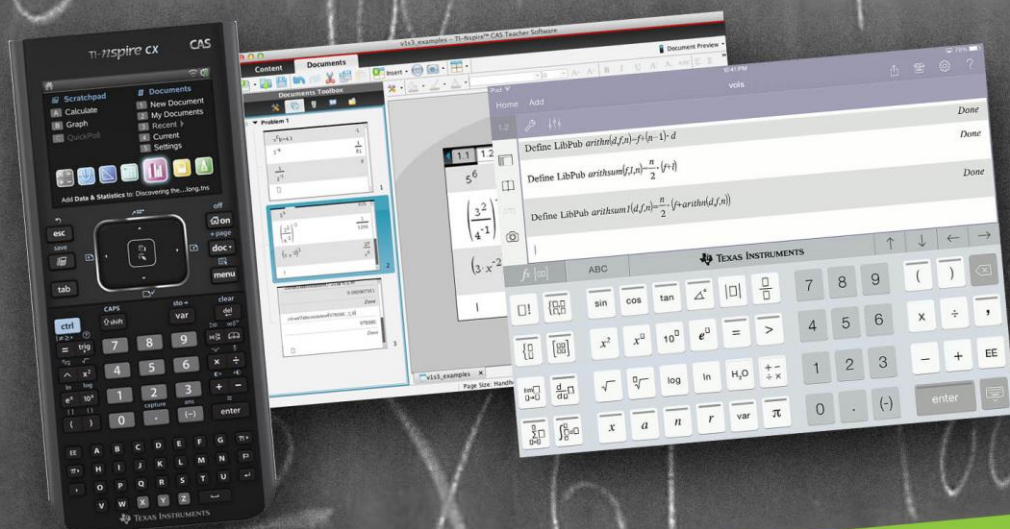


TI-Nspire Innovation in Education Series

TI-Nspire™ Guide

ALGEBRA FUNDAMENTALS



- Learn the Graphs and Calculator applications
- Add new math capabilities to your TI-Nspire and TI-Nspire CAS handheld and iPad app
- Use as a Rosetta Stone to show traditional and TI-Nspire or TI-Nspire CAS solutions
- Keep as a handy pocket reference for your phone

Larry Schroeder

TI- Nspire™ Guide

ALGEBRA FUNDAMENTALS

by Larry Schroeder



Computer Learning Service

Ti-Nspire™ Guide Algebra Fundamentals

TI-Nspire Innovation in Education Series

TI-Nspire Guide Algebra Fundamentals: TI-Nspire and TI-Nspire CAS Revealed and Extended

Published by
Computer Learning Service
1013 Woodbine Circle
Galesburg, IL 61401-2358
ComputerLearningService.com

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Version 1.0

ISBN: 978-0-915573-00-4

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About the Author



Brooks/Cole Math Tools Guide: Title Graphic (*Interactive Video Skillbuilder CD-ROM*)

Larry Schroeder is the author of *Brooks/Cole Math Tools Guide* for TI™-83/86/89/92 calculators and Microsoft Excel™ spreadsheets. The Guide can be found on the [Interactive Video Skillbuilder CD](#) that accompanies various editions of College Algebra, College Algebra and Trigonometry, Precalculus textbooks, and Tan series of *Finite Mathematics*, *Calculus*, *College Mathematics*, and *Applied Mathematics* texts *For the Managerial, Life, and Social Sciences* published by Cengage Learning/Thomson/Brooks/Cole (*Interactive Video Skillbuilder CD-ROM*).

Larry also authored the [WebTutor™ Advantage](#) for WebCT & Blackboard supplements for Thomson/Brooks/Cole Tan's *Finite Math*, *Calculus*, *College Mathematics*, and *Applied Mathematics* series of textbooks *For the Managerial, Life, and Social Sciences* (Schroeder and Tan).

