# BUSS 230 Managerial Economics Fall 2013-2014 <br> Dr. George G. Namur 

## Midterm Review Problems

## Problem 1

A firm's total cost and total revenue functions are as follows:

$$
\begin{gathered}
T R=100 Q-Q^{2} \\
T C=2 Q^{3}-5 Q^{2}+36 Q+4
\end{gathered}
$$

a) Write an expression for the profit function (as a function of $Q$ )
b) Determine the optimal quantity, $Q^{*}$, that maximizes profits

## Problem 2

A firm has the following marginal cost function

$$
M C=2 Q^{2}-26 Q+80
$$

Determine the quantity of output that will minimize its marginal cost.

## Problem 3

Determine the values for X and Y that maximize the following profit function:

$$
\pi=140 X-4 X^{2}-4 X Y+2 Y^{2}-20 Y
$$

## Problem 4

Using the substitution and Lagrangian methods, maximize the profit function in Problem 3 subject to the constraint $2 X+Y=40$

## Problem 5

The demand function faced by a firm is the following:

$$
Q_{X}=100-8 P_{X}+0.5 P_{Y}+2 I
$$

Where:
$\mathrm{Qx}_{\mathrm{x}}=$ Quantity demanded of product X
$P_{X}=$ Price of product $X$
$P_{Y}=$ Price of product $Y$
$\mathrm{I}=$ Income
Using $\mathrm{P}_{\mathrm{x}}=6, \mathrm{P}_{\mathrm{Y}}=12, \mathrm{I}=4.5$
a) Calculate the point price elasticity of demand $\mathrm{E}_{\mathrm{P}}$ at $\mathrm{P}_{\mathrm{X}}=6$.
b) Is demand elastic, inelastic or unitary elastic when $P_{x}=6$ ?
c) At this point on the demand for product $X$, what is the sign of the MR?
d) How would decreasing the price of $X$ from 6 to 5 affect TR? Explain
e) Calculate the arc price elasticity of demand between $P_{x}=6$ and $P_{x}=7$.
f) Calculate the cross price elasticity for the given values. Is $Y$ a substitute or a complementary product to X ?
g) Calculate the income elasticity. Is X a normal or an inferior good?

## Problem 6

The demand curve for haircuts at Terry Bernard's Hair Design is

$$
P=15-0.15 Q
$$

Where $Q$ is the number of cuts per week and $P$ is the price of a haircut. Terry is considering raising her price above the current price of $\$ 9$. Terry is unwilling to raise price if the price hike will cause revenues to fall.
a) Should Terry raise the price of haircuts above $\$ 9$. Why or why not?
b) Suppose demand for Terry's haircuts increases to $P=22-0.22 \mathrm{Q}$. At a price of $\$ 9$, should Terry raise the price of her haircuts? Why or why not?

## Problem 7

Consider the linear demand function $Q=20-0.5 P$
a) Write an expression for the inverse demand function.
b) Write an expression for the total revenue function.
c) Write an expression for the marginal revenue function.
d) Find the quantity, $Q^{*}$, at which total revenue is maximized.
e) Is revenue maximization (in part (d)) an example of a constrained optimization problem?

## Problem 8

The Johnson Robot Company's marketing manager estimates that the demand curve for the company's robots in 2008 is:

$$
P=3000-40 Q
$$

Where $P$ is the price of a robot and $Q$ is the number sold per month.
a) Derive the marginal revenue curve for the firm.
b) At what range of prices is the demand for the firm's product price elastic? Justify your answer carefully.
c) If the firm wants to maximize its total revenue, what price should it charge?

## Problem 9

Two goods have a cross price elasticity of +1.2
a) Would you describe these goods as substitutes or complements?
b) If the price of one of the goods increases by 5 percent, what will happen to the demand for the other product, holding constant the effects of all other factors?

## Problem 10

The demand for renting motorboats in a resort town has been estimated to be $Q_{d}=5,000-50 P$, where $Q_{d}$ is the quantity of boats demanded (boat-hours) and $P$ is the average price per hour to rent a motorboat. If this relationship holds true in the future:
a) How many boat-hours will be demanded at rental price of $\$ 10, \$ 20$, and $\$ 30$ per hour?
b) What is the arc price elasticity between $\$ 10$ and $\$ 20$ ? Between $\$ 20$ and $\$ 30$ ?
c) What is the point price elasticity at $\$ 10, \$ 20$, and $\$ 30$ ?

