

Building an Optimization Model with Solver

1. Components of an Optimization Model

An optimization model is a decision-making tool that helps you (as an individual or a company) determine the “best” **actions** to take in order to meet the established **objective(s)** while facing a number of **constraints** or “obstacles”. The Excel Solver add-in is one way to set up an optimization model.

Before we proceed with the discussion on the set up of an optimization model with Solver, you need to first understand the concepts behind the various components of an optimization model. As you can see, the model has three main components: objective, actions, and constraints.

- **Objective:** This is what you want to accomplish. Examples of objectives include minimizing cost, maximizing profit, minimizing number of staffs needed, etc. In general, the objective takes on either the *minimization* or *maximization* form.
- **Actions:** They are basically the solutions of an optimization model. They represent the strategies that need to be adopted in order to best achieve the objective. You can think of them as action items such as the number of tables and chairs to produce (in order to maximize the company’s profit).
- **Constraints:** They represent the “obstacles” the company faces. Examples of “obstacles” a company might encounter include limited resources and specific demands from the clients. These “obstacles” will limit (or constrain) the types of actions you can adopt to meet the objective.

2. An Example of an Optimization Model

There are a variety of optimization models, each catering to a different scenario, such as linear programming, non-linear programming, integer programming, binary programming, etc. We will focus on linear programming here. To help you understand the logic behind the setup of an optimization model using linear programming, we will use the following example as an illustration.

The Flair Furniture Company produces inexpensive tables and chairs. The production for each is similar in that both require a certain number of labor hours in the carpentry department and a certain number of labor hours in the painting department.

Each table takes 4 hours of carpentry work and 2 hours of painting work. Each chair requires 3 hours in carpentry and 1 hour in painting. During the current production period, 240 hours in carpentry time and 100 hours of painting time are available.

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The marketing personnel are confident that they can sell all the tables that are made. However, due to an existing inventory of chairs, they want Flair to make no more than 60 new chairs. Each table sold results in a profit contribution of \$7, and each chair sold yields a profit contribution of \$5.

You have been contacted by Flair Furniture to help them determine the best possible combination of tables and chairs to manufacture in order to attain the maximum profit. How would you help the company determine its product mix? What would be your recommendations?

In the example, the furniture company is trying to determine how many tables and how many chairs it should manufacture, given its limited resources and other constraints, in order to maximize its profit. We will go over how each of the three components is established for the optimization model.

- **Objective:** The furniture company's objective is to maximize its profit. Since it is manufacturing both tables and chairs, its total profit is defined as follows:

$$\text{Profit} = (\text{Profit per table} \times \text{No. of tables}) + (\text{Profit per chair} \times \text{No. of chairs})$$

We will use T and C to represent the numbers of tables and the number of chairs the company manufactures, respectively. Given that the company is earning a profit of \$7 per table and \$5 per chair, we can determine its total profit as follows:

$$\text{Profit} = 7T + 5C$$

- **Actions:** The furniture company needs to determine what T (i.e. the number of tables) and C (i.e. the number of chairs) should be in order to maximize its total profit.
- **Constraints:** There are limits to how many tables and chairs the furniture company can manufacture (i.e. its actions) because of its limited resources and other constraints. In this particular example, the company is facing three constraints: limited carpentry hours, limited painting hours, and a limit on the number of chairs to manufacture (due to existing inventory).

a. Limited carpentry hours

We know the company uses 4 and 3 carpentry hours to make a table and a chair, respectively. As a result, the total number of carpentry hours used by the company to make T tables and C chairs is $4T + 3C$. This total has to be less than 240 carpentry hours (which is the maximum amount currently available to the company).

$$\text{Constraint (a):} \quad 4T + 3C \leq 240$$

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b. Limited painting hours

Similarly, we know the company uses 2 and 1 painting hours on a table and a chair, respectively. As a result, the total number of painting hours used by the company on T tables and C chairs is $2T + 1C$. This total has to be less than 100 painting hours (which is the maximum amount currently available to the company).

