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BENDING AND SHEAR ANALYSIS AND DESIGN OF DUCTILE STEEL PLATE WALLS

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SUMMARY

For the past few decades global attention and interest has grown in the application of Ductile Steel Plate Walls (DSPW) for building lateral load resisting systems. Advantages of using DSPWs in a building as lateral force resisting system compromise stable hysteretic characteristics, high plastic energy absorption capacity and enhanced stiffness, strength and ductility. A significant number of experimental and analytical studies have been carried out to establish analysis and design methods for such lateral resisting systems, however, there is still a need for a general analysis and design methodology that not only accounts for the interaction of the plates and the framing system but also can be used to define the yield and ultimate resistance capacity of the DSPW in bending and shear combination. In this paper an analytical model of the DSPW that characterizes the structural capacity in the shear and bending interaction is presented and discussed. This proposed model provides a good understanding of how the different components of the system interact, and is able to properly represent the system's overall hysteretic characteristics. The paper also contributes to better understand the structural capacity of the DPSW and of the shear and bending interaction. The simplicity of the method permits it to be readily incorporated in practical non-linear dynamic analyses of buildings with DSPWs. To demonstrate the effectiveness of the proposed model, its predicted response is compared with results from experimental studies performed by various researchers.

INTRODUCTION

This paper presents a refined model for the shear and bending analysis of ductile steel plate walls (DSPW), which will be referred to here as the Modified Plate-Frame Interaction (M-PFI) model. As shown in Figure 1, the deformation of Ductile Steel Plate Walls (DSPW) is a combination of shear and bending deformations, both of which are considered in the M-PFI model. In this paper the model for pure shear analysis is addressed first, followed by the pure bending model. Finally, the shear and bending interaction for the M-PFI model is studied considering appropriate failure criteria. The discussion also includes the

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load-displacement behaviour of DSPWs. The M-PFI model explores three main phenomena occurring in the structure, namely elastic buckling, post-buckling and yielding behaviour of the DSPW.

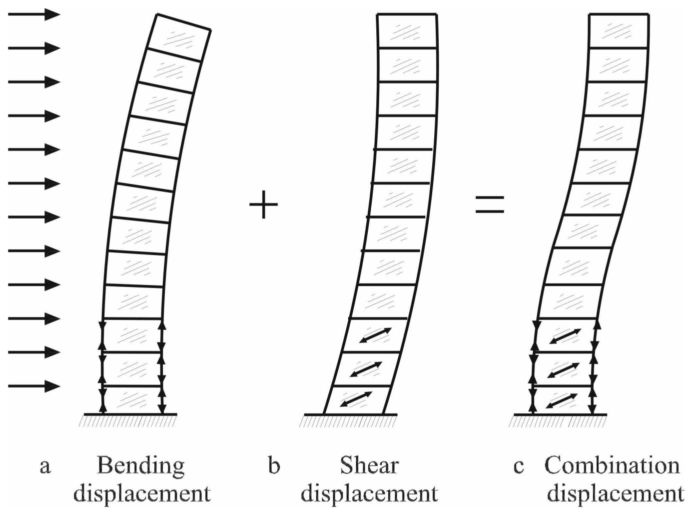

Figure 1. Bending and Shear Combination in DSPWs

SHEAR ANALYSIS OF DUCTILE STEEL PLATE WALLS

To begin with, the pure shear behaviour of a ductile steel plate wall is studied, with the following assumptions:

Basic Assumptions

A typical storey of a multi-storey structure with ductile steel plates wall can be represented as an isolated panel (Figure 2) for which the following assumptions can be made:

- The columns are assumed to be rigid enough so that their deformation can be neglected when calculating the shear deflection of the steel plate. It follows that a uniform tension field will develop across the entire steel plate.
- The difference in tension-field intensity in adjacent storeys is small and therefore bending of the floor beams due to the action of the tension field can be neglected.
- The steel plate can be considered as simply supported along its boundaries.
- The effect of global bending stresses on the shear buckling stress of the steel plate can be neglected.
- The behaviour of the steel plate and the steel frame can be treated as elastic-perfectly plastic.

Shear load-displacement relationships

For the steel shear wall model shown in Figure 2, the shear load-displacement diagrams for the steel plate and for the surrounding frame can be obtained separately. Then, by superimposing the two diagrams, the shear load-displacement of the DSPW panel can be obtained.

Shear load-displacement diagram of steel plate

A typical shear load-displacement diagram of a steel plate of height "*d*", width "*b*" and thickness "*t*" is shown in Figure 3. In this figure point *C* corresponds to the buckling limit, and point *D* corresponds to the yield point of the steel plate. Both points are yet to be determined.

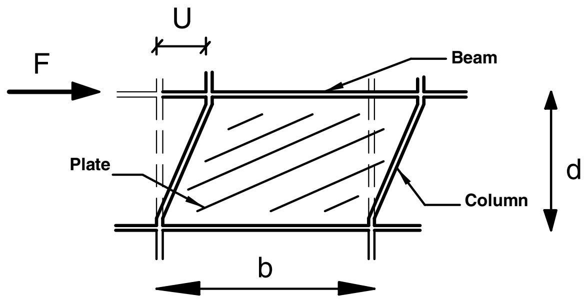

Figure 2. M-PFI model plate idealization for Shear Deformation

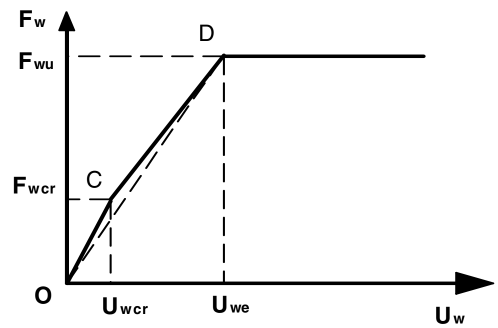

Figure 3. Shear load-displacement of steel plate only

The critical shear stress, τ_{cr} (see Figure 4.a) is given by

$$\tau_{cr} = \frac{K \cdot \pi^2 \cdot E}{12(1-\mu^2)} \cdot (\frac{t}{b})^2 \leq \tau_{wy} = \frac{\sigma_0}{\sqrt{3}} \tag{1}$$

in which t, E, μ, and σ_0 are the steel plate thickness, modulus of elasticity, Poisson's ratio and the uni-axial yield stress, respectively. K is obtained from

$$K = 5.35 + 4 \cdot (\frac{b}{d})^2 \qquad \text{for } (\frac{d}{b}) \geq 1 \tag{2}$$

$$K = 5.35 \cdot (\frac{b}{d})^2 + 4 \qquad \text{for } (\frac{d}{b}) \leq 1 \tag{3}$$