From *College Mathematics* 6th edition preface's Learning Aids, "WebTutor Advantage for WebCT & Blackboard by Larry Schroeder, Carl Sandburg College, is the most extensive WebCT or Blackboard cartridge available and contains expanded online study tools including: step-by-step lecture notes; student study guide with step-by-step TI-83/86/89/92 and Microsoft Excel explanations; a quick check interactive problem for each online example with accompanying step-by-step solution and step-by-step TI-83/86/89/92 solution; practice quizzes by chapter sections that can be used as electronically graded online exercises, and much more (Schroeder and Tan)."

Education - Background

Larry holds a Master of Science and Bachelor of Science in mathematics from Western Illinois University. His electronic training and experience from the military plus having taught at the elementary, secondary, and college level helped motivate his passion for incorporating educational technology so that all students, young or old, can meet their dreams and goals.

For 27 years Larry taught math and computer science at Carl Sandburg College. In addition to instruction, he coordinated the faculty technology efforts and oversaw the school's online courses. He was the chief designer of a faculty WebCT course that won international recognition as one of the top ten exemplary courses in the world. He is also proud of his involvement with the University of Illinois higher education's extending technology efforts and Carl Sandburg College's secondary and elementary outreach initiatives that assist other educators with their student use of technology in the learning process.

Now retired, he enjoys riding his bike, coaching his granddaughter's basketball team, and spending time in general with his family, especially his wife, children and grandchildren. He still loves keeping current with math technology advances and helping others improve their math skills. The advances with the TI-Nspire are what this book is about.

Dedication

To my wife and children for their love and support.

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Welcome and What We Are All About

Welcome to *TI-Nspire Guide Algebra Fundamentals*, *TI-Nspire Innovation in Education Series*.

There is no one road to the learning of mathematics. Different approaches for different learners are needed to take learners to where they want to go. The goal of this guide is to give you the flexibility of various approaches aided by the use of the TI-Nspire to reach your goals.

Manual explanations of math concepts in the guide are accompanied by TI-Nspire illustrations that can be used with the handheld, computer software, and iPad app. Techniques, examples, and exercises come from all three platforms with special instructions for the differences in their interfaces.

The *TI-Nspire Guide Algebra Fundamentals* emphasis is its attention on math standards that work. Significant time is spent on learning math by using methods that have shown to be successful in the classroom. There is additional emphasis given to building blocks topics.

TI-Nspire and TI-Nspire CAS Revealed and Extended feature of the guide is used to show how the TI-Nspire and TI-Nspire CAS commands and functions can be used to work individual problems as well as how they can be extended to help us understand complex math concepts or create a set of tools that can help with problem solving.

Upon mastery of the manual techniques apply the TI-Nspire and TI-Nspire CAS solutions to increase your efficiency and problem solving power. Concentrate on the solution without being bogged down with traditional labor-intensive steps. Embrace failure; use your additional time, to attempt more word and real world problems.

The *TI-Nspire Guide Algebra Fundamentals* is a digital learning environment. Every example is followed by an interactive reinforcement exercise. Hyperlinks are used to references earlier discussions of a topic, future discussions of a topic, websites, Table of Contents entries, and Appendix entries.

The guide is available on various eBook readers and eBook applications, be it a stand alone reader, on a phone, tablet, pc, or through a web-browser. The content uses mathematical notation, text, graphics, and TI-Nspire screenshots that takes advantage of the resolution of the display. The real-time access, anytime, anyplace nature of this guide allows a new way for you to gain the math knowledge and skills necessary to succeed in the classroom, at your job, and in your personal pursuit of learning.

An innovative approach is to use TI-Nspire CAS solutions as an aide in moving forward. In lessons requiring a concept that you have not yet mastered, use the TI-Nspire CAS solutions to assist with class assignments, allowing you to keep current. You keep moving forward with the new material, giving you additional time to

master the concept causing problems.

