BKK the EZ way

Ric Colacito, Max Croce, Steven Ho, Philip Howard

THE UNIVERSITY
of NORTH CAROLINA
at CHAPEL HILL

BKK the EZ way

Backus, Kehoe, and Kydland the Epstein and Zin way

Ric Colacito, Max Croce, Steven Ho, Philip Howard

THE UNIVERSITY
of NORTH CAROLINA
at CHAPEL HILL

Motivation

- Our goal: characterizing role of long-term risks in international macroeconomics
    - Current events: public concern about long-term world-wide growth prospects

Motivation

- Our goal: characterizing role of long-term risks in international macroeconomics
  - Current events: public concern about long-term world-wide growth prospects
- Should capital always flow toward the most productive countries?

Motivation

- Our goal: characterizing role of long-term risks in international macroeconomics
  - Current events: public concern about long-term world-wide growth prospects
- Should capital always flow toward the most productive countries? No

Motivation

- Our goal: characterizing role of long-term risks in international macroeconomics
  - Current events: public concern about long-term world-wide growth prospects
- Should capital always flow toward the most productive countries? No
- Novel BKK-based model with (1) recursive risk-sharing and (2) low investment home-bias
  1. positive short-run productivity shocks $\rightarrow$ inflow of investment goods ($NX_I \downarrow$ and $NX \downarrow$) standard

Motivation

- Our goal: characterizing role of long-term risks in international macroeconomics
  - Current events: public concern about long-term world-wide growth prospects
- Should capital always flow toward the most productive countries? No
- Novel BKK-based model with (1) recursive risk-sharing and (2) low investment home-bias
  1. positive short-run productivity shocks $\rightarrow$ inflow of investment goods ($NX_I \downarrow$ and $NX \downarrow$) standard
  2. positive long-run productivity shocks $\rightarrow$ outflow of investment goods ($NX_I \uparrow$ and $NX \uparrow$) NEW

Motivation

- Our goal: characterizing role of long-term risks in international macroeconomics
    - Current events: public concern about long-term world-wide growth prospects
- Should capital always flow toward the most productive countries? No
- Novel BKK-based model with (1) recursive risk-sharing and (2) low investment home-bias
    1. positive short-run productivity shocks $\rightarrow$ inflow of investment goods ($NX_I \downarrow$ and $NX \downarrow$) standard
    2. positive long-run productivity shocks $\rightarrow$ outflow of investment goods ($NX_I \uparrow$ and $NX \uparrow$) NEW
- Novel empirical evidence G7 countries support our model

Insight

- International allocation of resources depends on two channels:
  1. Productivity: resources are invested in the high-productivity country
  2. Risk-sharing: resources go to the low-productivity country

Insight

- International allocation of resources depends on two channels:
  1. Productivity: resources are invested in the high-productivity country
  2. Risk-sharing: resources go to the low-productivity country
- Add Epstein-Zin (EZ) to Backus-Kehoe-Kydland (BKK):
  1. Short-run shocks $\rightarrow$ productivity channel dominates
  2. Long-run shocks $\rightarrow$ risk-sharing channel dominates

Insight

- International allocation of resources depends on two channels:
  1. Productivity: resources are invested in the high-productivity country
  2. Risk-sharing: resources go to the low-productivity country
- Add Epstein-Zin (EZ) to Backus-Kehoe-Kydland (BKK):
  1. Short-run shocks $\rightarrow$ productivity channel dominates
  2. Long-run shocks $\rightarrow$ risk-sharing channel dominates
- Convincing quantitative results:
  1. Low investment home-bias $\rightarrow$ international quantities ✓
  2. Vintage capital $\rightarrow$ international asset prices✓

Resolved Puzzles

1. Quantity anomaly
    - Cross country correlations of GDP are higher than cross country correlations of consumption

Resolved Puzzles

1. Quantity anomaly
   - Cross country correlations of GDP are higher than cross country correlations of consumption
2. Equity premium puzzle
   - Excess return on equities over risk-free bonds is high from viewpoint of a large class of traditional models

Resolved Puzzles

1. Quantity anomaly
   - Cross country correlations of GDP are higher than cross country correlations of consumption
2. Equity premium puzzle
   - Excess return on equities over risk-free bonds is high from viewpoint of a large class of traditional models
3. Brandt Cochrane Santa-Clara puzzle
   - Disconnect between the volatility of exchange rate and co-movement of SDFs

