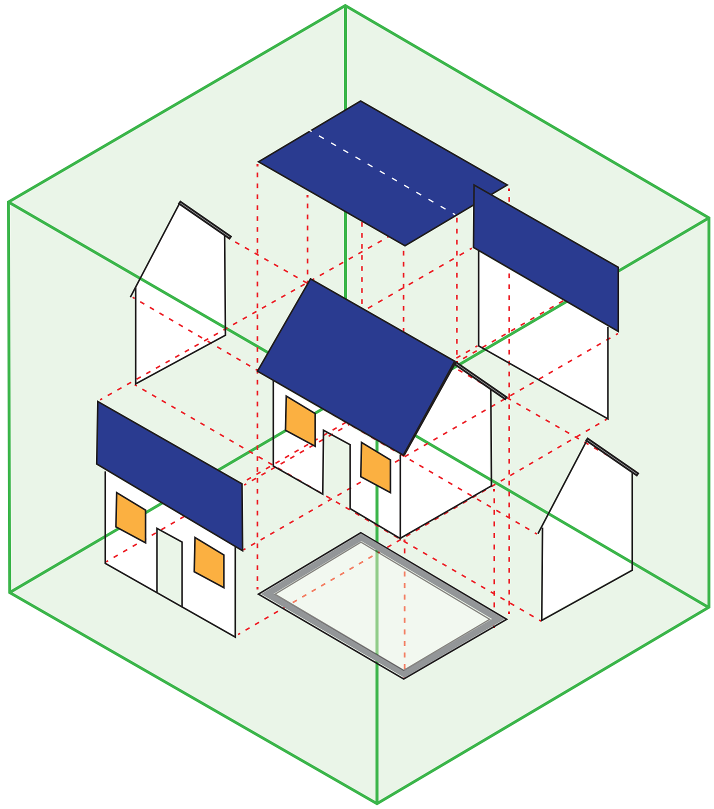
Build It, and They Will Come! Experiences in 3-D



What is one skill that architects, artists, physicists, designers, and engineers have in common? *Spatial reasoning*. This skill is the ability to visualize with the mind's eye. People in these professions are able to rotate an object mentally. They can examine a two-dimensional drawing or blueprint and imagine what the object will look like in three dimensions.

In this activity, you will explore cube constructions and isometric drawings as a means to develop your own spatial reasoning skills.

Four-Cube Structures

1. Using exactly 4 cubes, construct as many different structures as possible. As you build each structure, set it aside so you can later make comparisons.

2. During a class discussion, list the class rules for building structures as they emerge.

3. *Describe what it means for two structures to be congruent.*

4. In your own words, describe the terms *face*, *edge*, and *vertex* and how many of each are on a cube.

5. Match each drawing on Isometric Handout 1 to the structures you built for question 1.

From Isometric Drawings to Models: Constructing a 3-D Model from an Isometric Drawing

6. Examine your isometric drawing. How many blocks do you think are in your figure? Explain your reasoning.

7. Using interlocking cubes, build the model of the isometric drawing assigned to your group. When you are finished building your model, display the completed model on your table.

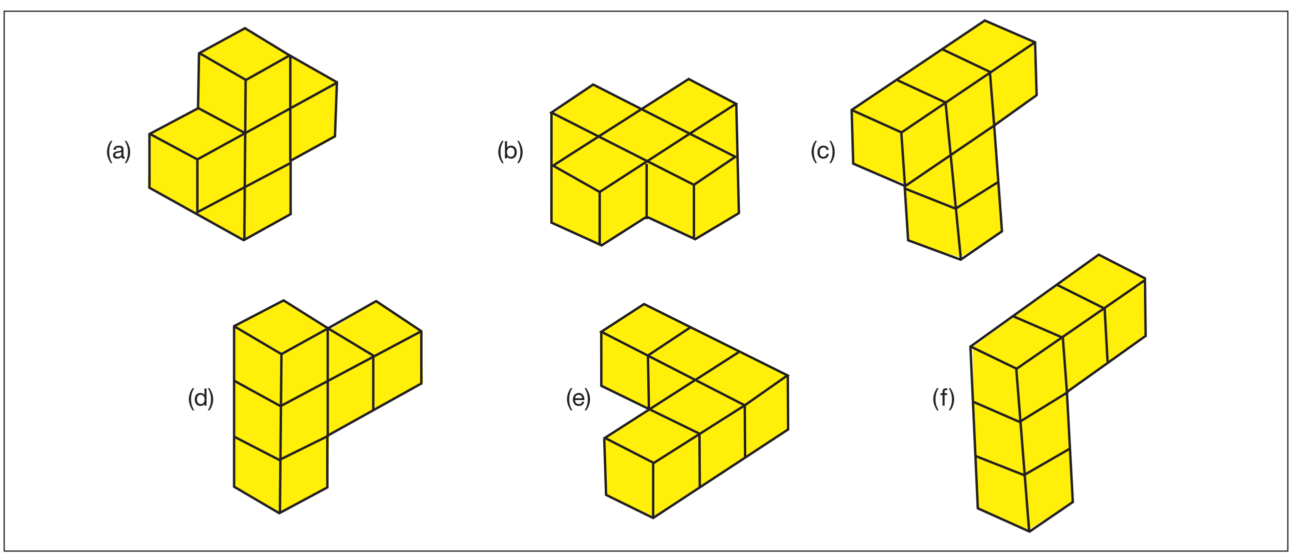
   a. How many total blocks did you actually need to build your assigned structure?

