Benford’s Law - Why And How To Use It

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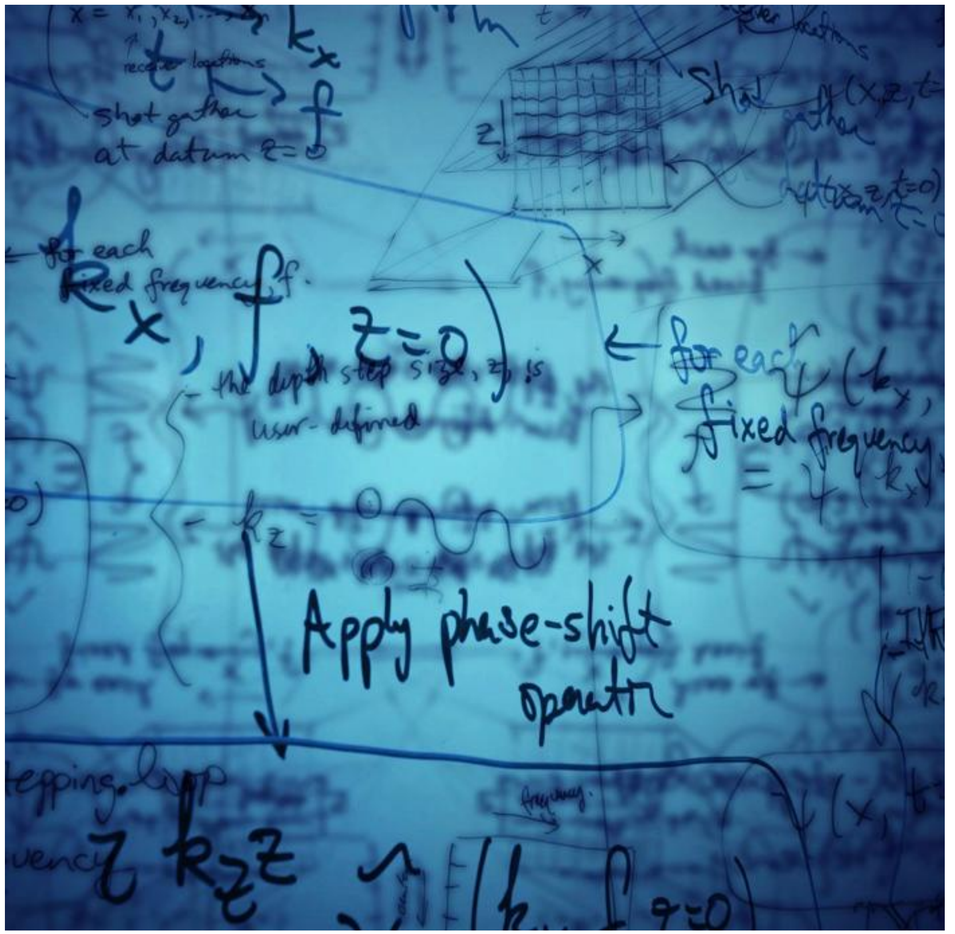
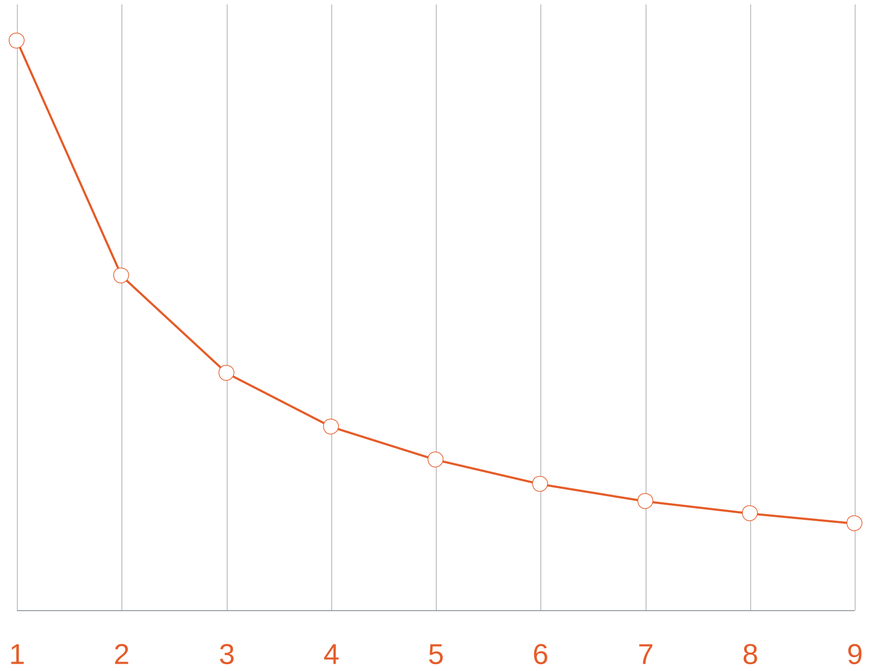
Definition

- Known as the “first digit law”, Benford’s Law states that in lists of numbers from many (but not all) real-life sources of data, the leading digit is distributed in a specific, non-uniform way.



Benford’s First Digit Chart



Expected Digit Frequency Percentages:

1 - 30.103%

2 - 17.609%

3 - 12.494%

4 - 9.691%

5 - 7.918%

6 - 6.695%

7 - 5.799%

8 - 5.115%

9 - 4.576%

Synopsis

- Efficient way to apply the smell test
- Easy to learn
- No need for special software
- Admissible in local, state, federal, and international criminal cases

- Disclaimer: Use together with other procedures

Early History

- **1881, Simon Newcomb**
  - initial discovery, article in American Journal of Mathematics
- **1938, Frank Benford**
  - initial testing took 6 years
  - total of 20,229 observations

More History

- **1961**: Pinkham, scale invariant
- **1988**: Carslaw, rounded numbers
- **1995**: Hill, mathematical proof
- **1996**: Nigrini, identified an accounting USE