Resolved Puzzles

1. Quantity anomaly
   - Cross country correlations of GDP are higher than cross country correlations of consumption
2. Equity premium puzzle
   - Excess return on equities over risk-free bonds is high from viewpoint of a large class of traditional models
3. Brandt Cochrane Santa-Clara puzzle
   - Disconnect between the volatility of exchange rate and co-movement of SDFs
4. Backus-Smith puzzle
   - Low correlation between consumption and exchange rate

Resolved Puzzles

1. Quantity anomaly
   - Cross country correlations of GDP are higher than cross country correlations of consumption
2. Equity premium puzzle
   - Excess return on equities over risk-free bonds is high from viewpoint of a large class of traditional models
3. Brandt Cochrane Santa-Clara puzzle
   - Disconnect between the volatility of exchange rate and co-movement of SDFs
4. Backus-Smith puzzle
   - Low correlation between consumption and exchange rate
5. Forward premium anomaly
   - High interest rate currencies continue to appreciate despite uncovered interest rate parity predictions
   - Our model will produce endogenous time-varying currency risk premia, however the overall amount is not sufficient to fully reconcile the anomaly

Agenda

- Economic Model
- Theoretical Predictions
- Empirical Results
- Summary

Preferences

$$\underbrace{C_t}_{\text{Consumption}} = \left[\underbrace{\lambda X_t^{1-\frac{1}{\beth}}}_{\substack{\text{Domestic}\\ \text{Good}}} + \underbrace{(1-\lambda) Y_t^{1-\frac{1}{\beth}}}_{\substack{\text{Foreign}\\ \text{Good}}} \right]^{\frac{1}{1-\frac{1}{\beth}}}$$

Preferences

$$C_t = \left[\lambda X_t^{1-\frac{1}{\beth}} + (1-\lambda) Y_t^{1-\frac{1}{\beth}} \right]^{\frac{1}{1-\frac{1}{\beth}}}$$

$$\tilde{C}_t = C_t - \varphi N_t^{1+\frac{1}{f}} A_{t-1} \qquad \text{(Raffo)}$$

Preferences

$$C_t = \left[\lambda X_t^{1-\frac{1}{\beth}} + (1-\lambda) Y_t^{1-\frac{1}{\beth}} \right]^{\frac{1}{1-\frac{1}{\beth}}}$$

$$\tilde{C}_t = C_t - \varphi N_t^{1+\frac{1}{f}} A_{t-1} \qquad \text{(Raffo)}$$

$$U_t = \frac{1-\beta}{1-\frac{1}{\psi}} \tilde{C}_t^{1-\frac{1}{\psi}} + \beta E_t \left[U_t^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \qquad \text{(EZ)}$$

EZ Risk-Sharing Motive

$$U_t \approx \underbrace{(1-\delta)\frac{\tilde{C}_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} + \delta E_t[U_{t+1}]}_{\text{CRRA Preferences}} - \underbrace{(\gamma - \frac{1}{\psi}) Var_t[U_{t+1}]\kappa_t}_{\text{Utility Variance}}$$

EZ Risk-Sharing Motive

$$U_t \approx \underbrace{(1-\delta)\frac{\tilde{C}_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} + \delta E_t[U_{t+1}]}_{\text{CRRA Preferences}} - \underbrace{(\gamma - \frac{1}{\psi}) Var_t[U_{t+1}]\kappa_t}_{\text{Utility Variance}}$$

$$M_{t+1} = \beta \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{E_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}$$

EZ Risk-Sharing Motive

$$U_t \approx \underbrace{(1-\delta)\frac{\tilde{C}_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} + \delta E_t[U_{t+1}]}_{\text{CRRA Preferences}} - \underbrace{(\gamma - \frac{1}{\psi}) Var_t[U_{t+1}]\kappa_t}_{\text{Utility Variance}}$$

$$M_{t+1} = \beta \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{E_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}$$

S_t is the relative distribution of wealth

- $\frac{S_t}{S_{t-1}} = \frac{\Delta C_t}{\Delta C_t^*} \frac{M_t}{M_t^*}$
- Good long-run news $\Rightarrow$ marginal utility $\downarrow \Rightarrow S_t \downarrow$

