Bending Analysis of Castellated Beams

By Sahar Elaiwi[*]
Boksun Kim[‡]
Long-Yuan Li[†]

Existing studies have shown that the load-carrying capacity of castellated beams can be influenced by the shear stresses particularly those around web openings and under the T-section, which could cause the beam to have different failure modes. This paper investigates the effect of web openings on the transverse deflection of castellated beams by using both analytical and numerical methods and evaluates the shear-induced transverse deflection of castellated beams of different lengths and flange widths subjected to uniformly distributed transverse load. The purpose of developing analytical solutions, which adopted the classical principle of minimum potential energy is for the design and practical use; while the numerical solutions are developed by using the commercial software ANSYS for the validation of the analytical solutions.

Keywords: *Castellated Beam, Deflection, Energy Method, Finite Element, Shear Effect.*

Introduction

Engineers and researchers have tried various methods to reduce the material and construction costs to help optimise the use of the steel structural members. The castellated beam is one of the steel members which uses less material, but has comparable performance as the I-beam of the same size (Altifillisch et al. 1957). An example is shown in Figure 1a. The castellated beam is fabricated from a standard universal I-beam or H-column by cutting the web on a half hexagonal line down the centre of the beam. The two halves are moved across by a half unit of spacing and then re-joined by welding. This process increases the depth of the beam and thus the bending strength and stiffness of the beam about the major axis are also enhanced without additional materials being added. This allows castellated beams to be used in long span applications with light or moderate loading conditions for supporting floors and roofs. In addition, the fabrication process creates openings on the web, which can be used to accommodate services. As a result, the designer does not need to increase the finished floor level. Thus, despite the increase in the beam depth the overall building height may actually be reduced.

When compared with a solid web solution where services are provided beneath the beam, the use of castellated beams could lead to savings in the cladding costs especially in recent years, the steel cost becomes higher. Owing to the fact that the steel materials have poor fire resistance, buildings made from steel

[*]PhD Student, Plymouth University, UK.
[‡]Plymouth University, UK.
[†]Plymouth University, UK.

structures require to use high quality fireproof materials to protect steel members from fire, which further increase its cost. Moreover, because of its lightweight the castellated beam is more convenient in transportation and installation than the normal I-beam.

Literature Review

For many years, the castellated beam have been used in construction because of its advantages when considering both the safety and serviceability while considering functional requirements according to the use for which the construction is intended. Extensive study has been done by researchers who are working in the construction field to identify the behaviour of castellated beams when they are loaded with different types of loads. It was found that the castellated beam could fail in various different modes depending on the dimensions of the beam and the type of loading as well as the boundary conditions of the beams. Kerdal and Nethercot (1984) informed the potential failure modes, which possibly take place in castellated beams. Also, they explained the reasons for the occurrence of these failure modes. For instance, shear force and web weld rupture cause a Vierendeel mechanism and web post-buckling. Additionally, they pointed out that any other failures whether caused by a flexural mechanism or a lateral-torsional instability is identical to the equivalent modes for beams without web opening.

The web openings in the castellated beam, however, may reduce the shear resistance of the beam. The saved evidence, that the method of analysis and design for the solid beam may not be suitable for the castellated beam (Boyer 1964, Kerdal and Nethercot 1984, Demirdjian 1999). Design guidance on the strength and stiffness for castellated beams is available in some countries. However, again, most of them do not take into account the shear effect. As far as the bending strength is concerned, neglecting the shear effect may not cause problems. However, for the buckling and the calculation of serviceability, the shear weakness due to web openings in castellated beams could affect the performance of the beams and thus needs to be carefully considered.

Experimental investigations (Aminian et al. 2012, Maalek 2004, Yuan et al. 2014, Yuan et al. 2016, Zaarour and Redwood, 1996) were carried out and finite elements methods (Hosain et al. 1974, Sherbourne and Van Oostrom 1972, Soltani et al. 2012, Sonck et al. 2015, Srimani and Das 1978, Wang et al. 2014) were also used to predict the deflection of castellated beams and/or to compare the predictions with the results from the experiments. The experimental findings (Zaarour and Redwood 1996) demonstrated the possibility of the occurrence of the buckling of the web posts between web openings. The shear deflection of the straight-sided tapering cantilever of the rectangular cross section (Maalek 2004) was calculated by using a theoretical method based on Timoshenko's beam theory and virtual work method. Linear genetic programming and integrated search algorithms (Aminian et al. 2012) showed that the use of the machine learning system is an active method to validate the failure load of castellated beams. A

numerical computer programme (Sherbourne and Van Oostrom 1972) was developed for the analysis of castellated beams considering both elastic and plastic deformations by using practical lower limit relationships for shear, moment and axial force interaction of plasticity. An analysis on five experimental groups of castellated beams (Srimani and Das 1978) was conducted to determine the deflection of the beam. It was demonstrated (Hosain et al. 1974) that the finite elements method is a suitable method for calculating the deflection of symmetrical section castellated beams. The effect of nonlinearity in material and/or geometry on the failure model prediction of castellated beams (Soltani et al. 2012) was done by using MSC/NASTRAN software to find out bending moments and shear load capacity, which are compared with those published in literature.

