# Dynamic Analysis of Bungee Jumping

A major qualifying project

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By


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# Abstract

The recent accident occurred in bungee jumping off the bridge at Victoria Falls, Zambia in the 2012 New Year's Eve has brought the safety issues of this popular extreme sport again to the public concern. This project is to simulate the dynamic process when a jumper dives off from a height and to determine some safety-related quantities such as the maximum diving distance and the maximum tension developed in the jumping cord. A single-degree-freedom mass-spring-damper model is adopted where the mass represents the jumper, the cord acts as a spring, and the damper simulates the air resistance. The motion of the jumper alternates between free fall and vibration depending on if the cord reaches its full length. Various spring and damping models were employed for a general discussion. Other health problems induced by bungee jumping were also considered and some precautions were provided to guarantee the jumpers' overall safety. The project may provide the industry with fundamental understanding of the bungee jumping process and a MATLAB based numerical tool to simulate the process on site. Important physical quantities such as the maximum tension in the cord can thus be determined to address some safety related issues.

# Executive Summary

As more and more people seek for excitement out of their routine lives, extreme sports have become a popular activity. Among which, bungee jumping is obviously one of the most famous and the most thrilling one. Since the modern promotion of the sport at 1980s, millions of people world-wide have taken the plunge into this extreme sport. Many safety standards and strict guidelines have set up to protect the public from casualties as much as possible. Unfortunately, even with all of these advances in this direction, bungee jumping accidents still occur. One of the most infamous deaths from bungee jumping occurred during the practice for the half-time show of Superbowl XXXI, in 1997. Laura Patterson died upon impact when she jumped from the top level of the Louisiana Superdome. Another accident happened in 2012 Victoria Falls, Zimbabwe; a lady named Erin Langworthy fell off from the 111-meter high bridge and plummeted in to the river full of crocodiles because of a snapped cord, though she luckily survived floating to the bank, the accident drew attention from all over the globe, and also brought the safety issues of this extreme sport to public concern. Except for direct collisions and fallings happened during jumping, other types of bungee jumping accidents can occur if strict safety guidelines are not followed.

The goal of this project was to simulate the physical process of bungee jumping using the knowledge of dynamics vibration. The problem can be investigated deeper by taking consideration of wind and temperature influence, the difference caused by air resistance coefficient and the difference induced from various jumping styles. Since the topic includes an overall safety analysis of the sport, besides building up the basic model for bungee jumping, the team also researched on how to reduce the happening risk of the eye trauma, back injury and other possible sequels resulting from bungee. Then when both the temporary safety and the subsequent health condition of the jumpers are carefully analyzed, the safety of this extreme sport is guaranteed.

The project can be divided into three parts; (1) Develop a dynamic model of bungee jumping; (2) Study environmental and peripheral influences on the dynamic system; (3) Investigate the overall safety of the jumpers (including their health condition after jumping). The first part is the foundation of the project: the resulting model is constituted mainly by two parts, a free-falling period when the cord is loose and a vibration phase when the cord is stretched. Moreover, a single degree of freedom mass-spring-damping system is used for the vibration period, where mass represents the jumper, spring for the cord and damping for the air resistance. In such case, the displacement and velocity of a preceding second will be used as the initial conditions for the following moment to exam its current status and apply the correct phase. However, far from being accurate, the model needs adjustment by calibrating the coefficients and refining the methodology. For example, the team first plugged in a default number from historical data as the stiffness of the cord. Later, for further accuracy, varies coefficients of stiffness and models of damping will be tested. This speculation turned out to be a good way to approach the real condition. Similar investigations were also made concerning air damping coefficient and transverse effect of the wind. MATLAB codes were written to simulate the dynamic process. Last but not the least, the health of the jumpers afterward was also taken considered of. Scientific journals and researches were read to find the influence of rope tied up at different area of human body. This can be determined by examining the jumping style and the way in which ropes were tied to the jumpers.

The result of the project will provide some preliminary advises and guidelines to bungee commercial organizations for reference. Since some bungee clubs were consulted on their equipment used in jumping, they reflected that the ropes and cords used for the clubs were usually homemade and therefore specially designed for each of their individual site. The ideal result of the project can be revised according to different jumping site and environment, plugging in the parameters that are compatible with the condition and using the appropriate cord equipment. The solutions and preventions for health problems afterward are also provided in the report for information. Note that different individuals at diverse age or shape have diverse health condition; medical consultations before jumping were highly suggested to fully ensure the health of the jumpers.

Moreover, there are still some other technical issues that need to be modified in bungee jumping. For example, the stiffness degradation of the cord should be considered as a severe problem. Since worn cord is extremely easy to induce snapped cord and therefore result in some fatal tragedies. However, the degradation cannot be measured unless strong evidence was collected over years, which requires of a long span of experiment period. Likewise, various body shapes might result in different effects of air resistance, while the topic of air resistance on irregular objects itself worth setting up another project.

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# Chapter 1: Introduction

## 1.1 Motivation

Bungee jumping is an extreme sport that's originated from a Pacific Island. Modern bungee started on the 1st of April (Fools' Day) 1979 when group of people from the Oxford University Sport Club, jumped from the 245-Clifton Suspension Bridge in Bristol, England. Up to this present time, bungee has become one of most popular and fashionable extreme sports. However, accidents accompanied with such sport have never die out. One famous example happened at the Victoria Falls in Zambia: An Australian woman plummeted into the crocodile-infested waters of the Zambezi River when the cord snapped. Fortunately, Ms. Langworthy swam to the Zimbabwe bank, surviving from the 111-meter fall and she only suffered from bruises and a fractured collar bone. Nevertheless, Zambia's tourism minister afraid that this accident would severely affect the tourist industry, therefore, his jumped from Victoria Falls himself to reassure visitors and added: "I myself will be engaging the operator on how we can make this exciting tourism event become totally incident-free." Yet, to assure the safety of this extreme-sport, we have to investigate using the scientific method. This method is closely related with our professional course of mechanical vibration. The approach examined by this project is to set up an initial value problem with given initial conditions, to calculate the limit for important parameters.

## 1.2 Project Statement

The purpose of this project is to use scientific method, to deliberately simulate the process of a real-world bungee practice. MATLAB is the major tool used in the simulation. Those important parameters, generally based on realistic cases, were controlled in the simulation for better running of the program. Since the eventual goal is to ensure the safety of the popular extreme sport, the numerical results that we were looking for are limits or maximum values for the series of parameters.

## 1.3 Report Organization

The Second Chapter-Background of this report will illustrate the origin, the popularity and some other basic information concerning bungee jumping, and therefore discuss the casualties caused by the sports. This section will provide information supporting the necessity of our project to prevent more people from getting injured.

Chapter 3 of Problem Statement will demonstrate details of a real-world bungee problem, including the formulation of basic model, the comparison for different stiffness coefficients and the comparison for various damping models. The analysis of acceleration along with surrounding health concerns is also covered.

Chapter 4 Results and Conclusions will explain how we could guarantee the safety of the sport upon knowing the numerical limits we got from section 3. We should also spread the analysis from our researching case to a general degree.