Since 1996

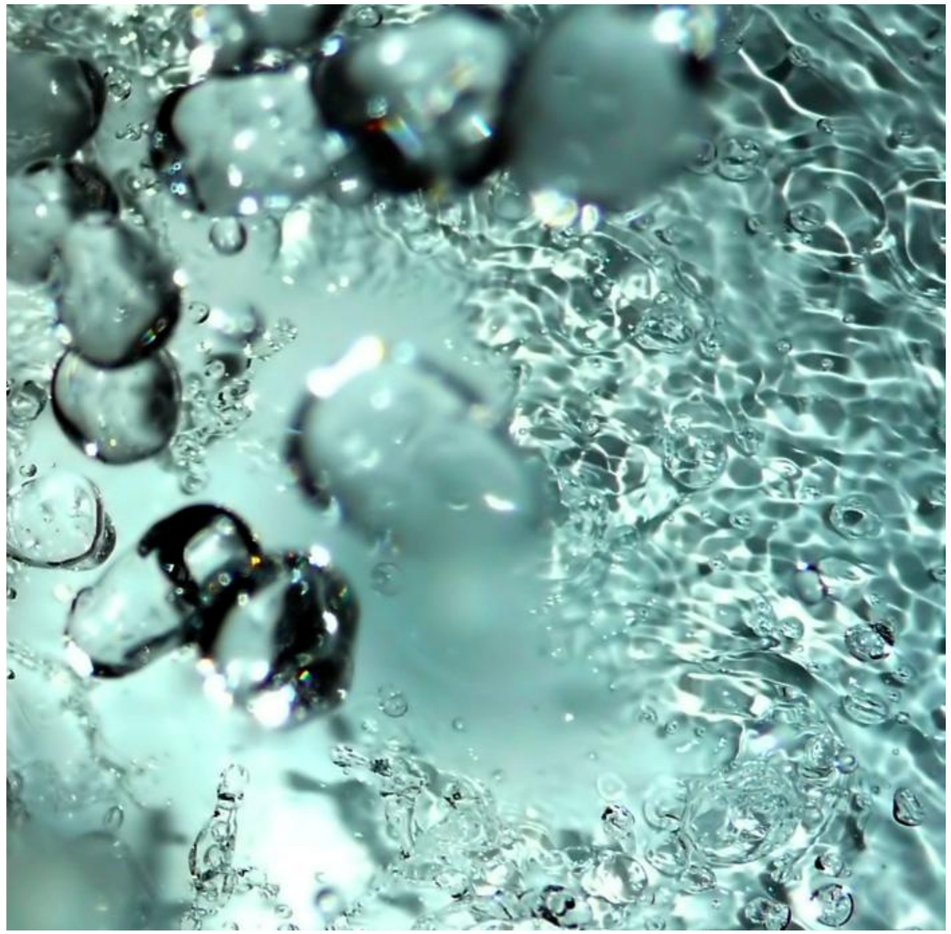
- **Publications**
  - Journal of Accountancy
  - New York Times
- **Proprietary Software**
  - ACL, IDEA, Microsoft Access
- **Major Users**
  - government authorities, litigators, bloggers and…

What It Does

- **Predicts the occurrence of digits**
  - Counts frequencies of digits
  - Improves sampling selection process
  - Digits 1-3 should be > 60% of first digits
- **Identifies amounts that do not conform to expectations**
  - The digit 9 should appear 4.5% of the time
  - Frauds that became big after starting small

Uses

- **To find fraud**
  - Percentages do not match expectations
- **To find inefficiency & errors**
  - Multiple checks for the same amount
  - Same amount, same invoice, different vendor
- **To find manipulative biases**
  - Management’s educated guesses



How: Five Tests

- **First Digit Test**
  - Count frequency of 1 – 9 as first digit
- **Second Digit Test**
  - What are we counting here?
- **First Two Digits Test**
- **First Three Digits Test**
- **Last Two Digits Test**

Examples

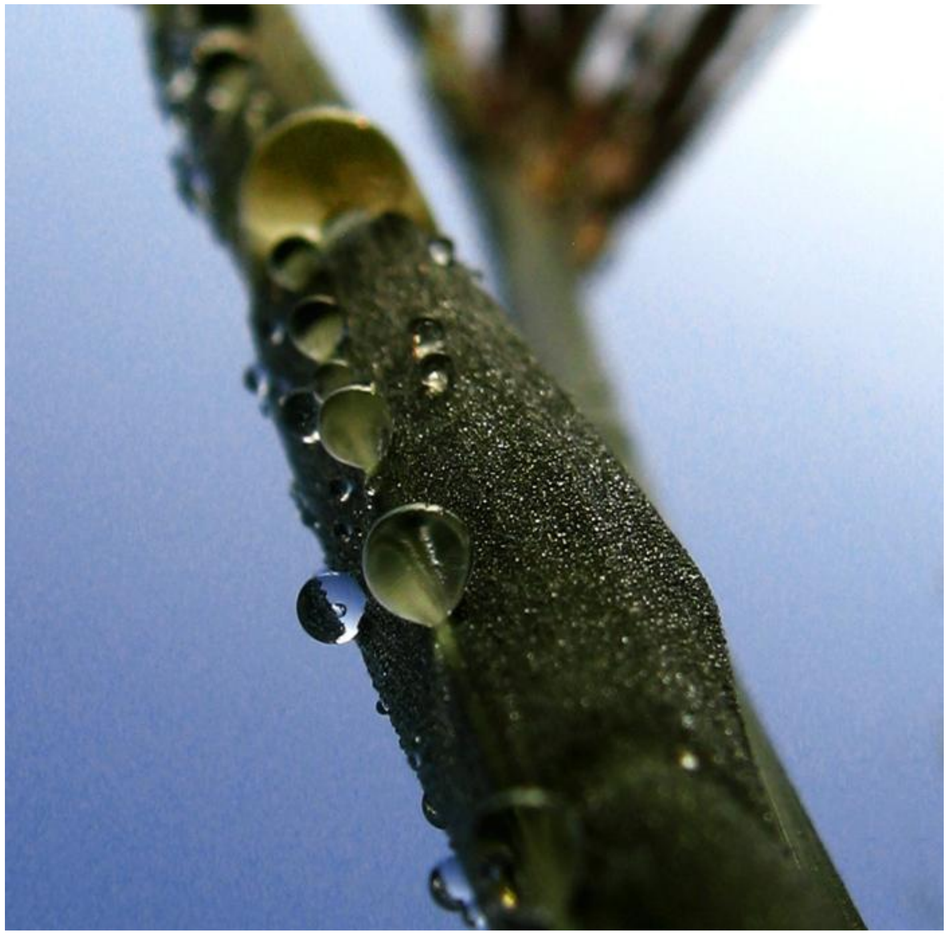
- **Benford's Law: "1" Appears More Often than Any Other Number**

