Bond Graph Methodology

- An **abstract** representation of a system where a collection of components interact with each other through energy ports and are placed in a system where energy is exchanged.

- A domain-independent graphical description of dynamic behavior c physical systems

- System models will be constructed using **a uniform notations for a types of physical system** based on energy flow

- Powerful tool for modeling engineering systems, especially when different physical domains are involved

- A form of object-oriented physical system modeling

Bond Graphs

- ✓Use analogous power and energy variables in all domains, but allow the special features of the separate fields to be represented.
- ✓The only physical variables required to represent all energetic systems are *power variables* [effort (e) & flow (f)] and *energy variables* [momentum p (t) and displacement q (t)].
- ✓Dynamics of physical systems are derived by the application of *instant-by-instant* energy conservation. Actual inputs are exposed.
- ✓Linear and non-linear elements are represented with the same symbols; non-linear kinematics equations can also be shown.
- ✓Provision for *active bonds*. Physical information involving information transfer, accompanied by negligible amounts of energy transfer are modeled as *active bonds*.

A Bond Graph's Reach

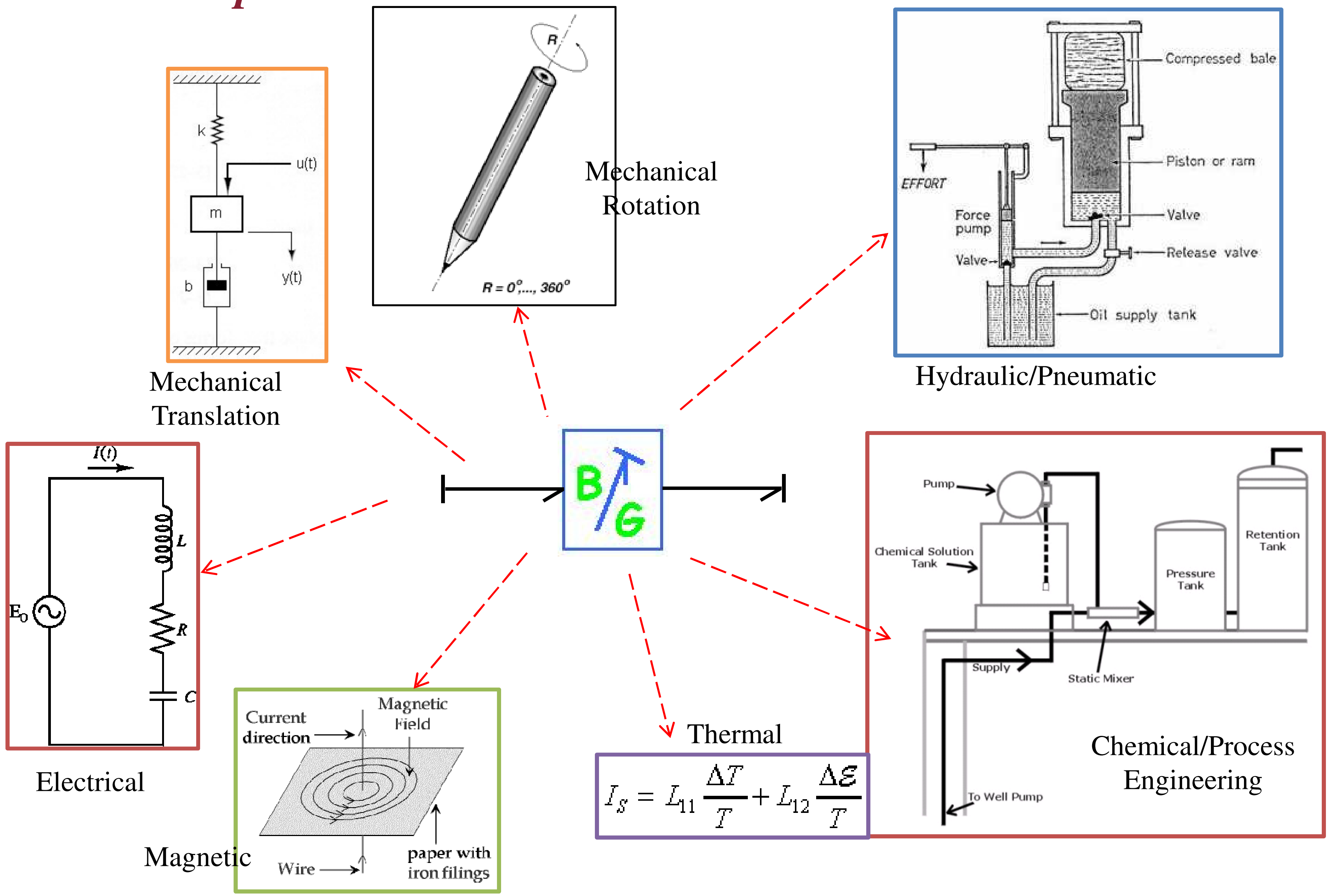


Figure 2. Multi-Energy Systems Modeling using Bond Graphs

- <u>**Introductory Examples**</u>
- **Electrical Domain**

Power Variables:

Electrical Voltage (u) & Electrical Current (i)

Power in the system: $P = u * i$

Constitutive Laws: $u_R = i * R$

$u_C = 1/C * (\int i \, dt)$

$u_L = L * (di/dt)$; or $i = 1/L * (\int u_L \, dt)$

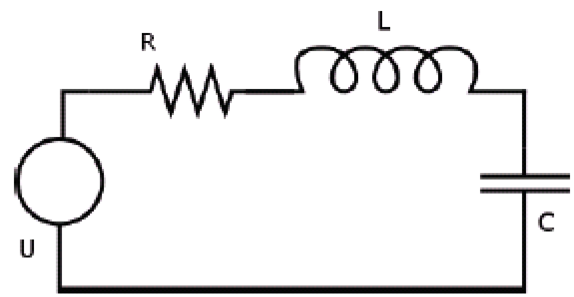


Fig 3. A series RLC circuit

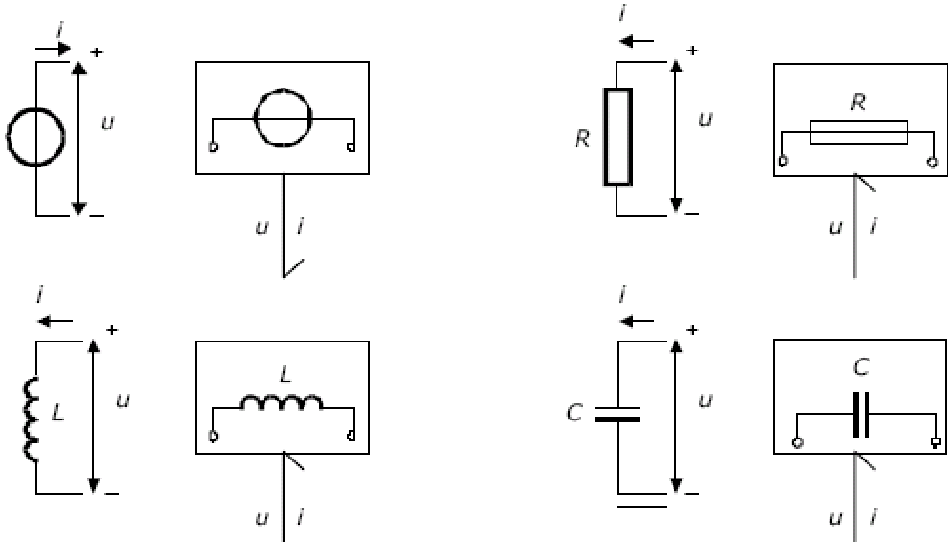


Fig. 4 Electric elements with power ports

Represent different elements with visible *ports* (*figure 4*)

To these ports, connect *power bonds* denoting energy exchange

The voltage over the elements are different

The current through the elements is the same

The R – L - C circuit

The common current becomes a “1-junction” in the bond graphs.

