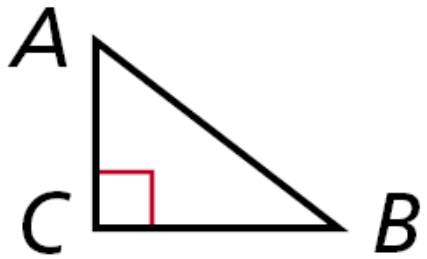
4-6 Triangle Congruence: ASA, AAS, and HL

Bellringer:

1. What are sides AC and BC called? Side AB?

legs; hypotenuse



2. Which side is in between $\angle A$ and $\angle C$?

$\overline{AC}$

3. Given ΔDEF and ΔGHI, if $\angle D \cong \angle G$ and $\angle E \cong \angle H$, why is $\angle F \cong \angle I$?

Third $\angle$s Thm.

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Objectives

Apply ASA, AAS, and HL to construct triangles and to solve problems.

Prove triangles congruent by using ASA, AAS, and HL.

Vocabulary

included side

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Participants in an orienteering race use a map and a compass to find their way to checkpoints along an unfamiliar course.

Directions are given by bearings, which are based on compass headings. For example, to travel along the bearing S 43° E, you face south and then turn 43° to the east.

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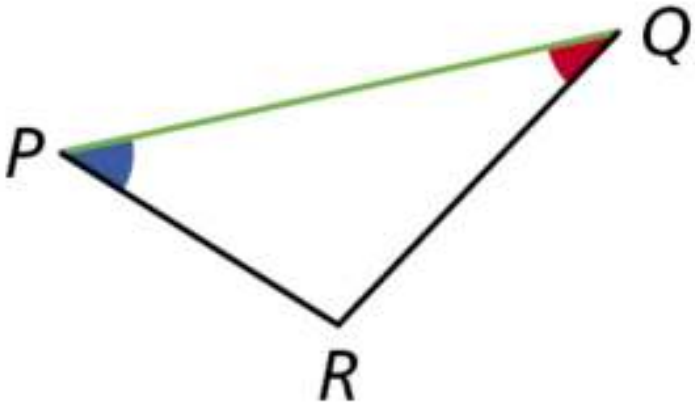
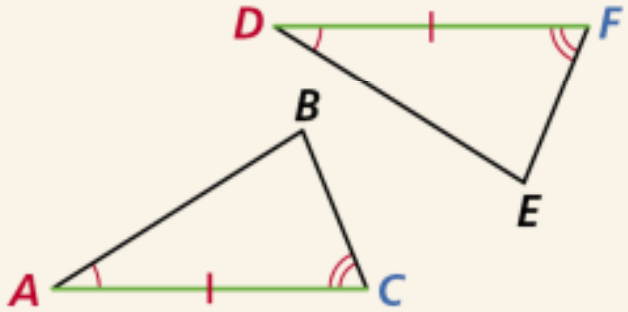
An **included side** is the common side of two consecutive angles in a polygon. The following postulate uses the idea of an included side.



$\overline{PQ}$ is the included side of $\angle P$ and $\angle Q$.

Postulate 4-5-1 Angle-Side-Angle (ASA) Congruence

POSTULATE	HYPOTHESIS	CONCLUSION
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.		$\triangle ABC \cong \triangle DEF$

Example 1: Problem Solving Application

A mailman has to collect mail from mailboxes at A and B and drop it off at the post office at C. Does the table give enough information to determine the location of the mailboxes and the post office?

	Bearing	Distance
A* to *B	N 65° E	8 mi
B* to *C	N 24° W	
C* to *A	S 20° W	

1 Understand the Problem

The **answer** is whether the information in the table can be used to find the position of points *A*, *B*, and *C*.

List the important information: The bearing from *A* to *B* is N 65° E. From *B* to *C* is N 24° W, and from *C* to *A* is S 20° W. The distance from *A* to *B* is 8 mi.

2 Make a Plan

Draw the mailman's route using vertical lines to show north-south directions. Then use these parallel lines and the alternate interior angles to help find angle measures of ΔABC.

