

trips also decrease the provisioning rate of the chicks, because adults cannot bring back more food in their stomachs to their brood just because they have been away longer.

Despite some promise in computer-enhanced automatic recognition systems based on individual markings⁸, researchers clearly must continue tagging animals for various reasons. Like non-tag marking methods, such as branding⁹ or clipping toes¹⁰, tags inevitably affect some aspect of an animal. The aspiration is that the nature of the beast is not changed by the process. However, given that selection can act on minute differences between individuals, this seems naive.

Instead, we should acknowledge that tags will impair animals. We can then strive to minimize the effects, quantifying them where possible so that we can put the resulting data into perspective. With such an informed background, proposed tagging programmes can consider whether the gain in (imperfect) knowledge

from a scientific viewpoint ethically justifies the harm inflicted. ■

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PARTICLE PHYSICS

Beyond Feynman's diagrams

Generations of physicists have spent much of their lives using Richard Feynman's famous diagrams to calculate how particles interact. New mathematical tools are simplifying the results and suggesting improved underlying principles.

NEIL TUROK

The world works in mathematical ways that we are occasionally privileged to discover. The laws of particle physics, for example, allow us to describe the basic constituents of the Universe, and their interactions, mathematically with astonishing precision and power. However, many important physical processes are so complicated that to perform the required calculations in traditional ways was, until recently, simply unfeasible. In near-simultaneous and complementary papers, Alday *et al.*¹ and Arkani-Hamed *et al.*² have introduced mathematical concepts that bring the calculations under control and provide insights of both immediate practical and deep theoretical importance.

The mathematical framework that we use to describe elementary particles such as electrons and photons, and their interactions, is known as quantum field theory. It was born from the synthesis of quantum mechanics with Maxwell's classical theory of electromagnetic fields and light. Unlike classical fields, quantum fields can be excited only in certain pre-specified, quantized packets of energy called particles. A photon, for example, is the elementary

particle of the quantized electromagnetic field. In the very simplest quantum field theories, the particles do not interact with each other; they merely travel singly through space at a fixed speed. But in more realistic quantum field theories, the particles collide, scatter off each other, and emit or absorb additional particles at rates that are governed by an overall parameter called the interaction coupling.

The physicist Richard Feynman developed a beautiful pictorial shorthand, called Feynman diagrams, for describing all of these processes. The diagrams show a number of initially widely separated particles moving towards each other, interacting, and flying apart again. To calculate the probability of any particular particle-interaction outcome, one draws all the contributing Feynman diagrams at each order of the interaction coupling, translates them into mathematical expressions using Feynman's rules, and adds all of the possible contributions together. This is a well-defined procedure, but at successive orders of the interaction coupling, the number of contributing diagrams grows rapidly and calculations quickly become arduous. Generations of physicists have spent large parts of their lives working out Feynman's formulae for many kinds of scattering processes, and then testing

those formulae in detailed experiments.

The work of Arkani-Hamed *et al.*² originates in a heroic, if mundane, computation undertaken in 1985 by two particle physicists at Fermilab in Batavia, Illinois. Parke and Taylor³ decided to compute all of the Feynman contributions to one of the simplest processes involving the strong nuclear force, whose elementary particle — the gluon — binds quarks together into protons and neutrons. They considered two incoming gluons colliding and producing four outgoing gluons. This is one of the most common processes: for example, in the Large Hadron Collider (LHC), located at CERN, near Geneva, Switzerland, it takes place 100,000 times per second and generates an enormous 'background' signal, which particle physicists must accurately predict and subtract as they search for signals indicating new physics.

The leading contribution to this six-gluon process involves no less than 220 Feynman diagrams, encoding tens of thousands of mathematical integrals. Yet Parke and Taylor found that they could express the final result in just three simple terms. This was the first indication that Feynman diagrams were somehow complicating the story, and that there might be a simpler and more efficient description of these scattering processes.

Further insight into this simplicity was gained by Bern, Dixon and Kosower⁴, and by Britto, Cachazo, Feng and Witten^{5,6}, who developed powerful new techniques — not involving Feynman diagrams — to infer higher-order scattering processes from lower-order ones. Their methods are not only of interest for experiments such as the LHC, but also for testing the mathematical consistency of theories such as supergravity, which is a candidate quantum field theory of gravity.

Work done over the past year has shown why these new methods are simpler than Feynman's. The formulation of quantum field theory used in Feynman's rules emphasizes locality, the principle that particle interactions occur at specific points in space-time; and unitarity, the principle that quantum-mechanical probabilities must sum to unity. However, the price of making these features explicit is that a huge amount of redundancy (technically known as gauge freedom) is introduced at intermediate steps, only to eventually cancel out in the final, physical result.

The calculations of Alday *et al.*¹ and Arkani-Hamed *et al.*² work differently. They assert relations between quantities in a new way, so that the relations are free of these redundancies and they turn out to be sufficient to define the theory. The first big surprise is that such relations exist, and the second is that they are expressed in quantities that are explicitly non-local — that is, quantities that are spread out over space and time.

Both sets of authors perform calculations within a particularly simple family of four-dimensional quantum field theories,

with interactions, known as $N = 4$ supersymmetric theories. These theories are not realistic descriptions of real-world particle physics, but they do have elementary particles such as gluons and quarks (and even Higgs bosons), and they provide a valuable testing ground for new calculational techniques.

Arkani-Hamed and colleagues² exploit a combination of twistor theory — a non-local description of space-time developed by Roger Penrose in the 1970s — and algebraic geometry to obtain a complete description of the scattering of all the elementary particles in these theories, in ascending powers of the interaction coupling. In doing so, the authors provide an excellent characterization of the scattering process when the interaction coupling is small. By contrast, Alday and colleagues¹ derive relations between non-local quantities known as Wilson loops, named after their inventor, the Nobel prizewinner Kenneth G. Wilson. The loops represent the flux of the strong nuclear-force fields through various geometrical areas. Using the powerful mathematical machinery of quantum integrability, Alday *et al.* are able to determine the behaviour of these fluxes in the limit at which the interaction coupling is large. The two sets of authors have therefore described the theory in its two opposite extreme limits — small and large coupling — and the hunt is now on for a complete description, one that is valid for any value of the interaction coupling.

