Behavior of compact L-shaped spandrel beams with alternative web reinforcement

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- Open web reinforcement provides a simpler option for detailing and producing prestressed concrete L-shaped spandrel beams compared with typical web reinforcement using closed stirrups.
- This paper discusses the full-scale testing of compact L-shaped spandrel beams to evaluate the performance of beams with alternative web reinforcement schemes.
- The test results were compared with the results of previous studies of slender L-shaped spandrel beams, and design recommendations are provided.

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Open web reinforcement has been shown to be an effective alternative to closed stirrups in the webs of slender precast concrete L-shaped spandrel beams subjected to combined shear and torsion.[1–6] For slender beams having a height-to-width ratio (aspect ratio) of 4.5 or greater, an open reinforcement scheme is a better alternative to the traditional closed stirrups mandated by the American Concrete Institute's (ACI's) *Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)*[7] because the beams are easier to produce with open reinforcement. While the behavior of slender L-shaped beams with open web reinforcement has been well documented, the use of alternatives to closed stirrup reinforcement for compact L-shaped cross sections (having aspect ratios less than 4.5) has not been investigated.

This paper presents an experimental study in which four full-scale, 46 ft (14 m) long, precast concrete, compact L-shaped beams were tested to failure. One of the test specimens served as a control and was designed with traditional closed stirrups. The remaining three beams were designed with alternative open and segmented reinforcement configurations.

The results of the study demonstrate the viability of replacing closed stirrups with alternative open and segmented web reinforcement in compact L-shaped spandrel beams. All four beams behaved satisfactorily at all loading stages and failed at loads much greater than the factored design loads. When failure occurred in the end region of the compact L-shaped beams with open reinforcement, no spiral cracking or face-shell spalling were observed, which is contrary to the failure mode associated with the torsion design concept of ACI 318-14. Rather, the observed failure planes in the compact L-shaped beams were similar to those observed in slender L-shaped beams, with skewed bending along a diagonal crack extending upward from the support. The results confirm the potential to simplify the design and detailing of compact L-shaped beams by using alternative transverse reinforcement details proportioned with a design approach based on failure in combined shear and torsion.

Precast, prestressed concrete L-shaped beams with compact cross sections are frequently used to support double-tee deck sections in parking structures when the top of the beam cannot extend above the top surface of the deck. For example, compact beams may be used at locations where traffic must pass over the end of a double tee, such as a ramp or a crossing point between bays. Compact L-shaped beams may also be used to support deck sections along the edges of a parking structure when a separate railing system will be installed. The primary purpose of the compact L-shaped beam is to transfer vertical loads from deck sections to columns.

Typical compact L-shaped beams are between 2 and 3 ft (0.6 and 0.9 m) deep and can have spans as large as 50 ft (15 m). These beams usually have a web thickness of 16 in. (400 mm) or more with a continuous ledge running along the bottom edge on one side of the beam, creating the L-shaped cross section. The ledge provides a bearing surface for the stems of the deck sections, so the compact L-shaped beam is subjected to a series of discrete eccentric loadings. The beams are simply supported at the columns, and lateral connections are provided to prevent rotation due to eccentric loading. Discrete connections between deck sections and the web of the spandrel beam provide lateral restraint along the length of a typical compact L-shaped beam.

Compact L-shaped beams are often subjected to heavy loads applied at high eccentricities. The amount of torsion developed in these members can be significant, and designs usually require closed stirrup reinforcement anchored by 135-degree hooks in accordance with section 25.7.1.6 of ACI 318-14, especially in the end regions. The required reinforcement causes severe congestion in the end regions where prestressing strands and longitudinal reinforcing bars must weave through closed stirrups that are closely spaced, as required by ACI 318-14 and the eighth edition of the *PCI Design Handbook: Precast and Prestressed Concrete*.[8] Precast, prestressed concrete L-shaped spandrel beams (both slender and compact) are typically designed according to a general procedure originally proposed by Zia and McGee[9] and later modified by Zia and Hsu.[10]

The early years of torsion research were largely focused on the behavior of rectangular beams subjected to applied twisting moments. Empirical design equations and sophisticated rational models were devised to reflect the observed response of such beams subjected to torsion.[10,11] The design formulas and their detailing requirements were incorporated into ACI 318 and were applied in practice to L-shaped beams. However, it was difficult to reconcile the need for such complex detailing with the observed behavior of precast concrete L-shaped beams subjected to eccentric vertical loading. Observations of beams in the laboratory and in the field by Logan,[6] Klein,[12] and Raths[13] showed that torsional failure of L-shaped spandrel beams did not result in face-shell spalling; instead, significant plate-bending effects in the web of slender L-shaped spandrel beams were observed.

The observed behavior raised questions about the need for closed stirrups in a slender (noncompact) L-shaped section. Hassan et al.[4] and Lucier et al.[5] demonstrated through full-scale tests and finite element analysis that open web reinforcement could be used safely and effectively in slender precast concrete L-shaped spandrel beams. Subsequent testing and analysis led to the development of rational design guidelines for slender precast concrete L-shaped beams.[1,2,5] These guidelines considered the torsion applied on an L-shaped section as two separate orthogonal components acting on an inclined failure plane (**Fig. 1**). One component of the torsion vector (the plate-bending component) causes out-of-plane bending of the beam web about a diagonal line extending upward from the support. The second orthogonal component of torsion (the twisting component) acts to twist the cross section about an axis perpendicular to the diagonal line.

Reinforcing steel in the web of the cross section was designed to resist the plate-bending component of torsion and a method for evaluating the resistance of the cross section to the twisting component was proposed. Tests confirmed the proposed guidelines were safe and effective for slender L-shaped beams with aspect ratios (web height divided by web thickness) of 4.5 or greater. The goal of the study presented in this paper was to examine the applicability of the previously developed design guidelines to beams with aspect ratios less than 4.5.

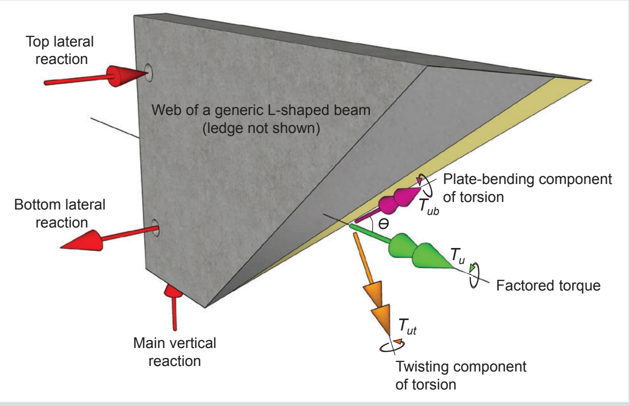


Figure 1. Components of torsion on a generic cross section. Note: θ = critical diagonal crack angle for region under consideration with respect to horizontal: 45 degrees for end regions, 30 degrees for transition regions, 0 degrees for flexure regions.

Objective and scope

The primary objective of this study was to examine the behavior of full-size compact L-shaped beams designed with open web reinforcement. Special attention was focused on the type of failure mode, the end-region capacity, the cracking pattern, and the crack angles.

The experimental program included four full-sized compact L-shaped beams. One of the four beams served as a control specimen and was designed using closed stirrups according to the guidelines in the eighth edition of the *PCI Design Handbook*.[8] The remaining three beams were designed using combinations of welded-wire reinforcement (WWR) and L- and C-shaped reinforcing bars as the torsion, shear, and ledge reinforcement. No closed stirrups were used for these three specimens, so the prestressing strands did not have to be threaded through stirrups prior to pretensioning. The L-shaped and C-shaped reinforcing bars, and WWR are much easier to produce and to install than a series of closed stirrups.

