Blue Pelican

Geometry Theorem Proofs

by Charles E. Cook; Refugio, Tx

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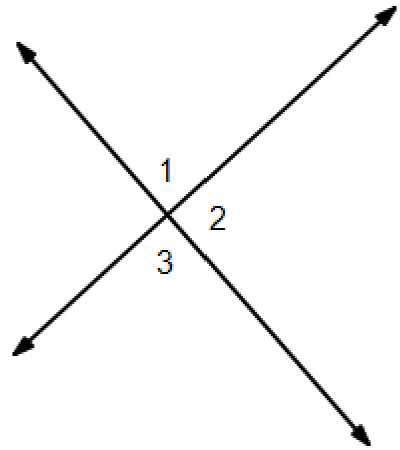
The theorems listed here are but a **few** of the total in this curriculum. The vast majority are presented in the lessons themselves.

Theorem Proof A

If two angles are vertical angles, then the angles are congruent.

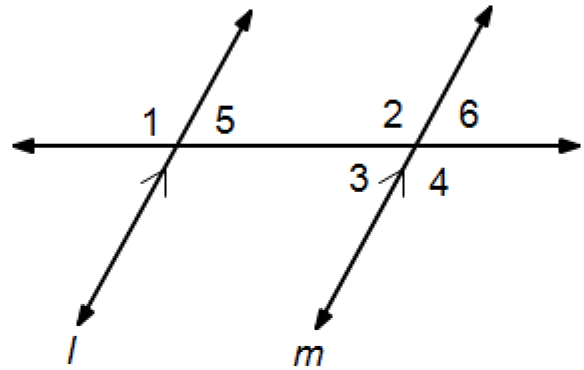


Proof:

Statement	**Reason**
1. ∠1 and ∠2 are supplementary ∠2 and ∠3 are supplementary	Linear Pair Postulate
2. $m\angle 1 + m\angle 2 = 180°$ $m\angle 2 + m\angle 3 = 180°$	Definition of supplementary
3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	Substitution
4. $m\angle 1 = m\angle 3$	Subtraction Property of Equality ($m\angle 2$ cancels from each side)
5. $\angle 1 \cong \angle 3$	Definition of congruence
∴ Vertical angles are congruent.	

Theorem Proof B

If two parallel lines are crossed by a transversal, then the alternate interior and alternate exterior angles are congruent.

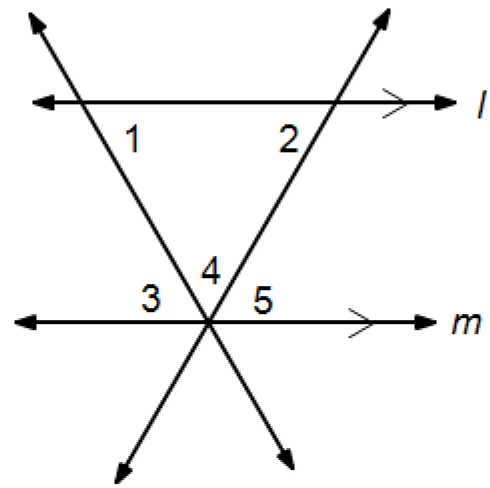


Proof:

Statement	**Reason**
1. $l \parallel m$	Given
2. $\angle 1 \cong \angle 2$ $\angle 5 \cong \angle 6$	Corresponding Angles Postulate
3. $\angle 2 \cong \angle 4$ $\angle 6 \cong \angle 3$	Vertical Angle Theorem (Theorem Proof A)
4. $\angle 1 \cong \angle 4$ $\angle 5 \cong \angle 3$	Substitution
∴ Alternate interior angles and alternate exterior angles are congruent.	

Theorem Proof C

If a polygon is a triangle, then the sum of its interior angles is 180°.