Quantum field theory is the most powerful mathematical formalism known to physics, successfully predicting, for example, the magnetic moment of the electron to one part in a trillion. The recent discovery of mathematical structures that are now seen to control quantum field theory is likely to be of enormous significance, allowing us not only to calculate complex physical processes relevant to real experiments, but also to tackle fundamental questions such as the quantum structure of space-time itself. The fact that the new formulations of the theory^{1,2} jettison much of the traditional language of quantum field theory, and yet are both simpler and more effective, suggests that an improved set of founding principles may also be at hand. ■

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STRUCTURAL BIOLOGY

Finding the wet spots

The functions of proteins are critically coupled to their interplay with water, but determining the dynamics of most water molecules at protein surfaces hasn't been possible. A new spectroscopic method promises to change that.

VINCENT J. HILSER

Proteins in cells are responsible for the vast majority of biological functions. Because life evolved in water, protein molecules are uniquely adapted to use their aqueous environments to facilitate their functions¹. Yet remarkably little is known about the interactions between solvent water and protein molecules, or how those interactions affect (or are affected by) the conformational changes at the heart of protein function. In *Nature Structural and Molecular Biology*, Nucci *et al.*² now report that nuclear magnetic resonance (NMR) spectroscopy of proteins encapsulated in reverse micelles³ — cell-like compartments in which nanometre-scale pools of water are surrounded by a membrane — can provide a comprehensive picture of how water molecules bind to proteins. This picture not only challenges current dogma about protein hydration, but also promises to illuminate key aspects of the relationship between protein and water dynamics, and of how proteins use water to perform their functions.

Early studies^{4,5} of protein–water interactions — the exchange of water molecules between a protein's surface and the surrounding bulk water — were performed in bulk solution using NMR. But because of ambiguities resulting from the timescale of the exchange process, as well as the inability to distinguish between that process and another in which labile hydrogens in the protein exchange with those in water⁶, direct experimental analysis of protein–water dynamics (hydration dynamics) was restricted to only the most long-lived of interactions. Attempts to rectify this have relied mostly on X-ray crystal structures of proteins to identify the locations of resolvable water molecules in the structure, which, in spite of well-documented reservations⁷, have generally been presumed to represent the 'hydration shell' of water molecules around the protein⁸ (Fig. 1a). Nucci and colleagues' new NMR approach² overcomes the previous experimental limitations, thus providing a comprehensive picture of the whole hydration shell around a test protein, ubiquitin.

The reverse-micelle technology used by Nucci *et al.*² was previously developed³ to overcome the protein-size limitation inherent to NMR studies — large proteins can't be studied by NMR because they tumble too slowly in solution. Encapsulation of large proteins in

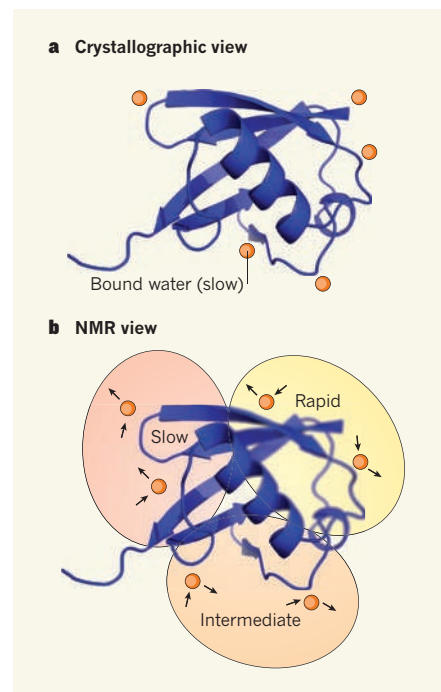


Figure 1 | Crystallographic versus NMR views of protein hydration. **a**, Certain sites at which water molecules associate with protein surfaces can be identified in X-ray crystal structures, as shown schematically here for ubiquitin. The sites are assumed to be those where water molecules reside for longest (that is, where the dynamics of water movement are slowest). **b**, This cartoon depicts the NMR view of hydration obtained by Nucci *et al.*², wherein a complete picture of the locations and dynamics of water molecules bound to ubiquitin was ascertained. They observed that water molecules cluster into regions corresponding to slow, intermediate and rapid average dynamics. Little correlation was found between the crystallographic and NMR views of hydration dynamics. Arrows indicate that the rates of water-molecule exchange between the protein's surface and the solvent are directly measured by NMR. By contrast, the X-ray picture is static, and exchange rates must be inferred or calculated. Images were created using PyMOL¹⁵.

reverse micelles dissolved in a low-viscosity fluid, however, allows them to tumble at rates similar to those of much smaller proteins. What's more, such encapsulation dramatically slows both the hydration dynamics and the hydrogen-exchange kinetics of proteins compared with the same quantities in bulk solvent. This is the cornerstone of Nucci and colleagues' advance², because it enables

Clearly, if one finds that a zillion Feynman diagrams add up to a simple expression which fits on one line, one becomes suspicious that something important might be going on...

What's special about amplitudes?

- Their simplicity !
 - ▶ calculation with textbook methods cumbersome, however final results are often strikingly simple
- Theorem:
 - ▶ if you have a simple result, there should be a simple way to get there!

Zvi Bern

Feynman Diagrams: Past, Present, Future

<http://www.youtube.com/watch?v=3SYW6eZFHwU>

Roger Penrose

Twistors and Quantum Non-Locality

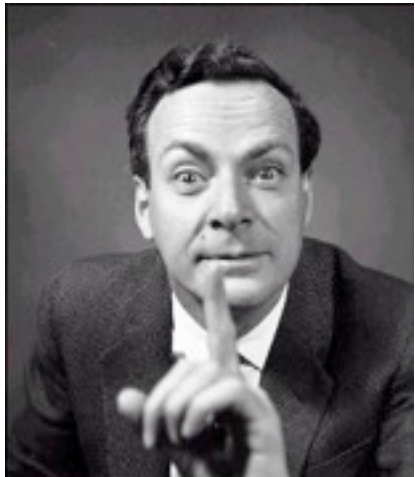
<http://www.youtube.com/watch?v=hAWyex1GKRU>

Textbook approach to amplitudes:



Calculate Feynman diagrams !

Textbook approach to amplitudes:



So, what's wrong with them ?

Unwanted complexity

Number of Feynman diagrams for $gg \rightarrow n g$ scattering: (tree level)



n	2	3	4	5	6	7	8
# of diagrams	4	25	220	2485	34300	559405	10525900

Gluon scattering



Result is:

$$\mathcal{A}(1^\pm, 2^+, \dots, n^+) =$$

[Faint, illegible text, likely bleed-through from the reverse side of the slide]

FullSimplify[

FullSimplify[...]