Note: the current through all connected bonds is the same, the voltages sum to zero

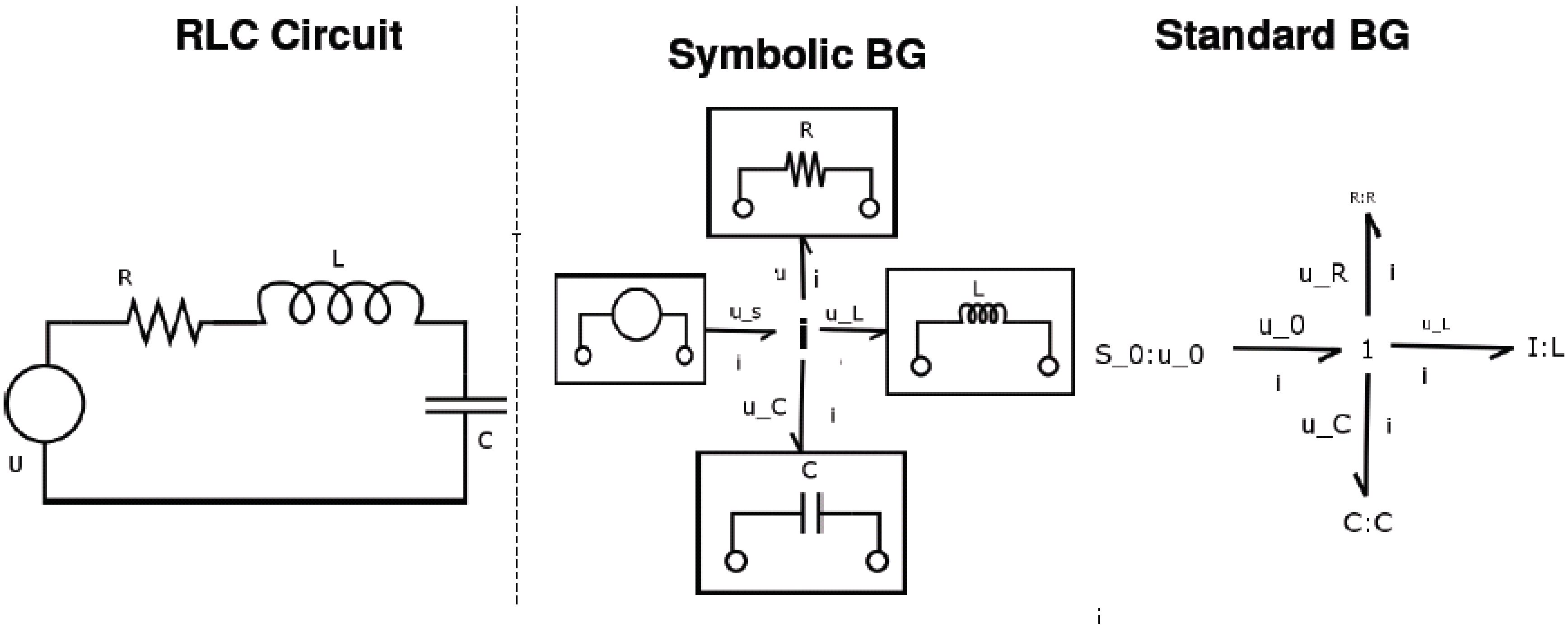


Fig 5. The RLC Circuit and its equivalent Bond Graph

Mechanical Domain

Mechanical elements like Force, Spring, Mass, Damper are similarly dealt with.

Power variables: Force (F) & Linear Velocity (v)

Power in the system: P = $F * v$

Constitutive laws: $F_d = \alpha * v$ $F_s = K_S * (\int v\,dt) = 1/C_S * (\int v\,dt)$

$F_m = m * (dv/dt)$; or $v = 1/m * (\int F_m\,dt)$; Also, F_a = force

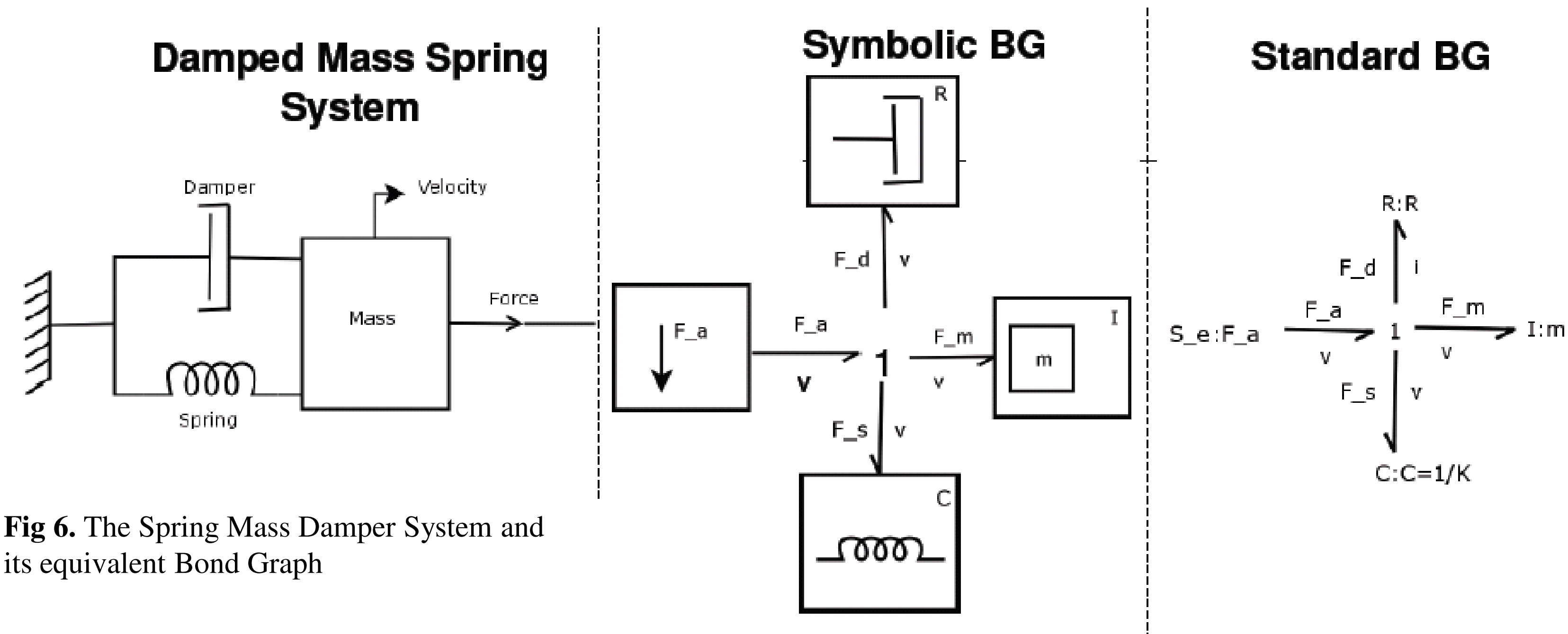


Fig 6. The Spring Mass Damper System and its equivalent Bond Graph

The common velocity becomes a “1-junction” in the bond graphs. Note: the velocity of all connected bonds is the same, the forces sum to zero)

Analogies Between The Mechanical And Electrical Elements

We see the following analogies

- The **Damper** is analogous to the **Resistor**.
- The **Spring** is analogous to the **Capacitor**, the mechanical *compliance* corresponds with the electrical *capacity*.
- The **Mass** is analogous to the **Inductor**.
- The **Force** source is analogous to the **Voltage** source.
- The common **Velocity** is analogous to the loop **Current**.

❑ . *Notice that the bond graphs of both the RLC circuit and the Spring-mass-damper system are identical*

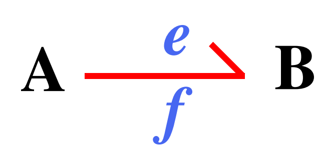
- Each of the various physical domains is characterized by a particular conserved quantity. *Table 1* illustrates these domains with corresponding flow (f), effort (e), generalized displacement (q), and generalized momentum (p).
- Note that *power = effort x flow* in each case.