## Problem 11

Given the following demand schedule:

| Price <br> $\mathbf{P ( \$ )}$ | Quantity <br> $\mathbf{Q}_{\boldsymbol{d}}$ (pounds <br> of steak) | Arc <br> Elasticity <br> $\mathbf{E}_{\mathbf{d}}$ | Total <br> Revenue <br> $\mathbf{( \$ )}$ | Marginal <br> Revenue <br> (\$/unit) |
| :---: | :---: | :---: | :---: | :---: |
| $\$ 12$ | 30 | NA |  | NA |
| $\$ 11$ | 40 |  |  |  |
| $\$ 10$ | 50 |  |  |  |
| $\$ 9$ | 60 |  |  |  |
| $\$ 8$ | 70 |  |  |  |
| $\$ 7$ | 80 |  |  |  |
| $\$ 6$ | 90 |  |  |  |
| $\$ 5$ | 100 |  |  |  |
| $\$ 4$ | 110 |  |  |  |

Compute the associated arc elasticity, total revenue and marginal revenue values.

## Problem 12

The general linear demand for good X is estimated to be

$$
Q=250,000-500 P-1.50 M-240 P_{R}
$$

Where $P$ is the price of good $X, M$ is average income of consumers who buy good X , and $\mathrm{P}_{\mathrm{R}}$ is the price of related good R . The values of $\mathrm{P}, \mathrm{M}$, and $\mathrm{P}_{\mathrm{R}}$ are expected to be $\$ 200, \$ 60,000$ and $\$ 100$, respectively. Use these values at this point on demand to make the following computations.
a) Compute the quantity of good X demanded for the given values of $\mathrm{P}, \mathrm{M}$, and $P_{R}$.
b) Calculate the price elasticity of demand $\mathrm{E}_{\mathrm{p}}$. At this point on the demand for X , is demand elastic, inelastic or unitary elastic? How would increasing the price of X affect total revenue? Explain.
c) Calculate the income of elasticity of demand Eм. Is good X normal or inferior? Explain how a 4 percent increase in income would affect demand for X , all other factors affecting the demand for X remaining the same.
d) Calculate the cross-price elasticity Exr. Are the goods X and R substitutes or complements? Explain how a 5 percent decrease in the price of related good R would affect demand for X , all other factors affecting the demand for X remaining the same.

## Problem 13

The estimated market demand for good X is:

$$
\widehat{Q}=70-3.5 P-0.6 M+4 P_{z}
$$

Where $\hat{Q}$ is the estimated number of units of good X demanded, P is the price of the good, M is income, and $\mathrm{P}_{\mathrm{z}}$ is the price of the related good Z . (all parameter estimates are statistically significant at the $1 \%$ level.)
a) Is X a normal or an inferior good? Explain.
b) Are X and Z substitutes or complements? Explain.
c) At $\mathrm{P}=10, \mathrm{M}=30$, and $\mathrm{P}_{2}=6$, compute estimates for the price ( $\mathrm{E}_{\mathrm{P}}$ ), income ( $\mathrm{E}_{\mathrm{M}}$ ) and cross-price elasticities ( $\mathrm{Exz}^{\mathrm{z}}$ ).

## Problem 14

A linear market demand function of the form

$$
Q=a+b P+c M+d P_{R}
$$

Was estimated using regression analysis. The results of this estimation are as follows:

| DEPENDENT VARIABLE: Q | R-SQUARE | F-RATIO | P-VALUE ON F |  |
| :--- | :--- | :--- | :--- | :--- |
| OBSERVATIONS: 24 | PARAMETER | STANDARD | T-RATIO | P-VALUE |
| VARIABLE | ESTIMATE | ERROR |  |  |
| INTERCEPT | 68.38 | 12.65 |  | 0.0001 |
| P | -6.50 | 3.15 |  | 0.0001 |
| M | 0.13926 | 0.0131 |  | 0.0001 |
| PR | -10.77 | 2.45 | 0.0002 |  |

a) Is the sign of as would be predicted theoretically? Why?
b) What does the sign of imply about the good?
c) What does the sign of imply about the relation between the commodity and the related good R ?
d) Test, at the $5 \%$ level, the significance of $\mathrm{P}, \mathrm{M}$ and $\mathrm{P}_{\mathrm{r}}$. Use both t -statistics and p-values to test the hypotheses.
e) Test, at the $5 \%$ level, the joint significance of the variables included in this regression. Use both the appropriate test statistic and its corresponding p-value to test the hypothesis and clearly state the testing procedure.
f) Using the values $P=225, M=24,000$ and $P_{R}=60$, calculate the estimates of:

1) The price elasticity of demand ( $E_{p}$ ).
2) The income elasticity of demand ( $\mathrm{E}_{\text {м }}$ ).
3) The cross price elasticity (Exr).