Axial compression buckling of castellated columns was investigated (Yuan et al. 2014), in which an analytical solution for critical load is derived based on stationary potential energy and considering the effect of the web shear deformations on the flexural buckling of simply supported castellated column. Recently, a parametric study on the large deflection analysis of castellated beams at high temperatures (Wang et al. 2014) was conducted by using finite element method to calculate the growth of the end reaction force, the middle span deflection, and the bending moments at susceptible sections of castellated beams. More recently, a comprehensive comparison between the deflection results of cellular and castellated beams obtained from numerical analysis (Sonck et al. 2015) was presented, which was obtained from different simplified design codes. The comparison showed that the design codes are not accurate for short span beams and conservative for long span beams. The principle of minimum potential energy was adopted (Yuan et al. 2016) to derive an analytical method to calculate the deflection of castellated/cellular beams with hexagonal/circular web openings, subjected to a uniformly distributed transverse load.

The previous research efforts show that there were a few of articles that dealt with the deflection analysis of castellated beams. Due to the geometric particulars of the beam, however, it was remarkable to note that most of the theoretical approximate methods are interested in calculating the deflection of the castellated beams for long span beams where the shear effect is negligible. However, the castellated beams/columns are used not only for long span beams/columns but also for short beams/columns. Owing to the complex of section profile of the castellated beams, the shear-effect caused by the web opening on the deflection calculation is not fully understood. There are no accurate calculation methods available in literature to perform these analyses. Thus it is important to know how the shear affects the deflection of the beam and on what kind of spans the shear effect can be ignored. In addition, researchers have adopted the finite elements method to predict the deflection of castellated beams by using different software programs such as MSC/NASTRAN, ABAQUS, and ANSYS. However, these programs need efficiency in use because any error could lead to significant distortions in results. European building standards do not have formulas for the calculation of deflections of castellated beams, which include shear deformations.

This paper presents the analytical method to calculate the elastic deflection of castellated beams. The deflection equation is to be developed based on the

principle of minimum potential energy. In order to improve the accuracy and efficiency of this method, shear rigidity factor is determined by using suitable numerical techniques. The analytical results were validated by using the numerical results obtained from the finite element analysis using ANSYS software.

Analytical Philosophy of Deflection Analysis of Castellated Beams

An approximate method of deflection analysis of castellated beams under a uniformly distributed transverse load is presented herein. The method is derived based on the principle of minimum potential energy of the structural system. Because of the presence of web openings, the cross-section of the castellated beam is now decomposed into three parts to calculate the deflection and bending stress, two of which represent the top and bottom T-sections, one of which represents the mid-part of the web. The analysis model is illustrated in Figure 1a, in which the flange width and thickness are b_f and t_f, the web depth and thickness are h_w and t_w, and the half depth of hexagons is a. The half of the distance between the centroids of the two T-sections is e. In this study, the cross-section of the castellated beam is assumed to be doubly symmetrical. Under the action of a uniformly distributed transverse load, the beam section will have axial and transverse displacement as shown in Figure 1b, where x is the longitudinal coordinate of the beam, z is the cross-sectional coordinate of the beam, (u_1, w) and (u_2, w) are the axial displacements and the transverse displacements of the centroids of the upper and lower T-sections. All points on the section are assumed to have the same transverse displacement because of the beam assumption used in the present approach (Yuan et al. 2014). The corresponding axial strains ε_{1x} in the upper T-section and ε_{2x} i in the lower T-section are linearly distributed and can be determined by using the strain-displacement relation as follows:

In the upper T-section： $-\left(\frac{h_w}{2} + t_f\right) \leq z \leq -a$

$$\varepsilon_{1x}(x,z) = \frac{du_1}{dx} - (z+e)\frac{d^2w}{dx^2} \tag{1}$$

In the lower T-section： $a \leq z \leq \left(\frac{h_w}{2} + t_f\right)$

$$\varepsilon_{2x}(x,z) = \frac{du_2}{dx} - (z-e)\frac{d^2w}{dx^2} \tag{2}$$

The shear strain γ_{xz} in the middle part between the two T-sections can also be determined using the shear strain-displacement relation as follows:

For the middle part between the two T-sections: $-a \leq z \leq a$

$$\gamma_{xz}(x,z) = \frac{du}{dz} + \frac{dw}{dx} = -\frac{u_1 - u_2}{2a} + \frac{e}{a}\frac{dw}{dx} \tag{3}$$

$$e = \frac{b_f t_f \left(\frac{h_w + t_f}{2}\right) + t_w \left(\frac{h_w}{2} - a\right)\left(\frac{h_w + 2a}{4}\right)}{b_f t_f + t_w \left(\frac{h_w}{2} - a\right)} \qquad (4)$$

Because the upper and lower T-sections behave according to Bernoulli's theory, the strain energy of the upper T-section U_1 and the lower T-section U_2 caused by a transverse load can be expressed as follows:

$$U_1 = \frac{Eb_f}{2} \int_0^l \int_{-(t_f + \frac{h_w}{2})}^{-\frac{h_w}{2}} \varepsilon_{1x}^2 \, dz dx + \frac{Et_w}{2} \int_0^l \int_{-(\frac{h_w}{2})}^{-a} \varepsilon_{1x}^2 \, dz dx$$

$$= \frac{1}{2}\int_0^l \left[EA_{tee}\left(\frac{du_1}{dx}\right)^2 + EI_{tee}\left(\frac{d^2 w}{dx^2}\right)^2 \right] dx \qquad (5)$$

$$U_2 = \frac{Et_w}{2} \int_0^l \int_a^{(\frac{h_w}{2})} \varepsilon_{2x}^2 \, dz dx + \frac{Eb_f}{2} \int_0^l \int_{\frac{h_w}{2}}^{(t_f + \frac{h_w}{2})} \varepsilon_{2x}^2 \, dz dx$$