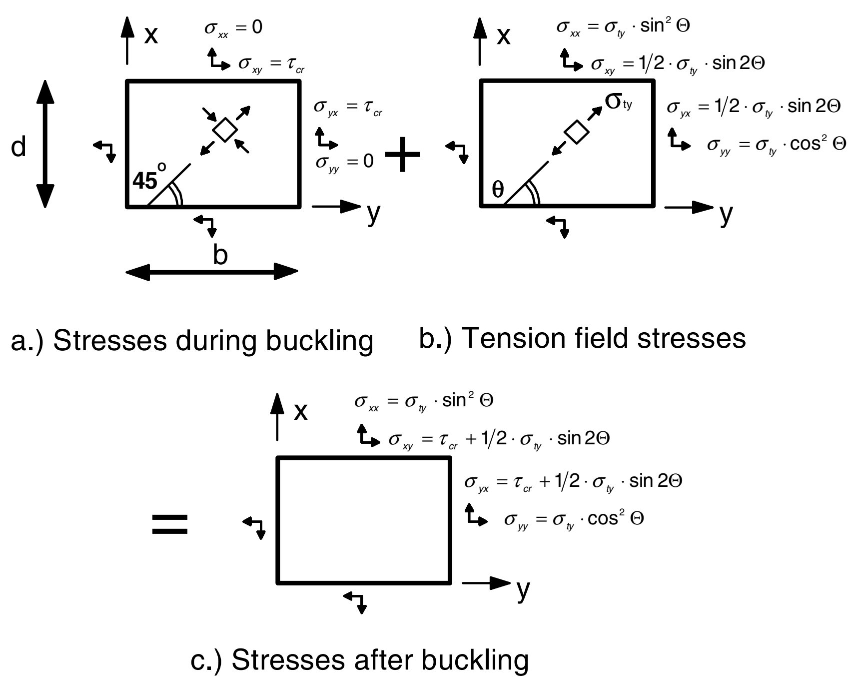

Figure 4. State of stresses in steel plate during and after buckling

The upper limit of τ_{cr} is the yield shear stress

$$\tau_{wy} = \frac{\sigma_0}{\sqrt{3}} \tag{4}$$

The critical shear force of the web plate, F_{wcr}, is therefore

$$F_{wcr} = \tau_{cr} \cdot b \cdot t \tag{5}$$

And the critical shear displacement, U_{wcr}, is obtained from

$$U_{wcr} = \frac{\tau_{cr}}{G} \cdot d \tag{6}$$

In which G is the shear modulus of the steel plate material. Once F_{wcr} and U_{wcr} are obtained from equations (5) and (6), respectively, point C can be defined in the shear load-displacement diagram (see Figure 3).

If it is assumed that during the post-buckling stage, a tension field inclined at an angle Θ with respect to the horizontal, as shown in Figure 4.b, gradually develops throughout the entire web plate. This assumed stress distribution provides a lower bound for the strength of the web plate, provided that the surrounding frame members are strong enough to sustain the normal boundary forces associated with the tension field. If σ_{ty} denotes the value of the tension field stress at which yielding occurs, the total state of stress in the plate at yield, shown in Figure 4.c is defined by

$$\sigma_{xx} = \sigma_{ty} \cdot \sin^2 \Theta \tag{7}$$

$$\sigma_{yy} = \sigma_{ty} \cdot \cos^2 \Theta \tag{8}$$

$$\sigma_{xy} = \sigma_{yx} = \tau_{cr} + \frac{1}{2}\sigma_{ty} \cdot \sin 2\Theta \tag{9}$$

According to the Von Mises yield criterion, yielding of the plate occurs when

$$(\sigma_{xx} - \sigma_{yy})^2 + \sigma_{yy}^2 + \sigma_{xx}^2 + 6 \cdot \sigma_{xy}^2 - 2 \cdot \sigma_0^2 = 0 \tag{10}$$

Substituting Eqs. (7), (8) and (9) into (10), the value of σ_{ty} at which yielding of the steel plate occurs, is defined by

$$3 \cdot \tau_{cr}^2 + 3 \cdot \tau_{cr} \cdot \sigma_{ty} \sin 2\Theta + \sigma_{ty}^2 - \sigma_0^2 = 0 \tag{11}$$

The shear strength of the web plate is now given by

$$F_{wu} = \sigma_{xy} \cdot b \cdot t = b \cdot t \cdot (\tau_{cr} + \frac{1}{2}\sigma_{ty} \cdot \sin 2\Theta) \tag{12}$$

The limiting elastic shear displacement U_{we}, is obtained by

$$U_{we} = U_{wcr} + U_{wpb} \tag{13}$$

U_{wcr} is the critical shear displacement as given in Eq. (6), and U_{wpb}, is the shear displacement from the post-buckling component of the shear forces. The latter is determined by equating the work done by the post buckled component of the shear forces to the strain energy of tension field. This leads to:

$$(\frac{1}{4}\sigma_{ty} \cdot \sin 2\Theta) \cdot t \cdot b \cdot U_{wpb} = \frac{\sigma_{ty}^2}{2 \cdot E} \cdot d \cdot b \cdot t \tag{14}$$

or

$$U_{wpb} = \frac{2 \cdot \sigma_{ty}}{E \cdot \sin 2\Theta} \cdot d \tag{15}$$

Substituting U_{wpb} from Eq. (15) and U_{wcr} from Eq. (6) in Eq. (13) gives

$$U_{we} = (\frac{\tau_{cr}}{G} + \frac{2 \cdot \sigma_{ty}}{E \cdot \sin 2\Theta}) \cdot d \tag{16}$$

Having determined F_{wu} from Eq. (12) and U_{we} from Eq. (16), point D is now defined in Figure 3.