   b. How did your prediction compare to the total number of blocks used to build the structure?

8. What strategies did you find helpful in building a model of the isometric drawing?

9. Five interlocking cubes are used to build each of the models in **figure 1**. Which models are congruent? Explain why.

Figure 1. Determine which figures are congruent.



Understanding Orthographic Views

10. a. Construct **figure 2**.

    b. Label the orthographic views in **figure 3** as they correspond to the sides of your isometric drawing in **figure 2**.

Build Models from Orthographic Views

11. Given the top, front, and right orthographic views in **figure 4**, use cubes to construct each model. Check your work with another student to see if your models are congruent.

Figure 2. Construct the figure.

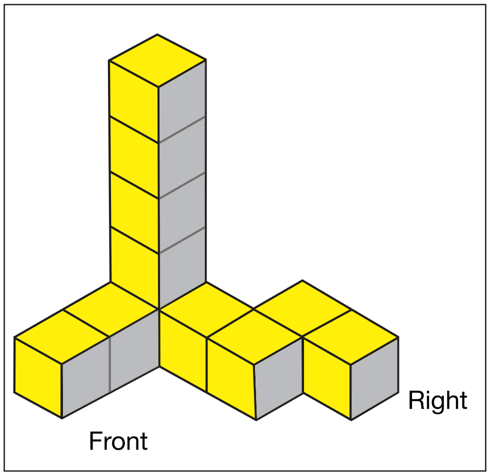


Figure 3. Identify the viewpoint.

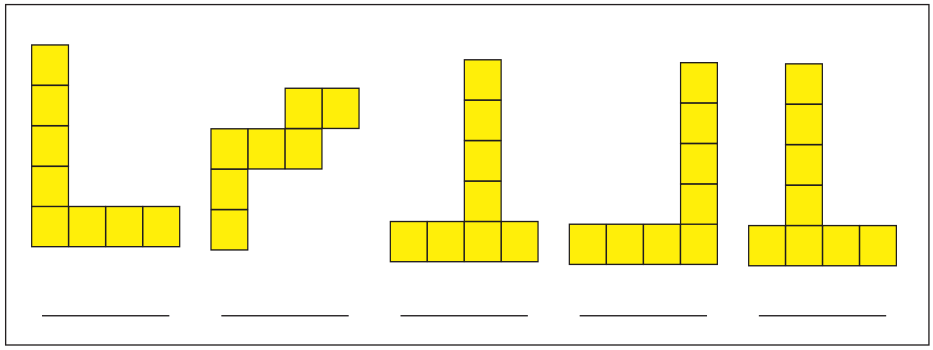
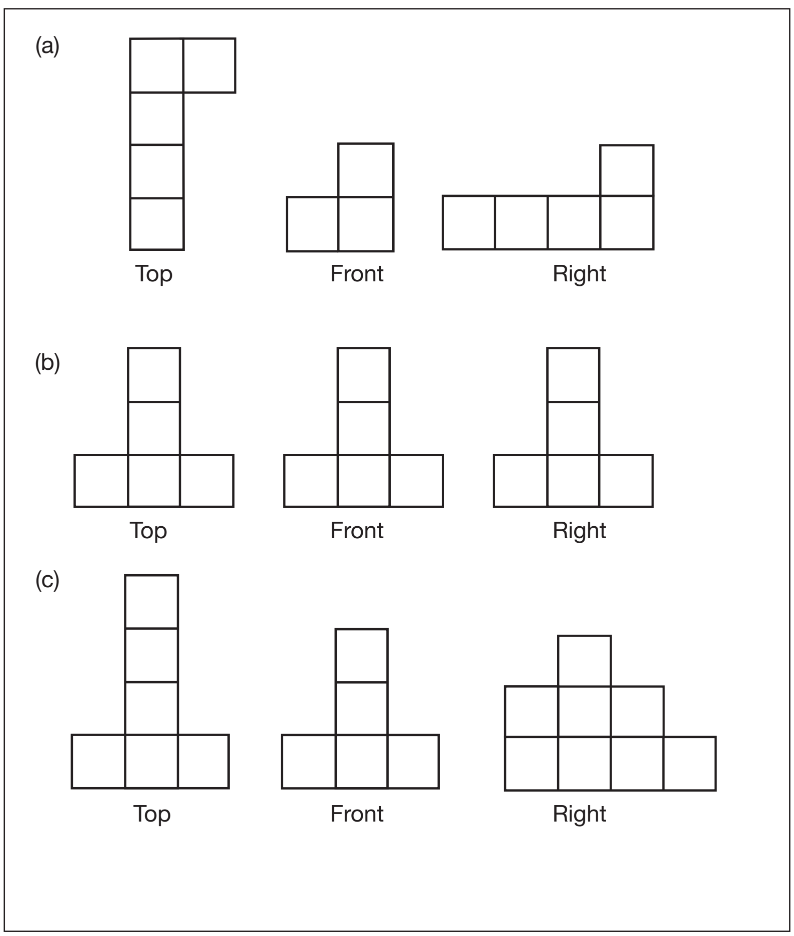


