BINOMIAL PAST PAPER QUESTIONS

1. The expansion of $(2 - px)^6$ in ascending powers of x, as far as the term in x^2, is

$$64 + Ax + 135x^2.$$

Given that $p > 0$, find the value of p and the value of A.

(Total 7 marks)

2. The first four terms, in ascending powers of x, of the binomial expansion of $(1 + kx)^n$ are

$$1 + Ax + Bx^2 + Bx^3 + \dots,$$

where k is a positive constant and A, B and n are positive integers.

(a) By considering the coefficients of x^2 and x^3, show that $3 = (n - 2)\,k$.

(4)

Given that $A = 4$,

(b) find the value of n and the value of k.

(4)
(Total 8 marks)

3. Find the first three terms, in ascending powers of x, of the binomial expansion of $(3 + 2x)^5$, giving each term in its simplest form.

(Total 4 marks)

4. (a) Write down the first 4 terms of the binomial expansion, in ascending powers of x, of $(1 + ax)^n$, $n > 2$.

(2)

Given that, in this expansion, the coefficient of x is 8 and the coefficient of x^2 is 30,

(b) calculate the value of n and the value of a,

(4)

(c) find the coefficient of x^3.

(2)
(Total 8 marks)

5. (a) Expand $(2 + \frac{1}{4}x)^9$ in ascending powers of x as far as the term in x^3, simplifying each term.

(4)

(b) Use your series, together with a suitable value of x, to calculate an estimate of $(2.025)^9$.

(2)
(Total 6 marks)

6. The first three terms in the expansion, in ascending powers of x, of $(1 + px)^n$, are $1 – 18x + 36p^2x^2$. Given that n is a positive integer, find the value of n and the value of p.

(Total 7 marks)

7. For the binomial expansion, in descending powers of x, of

$$\left(x^3 - \frac{1}{2x}\right)^{12},$$

(a) find the first 4 terms, simplifying each term.

(5)

(b) Find, in its simplest form, the term independent of x in this expansion.

(3)

(Total 8 marks)

8. (a) Write down the first three terms, in ascending powers of x, of the binomial expansion of $(1 + px)^{12}$, where p is a non-zero constant.

(2)

Given that, in the expansion of $(1 + px)^{12}$, the coefficient of x is $(-q)$ and the coefficient of x^2 is $11q$,

(b) find the value of p and the value of q.

(4)
(Total 6 marks)

1. $(2 - px)^6 = 2^6 +$

Coeff. of x or x^2

$= 64 + 6 \times 2^5(-px); \quad + 15 \times 2^4(-px)^2$ A1; A1

No $\binom{n}{r}$

$15 \times 16p^2 = 135 \quad \Rightarrow \quad p^2 = \frac{9}{16}$ or $\underline{p = \frac{3}{4}}$ (only) M1, A1

$-6.32p = A \quad \Rightarrow \quad \underline{A = -144}$ M1 A1 ft (their

Condone lost or extra '–' signs for M marks but A marks must be correct.
Final A1 ft is for –192x (their p > 0)

[7]

2. (a) $\frac{n(n-1)}{2!}k^2 = \frac{n(n-1)(n-2)}{3!}k^3$

One coefficient (no $\binom{n}{r}$*)* M1

A correct equation, no cancelling A1

e.g. $3k^2 = (n - 2)k^3$

Cancel at least n (n – 1) M1

$3 = (n - 2)k$ **(*)** A1 cso4

(b) $A = nk = 4$ B1

$3 = 4 - 2k$ M1

So $k = \frac{1}{2}$, and $n = 8$ A1, A14

[8]

3. $(3 + 2x)^5 = (3^5) + \binom{5}{1}3^4 \cdot (2x) + \binom{5}{2}3^3 (2x)^2 + \ldots$ M1

$= \underline{243, +810x, +1080x^2}$ B1, A1, A1

[4]

4. (a) $1 + nax, + \frac{n(n-1)}{2}(ax)^2 + \frac{n(n-1)(n-2)}{6}(ax)^3 + \ldots$ B1,B1 2 *accept 2!, 3!*

(b) $na = 8$, $\frac{n(n-1)}{2}a^2 = 30$ M1

both

$\frac{n(n-1)}{2}.\frac{64}{n^2} = 30$, $\frac{\frac{8}{a}\left(\frac{8}{a}-1\right)a^2}{2} = 30$ M1

either

$n = 16$, $a = \frac{1}{2}$ A1, A1 4

(c) $\frac{16.15.14}{6}.\left(\frac{1}{2}\right)^3 = 70$ M1 A1 2

[8]

5. (a) $\left(2+\frac{1}{4}x\right)^9 = 2^9 + 9 \times 2^8\left(\frac{1}{4}x\right) + \frac{9\times8}{2}(2^7)\frac{x^2}{16} + \frac{9\times8\times7}{6} \times 2^6 \times \frac{x^3}{64}$ M1 B1

(M1 for descending powers of 2 and ascending powers of *x*; B1 for coefficients 1, 9, 36, 84 in any form, as above)

$= 512 + 576x, + 288x^2 + 84x^3$ A1, A14

(b) $x = \frac{1}{10}$ gives

$(2.025)^9 = 512 + 57.6 + 2.88 + 0.084$ M1

$= 572.564$ A12

[6]

6. $(1 + px)^n \equiv 1 + npx, + \frac{n(n-1)p^2x^2}{2} + \ldots$ B1, B1

Comparing coefficients: $np = -18$, $\frac{n(n-1)}{2} = 36$ M1, A1

Solving $n(n-1) = 72$ to give $n = 9$; $p = -2$ M1 A1; A1 ft

[7]

7. (a) $(x^3)^{12}$; … + $\binom{12}{1}(x^3)^{11}\left((-)\frac{1}{2x}\right) + \binom{12}{2}(x^3)^{10}\left((-)\frac{1}{2x}\right)^2 + ...$ B1; M1

[For M1, needs binomial coefficients, ^nC_r form OK at least as far as shown]

Correct values for ^nC_rs : 12, 66, 220 used (may be implied) B1

$\{(x^3)^{12} + 12(x^3)^{11}\left(-\frac{1}{2x}\right) + 66(x^3)^{10}\left(-\frac{1}{2x}\right)^2 + 220(x^3)^9\left(-\frac{1}{2x}\right)^3 ...$

$= x^{36} - 6x^{32} + \frac{33}{2}x^{28} - \frac{55}{2}x^{24}$ A2(1,0) 5

(b) Term involving $(x^3)^3\left((-)\frac{1}{2x}\right)^9$; M1

coeff = $\frac{12.11.10}{3.2.1}\left((-)\frac{1}{2}\right)^9$ A1

$= -\frac{55}{128}$ (or –0.4296875) A1 3

[8]

8. (a) 1 + 12px, +66p^2x^2 B1, B1 2

accept any correct equivalent

(b) $12p = –q$, $66p^2 = 11q$ M1

Forming 2 equations by comparing coefficients

Solving for p or q M1

$p = –2$, $q = 24$ A1A1 4

[6]