In Figure 3, lines OC and CD can be substituted by a straight line OD, which simplifies the calculations with negligible effects on the shear load-displacement diagram. Thus, the slope of line OD in Figure 3, which is the stiffness of the steel plate, is given by

$$K_w = \frac{(\tau_{cr} + \frac{1}{2}\sigma_{ty} \cdot \sin 2\Theta)}{(\frac{\tau_{cr}}{G} + \frac{2 \cdot \sigma_{ty}}{E \cdot \sin 2\Theta})} \cdot \frac{b \cdot t}{d} \qquad (17)$$

As seen in Eq. (16), the limiting elastic shear displacement, U_{we}, is independent of the panel width, b, but directly dependent on the panel height, d.

The columns in DSPW systems are, in general, designed to carry gravity loads, and this is a useful characteristic for controlling uplift in the shear walls. The columns are normally assumed to be rigid enough so that a uniform tension field is developed throughout the entire steel plate, inclined at an angle Θ=45° to the horizontal, as shown in Figure 4. To ensure that the columns can sustain the normal boundary stresses associated with the tension field, and to make sure that a uniform tension field develops across the entire plate, the columns will need to have a minimum rigidity (this will be discussed further in the next section). For more detail on shear behaviour of web plates see Sabouri-Ghomi et al [1].

Shear load-displacement diagram of frame

For an internal storey ductile steel plate wall as shown in Figure 5, the shear load-displacement diagram is shown in Figure 6. It is assumed here that the beam-column connections are fixed and the beams behave as rigid elements. If point E is determined in Figure 6, then the load-displacement diagram of the frame will be defined. From Figure 5, the shear strength of the frame, F_{fu}, is

$$F_{fu} = \frac{4 \cdot M_{fp}}{d} \qquad (18)$$

in which M_{fp} is the plastic moment for the column. The limiting elastic shear displacement of the frame, U_{fe}, will be

$$U_{fe} = \frac{M_{fp} \cdot d^2}{6 \cdot E \cdot I_f} \cdot \frac{12 \cdot \rho + 4}{12 \cdot \rho + 1} \qquad (19)$$

in which I_f is the moment of inertia of the column and $\rho = \sum (EI_b / b) \Big/ \sum EI_f / d$. $\rho = \infty$ except for the last storey panel.

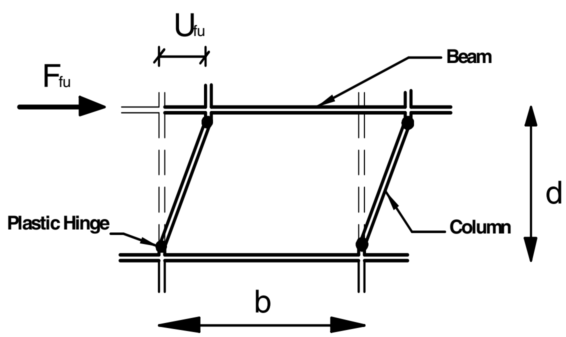

Figure 5. PFI model frame idealization

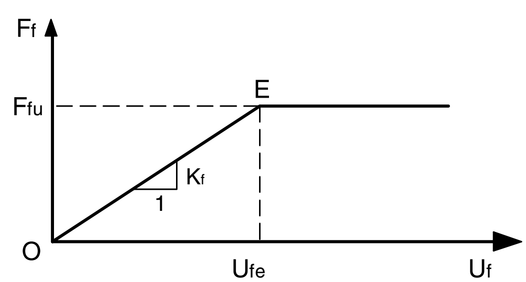

Figure 6. Shear load-displacement of frame only

Point E is defined in Figure 6 by obtaining F_{fu} and U_{fe} from Eqs. (18) and (19). The slope of line OE in Figure 6, which is the stiffness of the frame, is

$$K_f = \frac{24 \cdot E \cdot I_f}{d^3} \cdot \frac{12 \cdot \rho + 1}{12 \cdot \rho + 4} \qquad (20)$$

In Figure 7, the shear load-displacement of the panel is obtained by superimposing the diagrams shown in Figures 3 and 6. In this figure, *W*, *F* and *P* refer to the web steel plate, the frame and the steel plate panel, respectively. The steel plate panel is defined here as the combination of the plate and the frame elements. To ensure that the plate dissipates more energy than the frame, it is suggested that the steel plate walls be designed in such a way that the following expression is satisfied:

$$U_{fe} > U_{we} \tag{21}$$

As can be observed in Figure 7, by modelling the steel plate and frame separately, the designer has significant flexibility on the selection of the member sizes and properties.

