CHAPTER

12

BERNOULLI AND ENERGY EQUATIONS

This chapter deals with two equations commonly used in fluid mechanics: Bernoulli and energy equations. The *Bernoulli equation* is concerned with the conservation of kinetic, potential, and flow energies of a fluid stream and their conversion to each other in regions of flow where net viscous forces are negligible and where other restrictive conditions apply. The *energy equation* is a statement of the conservation of energy principle. In fluid mechanics, it is found convenient to separate *mechanical energy* from *thermal energy* and to consider the conversion of mechanical energy to thermal energy as a result of frictional effects as *mechanical energy loss*. Then the energy equation becomes the *mechanical energy balance*.

In this chapter we derive the Bernoulli equation by applying Newton's second law to a fluid element along a streamline and demonstrate its use in a variety of applications. We continue with the development of the energy equation in a form suitable for use in fluid mechanics and introduce the concept of *head loss*. Finally, we apply the energy equation to various engineering systems.

Objectives

The objectives of this chapter are to:

- Understand the use and limitations of the Bernoulli equation, and apply it to solve a variety of fluid flow problems.
- Work with the energy equation expressed in terms of heads, and use it to determine turbine power output and pumping power requirements.

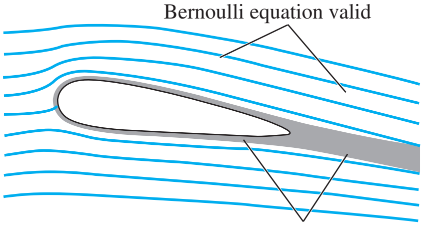


FIGURE 12–1

The *Bernoulli equation* is an *approximate* equation that is valid only in *inviscid regions of flow* where net viscous forces are negligibly small compared to inertial, gravitational, or pressure forces. Such regions occur outside of *boundary layers* and *wakes*.

12–1 ▪ THE BERNOULLI EQUATION

The **Bernoulli equation** is *an approximate relation between pressure, velocity, and elevation,* and is valid in *regions of steady, incompressible flow where net frictional forces are negligible* (Fig. 12–1). Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics. In this section, we derive the Bernoulli equation by applying the *conservation of linear momentum principle*, and we demonstrate both its usefulness and its limitations.

The key approximation in the derivation of the Bernoulli equation is that *viscous effects are negligibly small compared to inertial, gravitational, and pressure effects*. Since all fluids have viscosity (there is no such thing as an "inviscid fluid"), this approximation cannot be valid for an entire flow field of practical interest. In other words, we cannot apply the Bernoulli equation *everywhere* in a flow, no matter how small the fluid's viscosity. However, it turns out that the approximation *is* reasonable in certain *regions* of many practical flows. We refer to such regions as *inviscid regions of flow*, and we stress that they are *not* regions where the fluid itself is inviscid or frictionless, but rather they are regions where net viscous or frictional forces are negligibly small compared to other forces acting on fluid particles.

Care must be exercised when applying the Bernoulli equation since it is an approximation that applies only to inviscid regions of flow. In general, frictional effects are always important very close to solid walls (*boundary layers*) and directly downstream of bodies (*wakes*). Thus, the Bernoulli approximation is typically useful in flow regions outside of boundary layers and wakes, where the fluid motion is governed by the combined effects of pressure and gravity forces.

Acceleration of a Fluid Particle

The motion of a particle and the path it follows are described by the *velocity vector* as a function of time and space coordinates and the initial position of the particle. When the flow is *steady* (no change with time at a specified location), all particles that pass through the same point follow the same path (which is the *streamline*), and the velocity vectors remain tangent to the path at every point.

Often it is convenient to describe the motion of a particle in terms of its distance s along a streamline together with the radius of curvature along the streamline. The speed of the particle is related to the distance by $V = ds/dt$, which may vary along the streamline. In two-dimensional flow, the acceleration can be decomposed into two components: *streamwise acceleration* a_s along the streamline and *normal acceleration* a_n in the direction normal to the streamline, which is given as $a_n = V^2/R$. Note that streamwise acceleration is due to a change in speed along a streamline, and normal acceleration is due to a change in direction. For particles that move along a *straight path,* $a_n = 0$ since the radius of curvature is infinity and thus there is no change in direction. The Bernoulli equation results from a force balance along a streamline.

One may be tempted to think that acceleration is zero in steady flow since acceleration is the rate of change of velocity with time, and in steady flow there is no change with time. Well, a garden hose nozzle tells us that this understanding is not correct. Even in steady flow and thus constant mass flow rate, water accelerates through the nozzle (Fig. 12–2). *Steady* simply means *no change with time at a specified location,* but the value of a quantity may change from one location to another. In the case of a nozzle, the velocity of water remains constant at a specified point, but it changes from the inlet to the exit (water accelerates along the nozzle).

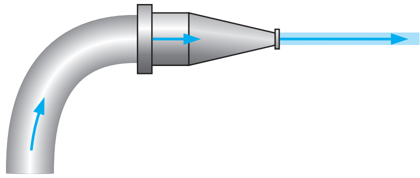


FIGURE 12–2
During steady flow, a fluid may not accelerate in time at a fixed point, but it may accelerate in space.

Mathematically, this can be expressed as follows: We take the velocity V of a fluid particle to be a function of s and t. Taking the total differential of $V(s, t)$ and dividing both sides by dt yield

$$dV = \frac{\partial V}{\partial s} ds + \frac{\partial V}{\partial t} dt \quad \text{and} \quad \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial t} \tag{12–1}$$

In steady flow $\partial V/\partial t = 0$ and thus $V = V(s)$, and the acceleration in the s-direction becomes

$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s} V = V\frac{dV}{ds} \tag{12–2}$$

where $V = ds/dt$ if we are following a fluid particle as it moves along a streamline. Therefore, acceleration in steady flow is due to the change of velocity with position.

Derivation of the Bernoulli Equation

Consider the motion of a fluid particle in a flow field in steady flow. Applying Newton's second law (which is referred to as the *linear momentum equation* in fluid mechanics) in the s-direction on a particle moving along a streamline gives

$$\sum F_s = ma_s \tag{12–3}$$

In regions of flow where net frictional forces are negligible, there is no pump or turbine, and there is no heat transfer along the streamline, the significant forces acting in the s-direction are the pressure (acting on both sides) and the component of the weight of the particle in the s-direction (Fig. 12–3). Therefore, Eq. 12–3 becomes

$$P\,dA - (P + dP)\,dA - W\sin\theta = mV\frac{dV}{ds} \tag{12–4}$$

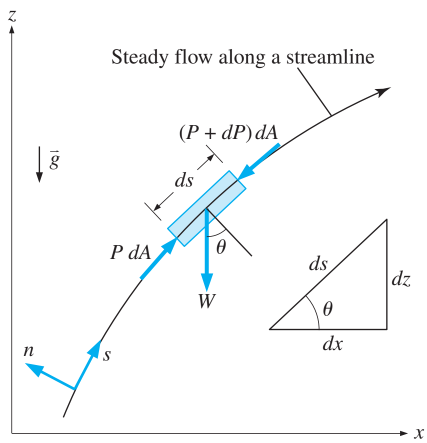


FIGURE 12–3
The forces acting on a fluid particle along a streamline.

where θ is the angle between the normal of the streamline and the vertical z-axis at that point, $m = \rho V = \rho\, dA\, ds$ is the mass, $W = mg = \rho g\, dA\, ds$ is the weight of the fluid particle, and $\sin\theta = dz/ds$. Substituting,

$$-dP\,dA - \rho g\,dA\,ds\frac{dz}{ds} = \rho\,dA\,ds\,V\frac{dV}{ds} \tag{12–5}$$

Canceling dA from each term and simplifying,

$$-dP - \rho g\,dz = \rho V\,dV \tag{12–6}$$

Noting that $V\,dV = \frac{1}{2}\,d(V^2)$ and dividing each term by ρ gives

$$\frac{dP}{\rho} + \tfrac{1}{2}\,d(V^2) + g\,dz = 0 \tag{12–7}$$

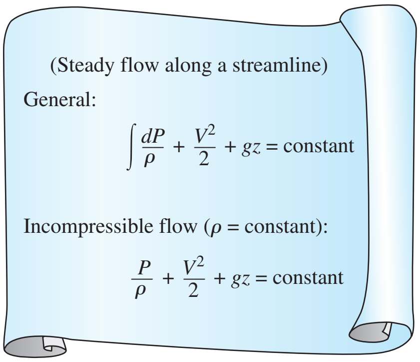


FIGURE 12–4
The incompressible Bernoulli equation is derived assuming incompressible flow, and thus it should not be used for flows with significant compressibility effects.

Integrating,

$$\textit{Steady flow:} \qquad \int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)} \tag{12–8}$$

since the last two terms are exact differentials. In the case of incompressible flow, the first term also becomes an exact differential, and integration gives

$$\textit{Steady, incompressible flow:} \quad \frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)} \tag{12–9}$$

This is the famous **Bernoulli equation** (Fig. 12–4), which is commonly used in fluid mechanics for steady, incompressible flow along a streamline in inviscid regions of flow. The Bernoulli equation was first stated in words by the Swiss mathematician Daniel Bernoulli (1700–1782) in a text written in 1738 when he was working in St. Petersburg, Russia. It was later derived in equation form by his associate Leonhard Euler (1707–1783) in 1755.

The value of the constant in Eq. 12–9 can be evaluated at any point on the streamline where the pressure, density, velocity, and elevation are known. The Bernoulli equation can also be written between any two points on the same streamline as

$$\textit{Steady, incompressible flow:} \qquad \frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2 \tag{12–10}$$

We recognize $V^2/2$ as *kinetic energy, gz* as *potential energy*, and P/ρ as *flow energy*, all per unit mass. Therefore, the Bernoulli equation can be viewed as an expression of *mechanical energy balance* and can be stated as follows (Fig. 12–5):

> The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when compressibility and frictional effects are negligible.

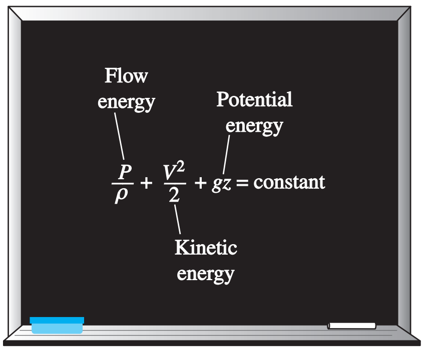


FIGURE 12–5
The Bernoulli equation states that the sum of the kinetic, potential, and flow energies (all per unit mass) of a fluid particle is constant along a streamline during steady flow.

The kinetic, potential, and flow energies are the mechanical forms of energy, and the Bernoulli equation can be viewed as the “conservation of mechanical energy principle.” This is equivalent to the general conservation of energy principle for systems that do not involve any conversion of mechanical energy and thermal energy to each other, and thus the mechanical energy and thermal energy are conserved separately. The Bernoulli equation states that during steady, incompressible flow with negligible friction, the various forms of mechanical energy are converted to each other, but their sum remains constant. In other words, there is no dissipation of mechanical energy during such flows since there is no friction that converts mechanical energy to sensible thermal (internal) energy.

Recall that energy is transferred to a system as work when a force is applied to the system through a distance. In the light of Newton’s second law of motion, the Bernoulli equation can also be viewed as: *The work done by the pressure and gravity forces on the fluid particle is equal to the increase in the kinetic energy of the particle.*

The Bernoulli equation is obtained from Newton’s second law for a fluid particle moving along a streamline. It can also be obtained from the *first law of thermodynamics* applied to a steady-flow system, as shown in Section 12–2.

Despite the highly restrictive approximations used in its derivation, the Bernoulli equation is commonly used in practice since a variety of practical fluid flow problems can be analyzed to reasonable accuracy with it. This is because many flows of practical engineering interest are steady (or at least steady in the mean), compressibility effects are relatively small, and net frictional forces are negligible in some regions of interest in the flow.

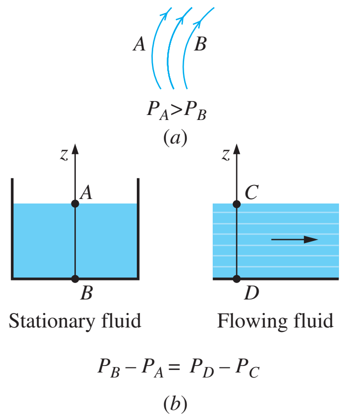


FIGURE 12–6
Pressure decreases towards the center of curvature when streamlines are curved (*a*), but the variation of pressure with elevation in steady, incompressible flow along a straight line (*b*) is the same as that in stationary fluid.

Force Balance across Streamlines

It is left as an exercise to show that a force balance in the direction n normal to the streamline yields the following relation applicable *across* the streamlines for steady, incompressible flow:

$$\frac{P}{\rho} + \int \frac{V^2}{R}\,dn + gz = \text{constant} \qquad \text{(across streamlines)} \qquad \textbf{(12–11)}$$

where R is the local radius of curvature of the streamline. For flow along curved streamlines (Fig 12–6*a*), the pressure *decreases* towards the center of curvature, and fluid particles experience a corresponding centripetal force and centripetal acceleration due to this pressure gradient.

For flow along a straight line, $R \to \infty$ and Eq. 12–11 reduces to $P/\rho + gz =$ constant or $P = -\rho gz +$ constant, which is an expression for the variation of hydrostatic pressure with vertical distance for a stationary fluid body. Therefore, the variation of pressure with elevation in steady, incompressible flow along a straight line in an inviscid region of flow is the same as that in the stationary fluid (Fig. 12–6*b*).

Unsteady, Compressible Flow

Similarly, using both terms in the acceleration expression (Eq. 12–3), it can be shown that the Bernoulli equation for *unsteady, compressible flow* is

$$\textit{Unsteady, compressible flow:} \qquad \int \frac{dP}{\rho} + \int \frac{\partial V}{\partial t}\,ds + \frac{V^2}{2} + gz = \text{constant} \qquad \textbf{(12–12)}$$

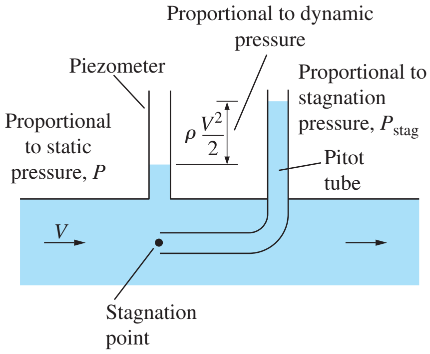
Static, Dynamic, and Stagnation Pressures

The Bernoulli equation states that the sum of the flow, kinetic, and potential energies of a fluid particle along a streamline is constant. Therefore, the kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change. This phenomenon can be made more visible by multiplying the Bernoulli equation by the density ρ,

$$P + \rho\frac{V^2}{2} + \rho gz = \text{constant (along a streamline)} \qquad \textbf{(12–13)}$$

Each term in this equation has pressure units, and thus each term represents some kind of pressure:

- P is the **static pressure** (it does not incorporate any dynamic effects); it represents the actual thermodynamic pressure of the fluid. This is the same as the pressure used in thermodynamics and property tables.
- $\rho V^2/2$ is the **dynamic pressure;** it represents the pressure rise when the fluid in motion is brought to a stop isentropically.



