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BIFURCATION ANALYSIS FOR A QUASI-LINEAR CENTRIFUGAL PENDULUM ABSORBER

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Abstract. *A vibrational pendulum absorber with trapezoidal suspension is a modification of the classical pendulum absorber, which allows not only its translational motion but some complex 2D-motion of the pendulum. It provides not only higher performances of the absorber, but also some new interesting features. About them it is a possibility for partial compensation of non-linearity. It allows avoiding the parasite super-harmonical respond of the pendulum. The bifurcation analysis of such a pendulum make possible to estimate effectiveness of a new type pendulum absorber.*
This paper is a continuation of previous investigations [1,2] devoted to a pendulum absorber problem. Main methods used here are methods based on the bifurcation theory [3-11] of nonlinear dynamical systems. Among them are the method of complete bifurcation groups with parameter (orbit) continuation, concepts of periodic skeletons with passports of stable and unstable orbits, unstable periodic infinitiums, determined chaotic regions and rare attractors. For numerical realization of the bifurcation theory for pendulum absorber problem, original software is used.

1 INTRODUCTION

The direct global bifurcation analysis of pendulum-like systems is presented by the main features and components of a new so-called bifurcation theory of nonlinear dynamics and chaos and its applications. The described part of bifurcation theory uses dynamical periodic systems, described by a model of ODE equations. Our approach is based on ideas of Poincaré, Andronov and other scientists' results concerning structural stability and bifurcations of different dynamical nonlinear systems and their topological properties [3-8]. For illustration of the advantages of the new bifurcation theory we use in this paper the pendulum vibration absorber model [1,2] with soft impact (Figure 1,a). We have found important unknown regular or chaotic attractors and new bifurcation groups with rare attractors. The method of complete bifurcation groups gives robust stability to the pendulum vibration absorber by changing the system parameters, that allows saving characteristics of the vibration absorber system and excluding other amazing regimes.

Recent efforts in nonlinear dynamics of pendulum systems show, that the behaviour of the driven damped pendula may be complex and sometimes with the unexpected phenomena such as stable hilltop vibrations, complex subharmonical and quasi-periodical vibrations, different rotations and other [9-11]. Our aim is doing complete bifurcation analysis for important parameter of pendulum vibration absorber systems by using the fundamental concepts of the bifurcation theory: a periodic skeleton, complete bifurcation group nT, subgroup of unstable periodic infinitium (UPI), concepts of rare attractors, complex protuberances, typical bifurcation topological groups and the topological structure of chaotic attractors and chaotic transients [3-8]. All results were obtained numerically, using software NLO [3] and SPRING [5], created in Riga Technical University.

2 MODEL OF THE PENDULUM ABSORBER

The studied nonlinear dynamical model of the pendulum vibration absorber [1,2] with soft impact (see Figure 1,a) is represented. System has trilinear dissipation and restoring forces (Figure 1,b). The equation of motion for studied pendulum system is such:

$$(1-\gamma)\ddot{\varphi}+f(\dot{\varphi})+f_1(\varphi)+h\omega\cos\omega t+\omega^2(1+d)^2(1+2h\sin\omega t)\sin\varphi=0, \qquad (1)$$

where φ – angle of the pendulum relative to basic disk (flywheel); $\dot{\varphi}$ – angular velocity of the pendulum relative to basic disk, where $\dot{\varphi}=d\varphi/dt$; b_1, b_2 – linear dissipation coefficients, accordingly to linear subregions of visco-elastic characteristic; c_r – stiffness coefficient of linear elastic characteristic at soft impact in coordinate $\varphi_r = \pm 15^{o}$:

$$f(\dot{\varphi})=\begin{cases} b_2\dot{\varphi} & if \quad \varphi<-\varphi_r \\ b_1\dot{\varphi} & if \quad -\varphi_r\le\varphi<\varphi_r \\ b_2\dot{\varphi} & if \quad \varphi\ge\varphi_r \end{cases};\quad f_1(\varphi)=\begin{cases} c_r(\varphi+\varphi_r) & if \quad \varphi<-\varphi_r \\ 0 & if \quad -\varphi_r\le\varphi\le\varphi_r \\ c_r(\varphi-\varphi_r) & if \quad \varphi>\varphi_r \end{cases}, \qquad (2)$$

γ – ratio of moments of inertia of the pendulum and disk (γ<1); h – moment from the motor (oscillating component) ($h \le 1.5$); ω – excitation order (ω = 2 or 3, respectively for 4-cylinder and 6-cylinder engine); d – error in setting up the pendulum vibration absorber at antiresonance ($-0.1 \le d \le 0.1$).