Included is a traditional Table of Contents. EBook readers and eBook application create their own logical Table of Contents. For their logical Table of Contents, some show a separate complete Table of Contents, others a separate Table of Contents containing section headings, and some link to the traditional Table of Contents. To always have a complete table of contents available a traditional Table of Contents is included as a physical part of the eBook.

Larry Schroeder

Getting Started - Working with the TI-Nspire Document Model

TI-Nspire Family

The TI-Nspire family includes handhelds, mac and pc software, and iPad apps plus accessories and classroom tools. Mac and pc software will be referred to as the computer software. Handhelds, computer software, and iPad apps have a TI-Nspire and TI-Nspire CAS version. The good part is that the document model is the same across all of the family and makes it easy to use any member of the family.

In *TI-Nspire Guide Algebra Fundamentals*, we illustrate how the handhelds, computer software, and iPad apps can be used to solve math problems. Our references to the TI-Nspire family in this guide will include everything except the accessories and classroom tools.

The handheld, computer software, and iPad app come in two models, TI-Nspire and TI-Nspire CAS. The computer software has two versions - student and teacher. For this guide's purposes both student and teacher versions of the computer software will work equally well.

In our illustrations, if the application only works for the TI-Nspire CAS model, the heading will be labeled as TI-Nspire CAS. If some or all parts of the illustration will work for the TI-Nspire as well, then the heading will say TI-Nspire Family.

With the handhelds there have been three generation's, Clickpad, Touchpad, and CX. By upgrading the operating system to OS 3.9 or current revision allowed, the older Clickpad and Touchpad models can be used as well.

Note: We will use handheld, iPad, and computer software screens for our illustrations of math concepts. However, the handheld, computer software, and iPad app functionality are identical. In the handheld, computer software, and iPad app only the user interface is different. The handheld and computer software have a ScratchPad option as well. We will create a calculator page or graph page on the iPad and use the **Discard My Changes** to simulate the ScratchPad on the iPad.

For all problems and solutions presented in this guide, the math input, solutions and are identical in content. The settings, options selected, and difference in screen sizes affect the format of how the solutions are displayed. For the iPad solutions to be readable on all devices it was necessary to use a graphics editor. The content is exactly the same for the edited screenshots. Please refer to the online reference manuals provided by TI for complete details about the user interface and display format for the handheld, computer software, and iPad applications.

The first instruction for using a TI-Nspire family member will be to "Begin by creating a new Document and adding a TI-Nspire page." Usually, the TI-Nspire page will be a "Calculator" or "Graphs" page. Use the necessary specific step-by-step procedure for iPad app, handheld, or computer software.

The last instruction for the new Document will say “**Keep/Save** the file as **vXsY-filename**” with X being the volume number and Y the section number. The Keep refers to **Keep My Changes** option of the iPad TI-Nspire app. The Save refers to handheld or computer software procedures for saving the file.

Subsequent examples or exercise instructions will be to add “TI-Nspire page,” normally a “Calculator” or “Graphs” page. The instructions will end with “**Keep My Changes/ctrl S** (Save) the file as **vXsY-filename**.” Here, since the instructions are straight forward, we are able to list the specific iPad and handheld instructions.

Before using the guide, we recommend that you become comfortable with your TI-Nspire device’s operation. Use Texas Instrument’s Professional Development, [Product Tutorials page links](#) to view your device’s tutorials. Texas Instruments teamed up with award winning training provider Atomic Learning™ to provide online video tutorials for learning how to operate their TI-Nspire handhelds, computer software, and iPad app. View the appropriate tutorials to become familiar with your device’s operation. The handhelds come with computer software, so handheld users should be sure to view the software tutorials as well (*Tutorials by Texas Instruments - US and Canada*). Then turn to this guide.

There are also second party sources that help you become comfortable with your device.