$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$

FIGURE 12–7
The static, dynamic, and stagnation pressures measured using piezometer tubes.

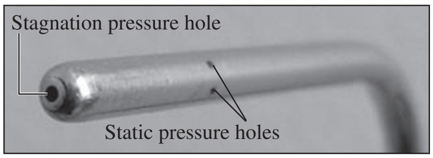


FIGURE 12–8
Close-up of a Pitot-static probe, showing the stagnation pressure hole and two of the five static circumferential pressure holes.
Photo by Po-Ya Abel Chuang. Used by permission.

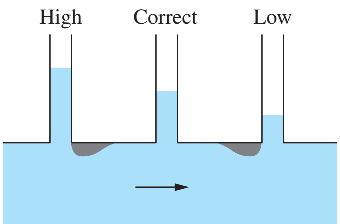


FIGURE 12–9
Careless drilling of the static pressure tap may result in an erroneous reading of the static pressure head.

- ρgz is the **hydrostatic pressure** term, which is not pressure in a real sense since its value depends on the reference level selected; it accounts for the elevation effects, i.e., fluid weight on pressure. (Be careful of the sign—unlike hydrostatic pressure ρgh which *increases* with fluid depth h, the hydrostatic pressure term ρgz *decreases* with fluid depth.)

The sum of the static, dynamic, and hydrostatic pressures is called the **total pressure**. Therefore, the Bernoulli equation states that *the total pressure along a streamline is constant*.

The sum of the static and dynamic pressures is called the **stagnation pressure**, and it is expressed as

$$P_{stag} = P + \rho \frac{V^2}{2} \qquad \text{(kPa)} \tag{12–14}$$

The stagnation pressure represents the pressure at a point where the fluid is brought to a complete stop isentropically. The static, dynamic, and stagnation pressures are shown in Fig. 12–7. When static and stagnation pressures are measured at a specified location, the fluid velocity at that location is calculated from

$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}} \tag{12–15}$$

Equation 12–15 is useful in the measurement of flow velocity when a combination of a static pressure tap and a Pitot tube is used, as illustrated in Fig. 12–7. A **static pressure tap** is simply a small hole drilled into a wall such that the plane of the hole is parallel to the flow direction. It measures the static pressure. A **Pitot tube** is a small tube with its open end aligned *into* the flow so as to sense the full impact pressure of the flowing fluid. It measures the stagnation pressure. In situations in which the static and stagnation pressure of a flowing *liquid* are greater than atmospheric pressure, a vertical transparent tube called a **piezometer tube** (or simply a **piezometer**) can be attached to the pressure tap and to the Pitot tube, as sketched in Fig. 12–8. The liquid rises in the piezometer tube to a column height (*head*) that is proportional to the pressure being measured. If the pressures to be measured are below atmospheric, or if measuring pressures in *gases*, piezometer tubes do not work. However, the static pressure tap and Pitot tube can still be used, but they must be connected to some other kind of pressure measurement device such as a U-tube manometer or a pressure transducer (Chap. 11). Sometimes it is convenient to integrate static pressure holes on a Pitot probe. The result is a **Pitot-static probe** (also called a **Pitot-Darcy probe**), as shown in Fig. 12–9 and discussed in more detail in Chap. 14. A Pitot-static probe connected to a pressure transducer or a manometer measures the dynamic pressure (and thus infers the fluid velocity) directly.

When the static pressure is measured by drilling a hole in the tube wall, care must be exercised to ensure that the opening of the hole is flush with the wall surface, with no extrusions before or after the hole (Fig. 12–9). Otherwise the reading would incorporate some dynamic effects, and thus it would be in error.

When a stationary body is immersed in a flowing stream, the fluid is brought to a stop at the nose of the body (the **stagnation point**). The flow streamline that extends from far upstream to the stagnation point is called the **stagnation streamline** (Fig. 12–10). For a two-dimensional flow in the *xy*-plane, the stagnation point is actually a *line* parallel to the *z*-axis, and the stagnation streamline is actually a *surface* that separates fluid that flows *over* the body from fluid that flows *under* the body. In an incompressible flow, the fluid decelerates nearly isentropically from its free-stream velocity to zero at the stagnation point, and the pressure at the stagnation point is thus the stagnation pressure.

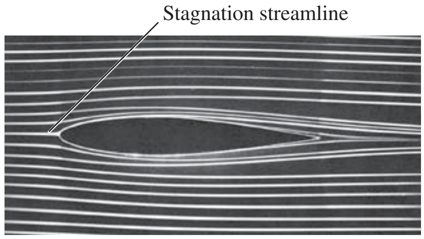


FIGURE 12–10
Streaklines produced by colored fluid introduced upstream of an airfoil; since the flow is steady, the streaklines are the same as streamlines and pathlines. The stagnation streamline is marked.

Courtesy ONERA. Photograph by Werlé.

Limitations on the Use of the Bernoulli Equation

The Bernoulli equation (Eq. 12–9) is one of the most frequently used and *misused* equations in fluid mechanics. Its versatility, simplicity, and ease of use make it a very valuable tool for use in analysis, but the same attributes also make it very tempting to misuse. Therefore, it is important to understand the restrictions on its applicability and observe the limitations on its use, as explained here:

1. **Steady flow** The first limitation on the Bernoulli equation is that it is applicable to *steady flow*. Therefore, it should not be used during the transient start-up and shut-down periods, or during periods of change in the flow conditions. Note that there is an unsteady form of the Bernoulli equation (Eq. 12–12), discussion of which is beyond the scope of the present text (see Panton, 1996).
2. **Negligible viscous effects** Every flow involves some friction, no matter how small, and *frictional effects* may or may not be negligible. The situation is complicated even more by the amount of error that can be tolerated. In general, frictional effects are negligible for short flow sections with large cross sections, especially at low flow velocities. Frictional effects are usually significant in long and narrow flow passages, in the wake region downstream of an object, and in *diverging flow sections* such as diffusers because of the increased possibility of the fluid separating from the walls in such geometries. Frictional effects are also significant near solid surfaces, and thus the Bernoulli equation is usually applicable along a streamline in the core region of the flow, but not along a streamline close to the surface (Fig. 12–11).

   A component that disturbs the streamlined structure of flow and thus causes considerable mixing and backflow such as a sharp entrance of a tube or a partially closed valve in a flow section can make the Bernoulli equation inapplicable.
3. **No shaft work** The Bernoulli equation was derived from a force balance on a particle moving along a streamline. Therefore, the Bernoulli equation is not applicable in a flow section that involves a pump, turbine, fan, or any other machine or impeller since such devices disrupt the streamlines and carry out energy interactions with the fluid particles. When the flow section considered involves any of these devices, the energy equation should be used instead to account for the shaft work input or output. However, the Bernoulli equation can still be applied to a flow section prior to or past a machine (assuming, of course, that the other restrictions on its use are satisfied). In such cases, the Bernoulli constant changes from upstream to downstream of the device.

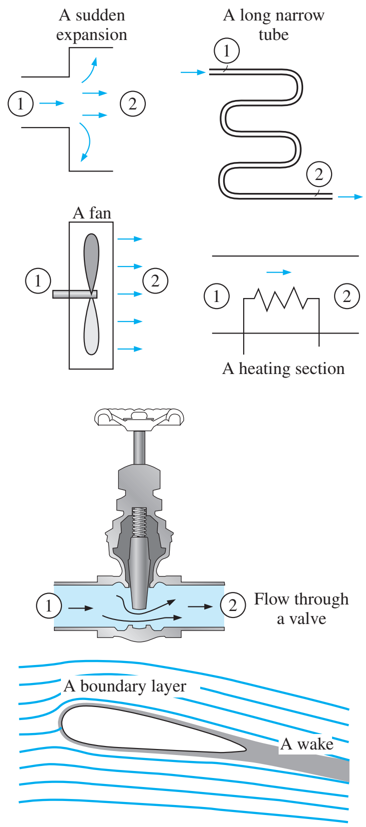


FIGURE 12–11
Frictional effects, heat transfer, and components that disturb the streamlined structure of flow make the Bernoulli equation invalid. It should *not* be used in any of the flows shown here.

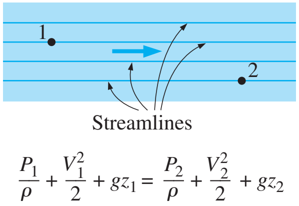


FIGURE 12–12
When the flow is irrotational, the Bernoulli equation becomes applicable between any two points along the flow (not just on the same streamline).

4. **Incompressible flow** One of the approximations used in the derivation of the Bernoulli equation is that ρ = constant and thus the flow is incompressible. This condition is satisfied by liquids and also by gases at Mach numbers less than about 0.3 since compressibility effects and thus density variations of gases are negligible at such relatively low velocities. Note that there is a compressible form of the Bernoulli equation (Eqs. 12–8 and 12–12).
5. **Negligible heat transfer** The density of a gas is inversely proportional to temperature, and thus the Bernoulli equation should not be used for flow sections that involve significant temperature change such as heating or cooling sections.
6. **Flow along a streamline** Strictly speaking, the Bernoulli equation $P/\rho + V^2/2 + gz = C$ is applicable along a streamline, and the value of the constant C is generally different for different streamlines. However, when a region of the flow is *irrotational* and there is no *vorticity* in the flow field, the value of the constant C remains the same for all streamlines, and the Bernoulli equation becomes applicable *across* streamlines as well (Fig. 12–12). Therefore, we do not need to be concerned about the streamlines when the flow is irrotational, and we can apply the Bernoulli equation between any two points in the irrotational region of the flow.

We derived the Bernoulli equation by considering two-dimensional flow in the xz-plane for simplicity, but the equation is valid for general three-dimensional flow as well, as long as it is applied along the same streamline. We should always keep in mind the approximations used in the derivation of the Bernoulli equation and make sure that they are valid before applying it.

Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)

It is often convenient to represent the level of mechanical energy graphically using *heights* to facilitate visualization of the various terms of the Bernoulli equation. This is done by dividing each term of the Bernoulli equation by g to give

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = \text{constant} \qquad \text{(along a streamline)} \qquad \textbf{(12–16)}$$

Each term in this equation has the dimension of length and represents some kind of "head" of a flowing fluid as follows:

- $P/\rho g$ is the **pressure head**; it represents the height of a fluid column that produces the static pressure P.
- $V^2/2g$ is the **velocity head**; it represents the elevation needed for a fluid to reach the velocity V during frictionless free fall.
- z is the **elevation head**; it represents the potential energy of the fluid.

Also, H is the **total head** for the flow. Therefore, the Bernoulli equation is expressed in terms of heads as: *The sum of the pressure, velocity, and elevation heads along a streamline is constant during steady flow when compressibility and frictional effects are negligible* (Fig. 12–13).

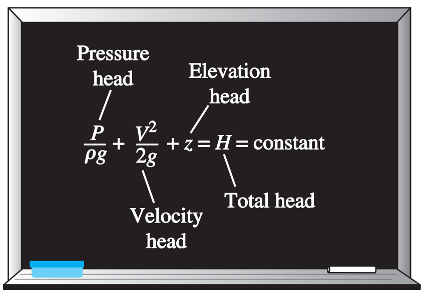


FIGURE 12–13
An alternative form of the Bernoulli equation is expressed in terms of heads as: *The sum of the pressure, velocity, and elevation heads is constant along a streamline.*

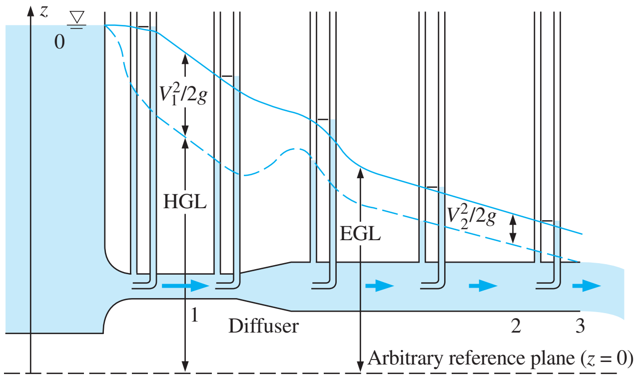


FIGURE 12–14
The *hydraulic grade line* (HGL) and the *energy grade line* (EGL) for free discharge from a reservoir through a horizontal pipe with a diffuser.

If a piezometer (which measures static pressure) is tapped into a pressurized pipe, as shown in Fig. 12–14, the liquid would rise to a height of $P/\rho g$ above the pipe center. The *hydraulic grade line* (HGL) is obtained by doing this at several locations along the pipe and drawing a curve through the liquid levels in the piezometers. The vertical distance above the pipe center is a measure of pressure within the pipe. Similarly, if a Pitot tube (measures static + dynamic pressure) is tapped into a pipe, the liquid would rise to a height of $P/\rho g + V^2/2g$ above the pipe center, or a distance of $V^2/2g$ above the HGL. The *energy grade line* (EGL) is obtained by doing this at several locations along the pipe and drawing a curve through the liquid levels in the Pitot tubes.

Noting that the fluid also has elevation head z (unless the reference level is taken to be the centerline of the pipe), the HGL and EGL are defined as follows: The line that represents the sum of the static pressure and the elevation heads, $P/\rho g + z$, is called the **hydraulic grade line**. The line that represents the total head of the fluid, $P/\rho g + V^2/2g + z$, is called the **energy grade line**. The difference between the heights of EGL and HGL is equal to the dynamic head, $V^2/2g$. We note the following about the HGL and EGL:

- For *stationary bodies* such as reservoirs or lakes, the EGL and HGL coincide with the free surface of the liquid. The elevation of the free surface z in such cases represents both the EGL and the HGL since the velocity is zero and the static (gage) pressure is zero.
- The EGL is always a distance $V^2/2g$ above the HGL. These two curves approach each other as the velocity decreases, and they diverge as the velocity increases. The height of the HGL decreases as the velocity increases, and vice versa.
- In an *idealized Bernoulli-type flow,* EGL is horizontal and its height remains constant. This would also be the case for HGL when the flow velocity is constant (Fig. 12–15).
- For *open-channel flow,* the HGL coincides with the free surface of the liquid, and the EGL is a distance $V^2/2g$ above the free surface.
- At a *pipe exit,* the pressure head is zero (atmospheric pressure) and thus the HGL coincides with the pipe outlet (location 3 on Fig. 12–14).
- The *mechanical energy loss* due to frictional effects (conversion to thermal energy) causes the EGL and HGL to slope downward in the direction of flow. The slope is a measure of the head loss in the pipe.