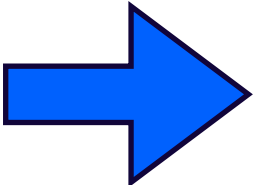
FullSimplify[...]

FullSimplify[...]

FullSimplify[...]

+ many more pages like this...]

$$= 0 !!$$



Why so simple?

Why zero?

The simplest amplitude

(Parke & Taylor, 1986; Berends, Giele 1987; Mangano, Parke, Xu 1988)

$$A_{\text{MHV}}(1^+, \dots, i^-, \dots, j^-, \dots, n^+) = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$

gluon helicities are a permutation of $--++\dots+$

Maximally Helicity Violating, or MHV

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where s is the corresponding pole and z is the momentum fraction. The result for particles 2 and 3 nearly parallel, Eq. (5), is only simple because $\mathcal{M}_{n-1}(-+++\dots)$ is zero to this order in g so that there is no interference term and therefore azimuthal averaging is not required.



The surprise about this result is that all denominators are simple dot products of two external momenta. The Feynman diagrams for n -gluon ($n > 5$) scattering contain propagators $(p_i + p_j + p_k)^2$, $(p_i + p_j + p_k + p_m)^2$, \dots . These propagators must cancel for Eq. (3) to be correct; this occurs for $n=6$. Of course, Altarelli and Parisi have taught us that many cancellations are expected.



We do not expect such a simple expression for the other helicity amplitudes. Also, we challenge the string theorists to prove more rigorously that Eq. (3) is correct.

Fermilab is operated by the Universities Research Association Inc. under contract with the United States

Department of Energy.

¹E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg, *Rev. Mod. Phys.* **56**, 579 (1984).

²M. T. Grisaru, H. N. Pendleton, and P. van Nieuwenhuizen, *Phys. Rev. D* **15**, 997 (1977); M. T. Grisaru and H. N. Pendleton, *Nucl. Phys.* **B124**, 81 (1977).

³S. J. Parke and T. R. Taylor, *Phys. Lett.* **157B**, 81 (1985).

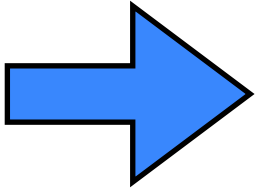
⁴T. Gottschalk and D. Sivers, *Phys. Rev. D* **21**, 102 (1980); F. A. Berends, R. Kleiss, P. de Causmacker, R. Gastmans, and T. T. Wu, *Phys. Lett.* **103B**, 124 (1981).

⁵S. J. Parke and T. R. Taylor, Fermilab Report No. Pub-85/118-T, 1985 (to be published); Z. Kunszt, CERN Report No. TH-4319, 1985 (to be published).

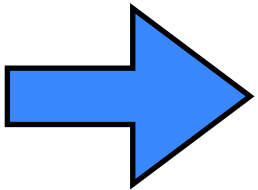
⁶Another numerical fact worth mentioning is that to leading order in g but to all orders in N , the amplitude $|\mathcal{M}_{n=6}(- - + + +)|^2$ is permutation symmetric apart from the factor $(p_1 \cdot p_2)^4$. This allows all permutations of this amplitude to be trivially calculated from one such permutation.

⁷G. Altarelli and G. Parisi, *Nucl. Phys.* **B126**, 298 (1977).

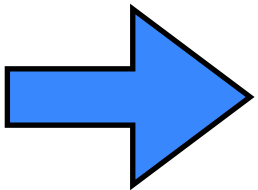
Back-of-envelope formula...



There are **hidden structures** in scattering amplitudes (and other physical observables)...



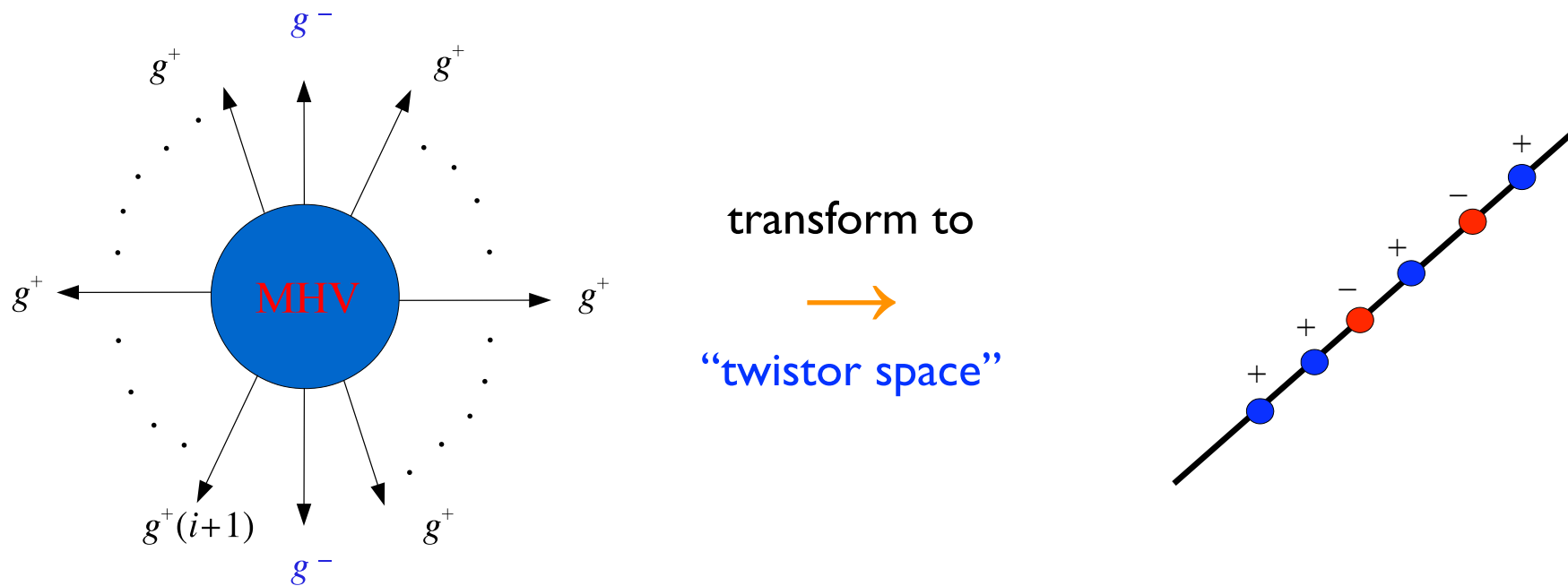
...which are not captured by Feynman diagrams