3 Solve

$m\angle CAB = 65° - 20° = 45°$

$m\angle CAB = 180° - (24° + 65°) = 91°$

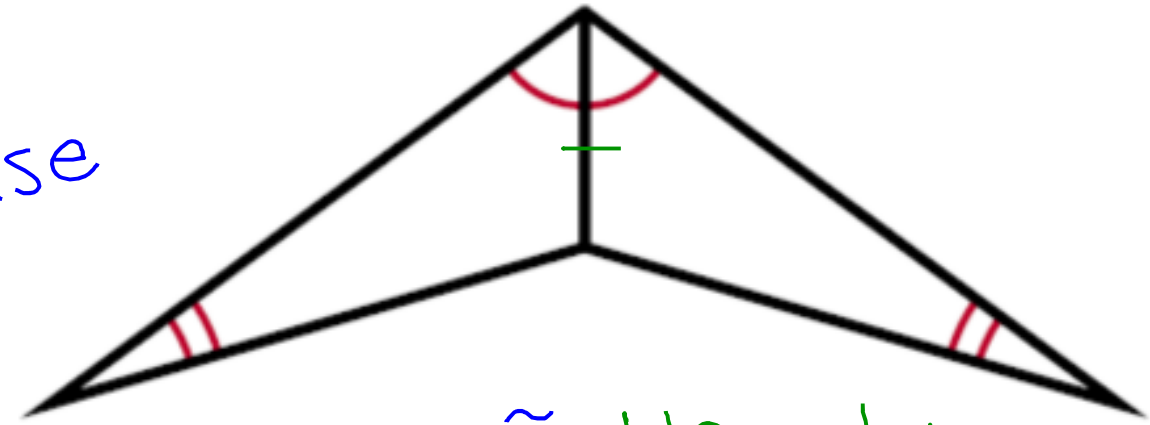
You know the measures of $m\angle CAB$ and $m\angle CBA$ and the length of the included side $\cancel{AB}$. Therefore by ASA, a unique triangle ABC is determined.

4 Look Back

One and only one triangle can be made using the information in the table, so the table does give enough information to determine the location of the mailboxes and the post office.

Example 2: Applying ASA Congruence

Determine if you can use ASA to prove the triangles congruent. Explain



We cannot use ASA b/c we do not know that the included sides are $\cong$. We only know the angles marked and the shared side

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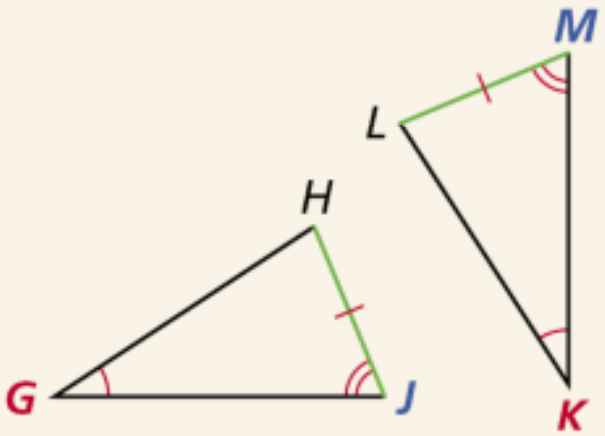
You can use the Third Angles Theorem to prove another congruence relationship based on ASA. This theorem is Angle-Angle-Side (AAS).

Theorem 4-5-2 Angle-Angle-Side (AAS) Congruence

THEOREM	HYPOTHESIS	CONCLUSION
If two angles and a nonincluded side of one triangle are congruent to the corresponding angles and nonincluded side of another triangle, then the triangles are congruent.		$\triangle GHJ \cong \triangle KLM$

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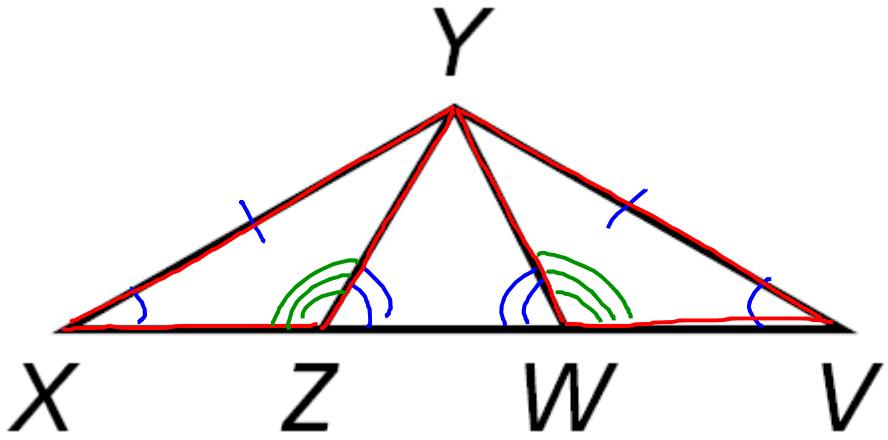
Example 3: Using AAS to Prove Triangles Congruent

Use AAS to prove the triangles congruent.