$$\text{Constraint (b):} \quad 2T + 1C \leq 100$$

c. Limit on number of chairs to manufacture

Due to existing inventory, the company does not want to manufacture more than 60 new chairs. As a result, the number of chairs to be manufactured, C, has to be less than or equal to 60.

$$\text{Constraint (c):} \quad C \leq 60$$

When we put the three components together for the furniture company (in the example), the optimization model will look as follows:

Objective: Maximize profit = $7T + 5C$

Actions: By determining T and C

Constraints: Subject to the limitations
 (a) $4T + 3C \leq 240$
 (b) $2T + 1C \leq 100$
 (c) $C \leq 60$

3. Installing and accessing Solver

- **Install Solver**

Office logo → **Excel Options** **File** → **Options**

→ Select **Add-Ins** → Click **Go...** next to the **Manage: Excel Add-ins** box

→ Check the **Solver Add-In** box → Click **OK**

- **Access Solver**

Data → **Analysis** → **Solver**

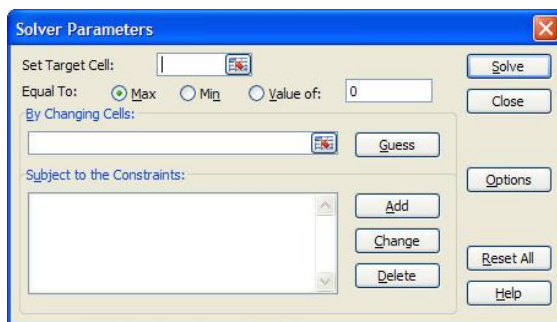
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4. Setting up Solver for an optimization model

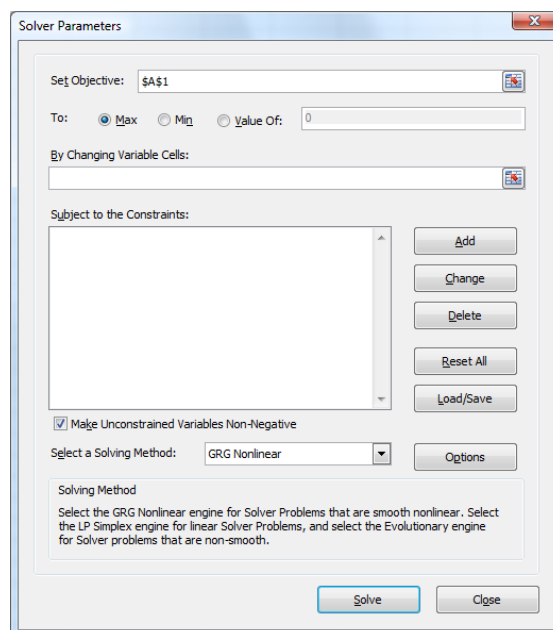
Excel 2010's Solver has a new interface, offers new solving methods, and generates new reports. Since not everyone has access to Excel 2010, we will look at how to set up Solver using both the new interface (in Excel 2010) and the old interface (in Excel 2007).

The main area of the Solver is the Solver Parameters. This is where you set up the objective, the actions, and the constraints of an optimization model. There are some small changes to the layouts of the Solver Parameters in Excel 2010, but the setups are very similar for both Excel 2007 and Excel 2010 (as shown below):

Excel 2007 Solver Parameters

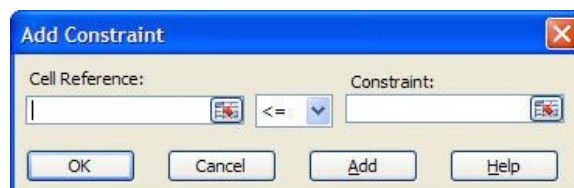


Excel 2010 Solver Parameters



The Add Constraint area is accessed by clicking **Add** next to the **Subject to the Constraints:** box and the layouts are identical for both Excel 2007 and Excel 2010 (as shown below):