Need **new framework** to describe and calculate them

Harmony of the MHV amplitude

(Witten '03)



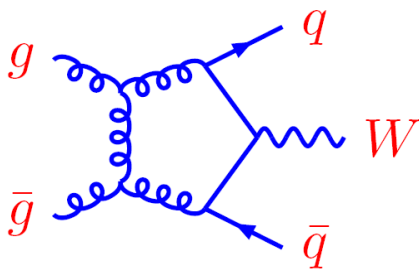
- **Harmony unveiled in Penrose's twistor space**
 - ▶ **MHV amplitude** → **line in twistor space**
 - ▶ **simplicity in terms of geometry**
 - **simplest scattering amplitude** corresponds to **simplest curve**

State-of-the-Art NLO QCD

Five point is *still* state-of-the art for QCD cross-sections:

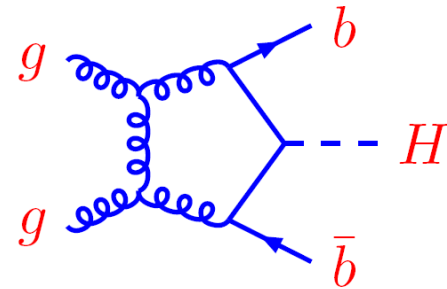
Typical examples:

$pp \rightarrow W, Z + 2 \text{ jets}$



Bern, Dixon, Kosower
Dixon, Kunszt, and Signer
Campbell and Ellis: MCFM

$pp \rightarrow \bar{b}bH$ or $pp \rightarrow \bar{t}tH$



Reina, Dawson, Jackson and Wackerth
Beenakker, Dittmaier, Kramer, Plumper, Spira

Brute force calculations give GB expressions – numerical stability?

Amusing numbers: 6g: 10,860 diagrams, 7g: 168,925 diagrams

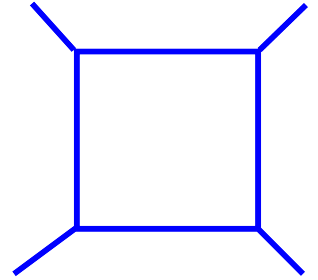
Much worse difficulty: integral reduction generates nasty dets.

$$\frac{1}{\det(k_i \cdot k_j)^n}$$

“Grim” determinant 7

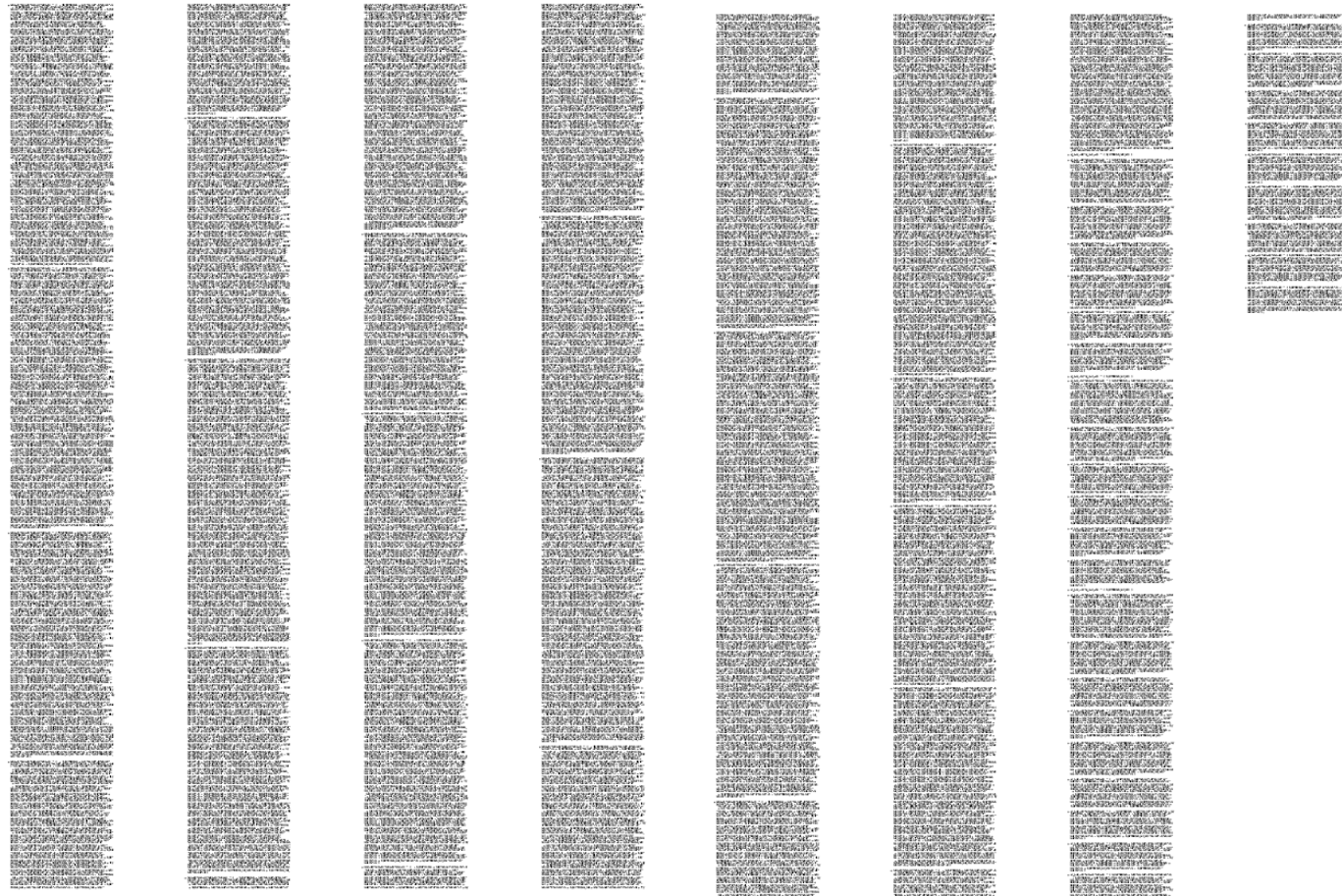
Consider an integral

$$\int \frac{d^{4-2\epsilon} \ell}{(2\pi)^{4-\epsilon}} \frac{\ell^\mu \ell^\nu \ell^\rho \ell^\lambda}{\ell^2 (\ell - k_1)^2 (\ell - k_1 - k_2)^2 (\ell + k_4)^2}$$



Evaluate this integral via Passarino-Veltman reduction. Result is ...

