## BEAULIEU COLLEGE

## PRELIMINARY EXAMINATIONS

GRADE 12

## MATHEMATICS

PAPER 2

| Time: | 3 Hours | 150 marks |
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| Date: | 29 July 2019 |  |
| Examiner: | Ms Smith | Moderator: | Mr Scholefield

EXAMINATION NUMBER:

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TEACHER: $\square$ Meikle
 Scholefield


## PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This paper consists of 26 pages and two pages with additional space for working. An Information Sheet of 2 pages (i-ii) is also provided. Please check that your paper is complete.
2. Write the last three digits of your examination number in the space provided above and answer all the questions on the question paper.
3. Please note that diagrams are not necessarily drawn to scale.
4. All necessary working details must be shown.
5. Round your answers off to TWO decimal place unless stated otherwise.
6. Approved non-programmable and non-graphical calculators may be used, unless otherwise stated.
7. Ensure that your calculator is in DEGREE mode.
8. It is in your own interest to write legibly and to present your work neatly.

## SECTION A

## QUESTION 1

In the diagram alongside, $A B C D$ is a quadrilateral having vertices $A(7 ; 1), B(-2 ; 9), C(-3 ;-4)$ and $D(8 ;-11)$.
$M$ is the midpoint of $B D$.

(a) Calculate the length of AC.
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(b) Calculate the gradient of AC.
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(c) Determine the equation of $A C$ in the form $y=m x+c$.
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(d) Determine whether M lies on AC .
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(e) Prove that $B D \perp A C$.
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(f) Calculate $\theta$, the inclination of BD.
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(g) Calculate the size of CBED.
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## QUESTION 2

In the diagram below, a circle with equation $x^{2}-8 x+y^{2}-4 y+15=0$, with centre $M$ is given. BE and EL are tangents to the circle at D and N respectively. The equation of tangent BE is $2 y-x=5$.

(a) Determine the coordinates of M and the length of the radius of the circle, showing ALL your calculations. (Leave your answer(s) in simplest surd form, if necessary.)
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(b) If it is given that the coordinates of $M$ are $(4 ; 2)$, determine the equation of $M D$.
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(c) Show that the coordinates of $D$ are $(3 ; 4)$.
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(d) If it is given that the coordinates of $L$ are $(9 ; 0)$, calculate the length of $L N$.
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## QUESTION 3

In the diagram below, the graph of the function $f(x)=\sin a x+b$ is drawn for $x \in\left[-180^{\circ} ; 180^{\circ}\right]$.

(a) Write down the values of $a$ and $b$.
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(b) On the set of axes given above, sketch the graph of $g(x)=-3 \cos x$ for $x \in\left[-180^{\circ} ; 180^{\circ}\right]$.
(c) Use the diagram to determine the maximum value of $h$ if $h(x)=g(x)-f(x)$ for $x \in\left[-180^{\circ} ; 180^{\circ}\right]$.
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## QUESTION 4

(a) Use the diagram below to prove the theorem that states:
"The opposite angles of a cyclic quadrilateral are supplementary."


Construction:

Required to prove:

Proof: $\qquad$
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(b) In the diagram below, PQRS is a cyclic quadrilateral. ST is a tangent to the circle at $S$ and chord $S R$ is produced to $V . P Q=Q R, \hat{S}_{1}=42^{\circ}$ and $\hat{S}_{2}=100^{\circ}$.


Determine, with reasons, the size of the following angles:
(1) $\quad \hat{Q}$
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(2) $\hat{R}_{2}$
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(3) $\hat{P}_{2}$
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(4) $\quad \hat{R}_{3}$
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(c) In the diagram below O is the centre of the circle going through points $\mathrm{A}, \mathrm{B}, \mathrm{C}$, $D$ and $E$. $E B \perp O A$ at $P$.


Prove, stating reasons:
(1) $\quad \hat{\mathrm{C}}_{1}=\hat{\mathrm{C}}_{2}$
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(2) EOFD is a cyclic quadrilateral.
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## QUESTION 5

The marks for 120 learners were recorded, grouped and then represented by the histogram given below:

(a) Write down the modal class of the learners' marks.
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(b) Determine an estimate for the mean of the learners' marks.
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(c) Determine an estimate for the standard deviation of the learners' marks.
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(d) The ten learners who scored between 0 and 20 checked their papers and found that the teacher neglected to mark one of their questions. After she marked the question, they all scored between 20 and 40 marks.
How will this influence the mean and the standard deviation? Explain.
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## SECTION B

## QUESTION 6

Below is a cumulative frequency graph showing the lengths of 60 characters in the Marvel Cinematic Universe (MCU) in centimetres.