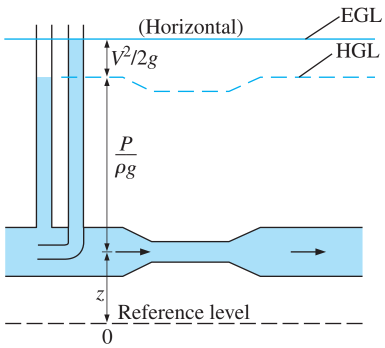


FIGURE 12–15
In an idealized Bernoulli-type flow, EGL is horizontal and its height remains constant. But this is not the case for HGL when the flow velocity varies along the flow.

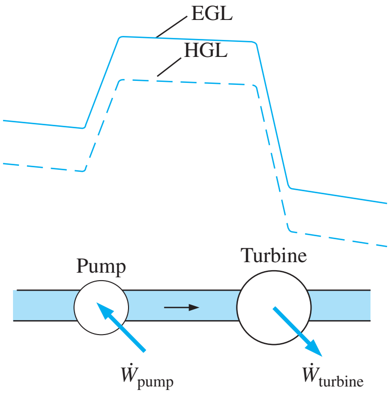


FIGURE 12–16
A *steep jump* occurs in EGL and HGL whenever mechanical energy is added to the fluid by a pump, and a *steep drop* occurs whenever mechanical energy is removed from the fluid by a turbine.

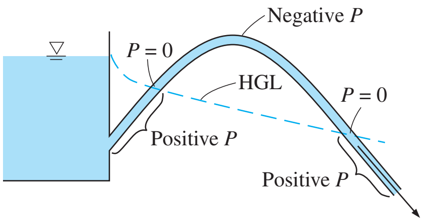


FIGURE 12–17
The gage pressure of a fluid is zero at locations where the HGL *intersects* the fluid, and the gage pressure is negative (vacuum) in a flow section that lies above the HGL.

A component that generates significant frictional effects such as a valve causes a sudden drop in both EGL and HGL at that location.

- A *steep jump* occurs in EGL and HGL whenever mechanical energy is added to the fluid (by a pump, for example). Likewise, a *steep drop* occurs in EGL and HGL whenever mechanical energy is removed from the fluid (by a turbine, for example), as shown in Fig. 12–16.
- The gage pressure of a fluid is zero at locations where the HGL *intersects* the fluid. The pressure in a flow section that lies above the HGL is negative, and the pressure in a section that lies below the HGL is positive (Fig. 12–17). Therefore, an accurate drawing of a piping system overlaid with the HGL can be used to determine the regions where the pressure in the pipe is negative (below atmospheric pressure).

The last remark enables us to avoid situations in which the pressure drops below the vapor pressure of the liquid (which may cause *cavitation*). Proper consideration is necessary in the placement of a liquid pump to ensure that the suction side pressure does not fall too low, especially at elevated temperatures where vapor pressure is higher than it is at low temperatures.

Now we examine Fig. 12–14 more closely. At point 0 (at the liquid surface), EGL and HGL are even with the liquid surface since there is no flow there. HGL decreases rapidly as the liquid accelerates into the pipe; however, EGL decreases very slowly through the well-rounded pipe inlet. EGL declines continually along the flow direction due to friction and other irreversible losses in the flow. EGL cannot increase in the flow direction unless energy is supplied to the fluid. HGL can rise or fall in the flow direction, but can never exceed EGL. HGL rises in the diffuser section as the velocity decreases, and the static pressure recovers somewhat; the total pressure does *not* recover, however, and EGL decreases through the diffuser. The difference between EGL and HGL is $V_1^2/2g$ at point 1, and $V_2^2/2g$ at point 2. Since $V_1 > V_2$, the difference between the two grade lines is larger at point 1 than at point 2. The downward slope of both grade lines is larger for the smaller diameter section of pipe since the frictional head loss is greater. Finally, HGL decays to the liquid surface at the outlet since the pressure there is atmospheric. However, EGL is still higher than HGL by the amount $V_2^2/2g$ since $V_3 = V_2$ at the outlet.

Applications of the Bernoulli Equation

So far, we have discussed the fundamental aspects of the Bernoulli equation. Now, we demonstrate its use in a wide range of applications through examples.

EXAMPLE 12–1 Spraying Water into the Air

Water is flowing from a garden hose (Fig. 12–18). A child places his thumb to cover most of the hose outlet, causing a thin jet of high-speed water to emerge. The pressure in the hose just upstream of his thumb is 400 kPa. If the hose is held upward, what is the maximum height that the jet could achieve?

Solution Water from a hose attached to the water main is sprayed into the air. The maximum height the water jet can rise is to be determined.

Assumptions **1** The flow exiting into the air is steady, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The surface tension effects are negligible. **3** The friction between the water and air is negligible. **4** The irreversibilities that occur at the outlet of the hose due to abrupt contraction are not taken into account.
Properties We take the density of water to be 1000 kg/m^3.
Analysis This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. The water height will be maximum under the stated assumptions. The velocity inside the hose is relatively low ($V_1^2 << V_j^2$, and thus $V_1 \cong 0$ compared to V_j) and we take the elevation just below the hose outlet as the reference level ($z_1 = 0$). At the top of the water trajectory $V_2 = 0$, and atmospheric pressure pertains. Then the Bernoulli equation along a streamline from 1 to 2 simplifies to

$$\frac{P_1}{\rho g} + \overbrace{\cancel{\frac{V_1^2}{2g}}}^{\approx 0} + \overbrace{\cancel{z_1}}^{0} = \frac{P_2}{\rho g} + \overbrace{\cancel{\frac{V_2^2}{2g}}}^{0} + z_2 \quad \rightarrow \quad \frac{P_1}{\rho g} = \frac{P_{atm}}{\rho g} + z_2$$

Solving for z_2 and substituting,

$$z_2 = \frac{P_1 - P_{atm}}{\rho g} = \frac{P_{1,\,gage}}{\rho g} = \frac{400 \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right)\left(\frac{1 \text{ kg·m/s}^2}{1 \text{ N}}\right)$$
$$= \mathbf{40.8\ m}$$

Therefore, the water jet can rise as high as 40.8 m into the sky in this case.
Discussion The result obtained by the Bernoulli equation represents the upper limit and should be interpreted accordingly. It tells us that the water cannot possibly rise more than 40.8 m, and, in all likelihood, the rise will be much less than 40.8 m due to irreversible losses that we neglected.

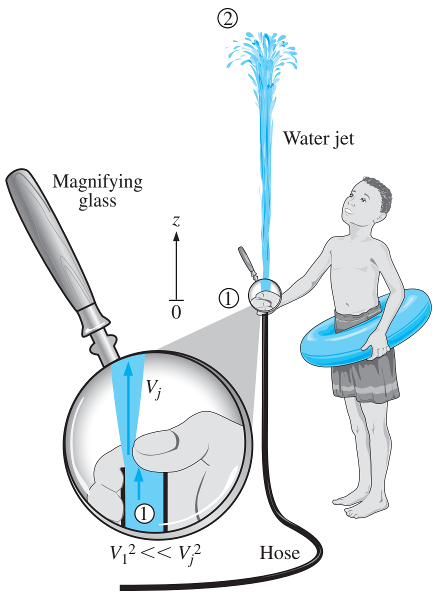


FIGURE 12–18
Schematic for Example 12–1. Inset shows a magnified view of the hose outlet region.

EXAMPLE 12–2 Water Discharge from a Large Tank

A large tank open to the atmosphere is filled with water to a height of 5 m from the outlet tap (Fig. 12–19). A tap near the bottom of the tank is now opened, and water flows out from the smooth and rounded outlet. Determine the maximum water velocity at the outlet.

Solution A tap near the bottom of a tank is opened. The maximum exit velocity of water from the tank is to be determined.
Assumptions **1** The flow is incompressible and irrotational (except very close to the walls). **2** The water drains slowly enough that the flow can be approximated as steady (actually quasi-steady when the tank begins to drain). **3** Irreversible losses in the tap region are neglected.
Analysis This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. We take point 1 to be at the free surface of water so that $P_1 = P_{atm}$ (open to the atmosphere), $V_1^2 << V_2^2$ and thus $V_1 \cong 0$ compared to V_2 (the tank is very large relative to the outlet), $z_1 = 5$ m and $z_2 = 0$ (we take the reference level at the center of the outlet). Also, $P_2 = P_{atm}$ (water discharges into the atmosphere). For flow along a streamline from 1 to 2, the Bernoulli equation simplifies to

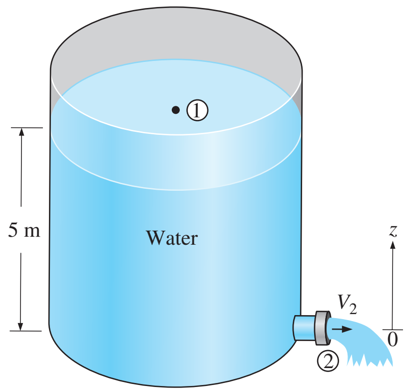


FIGURE 12–19
Schematic for Example 12–2.

$$\frac{\cancel{P_1}}{\rho g} + \frac{\cancel{V_1^2}^{\approx 0}}{2g} + z_1 = \frac{\cancel{P_2}}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2}^{0} \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

Solving for V_2 and substituting,

$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81\ \text{m/s}^2)(5\ \text{m})} = \mathbf{9.9\ m/s}$$

The relation $V = \sqrt{2gz}$ is called the **Torricelli equation**.

Therefore, the water leaves the tank with an initial maximum velocity of 9.9 m/s. This is the same velocity that would manifest if a solid were dropped a distance of 5 m in the absence of air friction drag. (What would the velocity be if the tap were at the bottom of the tank instead of on the side?)

Discussion If the orifice were sharp-edged instead of rounded, then the flow would be disturbed, and the average exit velocity would be less than 9.9 m/s. Care must be exercised when attempting to apply the Bernoulli equation to situations where abrupt expansions or contractions occur since the friction and flow disturbance in such cases may not be negligible. From conversion of mass, $(V_1/V_2)^2 = (D_2/D_1)^4$. So, for example, if $D_2/D_1 = 0.1$, then $(V_1/V_2)^2 = 0.0001$, and our approximation that $V_1^2 << V_2^2$ is justified.

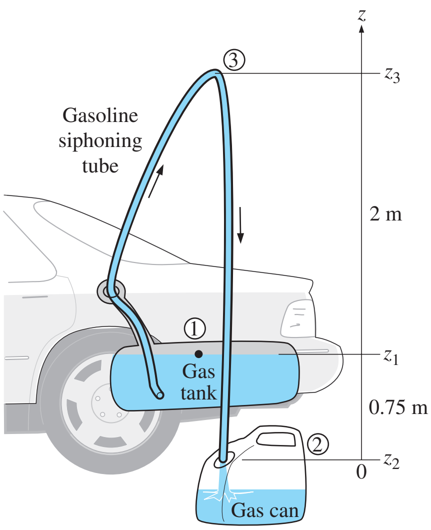


FIGURE 12–20
Schematic for Example 12–3.

EXAMPLE 12–3 Siphoning Out Gasoline from a Fuel Tank

During a trip to the beach (P_{atm} = 1 atm = 101.3 kPa), a car runs out of gasoline, and it becomes necessary to siphon gas out of the car of a Good Samaritan (Fig. 12–20). The siphon is a small-diameter hose, and to start the siphon it is necessary to insert one siphon end in the full gas tank, fill the hose with gasoline via suction, and then place the other end in a gas can below the level of the gas tank. The difference in pressure between point 1 (at the free surface of the gasoline in the tank) and point 2 (at the outlet of the tube) causes the liquid to flow from the higher to the lower elevation. Point 2 is located 0.75 m below point 1 in this case, and point 3 is located 2 m above point 1. The siphon diameter is 5 mm, and frictional losses in the siphon are to be disregarded. Determine (*a*) the minimum time to withdraw 4 L of gasoline from the tank to the can and (*b*) the pressure at point 3. The density of gasoline is 750 kg/m³.

Solution Gasoline is to be siphoned from a tank. The minimum time it takes to withdraw 4 L of gasoline and the pressure at the highest point in the system are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Even though the Bernoulli equation is not valid through the pipe because of frictional losses, we employ the Bernoulli equation anyway in order to obtain a *best-case estimate*. **3** The change in the gasoline surface level inside the tank is negligible compared to elevations z_1 and z_2 during the siphoning period.

Properties The density of gasoline is given to be 750 kg/m³.

Analysis (*a*) We take point 1 to be at the free surface of gasoline in the tank so that $P_1 = P_{atm}$ (open to the atmosphere), $V_1 \cong 0$ (the tank is large relative to the tube diameter), and $z_2 = 0$ (point 2 is taken as the reference level). Also, $P_2 = P_{atm}$ (gasoline discharges into the atmosphere). Then the Bernoulli equation simplifies to

$$\frac{\cancel{P_1}}{\rho g} + \frac{\cancel{V_1^2}^{\approx 0}}{2g} + z_1 = \frac{\cancel{P_2}}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2}^{0} \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

Solving for V_2 and substituting,

$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81 \text{ m/s}^2)(0.75 \text{ m})} = 3.84 \text{ m/s}$$

The cross-sectional area of the tube and the flow rate of gasoline are

$$A = \pi D^2/4 = \pi(5 \times 10^{-3} \text{ m})^2/4 = 1.96 \times 10^{-5} \text{ m}^2$$

$$\dot{V} = V_2 A = (3.84 \text{ m/s})(1.96 \times 10^{-5} \text{ m}^2) = 7.53 \times 10^{-5} \text{ m}^3/\text{s} = 0.0753 \text{ L/s}$$

Then the time needed to siphon 4 L of gasoline becomes

$$\Delta t = \frac{V}{\dot{V}} = \frac{4 \text{ L}}{0.0753 \text{ L/s}} = \mathbf{53.1 \text{ s}}$$

(*b*) The pressure at point 3 is determined by writing the Bernoulli equation along a streamline between points 3 and 2. Noting that $V_2 = V_3$ (conservation of mass), $z_2 = 0$, and $P_2 = P_{\text{atm}}$,

$$\frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2}^{\,0} = \frac{P_3}{\rho g} + \cancel{\frac{V_3^2}{2g}} + z_3 \quad \rightarrow \quad \frac{P_{\text{atm}}}{\rho g} = \frac{P_3}{\rho g} + z_3$$

Solving for P_3 and substituting,

$$\begin{aligned} P_3 &= P_{\text{atm}} - \rho g z_3 \\ &= 101.3 \text{ kPa} - (750 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.75 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg·m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right) \\ &= \mathbf{81.1 \text{ kPa}} \end{aligned}$$

Discussion The siphoning time is determined by neglecting frictional effects, and thus this is the *minimum time* required. In reality, the time will be longer than 53.1 s because of friction between the gasoline and the tube surface, along with other irreversible losses, as discussed in Chap. 14. Also, the pressure at point 3 is below the atmospheric pressure. If the elevation difference between points 1 and 3 is too high, the pressure at point 3 may drop below the vapor pressure of gasoline at the gasoline temperature, and some gasoline may evaporate (cavitate). The vapor then may form a pocket at the top and halt the flow of gasoline.