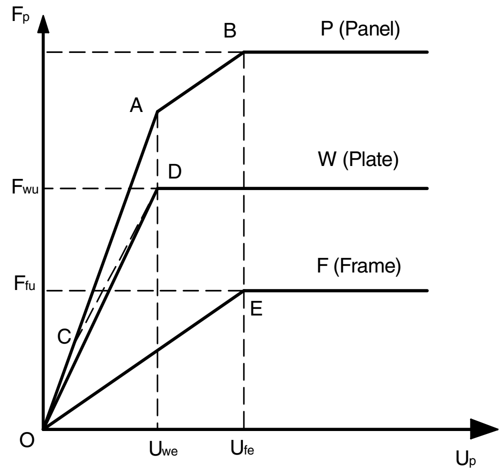

Figure 7. Components of PFI model shear load-displacement: frame only, plate only and combined effects in panel

Effect of rigidity of beams and columns on steel plate

To ensure that the frame members of the steel plate panel can sustain the normal boundary stresses associated with the tension field, and by assuming simple supports for the beams and columns, Eqs. (22) and (23) have to be satisfied (see Eqs. (7) and (8)).

$$M_{fp} \geq \frac{\sigma_{ty} \cdot t \cdot d^2}{8} \cdot \cos^2 \Theta \qquad \text{for columns} \tag{22}$$

$$M_{fp} \geq \frac{\sigma_{ty} \cdot t \cdot b^2}{8} \cdot \sin^2 \Theta \qquad \text{for beams} \tag{23}$$

When checking Eq. (22) the axial force on the columns needs to be considered. Eq. (23) has to be checked for the end beams of the steel plate panel. For internal storey beams, it is not always necessary to check Eq. (23) since the difference in tension-field intensity is usually negligible in the adjacent stories. Nevertheless, it may be desirable to check this equation for the beams, if this difference is considerable.

BENDING ANALYSIS OF DUCTILE STEEL PLATE WALLS

In this section, the M-PFI model for pure bending behaviour is considered. To model the DSPW for the analysis of pure bending, the plate and the frame are considered as one unit and the bending stresses in the unit section are studied. Chapman [2], Rockey and Jenkins [3], Basel and Thuerlimann [4], and many other researchers proposed different theories for the flexural analysis of steel webs of girders and tested several girders to verify their theoretical models. In this section these theories are applied for the bending behaviour, bending displacement, and bending stress distribution of DSPWs. For the bending analysis of the shear wall, the assumptions made are given next.

Basic Assumptions
A typical storey of a multi-storey structure with ductile steel plate walls in bending can be represented by an isolated panel (Figure 8), for which the following assumptions can be made:

- The slenderness of the columns is small enough that yielding or inelastic buckling in the steel plate takes place before any material yielding or buckling occurs in the columns.
- The steel plate can be considered as simply supported along its boundaries.
- The effect of global and inter-storey shear stresses on the bending and buckling stresses of the steel plate is neglected.
- The behaviour of the steel plate and the frame is elastic-perfectly plastic.

Bending load-displacement relationships
For the ductile steel wall model in pure bending (Figure 8), the combined load-displacement diagram for the steel plate and for the surrounding frame is obtained from the girder analysis theory. The moment capacity of the section is determined from the combined load-displacement diagram of the panel and the frame. A typical load-displacement diagram for bending of a unit DSPW of height "*d*", and width "*b*", which consists of a plate and a frame, is shown in Figure 9. In this figure point *C* corresponds to the buckling limit, and point *A* corresponds to the yield point of the steel plate. Point *B* in Figure 9 refers to the plastic capacity of the columns and consequently the frame. Point *B* is reached only when the columns does not undergo any buckling and have higher yield strength than the plate. These points are yet to be determined.

Critical buckling state of DSPW in pure bending
To determine the moment resistance of the DSPW at the stage of elastic critical plate buckling of the web, the critical bending stress, σ_{cr} (Figure 10.A), is defined as

$$\sigma_{cr} = \frac{K_b \cdot \pi^2 \cdot E}{12(1-\mu^2)} \cdot \left(\frac{t}{b}\right)^2 \leq \sigma_0 \qquad (24)$$

in which *t, E, μ,* and σ_0 are the steel plate thickness, modulus of elasticity, Poisson's ratio and the uni-axial yield stress, respectively. K_b is obtained from

$$K_b = 23.9 \qquad \text{for } 1 \leq \left(\frac{b}{d}\right) < 1.5 \qquad (25)$$

$$K_b = 15.87 + 1.87\left(\frac{b}{d}\right)^2 + 8.6\left(\frac{d}{b}\right)^2 \qquad \text{for } 1.5 \leq \frac{b}{d} \qquad (26)$$

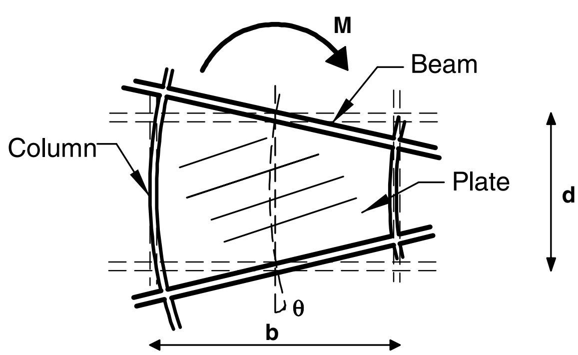

Figure 8. M-PFI model plate idealization for Bending Deformation

The upper limit of σ_{cr} is the yield bending stress

$$\sigma_{cr} = \sigma_0 \tag{27}$$

The critical moment force of the web plate, M_{cr}, is therefore

$$M_{cr} = \sigma_{cr} \cdot S_t \tag{28}$$

in which S_t is the total section modulus of the panel, which is obtained from the total moment of inertia of the wall, I_t, as shown below

$$S_t = \frac{2I_t}{b} = \frac{2}{b}\left(I_C + A_C (b/2)^2 + tb^3/12\right) \tag{29}$$

The critical bending displacement, U_{mcr}, is obtained from

$$U_{mcr} = \frac{M_{cr}}{EI_t} \cdot d^2 \tag{30}$$

Once M_{cr} and U_{mcr} are obtained from Eqs. (28) and (30), respectively, point C in the bending load-displacement diagram can be defined (Figure 9).