Table 1. Domains with corresponding flow, effort, generalized displacement and generalized momentum

	f ***flow***	e ***effort***	$q = \int f\,dt$ ***generalized displacement***	$p = \int e\,dt$ ***generalized momentum***
Electromagnetic	i *current*	u *voltage*	$q = \int i\,dt$ *charge*	$\lambda = \int u\,dt$ *magnetic flux linkage*
Mechanical Translation	v *velocity*	f *force*	$x = \int v\,dt$ *displacement*	$p = \int f\,dt$ *momentum*
Mechanical Rotation	ω *angular velocity*	T *torque*	$\theta = \int \omega\,dt$ *angular displacement*	$b = \int T\,dt$ *angular momentum*
Hydraulic / Pneumatic	φ *volume flow*	P *pressure*	$V = \int \varphi\,dt$ *volume*	$\tau = \int P\,dt$ *momentum of a flow tube*
Thermal	T *temperature*	F_S *entropy flow*	$S = \int f_S\,dt$ *entropy*	
Chemical	μ *chemical potential*	F_N *molar flow*	$N = \int f_N\,dt$ *number of moles*	

➢ Bonds and Ports

Power port **or** ***port***: The contact point of a sub model where an ideal connection will be connected; location in a system where energy transfer occurs



(directed bond from A to B)

Power bond **or** ***bond***: The connection between two sub models; drawn by a single line

Bond denotes ideal energy flow between two sub models; the energy entering the bond on one side immediately leaves the bond at the other side (*power continuity*).

- Energy flow along the bond has the physical dimension of power, being the product of two variables

Effort and ***Flow*** called power-conjugated variables

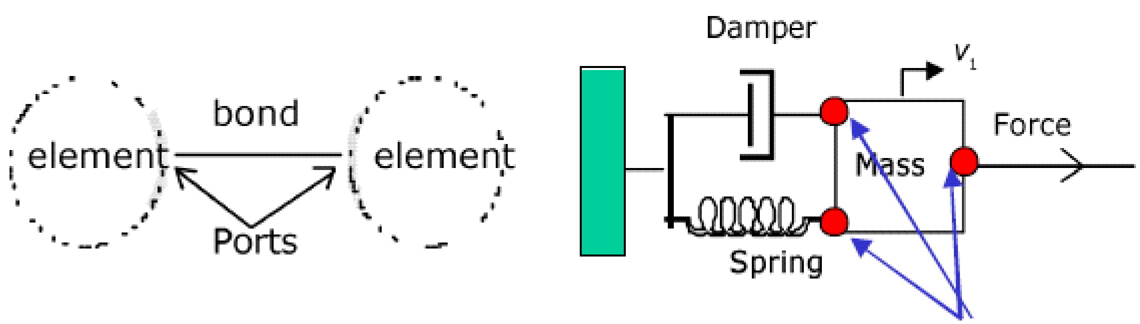


Fig. 7 Energy flow between two sub models represented by ports and bonds [4]

- **Bond Graph Elements**

Drawn as letter combinations (*mnemonic codes*) indicating the type of element.

C	storage element for a *q-type variable*, e.g. capacitor (stores charge), spring (stores displacement)
L	storage element for a *p-type variable*, e.g. inductor (stores flux linkage), mass (stores momentum)
R	resistor dissipating free energy, e.g. electric resistor, mechanical friction
Se, Sf	sources, e.g. electric mains (voltage source), gravity (force source), pump (flow source)
TF	transformer, e.g. an electric transformer, toothed wheels, lever
GY	gyrator, e.g. electromotor, centrifugal pump
0, 1	0 and 1 junctions, for ideal connection of two or more sub-models

Storage Elements

Two types; C – elements & I – elements; *q–type* and *p–type* variables are conserved quantities and are the result of an accumulation (or *integration*) process

C – element (capacitor, spring, etc.)

q is the conserved quantity, stored by accumulating the net flow, *f* to the storage element

Resulting balance equation ⟶ $dq/dt = f$

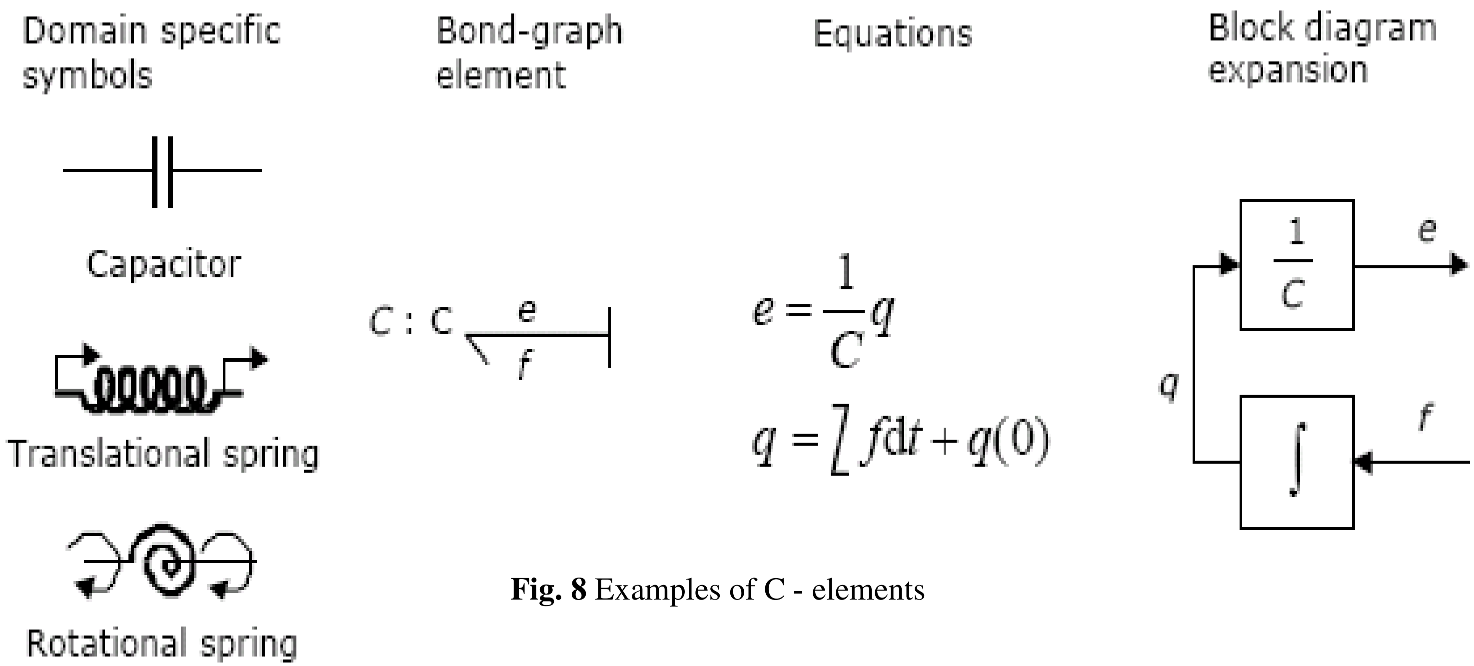


Fig. 8 Examples of C - elements

An element relates *effort* to the *generalized displacement*

1-port element that stores and gives up energy without loss

I – element (inductor, mass, etc.)

p is the conserved quantity, stored by accumulating the net effort, *e* to the storage element.