## Problem 15

Wilpen Company, a price-setting firm, produces nearly 80 percent of all tennis balls purchased in the United States. Wilpen estimates the U.S. demand for its tennis balls by using the following linear specification:

$$
Q=a+b P+c M+d P_{R}
$$

Where Q is the number of cans of tennis balls sold quarterly, P is the wholesale price Wilpen charges for a can of tennis balls, M is the consumers' average household income, and $\mathrm{P}_{\mathrm{R}}$ is the average price of tennis rackets. The regression results are as follows:

| DEPENDENT VARIABLE: Q | R-SQUARE | F-RATIO | P-VALUE ON F |  |
| :--- | :--- | :--- | :--- | :--- |
| OBSERVATIONS: 20 | 0.8435 |  | 0.001 |  |
|  | PARAMETER | STANDARD | T-RATIO | P-VALUE |
| VARIABLE | ESTIMATE | ERROR |  |  |
| INTERCEPT | 425120.0 | 220300.0 |  | 0.0716 |
| P | -37260.6 | 12587 |  | 0.0093 |
| M | 1.49 | 0.3651 |  | 0.0009 |
| PR | -1456.0 | 460.75 |  | 0.0060 |

a) Test, at the $5 \%$ level, the significance of the price, average household income and the price of tennis rackets in the above estimated regression. Use both t -statistics and p -values test the hypotheses.
b) Constructa $99 \%$ confidence interval for (each of) the parameters b, c and d. Use the constructed confidence interval to test the significance of the variables $\mathrm{P}, \mathrm{M}$ and $\mathrm{P}_{\mathrm{R}}$.
c) Are the variables included in this regression jointly significant? Clearly state the nulland alternative hypotheses, the test statistic used as well as the testing procedure (decision rule).
d) What proportion of the variation remains unexplained by the independent variables in this regression?
e) Compute the Adjusted $\mathrm{R}^{2}$ of this regression.

Wilpen plans to charge to charge a wholesale price of $\$ 1.65$ per can. The average price of a tennis racket is $\$ 110$, and the consumers' average household income is $\$ 24,600$.
f) What is the estimated number of cans of tennis balls demanded?
g) At the values of $P, M$ and $P_{R}$ given, what are the estimated vales of the price ( $E_{P}$ ), income ( $E_{M}$ ), and the cross price elasticity of demand ( EXR ).
h) What will happen, in percentage terms, to the number of cans of tennis balls demanded if the price of tennis balls decreases 15 percent?
i) What will happen, in percentage terms, to the number of cans of tennis balls demanded if average household income increases by 20 percent?
j) What will happen, in percentage terms, to the number of cans of tennis balls demanded if the average price of tennis rackets increases by 25 percent?

## Problem 16

The estimated market demand for good X is:

$$
\widehat{Q}=8000-25 P-0.12 M-30 P_{G}
$$

where $\widehat{Q}$ is the estimated number of units of good X demanded, P is the price of $\operatorname{good} \mathrm{X}, \mathrm{M}$ is income, and $\mathrm{P}_{\mathrm{G}}$ is the price of related good G . (All parameter estimates are statistically significant at the 1 percent level of significance.)
a) Is the sign of the coefficient associated with the price consistent with theory? Why or why not?
b) Given the above equation, is good X a normal or inferior good?
c) Are goods X and G complements or substitutes?
d) At $\mathrm{P}=\$ 12, \mathrm{M}=\$ 30,000$, and $\mathrm{P}_{\mathrm{G}}=\$ 50$, what is the predicted quantity of good X ?
e) At the values in part (d), compute the following elasticities:

1) Own Price elasticity Ep.
2) Cross-price elasticity Exg.
3) Income elasticity Ем.
f) The $R^{2}$ from this regression is 0.65 . What proportion of the variation in the demand for X is explained by the above regression? What proportion of variation in demand remains unexplained?
g) Given a sample size $\mathrm{n}=25$, compute the F -statistic for this demand equation.
h) Test, at the $5 \%$ level, the joint significance of all the variables included in the regression.
i) All else constant, what would be the effect (in percentage terms) of a $20 \%$ increase in income?
j) All else constant, what would the effect (in percentage terms) of 15\% increase in the price of the related product G ?