EXAMPLE 12–4 **Velocity Measurement by a Pitot Tube**

A piezometer and a Pitot tube are tapped into a horizontal water pipe, as shown in Fig. 12–21, to measure static and stagnation (static + dynamic) pressures. For the indicated water column heights, determine the velocity at the center of the pipe.

Solution The static and stagnation pressures in a horizontal pipe are measured. The velocity at the center of the pipe is to be determined.
Assumptions **1** The flow is steady and incompressible. **2** Points 1 and 2 are close enough together that the irreversible energy loss between these two points is negligible, and thus we can use the Bernoulli equation.
Analysis We take points 1 and 2 along the streamline at the centerline of the pipe, with point 1 directly under the piezometer and point 2 at the tip

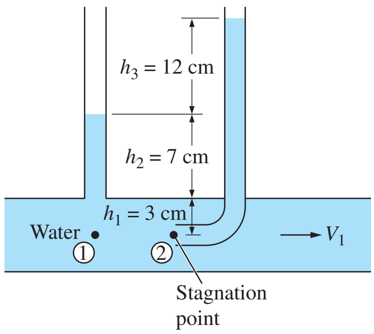


FIGURE 12–21
Schematic for Example 12–4.

of the Pitot tube. This is a steady flow with straight and parallel streamlines, and the gage pressures at points 1 and 2 can be expressed as

$$P_1 = \rho g(h_1 + h_2)$$
$$P_2 = \rho g(h_1 + h_2 + h_3)$$

Noting that $z_1 = z_2$, and point 2 is a stagnation point and thus $V_2 = 0$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \frac{\cancel{V_2^2}^{\,0}}{2g} + \cancel{z_2} \quad \rightarrow \quad \frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g}$$

Substituting the P_1 and P_2 expressions gives

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g} = \frac{\rho g(h_1 + h_2 + h_3) - \rho g(h_1 + h_2)}{\rho g} = h_3$$

Solving for V_1 and substituting,

$$V_1 = \sqrt{2gh_3} = \sqrt{2(9.81 \text{ m/s}^2)(0.12 \text{ m})} = \mathbf{1.53 \text{ m/s}}$$

Discussion Note that to determine the flow velocity, all we need is to measure the height of the excess fluid column in the Pitot tube compared to that in the piezometer tube.

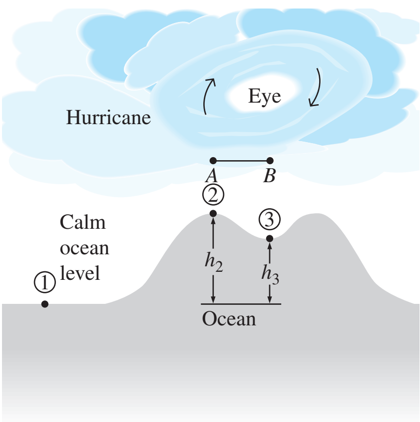


FIGURE 12–22
Schematic for Example 12–5. The vertical scale is greatly exaggerated.

EXAMPLE 12–5 The Rise of the Ocean Due to a Hurricane

A hurricane is a tropical storm formed over the ocean by low atmospheric pressures. As a hurricane approaches land, inordinate ocean swells (very high tides) accompany the hurricane. A Class-5 hurricane features winds in excess of 155 mph, although the wind velocity at the center "eye" is very low.

Figure 12–22 depicts a hurricane hovering over the ocean swell below. The atmospheric pressure 200 mi from the eye is 30.0 in Hg (at point 1, generally normal for the ocean) and the winds are calm. The atmospheric pressure at the eye of the storm is 22.0 in Hg. Estimate the ocean swell at (*a*) the eye of the hurricane at point 3 and (*b*) point 2, where the wind velocity is 155 mph. Take the density of seawater and mercury to be 64 lbm/ft^3 and 848 lbm/ft^3, respectively, and the density of air at normal sea-level temperature and pressure to be 0.076 lbm/ft^3.

Solution A hurricane is moving over the ocean. The amount of ocean swell at the eye and at active regions of the hurricane are to be determined.

Assumptions **1** The airflow within the hurricane is steady, incompressible, and irrotational (so that the Bernoulli equation is applicable). (This is certainly a very questionable assumption for a highly turbulent flow, but it is justified in the discussion.) **2** The effect of water sucked into the air is negligible.

Properties The densities of air at normal conditions, seawater, and mercury are given to be 0.076 lbm/ft^3, 64.0 lbm/ft^3, and 848 lbm/ft^3, respectively.

Analysis (*a*) Reduced atmospheric pressure over the water causes the water to rise. Thus, decreased pressure at point 2 relative to point 1 causes the ocean water to rise at point 2. The same is true at point 3, where the storm air velocity is negligible. The pressure difference given in terms of the mercury column height is expressed in terms of the seawater column height by

$$\Delta P = (\rho g h)_{\text{Hg}} = (\rho g h)_{\text{sw}} \rightarrow h_{\text{sw}} = \frac{\rho_{\text{Hg}}}{\rho_{\text{sw}}} h_{\text{Hg}}$$

Then the pressure difference between points 1 and 3 in terms of the seawater column height becomes

$$h_3 = \frac{\rho_{\text{Hg}}}{\rho_{\text{sw}}} h_{\text{Hg}} = \left(\frac{848 \text{ lbm/ft}^3}{64.0 \text{ lbm/ft}^3}\right)[(30 - 22) \text{ in Hg}]\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) = \mathbf{8.83 \text{ ft}}$$

which is equivalent to the storm surge at the *eye of the hurricane* (Fig. 12–23) since the wind velocity there is negligible and there are no dynamic effects.

(*b*) To determine the additional rise of ocean water at point 2 due to the high winds at that point, we write the Bernoulli equation between points *A* and *B*, which are on top of points 2 and 3, respectively. Noting that $V_B \cong 0$ (the eye region of the hurricane is relatively calm) and $z_A = z_B$ (both points are on the same horizontal line), the Bernoulli equation simplifies to

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + \cancel{z_A} = \frac{P_B}{\rho g} + \cancel{\frac{V_B^2}{2g}}^{\,0} + \cancel{z_B} \quad \rightarrow \quad \frac{P_B - P_A}{\rho g} = \frac{V_A^2}{2g}$$

Substituting,

$$\frac{P_B - P_A}{\rho g} = \frac{V_A^2}{2g} = \frac{(155 \text{ mph})^2}{2(32.2 \text{ ft/s}^2)}\left(\frac{1.4667 \text{ ft/s}}{1 \text{ mph}}\right)^2 = 803 \text{ ft}$$

where ρ is the density of air in the hurricane. Noting that the density of an ideal gas at constant temperature is proportional to absolute pressure and the density of air at the normal atmospheric pressure of 14.7 psia $\cong$ 30 in Hg is 0.076 lbm/ft^3, the density of air in the hurricane is

$$\rho_{\text{air}} = \frac{P_{\text{air}}}{P_{\text{atm air}}} \rho_{\text{atm air}} = \left(\frac{22 \text{ in Hg}}{30 \text{ in Hg}}\right)(0.076 \text{ lbm/ft}^3) = 0.056 \text{ lbm/ft}^3$$

Using the relation developed above in part (*a*), the seawater column height equivalent to 803 ft of air column height is determined to be

$$h_{\text{dynamic}} = \frac{\rho_{\text{air}}}{\rho_{\text{sw}}} h_{\text{air}} = \left(\frac{0.056 \text{ lbm/ft}^3}{64 \text{ lbm/ft}^3}\right)(803 \text{ ft}) = 0.70 \text{ ft}$$

Therefore, the pressure at point 2 is 0.70 ft seawater column lower than the pressure at point 3 due to the high wind velocities, causing the ocean to rise an additional 0.70 ft. Then the total storm surge at point 2 becomes

$$h_2 = h_3 + h_{\text{dynamic}} = 8.83 + 0.70 = \mathbf{9.53 \text{ ft}}$$

Discussion This problem involves highly turbulent flow and the intense breakdown of the streamlines, and thus the applicability of the Bernoulli equation in part (*b*) is questionable. Furthermore, the flow in the eye of the storm is *not* irrotational, and the Bernoulli equation constant changes across

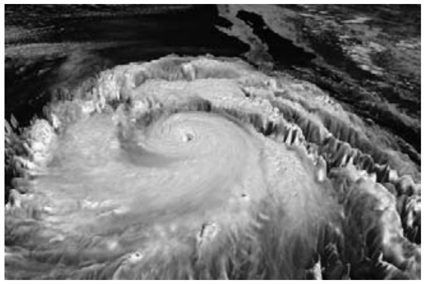


FIGURE 12–23
The eye of hurricane Linda (1997 in the Pacific Ocean near Baja California) is clearly visible in this satellite photo.

streamlines. The Bernoulli analysis can be thought of as the limiting, ideal case, and shows that the rise of seawater due to high-velocity winds cannot be more than 0.70 ft.

The wind power of hurricanes is not the only cause of damage to coastal areas. Ocean flooding and erosion from excessive tides is just as serious, as are high waves generated by the storm turbulence and energy.

EXAMPLE 12–6 Bernoulli Equation for Compressible Flow

Derive the Bernoulli equation when the compressibility effects are not negligible for an ideal gas undergoing (*a*) an isothermal process and (*b*) an isentropic process.

Solution The Bernoulli equation for compressible flow is to be obtained for an ideal gas for isothermal and isentropic processes.

Assumptions **1** The flow is steady and frictional effects are negligible. **2** The fluid is an ideal gas, so the relation $P = \rho RT$ is applicable. **3** The specific heats are constant so that P/ρ^k = constant during an isentropic process.

Analysis (*a*) When the compressibility effects are significant and the flow cannot be assumed to be incompressible, the Bernoulli equation is given by Eq. 12–8 as

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant} \qquad \text{(along a streamline)} \tag{1}$$

The compressibility effects can be properly accounted for by performing the integration $\int dP/\rho$ in Eq. 1. But this requires a relation between P and ρ for the process. For the *isothermal* expansion or compression of an ideal gas, the integral in Eq. 1 is performed easily by noting that T = constant and substituting $\rho = P/RT$,

$$\int \frac{dP}{\rho} = \int \frac{dP}{P/RT} = RT \ln P$$

Substituting into Eq. 1 gives the desired relation,

$$\textit{Isothermal process:} \qquad \mathbf{RT \ln P + \frac{V^2}{2} + gz = constant} \tag{2}$$

(*b*) A more practical case of compressible flow is the *isentropic flow of ideal gases* through equipment that involves high-speed fluid flow such as nozzles, diffusers, and the passages between turbine blades (Fig. 12–24). Isentropic (i.e., reversible and adiabatic) flow is closely approximated by these devices, and it is characterized by the relation $P/\rho^k = C$ = constant, where k is the specific heat ratio of the gas. Solving for ρ from $P/\rho^k = C$ gives $\rho = C^{-1/k}P^{1/k}$. Performing the integration,

$$\int \frac{dP}{\rho} = \int C^{1/k}P^{-1/k}\,dP = C^{1/k}\frac{P^{-1/k+1}}{-1/k+1} = \frac{P^{1/k}}{\rho}\frac{P^{-1/k+1}}{-1/k+1} = \left(\frac{k}{k-1}\right)\frac{P}{\rho} \tag{3}$$

Substituting, the Bernoulli equation for steady, isentropic, compressible flow of an ideal gas becomes

$$\textit{Isentropic flow:} \qquad \mathbf{\left(\frac{k}{k-1}\right)\frac{P}{\rho} + \frac{V^2}{2} + gz = constant} \tag{4a}$$

FIGURE 12–24
Compressible flow of a gas through turbine blades is often modeled as isentropic, and the compressible form of the Bernoulli equation is a reasonable approximation.
© Corbis RF.

or

$$\left(\frac{k}{k-1}\right)\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \left(\frac{k}{k-1}\right)\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 \qquad \textbf{(4b)}$$

A common practical situation involves the acceleration of a gas from rest (stagnation conditions at state 1) with negligible change in elevation. In that case we have $z_1 = z_2$ and $V_1 = 0$. Noting that $\rho = P/RT$ for ideal gases, P/ρ^k = constant for isentropic flow, and the Mach number is defined as $\text{Ma} = V/c$ where $c = \sqrt{kRT}$ is the local speed of sound for ideal gases, Eq. 4b simplifies to

$$\frac{P_1}{P_2} = \left[1 + \left(\frac{k-1}{2}\right)\text{Ma}_2^2\right]^{k/(k-1)} \qquad \textbf{(4c)}$$

where state 1 is the stagnation state and state 2 is any state along the flow.

Discussion It can be shown that the results obtained using the compressible and incompressible equations deviate no more than 2 percent when the Mach number is less than 0.3. Therefore, the flow of an ideal gas can be considered to be incompressible when $\text{Ma} \lesssim 0.3$. For atmospheric air at normal conditions, this corresponds to a flow speed of about 100 m/s or 360 km/h.

12–2 ▪ GENERAL ENERGY EQUATION

One of the most fundamental laws in nature is the **first law of thermodynamics**, also known as the **conservation of energy principle**, which provides a sound basis for studying the relationships among the various forms of energy and energy interactions. It states that *energy can be neither created nor destroyed during a process; it can only change forms.* Therefore, every bit of energy must be accounted for during a process.

A rock falling off a cliff, for example, picks up speed as a result of its potential energy being converted to kinetic energy (Fig. 12–25). Experimental data show that the decrease in potential energy equals the increase in kinetic energy when the air resistance is negligible, thus confirming the conservation of energy principle. The conservation of energy principle also forms the backbone of the diet industry: a person who has a greater energy input (food) than energy output (exercise) will gain weight (store energy in the form of fat), and a person who has a smaller energy input than output will lose weight. The change in the energy content of a system is equal to the difference between the energy input and the energy output, and the conservation of energy principle for any system can be expressed simply as $E_{\text{in}} - E_{\text{out}} = \Delta E$.

The transfer of any quantity (such as mass, momentum, and energy) is recognized *at the boundary* as the quantity *crosses the boundary*. A quantity is said to *enter* a system (or control volume) if it crosses the boundary from the outside to the inside, and to *exit* the system if it moves in the reverse direction. A quantity that moves from one location to another within a system is not considered as a transferred quantity in an analysis since it does not enter or exit the system. Therefore, it is important to specify the system and thus clearly identify its boundaries before an engineering analysis is performed.