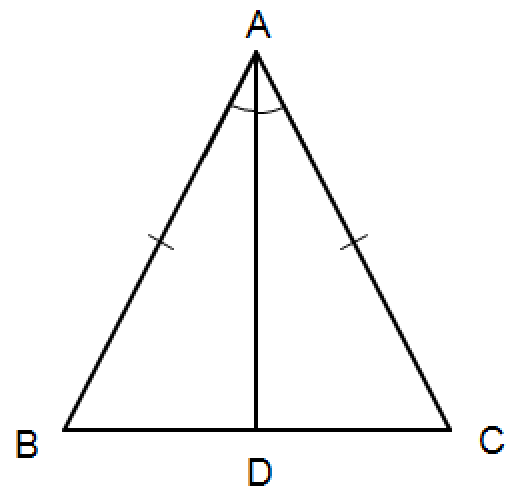


Proof:

Statement	**Reason**
1. $l \parallel m$	Given
2. ∠1 ≅ ∠3	Alternate Interior Angle Theorem (Theorem Proof B)
3. ∠2 ≅ ∠5	Alternate Interior Angle Theorem (Theorem Proof B)
4. m∠3 + m∠4 + m∠5 = 180°	Definition of straight angle
5. m∠1 + m∠4 + m∠2 = 180°	Substitution
∴ The sum of the interior angles of a triangle is 180°.	

Theorem Proof D

If a triangle is an isosceles triangle, then its base angles are congruent.

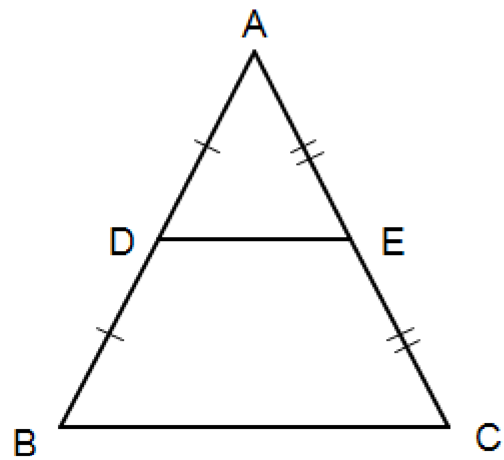


Proof:

Statement	**Reason**
1. AB ≅ AC AD is the angle bisector of ∠BAC	Given
2. ∠BAD ≅ ∠DAC	Definition of angle bisector
3. DA ≅ DA	Reflexive Property
4. ΔBAD ≅ ΔCAD	SAS Postulate
5. ∠B ≅ ∠C	CPCTC
∴ The base angles of an isosceles triangle are congruent.	

Theorem Proof E

If a segment of a triangle connects the midpoints of two sides, the segment is parallel to the third side and its length is half that of the third side.



Proof:

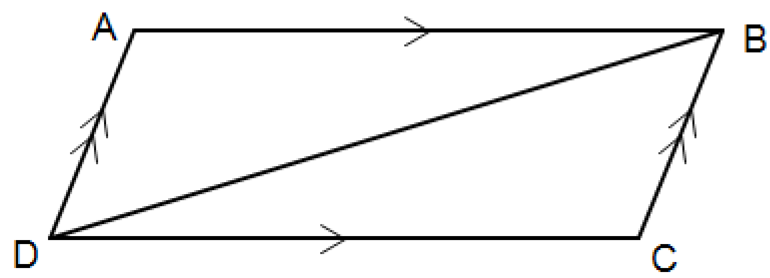
Statement	**Reason**
1. AD ≅ DB AE ≅ EC AB = 2(AD) AC = 2(AE)	Given
2. $\frac{AB}{AD} = 2$ $\quad \frac{AC}{AE} = 2$	Division Property of Equality
3. $\frac{AB}{AD} = \frac{AC}{AE}$	Substitution
4. ∠A ≅ ∠A	Reflexive Property
5. ΔADE ~ ΔABC	SAS Similarity
6. ∠ADE ≅ ∠B	Definition of Similarity
7. DE ∥ BC	Converse of Corresponding Angles Postulate
8. $\frac{BC}{DE} = 2$	Definition of Similarity

9. $\frac{BC}{2}$ = DE	Multiplication Property of Equality

∴ The midsegment of a triangle is parallel to the third side and half that of the third side.

Theorem Proof F

If a polygon is a parallelogram, then its opposite angles are congruent.