# Chapter 2: Background

## 2.1 The History of Bungee Jumping

Before bungee jumping came into being as a popular modern sport, it served as a religious ritual for a few hundred years. Examining the history of bungee jumping takes us to a small island in the South Pacific named Pentecost Island, one of 83 islands that make up the country of Vanuatu. During the history of bungee jumping, many Christian missionaries have attempted to change the culture of the inhabitants of Pentecost Island. However, even though most of the inhabitants of the island profess Christianity today, their ancient culture and rituals remain strong. Bungee jumping is one such ritual. The history of bungee jumping goes back to ancient times and beliefs about pleasing the gods in order to get good crops. The yam harvest is the principle event around which the naghol, the ancient predecessor of bungee jumping, takes place. The natives believed that if your jump was acceptable that the gods would grant you a good harvest. It is also a ceremony which marks the rite of passage from a boy's youth to manhood. They believed that the males who jump (it was only males, by the way), should not have sex the evening before their jump, and should wear no 'good luck charms'. Either of these was said to produce a bad jump. On an island without a hospital of any kind, any injury can become life threatening. Indeed, the history of bungee jumping has some very strange roots.

While examining the history of bungee jumping, it is intriguing to see how these ancient people practice this religious ceremony. Prior to the jump day, a wooden tower is built that is some seventy feet in height. Latched together with vines and no modern construction methods, it appears to be far from stable. To reduce the swaying of the tower from the wind, vines are used like guy wires. Groups of 20 or so men participate in the land diving ceremony. As the young men would jump, their mothers would toss an object from their childhood to the earth, symbolizing their transition from a child to a man.

Modern bungee jumping started with a jump from a suspension bridge in Bristol, England, on April 1, 1979. Obviously, it was no April fool's joke. From a height of 250 ft., four friends from the 'Dangerous Sports Club' leapt into the history books by taking the historic plunge. They were promptly arrested by the authorities shortly after completion of their activities. However, this did not stop their adventure. They moved their jumping activities to the United States, where they jumped from the Golden Gate Bridge in San Francisco, California. They also managed to secure sponsorship from the American television show 'That's Incredible' for their jump from the Royal Gorge Bridge in Colorado. Their perseverance helped bring modern bungee jumping to the spotlight of the media, and the masses.

## 2.2 The Popularity of Modern Bungee Jumping

Bungee jumping takes place around the world today, from Australia to the United States, to many countries in Europe. Though twenty-five years ago no one had heard of bungee jumping; today, it is everywhere. It is well respected as a dangerous and extreme sport. Despite its obvious danger, millions of people have successfully completed jumps since the onset of modern bungee jumping. Indeed, tens of thousands of bungee jumpers take the plunge each year, in an ever-increasing variety of ways and places.

Bungee jumping spread rapidly throughout the world, ever-growing in popularity. The idea of using a bungee cord was first spread through New Zealand. From here, Australia and France soon caught on and joined the sports. This new hobby, predominantly known and practiced only by skydivers, rock climbers, and other extremists caught the world's attention and spread like a wild fire. In America, bungee jumping was also proving popular. The first commercial bungee business began at San Diego, CA. Commercial sites in Colorado and Utah soon began to pop up. Expansions of these businesses now help to cover almost every western state. Bungee Jumping's popularity has helped it to become an officially recognized sport. National Freestyle Bungee Championships are held each year and are broadcasted on a number of channels including “ESPN 2".

Except places with long bungee history like USA, UK, Europe (especially Switzerland and Norway) and New Zealand, there are also lots of burgeoning sites around the globe, such as Nepal, Iceland, Zimbabwe, Chile and South Africa. In China, Macao Tower offers the highest bungee jumping in the world. There, jumpers can have the opportunity to throw themselves off of the Macau Tower, a height which is approximately 764 feet (233m). This is an experience which people will most likely never forget.

## 2.3 Locations and Types of Bungee Jumping

Probably the most common place to bungee jump from is the crane. In this scenario, a crane is used with a cage on the end. The cage is lowered to the ground, and the jumper is prepared, rigged up, and attached to the bungee cord. The bungee cord is then attached to the cage. The jumper is then raised up to the jump height using the crane. Often times the jump master will hold the coiled cords to prevent them from tangling up during the jump. Once the cage reaches the jump height, the jumper would jump from the cage, bouncing at the end of the jump. Upon completion of the jump, once the jumper is no longer bouncing, the cage is slowly lowered to the ground. The ground crew is prepared to catch the jumper as he is lowered and safely remove him from his harness. Once the jumper has cleared the platform, the bungee cords are recoiled and everything is inspected and prepared for the next jump participant.

Another prime location for a bungee jump is from a bridge. Many successful jumps have been done from a variety of bridge types as well. When jumping from a bridge, the jump team usually assembles a platform for the jumper to jump from. The jumper is then harnessed up, and attached to the bungee cords. The cords are then anchored to the bridge. Once everything is ready, the jumper jumps off the platform. A variation of this allows the jumper to jump from the rail of the bridge as well. The jumper will bounce around 2-4 times before coming to a stop. At this point the jump crew will usually lower down a secondary static line. When the jumper clips this line to their harness, it is then used to pull the jumper back onto the bridge. This is the safest recovery method used to get the jumper from the bottom jump position. Another method that is sometimes used is to pull the bungee cord up enough to un-hook it from the bridge, and then use the cord to lower the jumper to the ground.

There are six major styles of bungee jumping. Here we list them as following:

1. Jump backward, tie on the waist.

   A common style for novice, jumping backward from a platform, with the cord tied on his or her waist.

2. Jump forward, tie on the waist.

   Another basic move for novice, which resembles the first one, however, with the jumpers facing the ground, they could truly engage a visual thrilling.

3. Jump towards water, tied on the ankles.

   Coolest jumping style, with the cord firmly tied on the jumpers' ankles. Jumpers would face down to the ground, like diving athletes. When an assistant counting down to 0, the jumper would open his arms and dive sharply to the water.

4. Jump backward, tied on the ankles

   One of the most difficult styles, also tied on the ankles, the jumpers leap backward with their arms open.

5. Jump with tied chest.

   This is known having the closet feeling of death. The jumpers have their cord tied on the back, jump directly.

6. Duo Jump

   Cords would tie the couple tightly together while they bounce up and down in the air. This requires that one of then must be experienced, so that the jump could be both sweet and thrilling.


Figure 1: The numbered pictures show the corresponding jumping style.

## 2.4 Risks and Accidents in Bungee Jumping

Physicists regard bungee jumping as a dramatic demonstration of the conservation of energy. They know that gravitational potential energy at the top of the jump is converted to elastic potential energy at the bottom. The basic equations involved have been used for years to describe events in which loads are suddenly applied to springs. The bungee cord is simply a very weak spring yielding large spring deflections and rather small force magnitudes.

The accidents in Bungee usually caused fatal tragedy for the jumper. We can classify them into three major types: “accidents caused by errors made by clubs”, “post-bungee medical concerns” and “inappropriate way of jumping”.  The most serious are usually induced from bad preparation of bungee, which should be the errors made by clubs.

Accidents happened from such reasons are usually fatal. For example, a VA fast food worker assembles a cord nearly 70 feet and jumps from 70 foot railroad trestle. He did not take into account that a rope stretches, and therefore the tragedy happened. Bad attachment or bad facility quality can also be a potential danger.