Productivity growth

Symmetric specification across countries:

$$\begin{aligned}
\underbrace{\Delta a_t}_{\text{Productivity Growth}} &= \underbrace{\mu}_{\text{Deterministic Trend}} + \underbrace{\tau(a_{t-1} - a^*_{t-1})}_{\text{Co-integration Term}} + \underbrace{\sigma\varepsilon_{a,t}}_{\text{Short-run Risk}} + \underbrace{z_{x,t-1}}_{\text{Long-run Risk}} \\
z_{x,t} &= \rho z_{x,t-1} + \sigma_x \varepsilon_{x,t} \\
\varepsilon_{a,t}, \varepsilon_{x,t} &\overset{iid}{\sim} N(0,1)
\end{aligned}$$

Calibration: Croce (2008) and Colacito Croce (AER 2011, JPE 2011, JF 2012).

Production

$$\underbrace{X_t^{Tot}}_{GDP} = K_t^{\alpha}(A_t N_t)^{1-\alpha}$$

$$= \underbrace{X_t + I_{x,t}}_{\text{Domestic Use}} + \underbrace{X_t^* + I_{y,t}}_{\text{Foreign Use}}$$

Production

$$\underbrace{X_t^{Tot}}_{GDP} = K_t^{\alpha}(A_t N_t)^{1-\alpha}$$

$$= X_t + I_{x,t} + X_t^* + I_{y,t}$$

$$= \underbrace{X_t + Y_t P_t}_{\text{Consumption}} + \underbrace{I_{x,t} + I_{x,t}^* P_t}_{\text{Investment}} + \underbrace{X_t^* + I_{y,t}}_{\text{Exports}} - \underbrace{P_t(Y_t + I_{x,t}^*)}_{\text{Imports}}$$

P_t is the terms of trade

Capital & Investment

$$K_t = (1-\delta)K_{t-1} + e^{\omega_t} G_{t-1}$$

$$G_t = \left[\underbrace{\nu I_{x,t}^{1-\frac{1}{\xi}}}_{\text{Domestic Investment}} + \underbrace{(1-\nu) I_{x,t}^{*1-\frac{1}{\xi}}}_{\text{Foreign Investment}} \right]^{\frac{1}{1-\frac{1}{\xi}}} \tag{EGG}$$

Capital & Investment

$$K_t = (1-\delta)K_{t-1} + e^{\omega_t} G_{t-1}$$

$$G_t = \left[\underbrace{\nu I_{x,t}^{1-\frac{1}{\xi}}}_{\text{Domestic Investment}} + \underbrace{(1-\nu) I_{x,t}^{*1-\frac{1}{\xi}}}_{\text{Foreign Investment}} \right]^{\frac{1}{1-\frac{1}{\xi}}} \qquad \text{(EGG)}$$

$$\omega_{t+1} = -\left(\frac{1}{\alpha} - 1\right)(\Delta a_{t+1} - \mu) \qquad \text{(ACL)}$$

Capital & Investment

$$K_t = (1-\delta)K_{t-1} + e^{\omega_t} G_{t-1}$$

$$G_t = \left[\underbrace{\nu I_{x,t}^{1-\frac{1}{\xi}}}_{\text{Domestic Investment}} + \underbrace{(1-\nu) I_{x,t}^{*1-\frac{1}{\xi}}}_{\text{Foreign Investment}} \right]^{\frac{1}{1-\frac{1}{\xi}}} \quad \text{(EGG)}$$

$$\omega_{t+1} = -(\frac{1}{\alpha} - 1)(\Delta a_{t+1} - \mu) \quad \text{(ACL)}$$

$$\frac{\partial G_t}{\partial I_{x,t}}^{-1} = E_t \left[M_{t+1}^X (MPK_{t+1} + (1-\delta) Q_{K,t+1}) e^{\omega_{t+1}} \right]$$

$$\frac{\partial G_t^*}{\partial I_{y,t}}^{-1} = E_t \left[M_{t+1}^X (MPK_{t+1}^* + (1-\delta) Q_{K,t+1}^*) e^{\omega_{t+1}^*} \quad P_{t+1} \right]$$

