# Blocking in design of experiments

- **Blocking** is a technique for dealing with **nuisance factors**
- A **nuisance** factor is a factor that probably has some effect on the response, but it's of no interest to the experimenter…however, the variability it transmits to the response needs to be minimized
- Typical nuisance factors include batches of raw material, operators, pieces of test equipment, time (shifts, days, etc.), different experimental units
- Many industrial experiments involve blocking (or should)
- Failure to block is a common flaw in designing an experiment (consequences?)



# Dealing with nuisance variables

- If the nuisance variable is **known** and **controllable**, we use **blocking**
- If the nuisance factor is **known** and **uncontrollable**, sometimes we can use the **analysis of covariance** (see Chapter 15) to remove the effect of the nuisance factor from the analysis
- If the nuisance factor is **unknown** and **uncontrollable** (a **"lurking" variable)**, we hope that **randomization** balances out its impact across the experiment
- Sometimes several sources of variability are **combined** in a block, so the block becomes an aggregate variable

## Example: Hardness Testing

- We wish to determine whether 4 different tips produce different (mean) hardness reading on a Rockwell hardness tester
- Assignment of the tips to a test coupon (aka, the **experimental unit**)
- A completely randomized experiment
- The test coupons are a source of **nuisance variability**
- Alternatively, the experimenter may want to test the tips across coupons of various hardness levels



## Example (cont.)

- To conduct this experiment as a RCBD, assign all 4 tips to each coupon
- Each coupon is called a "**block**"
  - A more homogenous experimental unit on which to test the tips
  - Variability **between** blocks can be large, variability **within** a block should be relatively small
  - In general, a **block** is a specific level of the nuisance factor
  - A complete replicate of the basic experiment is conducted in each block
- A block represents a **restriction on randomization**
  - All runs **within** a block are **randomized**

## Example (cont.)

- Suppose that we use $b$ = 4 blocks:

■ TABLE 4.1
Randomized Complete Block Design for the Hardness Testing Experiment

| Test Coupon (Block) | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| Tip 3 | Tip 3 | Tip 2 | Tip 1 |
| Tip 1 | Tip 4 | Tip 1 | Tip 4 |
| Tip 4 | Tip 2 | Tip 3 | Tip 2 |
| Tip 2 | Tip 1 | Tip 4 | Tip 3 |

- Notice the **two-way structure** of the experiment
  - Once again, we are interested in testing the equality of treatment means, but now we have to remove the variability associated with the nuisance factor (the blocks)



## Extension of the ANOVA to RCBD

- Suppose that there are $a$ treatments (factor levels) and $b$ blocks
- A **statistical model** (effects model) for the RCBD is

$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \begin{cases} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, b \end{cases}$$

- The relevant (fixed effects) hypotheses are

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_a \text{ where } \mu_i = (1/b)\sum_{j=1}^{b}(\mu + \tau_i + \beta_j) = \mu + \tau_i$$

# Extension of the ANOVA to RCBD

ANOVA partitioning of total variability:

$$\sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ij}-\bar{y}_{..})^2 = \sum_{i=1}^{a}\sum_{j=1}^{b}[(\bar{y}_{i.}-\bar{y}_{..})+(\bar{y}_{.j}-\bar{y}_{..})$$

$$+(y_{ij}-\bar{y}_{i.}-\bar{y}_{.j}+\bar{y}_{..})]^2$$

$$= b\sum_{i=1}^{a}(\bar{y}_{i.}-\bar{y}_{..})^2 + a\sum_{j=1}^{b}(\bar{y}_{.j}-\bar{y}_{..})^2$$

$$+\sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ij}-\bar{y}_{i.}-\bar{y}_{.j}+\bar{y}_{..})^2$$

$$SS_T = SS_{Treatments} + SS_{Blocks} + SS_E$$



# Extension of the ANOVA to RCBD

The degrees of freedom for the sums of squares in

$$SS_T = SS_{Treatments} + SS_{Blocks} + SS_E$$

are as follows:

$$ab-1 = a-1+b-1+(a-1)(b-1)$$

Therefore, ratios of sums of squares to their degrees of freedom result in mean squares and the ratio of the mean square for treatments to the error mean square is an *F* statistic that can be used to test the hypothesis of equal treatment means

# ANOVA for the RCBD

■ **TABLE 4.2**
**Analysis of Variance for a Randomized Complete Block Design**

| Source of Variation | Sum of Squares | Degrees of Freedom | Mean Square | $F_0$ |
|---|---|---|---|---|
| Treatments | $SS_{\text{Treatments}}$ | $a-1$ | $\frac{SS_{\text{Treatments}}}{a-1}$ | $\frac{MS_{\text{Treatments}}}{MS_E}$ |
| Blocks | $SS_{\text{Blocks}}$ | $b-1$ | $\frac{SS_{\text{Blocks}}}{b-1}$ | |
| Error | $SS_E$ | $(a-1)(b-1)$ | $\frac{SS_E}{(a-1)(b-1)}$ | |
| Total | $SS_T$ | $N-1$ | | |



## EXAMPLE 4.1

A medical device manufacturer produces vascular grafts (artificial veins). These grafts are produced by extruding billets of polytetrafluoroethylene (PTFE) resin combined with a lubricant into tubes. Frequently, some of the tubes in a production run contain small, hard protrusions on the external surface. These defects are known as “flicks.” The defect is cause for rejection of the unit.