$$= \frac{1}{2}\int_0^l \left[EA_{tee}\left(\frac{du_2}{dx}\right)^2 + EI_{tee}\left(\frac{d^2 w}{dx^2}\right)^2 \right] dx \qquad (6)$$

where E is the Young's modulus of the two T-sections, G is the shear modulus, A_{tee} and I_{tee} are the area and the second moment of area of the T- section, which are determined in their own coordinate systems as follows:

$$A_{tee} = b_f t_f + t_w \left(\frac{h_w}{2} - a\right) \qquad (7)$$

$$I_{tee} = \frac{b_f t_f^3}{12} + b_f t_f \left(\frac{h_w + t_f}{2} - e\right)^2 + \frac{t_w}{12}\left(\frac{h_w}{2} - a\right)^3$$

$$+ t_w \left(\frac{h_w}{2} - a\right)\left(\frac{h_w + 2a}{4} - e\right)^2 \qquad (8)$$

The mid-part of the web of the castellated beam, which is illustrated in Figure 1a, is assumed to behave according to Timoshenko's theory (Yuan et al. 2014). Therefore, its strain energy due to the bending and shear can be expressed as follows:

$$U_b = \frac{1}{2}\sum K_b \, \Delta^2 \qquad (9)$$

where Δ is the relative displacement of the upper and lower T-sections due to a pair of shear forces and can be expressed as ($\Delta = 2a\gamma_{xz}$). While K_b is the

combined stiffness of the mid part of the web caused by the bending and shear, and is determined in terms of Timoshenko beam theory as follows,

$$\frac{1}{K_b} = \frac{3l_b}{2GA_b} + \frac{l_b^3}{12EI_b} \tag{10}$$

where $A_b = \sqrt{3}at_w$ is the equivalent cross-sectional area of the mid part of the web, $I_b = (\sqrt{3}a)^3 t_w/12$ is the second moment of area, and $l_b = 2a$ is the length of the Timoshenko beam; herein representing the web post length. Note that, the Young's modulus of the two T-sections is $E=2(1+v)G$ and the Poisson's ratio is taken as $v = 0.3$, the value of the combined stiffness of the mid part of the web caused by the bending and shear can be determined as fallow:

$$K_b = \frac{\sqrt{3}Gt_w}{4} \tag{11}$$

Thus, the shear strain energy of the web, U_{sh}, due to the shear strain γ_{xy} can be calculated as follows:

$$U_{sh} = \frac{\sqrt{3}}{2}Gt_w a^2 \sum_{k=1}^{n} \gamma_{xz}^2 \approx \frac{\sqrt{3}Gt_w a^2}{2 \times \frac{6a}{\sqrt{3}}} \int_0^l \gamma_{xz}^2 dx = \frac{Gt_w a}{4} \int_0^l \gamma_{xz}^2 dx \tag{12}$$

Let the shear rigidity factor $k_{sh} = 0.25$. Substituting Eqs. (3) into (12) gives the total shear strain energy of the mid-part of the web:

$$U_{sh} = \frac{Gt_w e^2 k_{sh}}{a} \int_0^l \left(\frac{dw}{dx} - \frac{u_\beta}{e}\right)^2 dx \tag{13}$$

Note that, in the calculation of shear strain energy of Eq. (12) one uses the concept of smear model, in which the shear strain energy was calculated first for web without holes. Then by assuming the ratio of the shear strain energies of the webs with and without holes is proportional to the volume ratio of the webs with and without holes, the shear strain energy of the web with holes was evaluated, in which $k_{sh} = 0.25$ was obtained (Kim et al. 2016). However, by using a two-dimensional linear finite element analysis (Yuan et al. 2016) the value of the combined stiffness of the mid part of the web of the castellated beam caused by the bending and shear, was found to be

$$K_b = 0.78 \times \frac{\sqrt{3}Gt_w}{4} \tag{14}$$

which is smaller than that above-derived from the smear model. This leads to the shear rigidity factor $k_{sh} = 0.78 \times 0.25$. The reason for this is probably due to

the smear model used for the calculation of the shear strain energy for the mid-part of the web in Eq. (12).