Post-bungee medical concerns include those who have been temporarily blinded after the jump. One of the most dangerous types of eye trauma associated with this sport is retinal hemorrhage; this presents a very real possibility of losing one's eyesight.

Bungee jumping also presents the possibility of orbital emphysema, which can also result in permanent loss of vision. On a slightly-lesser scale, but still noteworthy, are basic injuries to the eyes and their surrounding tissues. It might also produce joint problems. These medical problems are usually not taken good care of.

For those novice jumpers, a bad jumping method is also dangerous. Though it may not be fatal, the harm on the body is inevitable.

## 2.5 Single Degree of Freedom Vibration System

In our analysis, we divided the basic model into two phases, one of which is vibration that can be solved by employing single degree of freedom mass-spring-damper system. In this section, the background of single degree of freedom system is introduced. The simplest vibratory system can be described by a single mass connected to a spring. The mass is allowed to travel only along the spring elongation direction. Such systems are shown in the following figure,

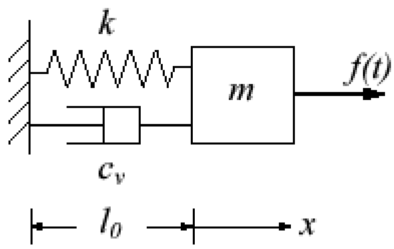


Figure 2: Single Degree of Freedom System

The general mathematical representation of a single degree of freedom system is expressed using Newton's second law. The analysis can be visualized with the following free body diagram.

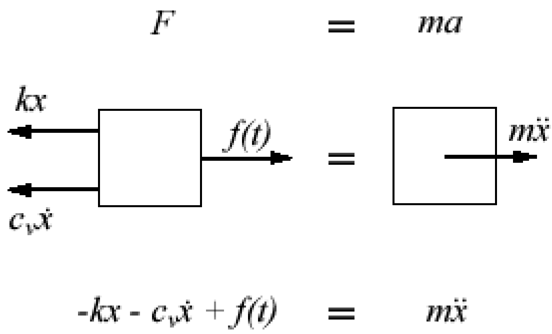


Figure 3: Free Body Diagram

The resulting equation of motion is a second order, non-homogeneous, ordinary differential equation:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \tag{2.1}$$

with the initial conditions,

$$x(t = 0) = x_0 \qquad \dot{x}(t = 0) = v_0$$

Since in our research, the external force, which is the weight of the jumper, is constant, we will only include the solution for SDOF under constant force. Letting $f(t) = f_0$, we have

$$m\ddot{x} + c\dot{x} + kx = f_0 \tag{2.2}$$

General solution is

$$\mathrm{x(t) = x_h(t) + x_p(t)} \tag{2.3}$$

where $\mathrm{x_h(t)}$ is the homogenous solution, and the particular solution, $\mathrm{x_p(t)}$, is a constant,

$$x_p(t) = x_0 \qquad \text{for all } t\,.$$

Plugging this constant solution into the differential equation, we get

$$m \cdot 0 + c \cdot 0 + kx_0 = f_0 \tag{2.4}$$

Hence,

$$x_0 = \frac{f_0}{k} \tag{2.5}$$

If the system is undamped, then

$$x(t) = x_h(t) + x_0 = C_1 \cos(\omega_0 t) + C_2 \sin(\omega_0 t) + x_0 \tag{2.6}$$

which tells us that the object is oscillating about $x = x_0$ . On the other hand, if the system is damped, then

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty}[x_h(t) + x_0] = 0 + x_0 \tag{2.7}$$

In this case, $x = x_0$ is where the object finally ends up. Either way, the effect of this constant force is to change the object's equilibrium point from $x = 0$ to $x = x_0$ . Accordingly, if *L* is the natural length of the spring, then we call $L + x_0$ the *equilibrium length* of the spring in this mass/spring system under the constant force $f_0$.

## 2.6 Runge-Kutta 4th Order Method

The method we took for the analysis is based on a step by step mode, instead of considering the bungee process as a whole. Moreover, our system is a non-linear system, where the ordinary differential equations need to be estimated using a low-error numerical method. Runge-kutta 4th order method has the advantage of easier computer simulation and tends to be accurate. This section introduces some background information of this method.

Consider the initial value problem:

$$\dot{y} = f(t, y), \ y(t_0) = y_0$$

Here, y is an unknown function of time t which we would like to approximate. $\dot{y}$ is a function of t and of y itself. At the initial time $t_0$ the corresponding y-value is $y_0$. The function f and the data $t_0$, $y_0$ are given.

One of the popular solutions used for the Runge-Kutta 4th Order Method is shown below:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h \text{ for } k = 1, 2, 3, \dots, n-1 \quad (2.8)$$

Where,

$$k_1 = hf(t_i, y_i) \quad (2.9)$$

$$k_2 = hf(t_i + \frac{h}{2}, y_i + \frac{k_1}{2}) \quad (2.10)$$

$$k_3 = hf(t_i + \frac{h}{2}, y_i + \frac{k_2}{2}) \quad (2.11)$$

$$k_4 = hf(t_i + h, y_i + k_3) \quad (2.12)$$

This is an approximate solution to the differential equation using the discrete set of points$\{(t_k, y_k)\}_{k=0}^{n}$.

# Chapter 3: Methodology

## 3.1 Development of the Dynamic Model

Because of the limitation of the project, we made some assumptions prior to our research to make the process easier. (1) Since human body is generally small compared with whole bungee environment, it is considered a point of mass in the basic model. (2) For easier calculation, the weight of the cord is ignored. (3)Because our analysis primarily deals with longitude motion, the transverse movement of the jumper is not considered.

Based on the above assumptions, the basic model can be divided into two phases by checking whether the cord have reached its original length.

Phase I: Free Fall

Before the rope reaches its original length, the jumper is a free-falling body.

$$m\ddot{x}(t) = mg - c\dot{x}(t) \tag{3.1}$$

$$x(0) = 0, \dot{x}(0) = 0$$

Phase II: When the rope exceeds its original length and it keeps stretching, the system is now vibration and can be considered as SDOF Mass-Spring-Damper System. It can be expressed by the following initial value problem (IVP):

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = W \tag{3.2}$$

Initial conditions (ICs):

$$x(0) = TBD, \dot{x}(0) = TBD$$

Where,

m … … mass of the jumper

c … … damping coefficient

$k \ldots \ldots$ stiffness coefficient

$W \ldots \ldots$ weight of the jumper

$x \ldots \ldots$ displacement of the human body

Later, the movement of jumper will keep switching between the two statuses, so determining which phase of movement it's at is a key issue in the analysis.

On the foundation of this two-phase method, the dynamic system can be simulated in MATLAB program. Since the problem is based on time derivative of distance traveled, the time eclipsed is the controlling factor and the distance is therefore the unknown variable. Since before the exact distance is examined, the part where it lies in have to be investigated. Therefore, the program should be run step by step with very small time interval. The distance and the velocity resulted in the previous step will be used as the initial value for the subsequent step. With the method provided, some pairs of data used at some designated heights are tested, and the graph resulted runs in a reasonable way. The MATLAB code is shown in the reference.