Capital & Investment

$$K_t = (1-\delta)K_{t-1} + e^{\omega_t} G_{t-1}$$

$$G_t = \left[\underbrace{\nu I_{x,t}^{1-\frac{1}{\xi}}}_{\text{Domestic Investment}} + \underbrace{(1-\nu) I_{x,t}^{*1-\frac{1}{\xi}}}_{\text{Foreign Investment}} \right]^{\frac{1}{1-\frac{1}{\xi}}} \quad \text{(EGG)}$$

$$\omega_{t+1} = -\left(\frac{1}{\alpha} - 1\right)(\Delta a_{t+1} - \mu) \quad \text{(ACL)}$$

$$\frac{\partial G_t}{\partial I_{x,t}}^{-1} = E_t \left[M_{t+1}^X (MPK_{t+1} + (1-\delta) Q_{K,t+1}) e^{\omega_{t+1}} \right]$$

$$\frac{\partial G_t^*}{\partial I_{y,t}}^{-1} = E_t \left[M_{t+1}^X (MPK_{t+1}^* + (1-\delta) Q_{K,t+1}^*) e^{\omega_{t+1}^*} \underbrace{P_{t+1}}_{\text{Terms of trade}} \right]$$

Results

Model	(4)	(5b)	(6)	Data
EZ ($IES = 1.1; RRA = 10$)	Y	Y	Y	
Long-run risk	Y	Y	Y	
Low Invest. home bias		Y	Y	
Vintage Capital			Y	
$E[\frac{I_y P}{I}]$	15	47✓	43✓	40
$E[\frac{Y \cdot P}{C}]$	15	3✓	3✓	5

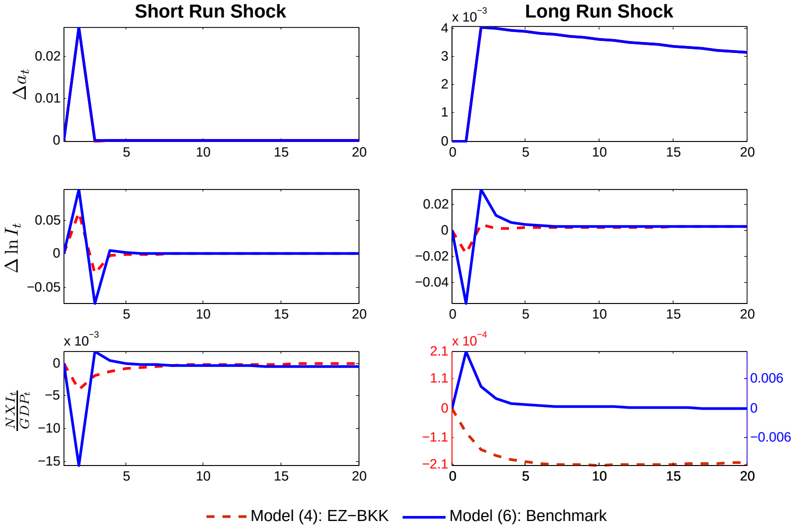
Results

Model	(4)	(5b)	(6)	Data
EZ ($IES = 1.1; RRA = 10$)	Y	Y	Y	
Long-run risk	Y	Y	Y	
Low Invest. home bias		Y	Y	
Vintage Capital			Y	
$E[\frac{I_y P}{I}]$	15	47✓	43✓	40
$E[\frac{Y \cdot P}{C}]$	15	3✓	3✓	5
$\rho(\Delta \frac{NXQ}{GDP}, \Delta GDP)$	-0.53	-0.06✓	-0.14✓	-0.27
$\sigma(\Delta e)$	0.54	9✓	10✓	11
$\rho(C) - \rho(GDP)$	0.10	-0.12✓	-0.06✓	-0.17

Results

Model	(4)	(5b)	(6)	Data
EZ ($IES = 1.1; RRA = 10$)	Y	Y	Y	
Long-run risk	Y	Y	Y	
Low Invest. home bias		Y	Y	
Vintage Capital			Y	
$E[\frac{I_y P}{I}]$	15	47✓	43✓	40
$E[\frac{Y \cdot P}{C}]$	15	3✓	3✓	5
$\rho(\Delta \frac{NXQ}{GDP}, \Delta GDP)$	-0.53	-0.06✓	-0.14✓	-0.27
$\sigma(\Delta e)$	0.54	9✓	10✓	11
$\rho(C) - \rho(GDP)$	0.10	-0.12✓	-0.06✓	-0.17
$E[r^{ex}]$	0.08	0.22	3.46✓	5.01
β_{UIP}	1.04	0.81	0.51✓	-0.72

