and tensile stress exceeds the tensile strength of the concrete" (ACI 318-95). Web-shear cracking "begins from an interior point in a member when the principal tensile stresses exceed the tensile strength of the concrete" (ACI 318-95).

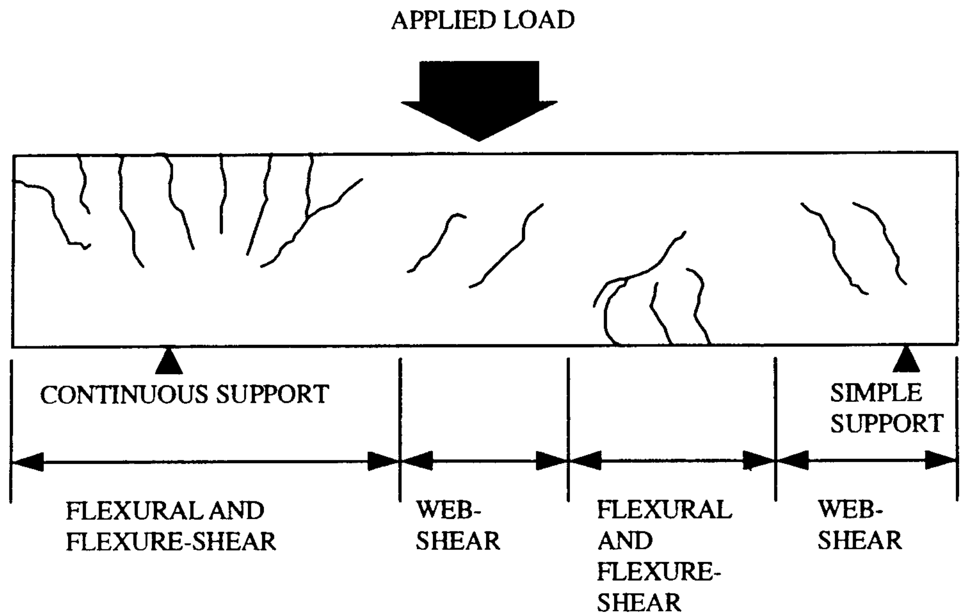

Figure 3. Types of Cracking in Concrete Beams (Adapted from ACI 318-95)

Over the years, more research was done to look specifically at the failure mechanisms along the diagonal cracks. Phenomenon such as "aggregate interlock" at the crack interface and “dowel action” of the reinforcing steel with the concrete became more important for continued research.

ACI 318-95 Method

ACI Committee 318 established empirical shear reinforcement equations based on the results of numerous beam tests. The model for the equations is based on a 45 degree truss analogy. The ACI 318-95 expression for the shear strength of non-prestressed concrete beams is:

$V_n = V_c + V_s = 2\sqrt{f_c'}b_w d + \dfrac{A_v f_y d}{s}$ where

$V_n =$ nominal total shear capacity of the section

$V_c =$ nominal shear capacity of the concrete

$V_s =$ nominal shear capacity of the steel reinforcement

$f_c' =$ concrete compressive strength

$b_w =$ effective web width of section

$d =$ effective depth of section from extreme compression fiber to centroid of longitudinal steel reinforcement

$A_v =$ cross sectional area of steel shear reinforcement

$f_y =$ yield strength of steel reinforcement

s = spacing of shear reinforcement

For prestressed concrete beams, the ACI equations are slightly more complex and will not be given here (see ACI 318-95).

AASHTO LRFD Method

The MCFT approach is based on the strain compatibility in the cracked concrete beam and allows for the fact that "even after cracking, tensile stresses in the concrete between the cracks can resist shear stresses." (Collins, 1996). In fact, Collins rationalizes that the "loss of tensile stresses in the concrete at the crack must be replaced by increased steel stresses" which are ultimately taken up by the longitudinal steel. This fact is shown in Figure 4 below:

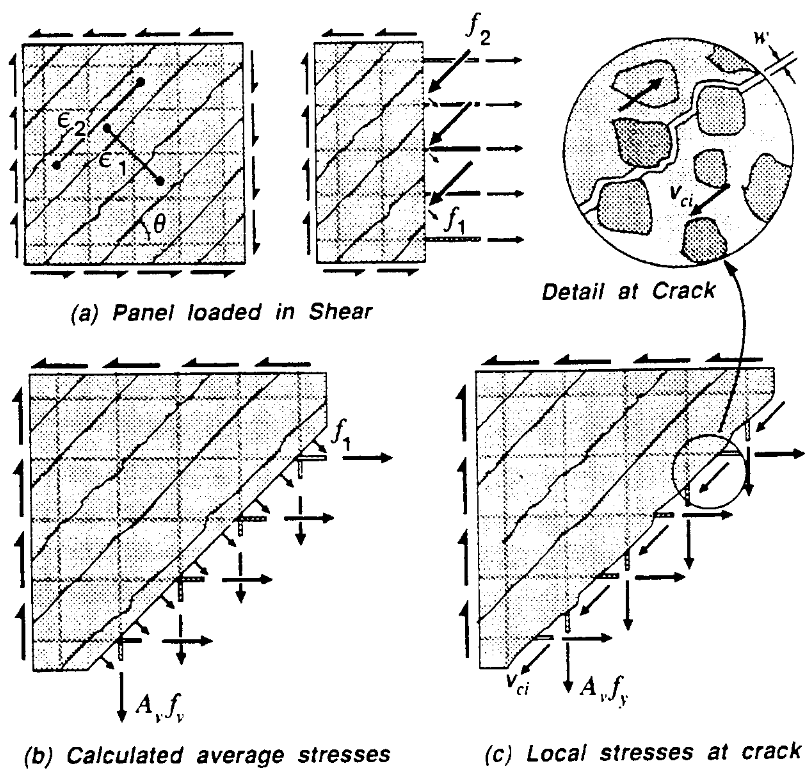

Figure 4. Reinforced concrete panels subjected to shear (Collins, 1996)

According to the MCFT, the nominal shear strength of a section is given by:

$$V_n = V_c + V_d + V_p$$

where V_p = shear load taken by a component of the prestressing steel

But, $V_s = \dfrac{A_v f_y d_v \cot\theta}{s}$ where θ = angle of principle compression

and $V_c = \beta\sqrt{f_c'}b_v d_v$ where β = tensile stress factor

However, β and θ are functions of the average longitudinal strain, ε_x, the shear stress, v, and the crack spacing, s_x, at the section such that:

$$\beta = \frac{\alpha_1\alpha_2 4\cot\theta}{1+\sqrt{500\varepsilon_1}} \le \frac{2.16}{0.3+\dfrac{24w_{cr}}{a+0.63}}$$ where

α_1 = reinforcement bonding characteristics (1.0 for deformed bars)

α_2 = load factor (1.0 for short term monotonic loading)

ε_1 = principle tensile strain

w_{cr} = crack width

a = maximum aggregate size

but we also must calculate the shear stress at the critical section using the expression

$$v = \frac{V_n - V_p}{b_v d_v}$$

and the average strain in the longitudinal steel reinforcement given by

$$\varepsilon_x = \frac{\dfrac{M_u}{d_v} + 0.5(N_u + V_u\cot\theta) - A_{ps}f_{po}}{E_s A_s + E_p A_p}$$

where

$M_u =$ ultimate bending moment load on the section

$N_u =$ ultimate axial load on section

$V_u =$ ultimate shear load on section

$A_{ps} =$ effective cross sectional area of prestressing steel

$f_{po} =$ stress in prestressed tendon when surrounding concrete is at zero stress

$E_s =$ elastic modulus of longitudinal reinforcement

$A_s =$ effective cross sectional area of longitudinal steel reinforcement

$E_p =$ elastic modulus of prestressing steel

$A_p =$ effective cross sectional area of prestressing steel

The significant equations from both methods are summarized in Figures 5 and 6 below:

ACI Method: $V_n = V_c + V_s$

Non-Prestressed Beams	$V_c = \left(1.9\sqrt{f_c'} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d$ but $\frac{V_u d}{M_u} \le 1.0$ $V_c \le 3.5\sqrt{f_c'} b_w d$ or $V_c = 2\sqrt{f_c'} b_w d$ $V_s = \frac{A_v f_y d}{s}$ $V_s \le 8\sqrt{f_c'} b_w d$
Prestressed Beams	$V_c = \left(0.6\sqrt{f_c'} + 700 \frac{V_u d}{M_u}\right) b_w d$ but $2\sqrt{f_c'} b_w d \le V_c \le 5\sqrt{f_c'} b_w d$ or $V_c = V_{ci} = 0.6\sqrt{f_c'} b_w d + V_d + \frac{V_i M_{cr}}{M_{max}}$ but $V_{ci} \ge 1.7\sqrt{f_c'} b_w d$ and $V_c \le V_{cw} = \left(3.5\sqrt{f_c'} + 0.3 f_{pc}\right) b_w d + V_p$ $V_s = \frac{A_v f_y d}{s} \le 8\sqrt{f_c'} b_w d$

Figure 5. ACI 318-95 Shear Design Equations (Collins, 1996)

General Method (AASHTO LRFD/MCFT): $V_n = V_c + V_s + V_p$

$V_c = \beta\sqrt{f_c'}b_v d_v$ $\quad V_s = \dfrac{A_v f_y}{s} d_v \cot\theta$ where β and θ are functions of the strain, ε_x,

shear stress, ν, and crack spacing, s_x

where $\nu = \dfrac{V_n - V_p}{b_v d_v}$ and $\varepsilon_x = \dfrac{\dfrac{M_u}{d_v} + 0.5(N_u + V_u \cot\theta) - A_{ps} f_{po}}{E_s A_s + E_p A_p}$

Note: All equations use U.S. units.

Figure 6. General Method Shear Design Equations (Collins, 1996)

To ease the design process and to insure that concrete crushing does not occur, Professor Collins developed two tables to use with the General method. These tables give values for θ and β based on the average strain in the longitudinal reinforcing steel, ε_x, and the shear stress, ν, at the critical section of the beam. The critical section for beams is located at a distance $d_v \cot\theta$ from the middle of the support. The tables are given for beams with and without steel reinforcement respectively. The table values shown are for U.S. units.

$\frac{v}{f_c'}$		Longitudinal strain $\varepsilon_x \times 1000$					
		≤ 0	≤ 0.25	≤ 0.50	≤ 1.00	≤ 1.50	≤ 2.00
≤ 0.050	θ deg	27.0	28.5	29.0	36.0	41.0	43.0
	β	4.88	3.49	2.51	2.23	1.95	1.72
≤ 0.075	θ deg	27.0	27.5	30.0	36.0	40.0	42.0
	β	4.88	3.01	2.47	2.16	1.90	1.65
≤ 0.100	θ deg	23.5	26.5	30.5	36.0	38.0	39.0
	β	3.26	2.54	2.41	2.09	1.72	1.45
≤ 0.150	θ deg	25.0	29.0	32.0	36.0	36.5	37.0
	β	2.55	2.45	2.28	1.93	1.50	1.24
≤ 0.200	θ deg	27.5	31.0	33.0	34.5	35.0	36.0
	β	2.45	2.33	2.10	1.58	1.21	1.00
≤ 0.250	θ deg	30.0	32.0	33.0	35.5	38.5	41.5
	β	2.30	2.01	1.64	1.40	1.30	1.25

Figure 7. Values of θ and β for members with web reinforcement (U.S. units).

(Collins, 1996)

s_x		Longitudinal strain $\varepsilon_x \times 1000$					
		≤ 0	≤ 0.25	≤ 0.50	≤ 1.00	≤ 1.50	≤ 2.00
≤ 5 in.	θ deg	27.0	29.0	31.0	34.0	36.0 deg	38.0 deg
	β	4.94	3.78	3.19	2.56	2.19	1.93
≤ 10 in.	θ deg	30.0	34.0	37.0	40.0	43.0	45.0 deg
	β	4.65	3.45	2.83	2.19	1.87	1.65
≤ 15 in.	θ deg	32.0	37.0	40.0	45.0	48.0	50.0 deg
	β	4.47	3.21	2.59	1.98	1.65	1.45
≤ 25 in.	θ deg	35.0	41.0	45.0	51.0	54.0	57.0 deg
	β	4.19	2.85	2.26	1.69	1.40	1.18
≤ 50 in.	θ deg	38.0	48.0	53.0	59.0	63.0	66.0 deg
	β	3.83	2.39	1.82	1.27	1.00	0.83
≤ 100 in.	θ deg	42.0	55.0	62.0	69.0	72.0	75.0 deg
	β	3.47	1.88	1.35	0.87	0.65	0.52

Figure 8. Values of θ and β for members without web reinforcement (U.S. units).

(Collins, 1996)

RESULTS

To test the difference between using the two methods, I chose two specific example problems to demonstrate the procedures for shear design. These design examples are contained in Appendix 1. In addition, I used the RESPONSE[17] computer program developed by Collins to analyze the two example problems. The results of this computer analysis are outlined in Appendix 2.

Both the ACI 318-95 and the AASHTO LRFD approach produce safe, economical designs. The MCFT approach better approximates experimental test results. In a study conducted by Xuan, Rizkalla and Maruyama[18] in 1987, the authors concluded that "ACI code underestimates the ultimate shear capacity for beams with shear reinforcement by 43 to 79 percent." They go on to state that this is due to the measured shallow angle, θ, of the diagonal cracks which measured about 25 degrees instead of the 45 degrees assumed by the ACI code.

The benefit of using the General method, according to Collins, is that "prestressed concrete beams, non-prestressed concrete beams and partially prestressed concrete beams having a wide variety of cross-sectional shapes can all be designed using the same basic expressions. The beneficial effects of prestressing are accounted for in the design by allowing lower values of theta to be used which will result in less transverse reinforcement."[19]

Durrani and Robertson conclude that "the observed concrete shear strengths are between 28 and 54 percent higher than the ACI Code predicted values" (Durrani, 1987). Their testing also resulted in cracks at $\theta \cong 30$ degrees to the longitudinal axis of the beams. The nominal shear failure loads that they observed were 37 to 55 percent greater than the ACI predicted values.

Perhaps the best comparison between the ACI 318-95 and the AASHTO LRFD shear design methods is done by Collins. Based on parameters such as member size and maximum aggregate size, Collins compares the results of the two approaches with his own experimental results as shown below in Figure 9.

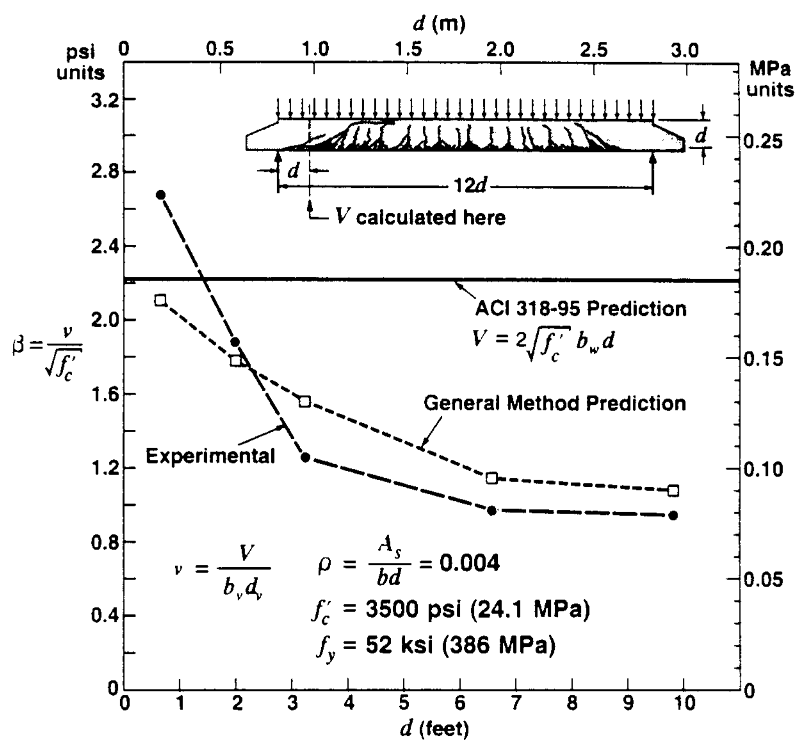

Figure 9. Influence of member size and maximum aggregate size on shear stresses at failure (Collins, 1996).

