

Big-O Notation and Recursion

Definition

- $f(n) = O(g(n))$ if for sufficiently large values of n , f is at most a positive constant multiple of g
- In other words, there exists a positive constant c and a natural number n_0 such that for every $n \geq n_0$ we have: $f(n) \leq c g(n)$.

Summations - Gauss's Formula

$$\sum_{i=1}^n i = \frac{n(n + 1)}{2}$$

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$$S = 1 + 2 + 3 + \dots + n$$

$$S = n + n-1 + n-2 + \dots + 1$$

$$2S = (n+1) + (n+1) + \dots + (n+1) = n(n+1)$$

Summations - Reindexing

- Geometric Series Sum: $\sum_{i=0}^n 2^i = 2^{n+1} - 1$
- How to solve? $4 + 8 + 16 + 32 + \dots + 2^n$

Summations - Reindexing

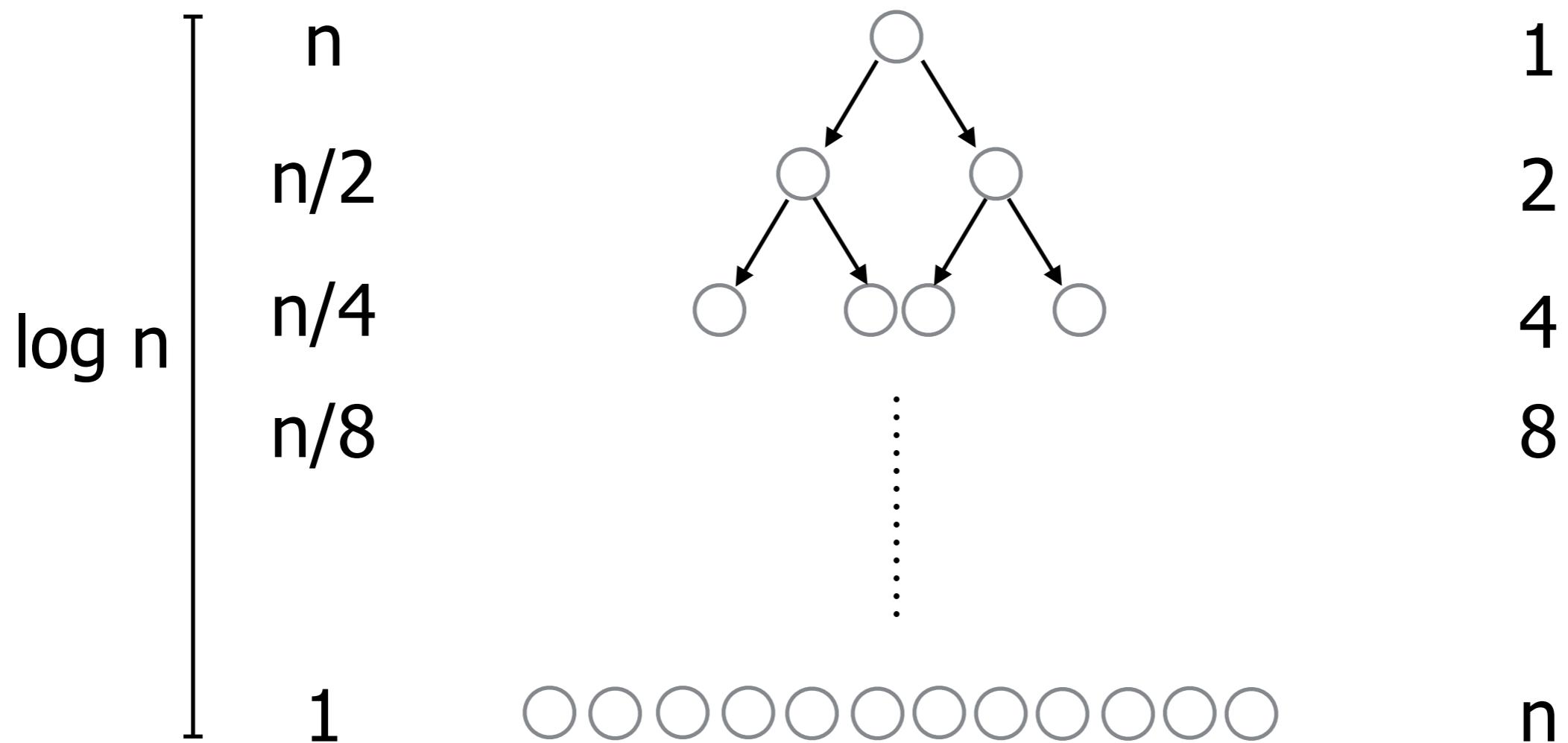
- Geometric Series Sum: $\sum_{i=0}^n 2^i = 2^{n+1} - 1$
- How to solve? $4 + 8 + 16 + 32 + \dots + 2^n$
- Same as: $4 \left(\sum_{i=0}^{n-2} 2^i \right)$
- Which equals: $4 \cdot 2^{n-1} - 4$

Summations - Reindexing

$$\sum_{i=0}^{n-1} i + 1 = \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

Here, changing the summation indices helps reduce the question to a known case.

$$T(n) = 2T(n/2)+1; T(1) = 1$$



$$\begin{aligned} \text{total} &= 1+2+4+\dots+n \\ &= O(n) \end{aligned}$$

$$T(n) = 8T(n/2) + n^2; T(1) = 1$$