- **$100 portfolio with a 10% rate of return**

- **Dow Jones: the next order of magnitude (a new "1"!) is reached faster and faster**

First & Second Digit Tests

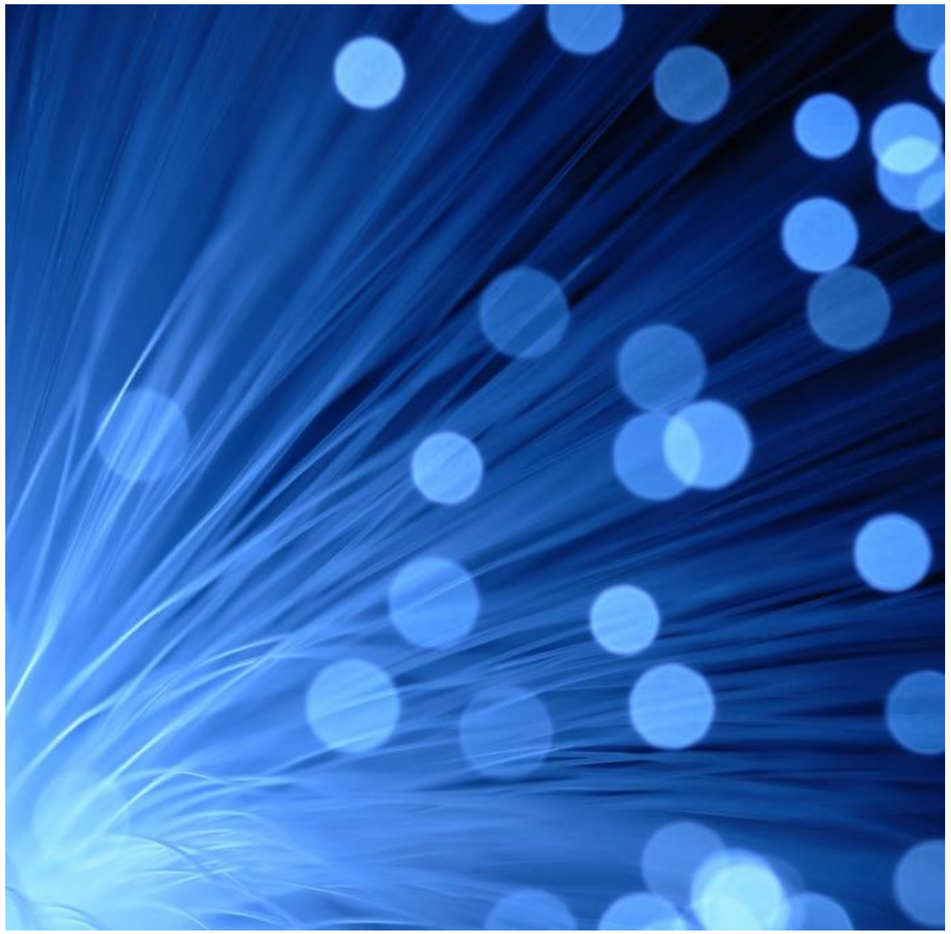
- Both are high level
- Both identify only obvious anomalies
- 1st digit checks reasonableness of data
- 2nd digit shows improper rounding