Test specimens

The four L-shaped spandrel beams tested in this program had identical cross sections of 28 × 34 in. (710 × 860 mm) with a continuous 8 × 8 in. (200 × 200 mm) ledge along the bottom edge of one web face. Beams were labeled LG1, LG2, LG3, and LG4 in the order they were tested. The ledge was held back 12 in. (300 mm) from each end of the beam, and all four spandrel beams were 46 ft (14 m) long. All four beams used a typical self-consolidating normalweight concrete with a measured average compressive strength (at the time of beam testing) of 8700 psi (60 MPa). The beams were prestressed and cast together on a longline casting bed, and 4 × 8 in. (100 × 200 mm) concrete control cylinders were prepared from the concrete used for each beam. The beams were delivered to the laboratory as needed for testing.

Two holes were provided through the web at each end of each beam to allow the beams to be connected to the test frame. These holes were located at 6 in. (150 mm) from the beam ends and 6 in. from the top and bottom of the beam. The holes were sized to accommodate the threaded rods used to bolt the beams to the test frame in a way that simulated discrete support conditions in the field. Embedded steel plates were provided along the top face of each spandrel beam to connect the L-shaped beam to the flanges of the double-tee deck sections.

To ensure end-region failures in this study, additional flexural and ledge reinforcement was provided in each beam. The same prestressing strand pattern was provided in each beam and partial-length mild steel reinforcement was added to provide additional moment capacity away from the end regions. A total of 20 prestressing strands were used for each beam: 17 strands were placed near the bottom of the web on a 2 in. (50 mm) grid, and the remaining 3 strands were placed near the top of the web. An initial prestressing force of 31,600 lb (140,500 N) (70% of the ultimate tensile strength of the prestressing strands) was applied to each strand. The minimum clear concrete cover was 1.75 in. (44 mm) for all strands.

The beams were reinforced with prestressing strands, WWR, and conventional deformed reinforcing bars. The prestressing strands were ½ in. (12.7 mm) special 7-wire, 270 ksi (1860 MPa), low-relaxation strands with a nominal cross-sectional area of 0.167 in.^{2} (108 mm^2). Conventional Grade 60 (414 MPa) no. 3 (10M), no. 4 (13M), no. 5 (16M), and no. 9 (29M) deformed reinforcing bars were also used in various locations.

Six 30 ft (9 m) long no. 9 (29M) reinforcing bars were placed at midspan as additional flexural reinforcement. This partial-length flexural reinforcement was used to increase the flexural capacity of the beam without affecting the end-region shear and torsion capacity. In addition to the partial-length flexural reinforcement, extra reinforcement was provided to strengthen the ledges at each loading point to prevent punching shear failure.

For specimens LG2 through LG4, shear and torsion reinforcement was provided on the inner web face (the face with the ledge) by various configurations of WWR in combination with open L- and C-shaped reinforcing bars. Shear reinforcement was provided on the outer web face of these beams (the face without a ledge) by WWR alone. As the actions of torsion and shear were largely offset on the outer web face, outer-face torsion reinforcement was not provided in specimens LG2, LG3, or LG4. **Table 1** and **Fig. 2** show the different transverse web reinforcement configurations used for each beam.

Specimen LG1 was designed with conventional closed stirrups to serve as a control specimen for the testing program.

Table 1. Web reinforcement schemes

Test specimen	Web reinforcement details
LG1	Control beam with closed stirrups in the web and ledge
LG2	WWR on the inner and outer web face, no transverse reinforcing steel crossing the top or bottom surface of the beam, L-shaped reinforcing bars provide ledge-to-web attachment
LG3	WWR on the inner and outer web faces, U-shaped reinforcing bars cross the top of the web, L-shaped reinforcing bars provide ledge-to-web attachment
LG4	C-shaped bars are used to provide steel on the inner web face. The shorter legs of each C-shaped bar cross the top and bottom faces of the web. WWR is used to provide steel on the outer face.

Note: WWR = welded-wire reinforcement.

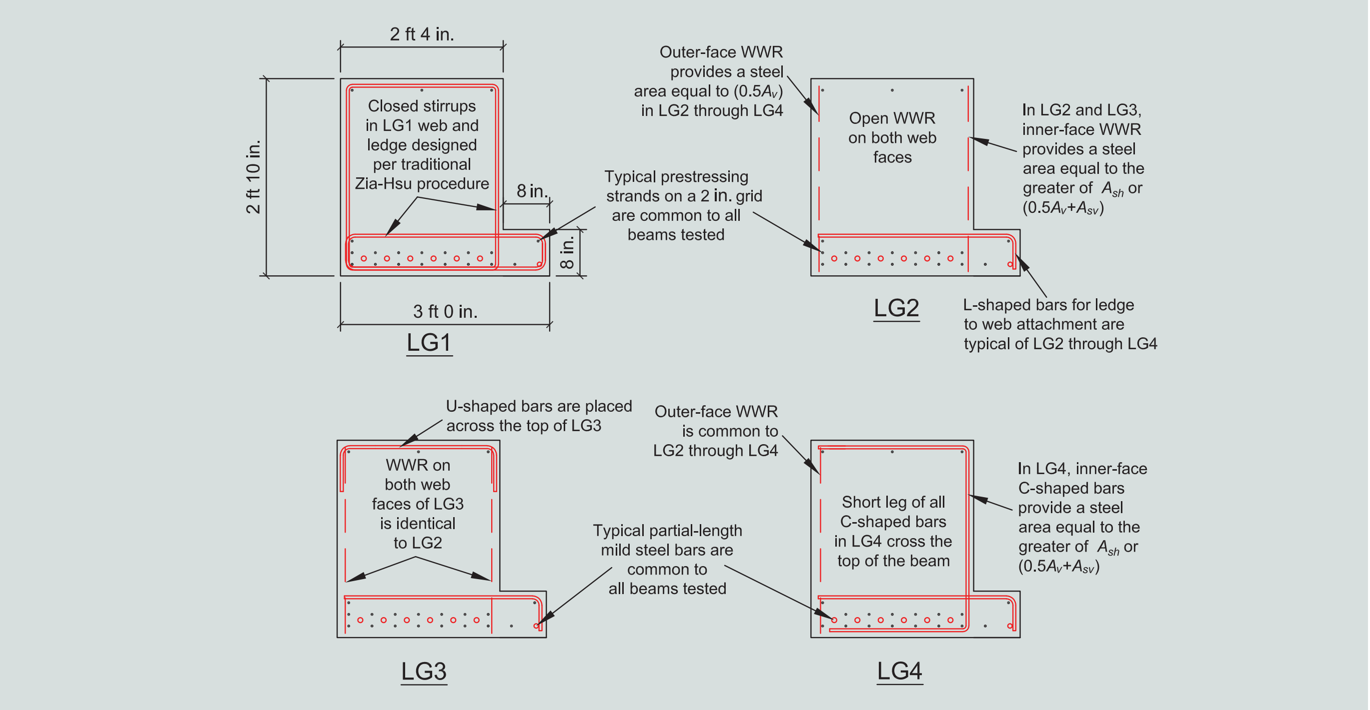


Figure 2. Cross sections showing web reinforcement details for tested beams. Note: Dimensions shown for LG1 are typical of all four beams. A_{sh} = required hanger steel; A_{sv} = total required area of steel crossing diagonal crack in vertical direction; A_v = area of shear reinforcement; WWR = welded-wire reinforcement. 1 in. = 25.4 mm; 1 ft = 0.305 m.