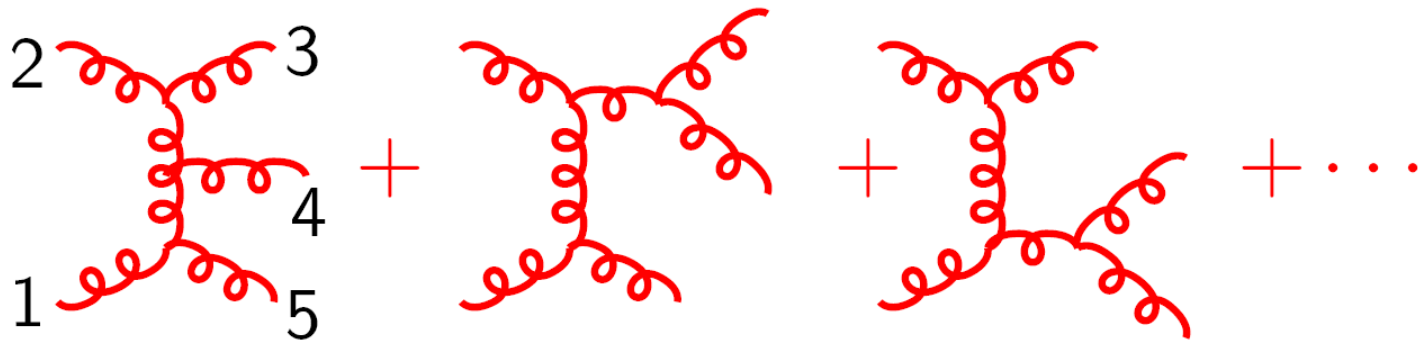
Result of performing the integration



Numerical stability is a key issue.
Clearly, there should be a better way

Tree-level example: Five gluons

Consider the five-gluon amplitude



If you evaluate this you find...

Result of evaluation (actually only a small part of it):

[[[["k1", "k4", "epsilon2", "k1", "epsilon1", "epsilon3", "epsilon4", "epsilon5"]]]]

[[[["k1", "k4", "epsilon2", "k1", "epsilon1", "epsilon3", "epsilon4", "epsilon5"]]]]

[[[["k1", "k4", "epsilon2", "k1", "epsilon1", "epsilon3", "epsilon4", "epsilon5"]]]]



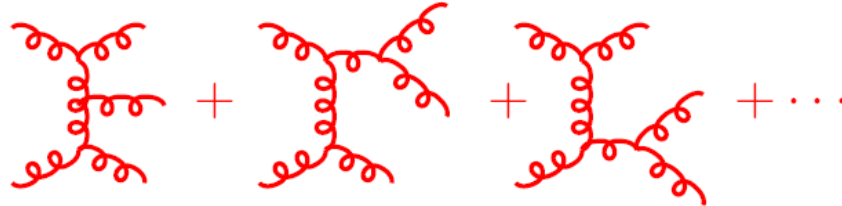
$$k_1 \cdot k_4 \epsilon_2 \cdot k_1 \epsilon_1 \cdot \epsilon_3 \epsilon_4 \cdot \epsilon_5$$

[[[["k1", "k4", "epsilon2", "k1", "epsilon1", "epsilon3", "epsilon4", "epsilon5"]]]]

[[[["k1", "k4", "epsilon2", "k1", "epsilon1", "epsilon3", "epsilon4", "epsilon5"]]]]

[[[["k1", "k4", "epsilon2", "k1", "epsilon1", "epsilon3", "epsilon4", "epsilon5"]]]]

Reconsider Five Gluon Tree



With spinor helicity:

$$A_5^{\text{tree}}(1^\pm, 2^+, 3^+, 4^+, 5^+) = 0$$

$$A_5^{\text{tree}}(1^-, 2^-, 3^+, 4^+, 5^+) = i \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

$$A_5^{\text{tree}}(1^-, 2^+, 3^-, 4^+, 5^+) = i \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

These are color stripped amplitudes:

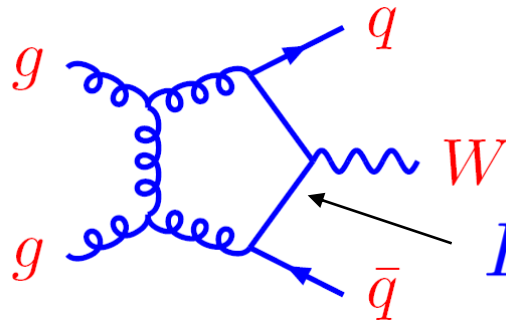
$$A_5 = \sum_{\text{perms}} \text{Tr}(T^{a_1} T^{a_2} T^{a_3} T^{a_4} T^{a_5}) A_5(1, 2, 3, 4, 5)$$

Why are Feynman diagrams difficult for high-loop or high-multiplicity processes?

- Vertices and propagators involve unphysical gauge-dependent off-shell states. An important origin of the complexity.



$$\int \frac{d^3\vec{p} dE}{(2\pi)^4}$$



Individual Feynman diagrams unphysical

$$E^2 - \vec{p}^2 \neq m^2$$

Einstein's relation between momentum and energy violated in the loops. **Unphysical states! Not gauge invariant.**

- **All steps should be in terms of gauge invariant on-shell physical states. On-shell formalism. Need to rewrite quantum field theory!**

Beyond Feynman Diagrams



- **Feynman diagrams** are **very general and powerful**
- However, for many applications, **on-shell methods based on analyticity** are a much more efficient way to get the same answer.
- They also give new insight into structure and properties of scattering amplitudes, not only in QCD