Figure 4. Use the top, front, and right orthographic views below to construct models.



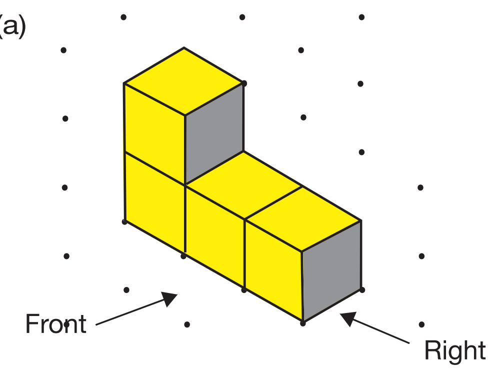
Mat Plans

In the previous activities, you worked with structures and developed various ways to represent those structures using isometric drawings and orthographic views. In the next activity, you will develop another way to represent a solid. This is called a *mat plan* or *mat diagram* (see **fig. 5**).

12. Using the isometric drawing in **figure 5**, explain what each number on the mat plan represents.

Figure 5. This shows an isometric drawing (a) and a mat plan (b) for a structure.

(a)



(b)

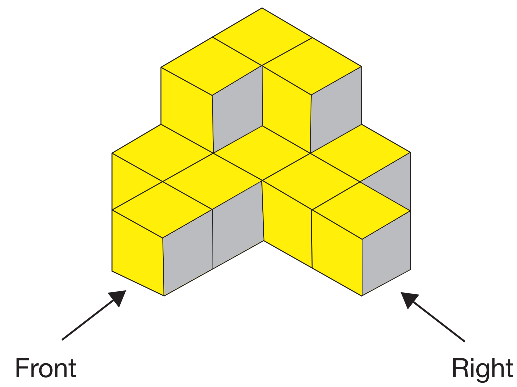
2	1	1

Right

Front

13. A mat plan (see **fig. 6**) for a structure has been started for you below. Analyze the drawing and complete the mat plan.

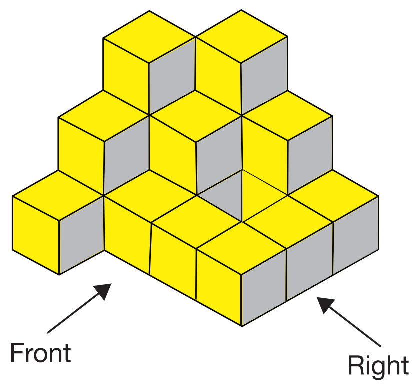
Figure 6. Complete the mat plan in **fig. 6**.



2	2	1	
	1		1
1			
	1		

14. Complete a mat plan for the given solid in **figure 7**.

Figure 7. More mat plan practice.



15. Build the structure that the mat plan in **figure 8** represents.

Figure 8. Build the structure represented by the mat plan below.