Ultimate yielding state of web plate in pure bending

In the post-buckling state the compression stress in the web is assumed not to increase beyond the critical buckling stress throughout the entire web plate, except for a very small portion close to the column in compression. For the tension stresses, during the post-buckling stage, the tension field stresses gradually develop parallel to the column in tension, as shown in Figure 10.B. The distribution of the stresses will not be linear any more and the neutral axis commences to move toward the tension column, which limits the web portion in tension to a smaller size than that in compression. This assumed stress distribution provides a lower bound for the strength of the web plate, provided that the surrounding frame members are strong enough to sustain the axial force associated with the bending and do not buckle. If σ_b denotes the tension field stress at which yielding occurs, the moment resistance at which the plate commences to yield will be

$$M_y = \sigma_b \cdot S_{eff} = \sigma_b \cdot I_{eff} / ((1-j)b) \tag{31}$$

where, j is the ratio of the distance between the neutral axis and the compression column to b, the width of the panel. S_{eff} and I_{eff} are the effective section modulus and the effective moment of inertia of the wall, respectively, which are as follows:

$$I_{eff} = A_C (j \cdot b)^2 + t_w (j \cdot b)^3 / 3 + A_c (1-j)^2 \cdot b^2 + \alpha \cdot t_w [(1-j)b - h_C/2 - \alpha/2]^2 \tag{32}$$

$$S_{eff} = I_{eff} / (1-j)b \tag{33}$$

where α is the length of web near the compression column, which is not buckling and is able to carry stresses higher than the critical buckling stress. Most ductile steel plate walls have a fish-plate configuration to which the web plate is welded during assembly. Since for thin web plates it is assumed that $\sigma_{cr} = 0$, the amount of α can be assumed equal to the width of the fish-plate. The corresponding deflection of the panel due to a bending moment of M_y is U_{my}, which is obtained from the following equation:

$$U_{my} = \frac{M_y}{EI_{eff}} \cdot d^2 \tag{34}$$

Once M_y and U_{my} are obtained from Eqs. (31) and (34), respectively, point A can be defined in the bending load-displacement diagram (see Figure 9). It is worthwhile to mention that the ultimate moment resistance of the wall could be much lower than the amount obtained from Eq. 31, if the columns buckle elastically or in-elastically before the web plate yields. This amount is shown by M_{yc} in the bending load-displacement diagram (see Figure 9). Since it is initially assumed that the columns will not buckle or yield before the web plate has yield or buckled, this option is not considered at this time.

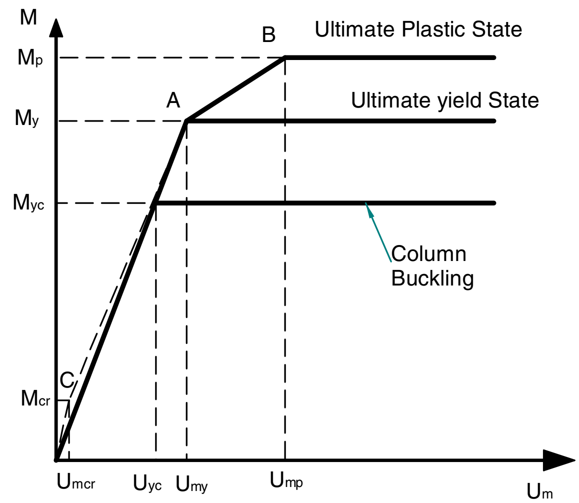

Figure 9. M-PFI bending model load-displacement of the web plate and the frame combined

If the yield stress of the columns is larger than that of the web plate and the slenderness of the columns is relatively low, so that the section is able to undergo axial plastification (yielding of the entire section) before it buckles, the ultimate plastic strength can be determined for the panel. Then the section is capable of providing a plastic hinge, which will have a plastic moment resistance of M_P as defined below:

$$M_P = \sigma_0 \cdot Z_P \tag{35}$$

in which, Z_P is the plastic section modulus of the wall considering only the contribution of the columns, as shown in Figure 10.C. The plastic section modulus, Z_P is as follow:

$$Z_P = b \cdot A_C \tag{36}$$

The corresponding deflection of the panel is U_{mp}, and is obtained from the following equation:

$$U_{mp} = \frac{M_P}{EI_P} \cdot d^2 \tag{37}$$

in which I_P is the plastic moment of inertia of the section, which does not consider the contribution of the web plate. Once M_P and U_{mp} are obtained from Eqs. (35) and (37), respectively, point *B* can be defined in the bending load-displacement diagram (see Figure 9). It is important to note that, to reach this point, very stocky columns are needed. In most practical cases the section will likely experience buckling before it reaches total material failure in the column sections.

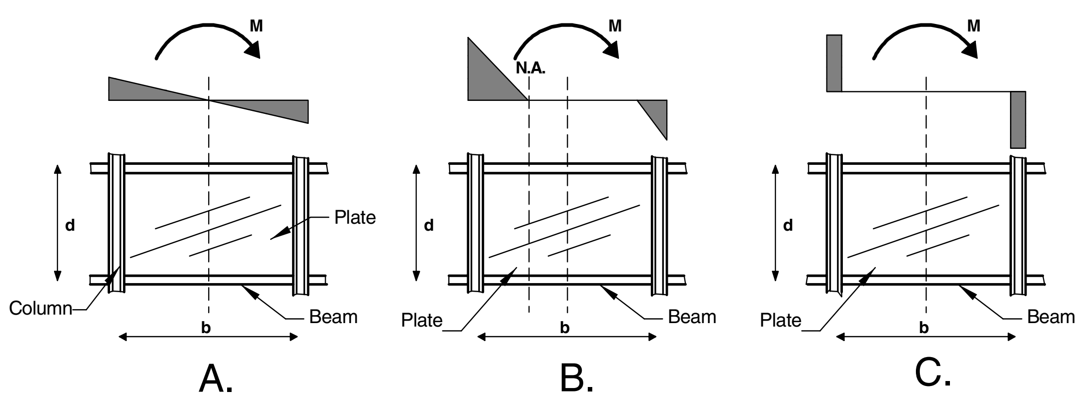

Figure 10. Stress distribution in the DSPW due to pure bending, A.) Critical buckling State, B.) Ultimate Yield Stress State of the Web plate, and C.) Ultimate Plastic Stress State.