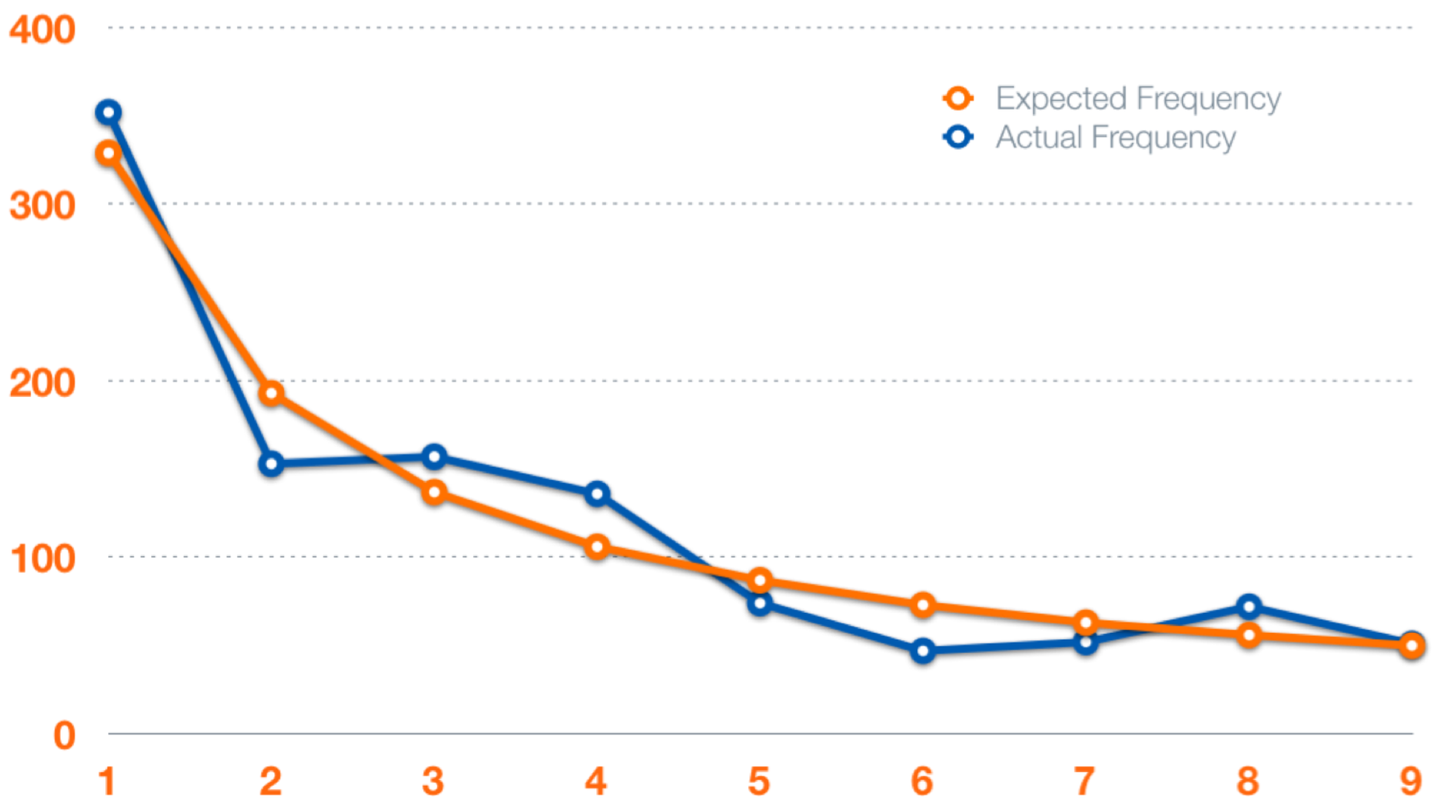


First Digit Test Table

First Digit	Actual Frequency	Expected Freq.	Variance #	Actual % Freq.	Expected % Freq.	Variance %
0	0	0	0	0%	0%	0%
1	352	329	23	32.176%	30.103%	2.073%
2	153	193	-40	13.985%	17.609%	-3.624%
3	157	137	20	14.351%	12.494%	1.857%
4	136	106	30	12.431%	9.691%	2.74%
5	74	87	-13	6.764%	7.918%	-1.154%
6	47	73	-26	4.296%	6.695%	-2.398%
7	52	63	-11	4.753%	5.799%	-1.046%
8	72	56	16	6.581%	5.115%	1.466%
9	51	50	1	4.662%	4.576%	0.086%

First Digit Test Chart

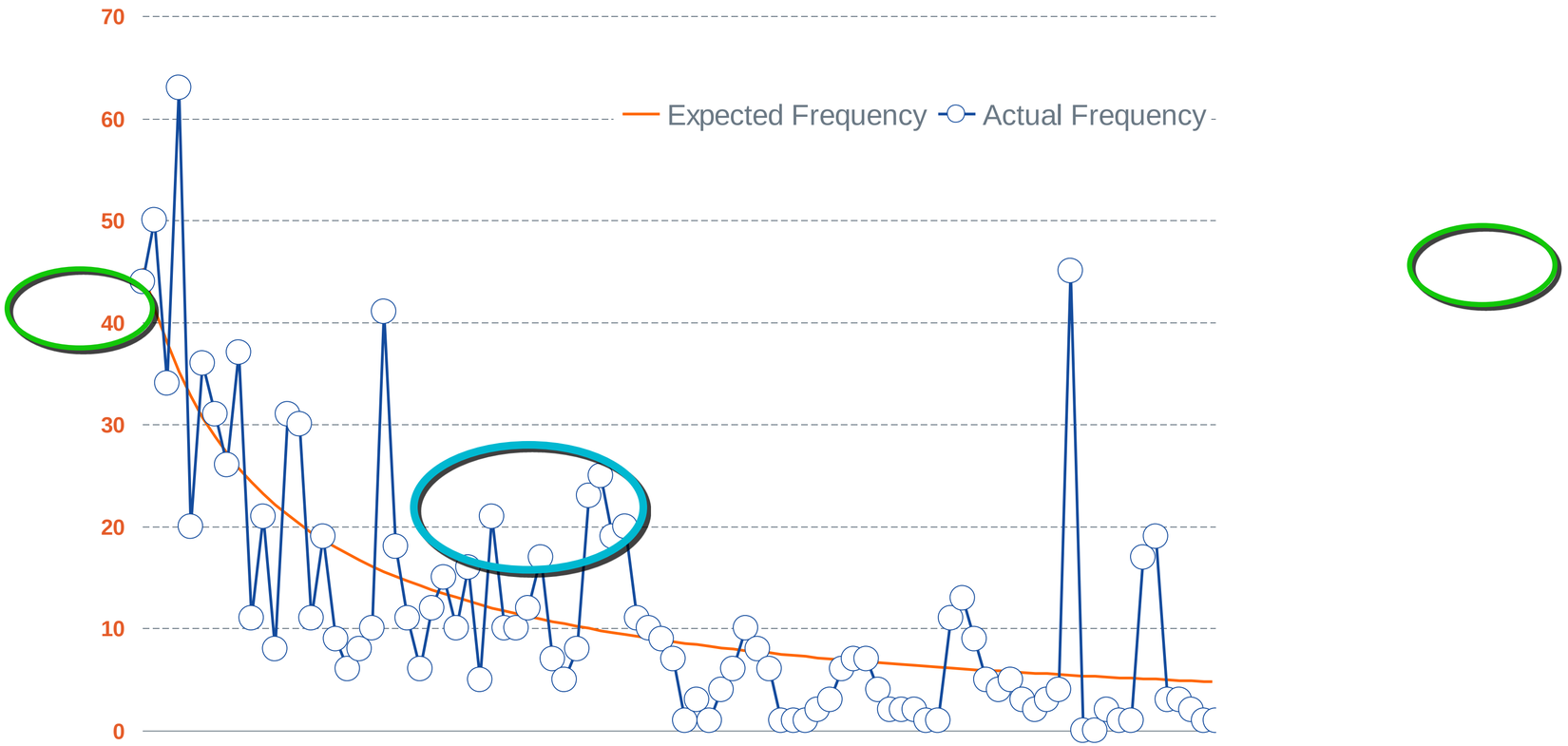




First Two Digits Test

- More focused
- Shows overused and underused digit patterns
- Provides an efficient audit sample for testing