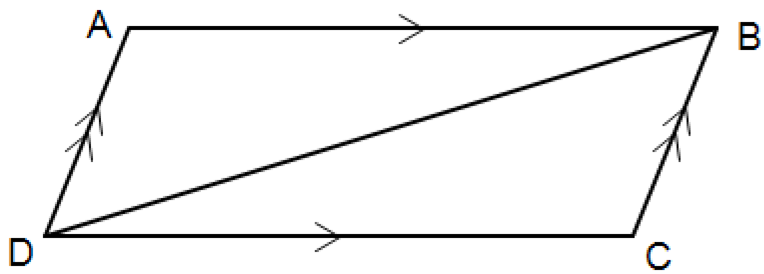


Proof:

Statement	**Reason**
1. AB ∥ DC AD ∥ BC	Given (ABCD is a parallelogram)
2. ∠ABD ≅ ∠BDC	Alternate Interior Angle Theorem (Theorem Proof B)
3. ∠ADB ≅ ∠DBC	Alternate Interior Angle Theorem (Theorem Proof B)
4. DB ≅ DB	Reflexive Property
5. ΔADB ≅ ΔCBD	ASA Postulate
6. ∠A ≅ ∠C	CPCTC
∴ The opposite angles of a parallelogram are congruent.	

Theorem Proof G

If a polygon is a parallelogram, then its opposite sides are congruent.

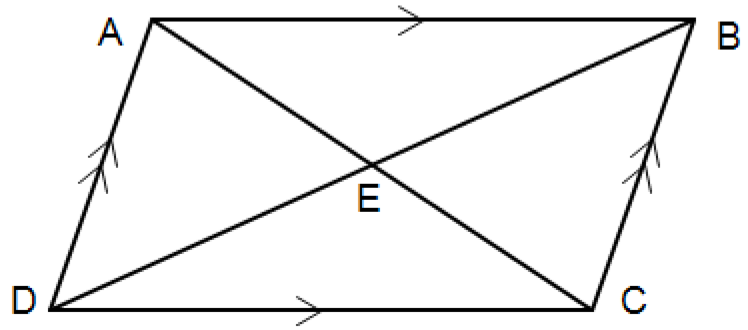


Proof:

Statement	**Reason**
1. AB ∥ DC AD ∥ BC	Given
2. ∠ABD ≅ ∠BDC	Alternate Interior Angle Theorem (Theorem Proof B)
3. ∠ADB ≅ ∠DBC	Alternate Interior Angle Theorem (Theorem Proof B)
4. DB ≅ DB	Reflexive Property
5. ΔADB ≅ ΔCBD	ASA Postulate
6. DC ≅ AB DA ≅ CB	CPCTC
∴ The opposite sides of a parallelogram are congruent.	

Theorem Proof H

If a polygon is a parallelogram, then its diagonals bisect each other.

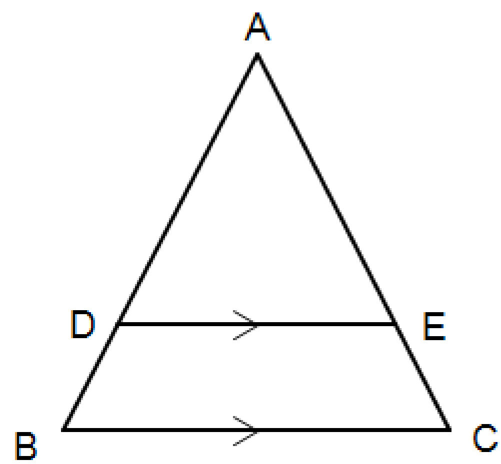


Proof:

Statement	**Reason**
1. AB ∥ DC AD ∥ BC	Given
2. ∠DAC ≅ ∠ACB	Alternate Interior Angle Theorem (Theorem Proof B)
3. ∠ADB ≅ ∠DBC	Vertical Angle Theorem (Theorem Proof A)
4. AD ≅ BC	Opposite sides of a parallelogram are congruent (Theorem Proof G)
5. ΔADE ≅ ΔCBE	ASA Postulate
6. AE ≅ EC DE ≅ EB	CPCTC
∴ The diagonals of a parallelogram bisect each other.	

Theorem Proof I

If a triangle contains a line parallel to one side, then that line divides the other two sides proportionally.

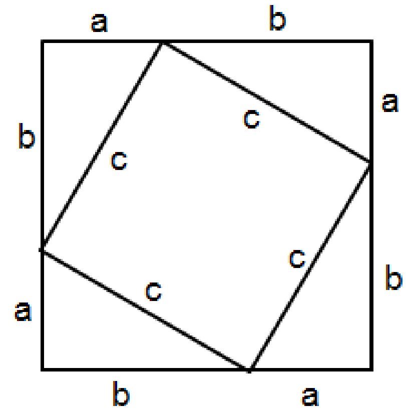


Proof:

Statement	**Reason**
1. DE ∥ BC	Given
2. ∠ADE ≅ ∠ABC	Corresponding Angles Postulate
3. ∠AED ≅ ∠ACB	Corresponding Angles Postulate
4. ΔADE ~ ΔABC	AA Similarity
5. $\frac{AD}{AB} = \frac{AE}{AC}$	Definition of Similarity
∴ A line in a triangle parallel to one side divides the other two sides proportionally.	