Closed stirrups were provided according to the approach recommended by Zia and Hsu,[10] as outlined in the *PCI Design Handbook*.[8] A total of 116 no. 3 (10M) closed stirrups were spaced along the web of LG1. Stirrups were spaced at 5 in. (130 mm) for a majority of the length and at 3 in. (75 mm) near the ends. The first 15 ft (4.6 m) of each end of LG1 had eight longitudinal no. 5 (16M) reinforcing bars placed around the perimeter of the section to meet the longitudinal torsion reinforcement requirement of the traditional Zia and Hsu design approach. As required by the design approach, this reinforcement was provided in addition to the reinforcement provided for flexure. These longitudinal bars provided for torsion were lapped with no. 5, U-shaped reinforcing bars at each end of the specimen to ensure development of the longitudinal torsion reinforcement at the ends of the beam. The ledge of specimen LG1 was reinforced with no. 3 closed stirrups spaced at 5 in. The ledge also included heavy welded reinforcements at each bearing point to prevent localized failure. These welded reinforcements were included on all four test specimens only for testing purposes.

Design of the web reinforcing steel in specimens LG2, LG3, and LG4 was based on two components of the applied torsion: a plate-bending component and a twisting component. The applied vertical shear was considered using conventional methods. First, vertical reinforcing steel was placed along the length of the web to satisfy requirements for shear reinforcement as specified in the *PCI Design Handbook*.[8] The vertical reinforcing steel required for shear was divided equally between the inner and outer web faces. Next, vertical reinforcing steel required to resist the plate-bending component of torsion was added to the inner web face in the end region. Additional longitudinal reinforcement was provided on the inner and outer web faces (in the form of U-shaped reinforcing bars) for plate bending. The reinforcement required for plate-bending was calculated using an assumed 45-degree failure plane. It was assumed that the twisting component of torsion would be resisted by the concrete section without considering the contribution of the reinforcement, and that failure would be controlled by twist. Additional details regarding the approach used to design the web reinforcement in these beams are provided in Lucier et al.[3] **Table 2** provides details of the web reinforcement provided in each beam.

The longitudinal reinforcing steel provided in the end regions of specimens LG2, LG3, and LG4 to resist plate bending was identical. Five U-shaped no. 5 (16M) reinforcing bars were placed longitudinally at each end of the web with the U shape in a horizontal plane. These reinforcing bars were placed at depths of 2, 12, 22, 30, and 32 in. (50, 300, 560, 760, and 810 mm) from the top surface of the beam. The top two U-shaped reinforcing bars were 4 ft 4 in. (1.32 m) long, and the bottom three U-shaped reinforcing bars were 2 ft 6 in. (0.76 m) long. A series of no. 3 (10M) L-shaped reinforcing bars spaced at 8 in. (200 mm) provided the reinforcement for ledge-to-web attachment for specimens LG2 through LG4, and a continuous sheet of WWR 6 × 6 W2.5 × W2.5 (150 × 150 MW16 × MW16) was provided for shear along the entire length on the outer web face (the face opposite the ledge) in these specimens.

The remaining web reinforcement was varied from beam to beam. Specimen LG2 was designed using a flat sheet of heavy WWR on the inner web face. WWR variable × 4 D4 × D10 (variable × 100 MD26 × MD65) with 4 in. (100 mm) spacing

<table>
<caption>**Table 2.** Details of reinforcement</caption>
<tr><th rowspan="2">Test specimen</th><th rowspan="2">Longitudinal web reinforcement for torsion at beam ends</th><th colspan="2">Vertical web reinforcement</th><th colspan="2">Reinforcement crossing top and bottom web surfaces</th></tr>
<tr><th>Inner face</th><th>Outer face</th><th>Top</th><th>Bottom</th></tr>
<tr><td>LG1</td><td>Eight no. 5 reinforcing bars × 15 ft lapped to four no. 5 U-shaped reinforcing bars × 2 ft 6 in.; three additional no. 5 × 2 ft 6 in. U-shaped reinforcing bars served to confine splitting stresses at the ends of the strands</td><td colspan="4">No. 3 closed stirrups at 3 in. for first 2 ft 0 in. each end, then at 5 in. for balance of span; area of reinforcement equals 0.44 in.^{2}/ft for first 2 ft 0 in. then 0.26 in.^{2}/ft for balance on each face (inner, outer, top, and bottom)</td></tr>
<tr><td>LG2</td><td rowspan="3">Two no. 5 × 4 ft 4 in. U-shaped reinforcing bars crossing the critical diagonal crack; three additional no. 5 × 2 ft 6 in. U-shaped reinforcing bars did not cross the diagonal crack and served to confine splitting stresses at the ends of the strands</td><td rowspan="2">WWR variable × 4 D4 × D10 provided 0.3 in.^{2}/ft vertical area of reinforcement plus WWR 6 × 6 W2.5 × W2.5 provided 0.05 in.^{2}/ft vertical area of reinforcement for the first 3 ft each end for a total vertical area of reinforcement of 0.35 in.^{2}/ft at the ends</td><td rowspan="3">WWR 6 × 6 W2.5 × W2.5 vertical area of reinforcement equals 0.05 in.^{2}/ft</td><td>None</td><td>None</td></tr>
<tr><td>LG3</td><td>No. 4 at 12 in. spacing, area of reinforcement equals 0.20 in.^{2}/ft</td><td>None</td></tr>
<tr><td>LG4</td><td>No. 4 C-shaped reinforcing bar at 6.5 to 8 in. spacing, area of reinforcement equals 0.37 to 0.30 in.^{2}/ft</td><td colspan="2">Short legs of inner face no. 4 C-shaped reinforcing bar at 6.5 to 8 in. spacing, area of reinforcement equals 0.37 to 0.30 in.^{2}/ft</td></tr>
<tr><td colspan="6">Note: LG2 and LG3 had additional longitudinal wires provided by inner face WWR that were not considered in the design. WWR 6 × 6 W2.1 × W2.1 = WWR 150 × 150 MW14 × MW14; WWR 6 × 6 W2.5 × W2.5 = WWR 150 × 150 MW16 × MW16; WWR variable × 4 D4 × D10 = WWR variable × 100 MD26 × MD65. WWR = welded-wire reinforcement. No. 3 = 10M; no. 4 = 13M; no. 5 = 16M; 1 in. = 25.4 mm; 1 ft = 0.305 m. 1 in.^{2}/ft = 2120 mm^2/m.</td></tr>
</table>

between vertical wires ran for the entire length of the beam. The continuous D4 × D10 WWR was supplemented by an additional piece of WWR 6 × 6 W2.5 × W2.5 for the first 3 ft (0.9 m) at each end of the specimen. All WWR extended the full depth of the web, and no reinforcement crossed over the top or bottom surfaces of the web in LG2. Specimen LG2 was used to evaluate the ability of the compact concrete cross section to resist the twisting component of torsion without reinforcement over the top and bottom surfaces of the web.

The reinforcement used for specimen LG3 was identical to that used in LG2 except that additional no. 4 (13M) U-shaped reinforcing bars were placed along the top of the web at 12 in. (300 mm) on center (Fig. 2). These U-shaped reinforcing bars were placed on top of the upper prestressing strands and hooked over the vertical WWR sheets on the faces of the web. This additional reinforcement across the top surface of the web was included to determine its effect on the torsional capacity and failure mode compared with the lack of top surface reinforcement in LG2.