First Two Digits Test Chart



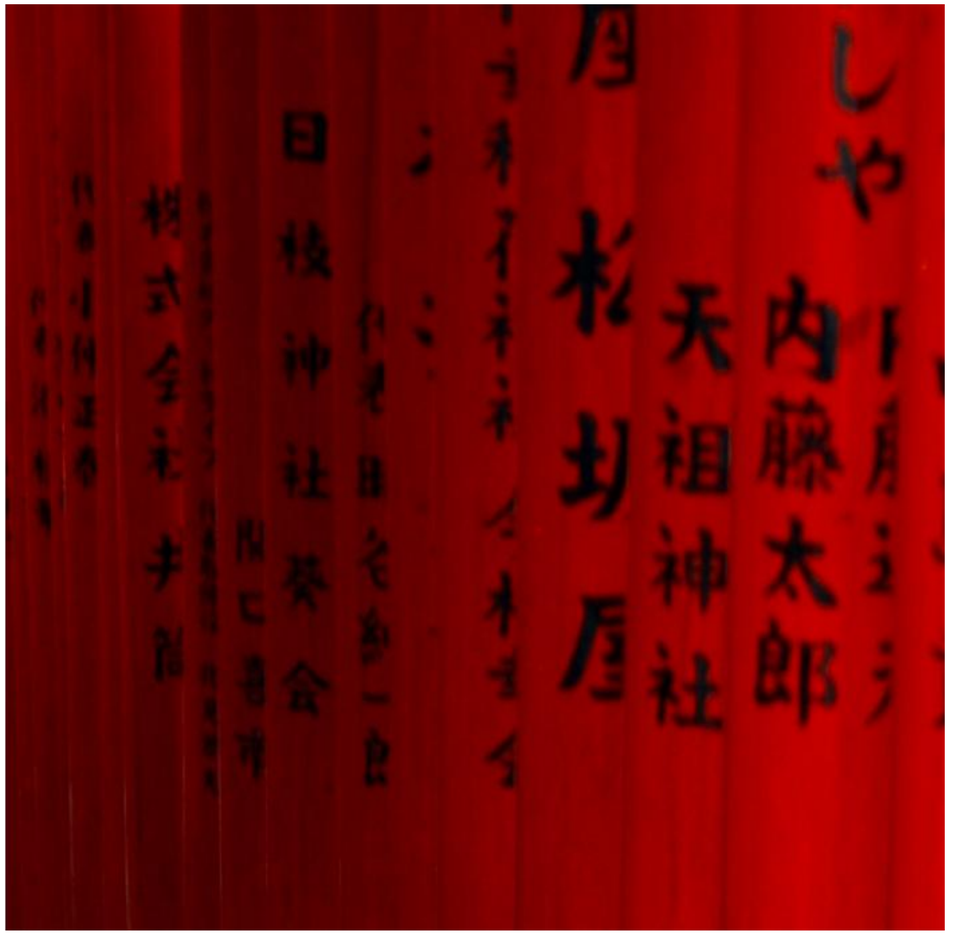
8	87	Sparkles INC	87.75	4132
8	87	Sparkles INC	87.75	4149
8	87	Sparkles INC	87.75	4167
8	87	Sparkles INC	87.75	4188
8	87	Sparkles	87.75	4204
8	87	Sparkles INC	87.75	4219
8	87	Sparkles INC	87.75	4226
8	87	Sparkles	87.75	4237
8	87	Sparkles	87.75	4239
8	87	Sparkles	87.75	4250
8	87	Sparkles	87.75	4259
8	87	Sparkles	87.75	4263
8	87	Sparkles	87.75	4296
8	87	Sparkles	87.75	4300
8	87	Sparkles	87.75	4318
8	87	Sparkles	87.75	4350
8	87	Sparkles	87.75	4375
8	87	Sparkles	87.75	4429
8	87	Sparkles	87.75	4528
8	87	Sparkles	87.75	4562
8	87	Sparkles INC	87.75	4643
8	87	Sparkles	87.75	4646
8	87	Sparkles	87.75	4648
8	87	Sparkles	87.75	4706
8	87	Sparkles	87.75	4707
8	87	Sparkles	87.75	4756
8	87	Sparkles	87.75	4773
8	87	Sparkles	87.75	4779
8	87	Sparkles	87.75	4803
8	87	Sparkles	87.75	4831
8	87	Sparkles	87.75	4837
8	87	Sparkles	87.75	4856
8	87	Sparkles	87.75	4878
8	87	Sparkles INC	87.75	4881
8	87	Sparkles INC	87.75	4888
8	87	Sparkles	87.75	4909
8	87	C Davis CO	87.75	4318
8	87	Sparkles	87.75	4976
8	87	Sparkles	87.75	4997
8	87	Sparkles	87.75	5027
8	87	Sparkles	87.75	5033
8	87	Sparkles INC	87.75	5080
8	87	Sparkles	8,775.44	5073
			12,460.94	

Relative Size Factor	100
Number of checks	43
Sparkles (incl "Inc" checks)	34
"Inc" checks	9

- *Multiple small payments for the same amount to the same vendor.*
- *Why is there a vendor with the same name without the “Inc”?*
- *Notice the single check to C Davis Co. for the same amount as the Sparkles checks.*
- *The $8,775 check - is it real?*