Resulting balance equation → $dp/dt = e$

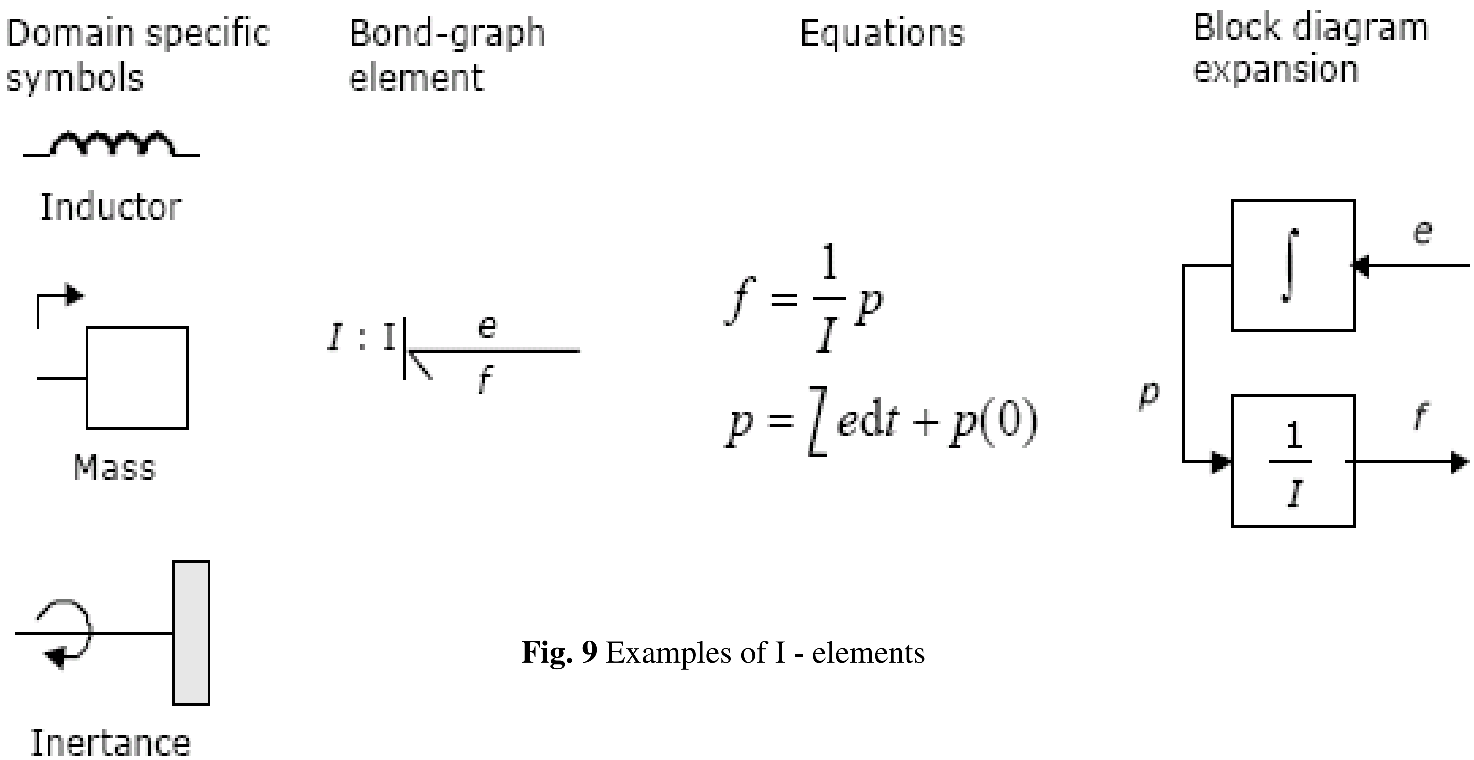


Fig. 9 Examples of I - elements

For an inductor, L [H] is the inductance and for a mass, m [kg] is the mass. For all other domains, an I – element can be defined.

R – element (electric resistors, dampers, frictions, etc.)

R – elements dissipate free energy and energy flow towards the resistor is always positive.

Algebraic relation between effort and flow: $\boldsymbol{e} = r * (\boldsymbol{f})$

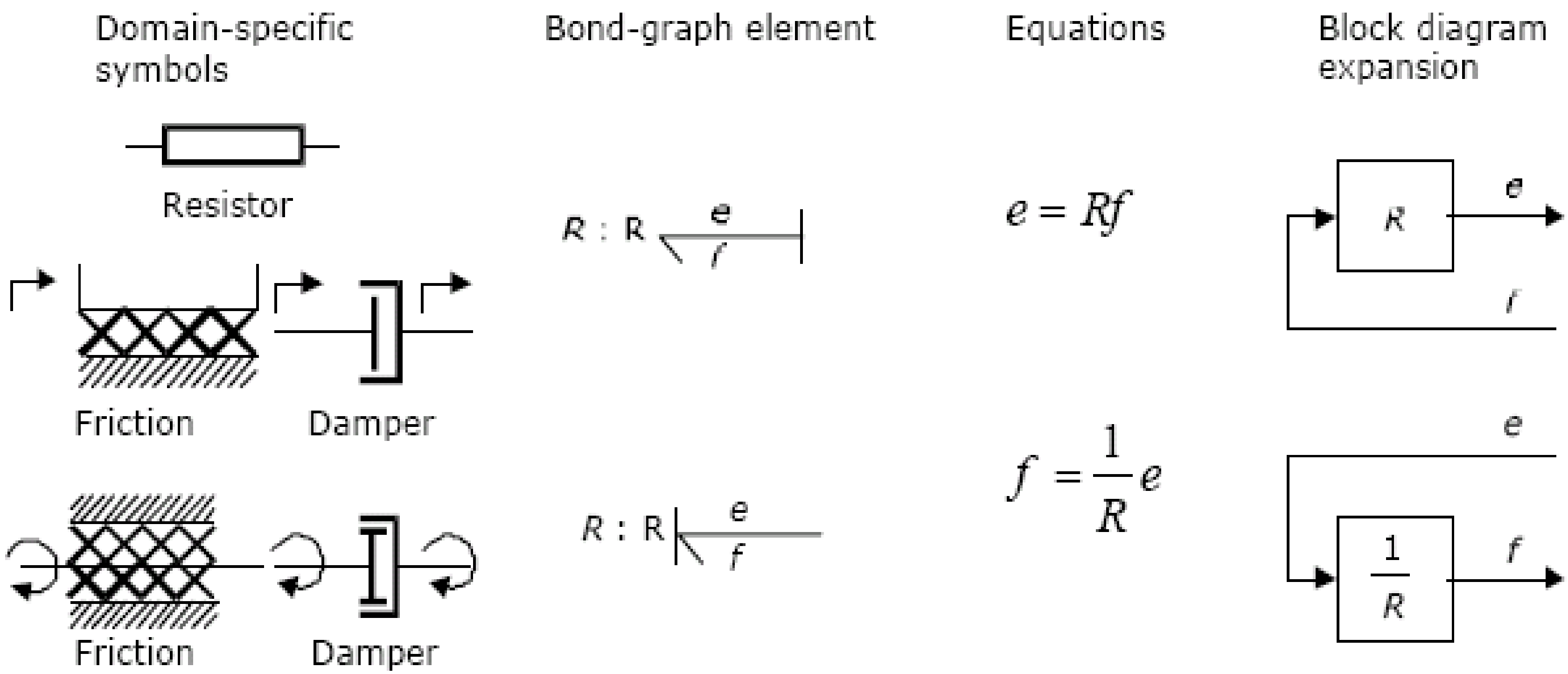


Fig. 10 Examples of Resistors

If the resistance value can be controlled by an external signal, the resistor is a modulated resistor, with mnemonic **MR**. E.g. hydraulic tap

Sources (voltage sources, current sources, external forces, ideal motors, etc.)

Sources represent the system-interaction with its environment. Depending on the type of the imposed variable, these elements are drawn as **Se** or **Sf**.