The variable parameter of the pendulum system Eq.1 is moment from the motor h = 0...1.5 and excitation order ω = 0...3.5. The base parameters for analysis are: γ = 0.1, b_1 = 0.2, b_2 = 5, c_r = 5000, d = 0, h = 0.5.

3 CONSTRUCTION OF PERIODIC SKELETON FOR BASE PARAMETERS OF THE PENDULUM ABSORBER

Before the construction of complete bifurcation diagrams, periodic skeleton for base parameters of the pendulum vibration absorber was build.

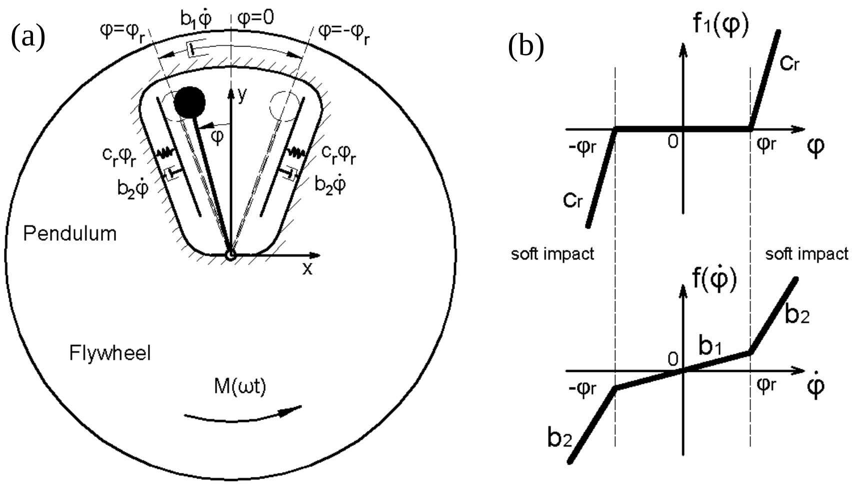

Figure 1: (a) A flywheel with pendulum vibration absorber with soft impact; (b) elastic dissipation forces characteristics and dependence of dissipation coefficient on state of system.

From the grid of 20×20 = 400 initial conditions inside the rectangle (-π/12; -1/ π/12; 1) we have obtained periodic skeleton [2] consisted from 17 regimes (one unstable P1 regime; one unstable P2 regime; four P4 regimes – two of them are stable; two unstable P5 regimes; two unstable P6 regimes; one unstable P7 regime; one unstable P8 regime; three unstable P9 regimes; one unstable P11 regime and one unstable) for h = 0.5 and ω = 3 with regime scanning for period from 1 till 16 by using the Newton-Kantorovich method.

By using the Poincaré map founded regimes can be specified by data of the solution (orbit) order, by the number of loops in the projection of the phase trajectories, the coordinates of the fixed point and stability characteristics. For example,

$$1\ (1/1),\ \text{Fixed point}\ (-0.185917,\ -0.502712),\ \rho_1 = -0.0781,\ \rho_2 = -4.5831.$$

This is the main passport of periodic regime (at a given point of the parameter space of the system). The examples of extended passport of periodic regimes with phase projections and time histories are shown in [2,11].

4 COMPLETE BIFURCATION ANALYSIS OF PENDULUM VIBRATION ABSORBER HEADINGS

The construction of bifurcation diagrams was done by the Runge-Kutta 4-th order with constant integration step: $\Delta t = T_\omega/2^k$, where the period of external force is $T_\omega = 2\pi/\omega$ and coefficient k = 7.

The results of complete bifurcation analysis of the pendulum vibration absorber model are represented in bifurcation diagrams of the fixed periodic points of the coordinates φ_p, $\dot{\varphi}_p$ and Am versus parameter h (Figure 2). There are two 1T bifurcations groups: 1T (with depth P1-P2) with complex protuberances and 1T isle. The pendulum system has many RA of different kinds. In this figure stable solutions are plotted by solid lines and unstable - by thin lines (reddish online). The branches of bifurcation diagram are not completed, because of problem of high instability.