The web of specimen LG4 was reinforced with a combination of WWR and conventional reinforcement. On the inner web face, no. 4 (13M) C-shaped reinforcing bars were provided at a spacing ranging from 6.5 to 8 in. (165 to 200 mm). The C-shaped reinforcing bars were placed so that the shorter legs of the C shape extended across the top and bottom of the web, fully developing the vertical leg of the reinforcing bar.

It should be noted that although the design of LG2, LG3, and LG4 followed the same principles developed by Lucier et al.[5] for slender L-shaped spandrel beams, there are some differences in the actual reinforcement detailing because the compact beams were designed before the guidelines for slender beams were finalized. The compact beams were fabricated with slightly more (about 10%) vertical web reinforcement than would be required had the approach for slender spandrel beams been followed. However, the compact beams had less longitudinal reinforcing steel (about 20% less) crossing the critical diagonal crack in the end region than would be required by the procedure for slender beams because the provided longitudinal reinforcing steel did not extend the full length of the end region.

Test setup

The compact L-shaped beams were tested to failure by applying loads to the beam ledge through the stems of short (12 ft [3.7 m] span) double-tee deck sections. Each L-shaped beam had a 45 ft (13.7 m) span and was supported by a 16 in. (410 mm) wide bearing pad centered with respect to the beam web. Teflon-coated bearing pads and stainless steel plates were used to remove as much friction as possible from the test setup. The main vertical reaction of each beam was measured at one end by two 200 kip (890 kN) load cells, also centered under the web. Using two load cells at this location not only measured the vertical reaction but also showed how the location of this reaction shifted with respect to the beam web during the test as the beam and bearing pads deformed.

Each L-shaped beam was supported laterally at both ends. Lateral strongbacks were provided by attaching a stiff steel beam (two channels) vertically to the inner face of the L-shaped beam through holes in the web that were precisely located during casting. The stiff steel beam extended above and below the top and bottom surfaces of the L-shaped beam. Threaded rods were used to connect the steel beam to supporting columns that were tied to the laboratory floor, providing torsional restraint to the beam web at the ends (**Fig. 3**). The forces in these threaded rods were measured using load cells.

Loads were applied to the L-shaped beam ledge by four 10 ft (3.0 m) wide double tees and one 5 ft (1.5 m) wide single tee (**Fig. 4**). All tee sections were 26 in. (660 mm) deep. Together, the double-tee and single-tee sections created a 45 ft (13.7 m) wide deck with a 12 ft (3.7 m) span. The tees were supported by the beam ledge at one end and by concrete support blocks on the other end. All deck sections were placed along the ledge of the beam with a 1 in. (25 mm) gap between the inner face of the web and the edge of the deck and were connected to the top of the L-shaped beam with a welded flange connection at midwidth of the deck section. Deck sections were not connected to adjacent deck sections (as would commonly be done in the field) to prevent the transfer of load between sections.

Hydraulic jacks and spreader beams were used to apply loads to the top surface of the double-tee and single-tee deck sections. The stems of the decks in turn applied load to the beam ledge representing the field condition. Teflon-coated bearing pads and stainless steel plates were used at each stem-to-ledge bearing location to reduce friction.

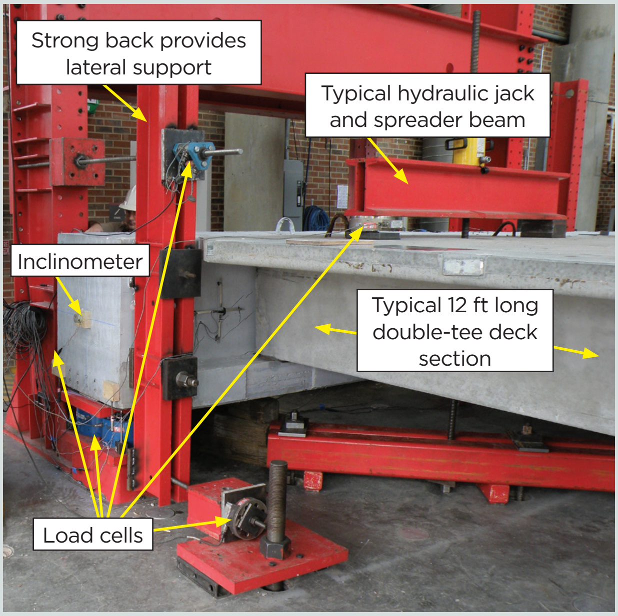


Figure 3. End region of a compact L-shaped beam in the test setup. Note: 1 ft = 0.305 m.

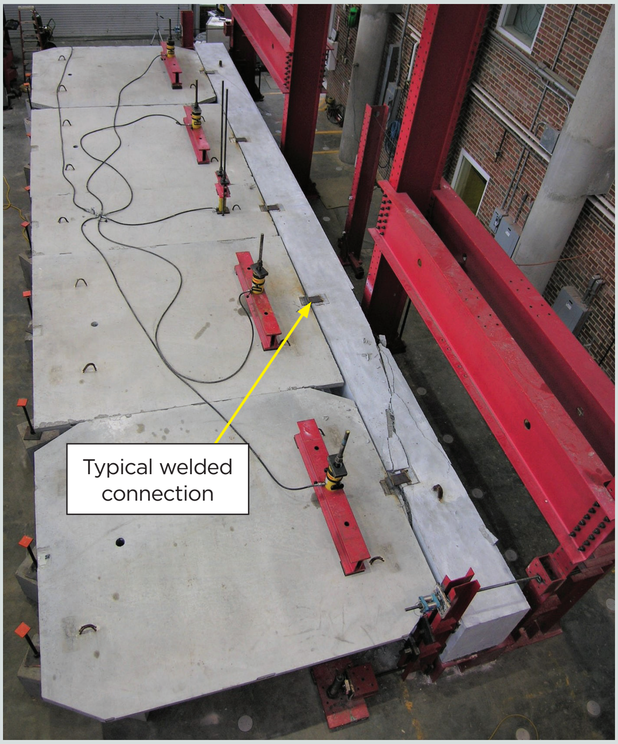


Figure 4. Overview of test setup.

Loading

Table 3 shows the design loads for the four L-shaped beams that were tested and the beam reactions for the selected load combinations based on the given design loads, respectively. The beams were designed and loaded as if they were supporting one end of a 45 ft (13.7 m) wide, 60 ft (18.3 m) span double-tee

Table 3. Selected load combinations for design and testing

Designation	Load combination	Beam reaction, kip
Dead load	DL	72.7
Service load	$DL + LL$	99.7
Reduced service load with snow	$1.0DL + 0.75LL + 0.75SL$	108.1
Service load with snow	$1.0DL + 1.0LL + 1.0SL$	120.0
Factored load	$1.2DL + 1.6LL + 0.5SL$	140.6
Nominal strength	$(1.2DL + 1.6LL + 0.5SL)/0.9$	156.6

Note: The self-weight of the 10DT26 deck was 71.6 lb/ft^2. The self-weight of the tested L-shaped beams was 1058 lb/ft. The live load considered was 40 lb/ft^2, and the snow load considered was 30 lb/ft^2. DL = dead load; LL = live load; SL = snow load. 1 kip = 4.448 kN. 1 lb/ft = 14.593 N/m; 1 lb/ft^2 = 47.880 N/m^2.

deck using 10DT26 sections. References to dead load within this testing program assume a full 60 ft span double-tee deck. Space limitations only allowed for a 45 ft wide, 12 ft (3.7 m) span double-tee deck to be used, so the hydraulic jacks were used to apply the extra dead load to simulate a longer span.