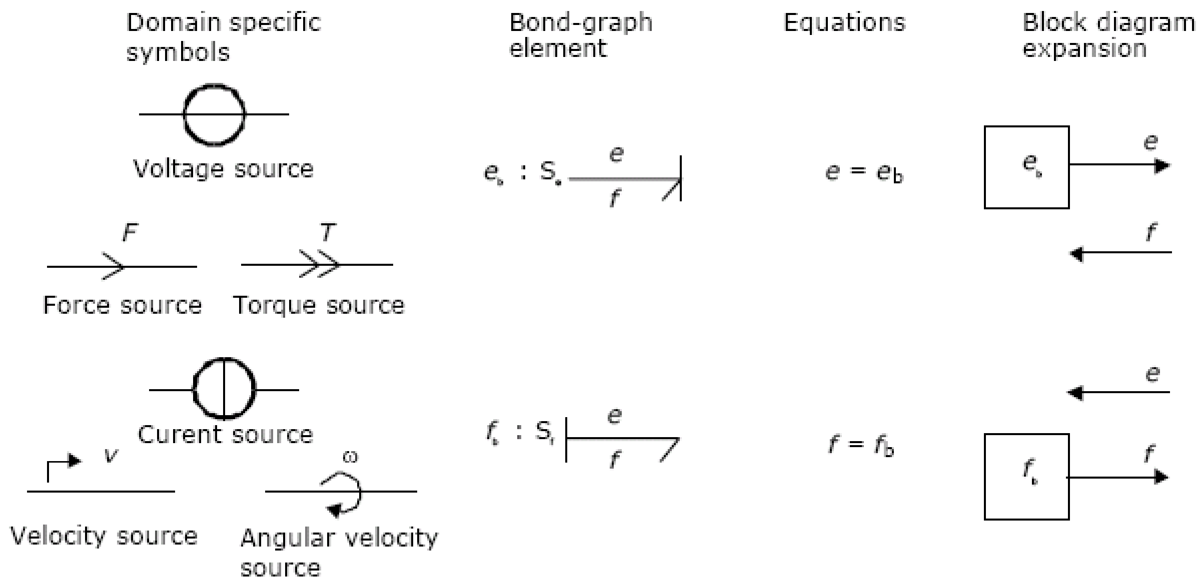


Fig. 12 Examples of Sources [4]

When a system part needs to be excited by a known signal form, the source can be modeled by a modulated source driven by some signal form (*figure 13*).

Fig. 13 Example of Modulated Voltage Source [4]

Transformers (toothed wheel, electric transformer, etc.)

An ideal transformer is represented by **TF** and is power continuous (i.e. no power is stored or dissipated). The transformations can be within the same domain (toothed wheel, lever) or between different domains (electromotor, winch).

$$\boldsymbol{e1} = n * \boldsymbol{e2} \qquad \& \qquad \boldsymbol{f2} = n * \boldsymbol{f1}$$

Efforts are transduced to efforts and flows to flows; *n* is the *transformer ratio*.

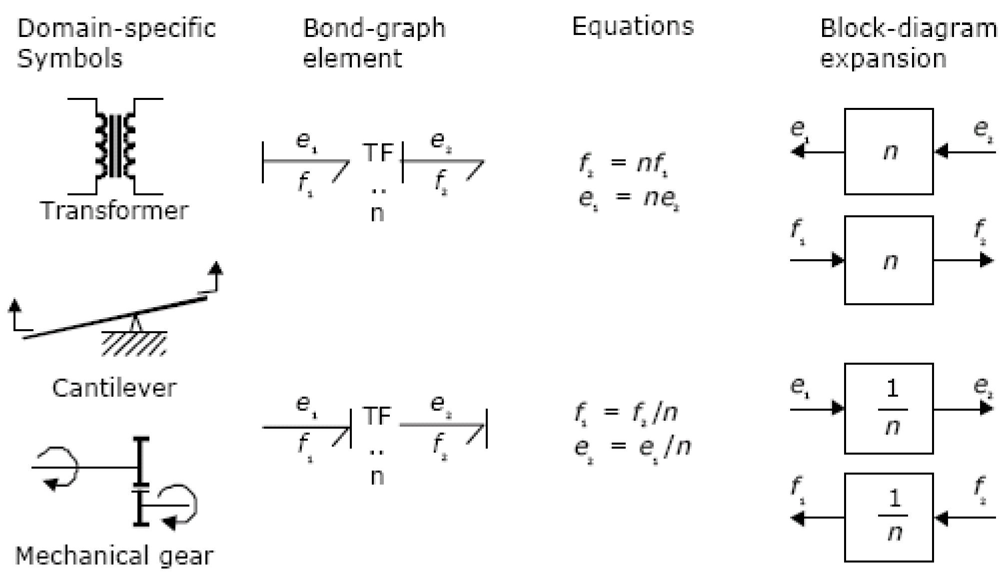


Fig. 14 Examples of Transformers [4]

Gyrators (electromotor, pump, turbine)

An ideal gyrator is represented by **GY** and is power continuous (i.e. no power is stored or dissipated). Real-life realizations of gyrators are mostly transducers representing a domain-transformation.

$$e1 = r * f2 \qquad \& \qquad e2 = r * f1$$

r is the gyrator ratio and is the only parameter required to describe both equations.

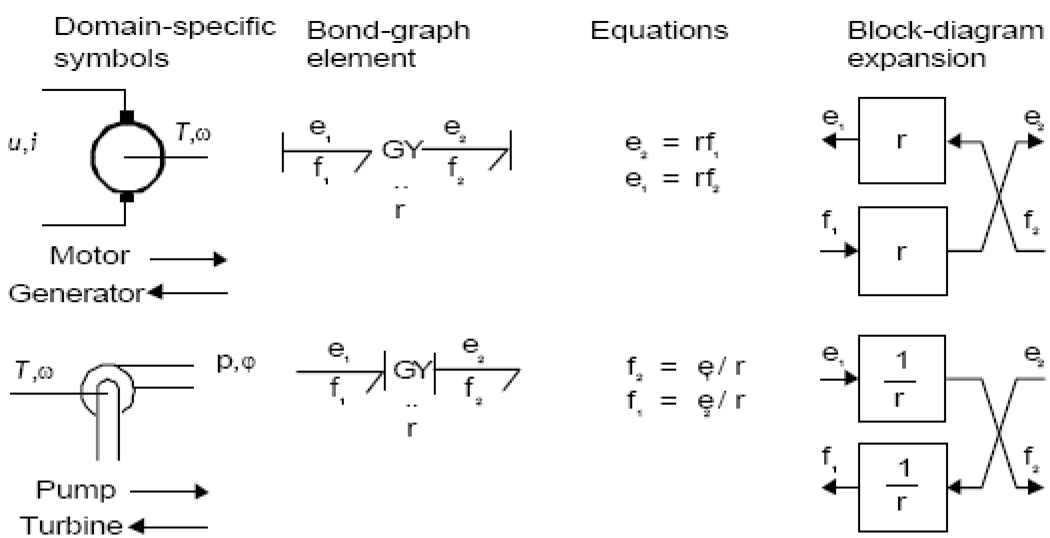


Fig. 15 Examples of Gyrators [4]

Junctions

Junctions couple two or more elements in a power continuous way; there is no storage or dissipation at a junction.

0 – junction

Represents a **node at which all efforts of the connecting bonds are equal**. E.g. a parallel connection in an electrical circuit.

The sum of flows of the connecting bonds is zero, considering the sign.

0 – junction can be interpreted as the generalized Kirchoff's Current Law.

Equality of efforts (like electrical voltage) at a parallel connection.

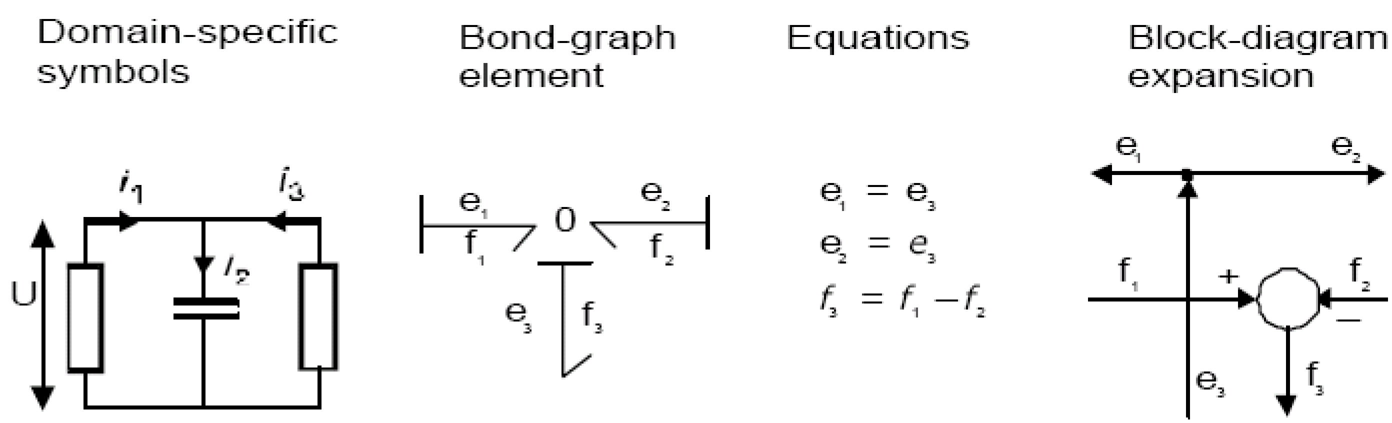


Fig. 16 Example of a 0-Junction [4]

1 – junction

Is the dual form of the 0-junction (roles of effort and flow are exchanged).