From preliminary research, the team decided to use the following data to test the basic model: L=20 m, K=40N/m, M=80kg, $g = 9.8\mathrm{m/s}^2$. However, a required factor, C, damping coefficient was not available from historical data; the team could not find it from online journals and essays either. Therefore, the method of terminal velocity was applied to determine damping coefficient.

Assume k equals to 0, the team could thus temporarily got rid of its influence. The free-falling body is only affected by damping:

$$m\ddot{x} + c\dot{x} = mg \qquad (3.3)$$

upon reaching the terminal velocity, acceleration $\ddot{x}$ became and $c = \frac{mg}{\dot{x}}$. Technical journals showed that the terminal velocity for a human body is about 120 mph (53.6m/s). Therefore, $c = \frac{80kg \times 9.8m/s^2}{53.6\ m/s} \approx 15kg/s$ , and the team would assume the damping coefficient to be 15kg/s for basic analysis.

Following diagram shows how the terminal velocity method was performed and how the damping coefficient was found.

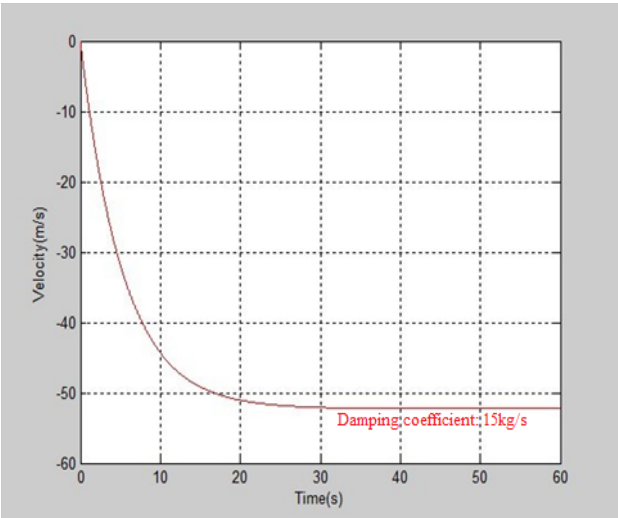


Figure 4: Terminal Velocity for Basic Model

Having all the required coefficients know for the model, the team could calculate the maximum displacement, maximum stress and maximum acceleration, which are shown in the follow graphs. Knowing these figures would help us better determine the jumping situation and draw an appropriate conclusion.

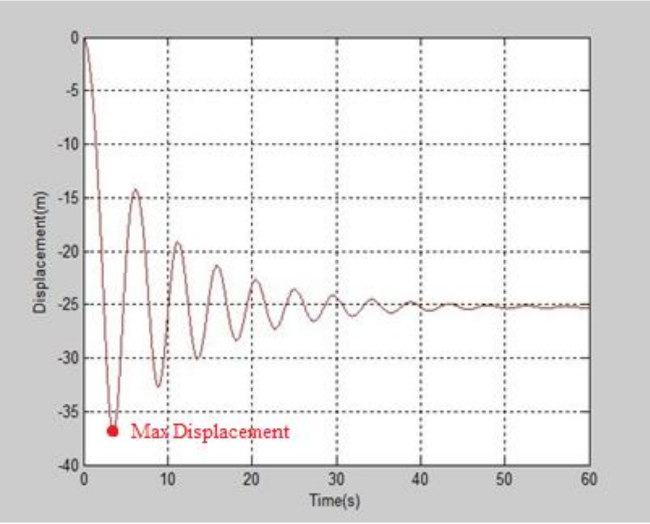


Figure 5: Time vs. Displacement for Basic Model

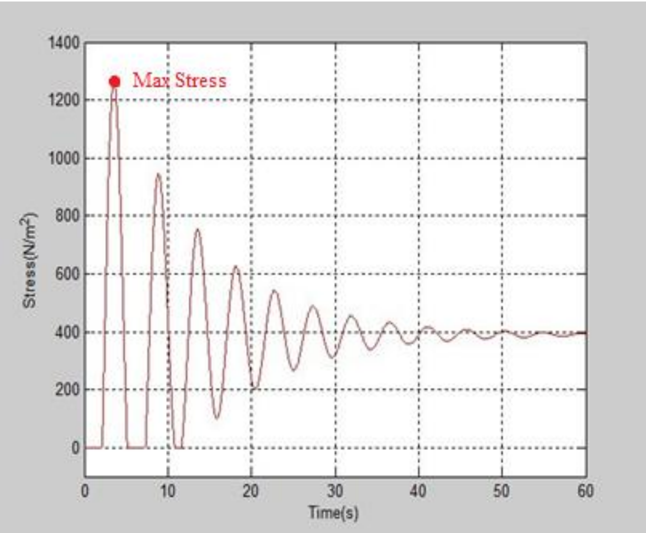


Figure 6: Time vs. Stress for Basic Model

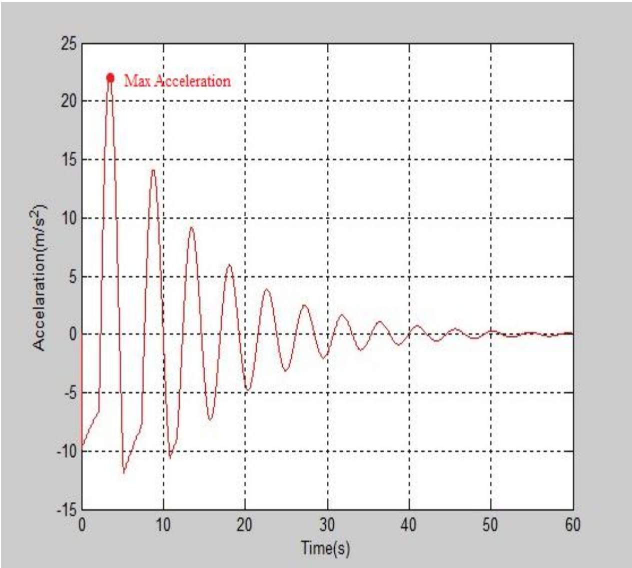


Figure 7: Time vs. Acceleration for Basic Model

## 3.2 Comparison of Various Coefficients of Stiffness

In the basic model, we used 150N/m stiffness for a given rope. However, the stiffness in real case, does not keep constant while stretching. It changes with the elongation of the cord. The next step of the team focuses on the function of how the stiffness coefficient changes with elongation.

Based on the research of following commercial specifications, the team assumes a linear function of stiffness coefficient. In this case, the team picked up the data of Medium Cord for calculation and the red curve in the middle represents the trend of tension with elongation of Median Cord. We completed a linear progression based on the two known stiffness values:

$$k = -5.57(x - L) + 204 \tag{3.4}$$

Table 1: Results from Denver CO.

| Results of calculations for three different jumps | | | | | |
|---|---|---|---|---|---|
| Jumper weight (N) | Cord | δ (m) | $F_{max}$ (N) | g | Jump height (m) |
| 1112 | Stiff | 17.9 | 3187 | 2.87 | 28.7 |
| 800 | Medium | 16.7 | 2311 | 2.99 | 27.5 |
| 490 | Soft | 13.8 | 1478 | 3.02 | 24.6 |

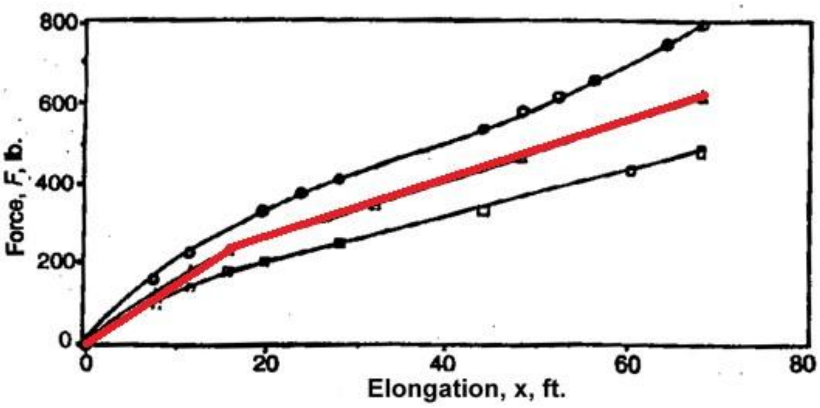


Figure 8: Force vs. Elongation for Linear Coefficient

The following plot shows the displacement and velocity for linear coefficient of stiffness.