BENDING – SHEAR INTERACTION FOR DUCTILE STEEL PLATE WALLS

In a multi-storey ductile steel plate wall (DSPW), a panel unit resists not only shear forces but also bending moments created by the overturning action of the entire wall. To obtain the capacity of the DSPW under both actions, the bending and shear interaction of the structure is introduced in the next section. The bending – shear interaction is derived for the panel and frame at various states, such as the buckling and the yielding state of the web plate, and the frame failure state. This interaction equation is used to adjust the load-displacement diagram for the shear forces to account for the flexural action.

Bending – Shear Interaction for Critical Buckling

The critical buckling shear load resistance of the wall panel is adjusted by considering the effect of the bending moment on the critical shear capacity of the web plate. This effect can be represented by the quadratic interaction equation:

$$\left(\frac{\tau_{pcr}'}{\tau_{cr}}\right)^2+\left(\frac{\sigma_{pcr}'}{\sigma_{cr}}\right)^2=1 \tag{38}$$

where τ_{pcr}' and σ_{pcr}' are the applied stresses on the web steel plate. The critical buckling shear load, F_{cr}, is then adjusted to account for the applied bending and a lower critical buckling value is obtained, which is $F_{cr}'=\tau_{pcr}'\cdot b\cdot t$. The total displacement of the panel due to the modified critical buckling shear stress is

$$U_{wcr}'=\frac{\tau_{cr}'}{G}\cdot d+\frac{M_b}{EI_t}\cdot d^2=\frac{\tau_{cr}'}{G}\cdot d+\frac{\sigma_{cr}'}{2E}\cdot bd^2 \tag{39}$$

where the new bending stress, σ_{cr}', is obtained from the applied moment, that is $\sigma_{cr}'=M_b/S_t$. By determining the elastic critical plate buckling shear load and the total displacement, point C is adjusted to point C' in Figure 11 to account for the flexural load effect.

Bending – Shear Interaction for Post-buckling Tension Field Stresses of the Plate

The tension field created in the buckled plate is part of the post-buckling characteristics of the steel plate. Since the global bending effects were ignored when deriving the pure shear capacity (as mentioned in the assumptions), the ultimate yield capacity of the steel plate in shear needs to be adjusted for the applied bending moment caused by the overturning effect. Therefore, the tension field stress, σ_{ty}, needs to be adjusted for the bending stress, σ_b, created by the bending moment in the web plate close to the tension column. As shown in Figure 12, the bending stress, σ_b, in the plate due to the bending moment is concentrated in the tension zone of the plate close to the tension column. This assumed stress distribution provides a lower bound for the strength of the web plate, provided that the surrounding frame members are strong enough to sustain the normal boundary forces associated with the tension field and axial forces associated with the bending moment. If σ_{ty} denotes the value of the tension field stress at which yielding occurs, the total state of stress in the plate at yielding, shown in Figure 12, is defined by

$$\sigma_{xx}=\sigma_b+\sigma_{ty}\cdot\sin^2\Theta \tag{40}$$

$$\sigma_{yy}=\sigma_{ty}\cdot\cos^2\Theta \tag{41}$$

$$\sigma_{xy}=\sigma_{yx}=\tau_{cr}+\frac{1}{2}\sigma_{ty}\cdot\sin 2\Theta \tag{42}$$

According to the Von Mises yield criterion, yielding of the plate occurs when

$$(\sigma_{xx}-\sigma_{yy})^2+\sigma_{yy}{}^2+\sigma_{xx}{}^2+6\cdot\sigma_{xy}{}^2-2\cdot\sigma_0{}^2=0 \tag{43}$$

Substituting Eqs. (40), (41) and (42) into (43) and assuming that for thin plates $\tau_{cr} = 0$, the modified value of σ'_{ty} at the yield state of the steel plate is defined by

$$\sigma_{ty}' = \frac{1}{2}\left[-\sigma_b(3\sin^2\Theta - 1) + \sqrt{(\sigma_b(3\sin^2\Theta - 1))^2 - 4(\sigma_b{}^2 - \sigma_0{}^2)}\right] \tag{44}$$

The bending stress, σ_b, is obtained from $\sigma_b = M_b / S_{eff}$. M_b is the applied bending moment at the ultimate yield state of the steel plate. The bending and shear interaction at the ultimate yield state of the steel plate is obtained by dividing both sides by $\sigma_{ty} = \sigma_0$, which results in the following:

$$\frac{\sigma_{ty}'}{\sigma_{ty}} = \frac{\frac{1}{2}\left[-\sigma_b(3\sin^2\Theta - 1) + \sqrt{(\sigma_b(3\sin^2\Theta - 1))^2 - 4(\sigma_b{}^2 - \sigma_0{}^2)}\right]}{\sigma_0} \tag{45}$$

The adjusted shear strength of the web plate is given by

$$F_{wu}' = \sigma_{xy} \cdot b \cdot t = b \cdot t \cdot (\tau_{cr} + \frac{1}{2}\sigma_{ty}' \cdot \sin 2\Theta) \tag{46}$$

For thin plates, it is assumed that $\tau_{cr} = 0$ thus the adjusted value for ultimate shear strength of the web plate is given by

$$F_{wu}' = b \cdot t \cdot (\frac{1}{2}\sigma_{ty}' \cdot \sin 2\Theta) \tag{47}$$