Represents **a node at which all flows of the connecting bonds are equal**. E.g. a series connection in an electrical circuit.

The efforts of the connecting bonds sum to zero.

1- junction can be interpreted as the generalized Kirchoff’s Voltage Law.

In the mechanical domain, 1-junction represents a *force-balance*, and is a generalization of Newton’ third law.

Additionally, equality of flows (like electrical current) through a series connection.

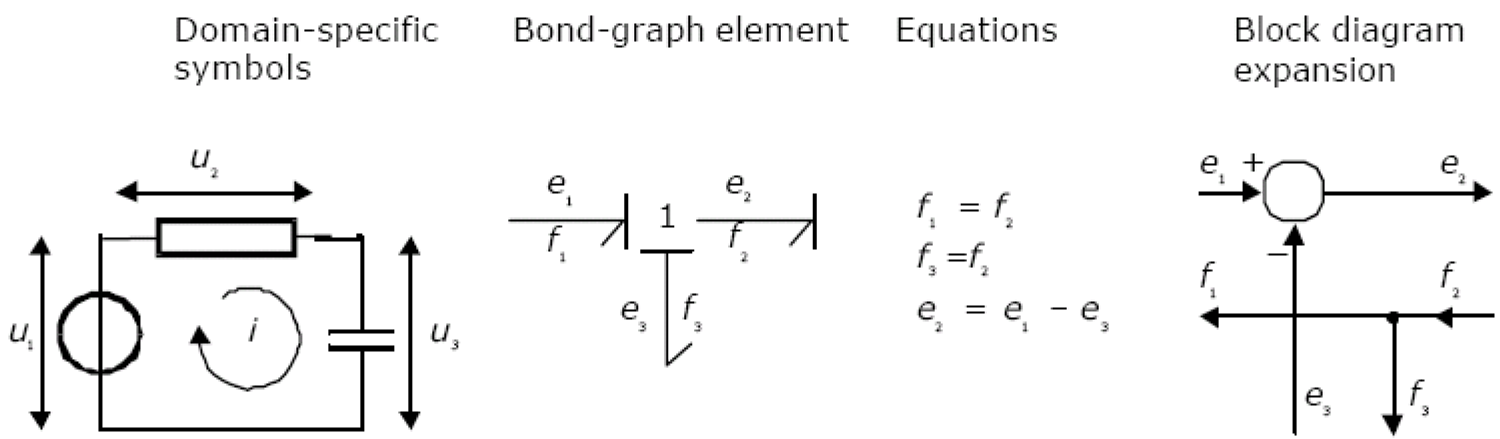


Fig. 17 Example of a 1-Junction [4]

Power Direction: The power is positive in the direction of the power bond. If power is negative, it flows in the opposite direction of the half-arrow.

Typical Power flow directions

R, C, and I elements have *an incoming bond* (half arrow towards the element)

Se, Sf: outgoing bond

TF– and GY–elements (transformers and gyrators): one bond incoming and one bond outgoing, to show the 'natural' flow of energy.

These are ***constraints*** on the model!

- **Causal Analysis**

  Causal analysis is the determination of the signal direction of the bonds

  Establishes the cause and effect relationships between the bonds

  Indicated in the bond graph by a *causal* stroke; the *causal stroke* indicates the direction of the effort signal.

  The result is a *causal bond graph*, which can be seen as a compact block diagram.

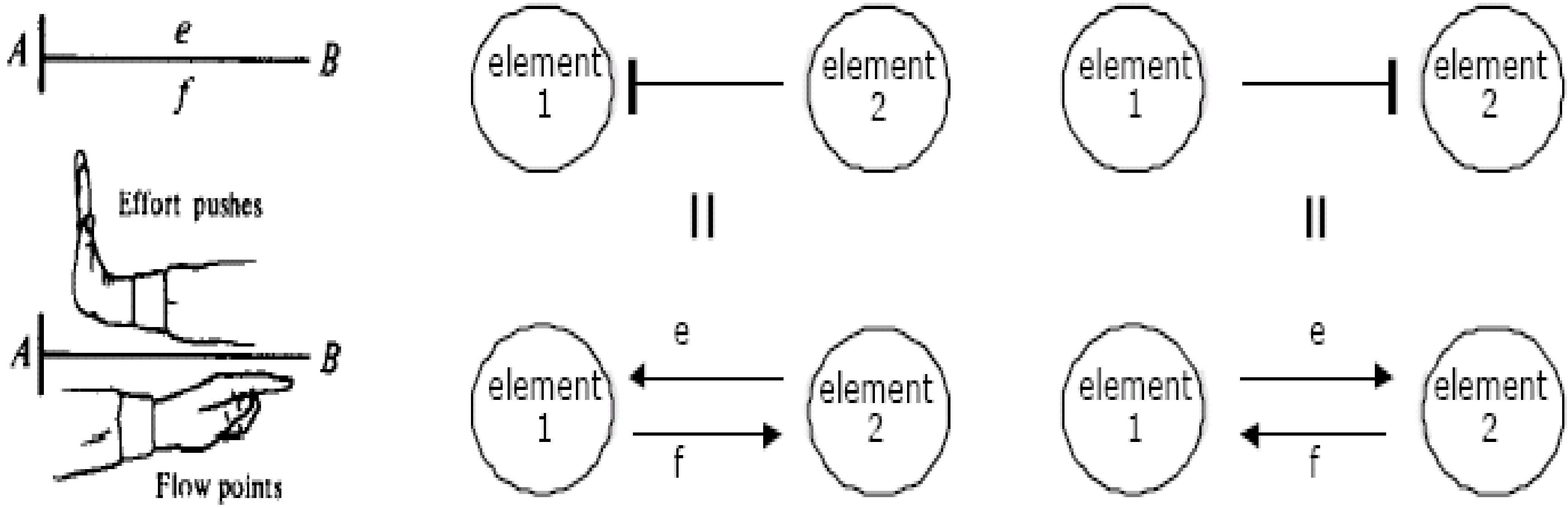
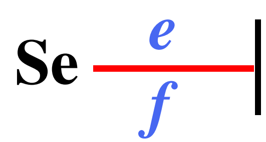
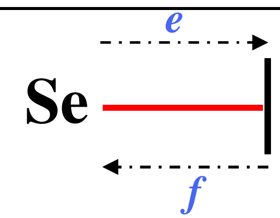
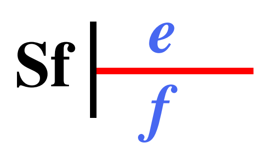
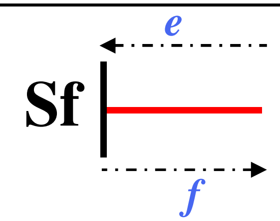
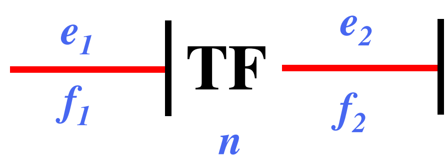
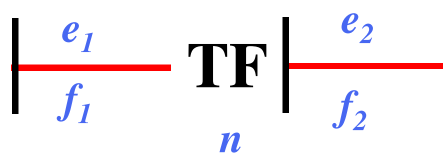
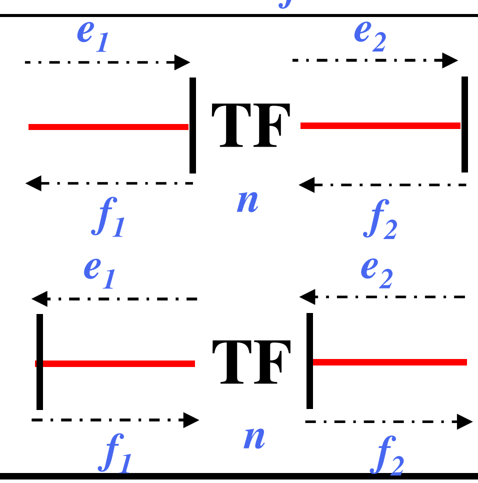
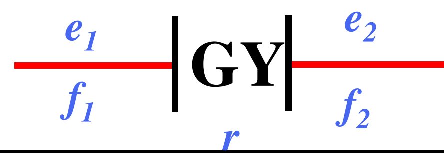
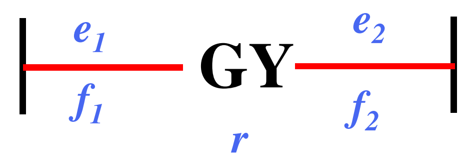
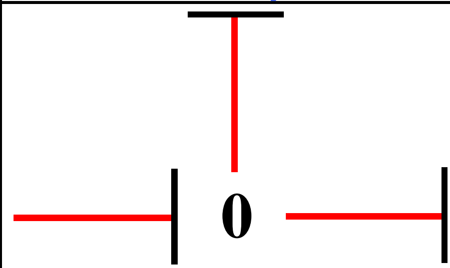
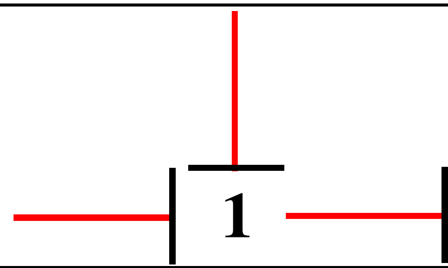
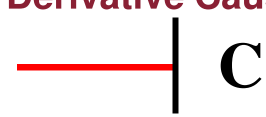
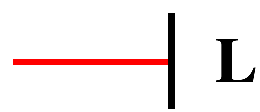
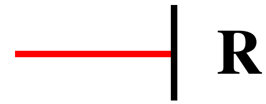
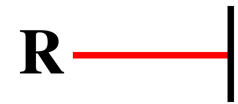