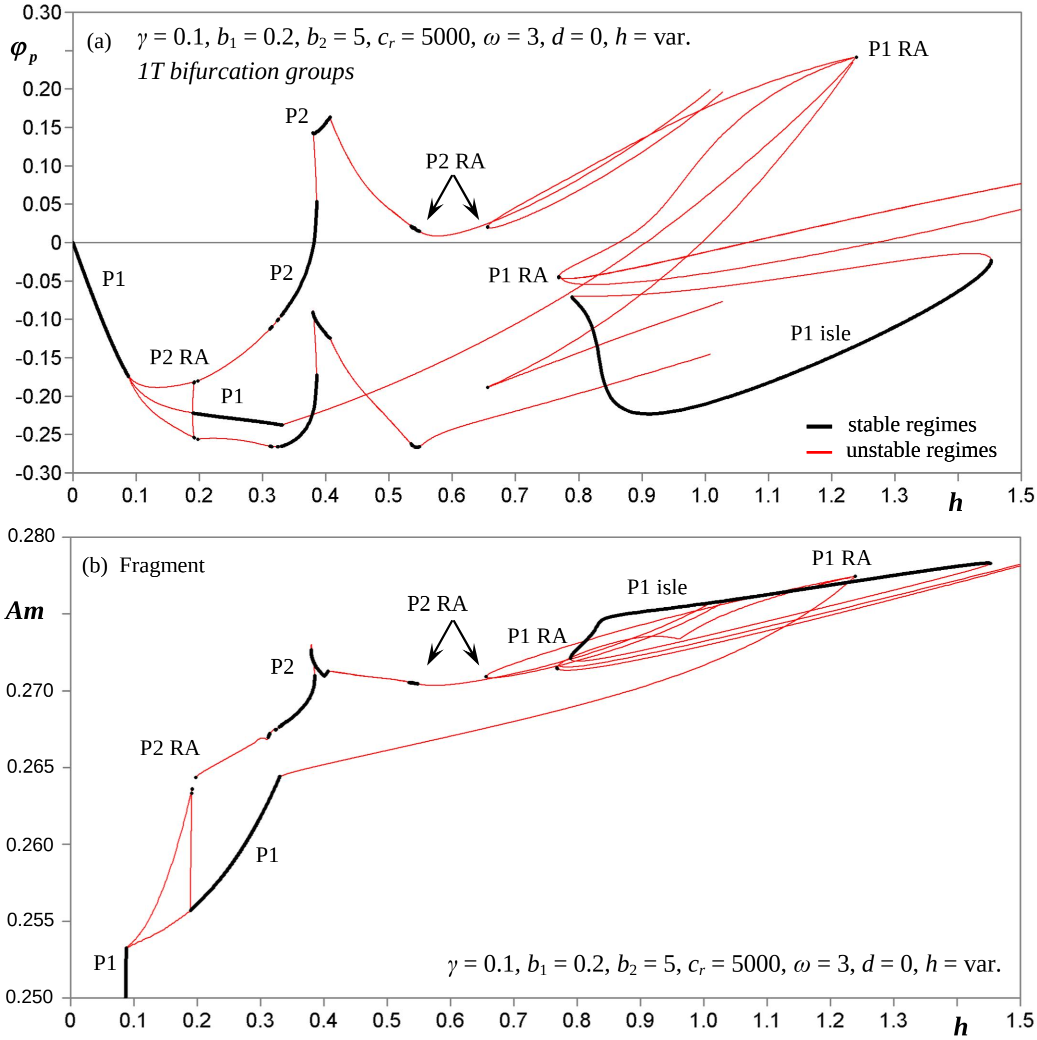

Figure 2: Pendulum vibration absorber with soft impact (see Eq.1 and Figure 1,a). Bifurcation diagrams of the fixed periodic points of the coordinates φ_p, $\dot{\varphi}_p$ and *Am* versus parameter *h*. There are two 1T bifurcations groups: 1T (with depth P1-P2) with complex protuberances and 1T isle. The pendulum system has many rare attractors of different kinds. Parameters: $\gamma = 0.1$, $b_1 = 0.2$, $b_2 = 5$, $c_r = 5000$, $\omega = 3$, $d = 0$, $h =$ var.

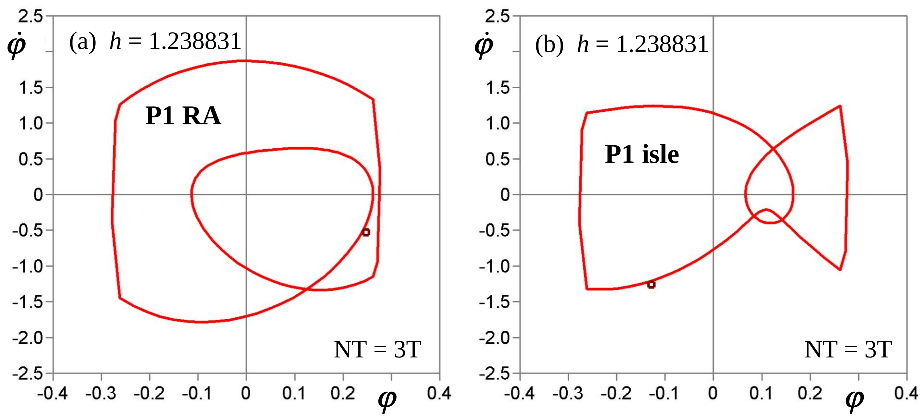

Figure 3: Coexistence of rare attractor P1 RA and P1 isle for $h = 1.238831$. Phase projections with NT = 3T.

