BEE701 POWER SYSTEM ANALYSIS

UNIT I

POWER SYSTEM COMPONENTS

Power system analysis

The evaluation of power system is called as power system analysis

Functions of power system analysis

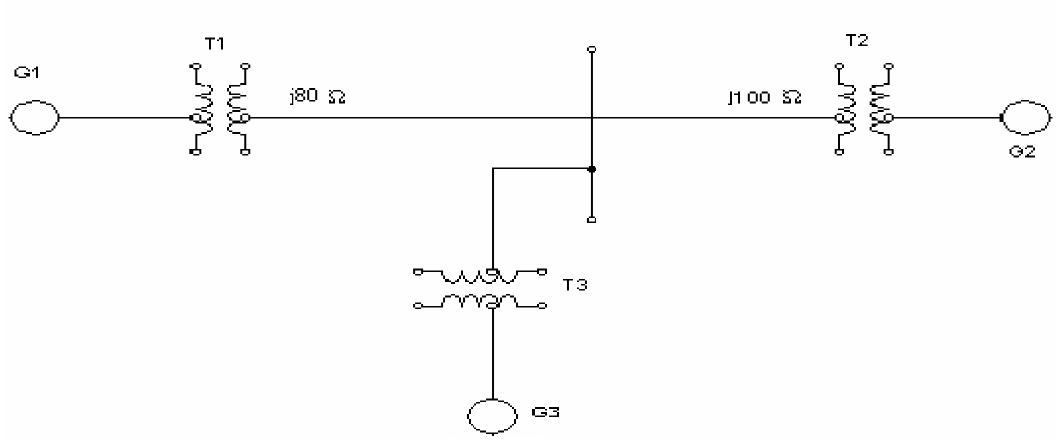
- To monitor the voltage at various buses, real and reactive power flow between buses.
- To design the circuit breakers.
- To plan future expansion of the existing system
- To analyze the system under different fault conditions
- To study the ability of the system for small and large disturbances (Stability studies)

COMPONENTS OF A POWER SYSTEM

1.Alternator
2.Power transformer
3.Transmission lines
4.Substation transformer
5.Distribution transformer
6.Loads

SINGLE LINE DIAGRAM

A single line diagram is diagrammatic representation of power system in which the components are represented by their symbols and interconnection between them are shown by a straight line9eventhough the system is three phase system0.The ratings and the impedances of the components are also marked on the single line diagram.



Purpose of using single line diagram

The purpose of the single line diagram is to supply in concise form of the significant information about the system.

Per unit value.

The per unit value of any quantity is defined as the ratio of the actual value of the any quantity to the base value of the same quantity as a decimal.

per unit=actual value/base value

Need for base values

The components or various sections of power system may operate at different voltage and power levels. It will be convenient for analysis of power system if the voltage, power, current and impedance rating of components of power system are expressed with reference to a common value called base value.

Advantages of per unit system

i. Per unit data representation yields valuable relative magnitude information.
ii. Circuit analysis of systems containing transformers of various transformation ratios is greatly simplified.
iii. The p.u systems are ideal for the computerized analysis and simulation of complex power system problems.
iv. Manufacturers usually specify the impedance values of equivalent in per unit of the equipments rating. If the any data is not available, it is easier to assume its per unit value than its numerical value.

v. The ohmic values of impedances are refereed to secondary is different from the value as referee to primary. However, if base values are selected properly, the p.u impedance is the same on the two sides of the transformer.
vi. The circuit laws are valid in p.u systems, and the power and voltages equations are simplified since the factors of √3 and 3 are eliminated.

Change the base impedance from one set of base values to another set

Let Z=Actual impedance ,Ω

Zb=Base impedance ,Ω

Per unit impedance of a circuit element= $\frac{Z}{Z_b} = \frac{Z}{\frac{(kVb)^2}{MVA_b}} = \frac{Z \times MVA_b}{(kVb)^2}$ (1)

The eqn 1 show that the per unit impedance is directly proportional to base megavoltampere and inversely proportional to the square of the base voltage.

Using Eqn 1 we can derive an expression to convert the p.u impedance expressed in one base value (old base) to another base (new base)

Let $kV_{b,old}$ and $MVA_{b,old}$ represents old base values and $kV_{b,new}$ and $MVA_{b,new}$ represent new base value

Let $Z_{p.u,old}$=p.u. impedance of a circuit element calculated on old base

$Z_{p.u,new}$=p.u. impedance of a circuit element calculated on new base

If old base values are used to compute the p.u.impedance of a circuit element ,with impedance Z then eqn 1 can be written as

$$Z_{p.u,old} = \frac{Z \times MVA_{b,old}}{(kV_{b,old})^2}$$

$$Z = Z_{p.u,old} \frac{(kV_{b,old})^2}{MVA_{b,old}} \qquad (2)$$

If the new base values are used to compute thep.u. impedance of a circuit element with impedance Z, then eqn 1 can be written as

$$Z_{p.u,new} = \frac{Z \times MVA_{b,new}}{(kV_{b,new})^2} \quad (3)$$

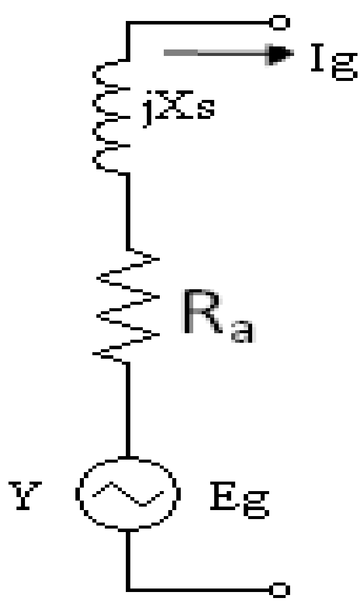
On substituting for Z from eqn 2 in eqn 3 we get

$$Z_{p.u,new} = Z_{p.u.old} \frac{(kV_{b,old})^2}{MVA_{b,old}} \times \frac{MVA_{b,new}}{(kV_{b,new})^2}$$

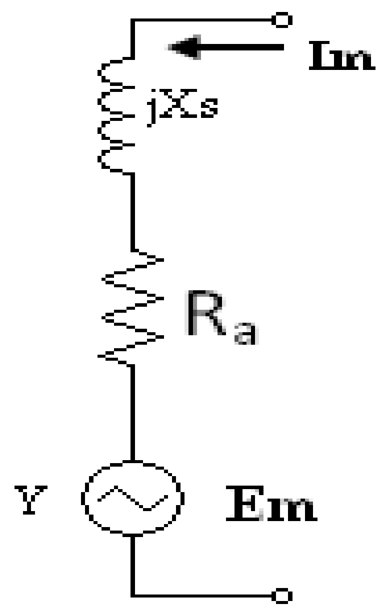
$$Z_{p.u,new} = Z_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right) \quad (4)$$

The eqn 4 is used to convert the p.u.impedance expressed on one base value to another base

MODELLING OF GENERATOR AND SYNCHRONOUS MOTOR

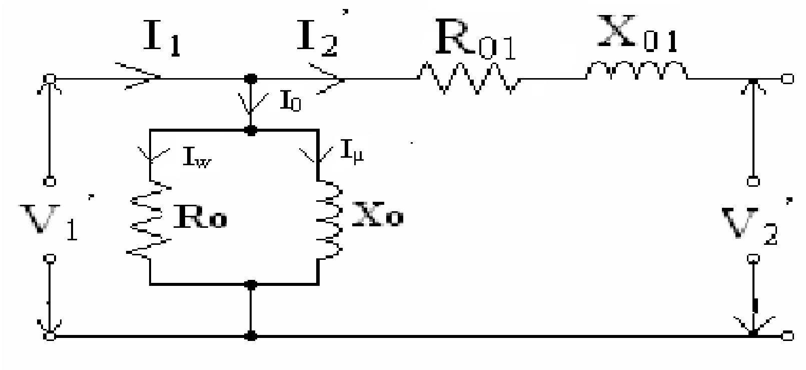


1Φ equivalent circuit of generator



1Φ equivalent circuit of synchronous motor

MODELLING OF TRANSFORMER

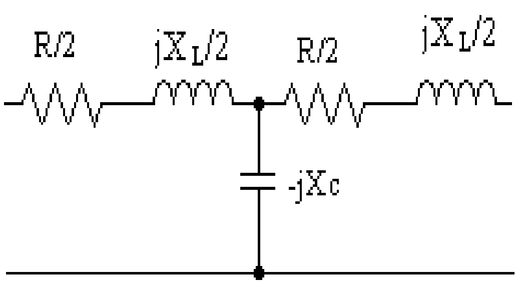


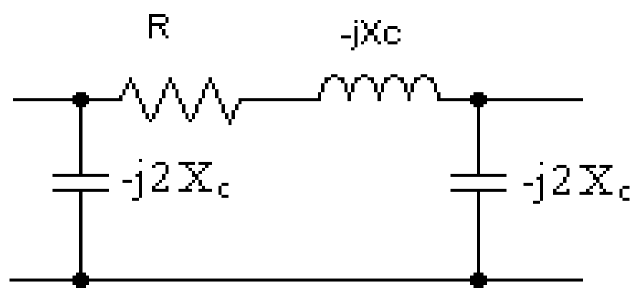
$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

$$R_{01} = R_1 + R_2^{'} = R_1 + \frac{R_2}{K^2}$$ =Equivalent resistance referred to 1°

$$X_{01} = X_1 + X_2^{'} = X_1 + \frac{X_2}{K^2}$$ =Equivalent reactance referred to 1°

MODELLING OF TRANSMISSION LINE





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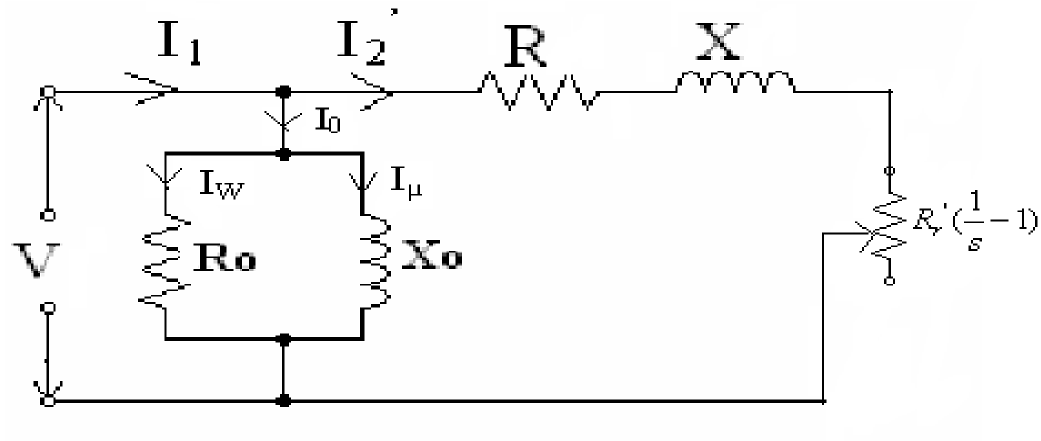
MODELLING OF INDUCTION MOTOR



$R_r'\left(\frac{1}{s}-1\right)$ =Resistance representing load

$R = R_S + R_r'$ =Equivalent resistance referred to stator

$X = X_S + X_r'$ =Equivalent reactance referred to stator

Impedance diagram & approximations made in impedance diagram

The impedance diagram is the equivalent circuit of power system in which the various components of power system are represented by their approximate or simplified equivalent circuits. The impedance diagram is used for load flow studies.

Approximation:

(i) The neutral reactances are neglected.

(ii) The shunt branches in equivalent circuit of transformers are neglected.

Reactance diagram & approximations made in reactance diagram

The reactance diagram is the simplified equivalent circuit of power system in which the various components of power system are represented by their reactances. The reactance diagram can be obtained from impedance diagram if all the resistive components are neglected. The reactance diagram is used for fault calculations.

Approximation:

(i) The neutral reactances are neglected.

(ii) The shunt branches in equivalent circuit of transformers are neglected.

(iii) The resistances are neglected.

(iv) All static loads are neglected.

(v) The capacitance of transmission lines are neglected.

PROCEDURE TO FORM REACTANCE DIAGRAM FROM SINGLE LINE DIAGRAM

1.Select a base power kVA_b or MVA_b

2.Select a base voltage kV_b

3. The voltage conversion is achieved by means of transformer kV_b on LT section= kV_b on HT section x LT voltage rating/HT voltage rating

4. When specified reactance of a component is in ohms

p.u reactance=actual reactance/base reactance

specified reactance of a component is in p.u

$$X_{p.u,new} = X_{p.u,old} * \frac{(kV_{b,old})^2}{(kV_{b,new})^2} * \frac{MVA_{b,new}}{MVA_{b,old}}$$

EXAMPLE

1. The single line diagram of an unloaded power system is shown in Fig 1.The generator transformer ratings are as follows.

G1=20 MVA, 11 kV, X''=25%

G2=30 MVA, 18 kV, X''=25%

G3=30 MVA, 20 kV, X''=21%

T1=25 MVA, 220/13.8 kV (Δ/Y), X=15%

T2=3 single phase units each rated 10 MVA, 127/18 kV(Y/Δ), X=15%

T3=15 MVA, 220/20 kV(Y/Δ), X=15%

Draw the reactance diagram using a base of 50 MVA and 11 kV on the generator1.