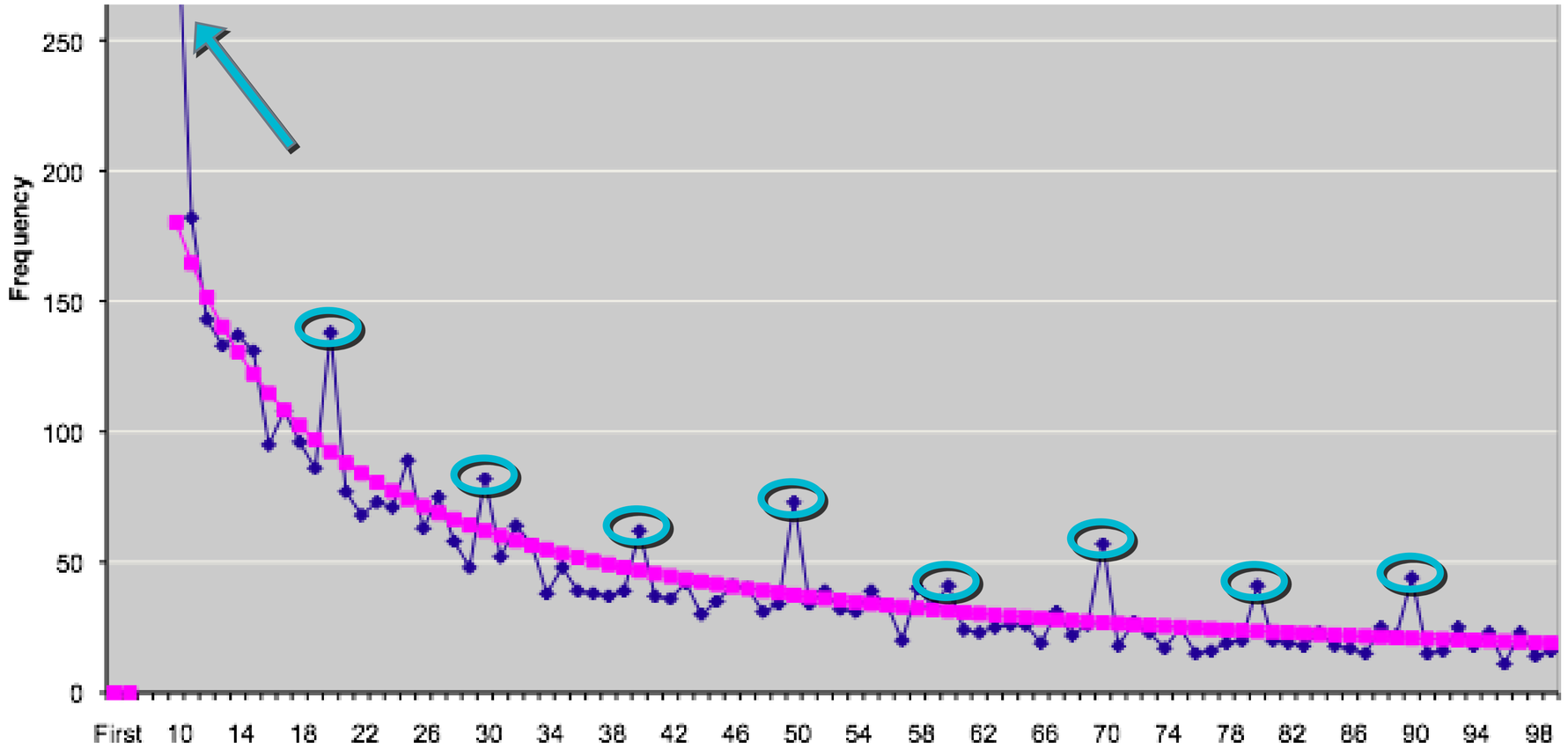
Rules for Data Sets

- Describe similar data
- No artificial minimums or maximums
- No pre-arranged numbers
- No aggregate totals
- One accounting period
- Large enough for patterns to manifest
- More small items and fewer large items