Load was applied to each beam in incremental cycles based on the load designations shown in Table 3. For each cycle, the specimen was loaded to the given level, observations were made, and the specimen was then unloaded. Load cycles were completed in this fashion up to the factored load level. Once the factored load was applied, it was then held on each beam for 24 hours. After completing the 24-hour sustained load test, the beam was unloaded and its recovery was monitored for 1 hour. At the end of the hour (provided the beam passed the ACI 318-14 chapter 27 recovery criterion), each spandrel beam was then loaded to failure in incremental cycles.

Instrumentation

Loads, strains, deflections, and rotations for each compact L-shaped beam test were measured using four types of instrumentation. Load cells measured the main vertical and lateral reactions and the loads being applied by the hydraulic jacks. Linear variable displacement transducers measured the vertical and lateral displacements of each L-shaped beam at several locations. Vertical displacement measurements were taken along the longitudinal centerline of the spandrel beam webs and at the intersection of the beam ledge and web at the beam midspan and quarter span. Vertical displacement measurements were also taken at one of the main vertical reactions for each spandrel beam to monitor support displacement. Lateral displacement at midspan and quarter spans was monitored at the top and bottom edges of the web. Similarly, lateral displacement was also monitored at the supports to record any support displacements.

Inclinometers measured the lateral rotation of each spandrel beam web at the midspan, quarter span, and support locations. Wire-arch clip gauges (pi gauges) measured concrete strains on the top, bottom, and inner faces of each spandrel beam in the end regions. Pi gauges also measured flexural concrete strains at the midspan and at one quarter-span location on the top and bottom surfaces of each member.

Summary of test results

All four test specimens carried their factored design loads for 24 hours, and all demonstrated ultimate capacities exceeding their respective factored design loads. In addition, all four specimens passed the 1-hour ACI 318-14 recovery criterion for the 24-hour load test for both the vertical and lateral deflections at the midspan. **Table 4** summarizes the maximum vertical reactions, measured vertical deflections at failure, and observed failure mode. The reactions represent the total force needed to support one end of the simply supported spandrel beam and include the self-weight of the L beam, double tees, and loading system.

Lateral reactions acting directly on the beam web (at the through holes) measured at the peak vertical reaction were determined from measurements made by the load cells included in the lateral restraint system.

The test for control specimen LG1 was stopped when the main vertical reaction reached approximately 220 kip (980 kN) to prevent damage to the loading system. End-region failure was not reached in the control specimen LG1, although extensive diagonal cracking was observed. The other three specimens ultimately exhibited end-region failure modes at vertical reactions of approximately 220 kip.

Cracking patterns

The cracking observed at the service load was minimal in all four beams. The inner face of all beams remained uncracked at the service level while some light flexural cracking was observed on the outer face of all beams near the midspan.

The cracking pattern observed up to failure was consistent for all four specimens and was similar to the pattern observed by others in slender L-shaped spandrel beams (Logan,[6] Klein,[12]

Table 4. Summary of test results

Test specimen	Web reinforcement	Maximum vertical reaction, kip	Maximum lateral reactions, kip		Midspan vertical deflection, in.	Description of failure mode
			Top left / Bottom left	Top right / Bottom right		
LG1	Closed stirrups anchored by 135-degree hooks	220.5	70.0 / 75.2	78.8 / 67.5	4.73	Beam did not fail. The test was stopped to avoid damage to the loading system. Spandrel beams showed diagonal cracking on the inner face and flexural cracking on the outer face.
LG2	Flat sheets of WWR only	220.1	89.6 / 97.6	98.9 / 103.5	4.90	Failure in the end region along a diagonal plane with smaller skew angle. Extensive diagonal cracking was observed on the inner face and moderate flexural cracking on the outer face.
LG3	Flat sheets of WWR plus U-shaped reinforcing bars	220.0	75.8 / 83.1	86.1 / 97.2	4.06	Failure in the end region along a diagonal plane with smaller skew angle. Cracking similar to LG2.
LG4	C-shaped reinforcing bars inner face, WWR outer face	220.3	94.1 / 107.1	84.3 / 85.1	6.19	Failure in the end region along a diagonal plane with larger skew angle. Cracking similar to LG2.

Note: WWR = welded-wire reinforcement. 1 in. = 25.4 mm; 1 kip = 4.448 kN.

Raths,[13] and Lucier et al.[5]). Such a cracking pattern indicates an interaction between torsion and shear stresses in an L-shaped beam. On the inner web face, the torsion and shear stresses act in the same direction, creating a high diagonal tension demand in the end regions. On the outer web face, the vertical shear stress counteracts the stresses developed in torsion. Thus, the diagonal tension demand on the outer web face is reduced and could even be opposite to the demand on the inner face depending on the relative magnitudes of the shear and torsional stresses.

Cracks on the inner face inclined upward from the support at an initial angle of approximately 45 degrees (**Fig. 5** and **6**). Moving away from the supports, the crack angle flattened and the inner face cracks began to propagate toward midspan. In addition, vertical flexural cracks extended upward from the bottom of the beam near midspan. Crack widths observed in the end regions of the beam with traditional closed stirrups (LG1) were not as wide as they were in beams with open web reinforcement, but there were more cracks. The cracking pattern observed for all beams was similar.

One important observation of the cracking pattern at the inner face for all beams was the tendency for a diagonal crack extending upward from the support to cross the top web surface at a skew, reflecting the effect of torsion. (**Fig. 7**). While the inner face cracking near the support exhibited an initial angle of about 45 degrees for all beams, the angle of the critical diagonal crack and the angle at which that crack skewed across the top web surface varied according to the quantity of reinforcement crossing the top web surface. Beams with little or no top reinforcement had cracks with

Figure 5. Inner face cracking observed after failure for a control compact L-shaped beam with closed web reinforcement (LG1). Note: Cracks are digitally enhanced.

Figure 6. Inner face cracking observed after failure for a compact L-shaped beam with open web reinforcement (LG4). Note: Cracks are digitally enhanced.

smaller skew angles than beams with more top reinforcement. The critical crack angle observed for beams LG1 and LG4 was approximately 45 degrees. Both beams had relatively high quantities of reinforcing steel crossing the top web surface. The critical crack and skew angles observed for beam LG2, having no top reinforcement, were closer to 32 degrees. The skew angle observed for beam LG3, having an intermediate amount of top reinforcement, was approximately 40 degrees.

The observed patterns of outer face cracking were also similar for all four specimens and matched the general pattern observed for slender L-shaped spandrel beams. Cracking on the outer face was minimal near the supports. Vertical cracks extending up from the bottom of the beam (due to vertical and lateral flexure) were observed near the midspan of all beams (**Fig. 8**). Between the end of the beam and the midspan, a limited number of inclined cracks were observed, indicating the effect of shear.

Failure modes

End-region failure modes were observed for three of the four compact L-shaped beams tested. Specimen LG1, the beam with closed stirrups, did not fail in the end region because the test was stopped to avoid damaging the testing system. In considering the end-region failure modes observed in the other three beams (LG2, LG3, and LG4), it is important to recall that all beams in this testing program had extra flexural and ledge reinforcement to prevent premature flexure or ledge punching failure. If any of these four beams had been reinforced with normal levels of flexural or ledge reinforcement,

LG1 (closed stirrups)

LG2 (no top steel)

LG3 (U-shaped reinforcing bars across the top)

LG4 (C-shaped inner-face reinforcement)

Figure 7. Diagonal cracks crossing the top web surface.

Figure 8. Typical outer face cracking in a compact L-shaped beam (LG1). Note: Cracks are digitally enhanced.