3	1	2
1	1	1
2		

Right

Front

Creating Your Own Isometric Drawings

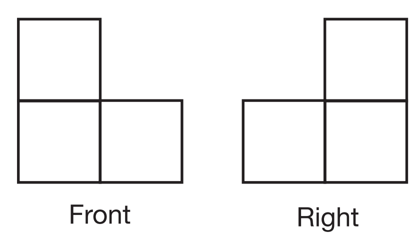
Visualization can be described as looking at a three-dimensional object and mentally seeing the appropriate orthographic views. It also can be described as looking at orthographic views and mentally visualizing the three-dimensional object. Visualization is important for anyone working in the fields of drafting, design, or engineering. Practice will improve one's ability to visualize.

Orthographic views are two-dimensional views of three-dimensional objects. Orthographic views represent the exact shape of an object as seen from one side at a time as you look at it from the perpendicular. Depth is not shown.

16. Set a single cube on your mat, and draw the front-right view on isometric dot paper.

17. Given the front and right orthographic views in **figure 9**, draw the structure on isometric grid paper.

Figure 9. This shows the front and right orthographic views. After you draw the structure on isometric grid paper, be sure to label your drawing.



18. Given the isometric drawing in **figure 10**, draw the orthographic views of the front, right, and top.

19. Given the isometric drawing in **figure 11**, create the orthographic views for the front, right, and top.

20. Use between 6 and 10 cubes to create a three-dimensional object. Place it on your mat plan, and draw two different views of the object. Identify the front and right views.

Figure 10. Draw and label the front, right, and top view.

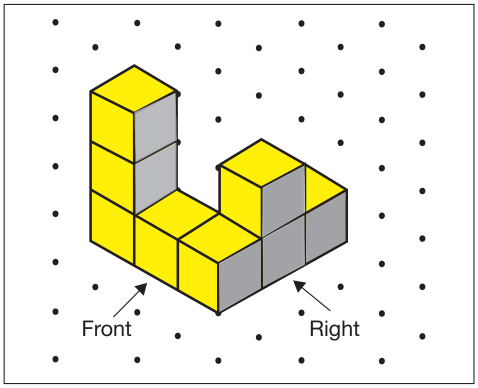


Figure 11. This shows an isometric drawing; draw the orthographic views of the front, right, and top.

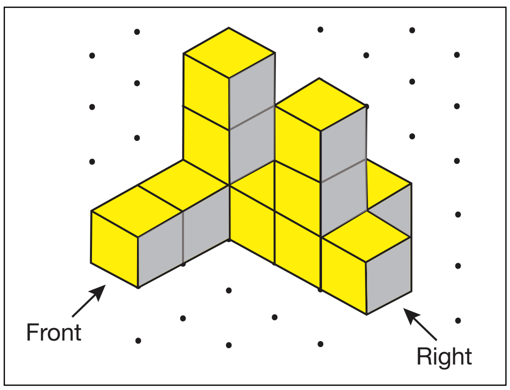


Figure 12. Create a rectangular prism with double the volume of the prism shown here.

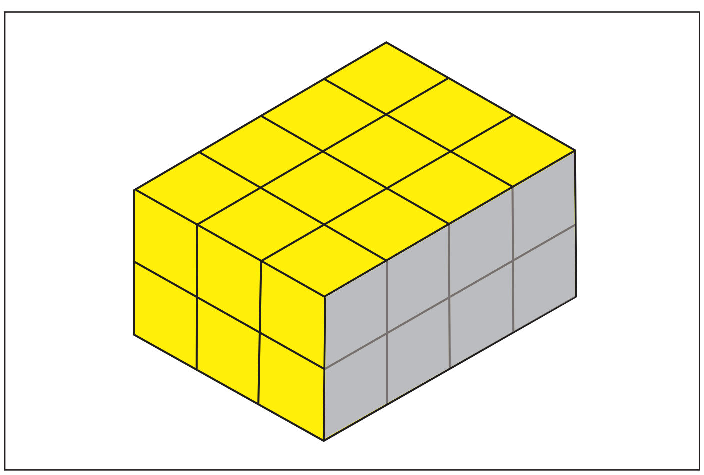


Table 1. Increasing the Volume

Factor Each Side Length Increases by	Way to Consider the New Side Lengths	Expression for New Volume

21. Create a rectangular prism with a volume that is double the volume of **figure 12** (see **table 1**). Explain your reasoning.

22. a. How will the volume of **figure 12** change as each dimension is doubled?

b. If each dimension is now "*n*" times the original dimensions, write the expression that would represent the new volume.