Figure 9 shows the predicted shear failure levels based on ACI 318-95, the General Method and the experimental results. On the vertical axis, the tensile stress factor, β, is plotted. Increasing beam depth is plotted on the horizontal axis. In this case, the ACI 318-95 method is unconservative, since it overestimates the concrete strength of the section. The General method, while also unconservative, gives a much better approximation of the experimental beam behavior. Collins summarizes his results with those done by several researchers in recent years as shown in Table 2. A total of over 528 beams were tested. Based on a number of parameters such as beam size, loading, and concrete strength, this table shows that the ACI approach can be very inaccurate and unconservative for large, lightly reinforced members and members under high axial compression. For members with uniform loads, members with inclined prestressing tendons and members under high axial tension, the ACI approach is unnecessarily conservative (Collins, 1996).

Reference	Date	Number and specimen type	Loading	Depth, in.	Concrete, psi	Stirrups $\frac{A_v f_y}{b_w s}$ psi	Experiment/predicted: ACI Mean	ACI Coefficient of variation, percent	General Mean	General Coefficient of variation, percent
Kani[19]	1979	68 rectangular beams	2 point loads on simple span	6 to 48	2230 to 5320	0	1.23	14.9	1.35	8.0
Kani[19]	1979	95 T-beams	2 point loads on simple span	12	2510 to 5550	0	1.60	11.5	1.63	10.1
Shioya[15]	1989	13 rectangular beams	Uniformly distributed load on simple span	5 to 124	2860 to 4130	0	0.86	42.9	0.98	25.1
Gupta[20]	1993	10 rectangular beams	End loads applying shear and compression	12	8700 to 9120	0 to 170	0.85	27.3	1.13	16.8
Adebar and Collins[21]	1996	7 rectangular columns	End loads applying shear and tension	12	6700 to 8500	0	2.75	51.4	0.90	12.8
Gregor and Collins[22]	1993	6 prestressed bridge girders	Uniformly distributed load on continuous span	36	6500 to 8400 psi	370 to 590	1.06	17.5	1.37	12.7
Collins and Végh[23]	1993	14 rectangular beams	Point loads on continuous span	11 to 36	7250 to 13,500	0 to 120	0.84	18.2	1.07	15.9
Griezic, Cook, and Mitchell[24]	1993	4 T-beams	Uniformly distributed load on simple span	16	5800	225 to 350	1.34	12.2	1.34	12.6
Haddadin, Hong, and Mattock[25]	1971	59 T-beams	Point loads on beams with tension or compression	18.5	1950 to 6500	0 to 700	1.61	32.3	1.45	18.7
Elzanaty, Nilson, and Slate[26]	1986	33 prestressed I-beams	2 point loads on simple span	14 and 18	6000 to 11,400	0 to 700	1.07	11.6	1.35	9.5
Pasley, Gogoi, Darwin, and McCabe[27]	1990	13 T-beams	Point loads on continuous span	18	4500	0 to 82	0.99	12.0	1.27	7.0
Mattock[28]	1969	31 rectangular beams	Point loads on beams with tension or compression	12	2200 to 8000	0	1.56	24.7	1.45	14.0
Bennett and Balasooriya[29]	1971	20 prestressed I-beams	2 point loads on simple span	10 and 18	4400 to 6460	630 to 1900	1.71	19.4	1.46	18.2
Bennett and Debaikey[29]	1974	22 prestressed I-beams	Point load on simple span	13	6000 to 10,500	103 to 5600	1.15	9.9	1.54	10.9
Moody, Viest, Elstner, and Hognestad[31]	1954	12 rectangular beams	Point load on simple span	12	880 to 4600	0	1.27	14.2	1.27	13.5
MacGregor[32]	1960	33 prestressed I-beams	Point load on simple span	12	2400 to 7000	0 to 470	1.09	25.8	1.54	22.5
Oleson, Sozen, and Siess[33]	1967	27 prestressed I-beams	Point load on simple span	12	2450 to 6700	0 to 350	1.06	18.8	1.59	15.3
Roller and Russell[34]	1990	10 rectangular beams	Point load on simple span	25 to 34	10,500 to 18,170	0 to 1176	1.05	20.0	1.19	13.5
Shahawy, Robinson, and Batchelor[35]	1993	39 full-size prestressed bridge girders	Point load on simple span	44	6000	165 to 1670	1.09	19.5	1.13	15.8
Yoon, Cook, and Mitchell[36]	1996	12 rectangular beams	Point load on simple span	30	5220 to 12,615	0 to 145	1.14	13.8	1.07	10.3
		528 beams				Average	1.32	33.7	1.39	19.7

Table 2. Experimental verification of ACI 318 vs. General Method (Collins, 1996)

Based on the results of Table 2 and Figure 10 below, it can easily be seen that the General method better represents the actual physical behavior of the beam under loading (Collins, 1996).

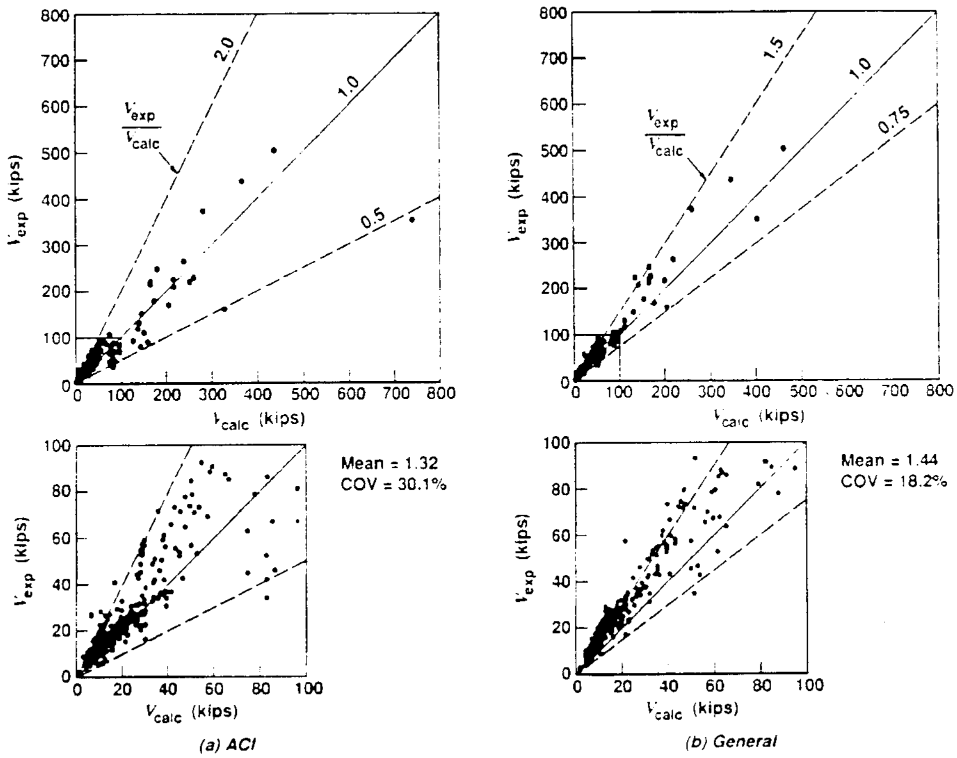

Figure 10. Experimental and predicted failure shears for 528 tests: ACI vs. General method (Collins, 1996).

Welded Wire Fabric

While the tensile strength of WWF turns out to be higher than that of conventional stirrups, Xuan notes that the ductility of the WWF is significantly less than that of the stirrups. He states that "the ultimate strain of the WWF is usually less than 2 percent while that of conventional stirrups is greater than 15 percent." (Xuan, 1988). However, he also states that WWF "used as shear reinforcement should exhibit adequate ductility to insure the overall ductility of the member." Xuan also concludes that anchorage by using two horizontal wires (as outlined in both the ACI 318 and AASHTO LRFD codes) at the top and the bottom of the vertical wires is sufficient to prevent premature beam failure (Xuan, 1988).