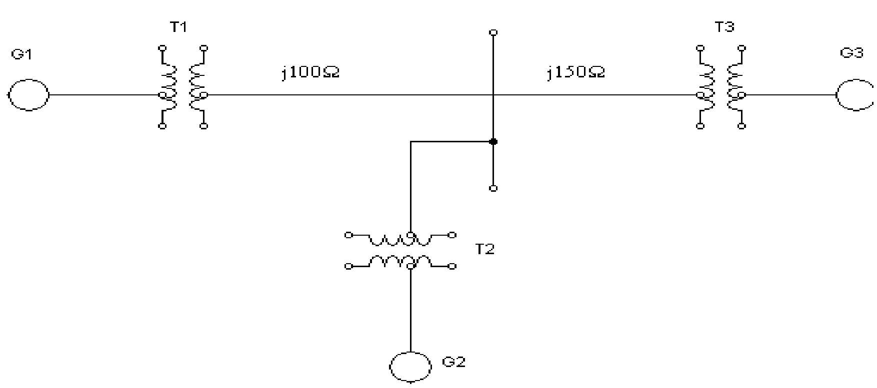


Fig 1

SOLUTION

Base megavoltampere, $MVA_{b,new}$ *=50 MVA*

Base kilovolt $kV_{b,new}$ *=11 kV (generator side)*

FORMULA

The new p.u. reactance $X_{pu,new} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$

Reactance of Generator G

$kV_{b,old}$ *=11 kV* $kV_{b,new}$ *=11 kV*

$MVA_{b,old}$ *= 20 MVA* $MVA_{b,new}$ *=50 MVA*

$X_{p.u,old}$ *=0.25p.u*

The new p.u. reactance of Generator G= $X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$

$= 0.25 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{50}{20}\right) = j0.625 p.u$

Reactance of Transformer T1

$kV_{b,old}$ *=11 kV* $kV_{b,new}$ *=11 kV*

$MVA_{b,old}$ *= 25 MVA* $MVA_{b,new}$ *=50 MVA*

$X_{p.u,old}$ *=0.15p.u*

The new p.u. reactance of Transformer T1= $X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$

$= 0.15 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{50}{25}\right) = j0.3 \ p.u$

Reactance of Transmission Line

It is connected to the HT side of the Transformer T1

Base kV on HT side of transformer T 1 $= Base\ kV\ on\ LT\ side \times \frac{HT\ voltage\ rating}{LT\ voltage\ rating}$

$= 11 \times \frac{220}{11} = 220\ kV$

Actual Impedance X_{actual} *= 100ohm*

Base impedance $X_{base} = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{220^2}{50} = 968\ ohm$

p.u reactance of 100 Ω transmission line= $\frac{Actual\ \ Reactance\ \ ,ohm}{Base\ \ Reactance\ \ ,ohm} = \frac{100}{968} = j0.103\ p.u$

p.u reactance of 150 Ω transmission line= $\frac{Actual\ \ Reactance\ \ ,ohm}{Base\ \ Reactance\ \ ,ohm} = \frac{150}{968} = j0.154\ p.u$

Reactance of Transformer T2

$kV_{b,old}$=127 * √3 kV =220 kV $kV_{b,new}$=220 kV

$MVA_{b,old}$= 10 * 3=30 MVA $MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.15p.u

$$\text{The new p.u. reactance of Transformer T2}=X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$=0.15 \times \left(\frac{220}{220}\right)^2 \times \left(\frac{50}{30}\right) = j0.25\ p.u$$

Reactance of Generator G2

It is connected to the LT side of the Transformer T2

$$\text{Base kV on LT side of transformer T 2} = \text{Base kV on HT side} \times \frac{LT\ voltage\ \ rating}{HT\ voltage\ \ rating}$$

$$=220 \times \frac{18}{220} = 18\ kV$$

$kV_{b,old}$=18 kV $kV_{b,new}$=18 kV

$MVA_{b,old}$= 30 MVA $MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.25 p.u

$$\text{The new p.u. reactance of Generator G 2}=X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$=0.25 \times \left(\frac{18}{18}\right)^2 \times \left(\frac{50}{30}\right) = j0.4167\ p.u$$

Reactance of Transformer T3

$kV_{b,old}$=20 kV $kV_{b,new}$=20 kV

$MVA_{b,old}$= 20 MVA $MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.15p.u

$$\text{The new p.u. reactance of Transformer T3} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{50}{30}\right) = j0.25 \text{ p.u}$$

Reactance of Generator G3

It is connected to the LT side of the Transformer T3

$$\text{Base kV on LT side of transformer T 3} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$= 220 \times \frac{20}{220} = 20 \text{ kV}$$

$kV_{b,old}$=20 kV

$kV_{b,new}$=20 kV

$MVA_{b,old}$= 30 MVA

$MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.21 p.u

$$\text{The new p.u. reactance of Generator G 3} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.21 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{50}{30}\right) = j0.35 \text{ p.u}$$

2) Draw the reactance diagram for the power system shown in fig 4 .Use a base of 50MVA 230 kV in 30 Ω line. The ratings of the generator, motor and transformers are

Generator = 20 MVA, 20 kV, X=20%

Motor = 35 MVA, 13.2 kV, X=25%

T1 = 25 MVA, 18/230 kV (Y/Y), X=10%

T2 = 45 MVA, 230/13.8 kV (Y/Δ), X=15%

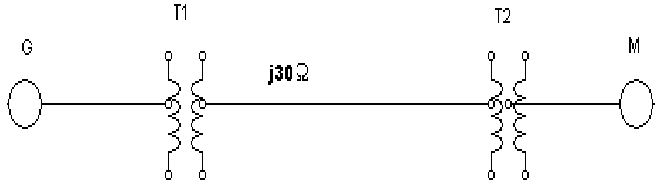


Fig 4

Solution

Base megavoltampere, $MVA_{b,new}$ *=50 MVA*

Base kilovolt $kV_{b,new}$ *=230 kV (Transmission line side)*

FORMULA

The new p.u. reactance $X_{pu,new} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$

Reactance of Generator G

It is connected to the LT side of the T1 transformer

Base kV on LT side of transformer T 1 $= Base\ kV\ on\ HT\ side \times \frac{LT\ voltage\ rating}{HT\ voltage\ rating}$

$$=230 \times \frac{18}{230} = 18\ kV$$

$kV_{b,old}$=20 kV $\qquad$ $kV_{b,new}$=18 kV

$MVA_{b,old}$= 20 MVA $\qquad$ $MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.2p.u

The new p.u. reactance of Generator G$= X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$

$$=0.2 \times \left(\frac{20}{18}\right)^2 \times \left(\frac{50}{20}\right) = j0.617\ p.u$$

Reactance of Transformer T1

$kV_{b,old}$=18 kV $\qquad$ $kV_{b,new}$=18 kV

$MVA_{b,old}$= 25 MVA $\qquad$ $MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.1p.u

The new p.u. reactance of Transformer T1$= X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$

$$=0.1 \times \left(\frac{18}{18}\right)^2 \times \left(\frac{50}{25}\right) = j0.2\ p.u$$

Reactance of Transmission Line

It is connected to the HT side of the Transformer T1

Actual Impedance $X_{actual} = j30\ ohm$

Base impedance $X_{base} = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{230^2}{50} = 1058\ ohm$

p.u reactance of j30 Ω transmission line $= \frac{Actual\ \ Reactance\ \ ,ohm}{Base\ \ Reactance\ \ ,ohm} = \frac{j30}{1058} = j0.028\ p.u$

Reactance of Transformer T2

$kV_{b,old} = 230\ kV$ $\quad$ $kV_{b,new} = 230\ \ kV$

$MVA_{b,old} = 45\ MVA$ $\quad$ $MVA_{b,new} = 50\ MVA$

$X_{p.u,old} = 0.15 p.u$

$$\text{The new p.u. reactance of Transformer T2} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{230}{230}\right)^2 \times \left(\frac{50}{45}\right) = j0.166\ p.u$$

Reactance of Motor M2

It is connected to the LT side of the Transformer T2

$$\text{Base kV on LT side of transformer T 2} = Base\ kV\ on\ HT\ side \times \frac{LT\ voltage\ \ rating}{HT\ voltage\ \ rating}$$

$$= 230 \times \frac{13.8}{230} = 13.8\ kV$$

$kV_{b,old} = 13.2\ kV$ $\quad$ $kV_{b,new} = 13.8\ \ kV$

$MVA_{b,old} = 35\ MVA$ $\quad$ $MVA_{b,new} = 50\ MVA$

$X_{p.u,old} = 0.25\ p.u$

$$\text{The new p.u. reactance of Generator G 2} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.25 \times \left(\frac{13.2}{13.8}\right)^2 \times \left(\frac{50}{35}\right) = j0.326\ p.u$$

BUS

The meeting point of various components in a power system is called a bus. The bus is a conductor made of copper or aluminum having negligible resistance. The buses are considered as points of constant voltage in a power system.

BUS IMPEDANCE MATRIX

The matrix consisting of driving point impedances and impedances of the network of a power system is called bus impedance matrix. It is given by the inverse of bus admittance matrix and it is denoted as Zbus . The bus impedance matrix is symmetrical.

BUS ADMITTANCE MATRIX

The matrix consisting of the self and mutual admittances of the network of a power system is called bus admittance matrix. It is given by the admittance matrix Y in the node basis matrix equation of a power system and it is denoted as Ybus . The bus admittance matrix is symmetrical.

EXAMPLE

1. Find the bus admittance matrix for the given network in Fig 2. Determine the reduced admittance matrix by eliminating node 4. The values are marked in p.u.

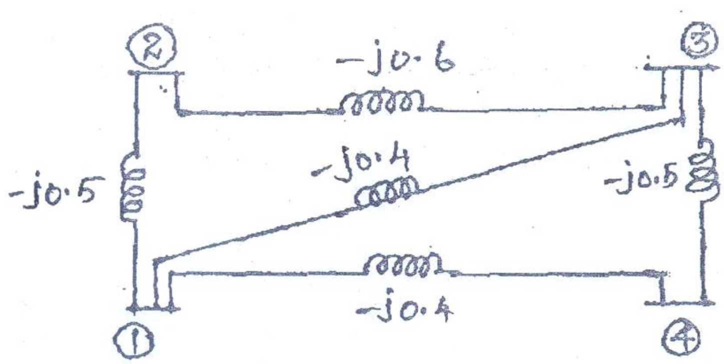


Fig 2

$$Y_{BUS} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

$$Y_{11} = y_{12} + y_{13} + y_{14} = -j0.5 - j0.4 - j0.4 = -j1.3$$

$$Y_{22} = y_{12} + y_{23} = -j0.5 - j0.6 = -j1.1$$

$$Y_{33} = y_{32} + y_{31} + y_{34} = -j0.6 - j0.4 - j0.5 = -j1.5$$

$$Y_{44} = y_{41} + y_{43} = -j0.4 - j0.5 = -j0.9$$

$$Y_{12} = -y_{12} = j0.5$$

$$Y_{13} = -y_{13} = j0.4$$

$$Y_{14} = -y_{14} = j0.4$$

$$Y_{21} = Y_{12} = j0.5$$

$$Y_{23} = -y_{23} = j0.6$$

$$Y_{24} = -y_{24} = 0$$

$$Y_{31} = Y_{13} = j0.4$$

$$Y_{32} = Y_{23} = j0.6$$

$$Y_{34} = -y_{34} = j0.5$$

$$Y_{41} = Y_{14} = j0.4$$

$$Y_{42} = Y_{24} = 0$$

$$Y_{43} = Y_{34} = j0.5$$

$$Y_{BUS} = \begin{bmatrix} -j1.3 & j0.5 & j0.4 & j0.4 \\ j0.5 & -j1.1 & j0.6 & 0 \\ j0.4 & j0.6 & -j1.5 & j0.5 \\ j0.4 & 0 & j0.5 & -j0.9 \end{bmatrix}$$

Elements of new bus admittance matrix after eliminating 4th row and 4th column

$$Y_{jk,new} = Y_{jk} - \frac{Y_{jn} Y_{nk}}{Y_{nn}}$$

N=4, j=1,2,3 k=1,2,3

$$Y_{11,new} = Y_{11} - \frac{Y_{14} Y_{41}}{Y_{44}} = -j1.3 - \frac{(j0.4)(j0.4)}{-j0.9} = -j1.12$$

$$Y_{12,new} = Y_{12} - \frac{Y_{14} Y_{42}}{Y_{44}} = j0.5 - \frac{(j0.4)(j0)}{-j0.9} = j0.5$$

$$Y_{13,new} = Y_{13} - \frac{Y_{14}Y_{43}}{Y_{44}} = j0.4 - \frac{(j0.4)(j0.5)}{-j0.9} = j0.622$$

$$Y_{21,new} = Y_{12,new} = j0.5$$

$$Y_{22,new} = Y_{22} - \frac{Y_{24}Y_{42}}{Y_{44}} = -j1.1 - \frac{(j0)(j0)}{-j0.9} = -j1.1$$

$$Y_{23,new} = Y_{23} - \frac{Y_{24}Y_{43}}{Y_{44}} = j0.6 - \frac{(j0)(j0.5)}{-j0.9} = j0.6$$

$$Y_{31,new} = Y_{13,new} = j0.622$$

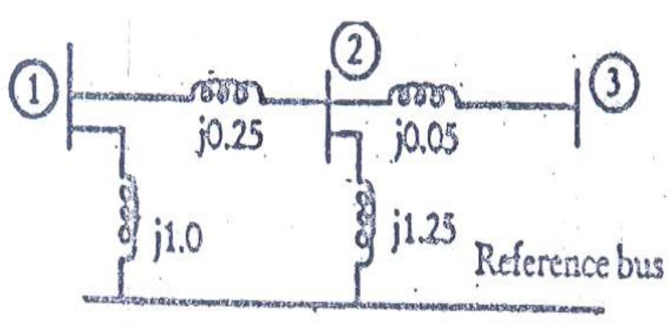
$$Y_{32,new} = Y_{23,new} = j0.6$$

$$Y_{33,new} = Y_{33} - \frac{Y_{34}Y_{43}}{Y_{44}} = -j1.5 - \frac{(j0.5)(j0.5)}{-j0.9} = -j1.22$$

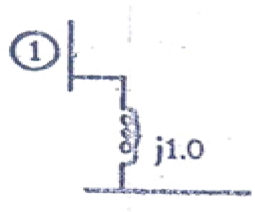
Reduced admittance matrix after eliminating 4 th row and 4^{th} column\

$$Y_{BUS} = \begin{bmatrix} -j1.12 & j0.5 & j0.622 \\ j0.5 & -j1.1 & j0.6 \\ j0.622 & j0.6 & -j1.222 \end{bmatrix}$$

2) Find the bus impedance matrix for the system whose reactance diagram is shown in fig 3. All the impedances are in p,u.