TI-Nspire Handheld Essentials

This guide is based on TI-Nspire CX OS 3.9. You should upgrade your OS, operating system, to the highest version allowed. The only exception would be if you were running some application that only works on an earlier version. The 3.9 in the OS name represent the version of the OS. Other numbers are included for various builds that fixed bugs. The Clickpad, Touchpad, and CX all have the same functionality. With the click pad model, there will be some differences in keystrokes.

All the generations of handhelds have both the TI-Nspire and TI-Nspire CAS models. We recommend the CX CAS model even though the ACT™ college entrance exam allows only the TI-Nspire model. SAT™ and advance placement exams allow both models. The older CAS models are slower, bulky, and have screens that are much harder to read.

Recommending the CX CAS is probably controversial. In the tutorial we show the manual work for the illustrations, examples, and exercises. In addition to using a TI-Nspire CX or TI-Nspire CX CAS, we recommend that you manually do or show all your work as well. Since the CX and CX CAS share all features except the Computer Algebra System, one could borrow a CX for ACT entrance exam.

TI-Nspire Computer Software Essentials

This guide is based on TI-Nspire OS 3.9. Again the same details apply as mentioned

for the handheld. With the one exception of running some application that only works on an earlier version we recommend that you use the most current version. We recommend that you keep all your platforms, handheld, computer software, and iPad app with the most currently allowed operating system.

There are two versions of the computer software, and each version comes in either the TI-Nspire and TI-Nspire CAS form. The first version is the TI-Nspire Student Software. The second version is the TI-Nspire Teacher Software.

The two versions are very similar. The teacher version has a Question application. The teacher version was designed with the classroom in mind. It has additional options for the classroom settings but can be used alone as well.

In either the student or teacher version, the Computer Software Workspace view takes many forms depending on the settings and options selected.