Excel 2007 and Excel 2010 Add Constraint



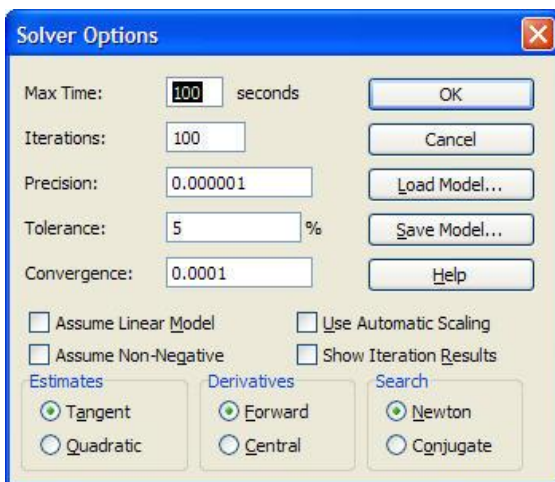
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The following table shows you where to set up the objective, the actions, and the constraints of a linear optimization model in Solver Parameters:

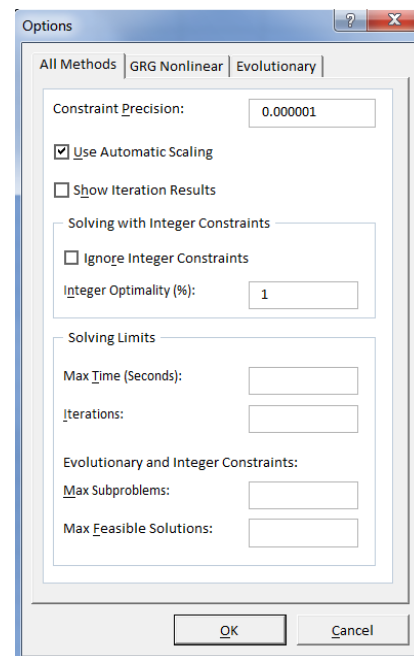
	Excel 2007	Excel 2010
1. Objective	Set Target Cell:	Set Objective:
2. Objective action	Equal:	To:
	Note: You can select <i>Max</i> (for maximization), <i>MIN</i> (for minimization), or specify a specific value for the objective in the Value of: box	
3. Actions (or decisions)	By Changing Cells:	By Changing Variable Cells:
4. Constraints	Subject to the Constraints:	Subject to the Constraints:
	Note: You click <i>Add</i> for each set of constraint(s), click <i>Change</i> to edit your selected set of constraint(s), and click <i>Delete</i> to delete your selected set of constraint(s)	
5. Others	Click Options ; select Assume Linear Model , Assume Non-Negative , and Use Automatic Scaling ; and click OK	Select Simplex LP in the Select a Solving Method : box
Click Solve when you are done with the setups		

Since you are building simple linear models, you will need to make appropriate selections in Solver Options if you are using Excel 2007 (but not if you are using Excel 2010).

Excel 2007 Solver Options



Excel 2010 (Solver) Options



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5. Using the SUMPRODUCT function

The SUMPRODUCT function is commonly used when setting up an optimization model with Solver. It is a combination of a SUM function and a PRODUCT function, i.e. it multiplies corresponding components in sets of arrays and then add up the results.

SUMPRODUCT (array1, [array2], [array3], ...)

The SUMPRODUCT function makes the setups of certain formulae much easier in an optimization model because you can complete the calculations with a single function rather than multiple functions. The following example shows how much easier it is to setup a formula with the use of the SUMPRODUCT function:

Example: Suppose a customer purchases 25 items of Product 1 (\$15/each), 12 items of Product 2 (\$20/each), and 20 items of Product 3 (\$10/each). The information regarding the purchases is set up in an Excel spreadsheet. What is the total cost of the purchases?

	A	B	C	D
1	Price (P)	Amount (Q)	P x Q	Formula
2	15	25	375	=A2*B2
3	12	20	240	=A3*B3
4	20	10	200	=A4*B4
5			815	
6				

Option 1: Regular setup

$$=(A2*B2)+(A3*B3)+(A4*B4)$$

Option 2: Using SUM function

$$=SUM(A2*B2,A3*B3,A4*B4)$$

Option 3: Using SUMPRODUCT function

$$=SUMPRODUCT(A1:A4,B1:B4)$$

As the number of items in the arrays increases, the benefit of using the SUMPRODUCT function greatly increases (compared to the other two options).