Figure 1. (a) *Notations used in Castellated Beams* (b) *Displacements* and (c) *Internal Forces*
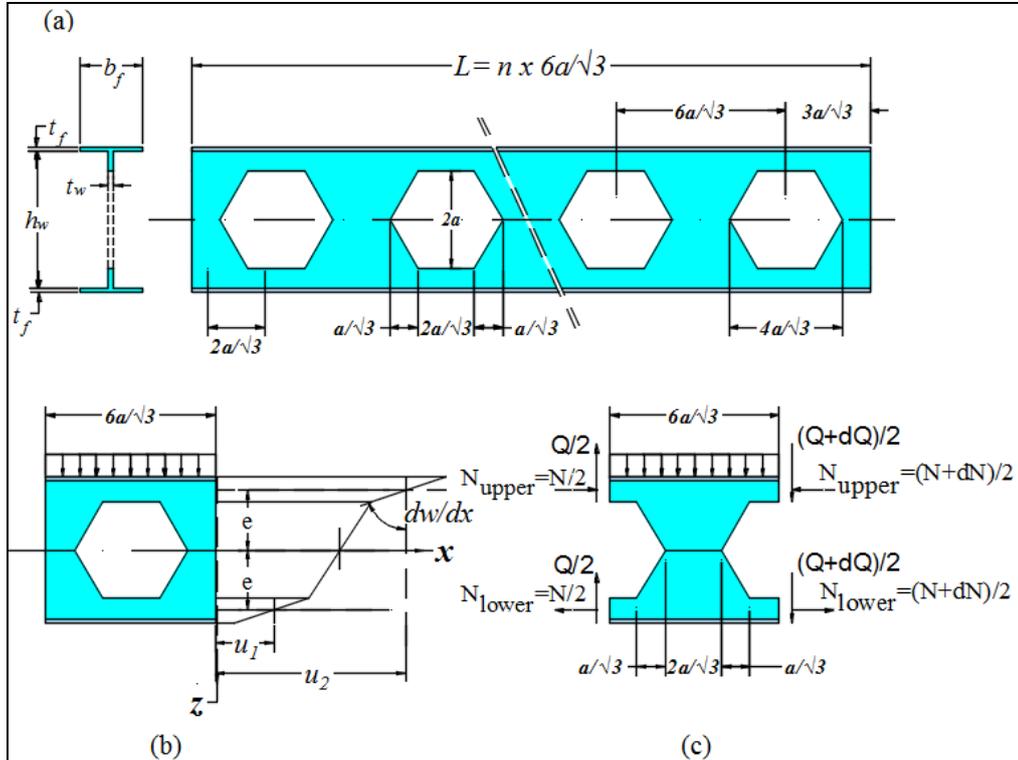
However, it should be mentioned that the factor of 0.78 in Eq. (14) was obtained for only one specific section of a castellated beam. It is not known whether this factor can also be applied to other dimensions of the beams. A finite element analysis model for determining the shear rigidity factor k_{sh} is therefore developed herein (see Figure 2c), in which the length and depth of the unit are (4a/√3) and (2a+a/2), respectively. In the unit the relative displacement Δ can be calculated numerically when a unit load F is applied (see Figure 2c). Hence, the combined rigidity $K_b=1/\Delta$ is obtained. Note that in the unit model all displacements and rotation of the bottom line are assumed to be zero, whereas the line where the unit load is applied is assumed to have zero vertical displacement. The calibration of the shear rigidity for beams of different section sizes shows that the use of the expression below gives the best results and therefore Eq. (15) is used in the present analytical solutions.

$$K_{sh} = \left(0.76 - \frac{b_f}{l}\right) \times \frac{1}{4} \tag{15}$$

where l is the length of the beam. Thus the total potential energy of the

castellated beam U_T is expressed as follows,

$$U_T = U_1 + U_2 + U_{sh} \tag{16}$$

For the simplicity of presentation, the following two new functions are introduced:

$$u_\alpha = \frac{u_1 + u_2}{2} \tag{17}$$

$$u_\beta = \frac{u_1 - u_2}{2} \tag{18}$$

By using Eqs. (17) and (18), the total potential energy of the castellated beam subjected to a uniformly distributed transverse load can be expressed as follows:

$$\Pi = EA_{tee} \int_0^l \left(\frac{du_\beta}{dx}\right)^2 dx + EI_{tee} \int_0^l \left(\frac{d^2w}{dx^2}\right)^2 dx \\ + \frac{Gt_w e^2 k_{sh}}{a} \int_0^l \left(\frac{dw}{dx} - \frac{u_\beta}{e}\right)^2 dx - W \tag{19}$$

where W is the potential of the uniformly distributed load q_{max} due to the transverse displacement, which can be expressed as follows:

$$W = q_{max} \int_0^l w\, dx \tag{20}$$

where q_{max} is the uniformly distributed load, which can be expressed in terms of design stress σ_y, as follows:

$$q_{max} = 16 \frac{\sigma_y I_{reduced}}{l^2 (h_w + 2t_f)} \tag{21}$$

$$I_{reduced} = \frac{b_f (h_w + 2t_f)^3}{12} - \frac{t_w a^3}{12} - \frac{(h_w)^3 (b_f - t_w)}{12} \tag{22}$$

Figure 2. *Shear Strain Energy Calculation Model: (a) Unit Considered, (b) Shear Deformation Calculation Model and (c) Finite Element Model of 4a/√3 Length Unit and (2a+a/2) Depth, Loaded by a Unite Force F*
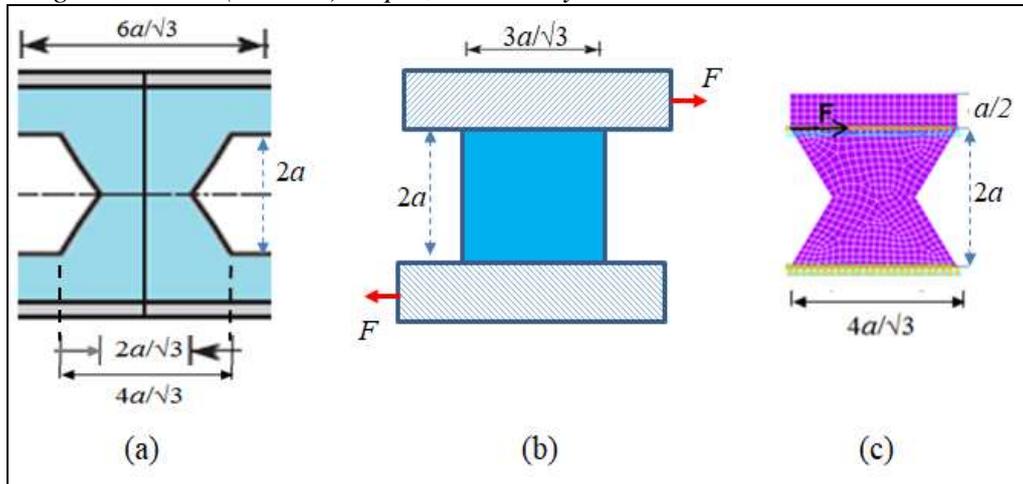
Deflection of Simply Supported Castellated Beam with Uniformly Distributed Transverse Loading