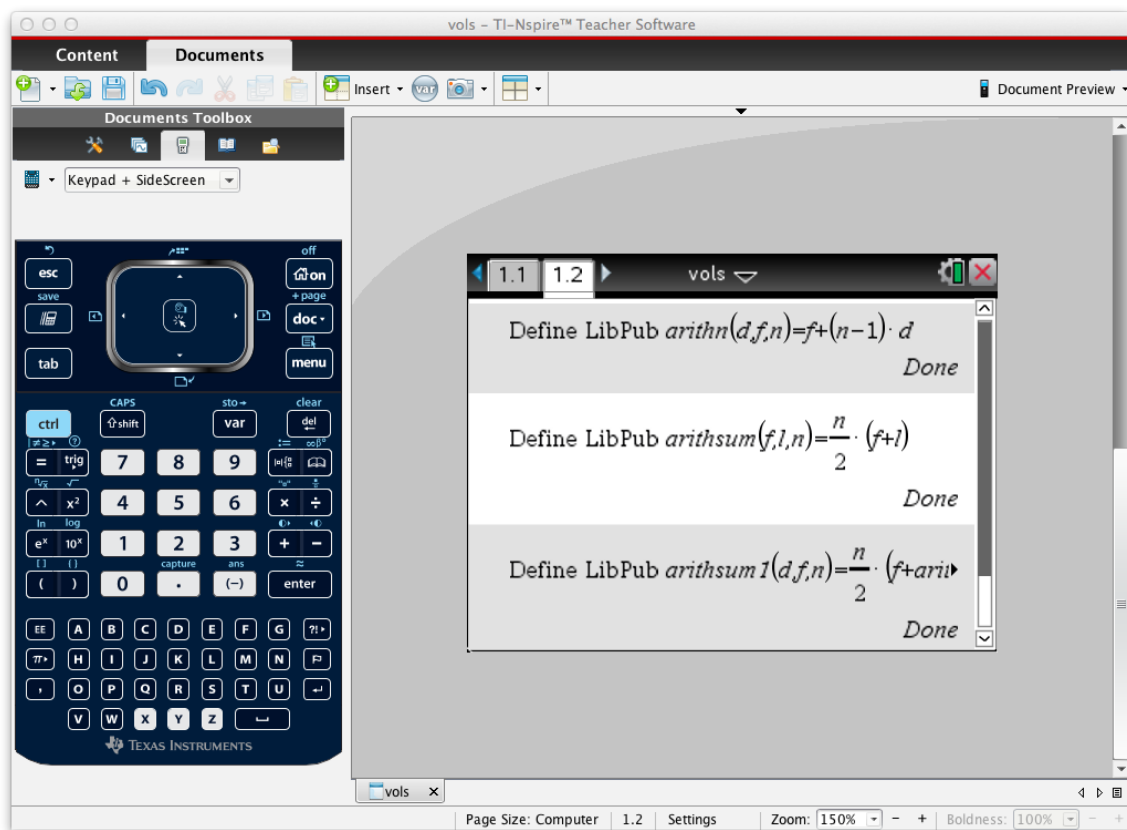


Figure 1 TI-Smartview of computer software

For classroom demonstrations, we recommend that the software be set to the Documents Tab. Use the Documents Toolbox to toggle between TI-SmartView (Keypad + SideScreen) and Page Sorter when the software is projected on an interactive whiteboards or digital projectors. This way your students can follow along using the same keystrokes on their handheld. Use Page Sorter to quickly display a different problem or page. Make sure the Document Preview in the upper

right hand corner is set to handheld. Otherwise, use all the options of the Documents Toolbox plus the other features of the software.

Ti-Nspire iPad App Essentials

This guide is based on TI-Nspire OS 3.11.0. Here we have a Release number for the build. At this time the handheld and computer software have OS 3.9 where the iPad is at 3.11.0.

One note is that current iPad screenshots are from iPad OS 3.8.1 and iPad OS 3.10.0. The only major differences are no page sorter in tutorial's current iPad screenshots and an application icon is shown. The application icon was dropped in iPad OS 3.11.0.

We edited the current iPad screenshots to only show the document bar and workspace entries inputs and results. This was done to give the appearance of enlarged fonts. This made the iPad screenshots entries readable on phone eBook reader apps, as well as standalone eBook readers, and tablet eBook reader apps.

The iPad app comes in both TI-Nspire and TI-Nspire CAS forms. The iPad tablet refers to its applications as apps. We will use iPad app when referring to the iPad version of the TI-Nspire family.

The interface for the TI-Nspire and TI-Nspire CAS apps is very nice. The Navigation Zone, Math & Numeric Zone, and Alpha Keys Zone of the handheld are gone. We now can toggle between a QWERTY keyboard and an integrated math keyboard. We have a document's bar with scrollable Tools icon, attributes icon, and other icons appropriate to the opened document.

With these features and a Page Sorter that we toggle on and off, the iPad Document Workspace is a joy to use. The toolbar at the top of the workspace completes the workspace interface.

Touching the **Home** label in the toolbar returns us to the TI-Nspire desktop and file system. Since the iPad has no standard file system, TI created its own. TI file solution is easy to use due to the beauty of its interface and the file system's simplicity. Computer like file operations are used to manage the folder and files. Touching the **Add** label allows to add an application page to the Document or add a new problem to the document.

We have used both the TI-Nspire iPad apps on the iPad mini, iPad mini retina, third & fourth generation iPads, and iPad Air. We especially like the mini retina because of its size, screen display, and power. We used the mini retina as if it were a touchscreen handheld. With our finger resting behind we could use our thumbs for easy entry.

The iPad retina displays with their screens and power spoiled us. Using a stylus and Bluetooth keyboard made us even more productive as we did not need to toggle the

integrated math keyboard.

Explanation 1.1 - Real Numbers

In this section we introduce exponential notation and look at order of operations. We then introduce the real numbers; look at subsets of the real numbers, and the real numbers on a number line.