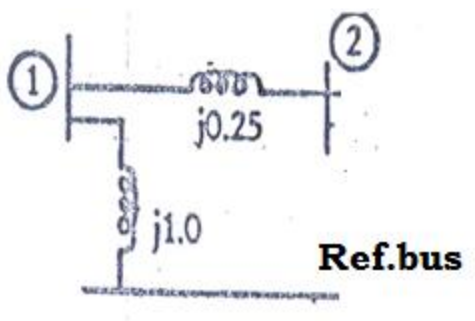


Step 1: connect bus 1 to ref bus through impedance j1.0



$$Z_{bus} = [j1.0]$$

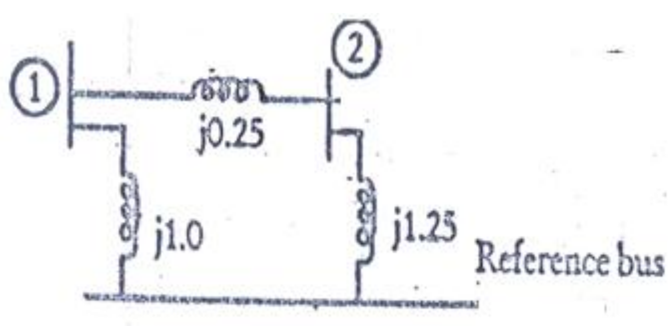
Step 2 connect bus 2 to the bus 1 through impedance j0.25



$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.0 + j0.25 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.25 \end{bmatrix}$$

Step 3 connect bus 2 to ref bus through impedance j1.25



$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j1.25 + j1.25 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j2.5 \end{bmatrix}$$

Number of buses is only 2.But matrix size is 3*3.The matrix size ir reduced by eliminating 3rd row and 3rd column

$$Z_{jk,ack} = Z_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

Where n=2 j=1,2 k=1,2

n=2 j=1 k=1

$$Z_{11,ack} = Z_{11} - \frac{Z_{13} Z_{31}}{Z_{33}}$$

$$Z_{11,ack} = j1.0 - \frac{j1.0 * j1.0}{j2.5} = j0.6$$

n=2 j=1 k=2

$$Z_{12,ack} = Z_{12} - \frac{Z_{13} Z_{32}}{Z_{33}}$$

$$Z_{12,ack} = j1.0 - \frac{j1.0 * j1.25}{j2.5} = j0.5$$

n=2 j=2 k=1

$$Z_{21,ack} = Z_{12,ack} = j0.5$$

n=2 j=2 k=2

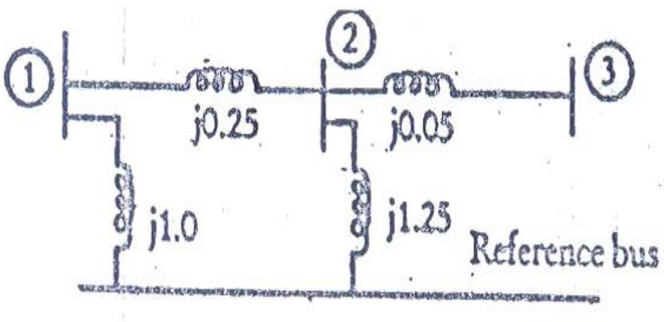
$$Z_{22,ack} = Z_{22} - \frac{Z_{23} Z_{32}}{Z_{33}}$$

$$Z_{22,ack} = j1.25 - \frac{j1.25 * j1.25}{j2.5} = j0.625$$

The reduced matrix

$$Z_{bus} = \begin{bmatrix} j0.6 & j0.5 \\ j0.5 & j.625 \end{bmatrix}$$

Step 4:connect bus 3 to bus 2 through impedance j0.0



$$Z_{bus} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j.625 & j.675 \end{bmatrix}$$

Symmetrical Components

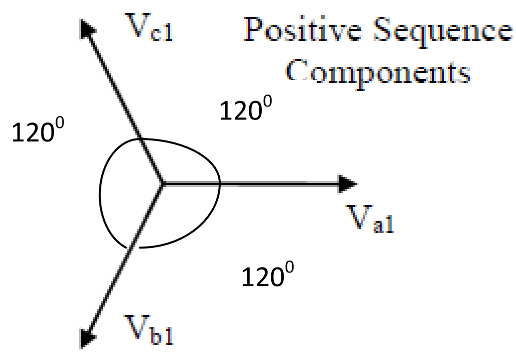
An unbalanced system of N related vectors can be resolved into N systems of balanced vectors. The N – sets of balanced vectors are called symmetrical components. Each set consists of N – vectors which are equal in length and having equal phase angles between adjacent vectors.

Sequence Impedance and Sequence Network

The sequence impedances are impedances offered by the devices or components for the like sequence component of the current .The single phase equivalent circuit of a power system consisting of impedances to the current of any one sequence only is called sequence network.

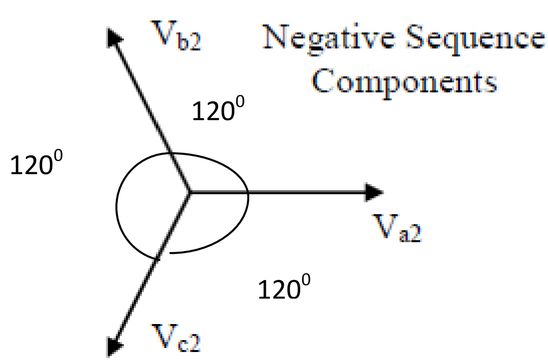
Positive Sequence Components

The positive sequence components are equal in magnitude and displayed from each other by 120_o with the same sequence as the original phases. The positive sequence currents and voltages follow the same cycle order of the original source. In the case of typical counter clockwise rotation electrical system, the positive sequence phasor are shown in Fig . The same case applies for the positive current phasors. This sequence is also called the “abc” sequence and usually denoted by the symbol “+” or “1”



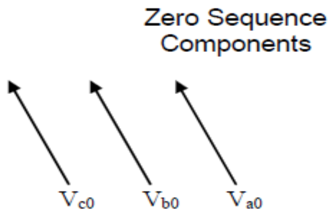
Negative Sequence Components

This sequence has components that are also equal in magnitude and displayed from each other by 120o similar to the positive sequence components. However, it has an opposite phase sequence from the original system. The negative sequence is identified as the “acb” sequence and usually denoted by the symbol “-” or “2” [9].The phasors of this sequence are shown in Fig where the phasors rotate anti- clockwise. This sequence occurs only in case of an unsymmetrical fault in addition to the positive sequence components,



Zero Sequence Components

In this sequence, its components consist of three phasors which are equal in magnitude as before but with a zero displacement. The phasor components are in phase with each other. This is illustrated in Fig . Under an asymmetrical fault condition, this sequence symbolizes the residual electricity in the system in terms of voltages and currents where a ground or a fourth wire exists. It happens when ground currents return to the power system through any grounding point in the electrical system. In this type of faults, the positive and the negative components are also present. This sequence is known by the symbol “0” .



EXAMPLE

1. The symmetrical components of a phase –a voltage in a 3-phase unbalanced system are $V_{a0} = 10\angle 180^0$ **V,** $V_{a1} = 50\angle 0^0$ **V and** $V_{a2} = 20\angle 90^0$ **V. Determine the phase voltages V_a ,V_b and V_c**

The phase voltages of $V_a, V_b\, and\, V_c$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_b = V_{a0} + a^2 V_{a1} + aV_{a2}$$

$$V_c = V_{a0} + aV_{a1} + a^2 V_{a2}$$

$V_{a0} = 10\angle 180^0 = -10 + j0$ V

$V_{a1} = 50\angle 0^0 = 50 + j0$ V

$V_{a2} = 20\angle 90^0 = 0 + j20$ V

a=1∠120^0 a^2= 1∠240^0

$a^2 V_{a1} = 1\angle 240^0 \times 50\angle 0^0 = 50\angle 240^0 = -25 - \text{j}43.30$

$aV_{a1} = 1\angle 120^0 \times 50\angle 0^0 = 50\angle 120^0 = -25 + \text{j}43.30$

$a^2 V_{a2} = 1\angle 240^0 \times 20\angle 90^0 = 20\angle 233 = 17.32 - \text{j}10$

$aV_{a2} = 1\angle 120^0 \times 20\angle 90^0 = 20\angle 210^0 = -17.32 - \text{j}10$

$$V_a = V_{a0} + V_{a1} + V_{a2} = (-10 + j0) + (50 + j0) + (0 + j20) = 40 + j20 = 44.72\angle 27^0\ V$$

$V_b = V_{a0} + a^2V_{a1} + aV_{a2} = (-10 + j0) + (-25 - \text{j}43.30) + (-17.32 - \text{j}10) = -52.32 - j53.90$
$= 74.69\angle{-134^0}\ \text{V}$

$V_c = V_{a0} + aV_{a1} + a^2V_{a2} = (-25 - \text{j}43.30) + (-25 + \text{j}43.30) + 17.32 - \text{j}10$ =-17.68+j33.3

= 37.70 $\angle{-118^0}\ V$

THREE-SEQUENCE IMPEDANCES AND SEQUENCE NETWORKS

Positive sequence currents give rise to only positive sequence voltages, the negative sequence currents give rise to only negative sequence voltages and zero sequence currents give rise to only zero sequence voltages, hence each network can be regarded as flowing within in its own network through impedances of its own sequence only.

In any part of the circuit, the voltage drop caused by current of a certain sequence depends on the impedance of that part of the circuit to current of that sequence.

The impedance of any section of a balanced network to current of one sequence may be different from impedance to current of another sequence.

The impedance of a circuit when positive sequence currents are flowing is called impedance, When only negative sequence currents are flowing the impedance is termed as negative sequence impedance. With only zero sequence currents flowing the impedance is termed as zero sequence impedance.

The analysis of unsymmetrical faults in power systems is carried out by finding the symmetrical components of the unbalanced currents.

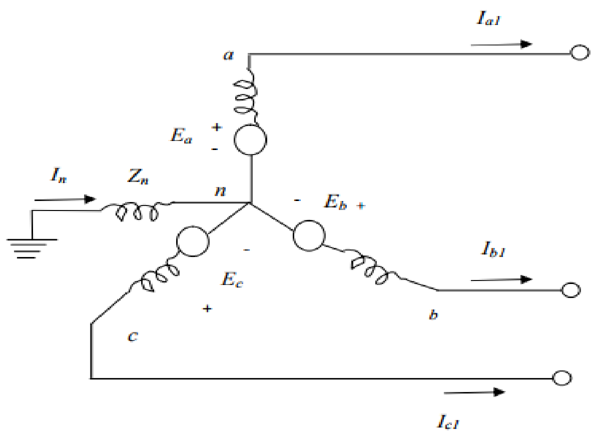
Since each sequence current causes a voltage drop of that sequence only, each sequence current can be considered to flow in an independent network composed of impedances to current of that sequence only.

The single phase equivalent circuit composed of the impedances to current of any one sequence only is

called the sequence network of that particular sequence. The sequence networks contain the generated emfs and impedances of like sequence. Therefore for every power system we can form three- sequence network s. These sequence networks, carrying current Ia1, Ia2 and Ia0 are then inter-connected to represent the different fault conditions.

SEQUENCE NETWORKS OF SYNCHRONOUS MACHINES

An unloaded synchronous machine having its neutral earthed through impedance, Zn, is shown in fig. below. A fault at its terminals causes currents I_a, I_b and I_c to flow in the lines. If fault involves earth, a current In flows into the neutral from the earth. This current flows through the neutral impedance Zn. Thus depending on the type of fault, one or more of the line currents may be zero. Thus depending on the type of fault, one or more of the line currents may be zero.



POSITIVE SEQUENCE NETWORK

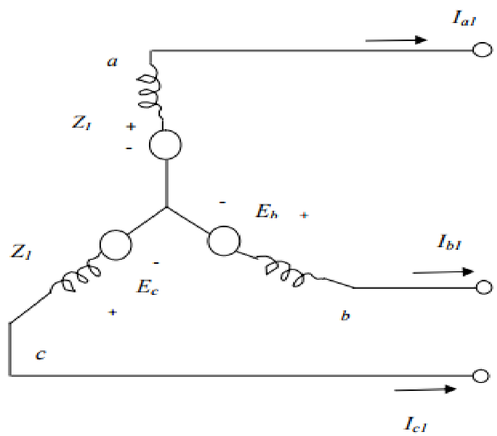
The generated voltages of a synchronous machine are of positive sequence only since the windings of a synchronous machine are symmetrical.

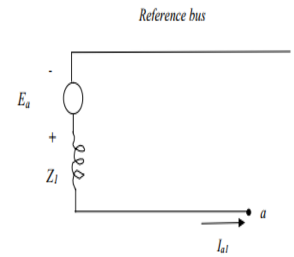
The positive sequence network consists of an emf equal to no load terminal voltages and is in series with the positive sequence impedance Z1 of the machine. Fig.2 (b) and fig.2(c) shows the paths for positive sequence currents and positive sequence network respectively on a single phase basis in the synchronous machine.

The neutral impedance Zn does not appear in the circuit because the phasor sum of I_{a1}, I_{b1} and I_{c1} is zero and no positive sequence current can flow through Zn. Since its a balanced circuit, the positive sequence N The reference bus for the positive sequence network is the neutral of the generator. The positive sequence impedance Z1 consists of winding resistance and direct axis reactance. The reactance is the sub-transient reactance X"d or transient reactance X'd or synchronous reactance Xd depending on whether sub-transient, transient or steady state conditions are being studied. From fig.2 (b) ,

the positive sequence voltage of terminal a with respect to the reference bus is given by:

$V_{a1} = E_a - Z_1 I_{a1}$





NEGATIVE SEQUENCE NETWORK

A synchronous machine does not generate any negative sequence voltage. The flow of negative sequence currents in the stator windings creates an mmf which rotates at synchronous speed in a direction opposite to the direction of rotor, i.e., at twice the synchronous speed with respect to rotor.

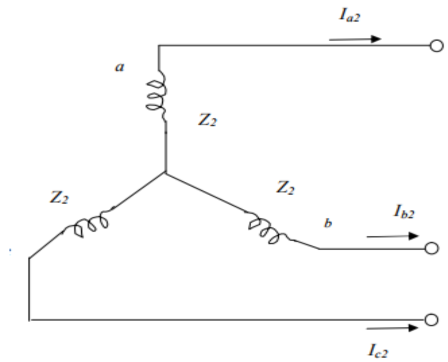
Thus the negative sequence mmf alternates past the direct and quadrature axis and sets up a varying armature reaction effect. Thus, the negative sequence reactance is taken as the average of direct axis and quadrature axis sub-transient reactance, i.e.,

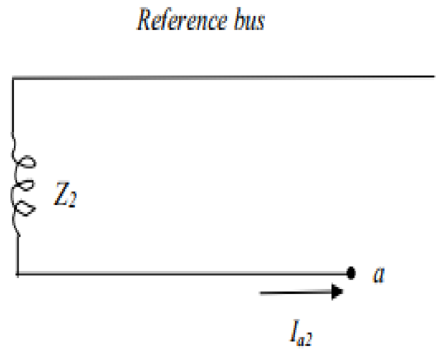
$X_2 = 0.5 (X''d + X''q)$.