For a simply supported castellated beam $u_\alpha(x)$, $u_\beta(x)$ and $w(x)$ can be assumed as follows:

$$u_\alpha(x) = \sum_{m=1,2,..} A_m \cos\frac{m\pi x}{l}$$

(23)

$$u_\beta(x) = \sum_{m=1,2,..} B_m \cos\frac{m\pi x}{l}$$

(24)

$$w(x) = \sum_{m=1,2,..} C_m \sin\frac{m\pi x}{l}$$

(25)

where A_m, B_m and C_m are the constants to be determined. It is obvious that the displacement functions assumed in Eqs. (23)-(25) satisfy the simply support boundary conditions, that are $w = \dfrac{d^2 w}{dx^2} = 0$ and $\dfrac{du_\alpha}{dx} = \dfrac{du_\beta}{dx} = 0$ at $x = 0$ and $x = l$, and m = 1,2,… is the integral number. Substituting Eqs. (23), (24) and (25) into (19) and (20) and according to the principle of minimum potential energy, it yields,

$$\delta(U_T + U_{sh} - W) = 0$$

(26)

The variation of Eq. (26) with respect to A_m, B_m and C_m results in following three algebraic equations:

$$EA_{tee} \left(\frac{m\pi x}{l}\right)^2 A_m = 0 \tag{27}$$

$$\left[EA_{tee} \left(\frac{m\pi}{l}\right)^2 + \frac{Gt_w k_{sh}}{a}\right] B_m - \left[\frac{Gt_w e k_{sh}}{a}\left(\frac{m\pi}{l}\right)\right] C_m = 0 \tag{28}$$

$$\left[EI_{tee} \left(\frac{m\pi}{l}\right)^4 + \frac{Gt_w e^2 k_{sh}}{a}\left(\frac{m\pi}{l}\right)^2\right] C_m - \left[\frac{Gt_w e k_{sh}}{a}\left(\frac{m\pi}{l}\right)\right] B_m$$
$$= \frac{[1-(-1)^m]q_{max}}{m\pi} \tag{29}$$

Mathematically Eqs. (27) -(29) lead to:

$$A_m = 0 \tag{30}$$

$$B_m = \frac{\left[\frac{Gt_w e k_{sh}}{a}\left(\frac{m\pi}{l}\right)\right]}{\left[EA_{tee}\left(\frac{m\pi}{l}\right)^2 + \frac{Gt_w k_{sh}}{a}\right]} C_m \tag{31}$$

$$C_m = \frac{1-(-1)^m}{(m\pi)^5} \frac{ql^4}{EI_{tee} + \dfrac{e^2 EA_{tee}}{1 + \dfrac{EA_{tee}\,a(m\pi)^2}{Gk_{sh}t_w l^2}}} \tag{32}$$

Therefore, the deflection of the castellated beam can be expressed as follows:

$$w(x) = \frac{ql^4}{E(I_{tee} + e^2 A_{tee})} \sum_{m=1,2,\ldots} \frac{2}{(m\pi)^5}\left[1 + \frac{e^2 A_{tee}}{I_{tee} + e^2 A_{tee}}\right.$$
$$\left. \times \frac{EA_{tee}\,a(m\pi)^2}{Gk_{sh}t_w l^2}\left(1 - \frac{EI_{tee}\,a(m\pi)^2}{Gk_{sh}t_w l^2 e^2}\right)\right] \sin\frac{m\pi x}{l} \tag{33}$$

The maximum deflection of the simply supported beam is at the mid of the beam, that is *x=l/2* and thus it can be expressed as follows:

$$w|_{x=l/2} = \frac{ql^4}{E(I_{tee} + e^2 A_{tee})} \left[\sum_{k=1,2,..} \frac{2}{\pi^5} \frac{(-1)^{k+1}}{(2k-1)^5} + \frac{e^2 A_{tee}}{I_{tee} + e^2 A_{tee}} \times \frac{EA_{tee}\, a}{Gk_{sh} t_w l^2} \right.$$

$$\left. \times \left(\sum_{k=1,2,..} \frac{2}{\pi^2} \frac{(-1)^{k+1}}{(2k-1)^3} - \frac{EI_{tee}\, a}{Gk_{sh} t_w l^2 e^2} \sum_{k=1,2,..} \frac{2}{\pi} \frac{(-1)^{k+1}}{(2k-1)} \right) \right]$$

$$\tag{34}$$

Note that, mathematically, the following equations hold,

$$\sum_{k=1,2,..} \frac{2}{\pi^5} \frac{(-1)^{k+1}}{(2k-1)^5} = \frac{5}{2 \times 384} \tag{35}$$

$$\sum_{k=1,2,..} \frac{2}{\pi^3} \frac{(-1)^{k+1}}{(2k-1)^3} = \frac{1}{16} \tag{36}$$

$$\sum_{k=1,2,..} \frac{2}{\pi} \frac{(-1)^{k+1}}{(2k-1)} = \frac{1}{2} \tag{37}$$