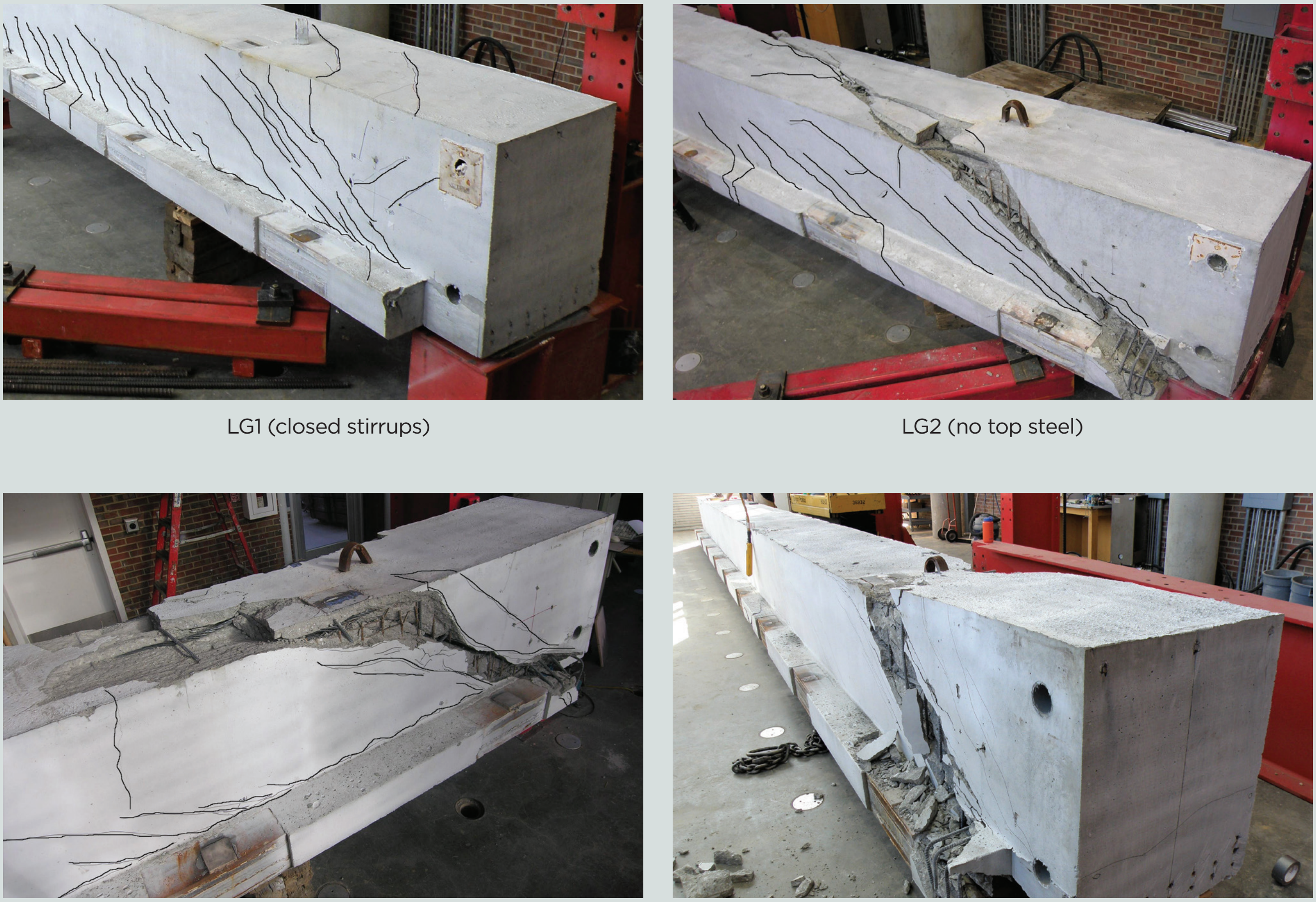
then flexural or ledge failure would have controlled the behavior long before end-region failures could be observed.

Figure 9 shows the observed failure modes for the four tested beams. The observed failure for LG2 and LG3 was skew bending about a critical diagonal crack extending upward from one support. When this crack reached the top surface of the web, it continued across the top of the beam at a skew. **Figure 10** shows the skewed crack plane and the separated failure surface of LG2 after testing.

The observed failure for beam LG4 also occurred in an end region; however, the skewed LG4 failure plane extended along the beam for a length much shorter than the lengths of the skewed failure planes of LG2 and LG3. For all specimens, a critical diagonal crack extended upward from one support and crossed over the top of the web. The failure plane for specimen LG4 crossed the top of the web in a direction nearly normal to the web face, compared with the highly skewed nature of the LG2 and LG3 failure planes. The difference between the LG2 and LG3 failure modes and the LG4 failure mode can be seen by comparing the outer web faces of the specimens after testing in **Fig. 11**.

Measured deflections

Figure 12 shows that the measured vertical load-deflection behavior at midspan for all four specimens was similar. The behavior of beams LG1, LG2, and LG3 was nearly identical through the 220 kip (980 kN) load level. The behavior of beam LG4 was similar to that of the other three beams; however, LG4 was less stiff at all load levels. The vertical load-deflection curves show the various loading cycles for each beam. Residual deflections can be observed at the end of each of these cycles, with subsequent cycles continuing from the



LG1 (closed stirrups)

LG2 (no top steel)

LG3 (U-shaped reinforcing bars across the top)

LG4 (C-shaped inner-face reinforcement)

Figure 9. Inner-face views of compact L-shaped beams after failure.

Figure 10. View of LG2 with separated failure surface (outer web face).

residual values. Horizontal plateaus in the load-deflection data correspond with the applied load being held to make observations and to mark cracks. The large plateau at the factored load level corresponds to the 24-hour sustained load test. It is important to note that the zero deflection reading occurs at a vertical reaction of approximately 37 kip (165 kN), a level equal to the self-weight of the beam and loading system in the laboratory but less than the full design dead load reaction.

Horizontal deflections were also measured at the top and bottom edge of the web at midspan for each tested beam. For all beams, the measured data indicate a tendency for the compact L-shaped beams to move outward (toward the applied load) at midspan, more so at the bottom of the beam than at the top. Thus, as load is applied to the ledge, the compact beam rotates toward the applied load but is restrained at its top surface by the welded deck connections. **Figure 13** shows a typical vertical load-lateral deflection curve for the top and bottom lateral deflections measured at midspan of specimen LG3.

Analysis for twisting resistance

The results from LG2, LG3, and LG4 can be evaluated and compared with the design procedure for slender L-shaped spandrel beams previously developed.[5] The slender spandrel beam design procedure independently considers three actions in the end region of an L-shaped beam: vertical shear, the plate-bending component of torsion, and the twisting component of torsion. The procedure predicts that the strengths of specimens LG2, LG3, and LG4 are all controlled by the twisting capacity.

The resistance of the concrete cross section to the twisting component of torsion, according to the slender spandrel beam design procedure,[5] is given by Eq. (1).

$$T_u \leq \phi_s\left(1.13\sqrt{f_c'}\,d_w h^2\right) \tag{1}$$

where

T_u = factored torsion demand

ϕ_s = strength reduction factor = 0.75

f_c' = average concrete compressive strength

d_w = effective web thickness

h = height of the web

For specimens LG2, LG3, and LG4, f_c' is 8700 psi (60 MPa), d_w is 26.25 in. (666.8 mm), and h is 34 in. (860 mm). The factored torsional strength, as predicted by Eq. (1), is 2399 kip-in. (271 kN-m). The nominal torsional strength $\frac{T_u}{\phi_s}$ is therefore equal to 3198 kip-in. (361 kN-m).