Fig. 18 Causality Assignment [4]

Causal Constraints: Four different types of constraints need to be discussed prior to following a systematic procedure for bond graph formation and causal analysis

Causality Type	Elements	Representation	Interpretation
Fixed	Se		
	Sf		
Constrained	TF	 OR 	

Causality Type	Elements	Representation
Constrained	GY	 OR 
	0 Junction	 **OR** any other combination where exactly one bond brings in the effort variable
	1 Junction	 **OR** any other combination where exactly one bond has the causal stroke away from the junction
Preferred	C	**Integral Causality (Preferred)**  **Derivative Causality** 
	L	**Integral Causality (Preferred)**  **Derivative Causality** 

Causality Type	Elements	Representation
Indifferent	R	 OR 

Some notes on **Preferred Causality** (C, I)

Causality determines whether an integration or differentiation w.r.t time is adopted in storage elements. ***Integration has a preference over differentiation*** because:

1. At integrating form, initial condition must be specified.

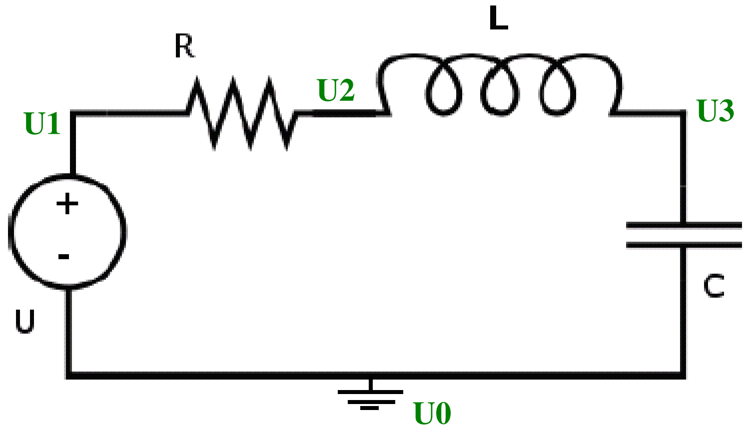
2. Integration w.r.t. time can be realized physically; Numerical differentiation is not physically realizable, since information at future time points is needed.

3. Another drawback of differentiation: When the input contains a step function, the output will then become infinite.

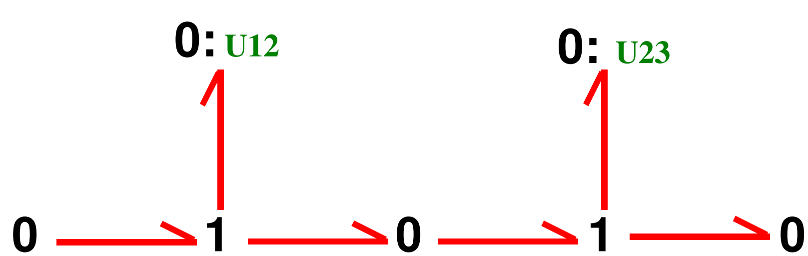
Therefore, integrating causality is the preferred causality. C-element will have effort-out causality and I-element will have flow-out causality

Examples

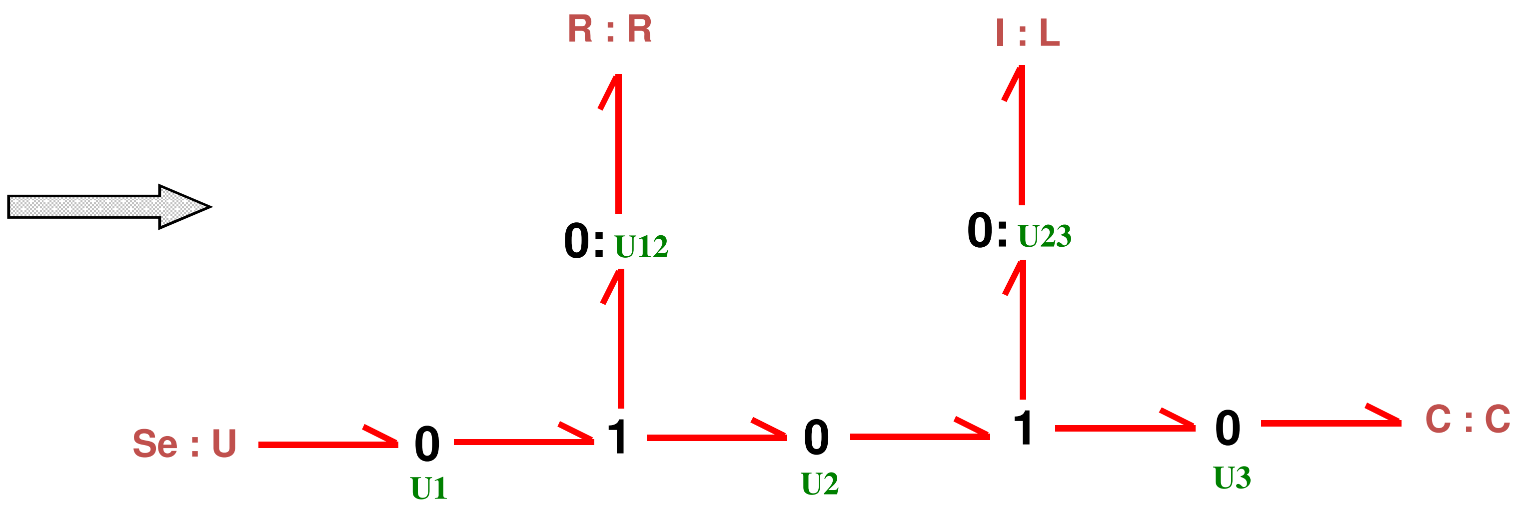
- **Electrical Circuit # 1 (R-L-C) and its Bond Graph model**