It not necessary to consider any time variation of X2 during transient conditions because there is no normal constant armature reaction to be effected. For more accurate calculations, the negative sequence resistance should be considered to account for power dissipated in the rotor poles or damper winding by double supply frequency induced currents. The fig.below shows the negative sequence currents paths and the negative sequence network respectively on a single phase basis of a synchronous machine. The reference bus for the negative sequence network is the neutral of the machine.

Thus, the negative sequence voltage of terminal a with respect to the reference bus is given by:

$V_{a2}= -Z_2I_{a2}$





ZERO SEQUENCE NETWORK

No zero sequence voltage is induced in a synchronous machine. The flow of zero sequence currents in the stator windings produces three mmf which are in time phase. If each phase winding produced a sinusoidal space mmf, then with the rotor removed, the flux at a point on the axis of the stator due to zero sequence current would be zero at every instant.

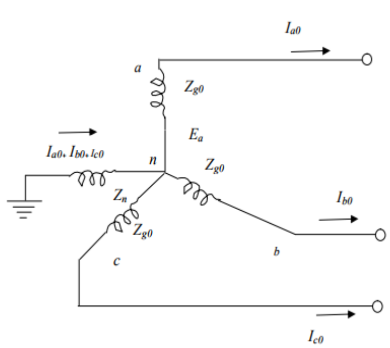
When the flux in the air gap or the leakage flux around slots or end connections is considered, no point in these regions is equidistant from all the three –phase windings of the stator.

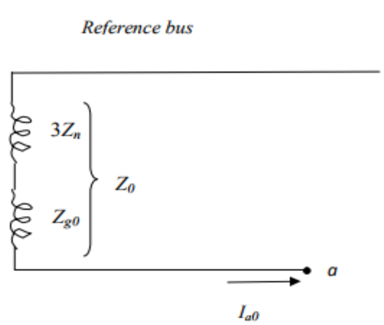
The mmf produced by a phase winding departs from a sine wave, by amounts which depend upon the arrangement of the winding.

The zero sequence currents flow through the neutral impedance Zn and the current flowing through this impedance is $3I_{a0}$

Fig.2(f) and fig.2(g) shows the zero sequence current paths and zero sequence network respectively, and as can be seen, the zero sequence voltage drop from point a to ground is $-3I_{a0}Z_n - I_{a0}Z_{g0}$ where Z_{g0} is the zero sequence impedance per phase of the generator.

Since the current in the zero sequence network is I_{a0} this network must have an impedance of 3Zn +Zg0. Thus, Z0 =3Zn +Zg0 The zero sequence voltage of terminal a with respect to the reference bus is thus: Va0 = -Ia0Z0





UNIT II

LOAD FLOW STUDIES

Load Flow Study (Or) Power Flow Study

The study of various methods of solution to power system network is referred to as load flow study. The solution provides the voltages at various buses, power flowing in various lines and line-losses.

The following work has to be performed for a load flow study.

(i) Representation of the system by single line diagrams.

(ii) Determining the impedance diagram using the information in single line diagram.

(iii) Formulation of network equations.

(iv) Solution of network equations.

Information's that are obtained from a load flow study

The information obtained from a load flow study is magnitude and phase angle of voltages, real and reactive power flowing in each line and the line losses. The load flow solution also gives the initial conditions of the system when the transient behavior of the system is to be studied.

Need for load flow study

The load flow study of a power system is essential to decide the best operation of existing system and for planning the future expansion of the system. It is also essential foe designing a new power system.

Quantities associated with each bus in a system

Each bus in a power system is associated with four quantities and they are real power (P), reactive power (Q), magnitude of voltage (V), and phase angle of voltage (δ).

Different types of buses n a power system

Types of bus	**Known or specified quantities**	**Unknown quantities or quantities to be determined.**
Slack or Swing or Reference bus	V, δ	P,Q
Generator or Voltage control or PV bus	P, V	Q, δ
Load or PQ bus	P, Q	V, δ

Need for slack bus

The slack bus is needed to account for transmission line losses. In a power system the total power generated will be equal to sum of power consumed by loads and losses. In a power system only the generated power and load power are specified for buses. The slack bus is assumed to generate the power required for losses. Since the losses are unknown the real and reactive power are not specified for slack bus.

Iterative methods to solve load flow problems

The load flow equations are non linear algebraic equations and so explicit solution as not possible. The solution of non linear equations can be obtained only by iterative numerical techniques.

Mainly used for solution of load flow study

- The Gauss seidal method,
- Newton Raphson method
- Fast decouple methods.

Flat voltage start

In iterative method of load flow solution, the initial voltages of all buses except slack bus assumed as 1+j0 p.u. This is refereed to as flat voltage start

Effect of acceleration factor in load flow study

Acceleration factor is used in gauss seidal method of load flow solution to increase the rate of convergence. Best value of A.F=1.6

Generator buses are treated as load bus

If the reactive power constraints of a generator bus violates the specified limits then the generator is treated as load bus.

Advantages and disadvantages of Gauss serial method

Advantages: Calculations are simple and so the programming task is lessees. The memory requirement is less. Useful for small systems;

Disadvantages: Requires large no. of iterations to reach converge .Not suitable for large systems. Convergence time increases with size of the system

Advantages and disadvantages of N.R method

Advantages: Faster, more reliable and results are accurate, require less number of iterations;

Disadvantages: Program is more complex, memory is more complex.

Compare the Gauss seidel and Newton raphson methods of load flow study

S.No	G.S	N.R	FDLF
1	Require large number of iterations to reach convergence.	Require less number of iterations to reach convergence.	Require more number of iterations than N.R method.
2	Computation time per iteration is less	Computation time per iteration is more	Computation time per iteration is less
3	It has linear convergence characteristics	It has quadratic convergence characteristics	------
4	The number of iterations required for convergence increases with size of the system	The number of iterations are independent of the size of the system	The number of iterations are does not dependent of the size of the system
5	Less memory requirements.	More memory requirements.	Less memory requirements than N.R.method.

Gauss-Seidel method

The step by step computational procedure for the Gauss-Seidel method of load flow studies

Algorithm when PV buses are present

1) Read the system data and formulate Y_{BUS} for the given power system network.

2) Assume a flat voltage profile (1+j0) for all the bus voltages except the slack bus. Let slack bus voltage be (a+j0) and it is not modified in any iteration.

3) Assume a suitable value of **€** called convergence criterion. Here **€** is a specified change in the bus voltage that is used to compare the actual change in bus voltage between and iteration. K^{th} and $(k+1)^{th}$ iteration

4) Set iteration count k= 0

5) Set bus count p=1.

6) Check for slack bus. If it is a slack bus then go to step (13), otherwise go to next step.

7) Check for generator bus. If it is a generator bus go to next step, otherwise go to step (9)

8) Replace the value of the voltage magnitude of generator bus in that iteration by the specified value. Keep the phase angle same as in that iteration. Calculate Q for generator bus.

The reactive power of the generator bus can be calculated by using the following equation

$$Q_{p,cal}^{k+1} = (-1) I.P.of \left\{ (V_P^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k \right] \right\}$$

The calculated reactive power may be within specified limits or it may violate the limits. If the calculated reactive power violates the specified limit for the reactive power then treat this bus as the load bus. The magnitude of the reactive power at this bus will correspond to the limit it has violated

$$\text{i.e. if } Q_{p,cal}^{k+1} < Q_{p,\min} \quad \text{then} \quad Q_p = Q_{p,\min}$$

$$\text{or if } Q_{p,cal}^{k+1} > Q_{p,\max} \text{ then } Q_p = Q_{p,\max}$$

Since the bus is treated as load bus, take actual value of Vp^k for (k+1) th iteration

i.e. | Vp^k | need not be replaced by $|V_p|_{sep}$ when the generator bus is treated as

load bus. Go to step (10).

9) For generator bus the magnitude of voltage does not change and so for all iterations the magnitude of bus voltage is the specified value only. The phase of the bus voltage can be calculated as shown below.

$$V_{p,temp}^{K+1} = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{(V_P^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{K+1} - \sum_{q=p}^{n} Y_{pq} V_q^K \right]$$

$$\delta_p^{k+1} = \tan^{-1} \left[\frac{I.P.\ of\ \ V_{p,temp}^{k+1}}{R.P.\ of\ \ V_{p,temp}^{k+1}} \right]$$

Now the (k+1)th iteration voltage of the generator bus is given by

$$V_p^{k+1} = |V_p|_{spe}\ \delta_p^{k+1}$$

Where $|V_p|_{spe}$ is magnitude of specified voltage.

After calculating V_p^{k+1} for generator bus go to step (12)

10) For the load bus the (k+1)th iteration value of load bus-p voltage, V_p^{k+1} can be calculated with the following equation.

$$V_p^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^K\right]$$

11) An acceleration factor ∝ can be used for faster convergence. If acceleration factor is specified then modify the (k+1)th iteration value of bus-p voltage using the following equation.

$$V_{p,acc}^{k+1} = V_p^k + \alpha(V_p^{k+1} - V_p^k)$$

Then set $V_p^{k+1} = V_{p,acc}^{k+1}$

12) Calculate the change in bus-p voltage, using the relation

$$\Delta V_p^k = V_p^{k+1} - V_p^k$$

Advance the bus count by 1 to evaluate other values of V_p^{k+1} and ΔV_p^k

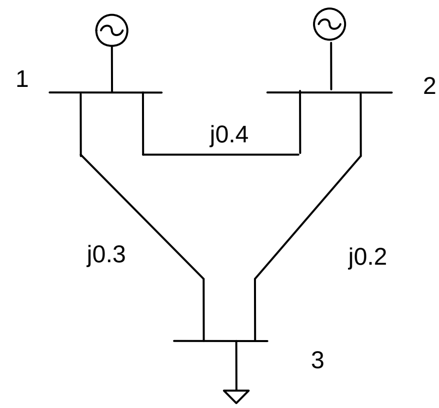
13) Check all the buses have been taken into account or not. If yes, go to the next step, Otherwise go back to step (6).

14) Determine the largest absolute value of change in voltage $|\Delta V|_{max}$

15) If $|\Delta V|_{max}$ is less than the pre specified tolerance €, then evaluate line flows and print the bus voltages and line flows. If not, advance the iteration count K= K+1 and go back to step (5).

EXAMPLE

1) Fig. shows a three bus power system.
Bus 1 : Slack bus, V= 1.05/00 p.u.
Bus 2 : PV bus, V = 1.0 p.u. Pg = 3 p.u.
Bus 3 : PQ bus, P_L = 4 p.u., Q_L = 2 p.u.
Carry out one iteration of load flow solution by Gauss Seidel method.



Neglect limits on reactive power generation.

Solution

Admittance of each line

$y_{12}=\frac{1}{Z_{12}}=\frac{1}{j0.4}=-j2.5\ p.u$

$y_{13}=\frac{1}{Z_{13}}=\frac{1}{j0.3}=-j3.333\ p.u$

$y_{23}=\frac{1}{Z_{23}}=\frac{1}{j0.2}=-j5\ p.u$

$$Y_{11}=y_{12}+y_{13}=-j2.5\ -j3.333=-j5.833\ p.u$$

$$Y_{22}=y_{12}+y_{23}=-j2.5-j5=-j7.5\ p.u$$

$$Y_{33}=y_{13}+y_{23}=\ -j3.333-j5=-j8.333\ p.u$$

$$Y_{12}=Y_{21}=-y_{12}=-(-j2.5\ )=j2.5\ p.u$$

$$Y_{13}=Y_{31}=-y_{13}=-(-j3.33\ )=j3.33\ p.u$$

$$Y_{23}=Y_{32}=-y_{23}=-(-j5\ )=j5\ p.u$$

The admittance matrix is given as

$$Y_{bus}=\begin{vmatrix} y_{12}+y_{13} & -y_{12} & -y_{13} \\ -y_{21} & y_{21}+y_{23} & -y_{23} \\ -y_{31} & -y_{32} & y_{32}+y_{31} \end{vmatrix}$$

$$= \begin{vmatrix} = -j5.833 & j2.5 & j3.33 \\ j2.5 & -j7.5 & j5 \\ j3.33 & j5 & -j8.333 \end{vmatrix}$$

Assume initial voltages to all buses

$V_1^{(0)}$= 1.05∠0^0=1.05+j0 p.u

$V_2^{(0)}$=1.0+j0 p.u

$V_3^{(0)}$=1.0+j0 p.u

Bus 1 is a slack bus

$V_1^{(1)}$= 1.05∠0^0=1.05+j0 p.u

Bus 2 is a generator bus

To calculate reactive power

$$Q_{p,cal}^{k+1} = (-1) \times Im\left\{\left(V_p^k\right)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k\right]\right\}$$

$$Q_{2cal}^1 = (-1) \times Im\{(V_2^0)^*[Y_{21}V_1^1 + Y_{22}V_2^0 + Y_{23}V_3^0]\}$$

$$=(-1) \times Im(1 - j0)[(j2.5)(1.05 + j0) + (-j7.5)(1 + j0) + (j5)(1 + j0)]\}$$

$$Q_{2cal}^1 = -0.125 \text{ p.u}$$

The phase of bus -2 voltage in first iteration is given by phase of $V_{p,temp}^{K+1}$

When p=3 $Q_2^1 = -0.125$ p.u and k=0

$$V_{P,temp}^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{\left(V_p^k\right)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right]$$

$$V_{2,temp}^{0+1} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - \sum_{q=1}^{2-1} Y_{2q} V_q^{0+1} - \sum_{q=2+1}^{3} Y_{2q} V_q^0\right]$$

$$V_{2,temp}^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0\right]$$

$$= \frac{1}{-j7.5}\left[\frac{3+j0.125}{1-j0} - (j2.5)(1.05 + j0) - (j5)(1 + j0)\right]$$

$$V_2^1 = \frac{1}{-j7.5}[3 - j7.5] = 1.077\angle 21.8^0 \text{ V}$$

$\delta_2^{1} = \angle V_{2,temp}^1 = 21.8^0$ V

$|\boldsymbol{V_2}^{\mathbf{1}}| = |\boldsymbol{V_2} \quad |\boldsymbol{spc}\angle\delta 2^1 = 1.0\angle 21.8^0$

$$|\boldsymbol{V_2}^{\mathbf{1}}| = \mathbf{0.928429} + \boldsymbol{j}\mathbf{0.3713}\ \boldsymbol{V}$$