Key Differences



Empirical Analysis

- Focus on G-7 countries

Empirical Analysis

- Focus on G-7 countries
- Estimate short-run shocks, $\varepsilon_{a,t}$:

$$\Delta \ln A_t = \underbrace{c + \beta_1 \cdot pd_{t-1} + \beta_2 \cdot rf_{t-1}}_{x_{t-1}} + \varepsilon_{a,t}$$

Empirical Analysis

- Focus on G-7 countries
- Estimate short-run shocks, $\varepsilon_{a,t}$:

$$\Delta \ln A_t = \underbrace{c + \beta_1 \cdot pd_{t-1} + \beta_2 \cdot rf_{t-1}}_{x_{t-1}} + \varepsilon_{a,t}$$

- Estimate long-run shocks, $\varepsilon_{x,t}$:

$$x_t = \rho x_{t-1} + \varepsilon_{x,t}$$

Empirical Analysis

- Focus on G-7 countries
- Estimate short-run shocks, $\varepsilon_{a,t}$:

$$\Delta \ln A_t = \underbrace{c + \beta_1 \cdot pd_{t-1} + \beta_2 \cdot rf_{t-1}}_{x_{t-1}} + \varepsilon_{a,t}$$

- Estimate long-run shocks, $\varepsilon_{x,t}$:

$$x_t = \rho x_{t-1} + \varepsilon_{x,t}$$

- Estimate response of investments

$$\Delta \ln I_t^{US} - \Delta \ln I_t^{World} = c + \beta_1 (\varepsilon_t^{US} - \varepsilon_t^{World}) + \beta_2 (\varepsilon_{x,t}^{US} - \varepsilon_{x,t}^{World})$$

Empirical Analysis

- Focus on G-7 countries
- Estimate short-run shocks, $\varepsilon_{a,t}$:

$$\Delta \ln A_t = \underbrace{c + \beta_1 \cdot pd_{t-1} + \beta_2 \cdot rf_{t-1}}_{x_{t-1}} + \varepsilon_{a,t}$$

- Estimate long-run shocks, $\varepsilon_{x,t}$:

$$x_t = \rho x_{t-1} + \varepsilon_{x,t}$$

- Estimate response of investments

$$\Delta \ln I_t^{US} - \Delta \ln I_t^{World} = c + \beta_1(\varepsilon_t^{US} - \varepsilon_t^{World}) + \beta_2(\varepsilon_{x,t}^{US} - \varepsilon_{x,t}^{World})$$

- Estimate response of NX of investments

$$\frac{NX_{I,t}}{GDP_t} = c + \beta_1(\varepsilon_t^{US} - \varepsilon_t^{World}) + \beta_2(\varepsilon_{x,t}^{US} - \varepsilon_{x,t}^{World})$$

Empirical Evidence

Panel A: Response of Investments

		Benchmark	pd	pd,rf	pr,rf,dc	pr,rf,di	pd,rf,dc,di
GDP	ε_a	2.53	2.17*** [0.58]	2.31*** [0.56]	2.39*** [0.50]	2.67*** [0.53]	2.84*** [0.53]
	ε_x	−0.85	−0.74 [4.01]	−1.42 [3.83]	−1.69 [1.53]	−3.69*** [1.07]	−4.10*** [1.02]
Investments	ε_a	2.53	1.91*** [0.55]	2.09*** [0.53]	2.25*** [0.49]	2.60*** [0.50]	2.70*** [0.50]
	ε_x	−0.85	0.37 [3.60]	−0.98 [3.29]	−1.88 [1.59]	−3.49*** [1.04]	−3.77*** [1]
Solow	ε_a	2.53	2.12*** [0.71]	2.16*** [0.70]	2.04*** [0.58]	2.23*** [0.76]	2.55*** [0.71]
	ε_x	−0.85	−2.69 [5.97]	−2.96 [6.10]	−2.49* [1.69]	−5.15*** [1.17]	−5.54*** [1.06]
System	ε_a	2.53	1.27*** [0.04]	1.27*** [0.04]	1.39*** [0.04]	1.44*** [0.04]	1.53*** [0.04]
	ε_x	−0.85	0.30 [0.25]	−1.07*** [0.27]	0.04 [0.23]	−3.82*** [0.11]	−3.31*** [0.11]