In testing completed by Pincheira, Rizkalla and Attiogbe[20] in 1988, the authors state that the "deformed WWF seems to provide a slightly better crack width control compared to conventional double-legged and single-legged stirrups." They also conclude that anchorage of the WWF by two horizontal wires at the top and bottom of the shear reinforcement was sufficient. However, they found that deformed "WWF is not as effective as conventional stirrups under cyclic loading" (Pincheira, 1989).

Durrani and Robertson indicate that the "observed shear strength contribution of the mesh reinforcement (WWF) was consistently higher than the ACI Code value by more than 50 percent (Durrani, 1987). They also conclude that two anchorage wires are sufficient for guarding against shear failure. Based on their testing, they conclude that both horizontal wires are "active" in taking the load and that premature failure of one wire did not lead to beam failure. However, in beams with only one horizontal wire for anchorage, there was premature failure. The authors directly relate the strength of the anchorage to the strength and quality of the welds in the fabric. They further state that deformed WWF forms a better bond with the concrete than that formed by smooth WWF (Durrani, 1987). Some details of WWF anchorage based on ACI 318-95 are shown in Figure 11:

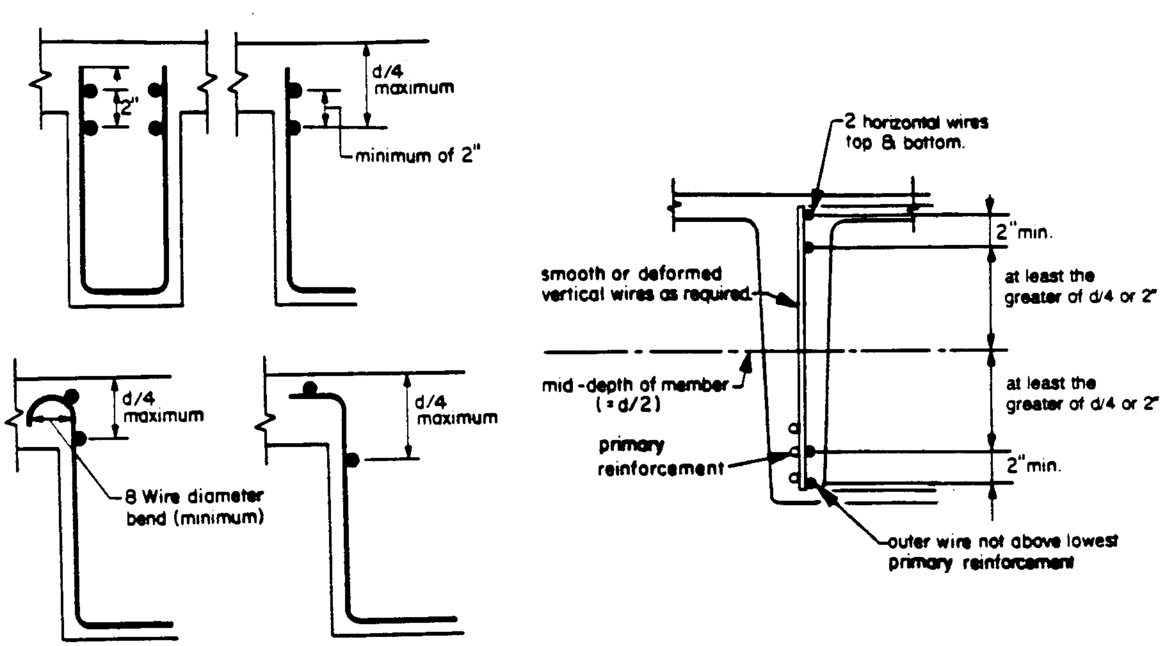

Figure 11. Anchorage of WWF in Concrete Beams (ACI 318-95)

Mitchell, Griezic and Cook compared the shear strength of WWF using both the ACI and General method approaches (Mitchell, 1994). In their study, they tested 4 beams using either WWF or conventional stirrups as shear reinforcement. They also kept the same steel ratio so that they could make a direct comparison of the results between the beams. The reinforcement details of two of the beams with deformed WWF shear reinforcement are shown in Figure 12:

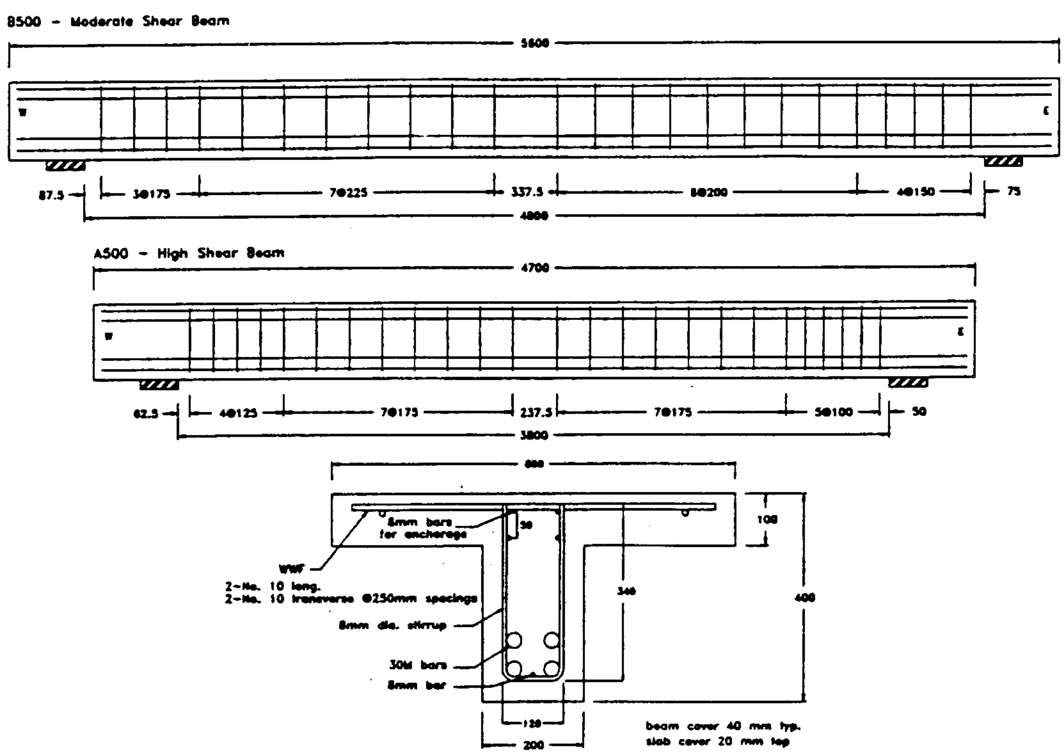

Figure 12. Detail of WWF Reinforcement in Beams A500 and B500 (Mitchell, 1994)

The cracking patterns of the four beams are shown in Figure 13:

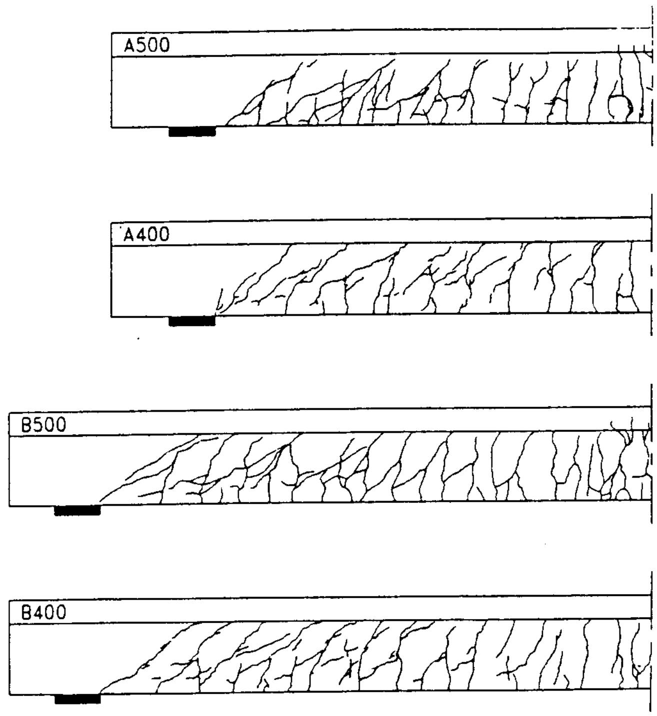

Figure 13. Cracking patterns of the four test beams (Mitchell, 1994).