Can You . . .

- determine the angle through which we drew the isometric drawings in **figure 1**?
- determine the maximum number of edges you can see in any structures in **figure 1**?
- determine the shapes that are made by a hexahedron when drawn on an isometric drawing grid?
- determine at which angle, as the object rotates from 0 degrees to 90 degrees, the square appears first to be a parallelogram and then a rhombus?

Did you know that . . .

- an isometric drawing, also called *isometric projection*, is a way of graphically representing three-dimensional objects? Such drawings are used by engineers, technical illustrators, and architects.
- isometric drawings are used in video games to create an illusion of three dimensions on a two-dimensional surface?

Source

NCTM Illuminations. 2013. "Isomentric Drawing Tool." http://illuminations.nctm.org/ActivityDetail.aspx?ID=125.

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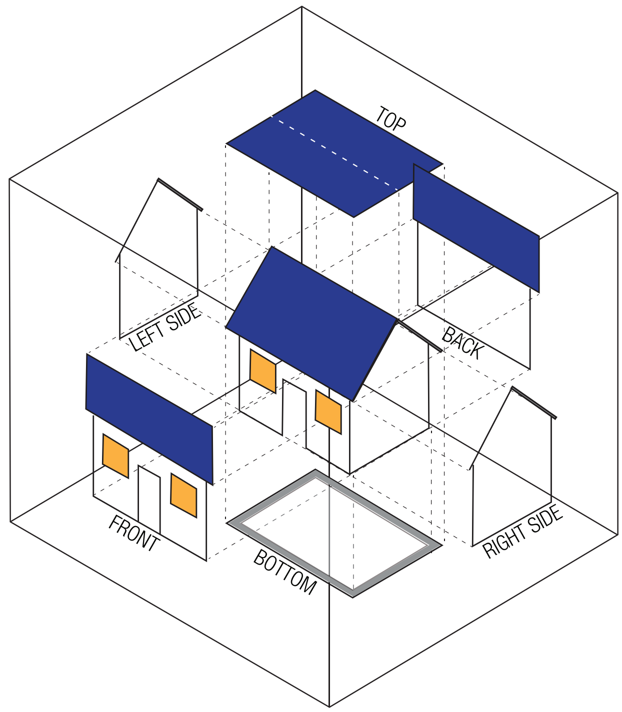
Defining Terms in Your Own Words