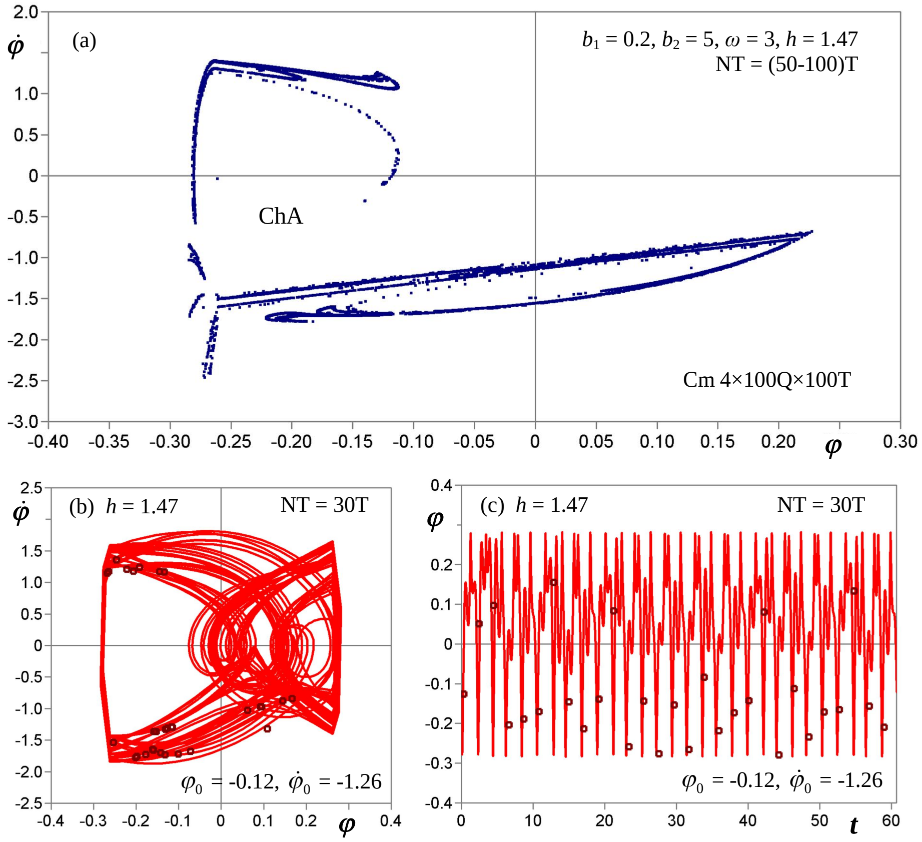

Figure 4: Chaotic attractor in pendulum absorber for h = 1.47: (a) Poincaré map from a contour Cm 4×100Q×(50-100)T; (b)-(c) phase projection and time histories with from initial conditions φ_0 = -0.12, $\dot{\varphi}_0$ = -1.26;.

The example of coexistence of rare attractor (P1 RA) with period-1 and P1 isle attractor for cross-section h = 1.238831 of bifurcation diagram (Figure 2) on phases projections is shown in Figure 3. Sometimes oscillation amplitudes of rare attractors are tenfold bigger than oscillating amplitudes of stable P1 regimes [2,10,11]. Rare attractor has higher velocity of oscillations than P1 isle. In our case oscillation amplitudes are limited by two elastic detents, which realise soft impacts.

The example of globally stable chaotic attractor for cross-section h = 1.47 of bifurcation diagram (Figure 2) obtained using the contour mapping Cm 4×100Q×(50-100)T form a contour (-0.4,-3/ 0.4,2), phase projection and time histories with NT = 30T from initial conditions φ_0 = -0.12, $\dot{\varphi}_0$ = -1.26, is shown in Figure 4.

5 CONCLUSIONS

The pendulum systems are widely used in the engineering, but their qualitative behavior hasn't been investigated enough. Therefore in this work the new nonlinear effects in pendulum vibration absorber, which can be used in dynamics of the machines and mechanisms, were shown. MCBG gives robust stability to the studied vibration absorber system by changing the system parameters, which allows saving characteristics of the vibration absorber system and excluding other amazing regimes, which can lead to small breakages of machine and mechanisms and may be to global technical catastrophes, because they are unexpected and usually have large amplitudes.

This work is preliminary, because it will be continued for the nonlinear dynamical systems with two or several degrees of freedom.

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