Panel B: Response of Net Exports of Investments

		Benchmark	pd	pd,rf	pr,rf,dc	pr,rf,di	pd,rf,dc,di
GDP	ε_a	−0.35	−0.19 [0.23]	−0.18 [0.22]	−0.20 [0.23]	−0.13 [0.33]	−0.08 [0.35]
	ε_x	0.35	0.66 [0.90]	0.62 [0.71]	1.14*** [0.42]	0.64*** [0.17]	0.47** [0.20]
Investments	ε_a	−0.35	−0.16 [0.22]	−0.15 [0.22]	−0.18 [0.22]	−0.12 [0.32]	−0.06 [0.34]
	ε_x	0.35	0.54 [0.94]	0.57 [0.66]	1.07*** [0.35]	0.60*** [0.16]	0.44*** [0.18]
Solow	ε_a	−0.35	−0.26 [0.25]	−0.23 [0.22]	−0.26 [0.22]	−0.17 [0.31]	−0.10 [0.34]
	ε_x	0.35	1.12 [1.02]	0.74 [0.81]	1.26*** [0.54]	0.73*** [0.19]	0.51** [0.22]
System	ε_a	−0.35	−0.24*** [0.01]	−0.17*** [0.02]	−0.19*** [0.02]	−0.12*** [0.02]	−0.07*** [0.01]
	ε_x	0.35	1.26*** [0.11]	0.93*** [0.05]	0.90*** [0.04]	0.43*** [0.04]	0.22*** [0.03]

Summary

- Empirical Contribution: investment outflows upon good long-run news

Summary

- Empirical Contribution: investment outflows upon good long-run news
- Model Contribution: international production economy with recursive risk-sharing and long-run risk
  - good news for the long (short) run $\rightarrow$ capital outflows (inflows)
  - quantity anomaly turned into a regularity

Summary

- Empirical Contribution: investment outflows upon good long-run news
- Model Contribution: international production economy with recursive risk-sharing and long-run risk
    - good news for the long (short) run $\rightarrow$ capital outflows (inflows)
    - quantity anomaly turned into a regularity
- Next step: incorporate frictions à la Stockman and Tesar 1995, Baxter and Crucini 1995, Kehoe and Perri 2002, Heathcote and Perri 2004, Bai and Zhang 2010, Petrosky-Nadeau 2011, Alessandria et al. 2011

Summary

- Empirical Contribution: investment outflows upon good long-run news
- Model Contribution: international production economy with recursive risk-sharing and long-run risk
  - good news for the long (short) run $\rightarrow$ capital outflows (inflows)
  - quantity anomaly turned into a regularity
- Next step: incorporate frictions à la Stockman and Tesar 1995, Baxter and Crucini 1995, Kehoe and Perri 2002, Heathcote and Perri 2004, Bai and Zhang 2010, Petrosky-Nadeau 2011, Alessandria et al. 2011
- Broader point: conveying the need of introducing long-run risk considerations in international investment theory

Relation to literature

Prior work by the authors:

- Colacito Croce (AER 2010, JPE 2011, JF 2012): International asset pricing with EZ preferences in endowment economy
  - $\hookrightarrow$ International production and investment flows
- Ai Croce Li (RFS 2012): Closed production economy with vintage capital to explain EP and VP & investment vintages
  - $\hookrightarrow$ International perspective

Previous papers:

- Backus Kehoe Kydland (JPE 1992, AER 1994): IRBC
- Raffo (JIE 2008): NX empirically driven by quantities (GHH preferences)
- Erceg Guerrieri Gust (JEDC 2008): Home bias is strong in consumption, mild in investment
  - $\hookrightarrow$ LRR-based AP perspective
- Tretvoll (2012): Robust-BKK with short-run risk only