The results of the comparison between the ACI empirical method and the General method are outlined below in Table 3:

Beam	Test results: V_{max} at d, kN	Test results: Failure mode	ACI 318M-89 predictions: V_n at d, kN	ACI 318M-89 predictions: Failure mode	ACI 318M-89 predictions: $\frac{V_{max}}{V_{ACI}}$	MCFT predictions: V_n at d, kN	MCFT predictions: Failure mode	MCFT predictions: $\frac{V_{max}}{V_{MCFT}}$
A500	332	Flexure	200	Shear	1.66	330	Flexure	1.01
A400	334	Flexure	212	Shear	1.58	316	Flexure	1.06
B500	291	Flexure	162	Shear	1.80	267	Flexure	1.09
B400	272	Flexure	169	Shear	1.61	266	Flexure	1.02

Table 3. Strength predictions using ACI 318 and General method (Mitchell, 1994)

As a result, Mitchell concludes that the smaller diameter WWF prevents larger cracks as compared to conventional stirrups with equivalent amounts of steel. This is a result of the smaller wires effectively "stapling" any diagonal cracks before they can open up. He also concludes that two horizontal wires are sufficient for anchorage. Mitchell states that "cold-rolled deformed welded wire fabric stirrups exhibited large strains and sufficient ductility to redistribute the stresses in the stirrups to avoid a sudden, brittle shear failure." He goes on to state that the "predictions of shear capacity using the modified compression field theory are more accurate that the predictions using ACI 318 expressions." In addition, he recommends using the full nominal yield stress of 500Mpa(about 72 ksi) for design calculations (Mitchell, 1994).

DISCUSSION AND CONCLUSIONS

Both the ACI 318-95 approach and the AASHTO LRFD (General method) approach to shear design produce safe beam designs. The empirical ACI 318 specifications are the result of years of testing and code refinement. While much can be said about the improvements in ACI shear design methods in the last 40 years, the code has been slow to change. One of the main reasons for this is that changes in the ACI 318 code specification must be approved by a voting committee. Another reason that the ACI 318 code is slow to change is that the current specifications have "passed the test of time." That is, they have been used successfully for many years. Despite the desire of the 1973 ACI 318 committee to have a "physically significant and rational method for shear design," the code has remained strictly empirical in its approach to shear design.

The General method adopted by the AASHTO LRFD specifications provides a "simplified, physically significant and rational" approach to beam shear design. This method offers several advantages:

- it is a rational approach based on the physical properties (stress-strain, equilibrium and compatibility conditions) of the beam

- it more accurately represents the "true condition" of the concrete beam by considering the strain in the longitudinal steel and the level of cracking in the beam. It accounts for the ability of the concrete between the cracks to carry some tensile load. It also accounts for the vertical component of the prestressing force to help carry some of the shear load.

- it uses fewer equations than the complex equations found in the ACI 318 specifications. The same General method equations cover all beam types, sizes and geometries.

The biggest obstacle faced by the General (MCFT) method is that it is relatively unproven in the construction industry. It will be interesting to see the level of acceptance of the AASHTO LRFD approach once many designers and steel detailers have a chance to become familiar with the procedure.

Welded Wire Fabric

Welded Wire Fabric greatly reduces the time, effort and material required for shear reinforcement design and placement. It can be an effective shear reinforcing material when proper attention is given to anchorage details and weld quality. Although WWF is less ductile than conventional cold rolled steel stirrups, its higher yield strength makes it a desirable material.

The two design examples done in Appendices 1 and 2 yield approximately the same answers using both the ACI 318-95 and the AASHTO LRFD approaches. In the first example problem, a 115 foot long, prestressed concrete bulb tee girder with composite slab is analyzed for shear reinforcement requirements. The web thickness of the girder is only 6 inches, thus making it a candidate for WWF shear reinforcement based on the confined work area for steel placement. The prestressing tendons are harped. The ACI 318-95 strength, minimum area and maximum spacing criteria are checked. In this case, the strength criteria controls and the steel wire spacing is selected at 1.5 inches. Next, the same girder is analyzed with the AASHTO LRFD procedure. After checking the maximum spacing, minimum reinforcement and strength criteria, we find that strength criteria controls. The steel wire spacing is selected at 4 inches.

The second example problem consists of a 75 foot long, prestressed concrete Type IV girder with composite slab. Once again, after checking the strength, minimum area and maximum spacing criteria according to ACI 318-95, we find that strength controls. The steel spacing is selected at 3 inches. Next, the same girder is analyzed with the AASHTO LRFD procedure. After checking the maximum spacing, minimum reinforcement and strength criteria, we find that minimum reinforcement controls. This time the spacing is selected at 3 inches, the same as determined for the ACI 318-95 method.

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APPENDIX 1: DESIGN EXAMPLES USING ACI 318-95 AND AASHTO LRFD GENERAL METHOD

Example IA: Prestressed Concrete Bulb Tee Girder with composite slab. Span length is 115 feet. Consider HS 25 loading.[21]

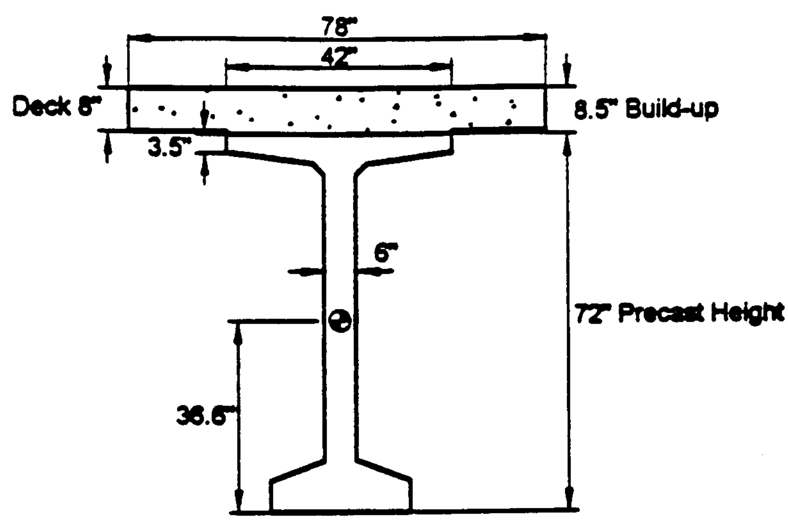

Figure 14. Girder Cross-Section Example IA (Feeser, 1994)

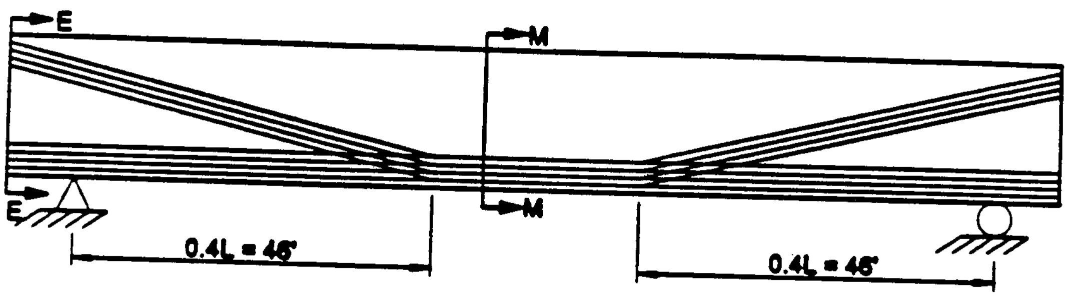

Figure 15. Prestressing Steel Harping Pattern Example IA (Feeser, 1994)