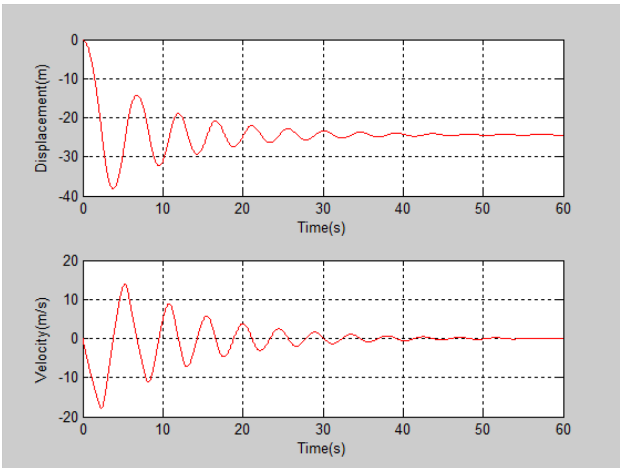


Figure 9: Displacement and Velocity of linear stiffness

Figure 8 also shows a clear turning point on the curve where the slope changes immediate. The team therefore modified it to a piecewise constant stiffness coefficient for comparison and easier analysis.

$$\text{If } (x-L) \leq 4.88, k = 204\text{; else, k=111.} \tag{3.5}$$

Table 2: Cord Stiffness

| Analysis of three bungee cords | | | |
|---|---|---|---|
| Cord | $K_1$ (N/m) | $x_1$ (m) | $K_2$ (N/m) |
| Stiff | 255 | 4.88 | 149 |
| Medium | 204 | 4.88 | 111 |
| Soft | 162 | 4.88 | 77 |
| *Cord stiffness data supplied by SkyTower Engineering, Denver CO* | | | |

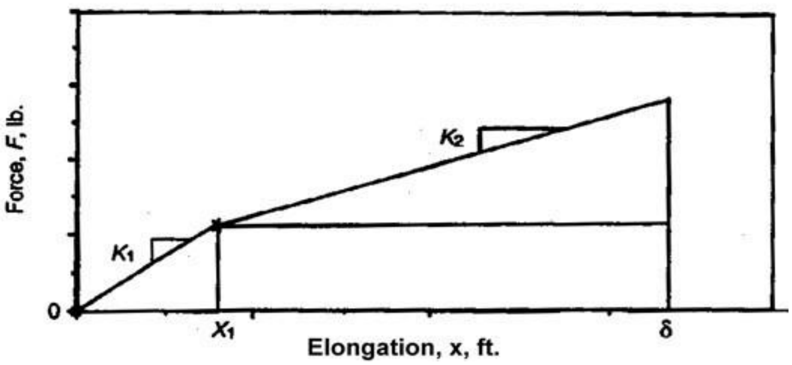

Figure 10: Force vs. Elongation for Piecewise Coefficient

The following plot shows the displacement and velocity for piecewise constant stiffness coefficient.

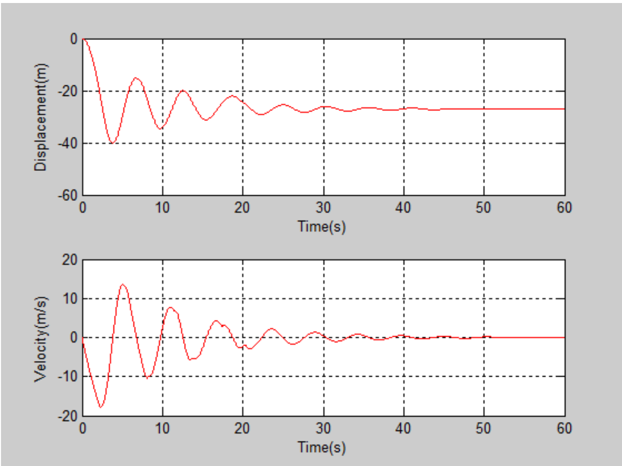

Figure 11: Displacement and Velocity for Piecewise Constant Stiffness

Though the two diagrams varied from each other on the maximum displacement, the difference was not large enough to create great error. Following diagram shows the displacement comparison for three different stiffness coefficients. Although they end up with slightly different final position, their tracks or moving tendencies largely resemble to each other.

## 3.3 Comparison of Various Models of Air Resistance

Another important factor in the equation is the air resistance. The team used a linear damping model of air resistance for the basic model, and the result is sound and reasonable. However, the quadratic model of air resistance is equally frequently used in similar investigations and studies. To manifest which model fits this project the best, the team worked on higher powers of velocity combining with a default coefficient. Generally, the damping model can be represented as

$$c_i \dot{x}^i$$

When i=1……basic model

i=2……quadratic model

i=3……cubic model

To find the coefficient, the team researched on historical data and finally decided on applying again the terminal velocity method for reliability. Since $m\ddot{x} + c_i \dot{x}^i = mg$ , where $c_i$, represents the damping coefficient in general. For quadratic model, upon reaching the terminal velocity of human body, $\dot{x} = 120mph(53.6m/s)$ , $\ddot{x} = 0$ , and $c_2 = \frac{mg}{x^2} = \frac{80\text{kg} \times 9.8\text{m/s}^2}{53.6\text{ m/s}}$ . The following graph shows how terminal velocity and p value is found.

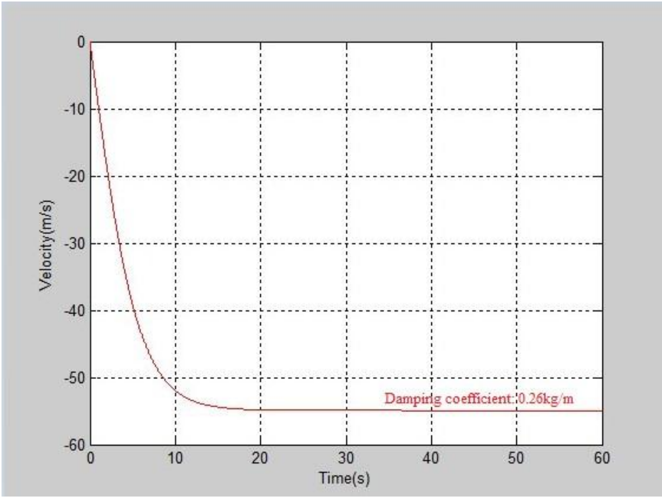

Figure 12: Terminal Velocity for Square Model

After calculation, the team found the $c_2$ value to be 0.26, and based on this $c_2$ value, the complete system equation became $80\ddot{x} + 0.26\dot{x}^2 + 150x = 80 \times 9.8$ . The following diagram represents how the system behaves in the new model of air resistance.