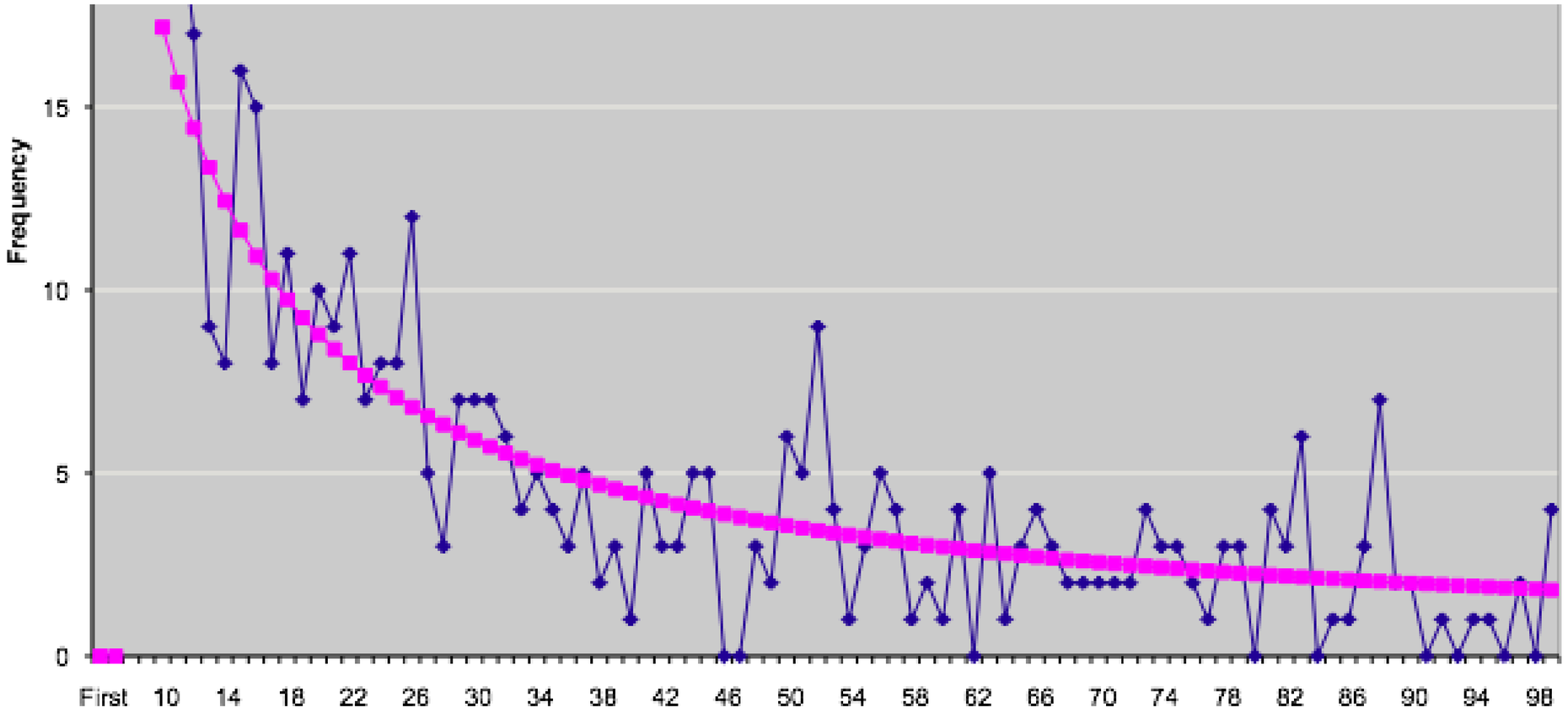


Two Concerns

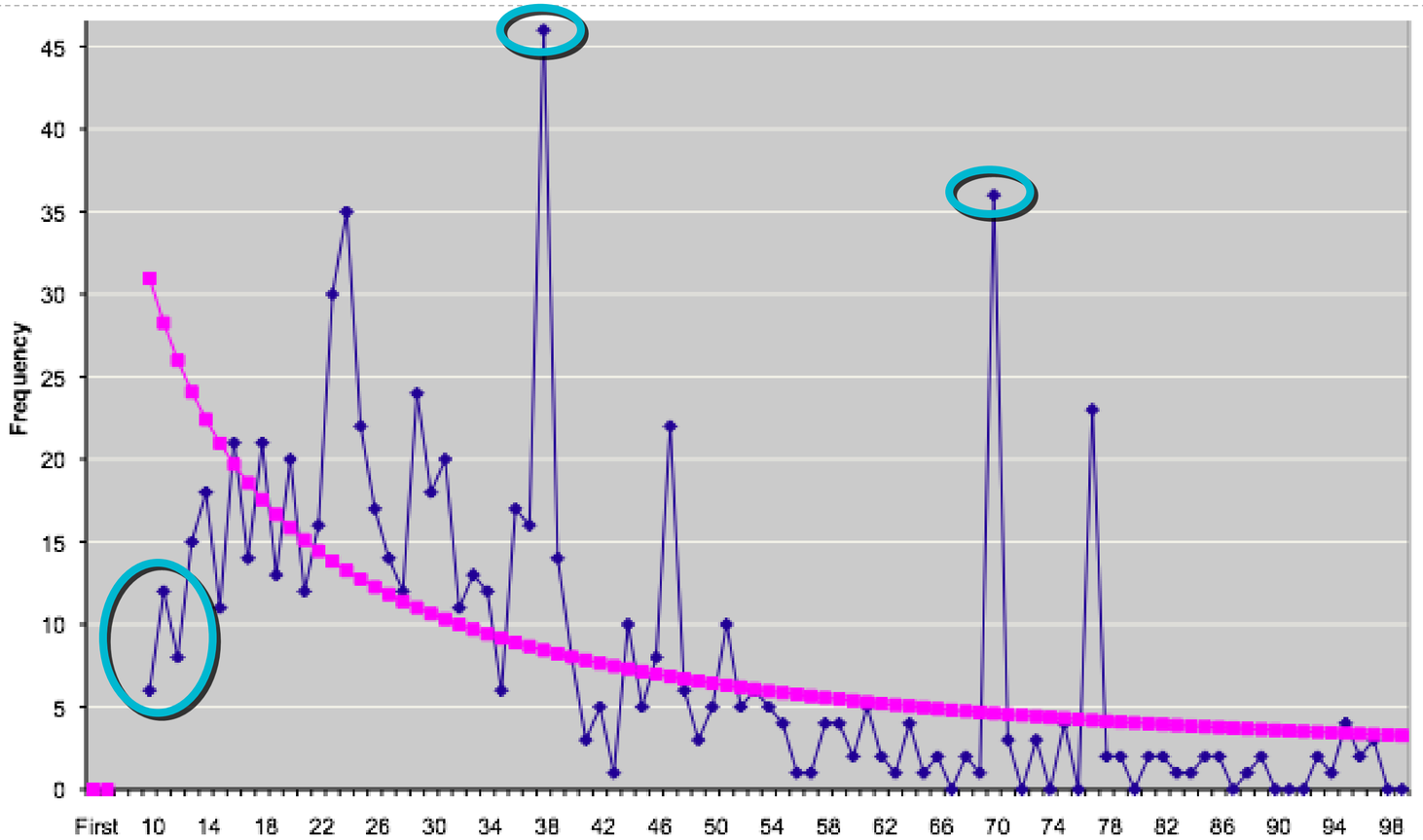
- **Intuitive**
  - A few aberrations will not trigger a significant departure from expectations
- **Statistical**
  - It takes smaller proportion of aberrations to trigger a departure when the data set has a large number of transactions