The total ultimate shear resistance of the panel at the yield state of the steel plate, F_{wv} is

$$F_{wv} = F_{wu}' + Kf \cdot U_{we} \tag{48}$$

The adjusted limiting elastic shear displacement U'_{we}, is obtained from

$$U_{we}' = U_{we} + \frac{M_b}{EI_{eff}} \cdot d^2 \tag{49}$$

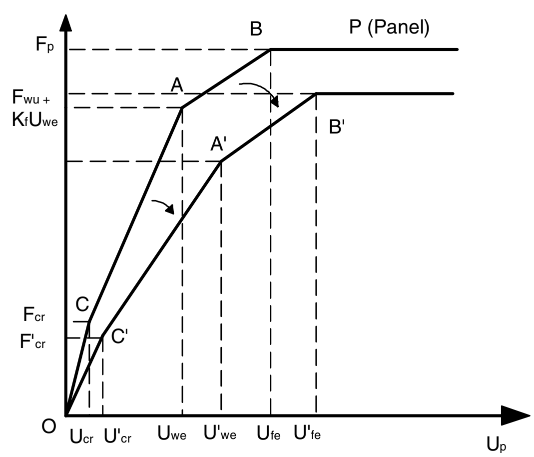

Figure 11. Modified load displacement diagram for shear resistance of the DSPW

Bending – Shear Interaction for Ultimate State

To capture the interaction of the interstorey shear and the global moment for the ultimate state of the frame, the interaction equation for bending and shear of the frame is mainly based on the axial and moment interaction of the column section. The axial and bending interaction equation of a section is usually given as a function of the following format:

$$\frac{M_{fN}}{M_{fp}} = f\left(\frac{N}{N_P}\right) \tag{50}$$

The ultimate shear resistance of the frame section is modified based on this interaction equation. Since the ultimate shear resistance of the frame is obtained form $F_{fu} = 4 \cdot M_{fp} / d$ and it is reasonable to assume that the ultimate plastic moment of the frame is obtained from $M_P = \sigma_0 \cdot Z_P$, and $Z_P = b \cdot A_C$ given that $N_P = A_C \cdot \sigma_0$, then $M_P = N_P \cdot b$, and Eq. 50 can be re-written as

$$\frac{F_{fu-N}}{F_{fu}} = f\left(\frac{M}{M_P}\right) \tag{51}$$

If the frame is made of I-section columns Eq. 50 becomes:

$$\frac{M_N}{M_P} = \left[1 - k_o \cdot \left(\frac{N}{N_P}\right)^2\right] \quad \text{for N.A. in the web of column} \tag{52}$$

$$\frac{M_N}{M_P} = \left[\frac{k_o t_w}{b_C}\left(1 - \frac{N}{N_P}\right) \cdot \left(\frac{2b_f d_c}{A_C} - \left(1 - \frac{N}{N_P}\right)\right)\right] \quad \text{for N.A. in the flange of column} \tag{53}$$

where d_c, t_w, and b_f are height, web thickness and flange width of the column respectively, and $k_o = d_c t_w / b$. Eqs. 63 and 64 become:

$$\frac{F_{fu-N}}{F_{fu}} = \left[1 - k_o \cdot \left(\frac{M}{M_P}\right)^2\right] \quad \text{for N.A. in the web of column} \tag{54}$$

$$\frac{F_{fu-N}}{F_{fu}} = \left[\frac{k_o t_w}{b_C}\left(1 - \frac{M}{M_P}\right) \cdot \left(\frac{2b_f d_c}{A_C} - \left(1 - \frac{M}{M_P}\right)\right)\right] \quad \text{for N.A. in the flange of column} \tag{55}$$

Using the modified shear resistance of the frame and adding it to the modified shear resistance of the steel plate results in the ultimate shear resistance of the panel, F_P:

$$F_P = F_{wu}' + F_{fu-N} \tag{56}$$

where F_{wu}' is the modified ultimate yield state of the steel plate and F_{fu-N} is the modified ultimate capacity of the frame, adjusted for bending – shear interaction. Accordingly, the total displacement U'_{fe} is modified to consider the moment effect, which results in

$$U_{fe}' = U_{fe} + \frac{M_p'}{EI_p} \cdot d^2 \tag{57}$$

The applied moment load at this stage is M_p', which is not necessarily equal to the total plastic moment capacity. With this information, the load-displacement curve *OCAB* is adjusted to *OC'A'B'* as shown in Figure 11. It is important to mention that for design purposes point *A* has to be determined since it is the yield point, whereas point *B* is used to determine the total shear resistance of the DSPW. Figure 13 shows the bending and shear interaction diagram for a given DSPW, which has a shear and bending capacity of F_P and M_P respectively.

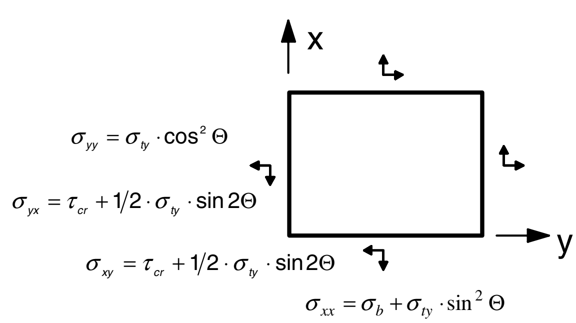

Figure 12. Bending and shear interaction of stresses in the steel panel

EFFECTIVENESS OF THE M-PFI METHOD

Results from tests conducted at the University of Alberta, Canada by Driver [5] and Behbahanifard [6] were used to evaluate the effectiveness of the M-PFI method. Details of the specimens tested are shown in Figure 14. The reason that these experimental studies were chosen is because they tested multi-storey DSPWs that experienced a significant amount of bending. For the large-scale steel plate wall shown in Figure 14 (see Driver et al. [5]), the M-PFI method leads to very satisfactory results when the moment contribution is accounted for, as shown in Figure 15.a. The wall tested by Behbahanifard [6], was Driver's specimen, excluding the first floor panel. For this specimen the M-PFI method also provides satisfactory results when the moment contribution is accounted for, as shown in Figure 15.b. From these comparisons it can be concluded that for steel plate walls made of steel plates welded to column and beam members, with rigid column to beam connections and adequate column capacity, the M-PFI method provides satisfactory results. Note that for the specimens investigated the angle of inclination for tension field was assumed to be 45° (Θ =45°).