Using Eqs. (35), (36) and (37), the maximum deflection of the beam can be simplified as follows:

$$w|_{x=l/2} = \frac{5ql^4}{384E(2I_{tee} + 2e^2 A_{tee})} + \frac{ql^2 a}{16Gk_{sh} t_w} \times \left(\frac{eA_{tee}}{I_{tee} + e^2 A_{tee}} \right)^2$$

$$\times \left(1 - \frac{2EI_{tee}\, a}{Gk_{sh} t_w l^2 e^2} \right) \tag{38}$$

It is clear from Eq. (38) that, the first part of Eq. (38) represents the deflection generated by the bending load, which is deemed as that given by Bernoulli-Euler beam, while the second part of Eq. (38) provides the deflection generated by the shear force. Moreover, Eq. (38) shows that the shear-induced deflection is proportional to the cross-section area of the two T-sections but inversely proportional to the beam length. This explains why the shear effect could be ignored for long span beams.

If the calculation does not consider the shear effect of web openings, Eq. (38) reduces to the following bending deflection equation.

$$w|_{x=l/2} = \frac{5ql^4}{384EI_{reduced}} \tag{39}$$

Numerical Study

In order to validate the abovementioned analytical solution numerical analysis using the finite element method is also carried out. The numerical computation

uses the ANSYS Programming Design Language (APDL). The FEA modelling of the castellated beams is carried out by using 3D linear Quadratic 4-Node thin shell elements (SHELL181). This element presents four nodes with six DOF per node, i.e., translations and rotations on the X, Y, and Z axis, respectively. Half-length of the castellated beams is used because of the symmetry in geometry. The lateral and transverse deflections and rotation are restrained (u_y=0, u_z=0 and θ_x=0) at the simply supported end, while the symmetrical boundary condition is applied at the other end by constraining the axial displacement and rotations around the two axes within the cross-section (u_x=0, θ_y=0 and θ_z=0). The material properties of the castellated beam are assumed to be linear elastic material with Young's modulus E = 210 GPa and Poisson's ratio v =0.3.

A line load effect is used to model applied uniformly distribution load, where the load is assumed acting on the junction of the flange and the web. The equivalent nodal load is calculated by multiply the distribution load with beam's half-length and then divided by the number of the nodes on the junction line of the flange and the web.

Discussion

Figure 3 shows a comparison of the maximum deflations between analytical solutions using different shear rigidity factors including one with zero shear factor and FEA numerical solution for four castellated beams of different flange widths. It can be seen from the figure that, the analytical solution using the proposed shear factor is closest to the numerical solution, whereas the analytical solutions using other shear factors is not as good as the present one. This demonstrates that the shear factor is also affected by the ratio of the flange width to the beam length. Also, it can be seen from the figure that, the longer the beam, the closer the analytical solution to the numerical solution; and the wider the flanges, the closer the analytical solution to the numerical solution.

Figure 3. *Maximum Deflections of Simply Supported Castellated Beams with Uniformly Distributed Load Obtained using Analytical Solution with Different Shear Rigidity Factors (Eqs. (38) and (39)) and FEA Numerical Solution for Four Castellated Beams of Different Flange Widths (a) bf=100mm (b) bf=150mm (c) bf=200mm (d) bf=250mm (h_w=300mm, t_f=10mm, t_w=8mm and a=100mm)*
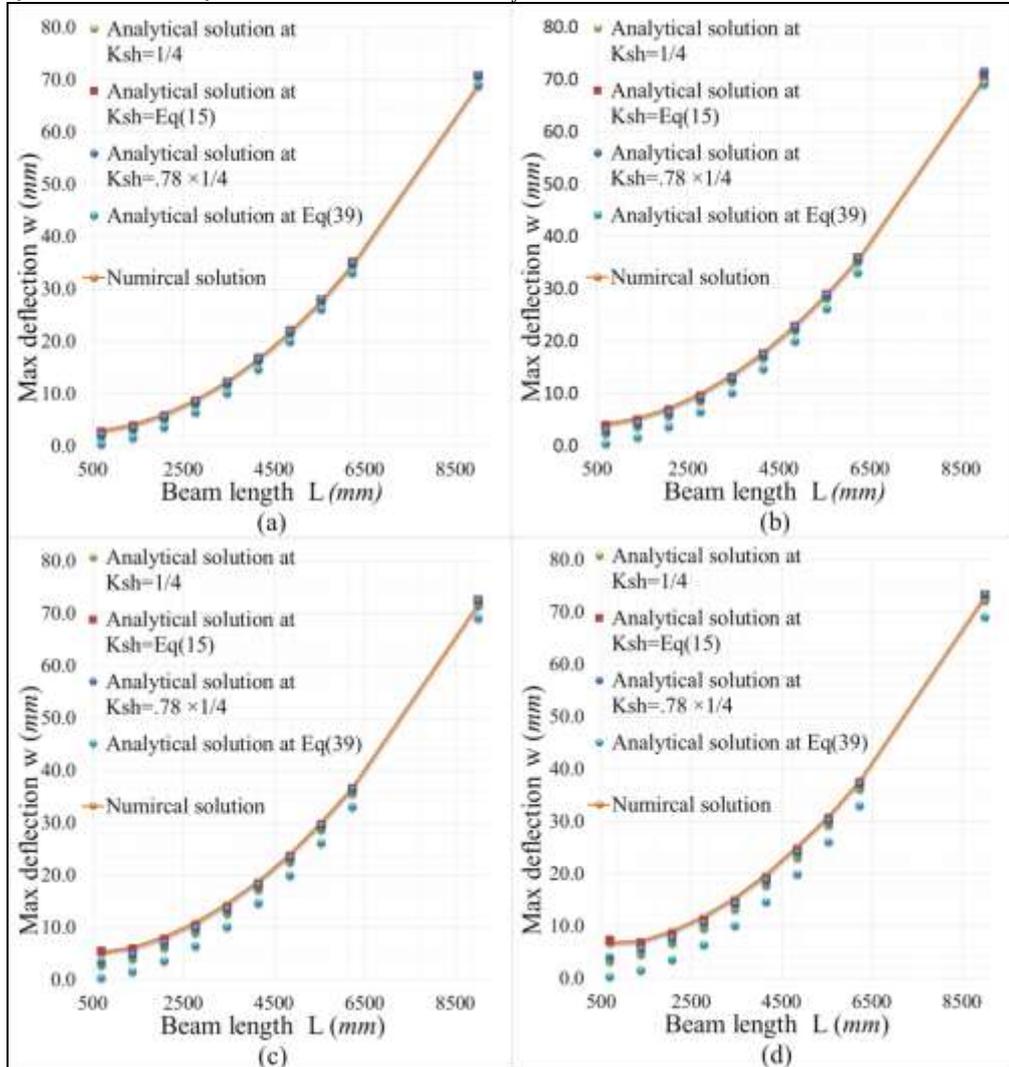