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6. Interpreting Solver reports

Once Solver found a solution for an optimization model, it will generate a number of reports. We will focus on two key reports, namely the *Answer Report* and the *Sensitivity Report*. The discussion on how to interpret various parts of the two reports will be based on the Flair Furniture Company example's answer and sensitivity reports. [Note: The *Limit Report* is not as important as the first two so we are ignoring it. Excel 2010 also introduces three new reports, namely the *Linearity Report*, the *Feasibility Report*, and the *Population Report* (but we are ignoring them for now).]

I. Answer Report

The *Answer Report* is made up of three parts and they are laid out the way the optimization model is set up, namely *Target Cell* (i.e. the objective), *Adjustable Cells* (i.e. the actions), and *Constraints*. The following is the answer report of the Flair Furniture Company example:

Target Cell (Max)

Cell	Name	Original Value	Final Value
\$C\$3	Maximize Profit	\$ -	\$ 410.00

Adjustable Cells

Cell	Name	Original Value	Final Value
\$C\$6	Number of units produced Tables	0	30
\$D\$6	Number of units produced Chairs	0	40

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$E\$11	Carpentry hours Used	240	\$E\$11<=\$G\$11	Binding	0
\$E\$12	Painting hours Used	100	\$E\$12<=\$G\$12	Binding	0
\$E\$13	Chair limits Used	40	\$E\$13<=\$G\$13	Not Binding	20

- a. **Target Cell:** The number reported under *Final Value* represents the optimal outcome for the objective. In the above report, the final value of the target cell is \$410.00. This means that the highest level of profit attainable by the company is \$410.00.
- b. **Adjustable Cells:** The numbers reported under *Final Value* represent the actions to take that will meet the objective (given the constraints). In the example, the final values of the two adjustable cells are 30 (for tables) and 40 (for chairs). This means that in order for the company to maximize its profit, it should make 30 tables and 40 chairs.

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- c. **Constraints:** The numbers reported under *Cell Value* represent the resources used. In the example, we know that the company used up 240 carpentry hours and 100 painting hours and it made 40 chairs. Has the company met the limits of its constraints? You can find the information reported under *Status*. *Binding* means that the limit of a constraint has been reached (e.g. the limited resource has been used up) and there is no slack; while *Not Binding* means that the limit of a constraint has not been reached (e.g. the limited resource has not been used up) and there is a slack. [Note: Slack is the difference between what is available and what is “used”.]

You should know by now that all the information contained in the *Answer Report* can also be found in the Solver model itself. Once you become acquainted with the Solver model, you will no longer need to generate the *Answer Report*.

II. Sensitivity Report

It is important that you know how to interpret the *Sensitivity Report* because it provides valuable information that will assist you in your decision-making process. The *Adjustable Cells* section of the report helps you determine when you should or should not change your actions and the *Constraints* section helps you determine when you should or should not change the environment (i.e. constraints). The following is the sensitivity report of the Flair Furniture Company example:

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$6	Number of units produced Tables	30	0	7	3	0.333333333
\$D\$6	Number of units produced Chairs	40	0	5	0.25	1.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$11	Carpentry hours Used	240	1.5	240	20	40
\$E\$12	Painting hours Used	100	0.5	100	20	10
\$E\$13	Chair limits Used	40	0	60	1E+30	20

- a. **Adjustable Cells:** This section helps you determine how your actions (i.e. strategies) will change when the objective coefficients change. Keep in mind that the objective coefficients are the non-action variables in your objective. In the example, the objective (i.e. maximizing profit) is determined by the numbers of tables and chairs produced (i.e. action items) and the profits per table and chair (i.e. non-action items). In this case, the profits per table and per chair represent the objective coefficients.

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Currently, the company's profit per table is \$7. Suppose the profit per table has increased to \$8 but the profit per chair remains at \$5. What should the company do? Just because the profit per table has increased (and the profit per chair has stayed the same), that does not mean that the company should shift its resources to manufacture more tables and less chairs. You need to pay attention to the numbers reported in the columns *Allowable Increase* and *Allowable Decrease*. These two columns will help you determine the range where the objective coefficient can fluctuate without affecting your original recommended action.