Given: $\angle X \cong \angle V$, $\angle YZW \cong \angle YWZ$, $\overline{XY} \cong \overline{VY}$

Prove: $\Delta XYZ \cong \Delta VYW$



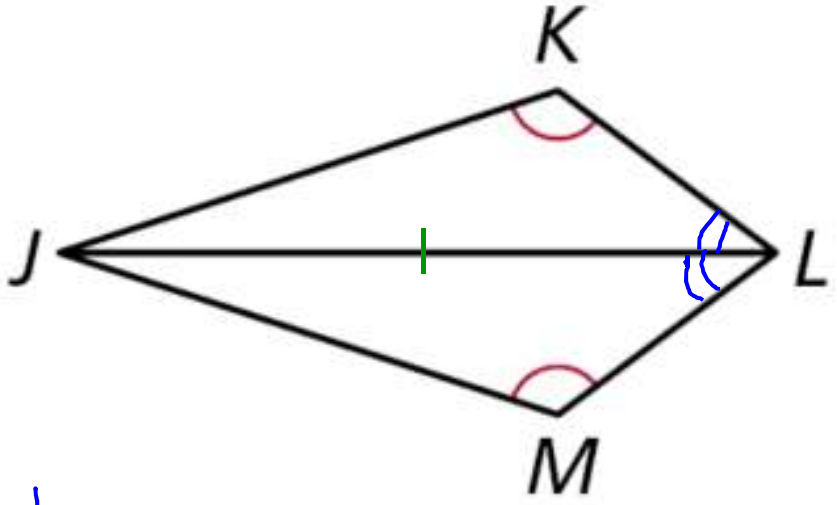
Statements	Reasons
1. $\angle X \cong \angle V$, $\angle YZW \cong \angle YWZ$, $\overline{XY} \cong \overline{VY}$	1. given
2. $\angle XZY \cong \angle VWY$	2. b/c they make a linear pair w/ congruent $\angle$s
3. $\Delta XYZ \cong \Delta VYW$	3. AAS

Check It Out! Example 3

Use AAS to prove the triangles congruent.

Given: $\overline{JL}$ bisects $\angle KLM$, $\angle K \cong \angle M$

Prove: $\Delta JKL \cong \Delta JML$



Statements	Reasons
1. $\overline{JL}$ bisects $\angle KLM$, $\angle K \cong \angle M$	1. given
2. $\angle KLJ \cong \angle MLJ$	2. def. of bisect
3. $\overline{JL} \cong \overline{JL}$	3. reflexive prop.
4. $\Delta JKL \cong \Delta JML$	4. AAS

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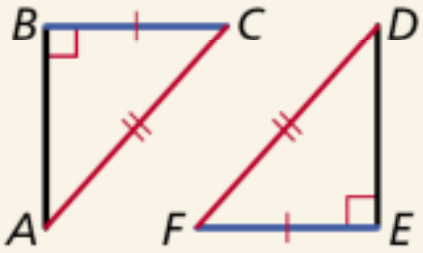
Theorem 4-5-3 Hypotenuse-Leg (HL) Congruence

THEOREM	HYPOTHESIS	CONCLUSION
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.		$\triangle ABC \cong \triangle DEF$

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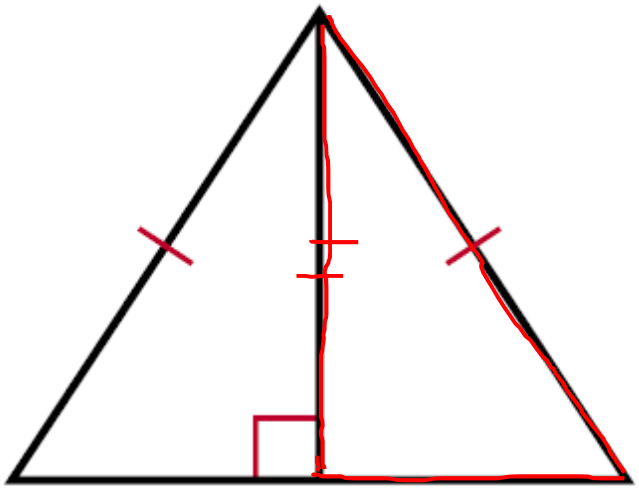
Example 4: Applying HL Congruence

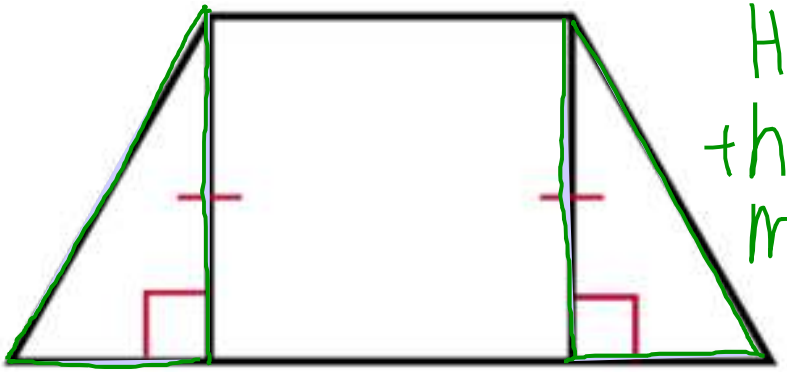
Determine if you can use the HL Congruence Theorem to prove the triangles congruent. If not, tell what else you need to know.

A.



HL can be used b/c the hypotenuses are marked congruent and there is a side (leg) that is shared.

B.



HL cannot be used b/c the hypotenuses are not marked congruent.