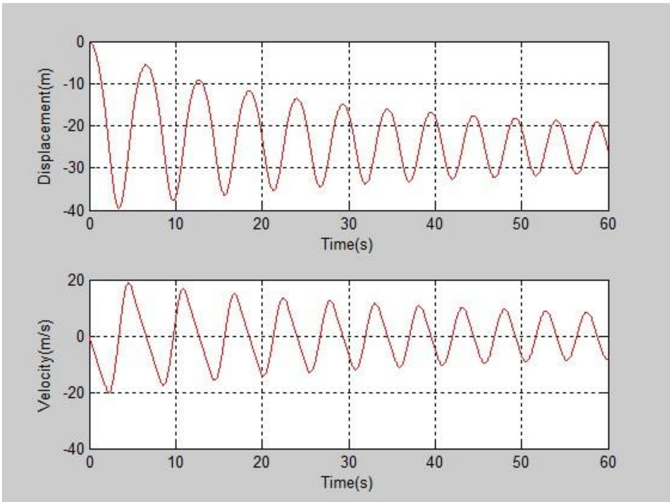

Figure 13: The Displacement and Velocity of Quadratic Model Damping

From the diagram, the team found that the frequency of motion seems reasonable; however, the movement of the object did not show any tendency of degradation, and the vibration seems to be going on forever. This conclusion is obviously absurd and thus the model needs further investigation for its validity.

With the experience of the quadratic model of air resistance, the team decided to model another relationship with velocity cube for reference. Same method of terminal velocity was applied to find the coefficient $c_3$ in equation $m\ddot{x} + c_3x^3 = mg$. Using the same terminal velocity method,

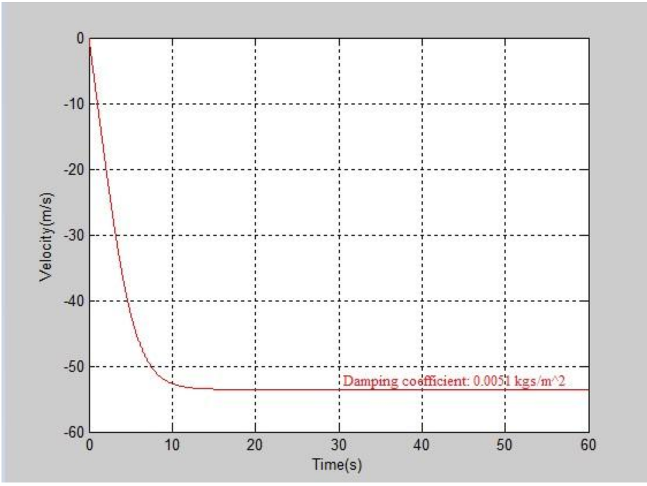


Figure 14: Terminal Velocity for Cubic Model

, the team found $c_3 = \frac{mg}{x^3}$ and is equal to 0.0051. Therefore, the complete equation for the third power air resistance model is $80\ddot{x} + 0.0051\dot{x}^3 + 150x = 80 \times 9.8$, and the following graph shows its movement track.

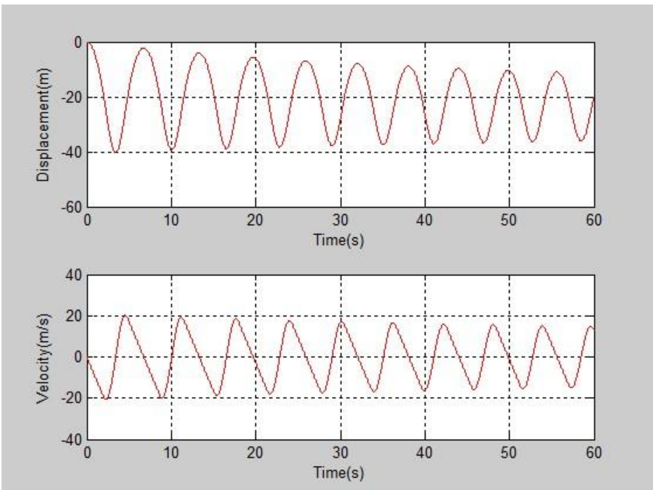


Figure 15: The Displacement and Velocity for Cubic Model Damping

Even though no obvious difference is found against the quadratic model, the degradation in displacement and velocity is even slower under this model. From the velocity graph, it can be seen that it remains more than 10 m/s after vibrating for 2 minutes in air, which is absolutely incompatible with real world cases.

The above models of air resistance, though frequently used in other scientific researches, appear to be out of track in this case. However, the original model of $cv$ represented air resistance showed the system became tranquil in 30 seconds, and the speed of the object follows a clear rate of degradation. Therefore, the team concluded that the linear model of air resistance in this bungee analysis is the only one fittest to the case.

## 3.4 Investigation on Health Condition after Jumping

Except the obvious falling danger of bungee, most people are not aware of the risks of other health problems. So far, there have been half dozen deaths and nearly as many critical injuries attributed to bungee jumping. While how well the bungee jumping equipment is maintained and overseen for its safety factors plays a large role in preventing deaths, there is a number of health risks associated with the sport itself.

Deaths and critical injuries generally occur as a result of improperly maintained equipment or miscalculations of cord length. A death from massive cranial trauma occurred in 1997, for example, from improperly handled cords. The victim was a female member of a professional bungee jumping team. However, even with the most stringent safety precautions, it is impossible to eliminate risks of injury, especially those which are actually associated with the sport of bungee jumping.

One of the most significant risks of the jumping particular to women is that of uterine prolapse. It is said that the speed and pressure of the jumping can cause the uterus to not only tip but, in some cases, slide out of its normal location and even out of the body itself. This, naturally, is very dangerous and potentially life-threatening.

Eye trauma is another very series health risk associated with this sport. One of the most dangerous types of eye trauma associated with jumping is retinal hemorrhage; this presents a significant possibility of losing one's eyesight. Another type of eye injury involved with this sport is the possibility of orbital emphysema, which can also result in permanent loss of version. On a slight-lesser scale, but still noteworthy risks of bungee jumping, are basic injuries to the eyes and their surrounding tissues.

Other health risks of bungee jumping vary in their severity. While such injuries as bruises and rope burn may be thought to be rather minor, dislocations and back injuries can range from moderate to disabling.

In general, the afterward health problems if not injured by direct collision can be listed as the following categories.

- Uterine Prolapse (Women Only)
- Eye Trauma
- Dislocations and Back Injuries

Uterine prolapse is a condition in which a woman's uterus (womb) sags or slips out of its normal position. The uterus may slip enough that it drops partway into the vagina (birth canal), creating a lump or bulge. This is called incomplete prolapse. In a more severe case – called complete prolapse – the uterus slips so far out of place that some of the tissue drops outside of the vagina.