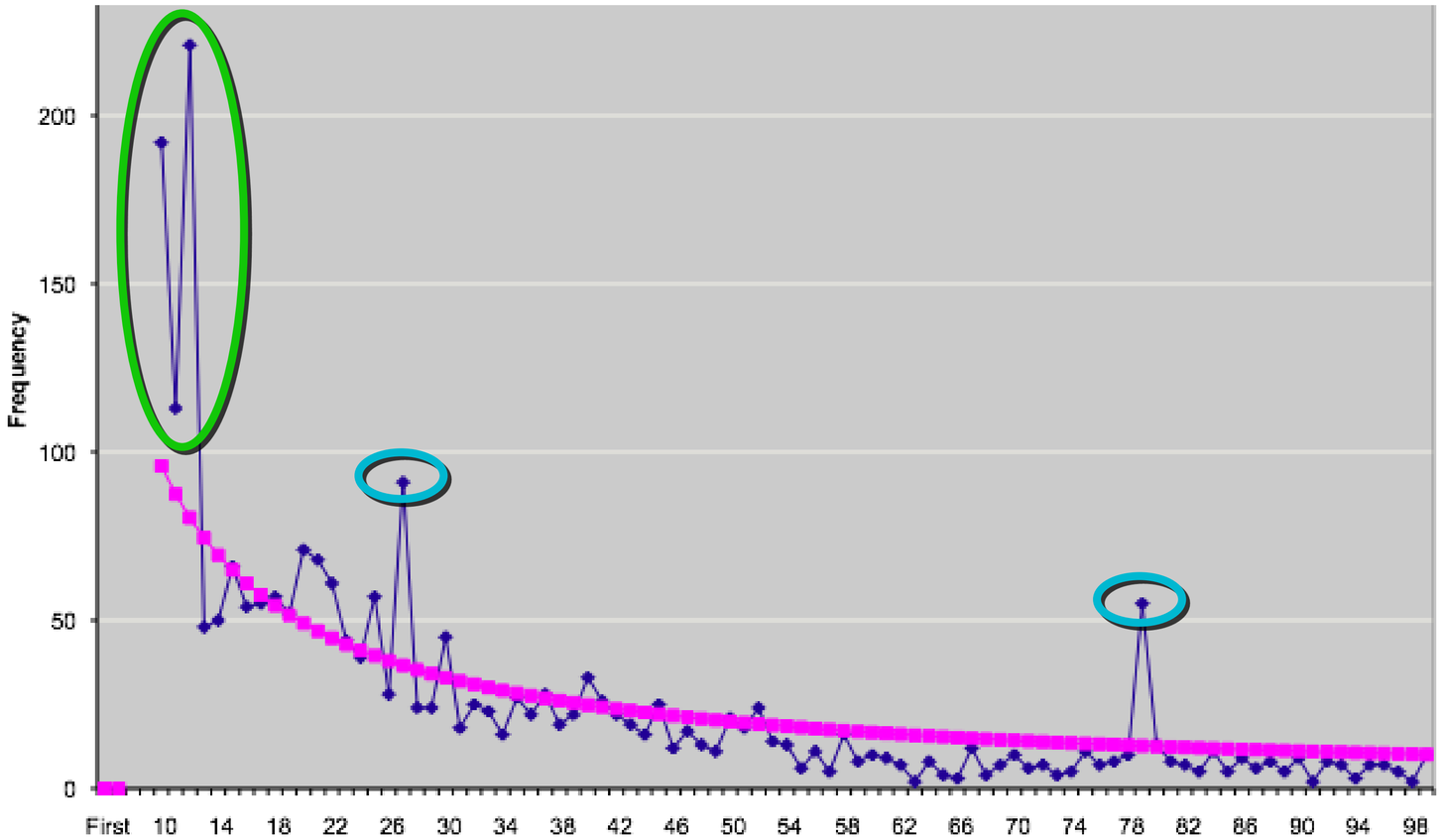
Example A: 4,356 Items



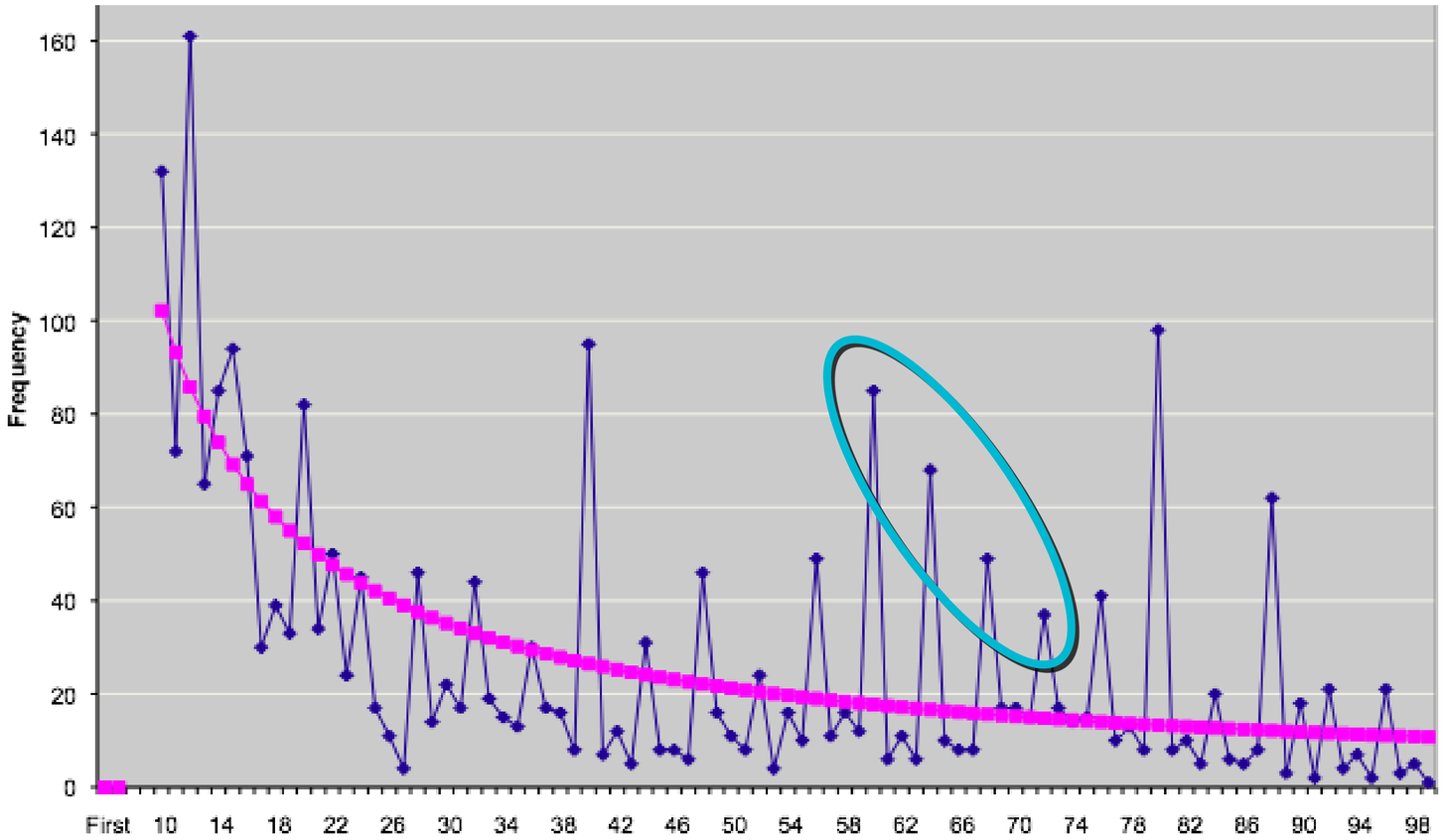
Example B: 415 Items



Example C: 748 Items



Example D: 2,316 Items



Example E: 2,469 Items

Good Uses

- Fraud inquiries
- Planning
- Individual financial statement accounts
- Scientific data, insurance claims, survey data, campaign financing …

Three A’s

- Adaptive Benford
- Almost Benford
- ANN

Additional Reading

- **Nigrini, Mark. Forensic Analystics: Methods and Techniques**; Wiley, 2011.
- **Ferraro, Eugene. Investigations in the Workplace**; Auerbach Publications, 2005.
- **Gibson, William. Pattern Recognition**; Berkeley, 2005.
- Numerous articles

Thank You!

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