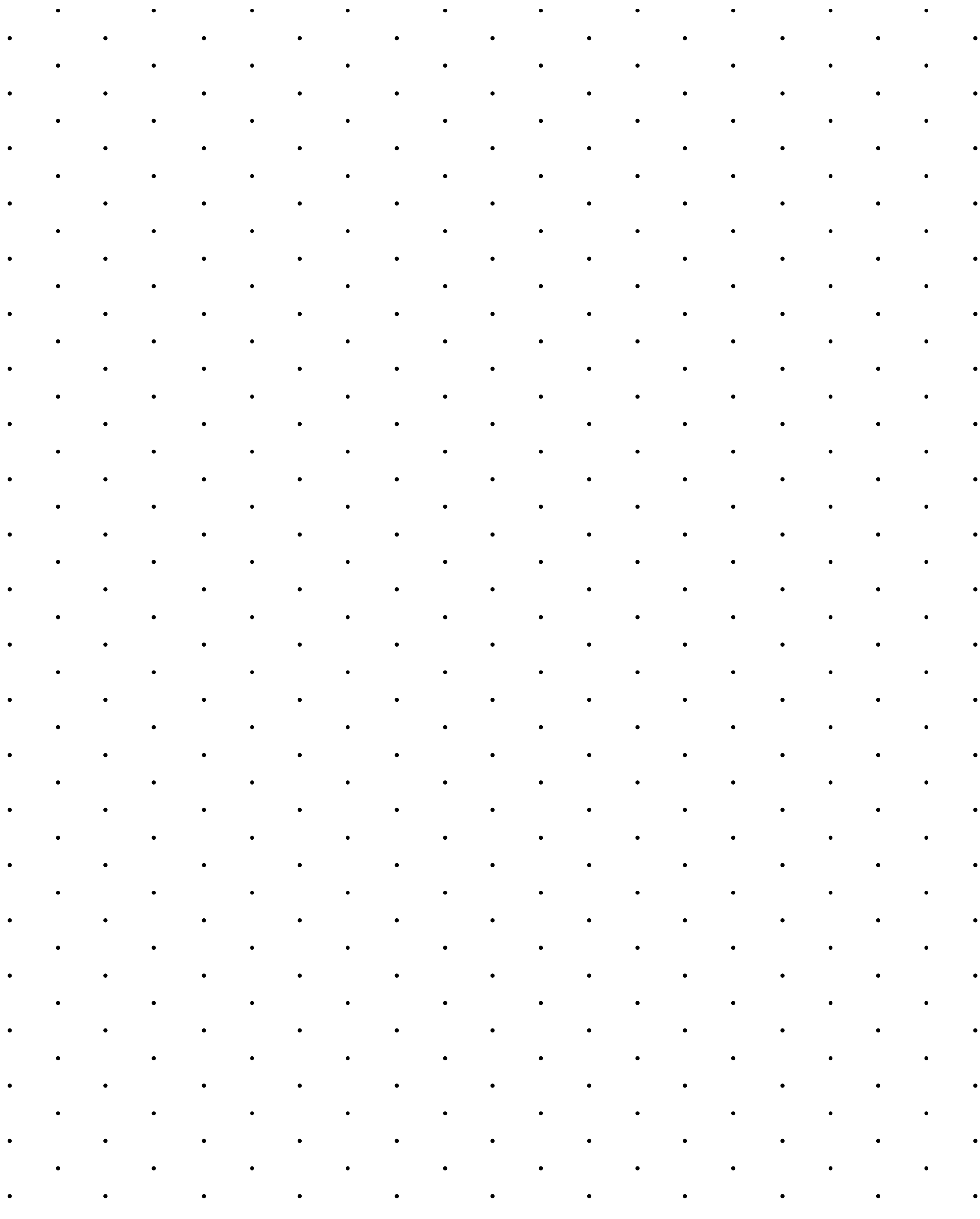


Face

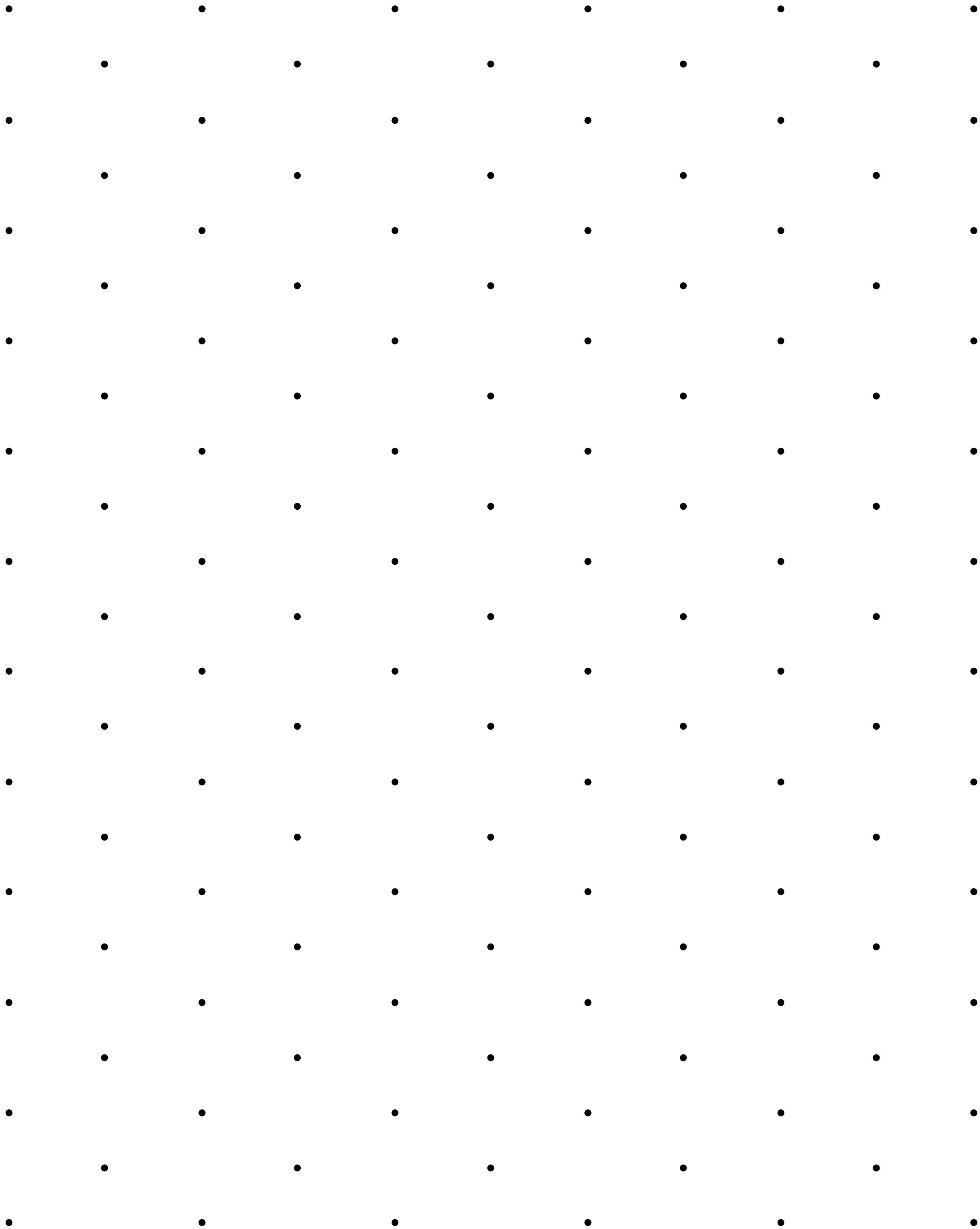
Edge

Vertex

Isometric Dot Paper (1 cm)



Isometric Dot Paper (2 cm)