## Problem 17

Rubax, a U.S. manufacturer of athletic shoes, estimates the following linear trend model for shoe sales:

$$
Q_{t}=a+b t+c_{1} D_{1}+c_{2} D_{2}+c_{3} D_{3}
$$

Where:
$\mathrm{Q}_{\mathrm{t}}=$ sales of athletic shoes in the quarter t
$\mathrm{t}=1,2, \ldots, 28$ [2001Q1,2001Q2, ....,2007Q4]
$D_{1}=1$ for the $1^{\text {st }}$ quarter of the year, 0 otherwise
$D_{2}=1$ for $2^{\text {nd }}$ quarter of the year, 0 otherwise
$D_{3}=1$ for the $3^{\text {rd }}$ quarter of the year, 0 otherwise
The regression analysis produces the following results:

| Observations | R-Square |
| :---: | :---: |
| 28 | 0.9651 |


| Variable | Parameter <br> Estimate | Standard <br> Error |
| :--- | ---: | ---: |
| Intercept | 184500 | 10310 |
| T | 2100 | 340 |
| $\mathrm{D}_{1}$ | 3280 | 1510 |
| $\mathrm{D}_{2}$ | 6250 | 2220 |
| $\mathrm{D}_{3}$ | 7010 | 1580 |

a) Is there sufficient statistical evidence of an upward trend in shoe sales?
b) Do these data indicate a statistically significant seasonal pattern of sales for Rubax shoes? If so, what is the seasonal pattern exhibited by the data? Carefully state the seasonal pattern, if any, and justify your answer.
c) Using the estimated forecast equation, forecast sales of Rubax shoes for 2008Q3 and 2009Q2.
d) How would you improve this forecast equation?

Problem 18
A firm experienced the demand shown in the following table:

| Year | Actual <br> Demand | 3-Year <br> Moving <br> Average | Exponential <br> Smoothing <br> (w=0.3) |
| :---: | ---: | :---: | :---: |
| 1995 | 800 | NA | NA |
| 1996 | 925 | NA |  |
| 1997 | 900 | NA |  |
| 1998 | 1,025 |  |  |
| 1999 | 1,150 |  |  |
| 2000 | 1,160 |  |  |
| 2001 | 1,200 |  |  |
| 2002 | 1,150 |  |  |

a) Compute the forecasts based on a 3-year moving average for 1998, $1999,2000,2001$ and 2002.
b) Compute exponential smoothing forecasts of demand using $\mathrm{w}=0.3$ for 1996, 1997, 1998, 1999, 2000, 2001, 2002. You may begin the exponential smoothing forecasting by assuming that the first forecast value ( $\mathrm{F}_{1}$ ) is equal to the actual (realized) value ( $\mathrm{A}_{1}$ ).
c) What criterion would you use to evaluate the accuracy of these forecasts? Write down the expression of the criterion you propose and define the elements of the expression carefully.
d) Compute the criterion you propose in part (c) for the 3-year moving average and the exponential smoothing forecasting methods. Which forecasting method does better?

## Problem 19

The demand for tea has been estimated as

$$
Q=7,000-550 P+210 I+425 P_{C}
$$

Where:
Q = thousands of pounds of tea sold
$\mathrm{P}=$ price per pound of tea
I = per capita disposable personal income in thousands of dollars
$\mathrm{P}_{\mathrm{C}}=$ price per pound of coffee
a) If next year's tea price is forecast to be $\$ 3$, per capita disposable personal income is estimated to be $\$ 15,000$ (i.e., 15), and the price per pound of coffee is estimated to be $\$ 4$, compute the expected quantity demanded for the comingyear.
b) Economic forecasters believe the probability of a major recession next year is high and would reduce per capita income to $\$ 13,000$ (13). In addition, a frost in Brazil is likely to increase the price of coffee to $\$ 7$ per round. What impact would these changes in the economic outlook have on the demand for tea?

## Problem 20

Savings-Mart (a chain of discount department stores) sells patio and lawn furniture. Sales are seasonal, with higher sales during the spring and summer quarters and lower sales during the fall and winter quarters. The company developed the following quarterly sales forecasting model:

$$
\hat{Y}=8.25+0.125 t-2.75 D_{1 t}+2.25 D_{2 t}+3.50 D_{3 t}
$$

Where:
$\hat{\mathrm{Y}}_{\mathrm{t}}=$ predicted sales (\$million) in quarter t
$8.25=$ quarterly sales ( $\$$ million) when $t=0$
$\mathrm{t}=$ time period (quarter) where fourth quarter of $2000=0$, first quarter of $2001=1$, second quarter of $2001=2, \ldots$
$\mathrm{D}_{1}=1$ for first-quarter observations, 0 otherwise
$\mathrm{D}_{2}=1$ for second-quarter observations, 0 otherwise
$\mathrm{D}_{3}=1$ for third-quarter observations, 0 otherwise
Forecast Savings-Mart's sales of patio and lawn furniture for each quarter of 2006.