Dist. Ft.	V_i kips	M_{max} k-ft	V_u/ϕ	V_{ci} kips	V_{cimin} kips	V_{cw} kips	M_{cr} k-ft
0.00	160.4	0.0	284.3	10000.0	50.9	128.7	1642.8
1.58	157.3	250.3	284.3	2814.0	50.9	208.8	4269.8
2.30	156.6	361.1	284.3	1940.0	50.9	210.8	4194.9
3.41	154.7	530.5	284.3	1309.2	50.9	213.7	4080.9
4.60	152.7	706.5	279.7	973.6	50.9	216.8	3962.8
6.90	148.9	1036.1	270.7	650.5	51.0	223.0	3743.1
9.20	145.0	1350.0	261.7	488.4	51.5	230.6	3535.8
11.50	141.1	1648.0	252.7	390.7	52.1	237.8	3340.8
23.00	121.7	2902.1	207.4	191.3	54.7	270.4	2551.8
28.75	111.9	3381.4	184.9	149.5	56.0	284.0	2273.3
34.50	102.0	3762.2	162.1	120.3	57.3	295.7	2072.3
46.00	71.9	4271.8	116.6	75.9	59.9	312.4	1902.3
57.50	51.7	4409.1	70.8	42.9	59.9	297.2	1723.7

Table 4. Critical locations and loads for Example IA (Feeser, 1994)

Method A: ACI 318-95:

Based on the table above, the critical point is located at a distance h/2 or 3.41 feet from the support. At this distance, the shear component required to be carried by the reinforcing steel, $V_s = 70.6$ kips. Since $4\sqrt{f'_c}b_w d \leq 70.6$ kips (Spec.11.5.4.3), the shear reinforcement spacing is either less than 3/4 of the height or 24" whichever is smaller. That is, $s \leq 0.75\,h$ or 24" (Spec.11.5.4.1). Checking the shear component for its maximum value, $V_s \leq 8\sqrt{f'_c}b_w d$ (Spec.11.5.6.8) is okay. We therefore need to check the shear reinforcement based on three criteria: strength (Spec.11.5.6.2), minimum area (Spec.11.5.5.3) and maximum spacing (Spec.11.5.4.1).

Strength: $s = \dfrac{A_v f_y d}{V_s} = \dfrac{(0.031\text{in}^2)(60\text{ksi})(63.56\text{in})}{70.6\text{kips}} = 1.67"$ based on the assumption that $A_v = 0.031\text{in}^2$ using D4.0 wire WWF. (Durrani, 1987).

Minimum Area: $s = \dfrac{A_v f_y}{50 b_w} = \dfrac{(0.031\text{in}^2)(60{,}000\text{psi})}{50(6\text{in})} = 6.2"$

Maximum Spacing: $s \leq \begin{Bmatrix} 0.75h = 0.75(80.5) = 60.4" \\ 24" \end{Bmatrix}$

Therefore, strength criteria controls and we would choose D4.0 wire shear reinforcement at approximately 1.67" (say 1.5") on center. We would then check anchorage requirements (Spec.12.13.2) and figure R12.13.2.4. We would also need to check development length requirements (Spec.12.7).

Method B: AASHTO LRFD:

The critical section using this approach is located at either a distance $0.5d_v \cot\,\theta$ or a distance d_v from the support. We assume $\theta \cong 30$ degrees and the strain in the longitudinal steel, $\varepsilon_x \cong 0$ near the support. Thus, $d_v = 0.9d_e$ or $0.72h = 67.05$." Therefore, $0.5d_v \cot\theta = 0.5(67.05)(\cot 30) = 58.06"$ and the previous value of $d_v = 67.05"$ controls.

Therefore, based on the critical loads shown in Table 4 and using Specification 5.8.3.4.2, the shear stress is approximately:

$$v = \frac{V_u - \phi V_p}{\phi b_v d_v} = \frac{(279.7)(0.85) - (0.85)(36)(0.153)(183.6)}{0.85(6")(67.05")} = 0.68\text{ksi}$$ assuming that we are using 1/2" diameter stress relieved strand prestressing steel. Assuming also that $f_c' = 5$ ksi, we have the following:

$$\frac{v}{f_c'} = \frac{0.68}{5\text{ksi}} = 0.136$$

Therefore, from Table 5.8.3.4.2-1 with $\varepsilon_x = 0$, we arrive at $\theta \cong 24\,\text{deg}$ and $\beta \cong 2.57$.

Using these new values, we calculate ε_x, according to Specification 5.8.3.4.2:

$$\varepsilon_x = \frac{\frac{M_u}{d_v} + 0.5N_u + 0.5V_u \cot\theta - A_{ps} f_{po}}{E_s A_s + E_p A_{ps}} \le 0.002$$

Substituting our values, we get:

$$\varepsilon_x = \frac{\frac{(3962.8\text{k} - \text{ft})(12'\text{ in / ft})}{67.05\text{in}} + 0.5(279.7)(0.85)(\cot 24\,\text{deg}) - 36(0.153)(184\text{ksi})}{29(2.65) + 28.5(5.508)} \cong -0.16$$

We now calculate the required steel reinforcement according to the maximum spacing (Spec.5.8.2.7), minimum reinforcement (Spec.5.8.2.5) and strength (Spec.5.8.3.3) criteria:

Maximum spacing: since $V_u \geq 0.1 f_c' b_v d_v$ then $s \leq \begin{Bmatrix} 0.4 d_v \\ 12" \end{Bmatrix}$

Substituting our values: $s \leq \begin{Bmatrix} 0.4(67.05\text{in}) = 26.8\text{in} \\ 12\text{in} \end{Bmatrix}$

Minimum reinforcement: $s = \dfrac{A_v f_y}{0.0316\sqrt{f_c'} b_v} = \dfrac{(0.031\text{in}^2)(60\text{ksi})}{0.0316(\sqrt{5\text{ksi}})(6\text{in})} = 4.39"$

Strength: $s = \dfrac{A_v f_y d_v \cot\theta}{V_s} = \dfrac{(0.031\text{in}^2)(60\text{ksi})(67.05\text{in})(\cot 24\,\text{deg})}{70.6\text{ kips}} = 3.97"$

Therefore, strength criteria controls and we would choose a spacing of 4 inches. We would use D4.0 wire WWF at 4" spacing.

Comparing the two methods, with ACI 318-95 we would use D4.0 wire WWF at 1.5" spacing. With AASHTO LRFD, we would use D4.0 wire WWF at 4" spacing.

In the AASHTO analysis, we started with a conservative estimate that $\theta \cong 30$ degrees. Based on the critical loading and the section properties, we found that $\theta = 24$ degrees. The computer analysis done in Appendix 2 shows that the maximum shear capacity of the section occurs at $\theta = 29$ degrees.

Example IB: Prestressed concrete Type IV girder with composite slab.

Slab length is 75 feet. Single span with two traffic lanes. HS-20 loading.[21]

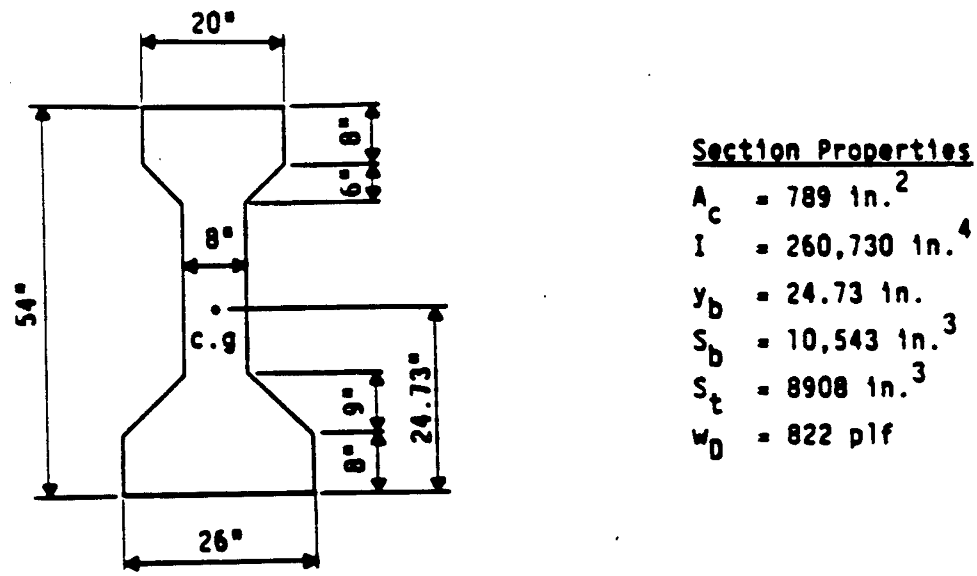

Figure 16. Cross Section of Girder Example IB (Feeser, 1994)