Bus 3 Load Bus

The specified powers are load powers and so they considered as negative powers

$P_3 = -P_L = -4$

$Q_3 = -Q_L = -2$

$$V_P^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{\left(V_p^k\right)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right]$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{\left(V_3^0\right)^*} - Y_{31}V_1^1 - Y_{32}V_2^1\right]$$

$$= \frac{1}{-j8.333}\left[\frac{-4+j2}{1-j0} - (j3.33)(1.05 + j0) - (j5)(\mathbf{0.928429} + \boldsymbol{j}\mathbf{0.3713}5)\right]$$

$$V_3^1 = 0.7806\angle -19.24^0$$

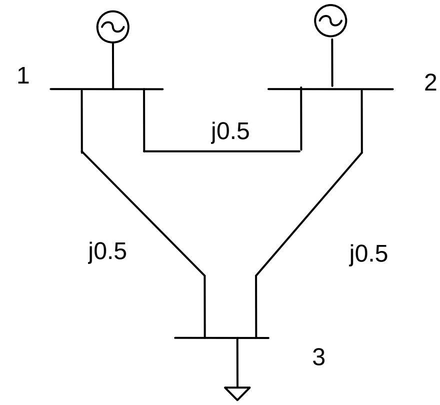
$$V_3^1 = 0.737046 - j0.25724\ p.u$$

2) Carry out one iteration of load flow analysis of the system given below by Gauss-Seidal method

Bus no	**Bus type**	**P**	**Q**	**V p.u**
1	**Slack**	**-**	**-**	**1.02**
2	**P-V**	**0.8**	**0.1 ≤ Q ≤ 1**	**1**
3	**P-Q**	**1.0**	**0.4**	**-**

Line reactance in p.u

Bus code	**Impedance**
1-2	**j0.5**
2-3	**j0.5**
3-1	**j0.5**



$y_{12}=\frac{1}{Z_{12}}=\frac{1}{j0.5}=-j2\ p.u$

$y_{13}=\frac{1}{Z_{13}}=\frac{1}{j0.5}=-j2\ p.u$

$y_{23}=\frac{1}{Z_{23}}=\frac{1}{j0.5}=-j2\ p.u$

$$Y_{11}=y_{12}+y_{13}=-j2-j2=-j4\ p.u$$
$$Y_{22}=y_{12}+y_{23}=-j2-j2=-j4\ p.u$$
$$Y_{33}=y_{13}+y_{23}=-j2-j2=-j4\ p.u$$
$$Y_{12}=Y_{21}=-y_{12}=-(-j2)=j2.p.u$$
$$Y_{13}=Y_{31}=-y_{13}=-(-j2)=j2.p.u$$
$$Y_{23}=Y_{32}=-y_{23}=-(-j2)=j2.p.u$$

The admittance matrix is given as

$$Y_{bus}=\begin{vmatrix} y_{12}+y_{13} & -y_{12} & -y_{13} \\ -y_{21} & y_{21}+y_{23} & -y_{23} \\ -y_{31} & -y_{32} & y_{32}+y_{31} \end{vmatrix}$$

$$=\begin{vmatrix} -j4 & j2 & j2 \\ j2 & -j4 & j2 \\ j2 & j2 & -j4 \end{vmatrix}$$

Assume initial voltages to all buses

$V_1^{(0)}= 1.02\angle 0^0=1.02+j0$ p.u

$V_2^{(0)}=1.0+j0$ p.u

$V_3^{(0)}=1.0+j0$ p.u

Bus 1 is a slack bus

$V_1^{(1)}$= 1.02∠0^0=1.02+j0 p.u

Bus 2 is a generator bus

To calculate reactive power

$$Q_{p,cal}^{k+1} = (-1) \times Im\left\{\left(V_p^k\right)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k\right]\right\}$$

$$Q_{2cal}^1 = (-1) \times Im\{(V_2^0)^*[Y_{21}V_1^1 + Y_{22}V_2^0 + Y_{23}V_3^0]\}$$

$$=(-1) \times Im(1 - j0)[(j2)(1.02 + \text{j}0) + (-j4)(1 + j0) + (j2)(1 + j0)]\}$$

$$Q_{2cal}^1 = -0.04\text{p.u}$$

This value is not with in the specified limit .so treat this bus as load bus
Q_2=0.1 P_2=0.3 and V_2^0=1.0+j0

$$V_P^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{\left(V_p^k\right)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0\right]$$

$$\frac{1}{-j4}\left[\frac{0.8 - j0.1}{1 - j0} - (j2)(1.02 + j0) - (j2)(1 + j0)\right]$$

=

$$\boldsymbol{\left|V_2{}^1\right| = 1.035 + j0.2 = 1.054\angle 10.93^0\ V}$$

Bus 3 Load Bus

The specified powers are load powers and so they considered as negative powers

P_3= −P_L= −1

Q_3= −Q_L= −0.4

$$V_P^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{\left(V_p^k\right)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right]$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1\right]$$

$$=\frac{1}{-j4}\left[\frac{-1+j0.4}{1-j0}-(j2)(1.02+j0)-(j2)(\mathbf{01.035+j0.2})\right]$$

$$V_3^1 =$$

LOAD FLOW USING NEWTON-RAPHSON METHOD

Newton-Raphson (NR) method is more efficient and practical for large power systems. Main advantage of this method is that the number of iterations required to obtain a solution is independent of the size of the problem and computationally it is very fast. Here load flow problem is formulated in polar form.

Rewriting eqn. (7.15) and (7.16)

$$P_i = \sum_{k=1}^{n} |V_i||V_k||Y_{ik}|\cos(\theta_{ik} - \delta_i + \delta_k) \qquad ...(7.50)$$

$$Q_i = -\sum_{k=1}^{n} |V_i||V_k||Y_{ik}|\sin(\theta_{ik} - \delta_i + \delta_k) \qquad ...(7.51)$$

Equations (7.50) and (7.51) constitute a set of nonlinear algebraic equations in terms of the independent variables, voltage magnitude in per unit and phase angles in radians, we can easily observe that two equations for each load bus given by eqn. (7.50) and (7.51) and one equation for each voltage controlled bus, given by eqn. (7.50). Expanding eqns. (7.50) and (7.51) in Taylor-series and neglecting higher-order terms. We obtain,

$$\therefore \begin{bmatrix} \Delta P_2^{(p)} \\ \vdots \\ \Delta P_n^{(p)} \\ \hline \Delta Q_2^{(p)} \\ \vdots \\ \Delta Q_n^{(p)} \end{bmatrix} = \left[\begin{array}{ccc|ccc} \left(\frac{\partial P_2}{\partial \delta_2}\right)^{(p)} & \cdots & \left(\frac{\partial P_2}{\partial \delta_n}\right)^{(p)} & \left(\frac{\partial P_2}{\partial |V_2|}\right)^{(p)} & \cdots & \left(\frac{\partial P_2}{\partial |V_n|}\right)^{(p)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \left(\frac{\partial P_n}{\partial \delta_2}\right)^{(p)} & \cdots & \left(\frac{\partial P_n}{\partial \delta_n}\right)^{(p)} & \left(\frac{\partial P_n}{\partial |V_2|}\right)^{(p)} & \cdots & \left(\frac{\partial P_n}{\partial |V_n|}\right)^{(p)} \\ \hline \left(\frac{\partial Q_2}{\partial \delta_2}\right)^{(p)} & \cdots & \left(\frac{\partial Q_2}{\partial \delta_n}\right)^{(p)} & \left(\frac{\partial Q_2}{\partial |V_2|}\right)^{(p)} & \cdots & \left(\frac{\partial Q_2}{\partial |V_n|}\right)^{(p)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \left(\frac{\partial Q_n}{\partial \delta_2}\right)^{(p)} & \cdots & \left(\frac{\partial Q_n}{\partial \delta_n}\right)^{(p)} & \left(\frac{\partial Q_n}{\partial |V_2|}\right)^{(p)} & \cdots & \left(\frac{\partial Q_n}{\partial |V_n|}\right)^{(p)} \end{array}\right] \begin{bmatrix} \Delta \delta_2^{(p)} \\ \vdots \\ \Delta \delta_n^{(p)} \\ \hline \Delta |V_2|^{(p)} \\ \vdots \\ \Delta |V_n|^{(p)} \end{bmatrix} \qquad ...(7.52)$$

In the above equation, bus-1 is assumed to be the slack bus.

Eqn. (7.52) can be written is short form i.e.,

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \qquad ...(7.53)$$

The diagonal elements of J_1 described by eqn. (7.57) may be written as:

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{k=1}^{n} |V_i|\,|V_k|\,|Y_{ik}|\,\sin(\theta_{ik} - \delta_i + \delta_k) - |V_i|^2\,|Y_{ii}|\,\sin\theta_{ii} \quad ...(7.65)$$

Using eqns. (7.65) and (7.51), we get

$$\frac{\partial P_i}{\partial \delta_i} = -Q_i - |V_i|^2\,|Y_{ii}|\,\sin\theta_{ii}$$

$$\therefore \qquad \frac{\partial P_i}{\partial \delta_i} = -Q_i - |V_i|^2 B_{ii} \quad ...(7.66)$$

where $B_{ii} = |Y_{ii}|\,\sin\theta_{ii}$ is the imaginary part of the diagonal elements of the bus admittance matrix. In a practical power system, $B_{ii} >> Q_i$ and hence we may neglect Q_i. Further simplification is obtained by assuming $|V_i|^2 \approx |V_i|$, which gives,

$$\frac{\partial P_i}{\partial \delta_i} = -|V_i|\,B_{ii} \quad ...(7.67)$$

Under normal operating conditions, $\delta_k - \delta_i$ is quite small. Therefore, $\theta_{ik} - \delta_i + \delta_k \approx \theta_{ik}$ and eqn. (7.58) reduces to

$$\frac{\partial P_i}{\partial \delta_k} = -|V_i|\,|V_k|\,B_{ik}$$

Assuming $\qquad |V_k| \approx 1.0$

$$\frac{\partial P_i}{\partial \delta_k} = -|V_i|\,B_{ik} \quad ...(7.68)$$

Similarly, the diagonal elements of J_4 as given by eqn. (7.59) may be written as:

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i|\,|Y_{ii}|\sin\theta_{ii} - \sum_{k=1}^{n} |V_i|\,|V_k|\,|Y_{ik}|\,\sin(\theta_{ik} - \delta_i + \delta_k) \quad ...(7.69)$$

Using eqns. (7.69) and (7.51), we get,

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i|\,|Y_{ii}|\sin\theta_{ii} + Q_i$$

$$\therefore \qquad \frac{\partial Q_i}{\partial |V_i|} = -|V_i|\,B_{ii} + Q_i \quad ...(7.70)$$

Again $B_{ii} >> Q_i$, Q_i may be neglected.

$$\therefore \qquad \frac{\partial Q_i}{\partial |V_i|} = -|V_i|\,B_{ii} \quad ...(7.71)$$

UNIT III

FAULT ANALYSIS-BALANCED FAULT

Symmetrical & Unsymmetrical Faults

Normally, a power system operates under balanced conditions. When the system becomes unbalanced due to the failures of insulation at any point or due to the contact of live wires, a short–circuit or fault, is said to occur in the line. Faults may occur in the power system due to the number of reasons like natural disturbances (lightning, high-speed winds, earthquakes), insulation breakdown, falling of a tree, bird shorting, etc.

Faults that occurs in transmission lines are broadly classified as

- Symmetrical faults
- Unsymmetrical faults

Symmetrical faults

In such types of faults, all the phases are short-circuited to each other and often to earth. Such fault is balanced in the sense that the systems remain symmetrical, or we can say the lines displaced by an equal angle (i.e. 120° in three phase line). It is the most severe type of fault involving largest current, but it occurs rarely. For this reason balanced short- circuit calculation is performed to determine these large currents.

Need for fault analysis

- To determine the magnitude of fault current throughout the power system after fault occurs.
- To select the ratings for fuses, breakers and switchgear.
- To check the MVA ratings of the existing circuit breakers when new generators are added into a system.

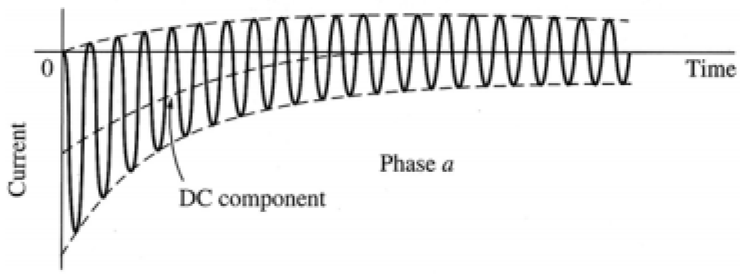
Common Power System Faults

Power system faults may be categorized as one of four types; in order of frequency of occurrence, they are: ·

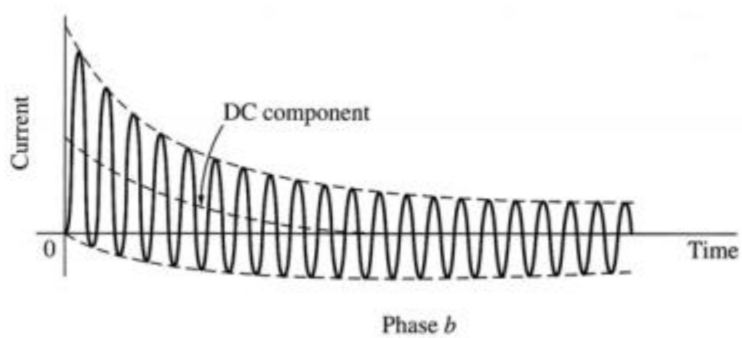
- Single line to ground fault
- Line to line fault
- Double line to ground fault
- Balanced three phase fault

3-Phase fault current transients in Phase fault current transients in synchronous generators synchronous generators

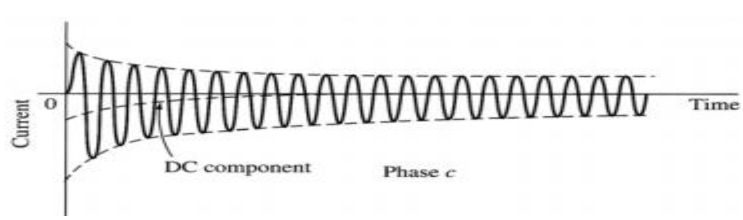
When a symmetrical 3-phase fault occurs at the terminals of a synchronous generator, the resulting current flow in the phases of the generator can appear as shown.