Uterine prolapse is easy to occur on women who did too much overload exercise, experts who experimented claimed that women that have normal uterine position tends to have the cervix shifted significantly when having a 20 kg load; the cervix would endure an obvious prolapse when having a 40 kg load. Therefore, bungee jumping is extremely easy to induce uterine prolapse.

While uterine prolapse may not be possible to prevent, there are steps that can be taken to help reduce the risk. Since the trouble is fairly common, and the risk of developing the condition increases with age, it is highly recommended that old women not to jump. To reduce risk, maintaining a heathy body weight and a good shape is extremely important. Exercise regularly would help build sound muscle structures and able to undertake the acceleration during bungee jumping.

Retinal hemorrhage, as an eye trauma, is a disorder of the eye in which bleeding occurs into the retentive tissue on the back wall of the eye. It may be occurred in case of bungee jumping because of the pressure increase and people high blood pressure tends to be vulnerable with the sickness. Even though mild retinal hemorrhages are not associated with chronic disease and will normally resorb without treatment, severe one need laser surgery to seal off damaged blood vessels in the retina. So the team suggests that people with hypertension not to try bungee jumping.

Orbital emphysema is typically a benign condition that occurs following forceful injection of air into the orbital soft tissue spaces. Rarely, the intraorbital air mass can cause central retinal artery occlusion. Because of the potential for severe visual loss, the rapid diagnosis and management of this condition are essential. Currently, there is no standard protocol for the treatment and management of severe orbital emphysema. The only suggestion is that patients with historical eye illness not to take sports with potential danger, including bungee jumping, because the patients might end up getting proptosis, diplopia or loss of vision.

The dislocations and back injuries induced by bungee jumping can be studied through physics of acceleration standings of human body. The acceleration is referred in units of g-force (gravitational force). The tolerance of G forces by human body depends on the magnitude of the g-force, the length of time for which it is applied, its direction and location of application and as well as the posture of the body.

During a vertical bungee jump when rope is tied on the feet, the jumper heads down for free fall, however, when the rope pulls him back, the acceleration at the instance could be much higher than 1 g. This posture drive blood to head and the limit is typically in the 2g to 3 g range, this jumping would cause capillaries in the eyes swell or burst under the increased blood pressure and the jumper’s vision turns red. Therefore, the team would not suggest the jumpers to jump heading down with their ankle tied, let alone the fact that it is exciting.

The human body is better at surviving g-forces that are perpendicular to the spine. In general when the acceleration is forwards, so that the g-force pushes the body backwards (colloquially known as "eyeballs in") a much higher tolerance is shown than when the acceleration is backwards, and the g-force is pushing the body forwards ("eyeballs out") since blood vessels in the retina appear more sensitive in the latter direction. Early experiments showed that untrained humans were able to tolerate 17 g eyeballs-in (compared to 12 g eyeballs-out) for several minutes without loss of consciousness or apparent long-term harm.

The team also researched on the standing limit of human skeleton. The hip joint can stand 3-4 times total human weight, knee joints can stand 5-6 times the total weight, while the Crus bones stand 700 kg tension and 300 kg distortion force. Physical science shows that the spine, along with the pelvis could carry the largest load within human limit.

Combining the previous knowledge, the team would suggest normal amateurs that like to try bungee jumping tie the cord around their back and waist, and jump facing downward to lay the body as horizontal as possible, since this posture and tying method is the best for human health and safety.

# Chapter 4: Conclusions and Discussions

## 4.1 Results and Outcomes

The project team focused on developing a comprehensive research based on the fundamental modulus. Therefore, the results and outcomes of the model play significant roles instructing us on controlling the coefficients and advising amateur jumpers.

The most important factor in the analysis is to investigate the maximum travelling distance of the jumper, because knowing the maximum displacement helps us determine the jumping height. For example, *figure 5* clearly exhibited the maximum displacement for the basic model the team set up in the previous chapter. In this situation, the height must exceed 47m much from the ground to prevent hitting the ground, since the team has to reserve a large value of difference to guarantee the safety.

Another crucial determinate quantity is the maximum stress on the cord. With the input of a certain weight into the program, the maximum stress on the cord would be shown. Usually, for a certain type of cord, the maximum standing stress would be given in the specification. *Figure 6* shows that in the our basic model, the human with 80kg of mass would induce at most 1270 $N/m^2$ stress, which we could use to see if it exceeds the maximum standing stress of the cord. If it does exceed, the team would try tougher and stiffer cord for human this heavy, and the team would gets to the conclusion of what is the maximum weight of jumper for this certain type of cord.

Likewise, the acceleration of human body was investigated as another safety related quantity. Like we specified in the study of health concern in the previous chapter, human body has a standing limit of g-force. If the acceleration in the jumping process reaches the limit, human body would have adverse reaction such as eye trauma or back injuries. Since according to our study, the minimum reacting g-force for common people is between 2-3 g for vertical jumping (horizontal jumping would have a higher limit), the team would consider any maximum acceleration under 3g to be a safe bungee jumping. As it is shown in *figure 7,* the maximum acceleration is about 2.2 g, which is considered safe with our criteria, though might be with minor congestion in the eyes.

## 4.2 Conclusions

With our analysis of the basic model, and the comparisons with some modified models, the team could draw a few conclusions for the project analysis.

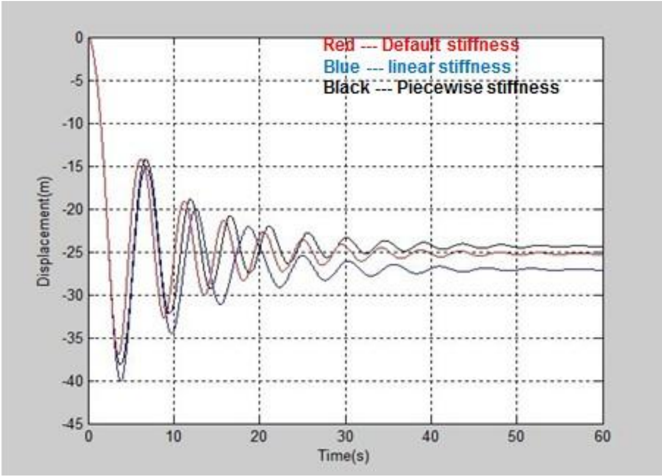

Figure 16: Displacements for Various Stiffness Coefficients

The above graph representing the displacements for various stiffness model we investigated in Chapter 3.2 shows similar trend for different models of stiffness coefficient. Whether it's default stiffness coefficient, linear form stiffness coefficient or piecewise constant stiffness coefficient, they all resembling each other in their maximum displacement and their term of degradation. Though there's a significant difference in their final position, as long as the maximum displacement is within the acceptable range, the safety of the jumper is garuanteed. However, through our investigating of the models, the modified piecewise constant coefficient of stiffness has the advantage of easier calculation and is wisely employed in the corresponding industry. Therefore, our final model would use the piecewise constant stiffness coefficient for a certain cord.