Just one QCD loop can be a challenge

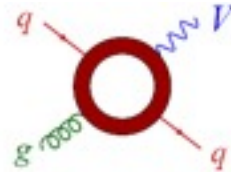
$pp \rightarrow W + n \text{ jets}$

(just amplitudes with most gluons)

of jets

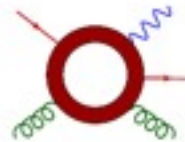
1-loop Feynman diagrams

1



11

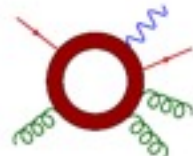
2



110

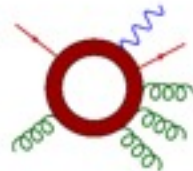
Current limit with Feynman diagrams

3



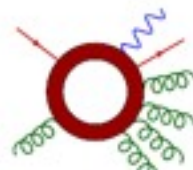
1,253

4



16,648

5



256,265

Current limit with on-shell methods

Master diagrams: One diagram to rule them all

ZB, Carrasco, Johansson (2010)

$N = 4$ super-Yang-Mills integrand

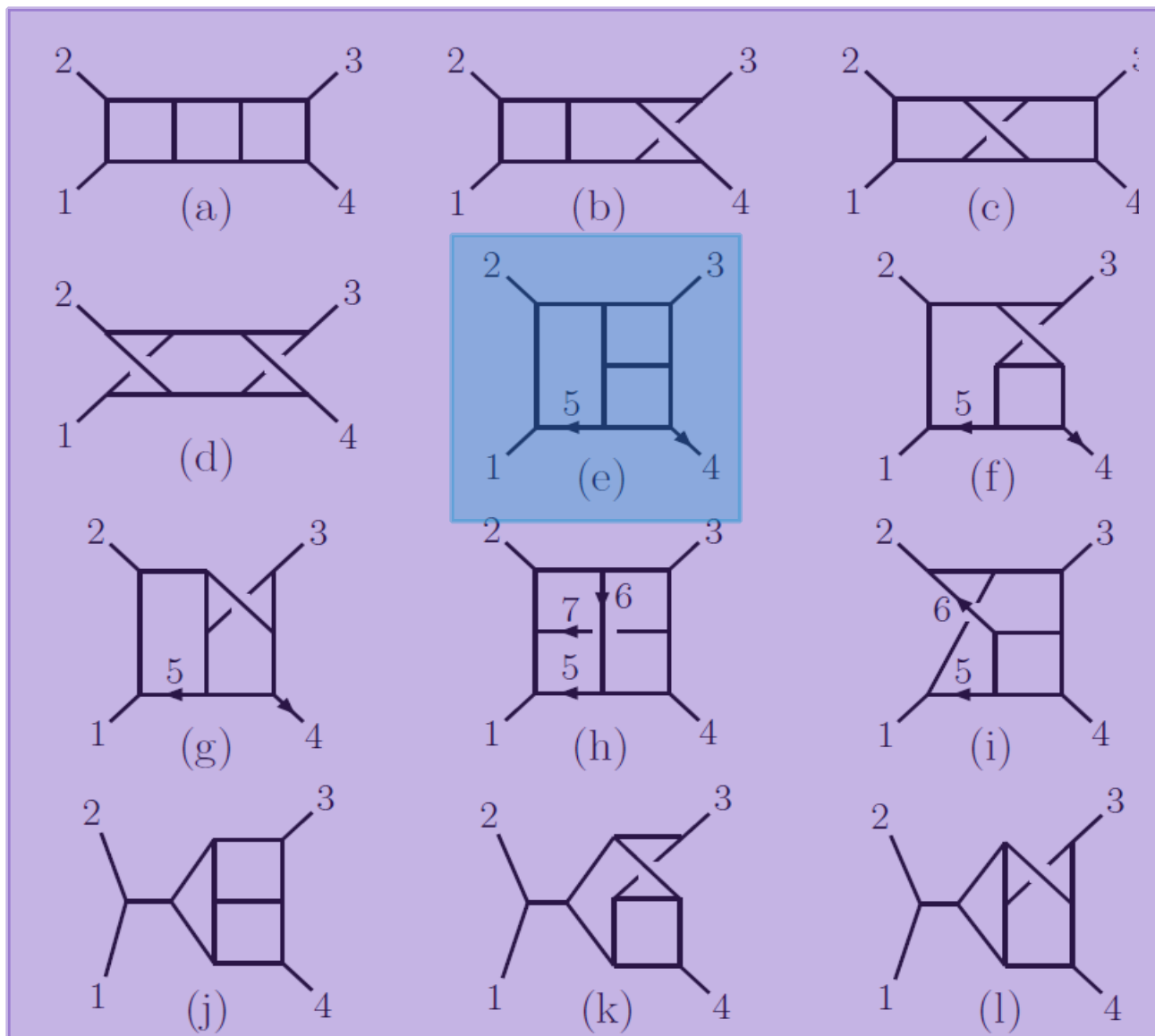


Diagram (e) is the master diagram.

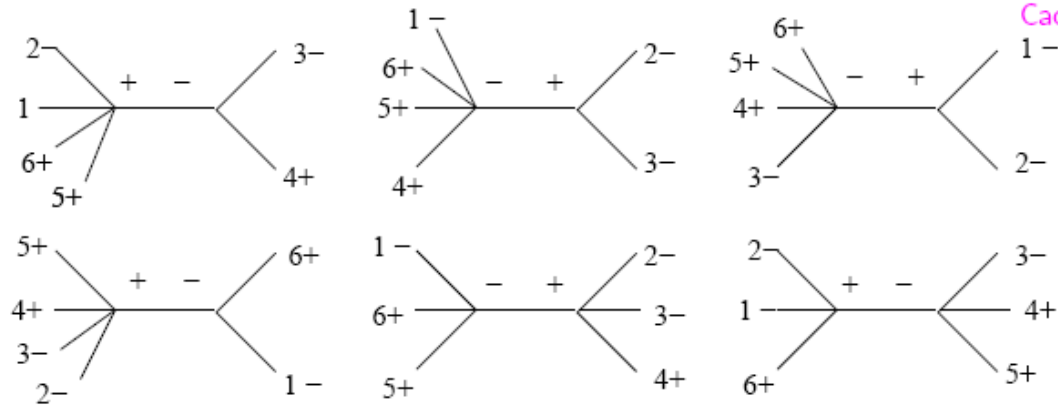
Determine the master numerator in proper form and duality gives all others.

$N = 8$ sugra given by double copy.