The current can be represented as a transient DC component added on top of a symmetrical AC component.



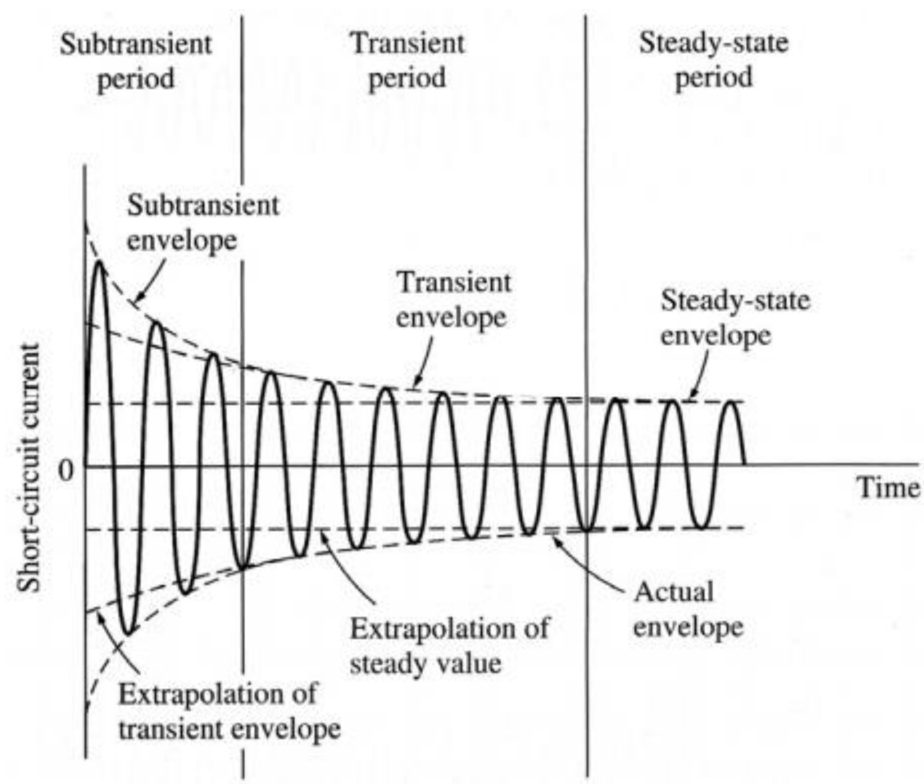
Therefore, while before the fault, only AC voltages and currents were present within the generator, immediately after the fault, both AC and DC currents are present.



Fault current transients in machines Fault current transients in machines

When the fault occurs, the AC component When the fault occurs, the AC component of current jumps to a very large value, but of current jumps to a very large value, but the total current cannot change instantly the total current cannot change instantly since the series inductance of the machine since the series inductance of the machine will prevent this from happening. will prevent this from happening.

The transient DC component of current is The transient DC component of current is just large enough such that the sum of the just large enough such that the sum of the AC and DC components just AC and DC components just after the fault the fault equals the AC current just equals the AC current just before the fault. the fault.Since the instantaneous values of current Since the instantaneous values of current at the moment of the fault are different in at the moment of the fault are different in each phase, the magnitude of DC each phase, the magnitude of DC components will be different in different components will be different in different phases.



There are three periods of time:

- Sub -transient period: first cycle or so after the fault transient period: first cycle or so after the fault – AC current is very AC current is very large and falls rapidly; large and falls rapidly;
- Transient period: current falls at a slower rate; Transient period: current falls at a slower rate;
- Steady -state period: current reaches its steady value. state period: current reaches its steady value.

It is possible to determine t It is possible to determine the time constants f he time constants for the sub or the sub -transient transient and transient periods . and transient periods

SHORT CIRCUIT CAPACITY

- It is the product of magnitudes of the prefault voltage and the post fault current.
- It is used to determine the dimension of a bus bar and the interrupting capacity of a circuit breaker.

Short Circuit Capacity (SCC)

$$|SCC| = |V^0||I_F|$$

$$|I_F| = \frac{|V_T|}{|Z_T|}$$

$$|SCC|_{1\phi} = \frac{|V_T|^2}{|Z_T|} = \frac{S_{b,1\phi}}{|Z_T|_{p.u}} MVA/\phi$$

$$|SCC|_{3\phi} = \frac{S_{b,3\phi}}{|Z_T|_{p.u}} MVA$$

$$I_f = \frac{|SCC|_{3\phi} * 10^6}{\sqrt{3} * V_{L,b} * 10^6}$$

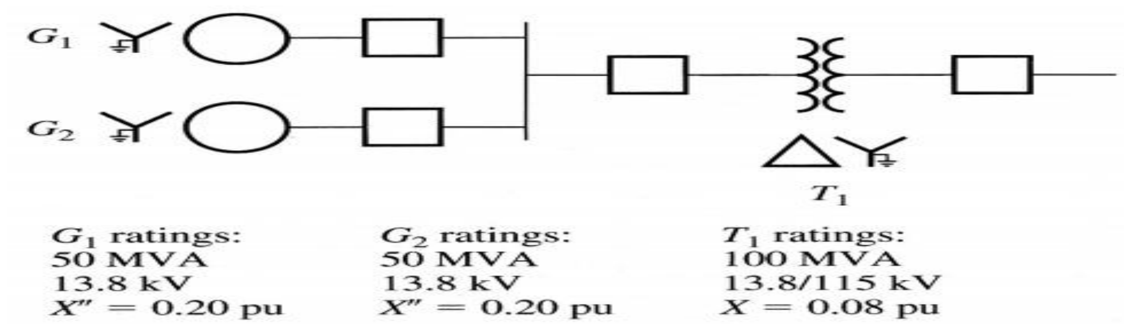
Procedure for calculating short circuit capacity and fault current

- Draw a single line diagram and select common base S_b MVA and kV
- Draw the reactance diagram and calculate the total p.u impedance from the fault point to source (Thevenin impedance Z_T)
- Determine SCC and I_f

EXAMPLE

Two generators are connected in parallel to the low voltage side of a transformer. Generators G1 and G 2 are each rated at 50 MVA, 13.8 kV, with a subtransient resistance of 0.2 pu. Transformer T1 is rated at 100 MVA, 13.8/115 kV with a series reactance of 0.08 pu and negligible resistance.

Assume that initially the voltage on the high side of the transformer is 120 kV, that the transformer is unloaded, and that there are no circulating currents between the generators. Calculate the subtransient fault current that will flow if a 3 phase fault occurs at the high-voltage side of transformer



Let choose the per-unit base values for this power system to be 100 MVA and 115 kV at the high-voltage side and 13.8 kV at the low-voltage side of the transformer. The subtransient reactance of the two generators to the system base is

$$X_{pu,new} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$X_1'' = X_2'' = 0.2 \times \left(\frac{13{,}800}{13{,}800}\right)^2 \times \left(\frac{100{,}000}{50{,}000}\right) = j0.4 p.u$$

The reactance of the transformer is already given on the system base, it will not change

$$X_T = 0.08\, p.u$$

The per-unit voltage on the high-voltage side of the transformer is

$$V_{pu} = \frac{Actual\ value}{Base\ value} = \frac{120{,}000}{115{,}000} = j1.044\, p.u$$

Since there is no load on the system, the voltage at the terminals of each generator, and the internal generated voltage of each generator must also be 1.044 pu. The per-phase per-unit equivalent circuit of the system is We observe that the phases of internal generated voltages are arbitrarily chosen as 0 0. The phase angles of both voltages must be the same since the generators were working in parallel

To find the subtransient fault current, we need to solve for the voltage at the bus 1 of the system. To find this voltage, we must convert first the per-unit impedances to admittances, and the voltage sources to equivalent current sources. The Thevenin impedance of each generator is ZTh = j0.4, so the short-circuit current of each generator is

$$I_{su} = \frac{V_{oc}}{Z_{th}} = \frac{1.044\angle 0^0}{j0.4} = 2.61\angle 90^0$$

Then the node equation for voltage V1

$$V_1(-j2.5) + V_1(-j2.5) + V_1(-j12.5) = 2.61\angle - 90^0 + 2.61\angle - 90^0$$

$$V_1 = \frac{5.22\angle 90^0}{-j17.5} = 0.298\angle 0^0$$

Therefore, the subtransient current in the fault is

$$I_F = V_1(-j12.5) = 3.729\angle{-90^0}\ p.u$$

Since the base current at the high-voltage side of the transformer is

$$I_{base} = \frac{S_{3\Phi,base}}{\sqrt{3}V_{LL,base}} = \frac{100{,}000{,}000}{\sqrt{3}115{,}000} = 502\ A$$

the subtransient fault current will be

$I_F = I_{F,p.u}\ I_{base} = 3.729 \times 502 = 1872$ A

ALGORITHM FOR SHORT CIRCUIT ANALYSIS USING BUS IMPEDANCE MATRIX

- Consider a n bus network. Assume that three phase fault is applied at bus k through a fault impedance z_f

Prefault voltages at all the buses are

$$V_{bus}(0) = \begin{bmatrix} V_1(0) \\ V_2(0) \\ . \\ V_k(0) \\ . \\ V_n(0) \end{bmatrix}$$

- Draw the Thevenin equivalent circuit i.e Zeroing all voltage sources and add voltage source at faulted bus k and draw the reactance diagram
- The change in bus voltage due to fault is

$$\Delta V_{bus} = \begin{bmatrix} \Delta V_1 \\ . \\ . \\ \Delta V_k \\ . \\ \Delta V_n \end{bmatrix}$$

- The bus voltages during the fault is

$$V_{bus}(F) = V_{bus}(0) + \Delta V_{bus}$$

- The current entering into all the buses is zero.the current entering into faulted bus k is –ve of the current leaving the bus k

$$\Delta V_{bus} = Z_{bus} I_{bus}$$

$$\Delta V_{bus} = \begin{pmatrix} Z_{11} & . & Z_{1k} & . & Z_{1n} \\ . & . & . & . & . \\ Z_{k1} & . & Z_{kk} & . & Z_{kn} \\ . & . & . & . & . \\ Z_{n1} & . & Z_{nk} & . & Z_{nn} \end{pmatrix} \begin{bmatrix} 0 \\ . \\ -I_k(F) \\ . \\ 0 \end{bmatrix}$$

$$V_k(F) = V_k(0) - Z_{kk} I_k(F)$$

$$V_k(F) = Z_f I_k(F)$$

$$I_k(F) = \frac{V_k(0)}{Z_{kk} + Z_f}$$

$$V_i(F) = V_i(0) - Z_{ik} I_k(F)$$

UNIT IV

FAULT ANALYSIS – UNBALANCED FAULTS

UNSYMMETRICAL FAULTS

- One or two phases are involved
- Voltages and currents become unbalanced and each phase is to be treated individually
- The various types of faults are

Shunt type faults

1.Line to Ground fault (LG)

2. Line to Line fault (LL)

3. Line to Line to Ground fault (LLG)

Series type faults

- Open conductor fault (one or two conductor open fault)

- Symmetrical components can be used to transform three phase unbalanced voltages and currents to balanced voltages and currents
- Three phase unbalanced phasors can be resolved into following three sequences

1.Positive sequence components

2. Negative sequence components

3. Zero sequence components

Single-Line-to-Ground Fault

Let a 1LG fault has occurred at node *k* of a network. The faulted segment is then as shown in Fig. 8where it is assumed that phase-a has touched the ground through an impedance Z_f . Since the system is unloaded before the occurrence of the fault we have

$$I_{fb} = I_{fc} = 0 \qquad (1)$$

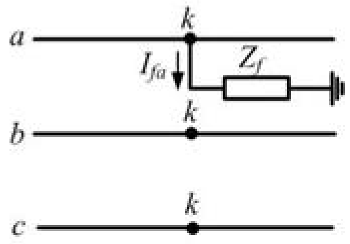


Fig. Representation of 1LG fault.

Also the phase-a voltage at the fault point is given by

From (1) we can write $$V_{ka} = Z_f I_{fa}$$ (2)

$$I_{fa012} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & a & a^2\\ 1 & a^2 & a\end{bmatrix}\begin{bmatrix}I_{fa}\\ 0\\ 0\end{bmatrix}$$ (3)

Solving (.3) we get

$$I_{fa0} = I_{fa1} = I_{fa2} = \frac{I_{fa}}{3}$$ (4)

his implies that the three sequence currents are in series for the 1LG fault. Let us denote the zero, positive and negative sequence Thevenin impedance at the faulted point as Z_{kk0} , Z_{kk1} and Z_{kk2} respectively.

$$\begin{aligned} V_{ka0} &= -Z_{kk0} I_{fa0}\\ V_{ka1} &= V_f - Z_{kk1} I_{fa1}\\ V_{ka2} &= -Z_{kk2} I_{fa2}\end{aligned}$$ (5)

Then from (4) and (5) we can write

$$\begin{aligned} V_{ka} &= V_{ka0} + V_{ka1} + V_{ka2}\\ &= V_f - (Z_{kk0} + Z_{kk1} + Z_{kk2}) I_{fa0}\end{aligned}$$ (6)

Again since

$$V_{ka} = Z_f I_{fa} = Z_f (I_{fa0} + I_{fa1} + I_{fa2}) = 3 Z_f I_{fa0}$$ (7)

The Thevenin equivalent of the sequence network is shown in Fig. 8.3.