Wages

$$W_t = MRS_t^{C,L}$$

$$W_t = \frac{\partial X_t^{Tot}}{\partial N_t}$$

$$MRS_t^{C,L} = \frac{\partial \tilde{C}_t / \partial L_t}{\partial \tilde{C}_t / \partial C_t}$$

$$(1-\alpha)\frac{X_t^{Tot}}{N_t} = \varphi\left(1+\frac{1}{f}\right) N_t^{\frac{1}{f}} A_{t-1} e^{\phi \Delta a_t}$$

Time varying Pareto weights

$$
\begin{aligned}
S_t =& S_{t-1} \frac{M_t}{M_t^*} \frac{C_t / C_{t-1}}{C_t^* / C_{t-1}^*} \\
=& \frac{1-\lambda}{\lambda} \left(\frac{C_t}{C_t^*} \right)^{1-\frac{1}{\beth}} \left(\frac{X_t}{X_t^*} \right)^{\frac{1}{\beth}} \\
=& \frac{\lambda}{1-\lambda} \left(\frac{C_t}{C_t^*} \right)^{1-\frac{1}{\beth}} \left(\frac{Y_t}{Y_t^*} \right)^{\frac{1}{\beth}}
\end{aligned}
$$

Terms of trade

The terms of trade is the price of imports over the price of exports:

$$P_t = \frac{1-\lambda}{\lambda}\left(\frac{X_t}{Y_t}\right)^{\frac{1}{\beth}}$$

Domestic SDF

$$M_{t+1} = \frac{\partial U_{t+1}/\partial C_{t+1}}{\partial U_t/\partial C_t}$$

$$= \beta \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma}$$

$$M_{t+1}^X = \frac{\partial U_{t+1}/\partial X_{t+1}}{\partial U_t/\partial X_t}$$

$$= \left(\frac{C_{t+1}}{C_t} \frac{X_t}{X_{t+1}} \right)^{\frac{1}{\beth}} M_{t+1}$$

EZ Prefs

Ordinally equivalent transformation: $V_t = \frac{U_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}}$

$$V_t = \frac{1-\beta}{1-\frac{1}{\psi}} \tilde{C}_t^{1-\frac{1}{\psi}} + \beta E_t \left[V_t^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}$$

$$U_t = \left[(1-\beta)\, \tilde{C}_t^{1-\frac{1}{\psi}} + \beta E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}}$$

Calibration

TABLE 2: Calibrated Parameter Values

Model:		(1)	(2)	(3)	(4)	(5)	(5b)	(6)
Subjective discount factor	β	0.985	0.985	0.985	0.985	0.985	0.985	0.9873
Risk aversion	γ	2	2	10	10	10	10	10
IES	ψ	0.5	0.5	0.5	1.1	1.1	1.1	1.1
Consumption home bias	λ	0.76	0.76	0.76	0.76	0.76	0.97	0.97
Consumption-bundle elasticity	$\beth$	1.5	1.5	1.5	1.5	1.5	1	1
Consumption-labor elasticity	f	1	1	1	1	1	1	1
Capital Income Share	α	0.36	0.36	0.36	0.36	0.3	0.3	0.3
Depreciation rate of capital	δ	0.1	0.1	0.1	0.1	0.06	0.06	0.06
Investment home bias	ν	0.76	0.76	0.76	0.76	0.76	0.53	0.57
Investment-bundle elasticity	ξ	1.5	1.5	1.5	1.5	1.5	1	1
Exposure of young vintages	ϕ_0	1	1	1	1	0	1	0
Long run mean of productivity	μ	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Persistence of long run shock	ρ	0.9859	0.9859	0.9859	0.9859	0.9859	0.9859	0.9859
Co-integration parameter	τ	5E-05	5E-05	5E-05	5E-05	5E-05	5E-05	5E-05
Short-run shock vol.	σ	0.027	0.027	0.027	0.027	0.027	0.027	0.027
Long-run shock vol.	σ_x	0	$.15\sigma$	$.15\sigma$	$.15\sigma$	$.15\sigma$	$.15\sigma$	$.15\sigma$
Short-run shocks correlation	ρ_{srr}	0	0.027	0.027	0.027	0.027	0.027	0.027
Long-run shocks correlation	ρ_{lrr}	–	0.85	0.85	0.85	0.85	0.85	0.85