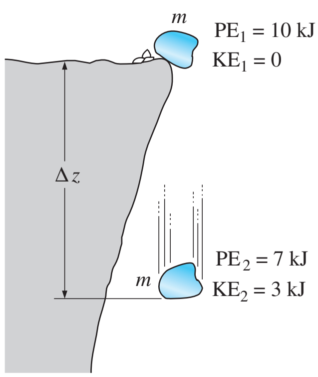


FIGURE 12–25
Energy cannot be created or destroyed during a process; it can only change forms.

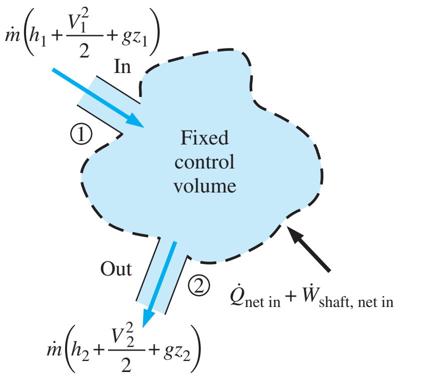


FIGURE 12–26
A control volume with only one inlet and one outlet and energy interactions.

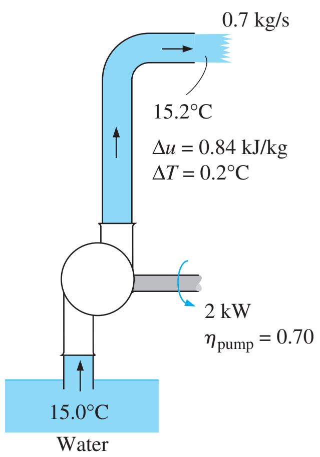


FIGURE 12–27
The lost mechanical energy in a fluid flow system results in an increase in the internal energy of the fluid and thus in a rise of fluid temperature.

where we used the definition of enthalpy $h = u + Pv = u + P/\rho$. The last two equations are fairly general expressions of conservation of energy, but their use is still limited to fixed control volumes, uniform flow at inlets and outlets, and negligible work due to viscous forces and other effects. Also, the subscript "net in" stands for "net input," and thus any heat or work transfer is positive if *to* the system and negative if *from* the system.

12–3 ▪ ENERGY ANALYSIS OF STEADY FLOWS

For steady flows, the time rate of change of the energy content of the control volume is zero, and the energy equation can be expressed as

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) \qquad \textbf{(12–17)}$$

It states that *during steady flow the net rate of energy transfer to a control volume by heat and work transfers is equal to the difference between the rates of outgoing and incoming energy flows by mass flow*.

Many practical problems involve just one inlet and one outlet (Fig. 12–26). The mass flow rate for such **single-stream devices** is the same at the inlet and outlet, and Eq. 12–17 reduces to

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right) \qquad \textbf{(12–18)}$$

where subscripts 1 and 2 refer to the inlet and outlet, respectively. The steady-flow energy equation on a unit-mass basis is obtained by dividing Eq. 12–18 by the mass flow rate $\dot{m}$,

$$q_{\text{net in}} + w_{\text{shaft, net in}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \qquad \textbf{(12–19)}$$

where $q_{\text{net in}} = \dot{Q}_{\text{net in}}/\dot{m}$ is the net heat transfer to the fluid per unit mass and $w_{\text{shaft, net in}} = \dot{W}_{\text{shaft, net in}}/\dot{m}$ is the net shaft work input to the fluid per unit mass. Using the definition of enthalpy $h = u + P/\rho$ and rearranging, the steady-flow energy equation can also be expressed as

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + (u_2 - u_1 - q_{\text{net in}}) \qquad \textbf{(12–20)}$$

where u is the *internal energy*, P/ρ is the *flow energy*, $V^2/2$ is the *kinetic energy*, and gz is the *potential energy* of the fluid, all per unit mass. These relations are valid for both compressible and incompressible flows.

The left side of Eq. 12–20 represents the mechanical energy input, while the first three terms on the right side represent the mechanical energy output. If the flow is ideal with no irreversibilities such as friction, the total mechanical energy must be conserved, and the term in parentheses ($u_2 - u_1 - q_{\text{net in}}$) must equal zero. That is,

Ideal flow (no mechanical energy loss): $$q_{\text{net in}} = u_2 - u_1 \qquad \textbf{(12–21)}$$

Any increase in $u_2 - u_1$ above $q_{\text{net in}}$ is due to the irreversible conversion of mechanical energy to thermal energy, and thus $u_2 - u_1 - q_{\text{net in}}$ represents the mechanical energy loss per unit mass (Fig. 12–27). That is,

Real flow (with mechanical energy loss): $$e_{\text{mech, loss}} = u_2 - u_1 - q_{\text{net in}} \qquad \textbf{(12–22)}$$

For single-phase fluids (a gas or a liquid), $u_2 - u_1 = c_v(T_2 - T_1)$ where c_v is the constant-volume specific heat.

The steady-flow energy equation on a unit-mass basis can be written conveniently as a **mechanical energy** balance,

$$e_{\text{mech, in}} = e_{\text{mech, out}} + e_{\text{mech, loss}} \tag{12–23}$$

or

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + e_{\text{mech, loss}} \tag{12–24}$$

Noting that $w_{\text{shaft, net in}} = w_{\text{pump}} - w_{\text{turbine}}$, the mechanical energy balance can be written more explicitly as

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 + w_{\text{pump}} = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + w_{\text{turbine}} + e_{\text{mech, loss}} \tag{12–25}$$

where w_{pump} is the mechanical work input (due to the presence of a pump, fan, compressor, etc.) and w_{turbine} is the mechanical work output (due to a turbine). When the flow is incompressible, either absolute or gage pressure can be used for P since P_{atm}/ρ would appear on both sides and would cancel out.

Multiplying Eq. 12–25 by the mass flow rate $\dot{m}$ gives

$$\dot{m}\left(\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}} \tag{12–26}$$

where $\dot{W}_{\text{pump}}$ is the shaft power input through the pump's shaft, $\dot{W}_{\text{turbine}}$ is the shaft power output through the turbine's shaft, and $\dot{E}_{\text{mech, loss}}$ is the *total* mechanical power loss, which consists of pump and turbine losses as well as the frictional losses in the piping network. That is,

$$\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech loss, pump}} + \dot{E}_{\text{mech loss, turbine}} + \dot{E}_{\text{mech loss, piping}}$$

By convention, irreversible pump and turbine losses are treated separately from irreversible losses due to other components of the piping system (Fig. 12–28). Thus, the energy equation is expressed in its most common form in terms of *heads* by dividing each term in Eq. 12–26 by $\dot{m}g$. The result is

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{\text{pump}, u} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{\text{turbine}, e} + h_L \tag{12–27}$$

where

- $h_{\text{pump}, u} = \dfrac{w_{\text{pump}, u}}{g} = \dfrac{\dot{W}_{\text{pump}, u}}{\dot{m}g} = \dfrac{\eta_{\text{pump}}\dot{W}_{\text{pump}}}{\dot{m}g}$ is the *useful head delivered to the fluid by the pump*. Because of irreversible losses in the pump, $h_{\text{pump}, u}$ is *less* than $\dot{W}_{\text{pump}}/\dot{m}g$ by the factor η_{pump}.
- $h_{\text{turbine}, e} = \dfrac{w_{\text{turbine}, e}}{g} = \dfrac{\dot{W}_{\text{turbine}, e}}{\dot{m}g} = \dfrac{\dot{W}_{\text{turbine}}}{\eta_{\text{turbine}}\dot{m}g}$ is the *extracted head removed from the fluid by the turbine*. Because of irreversible losses in the turbine, $h_{\text{turbine}, e}$ is *greater* than $\dot{W}_{\text{turbine}}/\dot{m}g$ by the factor η_{turbine}.
- $h_L = \dfrac{e_{\text{mech loss, piping}}}{g} = \dfrac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g}$ is the *irreversible head loss* between 1 and 2 due to all components of the piping system other than the pump or turbine.

FIGURE 12–28
A typical power plant has numerous pipes, elbows, valves, pumps, and turbines, all of which have irreversible losses.
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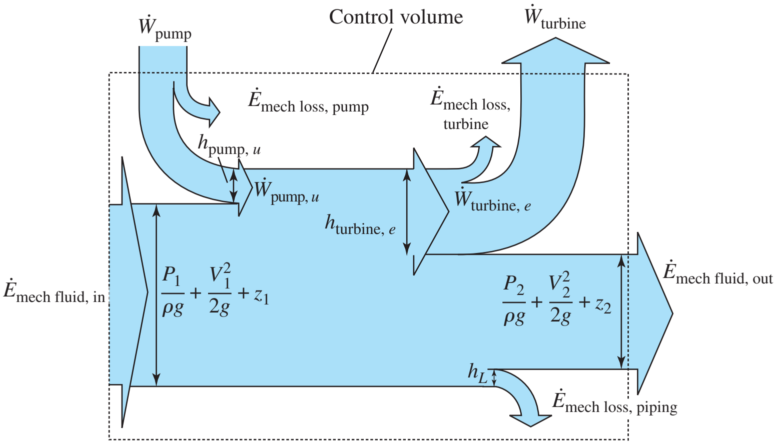


FIGURE 12–29
Mechanical energy flow chart for a fluid flow system that involves a pump and a turbine. Vertical dimensions show each energy term expressed as an equivalent column height of fluid, i.e., *head,* corresponding to each term of Eq. 12–27.

Note that the head loss h_L represents the frictional losses associated with fluid flow in piping, and it does not include the losses that occur within the pump or turbine due to the inefficiencies of these devices—these losses are taken into account by η_{pump} and $\eta_{turbine}$. Equation 12–27 is illustrated schematically in Fig. 12–29.

The *pump head* is zero if the piping system does not involve a pump, a fan, or a compressor, and the *turbine head* is zero if the system does not involve a turbine.

Special Case: Incompressible Flow with No Mechanical Work Devices and Negligible Friction

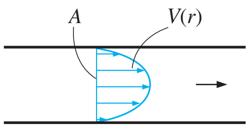
When piping losses are negligible, there is negligible dissipation of mechanical energy into thermal energy, and thus $h_L = e_{mech\ loss,\ piping}/g \cong 0$. Also, $h_{pump,\ u} = h_{turbine,\ e} = 0$ when there are no mechanical work devices such as fans, pumps, or turbines. Then Eq. 12–27 reduces to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad \text{or} \quad \frac{P}{\rho g}+\frac{V^2}{2g}+z=\text{constant} \tag{12–28}$$

which is the **Bernoulli equation** derived earlier using Newton's second law of motion. Thus, the Bernoulli equation can be thought of as a degenerate form of the energy equation.

Kinetic Energy Correction Factor, α

The average flow velocity V_{avg} was defined such that the relation $\rho V_{avg}A$ gives the actual mass flow rate. Therefore, there is no such thing as a correction factor for mass flow rate. However, as Gaspard Coriolis (1792–1843) showed, the kinetic energy of a fluid stream obtained from $V^2/2$ is not the same as the actual kinetic energy of the fluid stream since the square of a sum is not equal to the sum of the squares of its components (Fig. 12–30). This error can be corrected by replacing the kinetic energy terms $V^2/2$ in the energy equation by $\alpha V_{avg}^2/2$, where α is the **kinetic energy correction factor**. By using equations for the variation of velocity with the radial distance, it can be shown that the correction factor is 2.0 for fully developed laminar pipe flow, and it ranges between 1.04 and 1.11 for fully developed turbulent flow in a round pipe.



$\dot{m}=\rho V_{avg}A, \qquad \rho=\text{constant}$

$$\dot{KE}_{act}=\int \text{ke}\,\delta\dot{m}=\int_A \frac{1}{2}[V(r)]^2[\rho V(r)\,dA]$$
$$=\frac{1}{2}\rho\int_A [V(r)]^3\,dA$$
$$\dot{KE}_{avg}=\frac{1}{2}\dot{m}V_{avg}^2=\frac{1}{2}\rho A V_{avg}^3$$
$$\alpha=\frac{\dot{KE}_{act}}{\dot{KE}_{avg}}=\frac{1}{A}\int_A\left(\frac{V(r)}{V_{avg}}\right)^3 dA$$

FIGURE 12–30
The determination of the *kinetic energy correction factor* using the actual velocity distribution $V(r)$ and the average velocity V_{avg} at a cross section.

The kinetic energy correction factors are often ignored (i.e., α is set equal to 1) in an elementary analysis since (1) most flows encountered in practice are turbulent, for which the correction factor is near unity, and (2) the kinetic energy terms are often small relative to the other terms in the energy equation, and multiplying them by a factor less than 2.0 does not make much difference. When the velocity and thus the kinetic energy are high, the flow turns turbulent, and a unity correction factor is more appropriate. However, you should keep in mind that you may encounter some situations for which these factors *are* significant, especially when the flow is laminar. Therefore, we recommend that you always include the kinetic energy correction factor when analyzing fluid flow problems. When the kinetic energy correction factors are included, the energy equations for *steady incompressible flow* (Eqs. 12–26 and 12–27) become

$$\dot{m}\left(\frac{P_1}{\rho} + \alpha_1\frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2\frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}} \quad \text{(12–29)}$$

$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_{\text{pump, }u} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_{\text{turbine, }e} + h_L \quad \text{(12–30)}$$

If the flow at an inlet or outlet is fully developed turbulent pipe flow, we recommend using $\alpha = 1.05$ as a reasonable estimate of the correction factor. This leads to a more conservative estimate of head loss, and it does not take much additional effort to include α in the equations.

EXAMPLE 12–7 **Pumping Power and Frictional Heating in a Pump**

The pump of a water distribution system is powered by a 15-kW electric motor whose efficiency is 90 percent (Fig. 12–31). The water flow rate through the pump is 50 L/s. The diameters of the inlet and outlet pipes are the same, and the elevation difference across the pump is negligible. If the absolute pressures at the inlet and outlet of the pump are measured to be 100 kPa and 300 kPa, respectively, determine (*a*) the mechanical efficiency of the pump and (*b*) the temperature rise of water as it flows through the pump due to mechanical inefficiencies.

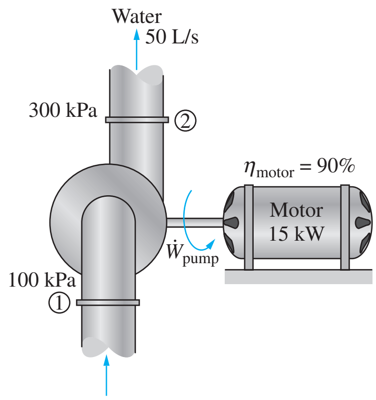


FIGURE 12–31
Schematic for Example 12–7.