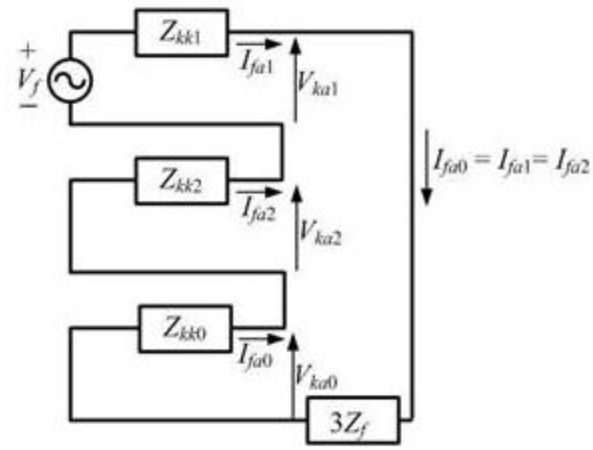
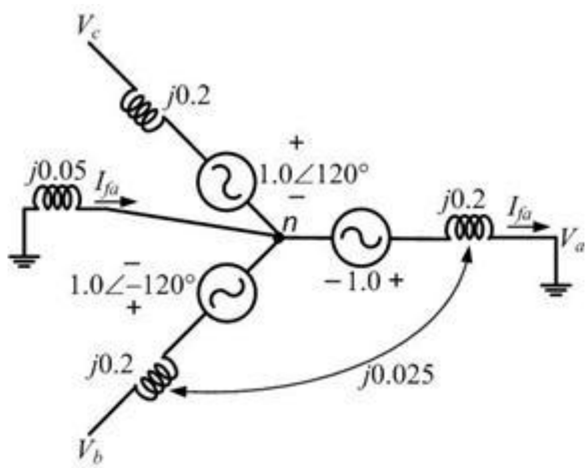


Fig. Thevenin equivalent of a 1LG fault.

Example 1

A three-phase Y-connected synchronous generator is running unloaded with rated voltage when a 1LG fault occurs at its terminals. The generator is rated 20 kV, 220 MVA, with subsynchronous reactance of 0.2 per unit. Assume that the subtransient mutual reactance between the windings is 0.025 per unit. The neutral of the generator is grounded through a 0.05 per unit reactance. The equivalent circuit of the generator is shown in Fig. We have to find out the negative and zero sequence reactances.



Since the generator is unloaded the internal emfs are

$$E_{an} = 1.0 \quad E_{bn} = 1.0\angle{-120^0} \quad E_{cn} = 1.0\angle 120^0$$

Since no current flows in phases b and c, once the fault occurs, we have from Fig.

$$I_{fa} = \frac{1}{j(0.2 + 0.05)} = 2 - j4.0$$

Then we also have

$$V_n = -X_n I_{fa} = -0.2$$

From Fig. we get

$$V_a = 0$$
$$V_b = E_{bn} + V_n + j0.025 I_{fa} = -0.6 - j0.866 = 1.0536\angle -124.72^0$$
$$V_c = E_{cn} + V_n + j0.025 I_{fa} = -0.6 + j0.866 = 1.0536\angle 124.72^0$$

Therefore

$$V_{a012} = C\begin{bmatrix} 0 \\ 1.0536\angle -124.72^0 \\ 1.0536\angle 124.72^0 \end{bmatrix} = \begin{bmatrix} -0.4 \\ 0.7 \\ -0.3 \end{bmatrix}$$
$$I_{fa1} = \frac{E_{an} - V_{a1}}{Z_1} = \frac{1 - 0.7}{j0.225} = -j1.333$$

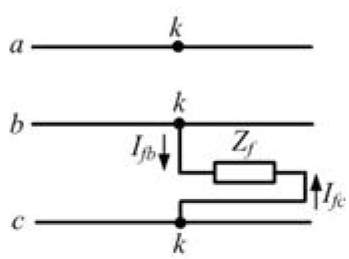
$$I_{fa0} = I_{fa1} = I_{fa2}$$
$$Z_{go} = \frac{-V_{a0}}{I_{a0}} - 3Z_n = j(0.3 - 0.15) = j0.15$$

$$Z_2 = \frac{-V_{a2}}{I_{a2}} = j0.225$$
$$I_{fa0} = \frac{1}{j(0.225 + 0.225 + 0.15 + 3 \times 0.05)} = -j1.333$$

Line-to-Line Fault

The faulted segment for an L-L fault is shown in Fig. where it is assumed that the fault has occurred at node *k* of the network. In this the phases b and c got shorted through the impedance Z_f. Since the system is unloaded before the occurrence of the fault we have

$$I_{fa} = 0 \qquad (1)$$



Also since phases b and c are shorted we have

$$I_{fb} = -I_{fc} \tag{2}$$

Therefore from (1) and (2) we have

$$I_{fa012} = C\begin{bmatrix} 0 \\ I_{fb} \\ -I_{fb} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 \\ (a-a^2)I_{fb} \\ (a^2-a)I_{fb} \end{bmatrix} \tag{3}$$

We can then summarize from (3)

$$I_{fa0} = 0$$
$$I_{fa1} = -I_{fa2} \tag{4}$$

herefore no zero sequence current is injected into the network at bus k and hence the zero sequence remains a dead network for an L-L fault. The positive and negative sequence currents are negative of each other.

Now from Fig. we get the following expression for the voltage at the faulted point

$$V_{kb} - V_{kc} = Z_f I_{fb} \tag{5}$$

Again

$$\begin{aligned} V_{kb} - V_{kc} &= V_{kb0} + V_{kb1} + V_{kb2} - V_{kc0} - V_{kc1} - V_{kc2} \\ &= (V_{kb1} - V_{kc1}) + (V_{kb2} - V_{kc2}) \\ &= (a^2 - a)V_{ka1} + (a - a^2)V_{ka2} \\ &= (a^2 - a)(V_{ka1} - V_{ka2}) \end{aligned} \tag{6}$$

Moreover since $I_{fa0} = I_{fb0} = 0$ and $I_{fa1} = - I_{fb2}$, we can write

$$I_{fb} = I_{fb1} + I_{fb2} = a^2 I_{fa1} + aI_{fb2} = (a^2 - a)I_{fa1} \tag{7}$$

Therefore combining (5) - (7) we get

$$V_{ka1} - V_{ka2} = Z_f I_{fa1} \tag{8}$$

Equations (5) and (8) indicate that the positive and negative sequence networks are in parallel. The sequence network is then as shown in Fig. From this network we get

$$I_{fa1} = -I_{fa2} = \frac{V_f}{Z_{kk1} + Z_{kk2} + Z_f}$$

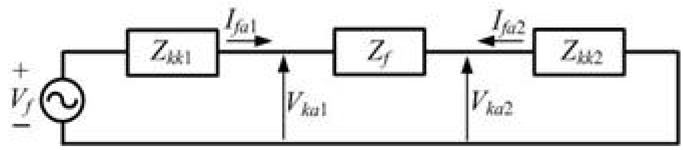


Fig. Thevenin equivalent of an LL fault.

Example 2

Let us consider the same generator as given in Example 1. Assume that the generator is unloaded when a bolted (Z_f = 0) short circuit occurs between phases b and c. Then we get from (2) $I_{fb} = -I_{fc}$. Also since the generator is unloaded, we have $I_{fa} = 0$.

$$V_{an} = E_{an} = 1.0$$
$$V_{bn} = E_{bn} - j0.225I_{fb} = 1.\angle -120^0 - j0.225I_{fb}$$

$$V_{cn} = E_{cn} - j0.225I_{fc} = 1.\angle 120^0 + j0.225I_{fb}$$

Also since $V_{bn} = V_{cn}$, we can combine the above two equations to get

$$I_{fb} = -I_{fc} = \frac{1\angle -120^0 - 1\angle 120^0}{j0.45} = -3.849$$

Then

$$I_{fa012} = C\begin{bmatrix} 0 \\ -3.849 \\ 3.849 \end{bmatrix} = \begin{bmatrix} 0 \\ -j2.2222 \\ j2.2222 \end{bmatrix}$$

We can also obtain the above equation from (9) as

$$I_{fa1} = -I_{fb2} = \frac{1}{j0.225 + j0.225} = -j2.222$$

Also since the neutral current I_n is zero, we can write $V_a = 1.0$ and

$$V_b = V_c = V_{bn} = V_{bn} = -0.5$$

Hence the sequence components of the line voltages are

$$V_{a012} = C\begin{bmatrix} 1.0 \\ -0.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}$$

Also note that

$$V_{a1} = 1.0 - j0.2251I_{fa1}$$
$$V_{a2} = -j0.2251I_{fa2} = 0.5$$

which are the same as obtained before.

Double- Line -to Ground Fault

The faulted segment for a 2LG fault is shown in Fig. where it is assumed that the fault has occurred at node *k* of the network. In this the phases b and c got shorted through the impedance Z_f to the ground.

$$I_{fa0} = \frac{1}{3}\left(I_{fa} + I_{fb} + I_{fc}\right) = \frac{1}{3}\left(I_{fb} + I_{fc}\right)$$
$$\Rightarrow 3I_{fa0} = I_{fb} + I_{fc} \qquad (1)$$

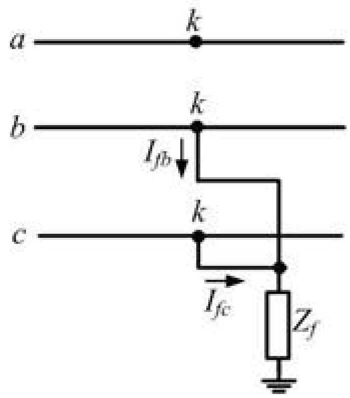


Fig. Representation of 2LG fault.

Also voltages of phases b and c are given by

$$V_{kb} = V_{kc} = Z_f\left(I_b + I_c\right) = 3Z_f I_{fa0} \qquad (2)$$

Therefore

$$V_{ka012} = C\begin{bmatrix} V_{ka} \\ V_{kb} \\ V_{kb} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} V_{ka} + 2V_{kb} \\ V_{ka} + \left(a + a^2\right)V_{kb} \\ V_{ka} + \left(a + a^2\right)V_{kb} \end{bmatrix} \qquad (3)$$

We thus get the following two equations from (3)

$$V_{ka1} = V_{ka2} = V_{ka0} - 3Z_f I_{fa0} \qquad (4)$$

$$V_{ka1} = V_{ka2}$$

$$3V_{ka0} = V_{ka} + 2V_{kb} = V_{ka0} + V_{ka1} + V_{ka2} + 2V_{kb} \qquad (5)$$

Substituting (8.18) and (8.20) in (8.21) and rearranging we get

$$V_{ka1} = V_{ka2} = V_{ka0} - 3Z_f I_{fa0} \qquad (6)$$

Also since $I_{fa} = 0$ we have

$$I_{fa0} + I_{fa1} + I_{fa2} = 0 \qquad (7)$$

The Thevenin equivalent circuit for 2LG fault is shown in Fig. 8.8. From this figure we ge

$$I_{fa1} = \frac{V_f}{Z_{kk1} + Z_{kk2} \| (Z_{kk0} + 3Z_f)} = \frac{V_f}{Z_{kk1} + \frac{Z_{kk2}(Z_{kk0} + 3Z_f)}{Z_{kk2} + Z_{kk0} + 3Z_f}} \quad (8)$$

$$I_{fa0} = -I_{fa1}\left(\frac{Z_{kk2}}{Z_{kk2} + Z_{kk0} + 3Z_f}\right) \quad (9)$$

$$I_{fa2} = -I_{fa1}\left(\frac{Z_{kk0} + 3Z_f}{Z_{kk2} + Z_{kk0} + 3Z_f}\right) \quad (10)$$

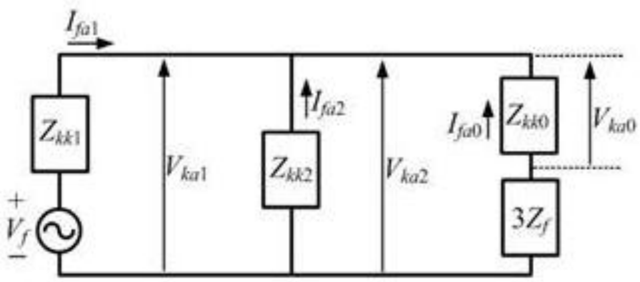


Fig. Thevenin equivalent of a 2LG fault.

Example 3

Let us consider the same generator as given in Examples 1 and 2. Let us assume that the generator is operating without any load when a bolted 2LG fault occurs in phases b and c. The equivalent circuit for this fault is shown in Fig. 8.9. From this figure we can write

$$E_{bn} + V_n = 1\angle -120° + V_n = j0.2I_{fb} - j0.025I_{fc}$$

$$E_{cn} + V_n = 1\angle 120° + V_n = j0.2I_{fc} - j0.025I_{fb}$$

$$V_n = -j0.05(I_{fb} + I_{fc})$$

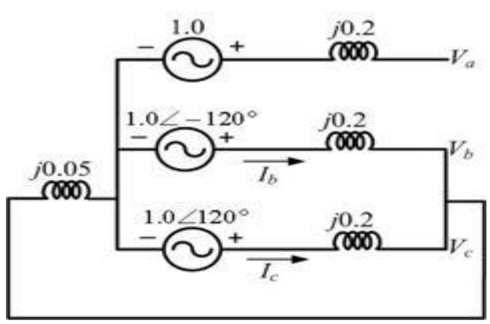


Fig. Equivalent circuit of the generator for a 2LG fault in phases b and c.

Combining the above three equations we can write the following vector-matrix form

$$j\begin{bmatrix} 0.25 & 0.025 \\ 0.025 & 0.25 \end{bmatrix}\begin{bmatrix} I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} 1\angle -120^\circ \\ 1\angle 120^\circ \end{bmatrix}$$

Solving the above equation we get

$I_{fb} = -3.849 + j1.8182$

$I_{fc} = 3.849 + j1.8182$

Hence

We can also obtain the above values using (8)-(10). Note from Example 1 that

$$Z_1 = Z_2 = j0.225, Z_0 = j(0.15 + 3 \times 0.05) = j0.3 \ \ and\ Z_f = 0$$

Then

$$I_{fa1} = \frac{1}{j0.225 + \left(\frac{j0.225 \times j0.3}{j0.525}\right)} = -j2.8283$$

$$I_{fa0} = -I_{fa1}\frac{j0.225}{j0.525} = j1.2121$$

Now the sequence components of the voltages are

$$V_{a1} = 1.0 - j0.225 I_{fa1} = 0.3636$$

$$V_{a2} = j0.225 I_{fa2} = 0.3636$$

$$V_{a0} = -j0.3 I_{fa0} = 0.3636$$

Also note from above Fig. that

$$V_a = E_{an} + V_n + j0.0225(I_{fb} + I_{fc}) = 1.0909$$

and Vb = Vc = 0. Therefore

$$V_{a012} = C\begin{bmatrix} 1.0909 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.3636 \\ 0.3636 \\ 0.3636 \end{bmatrix}$$

which are the same as obtained before.