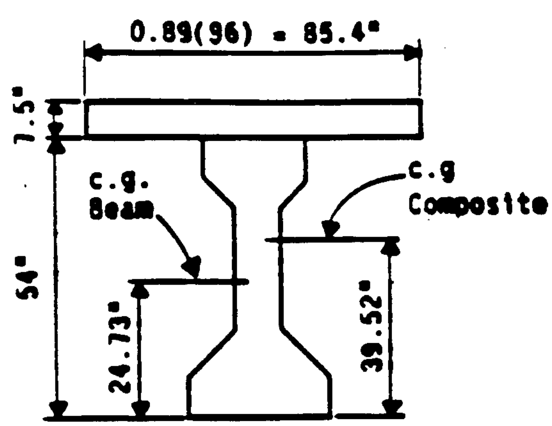

Figure 17. Transformed section of Girder Example IB (Feeser, 1994)

Based on HS-20 loading, we calculate the design loads at the critical section as:

$V_u = 99.95$ kips and $M_u = 2779$ kip – ft.

Method A: ACI 318-95. Using a similar procedure as in Example IA, the shear required to be carried by the steel reinforcement is found to be: $V_s = 35.12$ kips

Strength: $s = \dfrac{A_v f_y d}{V_s} = \dfrac{(0.031\text{in}^2)(60\text{ksi})(57.5\text{in})}{35.12\text{kips}} = 3.05"$

Minimum Area: $s = \dfrac{A_v f_y}{50 b_w} = \dfrac{(0.031\text{in}^2)(60{,}000\text{psi})}{50(8\text{in})} = 4.65"$

Maximum Spacing: $s \le \begin{Bmatrix} 0.75h = 0.75(61.5") = 46.1" \\ 24" \end{Bmatrix}$

Therefore, we would choose D4.0 wire at 3" on center. We would also need to check anchorage requirements (Spec.12.13.2) and development length requirements (Spec.12.7) as well.

Method B: AASHTO LRFD. Using the same method as Example IA, the critical section is located at $d_v = 57.5"$ from the support. We then calculate the shear stress at this point:

$$v = \frac{V_u - \phi V_p}{\phi b_v d_v} = \frac{99.95\text{kips}(0.85) - 0.85(36)(0.153)(183.6\text{ksi})}{0.85(8\text{in})(57.5\text{in})} = -1.98\text{ksi}$$

Assuming that f_c'=5 ksi, we calculate the ratio $\dfrac{v}{f_c'} = -0.40$

From Table 5.8.3.4.2-1 with $\varepsilon_x = 0, \theta \cong 27\,\text{deg}, \beta \cong 4.88$, we calculate the average longitudinal steel strain according to Specification 5.8.3.4.2:

$$\varepsilon_x = \frac{\frac{M_u}{d_v} + 0.5N_u + 0.5V_u \cot\theta - A_{ps}f_{po}}{E_sA_s + E_pA_{ps}} \cong 0$$

Now, calculate the reinforcing steel according to maximum spacing (Spec.5.8.2.7), minimum reinforcement (Spec.5.8.2.5) and strength (Spec. 5.8.3.3) criteria:

Maximum spacing: Since $V_u \leq 0.1 f'_c b_v d_v$ then

$$s \leq \begin{Bmatrix} 0.8d_v = 0.8(57.5\text{in}) = 46" \\ 24" \end{Bmatrix}$$

Minimum reinforcement: $s = \dfrac{A_v f_y}{0.0316\sqrt{f'_c}b_v} = \dfrac{(0.031\text{in}^2)(60\text{ksi})}{0.0316\sqrt{5\text{ksi}}\,(8\text{in})} = 3.29"$

Strength: $s = \dfrac{A_v f_y d_v \cot\theta}{V_s} = \dfrac{(0.031\text{in}^2)(60\text{ksi})(57.5\text{in})(\cot 27\,\text{deg})}{35.12\text{ kips}} = 5.98"$

Minimum reinforcement controls and we would use D4.0 wire WWF at 3" on center spacing.

Based on both ACI 318-95 and the AASHTO LRFD specifications, we would use D4.0 wire WWF at a spacing of 3" on center.

Once again, in the AASHTO analysis we started with a conservative estimate that $\theta \cong 30$ degrees. Based on the critical loading and the section properties, we found that $\theta = 27$ degrees. The computer analysis done in Appendix 2 shows that the maximum shear capacity of the section occurs at $\theta = 32$ degrees.

APPENDIX 2: DESIGN EXAMPLE ANALYSIS USING *RESPONSE* COMPUTER PROGRAM

EXAMPLE IA

Name of beam? Example IA

Web width in ? 6

Shear depth DV in ? 67.05

Total concrete area AC in2 ? 1314

Cylinder strength of concrete FCP psi e.g. 3500 ? 5000

Peak strain x 1000 ECP e.g. -2.2 ? -3

Cracking strength of concrete FCR psi ? 300

Maximum aggregate size MAGG in. e.g. 0.75 ? 0.75

Tension stiffening factors e.g. 1.0, 0.7 or 0.49 ? 1

Total area of longitudinal rebars ASX in2 ? 2.65

Yield strength of longitudinal rebars FYX ksi ? 60

Area of longitudinal tendons APX in2 ? 5.508

Ultimate strength of tendons FPU ksi ? 189

Modulus of tendons/1000 EP ksi e.g.29? 28.5

Ram-Os parameters of tendon A,B,C e.g. 0.025,118,10? 0.017,134,10

Strain difference of tendonx1000 DEP e.g. 6? 5.4

Area of stirrup legs AV in2 ? 0.031

Spacing of stirrups S in? 4

Yield strength of stirrups FYV ksi ? 60

Crack spacing controlled by long. reinf. SMX in ? 38.75

Crack spacing controlled by stirrups SMV in ? 31.07

****

SHEAR RESPONSE OF MEMBER Example IA

****

SECTION PROPERTIES

BV= 6 in DV= 67.05 in AC= 1314 in2 MAGG= .75 in

FCP= 5000 psi ECP=-3 FCR= 300 psi TSF= 1

ASX= 2.65 in2 FYX= 60 ksi

APX= 5.508 in2 FPU= 189 ksi A= .017 B= 134 C= 10

AV= .031 in2 FYV= 60 ksi

SMX= 38.75 in SMV= 31.07 in

****

If axial load constant type 1 if N/V constant type 2? 1

Axial load N kips ? 0

Axial Load N= 0 kips

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.01

THETA= 12.84 N= -0.0 kips V= 58.9 kips

ETx1000= -0.00 EXx1000= -0.19 GAMMAx1000= 0.09

F1= 33 psi F2= 642 psi F2MAX= 5000 psi

Crack spacing = 26.9 in Crack width = 0.000 in

Concrete tension limited by crack slipping.

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.02

THETA= 17.45 N= -0.0 kips V= 85.3 kips

ETx1000= -0.00 EXx1000= -0.19 GAMMAx1000= 0.13

F1= 67 psi F2= 675 psi F2MAX= 5000 psi

Crack spacing = 26.0 in Crack width = 0.001 in

Concrete tension limited by crack slipping.

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.03

THETA= 20.61 N= -0.0 kips V= 107.0 kips

ETx1000= -0.00 EXx1000= -0.19 GAMMAx1000= 0.16

F1= 100 psi F2= 707 psi F2MAX= 5000 psi

Crack spacing = 25.5 in Crack width = 0.001 in

Concrete tension limited by crack slipping.