Figure 4 shows the relative error of each analytical solution when it is compared with the finite element solution. From the figure it is evident that the error of the analytical solutions using the present shear rigidity factor does not exceed 6.0% for all of discussed four sections in all the beam length range (>3 meter). In contrast, the analytical solution ignoring the shear effect, or considering the shear effect by using smear model or by using the length-independent shear rigidity factor will have large error, particularly when the beam is short.

Figure 4. *Divergence of Maximum Deflections of Simply Supported Castellated Beams with Uniformly Distributed Load Obtained using Analytical Solution with Different Shear Rigidity Factors (Eqs. (38) and (39)) and FEA Numerical Solution for Four Castellated Beams of Different Flange Widths* (a) *bf=100mm* (b) *bf=150mm* (c) *bf=200mm* (d) *bf=250mm* *(h_w=300mm, t_f=10mm, t_w=8mm and a=100mm)*
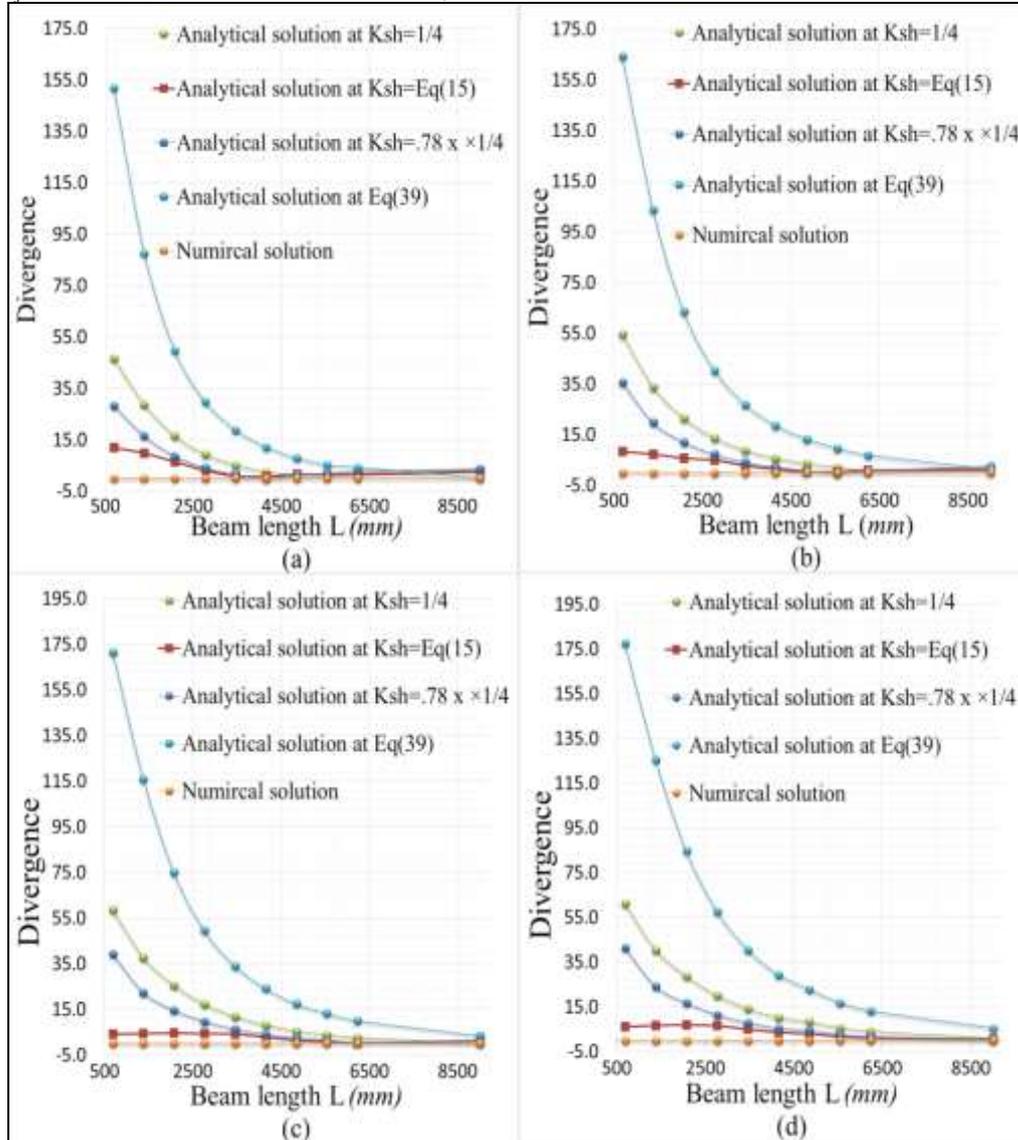
Conclusions