Solution The pressures across a pump are measured. The mechanical efficiency of the pump and the temperature rise of water are to be determined.
Assumptions **1** The flow is steady and incompressible. **2** The pump is driven by an external motor so that the heat generated by the motor is dissipated to the atmosphere. **3** The elevation difference between the inlet and outlet of the pump is negligible, $z_1 \cong z_2$. **4** The inlet and outlet diameters are the same and thus the average inlet and outlet velocities are equal, $V_1 = V_2$. **5** The kinetic energy correction factors are equal, $\alpha_1 = \alpha_2$.
Properties We take the density of water to be 1 kg/L = 1000 kg/m^3 and its specific heat to be 4.18 kJ/kg · °C.
Analysis (*a*) The mass flow rate of water through the pump is

$$\dot{m} = \rho\dot{V} = (1\text{ kg/L})(50\text{ L/s}) = 50\text{ kg/s}$$

The motor draws 15 kW of power and is 90 percent efficient. Thus the mechanical (shaft) power it delivers to the pump is

$$\dot{W}_{\text{pump, shaft}} = \eta_{\text{motor}}\dot{W}_{\text{electric}} = (0.90)(15\text{ kW}) = 13.5\text{ kW}$$

To determine the mechanical efficiency of the pump, we need to know the increase in the mechanical energy of the fluid as it flows through the pump, which is

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{E}_{\text{mech, out}} - \dot{E}_{\text{mech, in}} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2\right) - \dot{m}\left(\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1\right)$$

Simplifying it for this case and substituting the given values,

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{m}\left(\frac{P_2 - P_1}{\rho}\right) = (50 \text{ kg/s})\left(\frac{(300 - 100) \text{ kPa}}{1000 \text{ kg/m}^3}\right)\left(\frac{1 \text{ kJ}}{1 \text{ kP·m}^3}\right) = 10.0 \text{ kW}$$

Then the mechanical efficiency of the pump becomes

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{pump, } u}}{\dot{W}_{\text{pump, shaft}}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{pump, shaft}}} = \frac{10.0 \text{ kW}}{13.5 \text{ kW}} = \mathbf{0.741} \quad \text{or} \quad \mathbf{74.1\%}$$

(*b*) Of the 13.5-kW mechanical power supplied by the pump, only 10.0 kW is imparted to the fluid as mechanical energy. The remaining 3.5 kW is converted to thermal energy due to frictional effects, and this "lost" mechanical energy manifests itself as a heating effect in the fluid,

$$\dot{E}_{\text{mech, loss}} = \dot{W}_{\text{pump, shaft}} - \Delta \dot{E}_{\text{mech, fluid}} = 13.5 - 10.0 = 3.5 \text{ kW}$$

The temperature rise of water due to this mechanical inefficiency is determined from the thermal energy balance, $\dot{E}_{\text{mech, loss}} = \dot{m}(u_2 - u_1) = \dot{m}c\Delta T$. Solving for ΔT,

$$\Delta T = \frac{\dot{E}_{\text{mech, loss}}}{\dot{m}c} = \frac{3.5 \text{ kW}}{(50 \text{ kg/s})(4.18 \text{ kJ/kg·°C})} = \mathbf{0.017°C}$$

Therefore, the water experiences a temperature rise of 0.017°C which is very small, due to mechanical inefficiency, as it flows through the pump.

Discussion In an actual application, the temperature rise of water would probably be less since part of the heat generated would be transferred to the casing of the pump and from the casing to the surrounding air. If the entire pump and motor were submerged in water, then the 1.5 kW dissipated due to motor inefficiency would also be transferred to the surrounding water as heat.

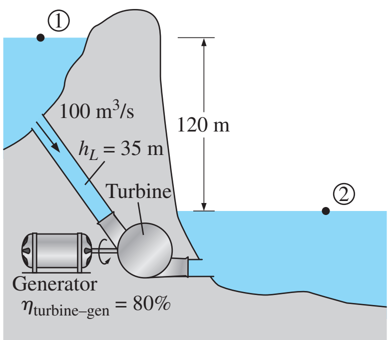


FIGURE 12–32
Schematic for Example 12–8.

EXAMPLE 12–8 Hydroelectric Power Generation from a Dam

In a hydroelectric power plant, 100 m^3/s of water flows from an elevation of 120 m to a turbine, where electric power is generated (Fig. 12–32). The total irreversible head loss in the piping system from point 1 to point 2 (excluding the turbine unit) is determined to be 35 m. If the overall efficiency of the turbine–generator is 80 percent, estimate the electric power output.

Solution The available head, flow rate, head loss, and efficiency of a hydroelectric turbine are given. The electric power output is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Water levels at the reservoir and the discharge site remain constant.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The mass flow rate of water through the turbine is

$$\dot{m} = \rho\dot{V} = (1000 \text{ kg/m}^3)(100 \text{ m}^3\text{/s}) = 10^5 \text{ kg/s}$$

We take point 2 as the reference level, and thus $z_2 = 0$. Also, both points 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{atm}$) and the flow velocities are negligible at both points ($V_1 = V_2 = 0$). Then the energy equation for steady, incompressible flow reduces to

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + h_{\text{pump},\,u}^{\nearrow 0} = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + \cancel{z_2}^{\nearrow 0} + h_{\text{turbine},\,e} + h_L$$

or

$$h_{\text{turbine},\,e} = z_1 - h_L$$

Substituting, the extracted turbine head and the corresponding turbine power are

$$h_{\text{turbine},\,e} = z_1 - h_L = 120 - 35 = 85 \text{ m}$$

$$\dot{W}_{\text{turbine},\,e} = \dot{m} g h_{\text{turbine},\,e} = (10^5 \text{ kg/s})(9.81 \text{ m/s}^2)(85 \text{ m})\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 83{,}400 \text{ kW}$$

Therefore, a perfect turbine–generator would generate 83,400 kW of electricity from this resource. The electric power generated by the actual unit is

$$\dot{W}_{\text{electric}} = \eta_{\text{turbine–gen}} \dot{W}_{\text{turbine},\,e} = (0.80)(83.4 \text{ MW}) = \mathbf{66.7 \text{ MW}}$$

Discussion Note that the power generation would increase by almost 1 MW for each percentage point improvement in the efficiency of the turbine–generator unit.

EXAMPLE 12–9 Fan Selection for Air Cooling of a Computer

A fan is to be selected to cool a computer case whose dimensions are 12 cm × 40 cm × 40 cm (Fig. 12–33). Half of the volume in the case is expected to be filled with components and the other half to be air space. A 5-cm-diameter hole is available at the back of the case for the installation of the fan that is to replace the air in the void spaces of the case once every second. Small low-power fan–motor combined units are available in the market and their efficiency is estimated to be 30 percent. Determine (*a*) the wattage of the fan–motor unit to be purchased and (*b*) the pressure difference across the fan. Take the air density to be 1.20 kg/m³.

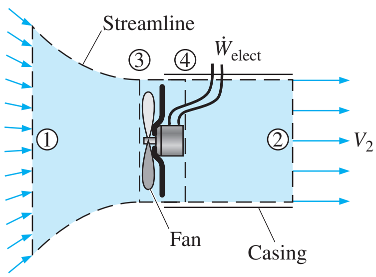


FIGURE 12–33
Schematic for Example 12–9.

Solution A fan is to cool a computer case by completely replacing the air inside once every second. The power of the fan and the pressure difference across it are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Losses other than those due to the inefficiency of the fan–motor unit are negligible. **3** The flow at the outlet is fairly uniform except near the center (due to the wake of the fan motor), and the kinetic energy correction factor at the outlet is 1.10.

Properties The density of air is given to be 1.20 kg/m³.

Analysis (*a*) Noting that half of the volume of the case is occupied by the components, the air volume in the computer case is

$$V = (\text{Void fraction})(\text{Total case volume})$$
$$= 0.5(12 \text{ cm} \times 40 \text{ cm} \times 40 \text{ cm}) = 9600 \text{ cm}^3$$

Therefore, the volume and mass flow rates of air through the case are

$$\dot{V} = \frac{V}{\Delta t} = \frac{9600\ \text{cm}^3}{1\ \text{s}} = 9600\ \text{cm}^3/\text{s} = 9.6 \times 10^{-3}\ \text{m}^3/\text{s}$$

$$\dot{m} = \rho\dot{V} = (1.20\ \text{kg/m}^3)(9.6 \times 10^{-3}\ \text{m}^3/\text{s}) = 0.0115\ \text{kg/s}$$

The cross-sectional area of the opening in the case and the average air velocity through the outlet are

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.05\ \text{m})^2}{4} = 1.96 \times 10^{-3}\ \text{m}^2$$

$$V = \frac{\dot{V}}{A} = \frac{9.6 \times 10^{-3}\ \text{m}^3/\text{s}}{1.96 \times 10^{-3}\ \text{m}^2} = 4.90\ \text{m/s}$$

We draw the control volume around the fan such that both the inlet and the outlet are at atmospheric pressure ($P_1 = P_2 = P_{\text{atm}}$), as shown in Fig. 12–33, where the inlet section 1 is large and far from the fan so that the flow velocity at the inlet section is negligible ($V_1 \cong 0$). Noting that $z_1 = z_2$ and frictional losses in the flow are disregarded, the mechanical losses consist of fan losses only and the energy equation (Eq. 12–29) simplifies to

$$\dot{m}\left(\cancel{\frac{P_1}{\rho}} + \alpha_1 \frac{\cancelto{0}{V_1^2}}{2} + \cancel{gz_1}\right) + \dot{W}_{\text{fan}} = \dot{m}\left(\cancel{\frac{P_2}{\rho}} + \alpha_2 \frac{V_2^2}{2} + \cancel{gz_2}\right) + \cancelto{0}{\dot{W}_{\text{turbine}}} + \dot{E}_{\text{mech loss, fan}}$$

Solving for $\dot{W}_{\text{fan}} - \dot{E}_{\text{mech loss, fan}} = \dot{W}_{\text{fan}, u}$ and substituting,

$$\dot{W}_{\text{fan}, u} = \dot{m}\alpha_2 \frac{V_2^2}{2} = (0.0115\ \text{kg/s})(1.10) \frac{(4.90\ \text{m/s})^2}{2} \left(\frac{1\ \text{N}}{1\ \text{kg·m/s}^2}\right) = 0.152\ \text{W}$$

Then the required electric power input to the fan is determined to be

$$\dot{W}_{\text{elect}} = \frac{\dot{W}_{\text{fan}, u}}{\eta_{\text{fan–motor}}} = \frac{0.152\ \text{W}}{0.3} = \mathbf{0.506\ W}$$

Therefore, a fan–motor rated at about a half watt is adequate for this job (Fig. 12–34). (*b*) To determine the pressure difference across the fan unit, we take points 3 and 4 to be on the two sides of the fan on a horizontal line. This time $z_3 = z_4$ again and $V_3 = V_4$ since the fan is a narrow cross section, and the energy equation reduces to

$$\dot{m}\frac{P_3}{\rho} + \dot{W}_{\text{fan}} = \dot{m}\frac{P_4}{\rho} + \dot{E}_{\text{mech loss, fan}} \quad \rightarrow \quad \dot{W}_{\text{fan}, u} = \dot{m}\frac{P_4 - P_3}{\rho}$$

Solving for $P_4 - P_3$ and substituting,

$$P_4 - P_3 = \frac{\rho \dot{W}_{\text{fan}, u}}{\dot{m}} = \frac{(1.2\ \text{kg/m}^3)(0.152\ \text{W})}{0.0115\ \text{kg/s}} \left(\frac{1\ \text{Pa·m}^3}{1\ \text{Ws}}\right) = \mathbf{15.8\ Pa}$$

Therefore, the pressure rise across the fan is 15.8 Pa.

Discussion The efficiency of the fan–motor unit is given to be 30 percent, which means 30 percent of the electric power $\dot{W}_{\text{electric}}$ consumed by the unit is converted to useful mechanical energy while the rest (70 percent) is "lost" and converted to thermal energy. Also, a more powerful fan is required in an actual system to overcome frictional losses inside the computer case. Note that if we had ignored the kinetic energy correction factor at the outlet, the required electrical power and pressure rise would have been 10 percent lower in this case (0.460 W and 14.4 Pa, respectively).

FIGURE 12–34
The cooling fans used in computers and computer power supplies are typically small and consume only a few watts of electrical power.

SUMMARY

This chapter deals with the Bernoulli and energy equations and their applications.

The *Bernoulli equation* is a relation between pressure, velocity, and elevation in steady, incompressible flow, and is expressed along a streamline and in regions where net viscous forces are negligible as

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

The Bernoulli equation can also be considered as an expression of mechanical energy balance, stated as: *The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when the compressibility and frictional effects are negligible*. Multiplying the Bernoulli equation by density gives

$$P + \rho\frac{V^2}{2} + \rho gz = \text{constant}$$

where P is the *static pressure*, which represents the actual pressure of the fluid; $\rho V^2/2$ is the *dynamic pressure*, which represents the pressure rise when the fluid in motion is brought to a stop; and ρgz is the *hydrostatic pressure*, which accounts for the effects of fluid weight on pressure. The sum of the static, dynamic, and hydrostatic pressures is called the *total pressure*. The Bernoulli equation states that *the total pressure along a streamline is constant*. The sum of the static and dynamic pressures is called the *stagnation pressure*, which represents the pressure at a point where the fluid is brought to a complete stop in a frictionless manner. The Bernoulli equation can also be represented in terms of "heads" by dividing each term by g,

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = \text{constant}$$

where $P/\rho g$ is the *pressure head*, which represents the height of a fluid column that produces the static pressure P; $V^2/2g$ is the *velocity head*, which represents the elevation needed for a fluid to reach the velocity V during frictionless free fall; and z is the *elevation head*, which represents the potential energy of the fluid. Also, H is the *total head* for the flow. The curve that represents the sum of the static pressure and the elevation heads, $P/\rho g + z$, is called the *hydraulic grade line* (HGL), and the curve that represents the total head of the fluid, $P/\rho g + V^2/2g + z$, is called the *energy grade line* (EGL).

The *energy equation* for steady, incompressible flow is expressed as

$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_{\text{pump, }u} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_{\text{turbine, }e} + h_L$$

where

$$h_{\text{pump, }u} = \frac{w_{\text{pump, }u}}{g} = \frac{\dot{W}_{\text{pump, }u}}{\dot{m}g} = \frac{\eta_{\text{pump}}\dot{W}_{\text{pump}}}{\dot{m}g}$$

$$h_{\text{turbine, }e} = \frac{w_{\text{turbine, }e}}{g} = \frac{\dot{W}_{\text{turbine, }e}}{\dot{m}g} = \frac{\dot{W}_{\text{turbine}}}{\eta_{\text{turbine}}\dot{m}g}$$

$$h_L = \frac{e_{\text{mech loss, piping}}}{g} = \frac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g}$$

$$e_{\text{mech, loss}} = u_2 - u_1 - q_{\text{net in}}$$

The Bernoulli and energy equations are two of the most fundamental relations in fluid mechanics.