Exponential Notation

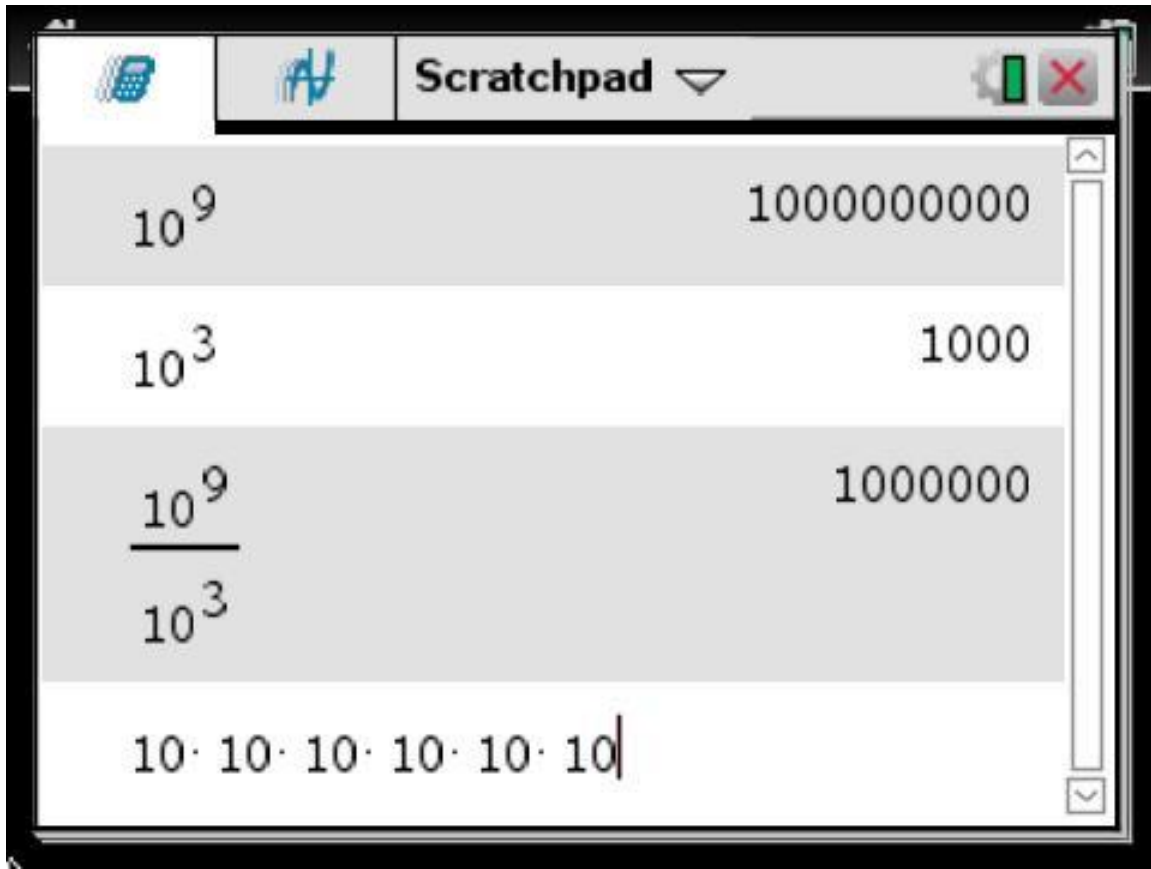
What is a billion divided by a thousand? Doing math with exponents is easier because you can subtract to do division and add to do multiplication.

The zeros in a problem can be used for its significant figures in measured numbers. For the trailing zeros in a billion and a thousand to be considered significant in measured numbers, they would have to be indicated. Various methods are used to identify significant figures in a number without a decimal point containing trailing zeros. Two common methods are bar above or under the last significant zero.

For multiplication or division the results number of significant figures is the least of its significant figures in the original measured numbers used in the calculation. Significant figures in arithmetic will be explained in section 1.3 discussion of [Scientific Notation](#).