All three specimens—LG2, LG3, and LG4—failed at a spandrel beam end reaction of approximately 220 kip (980 kN). Subtracting the 24.3 kip (108 kN) concentric self-weight of the beam itself from this reaction results in 195.7 kip (870.5 kN) of eccentrically applied ledge load at failure. With a load eccentricity of 20 in. (510 mm), the tested spandrel beams carried an applied torque of 3914 kip-in. (442 kN-m) at failure, which exceeds the predicted nominal torsional strength of 3198 kip-in. (361 kN-m) by 22%. Thus, the design provisions for the twist component developed for the slender spandrel beams are conservative when applied to the compact spandrel beams tested in this study. The failure of the three

LG3 (LG2 similar)

LG4

Figure 11. Outer-face views of compact L-shaped beams after failure.

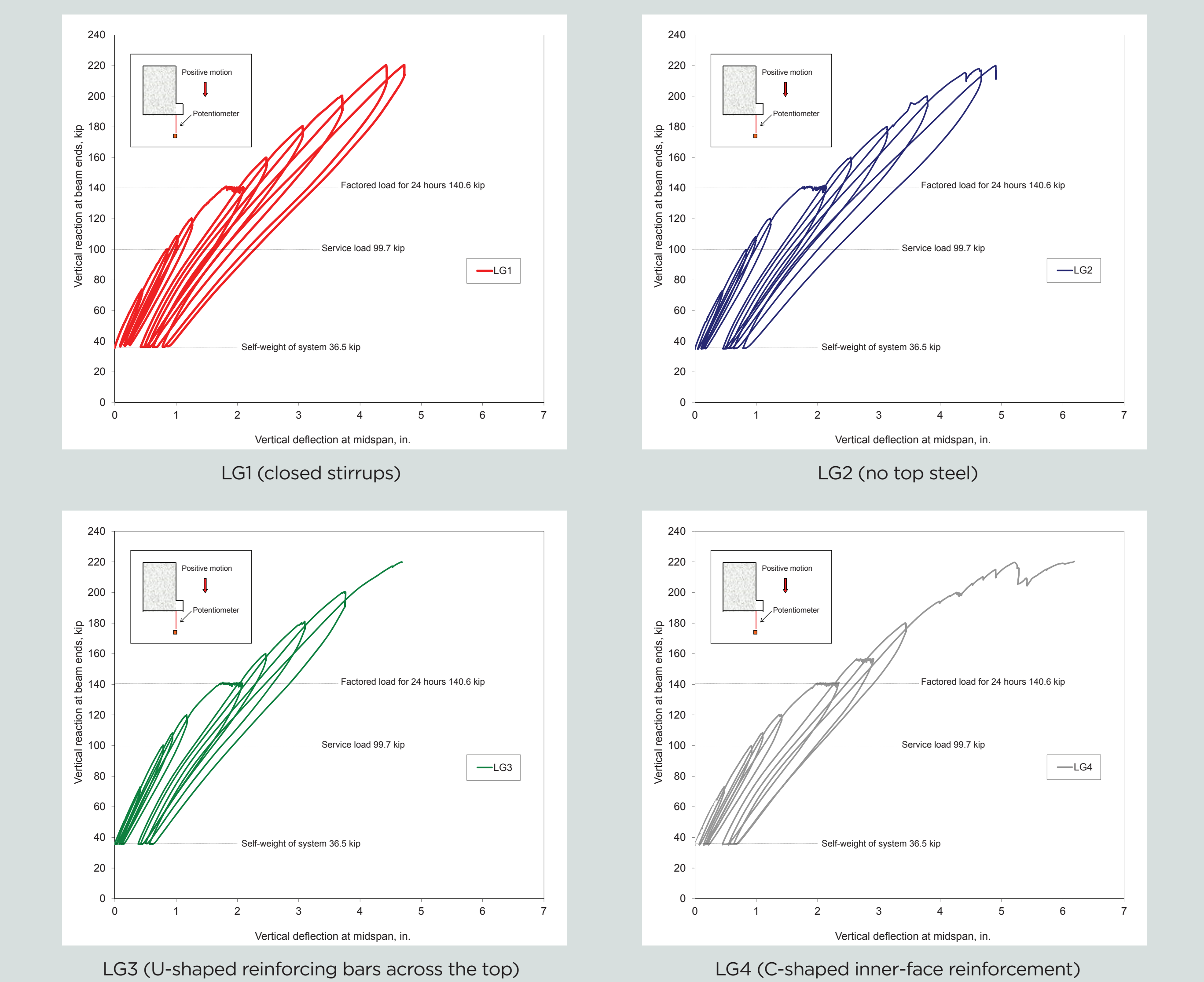


Figure 12. Measured vertical load-deflection data for individual beams at midspan. Note: 1 in. = 25.4 mm; 1 kip = 4.448 kN.

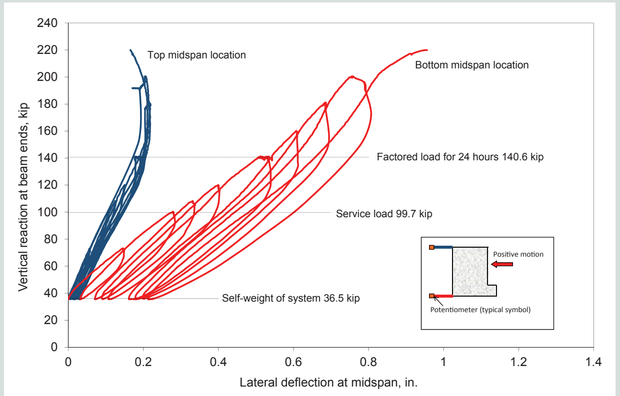


Figure 13. Typical measured vertical load–lateral deflection at midspan (LG3). Note: 1 in. = 25.4 mm; 1 kip = 4.448 kN.

specimens was controlled by twist or yielding of the transverse reinforcement on the inside face.

Discussion

In evaluating the observed end-region failures of LG2, LG3, and LG4, note that the design service load level end reaction for each beam was approximately 100 kip (440 kN). The test of LG1 was stopped at an end reaction of approximately 220 kip (980 kN), and the other three specimens failed in their end regions at end reactions of approximately 220 kip. All of the specimens carried more than twice the service load. Other failure modes (probably ledge punching) would have caused beam failure at a reduced strength in all four cases if additional flexure and ledge reinforcement had not been provided.

Based on the failure modes and test data, it is clear that the performance of the end region reinforced with closed stirrups and traditional torsional detailing (LG1) was superior to the performance of the end regions of the other three specimens with open reinforcement. There is no doubt that closed stirrups and longitudinal torsion reinforcement provide excellent crack control and strong end regions, especially at higher load levels. What is debatable, however, is whether the expense and complexity of a closed reinforcement scheme can be justified given that the service load behavior was identical for specimens with open and closed reinforcement and that the strength of all end regions far exceeded the capacity of other failure modes (flexure or ledge punching) that would develop in beams without special enhancements and would typically control the beam capacity.

The ACI 318-14 requirements for torsion detailing with closed ties are based on beams tested mostly in pure torsion. None of the compact beams tested exhibited classical torsional behaviors; that is, spalling of the concrete cover was not observed in any of the beams nor was a distinctive spiral cracking pattern. Rather, the behavior of the compact L-shaped sections was similar to that previously observed in slender L-shaped spandrel beams.[1] The distinctive tied-arch-type cracking pattern, with torsional and shear stresses acting together on the inner web face and acting to oppose one another on the outer web face, was evident in all four of the tested beams.