Other information given:

- The length of the shortest character is 164 cm .
- The length of the tallest character is 248 cm .
(a) Sketch a box and whisker plot to represent the lengths of the characters in the MCU .

(b) Describe the skewness of the data. (Give a reason for your answer.)
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(c) How many characters are taller than 205 cm ?
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(d) Due to a disastrous event called "the snap", the fifteen shortest MCU characters as well as the fifteen tallest MCU characters disappear from existence.
How does this affect the median? (Explain your answer.)
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## QUESTION 7

(a) If $\sin \theta=\frac{3}{5}$ and $\cos \theta<0$, determine WITHOUT using a calculator, the value of:

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\begin{equation*}
\text { (1) } \tan \theta \tag{3}
\end{equation*}
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(2) $\sin 2 \theta$
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(b) If $\sin A \cdot \cos A=p$, determine the value of the following expression in terms of $p$ :

$$
\begin{equation*}
\cos \left(180^{\circ}-A\right) \cdot \sin \left(270^{\circ}-A\right)-\sin ^{2}\left(180^{\circ}+A\right) \tag{5}
\end{equation*}
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(c) Prove the following identity: $\frac{2 \tan x-\sin 2 x}{2 \sin ^{2} x}=\tan x$
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(d) Determine the general solution of $\cos 4 B \cdot \cos 20^{\circ}-\sin 4 B \cdot \sin 20^{\circ}=-1$
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## QUESTION 8

In the diagram below, two circles touch internally at point N. QNP is a common tangent to both circles. MRL is a tangent to the smaller circle. KM, KN, KL and MN are chords of the larger circle. RS, RN and SN are chords of the smaller circle with $R$ and $S$ points on $K N$ and $M N$ respectively.


Prove, giving reasons, that:
(a) $\frac{\mathrm{KR}}{\mathrm{RN}}=\frac{\mathrm{MS}}{\mathrm{SN}}$
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(b) $\quad M R^{2}=M S . M N$
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## QUESTION 9

In the diagram alongside, a circle passes through the points $\mathrm{P}(-2 ; 1)$, $Q(14 ; 13)$ and $R(20 ; k)$, where $k \in \mathbb{R} . \mathrm{PQ} \perp \mathrm{QR}$.


Determine the equation of the circle in the form $(x-a)^{2}+(y-b)^{2}=r^{2}$.
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## QUESTION 10

(a) In the diagram below, $\triangle A B C$ is given. $M$ is a point on $B C$ such that $B M=5 \mathrm{~cm}$, $M C=6 \mathrm{~cm}, A B=7 \mathrm{~cm}$ and $A M=3 \mathrm{~cm}$.


Determine the area of $\triangle A B C$.
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(b) A corner of a rectangular block of wood is cut off and shown in the diagram below. The inclined plane, that is $\triangle A C D$, is an isosceles triangle with $A \hat{D} C=A \hat{C} D=\theta . A \hat{C} B=\frac{1}{2} \theta, A C=3 k, C D=2 k$ and $A B=12 \mathrm{~cm}$.

(1) Write down CÂD in terms of $\theta$.
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(2) Determine the value of $k$ if it is given that $k \neq 0$ and $\theta \in\left(0^{\circ} ; 90^{\circ}\right)$.
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## QUESTION 11

(a) $\quad \triangle A B C$ is drawn below. $A B\|F D, A F\| D E$ and $F E: E C=3: 4$.


Determine, giving reasons:
(1) $\frac{A D}{D C}$
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(2) $E C: B C$
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(b) In the diagram below two identical larger circles with centres $K$ and $L$ touch externally at N . A smaller circle with centre M touches the two larger circles externally at $P$ and $Q$. KL and $P Q$ are drawn. $M N$ is a common tangent to the two larger circles that bisects $P Q$ at $R$ so that $P R=R Q . M R: R N=2: 5$ and $K L=25$ units.


Determine $\frac{\text { Area of smaller circle }}{\text { Area of larger circle }}$, showing ALL your calculations and supplying reasons where necessary.
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EXAMINATION NUMBER: $\square$

MARKING GRID

| Question | Analytical Geometry | Statistics | Trigonometry | Euclidean Geometry |
| :---: | :---: | :---: | :---: | :---: |
| 1 | /18 |  |  |  |
| 2 | /17 |  |  |  |
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| TOTAL PER TOPIC | 143 | $/ 19$ | 141 | 147 |
| TOTAL MARK |  |  |  |  |
|  |  |  |  | 1150 |