The product developer responsible for the vascular grafts suspects that the extrusion pressure affects the occurrence of flicks and therefore intends to conduct an experiment to investigate this hypothesis. However, the resin is manufactured by an external supplier and is delivered to the medical device manufacturer in batches. The engineer also suspects that there may be significant batch-to-batch variation, because while the material should be consistent with respect to parameters such as molecular weight, mean particle size, retention, and peak height ratio, it probably isn’t due to manufacturing variation at the resin supplier and natural variation in the material. Therefore, the product developer decides to investigate the effect of four different levels of extrusion pressure on flicks using a randomized complete block design considering batches of resin as blocks. The RCBD is shown in Table 4.3. Note that there are four levels of extrusion pressure (treatments) and six batches of resin (blocks). Remember that the order in which the extrusion pressures are tested within each block is random. The response variable is yield, or the percentage of tubes in the production run that did not contain any flicks.

# Example: Vascular Graft

- To conduct this experiment as a RCBD, all 4 pressures are assigned to each of the 6 batches of resin
- Each batch of resin is called a "**block**"; that is, it's a more homogenous experimental unit on which to test the extrusion pressures

■ TABLE 4.3
Randomized Complete Block Design for the Vascular Graft Experiment

| Extrusion Pressure (PSI) | Batch of Resin (Block) 1 | 2 | 3 | 4 | 5 | 6 | Treatment Total |
|---|---|---|---|---|---|---|---|
| 8500 | 90.3 | 89.2 | 98.2 | 93.9 | 87.4 | 97.9 | 556.9 |
| 8700 | 92.5 | 89.5 | 90.6 | 94.7 | 87.0 | 95.8 | 550.1 |
| 8900 | 85.5 | 90.8 | 89.6 | 86.2 | 88.0 | 93.4 | 533.5 |
| 9100 | 82.5 | 89.5 | 85.6 | 87.4 | 78.9 | 90.7 | 514.6 |
| Block Totals | 350.8 | 359.0 | 364.0 | 362.2 | 341.3 | 377.8 | $y_{..} = 2155.1$ |



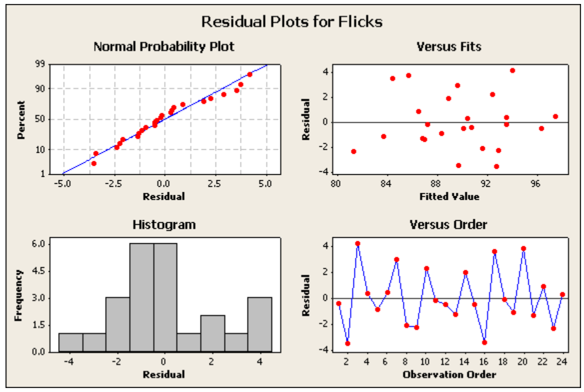
# Minitab Output

**Two-way ANOVA: Flicks versus Pressure, Batch**

| Source | DF | SS | MS | F | P |
|---|---|---|---|---|---|
| Pressure | 3 | 178.171 | 59.3904 | 8.11 | 0.002 |
| Batch | 5 | 192.252 | 38.4504 | 5.25 | 0.006 |
| Error | 15 | 109.886 | 7.3257 | | |
| Total | 23 | 480.310 | | | |

S = 2.707   R-Sq = 77.12%   R-Sq(adj) = 64.92%

# Residual analysis






# Residual Analysis for the Vascular Graft Example

- Basic residual plots indicate that **normality**, **constant variance** assumptions are satisfied
  - No obvious problems with **randomization**
  - No patterns in the residuals vs. block
- Can also plot residuals versus the pressure (residuals by factor)
  - These plots provide more information about the constant variance assumption, possible outliers

## Your turn: Which pressures are different?



## Other aspects of the RCBD

- The RCBD utilizes an *additive model*
  - Can't explore interaction between treatments and blocks
- Treatments and/or blocks as random effects
  - Analysis is the same but interpretation is different
    - If blocks are random (e.g., selection of raw material batches) then assume treatment effect is the same throughout the *population* of blocks.
    - Any interaction between treatments and blocks are will not affect the test on treatment means (interaction will affect both treatment and error mean squares)

## What about missing values?

- What happens if one of the measurements in your experiment is missing?
  - An error in measurement gives a result you know isn't right
  - Damage to a machine prevents you from completing a test
  - Etc.
- Inexact method – estimate the missing value and go on with the analysis
  - Reduce error degrees of freedom by 1 for each missing value
  - Danger – increase in "false" significance
- Exact method – general regression significance test
  - Test on *unbalanced* data (treatment and block not *orthogonal*)
  - Use Minitab or other computer application
    - in Minitab, use GLM model in ANOVA menu (2-Way ANOVA requires balanced designs)