Isometric Handout 1

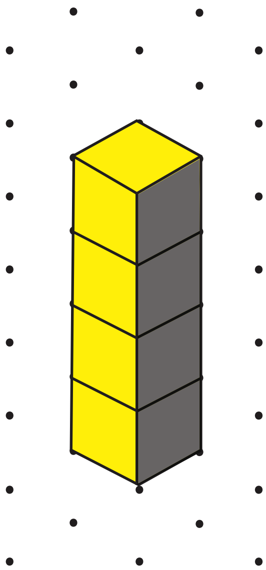


Figure 1

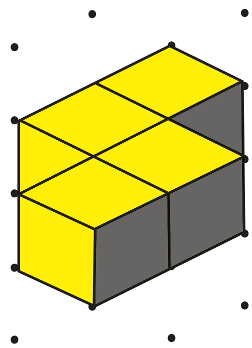


Figure 2

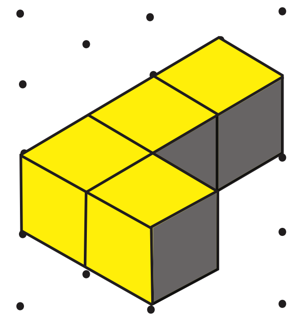


Figure 3

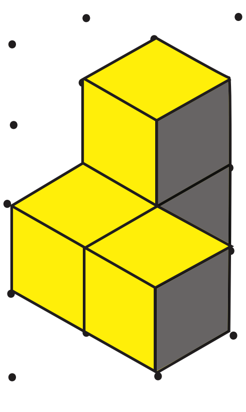


Figure 4

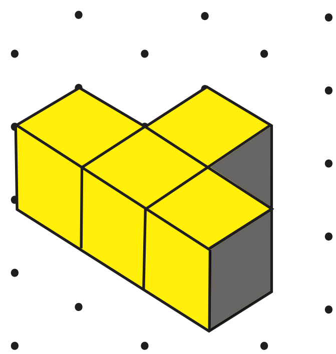


Figure 5

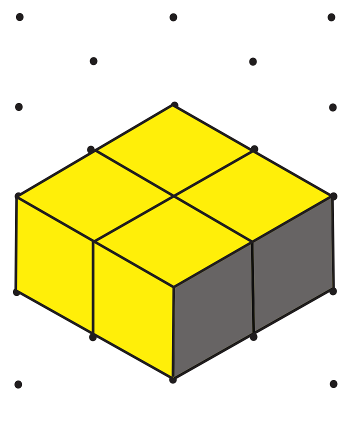


Figure 6

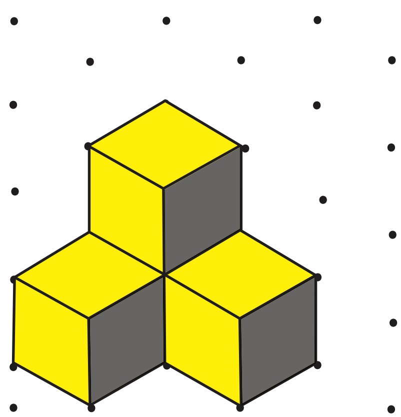


Figure 7

Isometric Handout 2

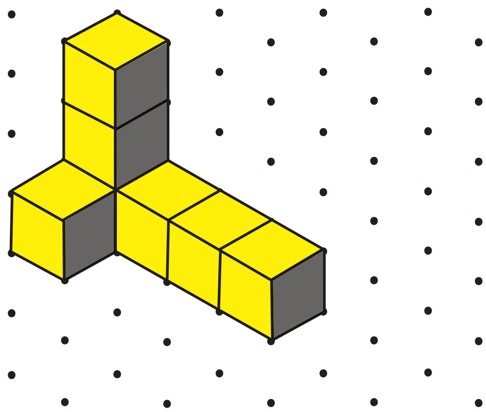


Figure 1

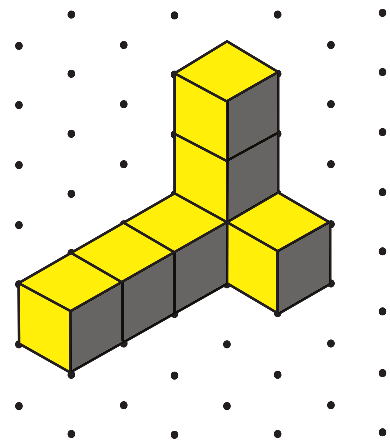


Figure 2

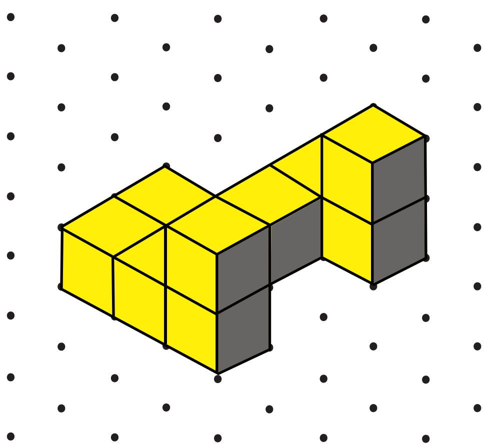


Figure 3

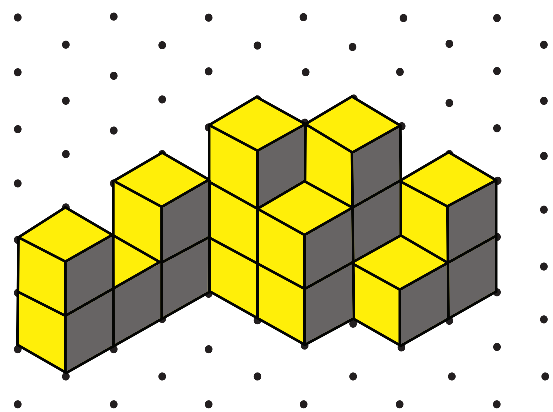


Figure 4

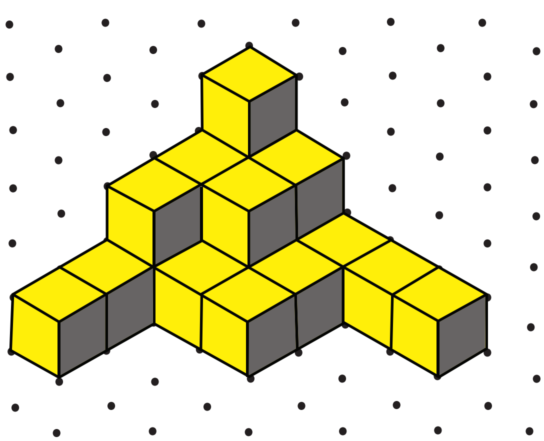


Figure 5

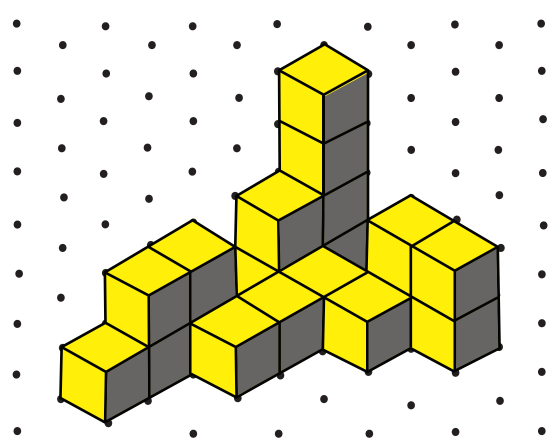


Figure 6