This study has reported the theoretical and numerical solutions for calculating the deflection of hexagonal castellated beams with simply supported boundary condition, subjected to a uniformly distributed transverse load. The analysis is based on the total potential energy method, by taking into account the influence of web shear deformations. The main novelty of the present analytical solution for

the calculation of deflection is it considers the shear effect of web openings more accurately. Both the analytical and numerical solutions are employed for a wide spectrum of geometric dimensions of I-shaped castellated beams in order to evaluate the analytical results. From the present study, the main conclusions can be summarized as follows:

1. The present analytical results are in excellent agreement with those obtained from the finite element analysis, which demonstrates the appropriateness of proposed approach.
2. Shear effect on the deflection of castellated beams is very important, particularly for short and medium length beams with narrow or wide section. Ignoring the shear effect could lead to an under-estimation of the deflection.
3. Divergence between analytical and numerical solutions does not exceed 6.0% even for short span castellated beam with narrow or wide section.
4. The effect of web shear on the deflection reduces when castellated beam length increases.
5. Despite that the numerical solution based on FEA has been widely used in the analysis of castellated beams; it is usually time-consuming and limited to specific geometrical dimensions. Thus, a simplified calculation solution that is able to deliver reasonable results but requires less computational effort would be helpful for both researchers and designers.

Acknowledgments

References

Altifillisch MD, Cooke RB, Toprac AA (1957) An Investigation of Open Web Expanded Beams. *Welding Research Council Bulletin, New York* 47: 307-320.

Aminian P, Niroomand H, Gandomi AH, Alavi AH, Arab Esmaeili M (2012) New Design Equations for Assessment of Load Carrying Capacity of Castellated Steel Beams: A Machine Learning Approach. *Neural Computing and Applications* 23(1): 119-131. http://doi:10.1007/s00521-012-1138-4.

Boyer JP (1964) Castellated Beam- A New Development Castellated Beams-New Developments. *AISC Engineering* 1(3): 104-108.

Demirdjian S (1999) *Stability of Castellated Beam Webs.* (PhD), McGill University Montreal, Canada.

Hosain M, Cheng W, Neis V (1974) Deflection Analysis of Expanded Open-Web Steel Beams. *Computers & Structures* 4(2): 327-336.

Kerdal D, Nethercot D (1984) Failure Modes for Castellated Beams. *Journal of Constructional Steel Research* 4(4): 295-315.

Kim B, Li L-Y, Edmonds A (2016) Analytical Solutions of Lateral–Torsional Buckling of Castellated Beams. *International Journal of Structural Stability and Dynamics*, 1550044. http://doi:10.1142/s0219455415500443.

Maalek S (2004) Shear Deflections of Tapered Timoshenko Beams. *International Journal of Mechanical Sciences* 46(5): 783-805. http://doi:10.1016/j.ijmecsci. 2004.05.003.

Sherbourne A, Van Oostrom J (1972) Plastic Analysis of Castellated Beams—Interaction of Moment, Shear and Axial Force. *Computers & Structures* 2(1): 79-109.

Soltani MR, Bouchaïr A, Mimoune M (2012) Nonlinear FE Analysis of the Ultimate Behavior of Steel Castellated Beams. *Journal of Constructional Steel Research* 70: 101-114. http://doi:10.1016/j.jcsr.2011.10.016.

Sonck D, Kinget L, Belis J (2015) *Deflections of Cellular and Castellated Beams.* Paper presented at the Future Visions (International Association for Shell and Spatial Structures) (IASS2015).

Srimani SS, Das P (1978) Finite Element Analysis of Castellated Beams. *Computers & Structures* 9(2): 169-174.

Wang P, Wang X, Ma N (2014) Vertical Shear Buckling Capacity of Web-Posts in Castellated Steel Beams with Fillet Corner Hexagonal Web Openings. *Engineering Structures* 75: 315-326. http://doi:10.1016/j.engstruct.2014.06.019.

Yuan W-B, Kim B, Li L-Y (2014) Buckling of Axially Loaded Castellated Steel Columns. *Journal of Constructional Steel Research* 92: 40-45. http://doi:10.1016/ j.jcsr.2013.10.013.

Yuan W-B, Yu N-T, Bao Z-S, Wu L-P (2016) Deflection of Castellated Beams Subjected to Uniformly Distributed Transverse Loading. *International Journal of Steel Structures* 16(3): 813-821.

Zaarour W, Redwood R (1996) Web Buckling in Thin Webbed Castellated Beams. *Journal of Structural Engineering* 122(8): 860-866.