Theorem Proof J

If a triangle is a right triangle, then the sum of the legs squared equals the square of the hypotenuse.

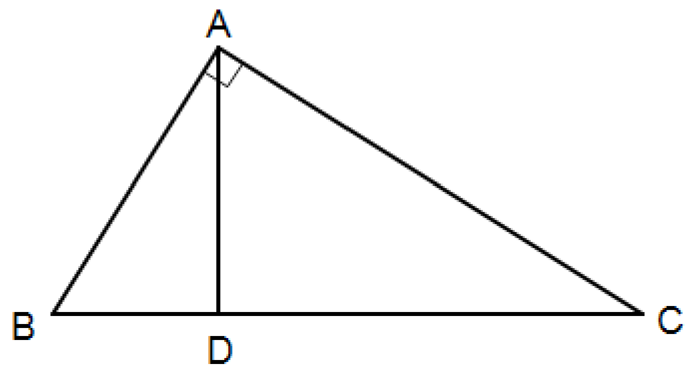


Proof:

Statement	Reason
1. $A_{square} = (a+b)^2$	Square formula: s^2
2. $A_{square} = 4\left(\frac{1}{2}ab\right) + c^2$	Alternate square formula – summation of the interior areas
3. $(a+b)^2 = 4\left(\frac{1}{2}ab\right) + c^2$	Substitution
4. $a^2 + 2ab + b^2 = 2ab + c^2$	Simplification
5. $a^2 + b^2 = c^2$	Subtraction Property of Equality
∴ The square of the hypotenuse of a right triangle is equal to the sum of the squares of its legs.	

See the next page for an alternate proof using similarity.

If a triangle is a right triangle, then the sum of the legs squared equals the square of the hypotenuse.



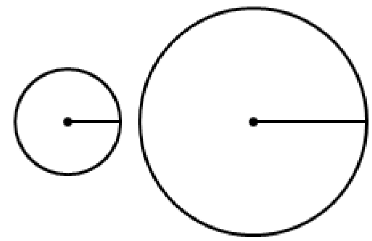
Proof:

Statement	**Reason**
1. ΔBAC is a right triangle AD is an altitude of ΔBAC	Given
2. BD + CD = BC	Segment Addition Postulate
3. ∠BDA and ∠ADC are right angles	Definition of an altitude
4. ∠BDA ≅ ∠ADC ≅ ∠BAC	Definition of congruence
5. ∠B ≅ ∠B ∠C ≅ ∠C	Reflexive Property
6. ΔBDA ~ ΔBAC ΔBAC ~ ΔADC	AA Similarity
7. ΔBDA ~ ΔBAC ~ ΔADC	Substitution
8. $\frac{\mathrm{AB}}{\mathrm{BC}} = \frac{\mathrm{BD}}{\mathrm{AB}} \qquad \frac{\mathrm{AC}}{\mathrm{BC}} = \frac{\mathrm{CD}}{\mathrm{AC}}$	Definition of Similarity
9. $(\mathrm{AB})^2 = \mathrm{BC} * \mathrm{BD}, \quad (\mathrm{AC})^2 = \mathrm{BC} * \mathrm{CD}$	Cross-multiplication

10. $(AB)^2 + (AC)^2 = BC * BD + BC * CD$	Addition Property of Equality
11. $(AB)^2 + (AC)^2 = BC(CD + BD)$	Factoring
12. $(AB)^2 + (AC)^2 = (BC)^2$	Substitution
$\therefore$ The square of the hypotenuse of a right triangle is equal to the sum of the squares of its legs.	

Theorem Proof K

If two shapes are circles, then the shapes are similar.



Proof:

By definition, a circle is the set of all points at a given distance (the radius) from a given point.

The radius can be increased or decreased by any scale factor; therefore, the radii of any two circles are proportional.

Therefore, since the radii of any two circles are proportional, the two circles are similar and it follows that all circles are similar.