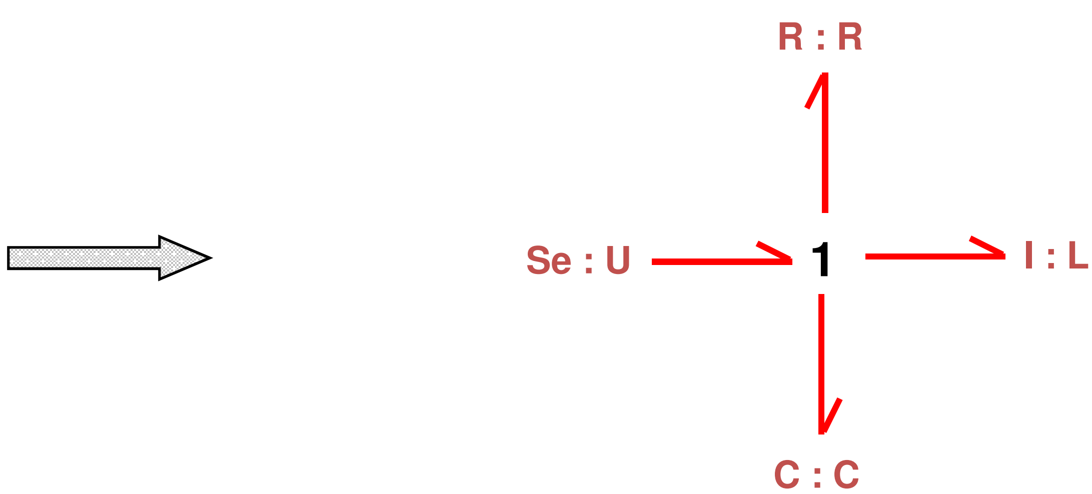


0 U1   0 U2   0 U3



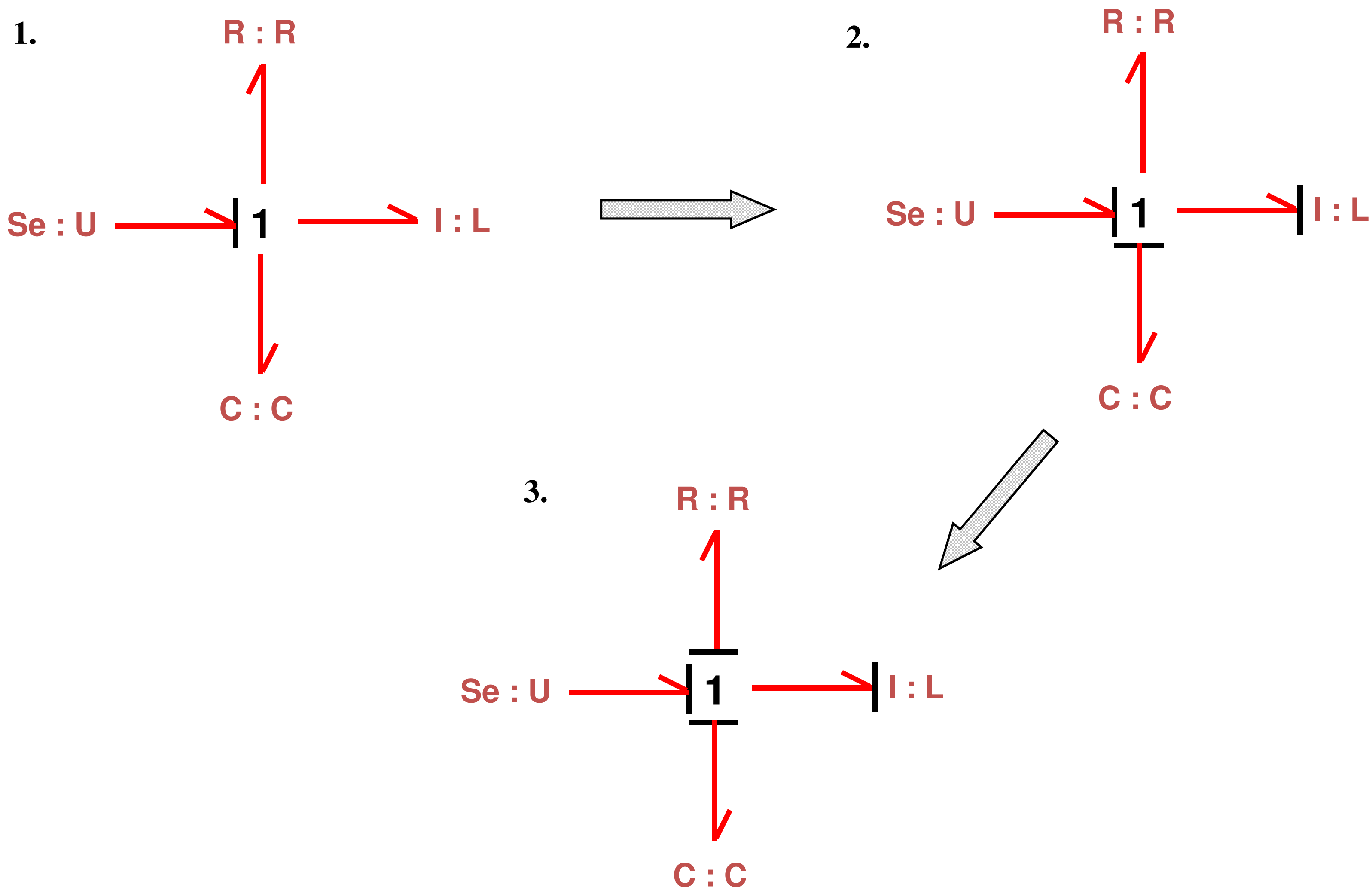
Examples (contd..)





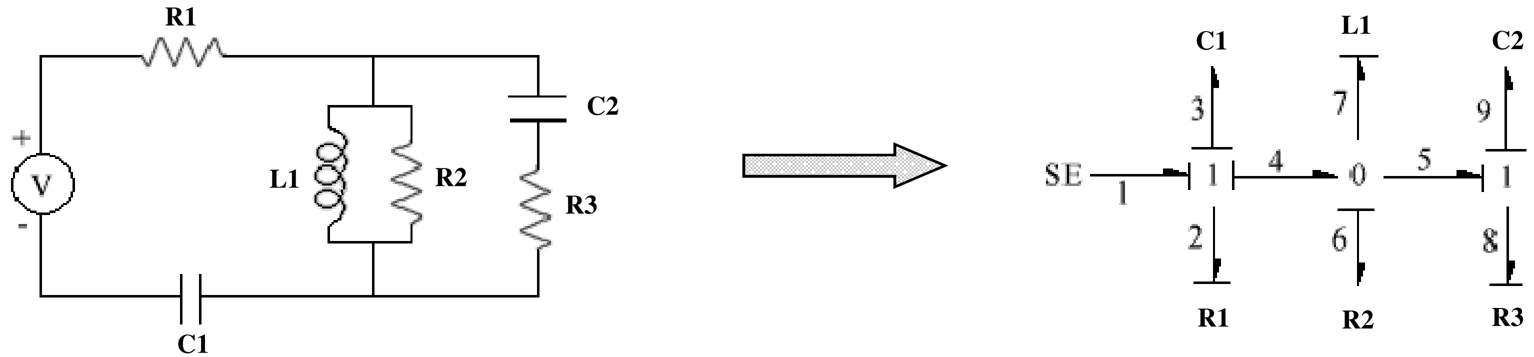
Examples (contd..)

The Causality Assignment Algorithm:



Examples (contd..)

- Electrical Circuit # 2 and its Bond Graph model



- A DC Motor and its Bond Graph model