REFERENCES AND SUGGESTED READING

1. R. C. Dorf, ed. in chief. *The Engineering Handbook*. Boca Raton, FL: CRC Press, 1995.
2. R. L. Panton. *Incompressible Flow*, 2nd ed. New York: Wiley, 1996.
3. M. Van Dyke. *An Album of Fluid Motion*. Stanford, CA: The Parabolic Press, 1982.

PROBLEMS*

Bernoulli Equation

12–1C What is streamwise acceleration? How does it differ from normal acceleration? Can a fluid particle accelerate in steady flow?

12–2C Express the Bernoulli equation in three different ways using (*a*) energies, (*b*) pressures, and (*c*) heads.

12–3C What are the three major assumptions used in the derivation of the Bernoulli equation?

12–4C Define static, dynamic, and hydrostatic pressure. Under what conditions is their sum constant for a flow stream?

12–5C What is stagnation pressure? Explain how it can be measured.

12–6C Define pressure head, velocity head, and elevation head for a fluid stream and express them for a fluid stream whose pressure is P, velocity is V, and elevation is z.

12–7C What is the hydraulic grade line? How does it differ from the energy grade line? Under what conditions do both lines coincide with the free surface of a liquid?

12–8C How is the location of the hydraulic grade line determined for open-channel flow? How is it determined at the outlet of a pipe discharging to the atmosphere?

12–9C In a certain application, a siphon must go over a high wall. Can water or oil with a specific gravity of 0.8 go over a higher wall? Why?

12–10C Explain how and why a siphon works. Someone proposes siphoning cold water over a 7-m-high wall. Is this feasible? Explain.

12–11C A glass manometer with oil as the working fluid is connected to an air duct as shown in Fig. P12–11C. Will the oil levels in the manometer be as in Fig. P12–11C*a* or *b*? Explain. What would your response be if the flow direction is reversed?

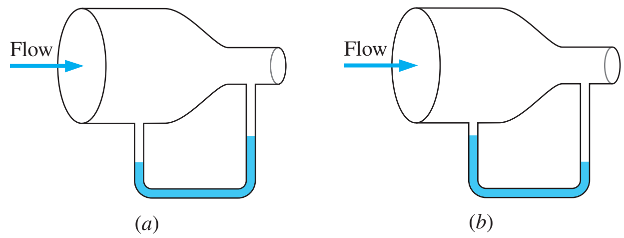


FIGURE P12–11C

12–12C The velocity of a fluid flowing in a pipe is to be measured by two different Pitot-type mercury manometers shown in Fig. P12–12C. Would you expect both manometers to predict the same velocity for flowing water? If not, which would be more accurate? Explain. What would your response be if air were flowing in the pipe instead of water?

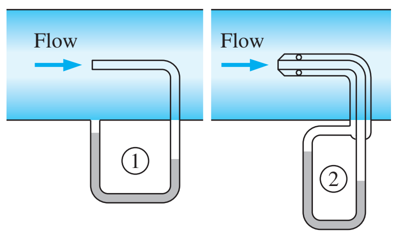


FIGURE P12–12C

12–13C The water level of a tank on a building roof is 20 m above the ground. A hose leads from the tank bottom to the ground. The end of the hose has a nozzle, which is pointed straight up. What is the maximum height to which the water could rise? What factors would reduce this height?

12–14C A student siphons water over a 8.5-m-high wall at sea level. She then climbs to the summit of Mount Shasta (elevation 4390 m, P_{atm} = 58.5 kPa) and attempts the same experiment. Comment on her prospects for success.

12–15 In a hydroelectric power plant, water enters the turbine nozzles at 800 kPa absolute with a low velocity. If the nozzle outlets are exposed to atmospheric pressure of 100 kPa, determine the maximum velocity to which water can be accelerated by the nozzles before striking the turbine blades.

12–16 A Pitot-static probe is used to measure the speed of an aircraft flying at 3000 m. If the differential pressure reading is 3 kPa, determine the speed of the aircraft.

* Problems designated by a “C” are concept questions, and students are encouraged to answer them all. Problems designated by an “E” are in English units, and the SI users can ignore them. Problems with the icon are solved using EES, and complete solutions together with parametric studies are included on the text specific web site. Problems with the icon are comprehensive in nature and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

12–17 While traveling on a dirt road, the bottom of a car hits a sharp rock and a small hole develops at the bottom of its gas tank. If the height of the gasoline in the tank is 40 cm, determine the initial velocity of the gasoline at the hole. Discuss how the velocity will change with time and how the flow will be affected if the lid of the tank is closed tightly. *Answer:* 2.80 m/s

12–18E The drinking water needs of an office are met by large water bottles. One end of a 0.25-in-diameter plastic hose is inserted into the bottle placed on a high stand, while the other end with an on/off valve is maintained 2 ft below the bottom of the bottle. If the water level in the bottle is 1.5 ft when it is full, determine how long it will take at the minimum to fill an 8-oz glass (= 0.00835 ft^3) (*a*) when the bottle is first opened and (*b*) when the bottle is almost empty. Neglect frictional losses.

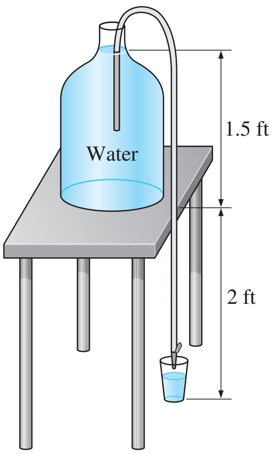


FIGURE P12–18E

12–19 A piezometer and a Pitot tube are tapped into a 4-cm-diameter horizontal water pipe, and the height of the water columns are measured to be 26 cm in the piezometer and 35 cm in the Pitot tube (both measured from the top surface of the pipe). Determine the velocity at the center of the pipe.

12–20 The diameter of a cylindrical water tank is D_o and its height is H. The tank is filled with water, which is open to the atmosphere. An orifice of diameter D with a smooth entrance (i.e., negligible losses) is open at the bottom. Develop a relation for the time required for the tank (*a*) to empty halfway and (*b*) to empty completely.

12–21E A siphon pumps water from a large reservoir to a lower tank that is initially empty. The tank also has a rounded orifice 20 ft below the reservoir surface where the water leaves the tank. Both the siphon and the orifice diameters are 2 in. Ignoring frictional losses, determine to what height the water will rise in the tank at equilibrium.

12–22 Water enters a tank of diameter D_T steadily at a mass flow rate of $\dot{m}_{in}$. An orifice at the bottom with diameter D_o allows water to escape. The orifice has a rounded entrance, so the frictional losses are negligible. If the tank is initially empty, (*a*) determine the maximum height that the water will reach in the tank and (*b*) obtain a relation for water height z as a function of time.

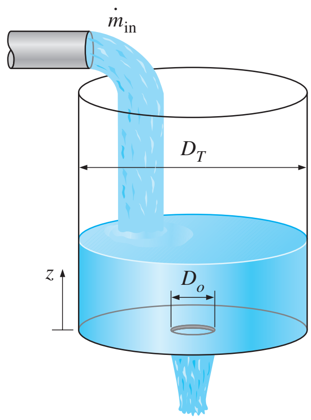


FIGURE P12–22

12–23E Water flows through a horizontal pipe at a rate of 1 gal/s. The pipe consists of two sections of diameters 4 in and 2 in with a smooth reducing section. The pressure difference between the two pipe sections is measured by a mercury manometer. Neglecting frictional effects, determine the differential height of mercury between the two pipe sections. *Answer:* 0.52 in

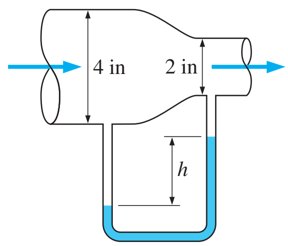


FIGURE P12–23E

12–24 An airplane is flying at an altitude of 12,000 m. Determine the gage pressure at the stagnation point on the nose of the plane if the speed of the plane is 300 km/h. How would you solve this problem if the speed were 1050 km/h? Explain.

12–25 The air velocity in the duct of a heating system is to be measured by a Pitot-static probe inserted into the duct parallel to the flow. If the differential height between the water columns connected to the two outlets of the probe is 2.4 cm, determine (*a*) the flow velocity and (*b*) the pressure rise at the tip of the probe. The air temperature and pressure in the duct are 45°C and 98 kPa, respectively.

12–26 The water in a 8-m-diameter, 3-m-high aboveground swimming pool is to be emptied by unplugging a 3-cm-diameter, 25-m-long horizontal pipe attached to the bottom of the pool. Determine the maximum discharge rate of water through the pipe. Also, explain why the actual flow rate will be less.

12–27 Reconsider Prob. 12–26. Determine how long it will take to empty the swimming pool completely. *Answer:* 15.4 h

12–28 Reconsider Prob. 12–27. Using EES (or other) software, investigate the effect of the discharge pipe diameter on the time required to empty the pool completely. Let the diameter vary from 1 to 10 cm in increments of 1 cm. Tabulate and plot the results.

12–29 Air at 110 kPa and 50°C flows upward through a 6-cm-diameter inclined duct at a rate of 45 L/s. The duct diameter is then reduced to 4 cm through a reducer. The pressure change across the reducer is measured by a water manometer. The elevation difference between the two points on the pipe where the two arms of the manometer are attached is 0.20 m. Determine the differential height between the fluid levels of the two arms of the manometer.

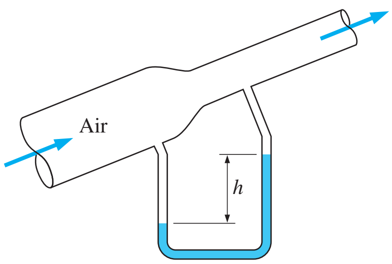


FIGURE P12–29

12–30E Air is flowing through a venturi meter whose diameter is 2.6 in at the entrance part (location 1) and 1.8 in at the throat (location 2). The gage pressure is measured to be 12.2 psia at the entrance and 11.8 psia at the throat. Neglecting frictional effects, show that the volume flow rate can be expressed as

$$\dot{V} = A_2\sqrt{\frac{2(P_1 - P_2)}{\rho(1 - A_2^2/A_1^2)}}$$

and determine the flow rate of air. Take the air density to be 0.075 lbm/ft^3.

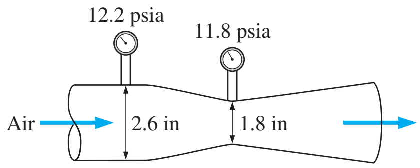


FIGURE P12–30E

12–31 The water pressure in the mains of a city at a particular location is 350 kPa gage. Determine if this main can serve water to neighborhoods that are 50 m above this location.

12–32 A pressurized tank of water has a 10-cm-diameter orifice at the bottom, where water discharges to the atmosphere. The water level is 2.5 m above the outlet. The tank air pressure above the water level is 250 kPa (absolute) while the atmospheric pressure is 100 kPa. Neglecting frictional effects, determine the initial discharge rate of water from the tank. *Answer:* 0.147 m^3/s

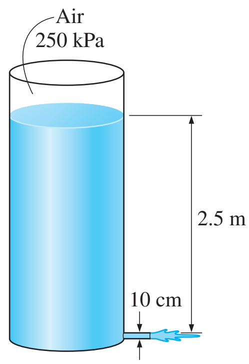


FIGURE P12–32

12–33 Reconsider Prob. 12–32. Using EES (or other) software, investigate the effect of water height in the tank on the discharge velocity. Let the water height vary from 0 to 5 m in increments of 0.5 m. Tabulate and plot the results.

12–34 A handheld bicycle pump can be used as an atomizer to generate a fine mist of paint or pesticide by forcing air at a high velocity through a small hole and placing a short tube between the liquid reservoir and the high-speed air jet. The pressure across a subsonic jet exposed to the atmosphere is nearly atmospheric, and the surface of the liquid in the reservoir is also open to atmospheric pressure. In light of this, explain how the liquid is sucked up the tube. *Hint:* Read Sec. 12–1 carefully.

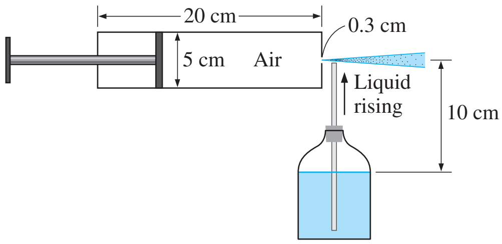


FIGURE P12–34

12–35 The water level in a tank is 15 m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank cover is airtight, and the air pressure above the water surface is 3 atm gage. The system is at sea level. Determine the maximum height to which the water stream could rise. *Answer:* 46.0 m

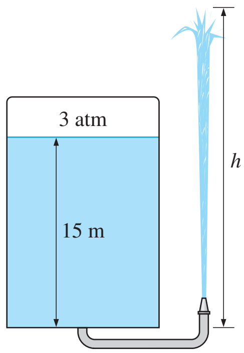


FIGURE P12–35

12–36 A Pitot-static probe connected to a water manometer is used to measure the velocity of air. If the deflection (the vertical distance between the fluid levels in the two arms) is 7.3 cm, determine the air velocity. Take the density of air to be 1.25 kg/m^3.

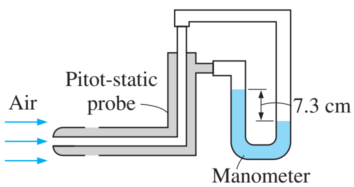


FIGURE P12–36

12–37E The air velocity in a duct is measured by a Pitot-static probe connected to a differential pressure gage. If the air is at 13.4 psia absolute and 70°F and the reading of the differential pressure gage is 0.15 psi, determine the air velocity. *Answer:* 143 ft/s

12–38 In cold climates, water pipes may freeze and burst if proper precautions are not taken. In such an occurrence, the exposed part of a pipe on the ground ruptures, and water shoots up to 42 m. Estimate the gage pressure of water in the pipe. State your assumptions and discuss if the actual pressure is more or less than the value you predicted.

Energy Equation

12–39C What is irreversible head loss? How is it related to the mechanical energy loss?

12–40C What is useful pump head? How is it related to the power input to the pump?

12–41C What is the kinetic energy correction factor? Is it significant?

12–42C The water level in a tank is 20 m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The water stream from the nozzle is observed to rise 25 m above the ground. Explain what may cause the water from the hose to rise above the tank level.

12–43C A person is filling a knee-high bucket with water using a garden hose and holding it such that water discharges from the hose at the level of his waist. Someone suggests that the bucket will fill faster if the hose is lowered such that water discharges from the hose at the knee level. Do you agree with this suggestion? Explain. Disregard any frictional effects.

12–44 A 3-m-high tank filled with water has a discharge valve near the bottom and another near the top. (*a*) If these two valves are opened, will there be any difference between the discharge velocities of the two water streams? (*b*) If a hose whose discharge end is left open on the ground is first connected to the lower valve and then to the higher valve, will there be any difference between the discharge rates of water for the two cases? Disregard any frictional effects.