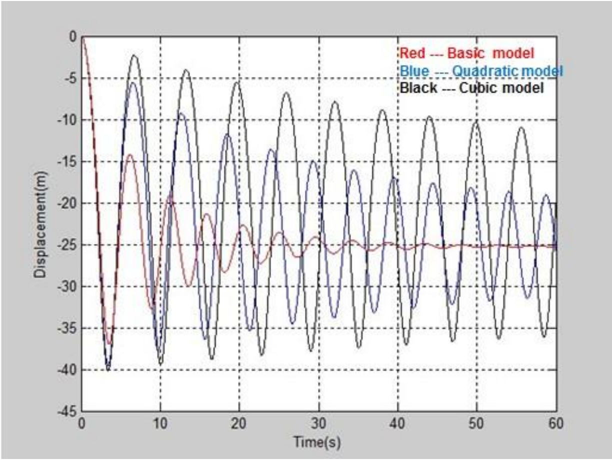

Figure 17: Displacements for Various Damping Models

The above diagram shows the track of various damping models. Obviously, beside the basic model, the square model and the cubic model both show unreasonable tendencies. According to our research on existing online videos, the number vibrating cycles would not exceed 7 from human visional perspective. The basic completely fits the condition, whereas the other two seem to be vibrating forever. The observation was analysed and we considered it is because of the following reasons.

First, the calibrated C is not accurate. We applied a method of terminal velocity of calibrate the damping coefficient c for all three models. However, this method is based on the assumption that the body reaches terminal velocity and the thus the acceleration would become 0 at that point. Yet, human body never hit terminal velocity in the simulation process. So the method needs further improvement.

Moreover, when absolute velocity is less than 1, for square model and cubic model, the damping force will be smaller and smaller due to mathematic reason. We believe that's the major factor that cause these two models vibrating forever.

Anyway, the existing problems and the past experience both demonstrated that basic model, which is the linear model of damping is the most effective for our simulated program.

In conclusion, we ended up using the basic model for damping force and the piecewise constant equation for stiffness coefficient. The modified model simulating the complete process provides maximum displacement, maximum stress and maximum acceleration for each individual condition, as long as the required information were given, the result our project could from these aspects, generally instruct preliminary bungee jumping.

## 4.3 Future Work and Discussions

Since our project is completely numerical analysis without using a prototype, our calculations and conclusions are largely based on some assumptions. Some of the assumptions, however, conflict with real world cases and therefore result in error of the result.

First of all, model automatic ignores the weight of cord to simplify the problem. However, the weight of cord is actually significant compared with human body weight, especially when the cord is extreme long, the mass of the cord might even exceed the mass of the jumper. Yet, the cord weight spread out all over length, we might only consider it at one end, where it is joined by human weight.

Another assumption we did was to set view human body as a single point of mass instead of a rigid body. If rigid body movement is considered, the problem would be more complicated as it involves force distribution and body rotation.

We also ignore the transverse motion in the process. It's not big difference for bungee site like bridges, but for sites like dams, overwhelming transverse motion might result in colliding on the wall. Since our concerns generally apply to longitudinal movement of the body, we would put this topic as a future discussion problem.

There is also something that we cannot fully investigate because of limited resources and restricted time. For example, the reason why Ms Langworthy plummeted into the river is probably due to aged or worn cord, because aged and worn cord is extremely easy to induce fracture. However, the degradation of cord cannot be measured unless strong evidence was collected over years and thousand trials for the a dozen cords were tested. The situation obviously wouldn't allow us to complete such measurement.

Above all, though our project analysis gave us a basic understanding of the process, and summarized some important factors in the jumping, there are a lot remaining for future effort. Throughout these few months' work, we developed deep interest in the extreme sport, and we are eager to try ourselves.

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# Appendix

## A1 MATLAB Codes:

```
% bungee MQP matlab program
% plot the displacement and velocity of the body through bungee process
w(1,1)=0; % displacement of the body
w(2,1)=0; % velocity of the body
ti=0; %initial time
tf=60; % final time
n=100000; % number of steps
h=(tf-ti)/n; % change of time for each step
length=20; % string length
T=0:(tf/n):tf; % time
for i=1:n;
    if w(1,i)<length; % dynamic part
        t=ti+h*(i-1); % RungeKutta method of order four
        k11=h*fun1(t,w(1,i),w(2,i));
        k12=h*fun2(t,w(1,i),w(2,i));
        k21=h*fun1(t+h/2,w(1,i)+k11/2,w(2,i)+k12/2);
        k22=h*fun2(t+h/2,w(1,i)+k11/2,w(2,i)+k12/2);
        k31=h*fun1(t+h/2,w(1,i)+k21/2,w(2,i)+k22/2);
        k32=h*fun2(t+h/2,w(1,i)+k21/2,w(2,i)+k22/2);
        k41=h*fun1(t+h,w(1,i)+k31,w(2,i)+k32);
        k42=h*fun2(t+h,w(1,i)+k31,w(2,i)+k32);
        w(1,i+1)=w(1,i)+1/6*(k11+2*k21+2*k31+k41);
        w(2,i+1)=w(2,i)+1/6*(k12+2*k22+2*k32+k42);
    else % vibration part
        t=ti+h*(i-1);
        k11=h*fun3(t,w(1,i),w(2,i));
        k12=h*fun4(t,w(1,i),w(2,i));
        k21=h*fun3(t+h/2,w(1,i)+k11/2,w(2,i)+k12/2);
        k22=h*fun4(t+h/2,w(1,i)+k11/2,w(2,i)+k12/2);
        k31=h*fun3(t+h/2,w(1,i)+k21/2,w(2,i)+k22/2);
        k32=h*fun4(t+h/2,w(1,i)+k21/2,w(2,i)+k22/2);
        k41=h*fun3(t+h,w(1,i)+k31,w(2,i)+k32);
        k42=h*fun4(t+h,w(1,i)+k31,w(2,i)+k32);
        w(1,i+1)=w(1,i)+1/6*(k11+2*k21+2*k31+k41);
        w(2,i+1)=w(2,i)+1/6*(k12+2*k22+2*k32+k42);
    end;
end;
subplot(2,1,1); plot(T,-w(1,:),'r'), grid; xlabel('Time(s)');
```

```
ylabel('Displacement(m)');hold on % plot displacement
subplot(2,1,2); plot(T,-w(2,:),'r'), grid; xlabel('Time(s)');
ylabel('Velocity(m/s)'); hold on % plot velocity

for i=1:n+1;
    if (w(1,i)-length)<=4.88
        k(i)=150;
    else
        k(i)=150;
    end;
end;

for i=1:n+1;
    if w(1,i)<=20;
        extend(i)=0;
    else
        extend(i)=w(1,i)-length;
    end;
end;
stress=times(k,extend)/area;
plot(T,stress,'r'), grid; xlabel('Time(s)'); ylabel('Stress(N/m^2)');
hold on % plot stress

function f=fun1(t,x1,x2)
f=x2;
end

function f=fun2(t,x1,x2)
m=80;
g=9.8;
c=15;
if x2>=0
    f=(m*g-c*x2^1)/m;
else
    f=(m*g-c*x2^1)/m;
end

function f=fun3(t,x1,x2)
f=x2;
end

function f=fun4(t,x1,x2)
m=80;
g=9.8;
```

```
c=15;
length=20;

if (x1-length)<=4.88
    k=150;
else
    k=150;
end

if x2>=0
    f=(m*g-c*x2^1-k*(x1-length))/m;
else
    f=(m*g-c*x2^1-k*(x1-length))/m;
end
```