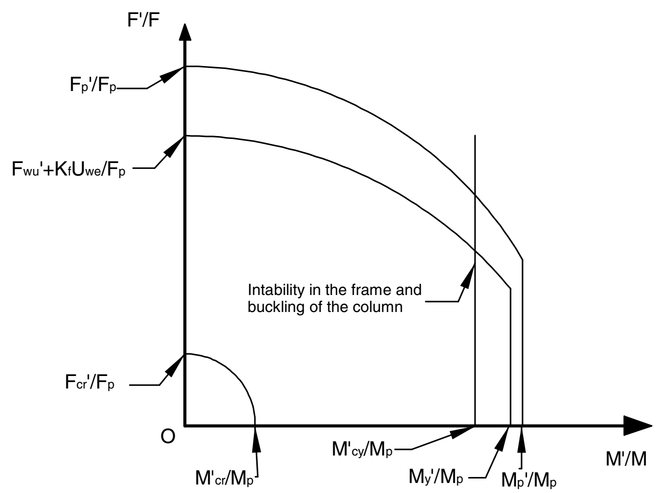

Figure 13. Modified load displacement diagram for shear resistance of the DSPW

CONCLUSIONS

In this paper an analytical model, the Modified Plate-Frame Interaction (M-PFI) method, has been introduced, and it has been demonstrated that the method predicts accurately the structural behavior of multi-storey steel plate walls. A significant advantage of this method is that many design parameters, such as the shear load-displacement values, strength, stiffness and limiting elastic displacement for the steel plate, and plate-frame interaction can be evaluated individually, and their effect on the overall wall capacity can easily be determined. It is also important to mention that the M-PFI model considers the behavior of DSPW not only for shear forces but also for overturning moments. This provides the designer with great flexibility for the design of ductile steel plate walls. An added benefit is that the method is suitable for incorporation in practical seismic design provisions.

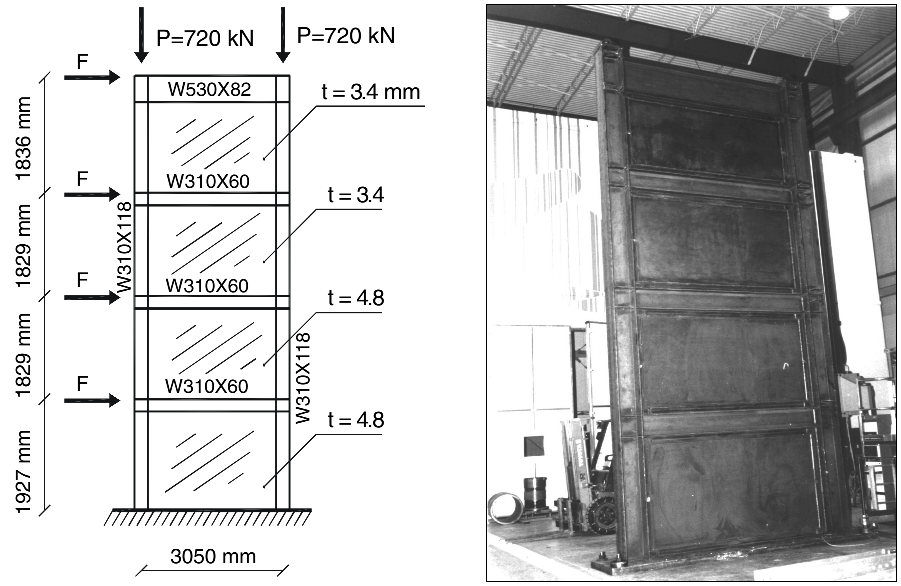

Figure 14. Details of specimens tested at University of Alberta test by Driver et al. (1997) (Photo courtesy of R. Driver)

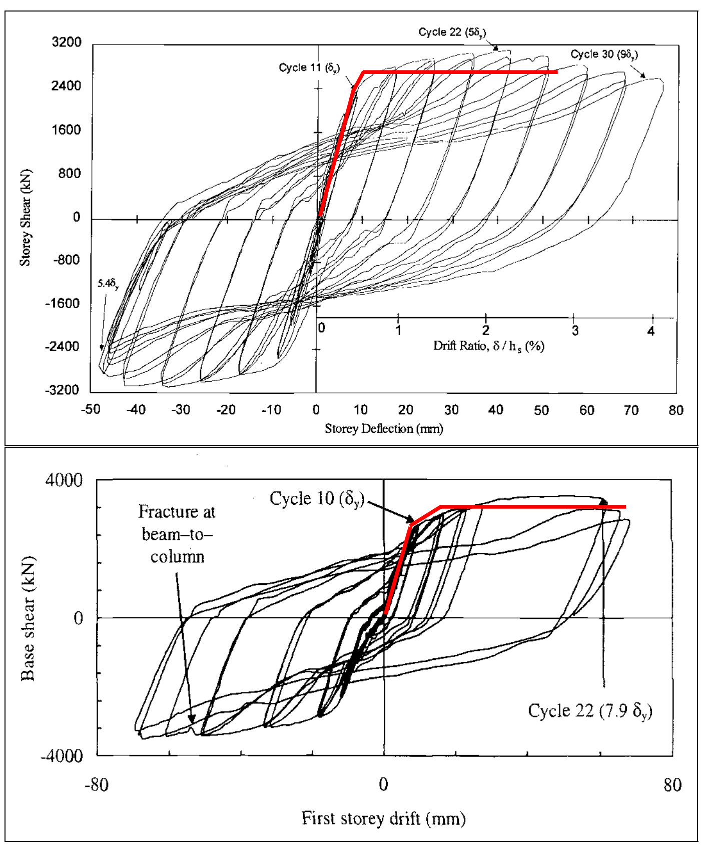

Figure 15. Comparison of M-PFI model prediction and experimental results from tests conducted at University of Alberta, a).after Driver et al. (1997) (top) and b). after Behbahanifrad (2003) (bottom)

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