## Sample size

- Sample sizing in the RCBD refers to the number of blocks to run
- Can use Minitab sample size calculator with:
  - number of levels = treatment level
  - sample size = number of blocks
- Example 4.2, pg. 134
  - note the difference between the results using the OC curve approach and Minitab

# The Latin square design

- Latin square designs are used to simultaneously control (or eliminate) two sources of nuisance variability
- Latin squares are not used as much as the RCBD in industrial experimentation
- A significant assumption is that the three factors (treatments, nuisance factors) do not interact
  - If this assumption is violated, the Latin square design will not produce valid results



# The rocket propellant problem – A Latin square design

**TABLE 4.8**
Latin Square Design for the Rocket Propellant Problem

| Batches of Raw Material | Operators 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | $A = 24$ | $B = 20$ | $C = 19$ | $D = 24$ | $E = 24$ |
| 2 | $B = 17$ | $C = 24$ | $D = 30$ | $E = 27$ | $A = 36$ |
| 3 | $C = 18$ | $D = 38$ | $E = 26$ | $A = 27$ | $B = 21$ |
| 4 | $D = 26$ | $E = 31$ | $A = 26$ | $B = 23$ | $C = 22$ |
| 5 | $E = 22$ | $A = 30$ | $B = 20$ | $C = 29$ | $D = 31$ |

- This is a *5x5 Latin square design*
- Latin letters (*A, B, C, D, E*) are the treatment levels
- The experiment is designed such that every treatment level is tested once at each combination of nuisance factors

## Other Latin squares designs …

- 4 different fonts (treatments) tested for reading speed on different computer screens and in different ambient light levels (4 levels each)
- 3 different material types (treatments) tested for strength at different material lengths and measuring device (3 levels each)
- Note that once the design is complete, the order of the trials in the experiment is *randomized*



## Statistical analysis of the Latin square design

- The statistical (effects) model is

$$y_{ijk} = \mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk} \begin{cases} i = 1, 2, \ldots, p \\ j = 1, 2, \ldots, p \\ k = 1, 2, \ldots, p \end{cases}$$

- The statistical analysis (ANOVA) is much like the analysis for the RCBD.
- See the ANOVA table, page 140
- Using Minitab (GLM) for the analysis …

■ **TABLE 4.9**
**Analysis of Variance for the Latin Square Design**

| Source of Variation | Sum of Squares | Degrees of Freedom | Mean Square | $F_0$ |
|---|---|---|---|---|
| Treatments | $SS_{\text{Treatments}} = \frac{1}{p}\sum_{j=1}^{p} y_{.j.}^2 - \frac{y_{...}^2}{N}$ | $p-1$ | $\frac{SS_{\text{Treatments}}}{p-1}$ | $F_0 = \frac{MS_{\text{Treatments}}}{MS_E}$ |
| Rows | $SS_{\text{Rows}} = \frac{1}{p}\sum_{i=1}^{p} y_{i..}^2 - \frac{y_{...}^2}{N}$ | $p-1$ | $\frac{SS_{\text{Rows}}}{p-1}$ | |
| Columns | $SS_{\text{Columns}} = \frac{1}{p}\sum_{k=1}^{p} y_{..k}^2 - \frac{y_{...}^2}{N}$ | $p-1$ | $\frac{SS_{\text{Columns}}}{p-1}$ | |
| Error | $SS_E$ (by subtraction) | $(p-2)(p-1)$ | $\frac{SS_E}{(p-2)(p-1)}$ | |
| Total | $SS_T = \sum_i\sum_j\sum_k y_{ijk}^2 - \frac{y_{...}^2}{N}$ | $p^2-1$ | | |



# Checking for model adequacy

- The residuals in the Latin square are given by:

- As with any design, check the adequacy by plotting the appropriate residuals

## Replication of Latin squares

- Small Latin squares provide a relatively small number of error degrees of freedom
  - makes seeing differences in treatment effects difficult
- Replication of Latin squares can be done in three ways:
  - Repeat each combination of row and column variable in each replication
  - Change 1 nuisance variable but keep the other the same for each replication
  - Use different levels of both nuisance variables
- Note, now the number of trials is $N = np^2$



## Other designs

- Crossover designs
  - Repeated Latin squares in an experiment in which order (or time period) matters
  - See figure 4.7, pg. 145
- Graeco-Latin squares
  - Extension of the Latin squares design with 3 nuisance variables
    - Let each Greek letter indicate the 3rd nuisance factor level
    - Each combination of row variable, column variable,Greek letter, and Latin letter appears once and only once.
    - See example 4.4, pg. 147

# Homework: Due Wednesday, 9/12

- 3.17
- 3.19
- 3.24

- 4.7
- 4.19
- 4.20
- 4.29