UNIT V

STABILITY ANALYSIS

STABILITY

- ❖ The tendency of a power system to develop restoring forces equal to or greater than the disturbing forces to maintain the state of equilibrium.
- ❖ Ability to keep the machines in synchronism with another machine

CLASSIFICATION OF STABILITY

- ❖ Steady state stability
  Ability of the power system to regain synchronism after small and slow disturbances (like gradual power changes)
- ❖ Dynamic stability
  Ability of the power system to regain synchronism after small disturbances occurring for a long time (like changes in turbine speed, change in load)
- ❖ Transient stability
  This concern with sudden and large changes in the network conditions i.e. . sudden changes in application or removal of loads, line switching operating operations, line faults, or loss of excitation.
- ❖ Steady state limit is the maximum power that can be transferred without the system become unstable when the load in increased gradually under steady state conditions.
- ❖ Transient limit is the maximum power that can be transferred without the system becoming unstable when a sudden or large disturbance occurs.

Rotor Angle Stability

- It is the ability of interconnected synchronous machines of a power system to maintain in synchronism. The stability problem involves the study of the electro mechanical oscillations inherent in power system.
- Types of Rotor Angle Stability
  1. Small Signal Stability (or) Steady State Stability

2. Transient stability

Voltage Stability

- It is the ability of a power system to maintain steady acceptable voltages at all buses in the system under normal operating conditions and after being subjected to a disturbance.
- The major factor for instability is the inability of the power system to meet the demand for reactive power.
- Mid Term Stability

  It represents transition between short term and long term responses.

  Typical ranges of time periods.

  1. Short term : 0 to 10s
  2. Mid Term : 10 to few minutes
  3. Long Term : a few minutes to 10's of minutes
- Long Term Stability

  Usually these problem be associated with

  1. Inadequacies in equipment responses.
  2. Poor co-ordination of control and protection equipment.
  3. Insufficient active/reactive power reserves.

Swing Equation

Let us consider a three-phase synchronous alternator that is driven by a prime mover. The equation of motion of the machine rotor is given by

$$J\frac{d^2\theta}{dt^2} = T_m - T_e = T_a \tag{1}$$

where

J is the total moment of inertia of the rotor mass in kgm^2

T_m is the mechanical torque supplied by the prime mover in N-m

T_e is the electrical torque output of the alternator in N-m

θ is the angular position of the rotor in rad

Neglecting the losses, the difference between the mechanical and electrical torque gives the net accelerating torque T_a . In the steady state, the electrical torque is equal to the mechanical torque, and hence the accelerating power will be zero. During this period the rotor will move at **synchronous speed** ω_s in rad/s.

The angular position θ is measured with a stationary reference frame. To represent it with respect to the synchronously rotating frame, we define

$$\theta = \omega_s t + \delta \quad (2)$$

where δ is the angular position in rad with respect to the synchronously rotating reference frame. Taking the time derivative of the above equation we get

$$I_S = \frac{V_1\angle\delta - V_2}{jX} = \frac{V_1\cos\delta - V_2 + jV_1\sin\delta}{jX} \quad (3)$$

Defining the angular speed of the rotor as

we can write (3) as

$$\omega_r = \frac{d\theta}{dt}$$

$$\omega_r - \omega_s = \frac{d\delta}{dt} \quad (4)$$

We can therefore conclude that the rotor angular speed is equal to the synchronous speed only when $d\delta/dt$ is equal to zero. We can therefore term $d\delta/dt$ as the error in speed. Taking derivative of (3), we can then rewrite (1) as

$$J\frac{d^2\delta}{dt^2} = T_m - T_e = T_a \quad (5)$$

Multiplying both side of (5) by ω_m we get

$$J\omega_r\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \quad (6)$$

where P_m , P_e and P_a respectively are the mechanical, electrical and accelerating power in MW.

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \text{ per unit} \quad (7)$$

We now define a normalized inertia constant as

$$H = \frac{\text{Stored kinetic energy at synchronous speed in mega - joules}}{\text{Generator MVA rating}} = \frac{J\omega_s^2}{2S_{rated}}$$

Substituting the above in (6) we get

$$2H\frac{S_{rated}}{\omega_s^2}\omega_r\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \tag{8}$$

In steady state, the machine angular speed is equal to the synchronous speed and hence we can replace ω_r in the above equation by ω_s. Note that in (8) P_m , P_e and P_a are given in MW. Therefore dividing them by the generator MVA rating S_{rated} we can get these quantities in per unit. Hence dividing both sides of (8) by S_{rated} we get

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \quad \text{per unit} \tag{9}$$

Equation (9) describes the behaviour of the rotor dynamics and hence is known as the swing equation. The angle δ is the angle of the internal emf of the generator and it dictates the amount of power that can be transferred. This angle is therefore called the **load angle** .

Equal Area Criterion

The real power transmitted over a lossless line is given by (9.4). Now consider the situation in which the synchronous machine is operating in steady state delivering a power P_e equal to P_m when there is a fault occurs in the system. Opening up of the circuit breakers in the faulted section subsequently clears the fault. The circuit breakers take about 5/6 cycles to open and the subsequent post-fault transient last for another few cycles. The input power, on the other hand, is supplied by a prime mover that is usually driven by a steam turbine. The time constant of the turbine mass system is of the order of few seconds, while the electrical system time constant is in milliseconds. Therefore, for all practical purpose, the mechanical power is remains constant during this period when the electrical transients occur. The transient stability study therefore concentrates on the ability of the power system to recover from the fault and deliver the constant power P_m with a possible new load angle δ .

Consider the power angle curve shown in Fig.. Suppose the system of Fig. is operating in the steady state delivering a power of P_m at an angle of δ_0 when due to malfunction of the line, circuit breakers open reducing the real power transferred to zero. Since P_m remains constant, the accelerating power P_a becomes equal to P_m . The difference in the power gives rise to the rate of change of stored kinetic energy in the rotor masses. Thus the rotor will accelerate under the constant influence of non-zero accelerating power and hence the load angle will increase. Now suppose the circuit breaker re-closes at an angle δ_c. The power will then revert back to the normal operating curve. At that point, the electrical power will be more than the mechanical power and the accelerating power will be negative. This will cause the machine decelerate. However, due to the inertia of the rotor masses, the load angle will still keep on increasing. The increase in this angle may eventually stop and the rotor may start decelerating, otherwise the system will lose synchronism.

Note that

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = 2\left(\frac{d\delta}{dt}\right)\left(\frac{d^2\delta}{dt^2}\right)$$

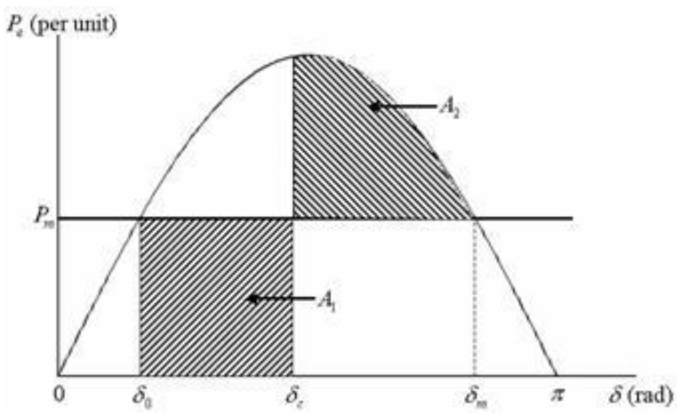


Fig. Power-angle curve for equal area criterion.

Hence multiplying both sides of (9.14) by and rearranging we get

$$\frac{H}{\omega_s}\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \left(P_m - P_e\right)\frac{d\delta}{dt}$$

Multiplying both sides of the above equation by dt and then integrating between two arbitrary angles δ_0 and δ_c we get

$$\frac{H}{\omega_s}\left(\frac{d\delta}{dt}\right)^2\Bigg|_{\delta_0}^{\delta_c} = \int_{\delta_0}^{\delta_c}\left(P_m - P_e\right)d\delta \qquad (1)$$

Now suppose the generator is at rest at δ_0. We then have $d\delta / dt = 0$. Once a fault occurs, the machine starts accelerating. Once the fault is cleared, the machine keeps on accelerating before it reaches its peak at δ_c , at which point we again have $d\delta / dt = 0$. Thus the area of accelerating is given from (1) as

$$A_1 = \int_{\delta_0}^{\delta_c} (P_m - P_e) d\delta = 0 \quad (2)$$

In a similar way, we can define the area of deceleration. In Fig., the area of acceleration is given by A_1 while the area of deceleration is given by A_2 . This is given by

$$A_2 = \int_{\delta_c}^{\delta_m} (P_e - P_m) d\delta = 0 \quad (3)$$

Now consider the case when the line is reclosed at δ_c such that the area of acceleration is larger than the area of deceleration, i.e., $A_1 > A_2$. The generator load angle will then cross the point δ_m , beyond which the electrical power will be less than the mechanical power forcing the accelerating power to be positive. The generator will therefore start accelerating before is slows down completely and will eventually become unstable. If, on the other hand, $A_1 < A_2$, i.e., the decelerating area is larger than the accelerating area, the machine will decelerate completely before accelerating again. The rotor inertia will force the subsequent acceleration and deceleration areas to be smaller than the first ones and the machine will eventually attain the steady state. If the two areas are equal, i.e., $A_1 = A_2$, then the accelerating area is equal to decelerating area and this is defines the **boundary of the stability limit.** The clearing angle δ_c for this mode is called the **Critical Clearing Angle** and is denoted by δ_{cr}. We then get from Fig. by substituting $\delta_c = \delta_{cr}$

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_e) d\delta = \int_{\delta_{cr}}^{\delta_m} (P_e - P_m) d\delta$$

We can calculate the critical clearing angle from the ab move equation. Since the critical clearing angle depends on the equality of the areas, this is called the **equal area criterion**.

Techniques for stability Improvement

Transient Stability Improvement methods:

- Transient stability of the system can be improved by increasing the system voltage. Increase in the voltage profile of the system implies increase in the power transfer ability.
- This helps in increasing the difference between initial load angle and critical clearance angle. Hence increase in power allows the machine to allow to rotate through large angle before reaching critical clearance angle.

- Increase in the X/R ratio in the power system increases the power limit of the line. Thus helps to improve the stability
- High speed circuit breakers helps to clear the fault as quick as possible. The quicker the breaker operates, the faster the fault cleared and better the system restores to normal operating conditions
- By Turbine fast valving: One of the main reason for the instability in the power system is due to the excess energy supplied by the turbine during the fault period. Fast Valving helps in reducing the mechanical input power when the generator is under acceleration during the fault and hence improves the stability of the system
- Use of Auto Re-closing: Majority of the faults in the power system will be momentary and can be self-cleared. Hence circuit breakers employed for fault clearance opens in sensing the fault with time delay of 2 cycles and re-closes after particular time to determine whether the fault is cleared.
- Use of Auto Re-closing: Majority of the faults in the power system will be momentary and can be self-cleared. Hence circuit breakers employed for fault clearance opens in sensing the fault with time delay of 2 cycles and re-closes after particular time to determine whether the fault is cleared.

Some of the other ways to improve the transient stability are by employing lightning arresters, high neutral grounding impedance, single pole switching, quick Automatic Voltage Regulators (AVRs

: *A 60 Hz, 4 pole turbogenerator rated 100 MVA, 13.8 KV has an inertia constant of 10 MJ / MVA.*

(*a*) Find the stored energy in the rotor at synchronous speed.
(*b*) If the input to the generator is suddenly raised to 60 MW for an electrical load of 50 MW, find rotor acceleration.
(*c*) If the rotor acceleration calculated in part (b) is maintained for 12 cycles, find the change in torque angle and rotor speed in rpm at the end of this period.

Solution:

(*a*) Stored energy = *GH*

$$G = 100 \text{ MVA}, H = 10 \text{ MJ/MVA}.$$

$\therefore$ Stored energy = 100 × 10 = 1000 *MJ.*

(*b*) $P_a = P_i - P_e = 60 - 50 = 10$ MW.

we know, $$M = \frac{GH}{180f} = \frac{100 \times 10}{180 \times 60} = \frac{5}{54} \text{ MJ-Sec/elect deg.}$$

Now $$M.\frac{d^2\delta}{dt^2} = P_a$$

$$\therefore \quad \frac{5}{54}\,\frac{d^2\delta}{dt^2} = 10$$

$$\therefore \quad \frac{d^2\delta}{dt^2} = \frac{10 \times 54}{5} = 108 \text{ elect-deg/Sec}^2$$

$$\therefore \quad \alpha = \text{acceleration} = 108 \text{ elect-deg/Sec}^2$$ **Ans.**

(*c*) $12 \text{ cycles} = \frac{12}{60} = 0.2$ sec.

Change in $$\delta = \frac{1}{2}\,\alpha.(\Delta t)^2 = \frac{1}{2} \times 108 \times (0.2)^2 \text{ elect-degree.}$$

$$= 2.16 \text{ elect-degree.}$$

Now $$\alpha = 108 \text{ elect-deg/Sec}^2$$

$$\therefore \quad \alpha = 60 \times \frac{108}{360^\circ} \text{ rpm/Sec} = 18 \text{ rpm/Sec.}$$

$\therefore$ Rotor speed at the end of 12 cycles

$$= \frac{120f}{P} + \alpha\,(\Delta t)$$

$$= \left(\frac{120 \times 60}{4} + 18 \times 0.2\right) \text{ rpm}$$

$$= 1803.6 \text{ rpm}$$ **Ans.**

A 400 MVA synchronous machine has H_1 = 4.6 MJ/MVA and a 1200 MVA machine has H_2 = 3.0 MJ/MVA. The two machines operate in parallel in a power plant. Find out H_{eq}, relative to a 100 MVA base.

Solutions:

Total kinetic energy of the two machines is

$$KE = 4.6 \times 400 + 3 \times 1200 = 5440 \text{ MJ.}$$

Using the formula given in eqn. (11.28),

$$H_{eq} = \left(\frac{400}{100}\right) \times 4.6 + \left(\frac{1200}{100}\right) \times 3$$

$\therefore$ $$H_{eq} = 54.4 \text{ MJ/MVA}$$

or, equivalent inertia relative to a 100 MVA base is

$$H_{eq} = \frac{\text{KE}}{\text{System base}} = \frac{5440}{100} = 54.4 \text{ MJ/MVA} \quad \textbf{Ans.}$$