The end-region failures observed in the compact beams were in the form of combined shear and torsion. Comparing the failure loads with the predicted twist resistance capacity of the compact section indicates that the twist resistance based on the previously proposed procedure for slender spandrel beams[5] is conservative for the tested compact sections and that the proposed procedure is applicable for both cases. It should be noted, however, that postfailure analysis of the available experimental results indicates that the concrete contribution to shear resistance V_c was not reliable above a level equal to $2.5\sqrt{f_c'}$. As such, this level of concrete contribution to shear resistance should be considered as an upper limit.

The cracking patterns and failure modes observed in all four tests indicate that while web reinforcement on the inner and outer web faces is critical in an L-shaped section, web reinforcement crossing the top surface of the beam also influences behavior. The skewed angle at which a crack crosses the top surface of a compact L-shaped beam is strongly influenced by the quantity and spacing of top surface reinforcing steel. The angle of the critical diagonal skewed-bending crack was observed to be approximately 45 degrees when a relatively large amount of reinforcing steel was provided across the top surface of the beam. The angle of this crack flattened to approximately 30 degrees when no reinforcing steel crossed the top surface. Because the design procedure for open reinforcement assumes a crack angle of 45 degrees in the end region, top reinforcing steel would be required to ensure that the critical diagonal crack develops at the assumed angle. In addition, the presence of top reinforcing steel provides additional benefits, such as control of crack widths and enhanced aggregate interlock. Top reinforcing steel should be included in the design of compact sections and should be spaced at a center-to-center distance of no more than half of the web thickness to be effective.

Additional study and further testing would help to better understand how the beam behavior is influenced by top reinforcing steel.

Conclusion

Based on the results of this investigation, the following conclusions are drawn:

- The behavior of the four test specimens was similar up to the 220 kip (980 kN) end reaction level. The beam behavior was satisfactory at all loading conditions, including load levels well above the factored load condition. The specimens also easily met the recovery criteria prescribed by chapter 27 of ACI 318-14 following a 24-hour sustained load test at the factored load level.

- Specimen LG1 had traditional closed stirrups and a larger amount of reinforcing steel than the other specimens and was observed to have the highest end-region strength of all beams tested. End-region failure of this beam could not be observed because testing was stopped before failure to prevent damage to the loading system.

- The failure load of each test specimen exceeded the factored design load, the nominal load, and the nominal torsional strength by substantial margins.

- The failure mode of the compact L-shaped specimens with open web reinforcement was combined shear and torsion along a skewed diagonal plane, similar to the slender L-shaped beams studied in previous investigations.[1,5,6,12,13] The spiral cracking and face-shell spalling associated with members loaded in pure torsion and the torsion design methods in ACI 318-14 were not observed. Spiral cracking and face-shell spalling is not expected for members with open web reinforcement.

- These tests confirm the validity of designing for torsion by considering the applied torque as two independent orthogonal components (bending and twisting), thus greatly simplifying torsion design and detailing. Analysis of the test data using the design equations developed previously for slender L-shaped beams indicated that the equations are applicable to compact L-shaped beams.[5]

- Comparing the failure loads with the predicted twist resistance capacity of the compact section indicated that the twist resistance model proposed for slender spandrel beams is applicable and conservative for compact spandrel beam sections.

- The tests also confirmed that performance of compact spandrel beams with open and segmented web reinforcement was satisfactory, as in the case of slender L-shaped spandrel beams. Using these alternative details greatly simplifies placement of the reinforcement.

- The design procedure used for transverse reinforcement assumes a crack angle of 45 degrees in the end region across the top of the beam.[5] A minimum amount of reinforcing steel should cross the top of the beam web to avoid cracks at shallow angles along the top of the section, as observed in LG2, which had no reinforcing steel across the top of the section. Top reinforcing steel provides additional benefits, such as control of crack widths and enhanced aggregate interlock, so it should be included in the design of a compact section. Top reinforcing steel may be provided as inverted U-shaped reinforcing bars (as with LG3) or as the top leg of C-shaped reinforcing steel (as with LG4) and should be spaced at a center-to-center distance of no more than half of the web thickness to be effective. The recommended minimum reinforcement across the top of the beam is no. 3 (10M) reinforcing bars at 6 in. (150 mm) center-to-center spacing or equivalent. Additional study and further testing are recommended to better understand how the failure mode is influenced by top reinforcing steel and to provide better guidance on the amount of reinforcing steel required.

- If compact L-shaped beams are designed for torsion assuming a skewed diagonal failure plane, then an upper limit of $2.5\sqrt{f'_c}$ should be considered for the concrete contribution to shear capacity.

In conclusion, compact L-shaped beams with alternative transverse reinforcement schemes demonstrate adequate performance when compared to similar beams with closed stirrups that comply with ACI 318-14. A nominal amount of reinforcement across the top of the section is recommended to control cracking.

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Notation

A_{sh} = required hanger steel

A_{sv} = total required area of steel crossing diagonal crack in vertical direction

A_v = area of shear reinforcement

d_w = effective depth from outer surface of web to centroid of combined horizontal and vertical steel reinforcement of web, usually taken as web thickness less concrete cover less diameter of inner face vertical steel bars

D_L = dead load

f'_c = specified concrete compressive strength

h = height of web

L_L = live load

S_L = snow load

T_u = factored torque

T_{ub} = plate-bending component of torsion

T_{ut} = twisting component of torsion

V_c = nominal concrete shear strength as given by ACI 318-14

ϕ_s = strength reduction factor for shear = 0.75

θ = critical diagonal crack angle for region under consideration with respect to horizontal: 45 degrees for end regions, 30 degrees for transition regions, 0 degrees for flexure regions

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Abstract

Open web reinforcement has been shown to be an effective alternative to closed stirrups in the webs of slender precast concrete L-shaped spandrel beams subjected to combined shear and torsion.[1–6] For slender beams, an open reinforcement scheme is a better alternative to the traditional closed stirrups mandated by the American Concrete Institute's (ACI's) *Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)*[7] because the beams are easier to produce with open reinforcement. Although the behavior of slender L-shaped beams (having aspect ratios of 4.5 or greater) with open web reinforcement has been well documented, the use of alternatives to closed stirrup reinforcement for compact L-shaped cross sections having aspect ratios much less than 4.5 has not been investigated previously.

This paper presents an experimental study in which four full-scale, 46 ft (14 m) long, precast concrete, compact L-shaped beams were tested to failure. One of the test specimens served as a control and was designed with traditional closed stirrups. The remaining three beams were designed with alternative open and segmented reinforcement configurations.

The results of the study demonstrate the viability of replacing closed stirrups with alternative open and segmented web reinforcement in compact L-shaped spandrel beams. All four beams behaved satisfactorily at all loading stages and failed at loads much greater than the factored design loads. When failure occurred in the end region of the compact L-shaped beams with open reinforcement, no spiral cracking or face-shell spalling were observed, which is contrary to the failure mode associated with the torsion design concept of ACI 318-14.[7] Rather, the observed failure planes in the compact L-shaped beams were similar to those observed in slender L-shaped beams, with skewed bending along a diagonal crack extending upward from the support. The results confirm the potential to simplify the design and detailing of compact L-shaped beams by using alternative transverse reinforcement details proportioned with a design approach based on failure in combined shear and torsion.

Keywords

Compact L-shaped beam, open reinforcement, skew bending, torsion, twist.

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