12–45E In a hydroelectric power plant, water flows from an elevation of 240 ft to a turbine, where electric power is generated. For an overall turbine–generator efficiency of 83 percent, determine the minimum flow rate required to generate 100 kW of electricity. *Answer:* 370 lbm/s

12–46E Reconsider Prob. 12–45E. Determine the flow rate of water if the irreversible head loss of the piping system between the free surfaces of the source and the sink is 36 ft.

12–47 A fan is to be selected to ventilate a bathroom whose dimensions are 2 m × 3 m × 3 m. The air velocity is not to exceed 8 m/s to minimize vibration and noise. The combined efficiency of the fan–motor unit to be used can be taken to be 50 percent. If the fan is to replace the entire volume of air in 10 min, determine (*a*) the wattage of the fan–motor unit to be purchased, (*b*) the diameter of the fan casing, and (*c*) the pressure difference across the fan. Take the air density to be 1.25 kg/m^3 and disregard the effect of the kinetic energy correction factors.

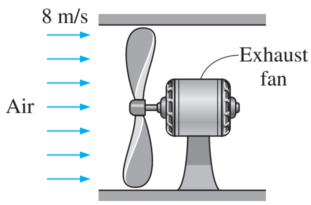


FIGURE P12–47

12–48 Water is being pumped from a large lake to a reservoir 25 m above at a rate of 25 L/s by a 10-kW (shaft) pump. If the irreversible head loss of the piping system is 5 m, determine the mechanical efficiency of the pump. *Answer:* 73.6 percent

12–49 Reconsider Prob. 12–48. Using EES (or other) software, investigate the effect of irreversible head loss on the mechanical efficiency of the pump. Let the head loss vary from 0 to 15 m in increments of 1 m. Plot the results, and discuss them.

12–50 A 7-hp (shaft) pump is used to raise water to a 15-m higher elevation. If the mechanical efficiency of the pump is 82 percent, determine the maximum volume flow rate of water.

12–51 Water flows at a rate of 0.035 m^3/s in a horizontal pipe whose diameter is reduced from 15 cm to 8 cm by a reducer. If the pressure at the centerline is measured to be 480 kPa and 445 kPa before and after the reducer, respectively, determine the irreversible head loss in the reducer. Take the kinetic energy correction factors to be 1.05. *Answer:* 1.18 m

12–52 The water level in a tank is 20 m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank is at sea level, and the water surface is open to the atmosphere. In the line leading from the tank to the nozzle is a pump, which increases the pressure of water. If the water jet rises to a height of 27 m from the ground, determine the minimum pressure rise supplied by the pump to the water line.

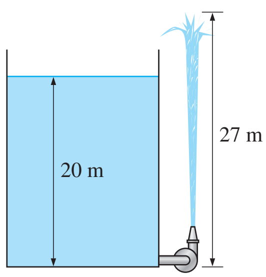


FIGURE P12–52

12–53 A hydraulic turbine has 85 m of head available at a flow rate of 0.25 m^3/s, and its overall turbine–generator efficiency is 78 percent. Determine the electric power output of this turbine.

12–54 An oil pump is drawing 25 kW of electric power while pumping oil with $\rho = 860$ kg/m^3 at a rate of 0.1 m^3/s. The inlet and outlet diameters of the pipe are 8 cm and 12 cm, respectively. If the pressure rise of oil in the pump is measured to be 250 kPa and the motor efficiency is 90 percent, determine the mechanical efficiency of the pump. Take the kinetic energy correction factor to be 1.05.

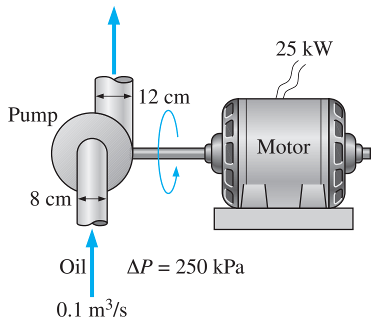


FIGURE P12–54

12–55 Water flows at a rate of 20 L/s through a horizontal pipe whose diameter is constant at 3 cm. The pressure drop across a valve in the pipe is measured to be 2 kPa, as shown in Fig P12–55. Determine the irreversible head loss of the valve, and the useful pumping power needed to overcome the resulting pressure drop. *Answers:* 0.204 m, 40 W

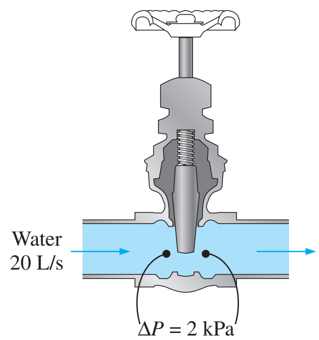


FIGURE P12–55

12–56E The water level in a tank is 66 ft above the ground. A hose is connected to the bottom of the tank at the ground level and the nozzle at the end of the hose is pointed straight up. The tank cover is airtight, but the pressure over the water surface is unknown. Determine the minimum tank air pressure (gage) that will cause a water stream from the nozzle to rise 90 ft from the ground.

12–57 A large tank is initially filled with water 5 m above the center of a sharp-edged 10-cm-diameter orifice. The tank water surface is open to the atmosphere, and the orifice drains to the atmosphere. If the total irreversible head loss in the system is 0.3 m, determine the initial discharge velocity of water from the tank. Take the kinetic energy correction factor at the orifice to be 1.2.

12–58 Water enters a hydraulic turbine through a 30-cm-diameter pipe at a rate of 0.6 m^3/s and exits through a 25-cm-diameter pipe. The pressure drop in the turbine is measured by a mercury manometer to be 1.2 m. For a combined turbine–generator efficiency of 83 percent, determine the net electric power output. Disregard the effect of the kinetic energy correction factors.

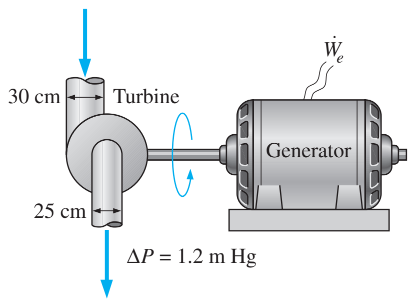


FIGURE P12–58

12–59 The velocity profile for turbulent flow in a circular pipe is approximated as $u(r) = u_{max}(1 - r/R)^{1/n}$, where $n = 7$. Determine the kinetic energy correction factor for this flow. *Answer:* 1.06

12–60 Water is pumped from a lower reservoir to a higher reservoir by a pump that provides 20 kW of useful mechanical power to the water. The free surface of the upper reservoir is 45 m higher than the surface of the lower reservoir. If the flow rate of water is measured to be 0.03 m^3/s, determine the irreversible head loss of the system and the lost mechanical power during this process.

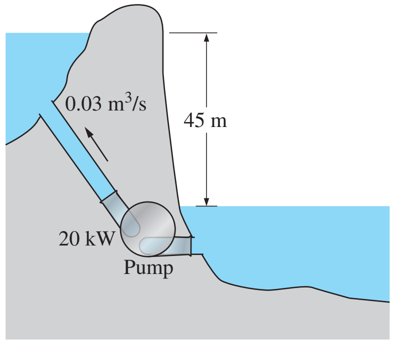


FIGURE P12–60

12–61 Water in a partially filled large tank is to be supplied to the roof top, which is 8 m above the water level in the tank, through a 2.5-cm-internal-diameter pipe by maintaining a constant air pressure of 300 kPa (gage) in the tank. If the head loss in the piping is 2 m of water, determine the discharge rate of the supply of water to the roof top.

12–62 Underground water is to be pumped by a 78 percent efficient 5-kW submerged pump to a pool whose free surface is 30 m above the underground water level. The diameter of the pipe is 7 cm on the intake side and 5 cm on the discharge side. Determine (*a*) the maximum flow rate of water and (*b*) the pressure difference across the pump. Assume the elevation difference between the pump inlet and the outlet and the effect of the kinetic energy correction factors to be negligible.

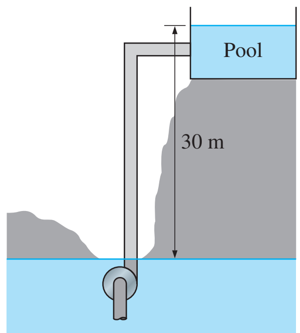


FIGURE P12–62

12–63 Reconsider Prob. 12–62. Determine the flow rate of water and the pressure difference across the pump if the irreversible head loss of the piping system is 4 m.

12–64 A fireboat is to fight fires at coastal areas by drawing seawater with a density of 1030 kg/m^3 through a 20-cm-diameter pipe at a rate of 0.1 m^3/s and discharging it through a hose nozzle with an exit diameter of 5 cm. The total irreversible head loss of the system is 3 m, and the position of the nozzle is 3 m above sea level. For a pump efficiency of 70 percent, determine the required shaft power input to the pump and the water discharge velocity. *Answers:* 199 kW, 50.9 m/s

FIGURE P12–64

Review Problems

12–65 Air flows through a pipe at a rate of 170 L/s. The pipe consists of two sections of diameters 18 cm and 10 cm with a smooth reducing section that connects them. The pressure difference between the two pipe sections is measured by a water manometer. Neglecting frictional effects, determine the differential height of water between the two pipe sections. Take the air density to be 1.20 kg/m^3. *Answer:* 2.60 cm

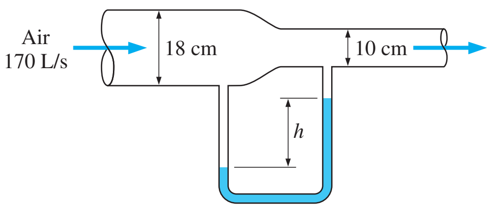


FIGURE P12–65

12–66 Air at 100 kPa and 25°C flows in a horizontal duct of variable cross section. The water column in the manometer that measures the difference between two sections has a vertical displacement of 8 cm. If the velocity in the first section is low and the friction is negligible, determine the velocity at the second section. Also, if the manometer reading has a possible error of ±2 mm, conduct an error analysis to estimate the range of validity for the velocity found.

12–67 A very large tank contains air at 102 kPa at a location where the atmospheric air is at 100 kPa and 20°C. Now a 2-cm-diameter tap is opened. Determine the maximum flow rate of air through the hole. What would your response be if air is discharged through a 2-m-long, 4-cm-diameter tube with a 2-cm-diameter nozzle? Would you solve the problem the same way if the pressure in the storage tank were 300 kPa?

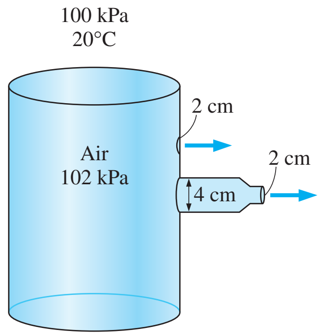


FIGURE P12–67

12–68 Water is flowing through a Venturi meter whose diameter is 7 cm at the entrance part and 4 cm at the throat. The pressure is measured to be 380 kPa at the entrance and 150 kPa at the throat. Neglecting frictional effects, determine the flow rate of water. *Answer:* 0.0285 m^3/s

12–69 Water flows at a rate of 0.025 m^3/s in a horizontal pipe whose diameter increases from 6 to 11 cm by an enlargement section. If the head loss across the enlargement section is 0.45 m and the kinetic energy correction factor at both the inlet and the outlet is 1.05, determine the pressure change.

12–70 A 3-m-high large tank is initially filled with water. The tank water surface is open to the atmosphere, and a sharp-edged 10-cm-diameter orifice at the bottom drains to the atmosphere through a horizontal 80-m-long pipe. If the total irreversible head loss of the system is determined to be 1.5 m, determine the initial velocity of the water from the tank. Disregard the effect of the kinetic energy correction factors. *Answer:* 5.42 m/s

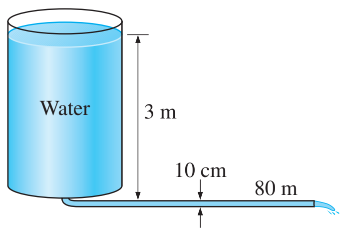


FIGURE P12–70

12–71 Reconsider Prob. 12–70. Using EES (or other) software, investigate the effect of the tank height on the initial discharge velocity of water from the completely filled tank. Let the tank height vary from 2 to 15 m in increments of 1 m, and assume the irreversible head loss to remain constant. Tabulate and plot the results.

12–72 Reconsider Prob. 12–70. In order to drain the tank faster, a pump is installed near the tank exit. Determine the pump head input necessary to establish an average water velocity of 6.5 m/s when the tank is full.

12–73E The water level in a tank is 120 ft above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank is at sea level, and the water surface is open to the atmosphere. In the line leading from the tank to the nozzle is a pump, which increases the water pressure by 10 psia. Determine the maximum height to which the water stream could rise.

12–74 A wind tunnel draws atmospheric air at 20°C and 101.3 kPa by a large fan located near the exit of the tunnel. If the air velocity in the tunnel is 80 m/s, determine the pressure in the tunnel.

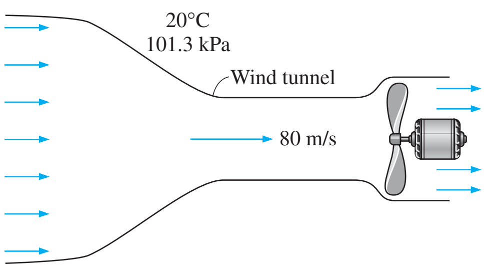


FIGURE P12–74

Design and Essay Problems

12–75 Computer-aided designs, the use of better materials, and better manufacturing techniques have resulted in a tremendous increase in the efficiency of pumps, turbines, and electric motors. Contact one or more pump, turbine, and motor manufacturers and obtain information about the efficiency of their products. In general, how does efficiency vary with rated power of these devices?

12–76 Using a handheld bicycle pump to generate an air jet, a soda can as the water reservoir, and a straw as the tube, design and build an atomizer. Study the effects of various parameters such as the tube length, the diameter of the exit hole, and the pumping speed on performance.

12–77 Using a flexible drinking straw and a ruler, explain how you would measure the water flow velocity in a river.

12–78 The power generated by a wind turbine is proportional to the cube of the wind velocity. Inspired by the acceleration of a fluid in a nozzle, someone proposes to install a reducer casing to capture the wind energy from a larger area and accelerate it before the wind strikes the turbine blades, as shown in Fig. P12–78. Evaluate if the proposed modification should be given a consideration in the design of new wind turbines.

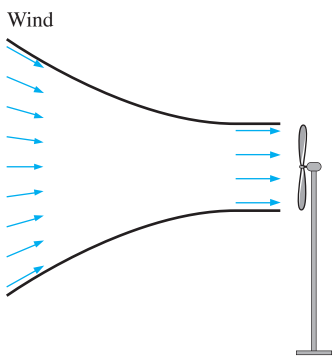


FIGURE P12–78