Six gluon example

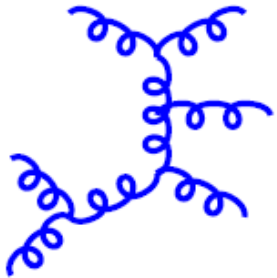
QCD gluon
scattering
amplitude

-- -- -- + + +



Cachazo, Svrcek and Witten

$$\begin{aligned}
 A_6(1^-, 2^-, 3^-, 4^+, 5^+, 6^+) = & \frac{\langle 12 \rangle^3}{\langle 56 \rangle \langle 61 \rangle \langle 2 | 5 + 6 + 1 | q \rangle \langle 5 | 6 + 1 + 2 | q \rangle} \times \frac{1}{s_{34}} \times \frac{\langle 3 | 4 | q \rangle^3}{\langle 34 \rangle \langle 4 | 3 | q \rangle} \\
 & + \frac{\langle 1 | 4 + 5 + 6 | q \rangle^3}{\langle 45 \rangle \langle 56 \rangle \langle 61 \rangle \langle 4 | 5 + 6 + 1 | q \rangle} \times \frac{1}{s_{23}} \times \frac{\langle 23 \rangle^3}{\langle 3 | 2 | q \rangle \langle 2 | 3 | q \rangle} \\
 & + \frac{\langle 3 | 4 + 5 + 6 | q \rangle^3}{\langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 6 | 3 + 4 + 5 | q \rangle} \times \frac{1}{s_{12}} \times \frac{\langle 12 \rangle^3}{\langle 2 | 1 | q \rangle \langle 1 | 2 | q \rangle} \\
 & + \frac{\langle 23 \rangle^3}{\langle 34 \rangle \langle 45 \rangle \langle 5 | 2 + 3 + 4 | q \rangle \langle 2 | 3 + 4 + 5 | q \rangle} \times \frac{1}{s_{61}} \times \frac{\langle 1 | 6 | q \rangle^3}{\langle 61 \rangle \langle 6 | 1 | q \rangle} \\
 & + \frac{\langle 1 | 5 + 6 | q \rangle^3}{\langle 56 \rangle \langle 61 \rangle \langle 5 | 6 + 1 | q \rangle} \times \frac{1}{s_{561}} \times \frac{\langle 23 \rangle^3}{\langle 34 \rangle \langle 4 | 2 + 3 | q \rangle \langle 2 | 3 + 4 | q \rangle} \\
 & + \frac{\langle 12 \rangle^3}{\langle 61 \rangle \langle 2 | 6 + 1 | q \rangle \langle 6 | 1 + 2 | q \rangle} \times \frac{1}{s_{612}} \times \frac{\langle 3 | 4 + 5 | q \rangle^3}{\langle 34 \rangle \langle 45 \rangle \langle 5 | 3 + 4 | q \rangle}
 \end{aligned}$$



220 Feynman diagrams



A “perfect” calculation

Twistor theory

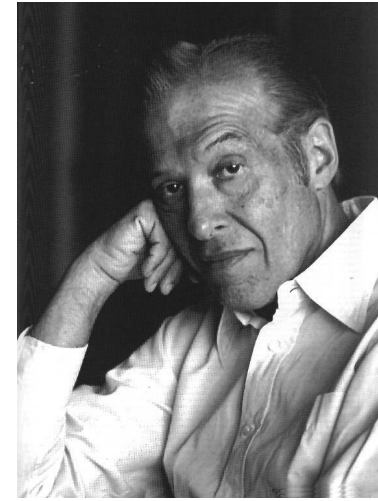
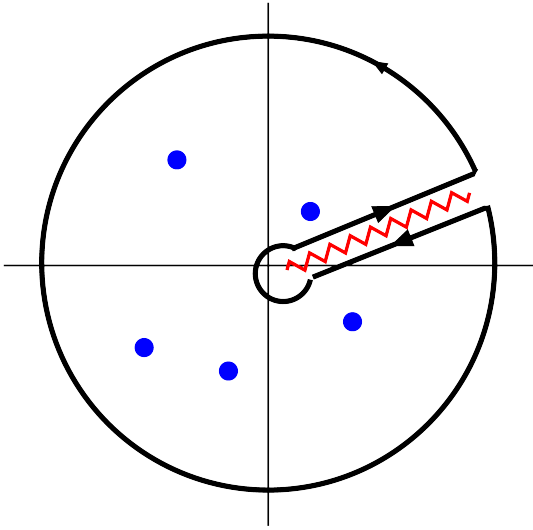
From Wikipedia, the free encyclopedia

"Twistor" redirects here. Not to be confused with "Twister"

In [theoretical](#) and [mathematical physics](#), **twistor theory** maps the [geometric](#) objects of conventional 3+1 [space-time](#) ([Minkowski space](#)) into geometric objects in a 4 dimensional space with [metric signature](#) (2,2). This space is called [twistor space](#), and its [complex](#) valued coordinates are called "twistors."

Twistor theory was first proposed by [Roger Penrose](#) in 1967,^[1] as a possible path to a theory of [quantum gravity](#). The twistor approach is especially natural for solving the equations of motion of [massless fields](#) of arbitrary [spin](#).

In 2003, [Edward Witten](#)^[2] proposed to unite twistor and [string theory](#) by embedding the [topological B model](#) of string theory in [twistor space](#). His objective was to model certain [Yang-Mills amplitudes](#). The resulting model has come to be known as twistor string theory (read below). [Simone Speziale](#) and collaborators have also applied it to [loop quantum gravity](#).



“One of the most remarkable discoveries in elementary particle physics has been that of the existence of the complex plane.”

J. Schwinger in “Particles, Sources and Fields” Vol 1

Twistor Space \mathbb{C}^4

Four-dimensional complex space

Minkowski space \mathbb{R}^4

Four-dimensional real space

$$\mathbf{R^4 \Leftrightarrow C^4}$$

light rays \Leftrightarrow points

light cones \Leftrightarrow circles

point \Leftrightarrow sphere

angles preserved

Zvi Bern

Feynman Diagrams: Past, Present, Future

<http://www.youtube.com/watch?v=3SYW6eZFHwU>

Roger Penrose

Twistors and Quantum Non-Locality

<http://www.youtube.com/watch?v=hAWyex1GKRU>

we can think of the points (x, y) as lying in a two-dimensional plane, on which the unbroken $SL(2)$ acts. The area of the triangle associated with W^1, W^2, W^3 is the $SL(2)$ invariant given by

$$\text{Area}(W^1, W^2, W^3) = \frac{1}{2} \begin{vmatrix} x^1 & x^2 & x^3 \\ y^1 & y^2 & y^3 \\ 1 & 1 & 1 \end{vmatrix} \quad (23)$$

which we can write in a projectively invariant way as

$$\text{Area}(W^1 W^2 W^3) = \frac{1}{2} \frac{\langle W^1 W^2 W^3 \rangle}{(Z_A \cdot W^1)(Z_A \cdot W^2)(Z_A \cdot W^3)}. \quad (24)$$

Note that this is *not* invariant under rescaling the reference twistor Z_A , which is appropriate, since Z_A defines the plane in which the area is defined and the area is not dimensionless.