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.04

THETA= 23.01 N= 0.1 kips V= 126.3 kips

ETx1000= -0.00 EXx1000= -0.19 GAMMAx1000= 0.19

F1= 133 psi F2= 739 psi F2MAX= 5000 psi

Crack spacing = 25.2 in Crack width = 0.001 in

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.05

THETA= 24.93 N= -0.0 kips V= 144.3 kips

ETx1000= -0.00 EXx1000= -0.19 GAMMAx1000= 0.22

F1= 167 psi F2= 772 psi F2MAX= 5000 psi

Crack spacing = 25.0 in Crack width = 0.001 in

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.06

THETA= 26.51 N= 0.0 kips V= 161.3 kips

ETx1000= -0.00 EXx1000= -0.19 GAMMAx1000= 0.25

F1= 200 psi F2= 804 psi F2MAX= 5000 psi

Crack spacing = 24.8 in Crack width = 0.001 in

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.07

THETA= 27.85 N= 0.0 kips V= 177.7 kips

ETx1000= -0.00 EXx1000= -0.19 GAMMAx1000= 0.27

F1= 233 psi F2= 836 psi F2MAX= 5000 psi

Crack spacing = 24.7 in Crack width = 0.002 in

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.08

THETA= 28.99 N= 0.0 kips V= 193.6 kips

ETx1000= -0.00 EXx1000= -0.19 GAMMAx1000= 0.30

F1= 267 psi F2= 868 psi F2MAX= 5000 psi

Crack spacing = 24.6 in Crack width = 0.002 in

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.09

THETA= 28.27 N= 0.0 kips V= 185.5 kips

ETx1000= 0.01 EXx1000= -0.19 GAMMAx1000= 0.30

F1= 247 psi F2= 858 psi F2MAX= 5000 psi

Crack spacing = 24.6 in Crack width = 0.002 in

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.1

THETA= 28.13 N= 0.0 kips V= 185.1 kips

ETx1000= 0.02 EXx1000= -0.19 GAMMAx1000= 0.31

F1= 245 psi F2= 861 psi F2MAX= 5000 psi

Crack spacing = 24.7 in Crack width = 0.002 in

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.5

THETA= 25.04 N= -0.1 kips V= 183.3 kips

ETx1000= 0.35 EXx1000= -0.17 GAMMAx1000= 0.62

F1= 200 psi   F2= 989 psi   F2MAX= 5000 psi

Crack spacing = 24.9 in   Crack width = 0.012 in

Concrete tension limited by crack slipping.

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 100

EXAMPLE IB

Name of beam? Example IB

Web width in ? 8

Shear depth DV in ? 57.75

Total concrete area AC in2 ? 1429

Cylinder strength of concrete FCP psi e.g. 3500 ? 5000

Peak strain x 1000 ECP e.g. -2.2 ? -3

Cracking strength of concrete FCR psi ? 300

Maximum aggregate size MAGG in. e.g. 0.75 ? 0.75

Tension stiffening factors e.g. 1.0, 0.7 or 0.49 ? 1

Total area of longitudinal rebars ASX in2 ? 2.65

Yield strength of longitudinal rebars FYX ksi ? 60

Area of longitudinal tendons APX in2 ? 3.672

Ultimate strength of tendons FPU ksi ? 189

Modulus of tendons/1000 EP ksi e.g.29? 28.5

Ram-Os parameters of tendon A,B,C e.g. 0.025,118,10? 0.017,134,10

Strain difference of tendonx1000 DEP e.g. 6? 5.4

Area of stirrup legs AV in2 ? 0.031

Spacing of stirrups S in? 3

Yield strength of stirrups FYV ksi ? 60

Crack spacing controlled by long. reinf. SMX in ? 34.48

Crack spacing controlled by stirrups SMV in ? 27.34

****

SHEAR RESPONSE OF MEMBER exampleIB

****

SECTION PROPERTIES

BV= 8 in DV= 57.75 in AC= 1429 in2 MAGG= .75 in

FCP= 5000 psi ECP=-3 FCR= 300 psi TSF= 1

ASX= 2.65 in2 FYX= 60 ksi

APX= 3.672 in2 FPU= 189 ksi A= .017 B= 134 C= 10

AV= .031 in2 FYV= 60 ksi

SMX= 34.48 in SMV= 27.34 in

****

If axial load constant type 1 if N/V constant type 2? 1

Axial load N kips ? 0

Axial Load N= 0 kips

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.01

THETA= 15.88 N= 0.1 kips V= 54.2 kips

ETx1000= -0.00 EXx1000= -0.12 GAMMAx1000= 0.07

F1= 33 psi F2= 412 psi F2MAX= 5000 psi

Crack spacing = 23.2 in Crack width = 0.000 in

Concrete tension limited by crack slipping.

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.02

THETA= 21.16 N= 0.0 kips V= 79.6 kips

ETx1000= -0.00 EXx1000= -0.12 GAMMAx1000= 0.11

F1= 67 psi F2= 445 psi F2MAX= 5000 psi

Crack spacing = 22.4 in Crack width = 0.000 in

Concrete tension limited by crack slipping.

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.03

THETA= 24.59 N= 0.1 kips V= 101.0 kips

ETx1000= -0.00 EXx1000= -0.12 GAMMAx1000= 0.13

F1= 100 psi F2= 478 psi F2MAX= 5000 psi

Crack spacing = 22.1 in Crack width = 0.001 in

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.04

THETA= 27.07 N= -0.0 kips V= 120.6 kips

ETx1000= -0.00 EXx1000= -0.12 GAMMAx1000= 0.16

F1= 133 psi F2= 511 psi F2MAX= 5000 psi

Crack spacing = 21.8 in Crack width = 0.001 in

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.05

THETA= 28.99 N= 0.0 kips V= 139.0 kips

ETx1000= -0.00 EXx1000= -0.12 GAMMAx1000= 0.18

F1= 167 psi F2= 543 psi F2MAX= 5000 psi

Crack spacing = 21.7 in Crack width = 0.001 in

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.06

THETA= 30.52 N= 0.0 kips V= 156.8 kips

ETx1000= -0.00 EXx1000= -0.12 GAMMAx1000= 0.21

F1= 200 psi F2= 576 psi F2MAX= 5000 psi

Crack spacing = 21.6 in Crack width = 0.001 in

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.07

THETA= 31.78 N= 0.0 kips V= 174.1 kips

ETx1000= -0.00 EXx1000= -0.12 GAMMAx1000= 0.23

F1= 233 psi F2= 608 psi F2MAX= 5000 psi

Crack spacing = 21.6 in Crack width = 0.002 in

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.08

THETA= 32.83 N= 0.0 kips V= 191.0 kips

ETx1000= -0.00 EXx1000= -0.12 GAMMAx1000= 0.25

F1= 267 psi F2= 641 psi F2MAX= 5000 psi

Crack spacing = 21.5 in   Crack width = 0.002 in

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.09

THETA= 31.89 N= 0.0 kips V= 180.5 kips

ETx1000= 0.01 EXx1000= -0.12 GAMMAx1000= 0.26

F1= 243 psi F2= 628 psi F2MAX= 5000 psi

Crack spacing = 21.6 in   Crack width = 0.002 in

Longitudinal reinforcement yielding at crack.

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.1

THETA= 31.70 N= 0.0 kips V= 180.3 kips

ETx1000= 0.02 EXx1000= -0.11 GAMMAx1000= 0.26

F1= 240 psi F2= 633 psi F2MAX= 5000 psi

Crack spacing = 21.6 in   Crack width = 0.002 in

Longitudinal reinforcement yielding at crack.

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 0.5

THETA= 27.15 N= 0.0 kips V= 181.2 kips

ETx1000= 0.35 EXx1000= -0.09 GAMMAx1000= 0.60

F1= 188 psi F2= 778 psi F2MAX= 5000 psi

Crack spacing = 21.8 in   Crack width = 0.011 in

Longitudinal reinforcement yielding at crack.

****

Value of prin.tens.str.x1000 E1 e.g. 2 Input 99 to change N.Input 100 to end. ? 100