In the example, we know the initial recommendation for the number of tables to produce is 30 (given a profit per table of \$7). The sensitivity report indicates that the allowable increase and decrease are 3 and 0.33, respectively, for the objective coefficient associated with table (i.e. profit per table). This means that as long as the profit per table remains in the range of \$6.67 ($=7-0.33$) and \$10 ($=7+3$), the company would still maximize its profit by manufacturing 30 tables. Any deviation from this recommendation will mean that the company is no longer maximizing its profit. [Note: You can verify this by changing the profit per table from \$7 to \$8 and rerun the Solver model. You will notice that your actions (i.e. the solutions) remain the same.]

When should the company deviate from this recommendation? The company will have to change the number of tables to produce if the profit per table either falls below \$6.67 or rise above \$10 (assuming that the profit per chair remains unchanged). When this situation arises, you will have to rerun Solver to determine the new recommended actions.

You should also be aware of the significance of the *Reduced Cost* column when one of your action items is 0 (which means that you are recommended not to take any action for that particular item). The numbers reported in this column show the impacts on the objective if you deviate from the no-action recommendation. In the example, the Solver model recommends the manufacture of both tables and chairs (i.e. the numbers listed under the *Final Value* column are not zeros), so the numbers reported in the *Reduced Cost* are zeros. Suppose the Solver model recommends that the company not produces any chair (i.e. Final Value of 0) and a reduced cost of -5. This means that if the company insists on manufacturing chairs, its profit will drop by \$5 for every chair it produces. [Note: A positive reduced cost indicates how the objective will increase and a negative reduced cost indicates how the objective will decrease for every unit increase in the associated action.]

- b. **Constraints:** This section tells you how your objective will be affected when you change the constraints. The numbers reported under the *Shadow Price* column represent the impacts on the objective when the constraints fluctuate within given ranges.

In the example, the shadow price for the carpentry hours is 1.5. This means that for every additional carpentry hour made available, it will increase the company's profit by \$1.50; and for every additional carpentry hour taken away, it will decrease the company's profit by \$1.50. However, please be aware that the impact indicated by the shadow price is

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relevant only if the constraint fluctuates within a certain range. In this particular example, the current constraint on carpentry hours is 240 hours with an allowable increase of 20 and an allowable decrease of 40. This means that the shadow price (i.e. impact) of 1.5 is relevant only if the number of carpentry hours fluctuate between 200 ($=240-40$) and 260 ($=240+20$) hours. The shadow price of 1.5 is no longer relevant if the company changes the carpentry hours available below 200 hours or above 260 hours.

What are the general rules for interpreting the shadow price?

- A more (less) restrictive constraint makes the objective worse (better)
- If the objective is to maximize (minimize) then worse means decrease (increase) and better means increase (decrease)

By the way, you will only see a shadow price for a constraint when the limit for that constraint has been reached (i.e. no slack or binding for that constraint in the *Answer Report*). In the example, the shadow price on the chair production constraint is 0 and the impact is relevant as long as the limit on chair production is between 40 ($=60-20$) and infinity ($=60+1E+30$). This makes sense because there is a slack associated with the chair production constraint. The company's current recommended chair production of 40 has not met its limit of 60 yet. If the company changes the limit to somewhere between 40 and infinity, it will have no impact on its profit because the recommended chair production is still below that limit. It will only affect its profit if it changes the limit on chair production to below 40 (i.e. the lower end of the range). In that case, we will need to rerun the Solver model to determine the new shadow price. [Note: In this example, we are not concerned with the upper end of the range because you cannot go higher than infinity.]

How can the shadow prices help the company make better decisions? Suppose the company plans on hiring a part-time employee (who can do both carpentry and painting) for 10 hours of work. Where should the company place the part-time employee? Given the carpentry hours has a shadow price of 1.5 and the painting hours has a shadow price of 0.5, the part-time employee should be assigned carpentry work because that will have a bigger impact on the company's profit. On the other hand, if the company plans on cutting 8 hours of work time, it should reduce the painting hours because doing so has a smaller impact on its profit.