Now, suppose we are given instead three points in the original space, Z_a^I, Z_b^I, Z_c^I . Each of these points is associated with a line in the W space, with e.g. the point a defining the line $Z_a^I W_I = 0$. The lines a and b intersect at the point (ab) in W space, with co-ordinate $W_I^{(ab)} = \epsilon_{IJK} Z_a^J Z_b^K$. Thus, the area of this dual triangle is

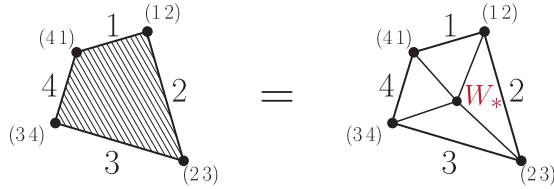
$$\text{Area}(\widetilde{[abc]}) = \frac{1}{2} \frac{\langle (ab)(bc)(ca) \rangle}{\langle Aab \rangle \langle Abc \rangle \langle Aca \rangle} = \frac{1}{2} \frac{\langle abc \rangle^2}{\langle Aab \rangle \langle Abc \rangle \langle Aca \rangle} = [abc]. \quad (25)$$

With these elementary facts in hand, it is easy to identify the triangulations of the polygon associated with the BCFW/CSW representations of the amplitude, which correspond to triangulating \widetilde{L}_n , with the dual triangles $[\ast i i + 1]$. An example of a BCFW triangulation for the 4-particle amplitude is shown below:

$$\text{Area} = [234] + [241]$$

Note that the BCFW triangulation is characterized by not introducing any new *lines*, but certainly introduces new vertices. However, we have an even more obvious triangulation of the same object, introducing a dual reference point W_* , and triangulating directly using the

vertices as



For a general polygon, the area can be triangulated using the triangles with vertices $(W_*, (i-1i), (ii+1))$. We can compute the area of this triangle using eqn. (24), giving an n -term expression for the amplitude

$$A_n = \frac{1}{2} \sum_i \frac{\langle W_* (i-1i) (ii+1) \rangle}{Z_A \cdot W_* \langle A i-1i \rangle \langle A ii+1 \rangle} = \sum_i \frac{(Z_i \cdot W_*) \langle i-1ii+1 \rangle}{(Z_A \cdot W_*) \langle A i-1i \rangle \langle A ii+1 \rangle}. \quad (26)$$

Note that in this form, *all* the poles involving the Z_i are manifestly “physical”. This is obvious, since we have triangulated the polygon only using its vertices, and the divergences of the amplitude can only occur if some vertex $(kk+1)$ moves off to infinity, making the area diverge. By contrast, the BCFW/CSW representations introduce new points in W space, with associated spurious pole which cancel in the sum. Note also that this triangulation involved a natural reference point W_* , analogous to the reference point Z_* in the CSW representation of the amplitude. The result is independent of W_* , but term-by-term has a “spurious pole” $(Z_A \cdot W_*)$. We can choose W_* to coincide with one of the external points, say $W_* = (kk+1)$, giving an $(n-2)$ term expression with manifestly physical poles which is however no longer manifestly cyclic invariant.

We close this warm-up section with a few comments. We have drawn pictures of our polygons on a real 2-dimensional plane, but of course the functions are all holomorphic and defined on \mathbb{CP}^2 . The complex areas have a very nice interpretation in terms of contour integrals in \mathbb{CP}^2 with boundaries on the polygon \tilde{L}_n [4]. It is perhaps easiest to get a feeling for such contour integrals with boundary by considering the simplest case of a standard integral over one complex variable z , thought of as a projective integral over \mathbb{CP}^1 . Let’s introduce a variable $w_I = (x, y)$ in \mathbb{C}^2 . Consider an integral with boundaries on $z_a^I w_I = 0, z_b^I w_I = 0$:

$$\int \frac{Dw}{(z_c \cdot w)^2} \quad (27)$$

$$z_a \cdot w = 0$$

$$z_b \cdot w = 0$$

here, we use

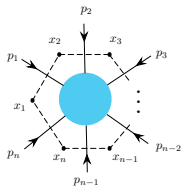
$$D^{n-1}w \equiv \frac{d^n w}{\text{volGL}(1)} \quad (28)$$

Good variables

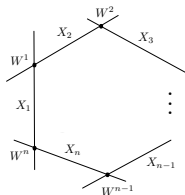
- 1 spinor helicity $p_i = |i\rangle[i]$ makes null condition $p_i^2 = 0$ manifest.
- 2 region variables make mom. cons. manifest, but requires $(x_i - x_{i+1})^2 = p_i^2 = 0$.

Wouldn't it be nice to have unconstrained variables in which dual conformal symmetry is manifest!!?

This is what **momentum twistors** do! [Hodges'09]



from [Mason, Skinner'09]



$x_i \rightarrow X_i$ line in \mathbb{CP}^3

$$(x_i - x_{i+1})^2 = 0$$

$\rightarrow X_i$ and X_{i+1} intersect

The intersection points W_i are the momentum twistors

$$\text{Define } \langle ijkl \rangle = \epsilon_{ABCD} W_i^A W_j^B W_k^C W_l^D$$

Fads come and go

Today our field is one of the hottest ones around.
Yesterday's impossible problems are today's trivialities.

Is our field just another (albeit long lasting) fad?

- Ringwaldmania (1989) .
 - M theory as a matrix model (1996)
 - Noncommutative field theory.
 - Dijkgraaf –Vafa. (2002)
 - String based model building (1986-1989).
- etc.

Some disappear completely and some have tails that fade in time

Question:

What should we do to ensure that we have a long-lasting impact, so people care about what we are doing here 10 years from now?