Note: When we are using a measuring tool to determine a quantity we will use measured numbers. Exact numbers are another important type of number used in measurement problems. Exact numbers result from a count or are defined numbers. We will assume in our discussion, examples, and exercises unless measurement or exact numbers are involved that we are dealing with numbers in general and no reference will be made to significant figure or decimal places in our solutions.

TI-Nspire Family Using Exponents



Begin by selecting the **Scratchpad** key on the handheld. Key in as shown. Use the **exponent key** and **fraction template**. For the last entry, press **enter** to verify that there are six factors of ten in the answer of a million.

If the Scratchpad has computations or graphs, clear the workspace first. To clear use the handheld touchpad like a computer touchpad by moving your fingertip in a sliding motion on the center area of the Touchpad to activate and move the mouse pointer. Select the Down arrow icon to right of word Scratchpad at top of screen and select **Clear Scratchpad** from the menu. An alternative and simpler technique is to clearing the Scratchpad's workspaces, is to clear only the handheld's calculation work area. Select **menu > Actions > Clear History**. Note that on the handheld the **menu** key is used to display the **Actions** and **Clear History** menu and submenu items.

The iPad app has no **Scratchpad** feature. From the Home screen, begin by adding a new document. Select the **New** menu, Add Document **Calculator** option. When done with the calculations, press the **Home** label on the Calculator page and select **Discard My Changes**. Subsequently in this guide when we use the handheld Scratchpad we will assume that with the iPad app we follow this procedure and **Discard My Changes**.

We start by using exponents to write the product where the factors are the same. **Factors** are numbers or expressions that when multiplied represent a number or a product. In general, we have the following definition of Exponential Notation. Later, in this section discussion of [The Set of Real Numbers](#), we will explain what is meant by a being real and n being a natural number. We will expand upon the role of exponents in section 1.3 – [Integer Exponents](#) and section 1.4 [Radicals and Rational Exponents](#) of the tutorial.

Exponential Notation

For any real number a and natural number n

$$a^n = a \cdot a \cdot \dots \cdot a \quad n \text{ factors}$$

The number a is called the base and n is the exponent.

Example 1 Simplify Expressions

(a) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

(b) $4^3 = 4 \cdot 4 \cdot 4 = 64$

TI-Nspire Family Using Exponents - Creating the Example File



Begin by creating a new Document and adding a Calculator page. Key in the expressions as shown followed by **enter**.

Keep/Save the file as **v1s1-examples**.

Exercise 1 Exponential Notation

Evaluate:

$$-2^4$$

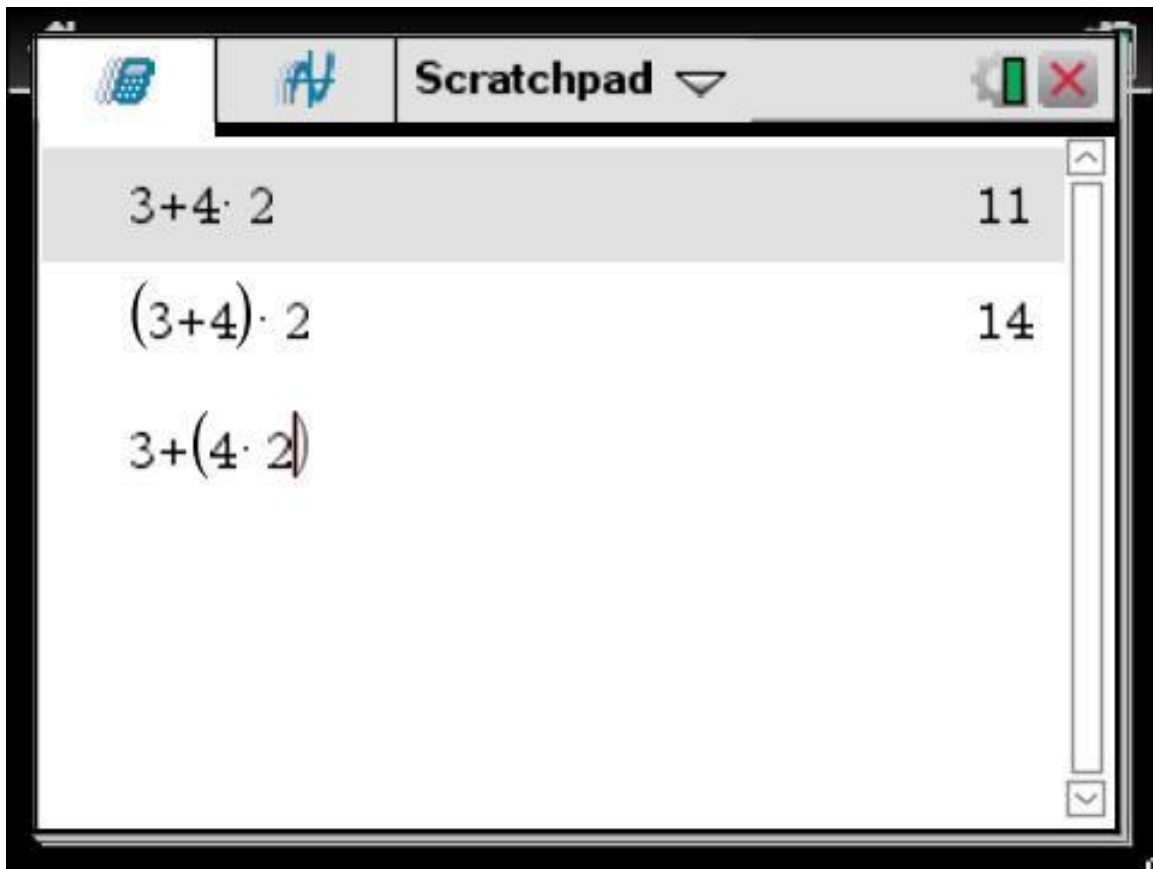
[Solution](#) >>

Order of Operations

What order would we do $3 + 4 \times 2$? If we add 3 and 4 and multiply by 2, we get 14. If we add 3 to the product of 4 and 2, we get 11. As we can see, the order makes a difference. Calculate in the wrong order and we will get the wrong result. Long ago, people agreed to follow rules when doing calculations.

If we put in grouping symbols, $(3 + 4) \times 2$ would give us 14 and $3 + (4 \times 2)$ would give us 11. We do all the calculations within the grouping symbols first.

TI-Nspire Family Order of Operations



Begin by selecting the **Scratchpad** key on the handheld. On the iPad create a document, calculator page, do the entries, and discard when done.

Use the handheld touchpad to activate and move the mouse pointer, select the Down arrow icon to

right of word Scratchpad at top of screen and select **Clear Scratchpad** from the Scratchpad's menu or using the **menu** key select **menu > Actions > Clear History** to clear the handheld's calculation workspace.

We can also use the conceptual menu, **ctrl menu**, and **Clear History**. We will only list the **menu > Actions > Clear History** option in our subsequent instructions, but the **ctrl menu**, and **Clear History** option could be used as well.

The conceptual menu is very handy and as the name implies, the submenu items will be related to the concept being used. This means the submenu items are the same for each individual concept but submenu items vary between concepts.

Key in as shown. Press **enter** to see the last result. Notice that parenthesis in this entry are now gone since they were not necessary.

For grouping symbols we use parenthesis (), brackets [], or braces {}. Generally we will use them in that order. The TI-Nspire will use only nested parentheses, not brackets and braces.

Sometimes the following rule minus the unary step is referred to by the acronym *PEMDAS* (Parenthesis, Exponent, multiplication, division, addition, subtraction).

Order of Operations

1. Simplify all operations within any grouping symbols.

Simplify all operations above and below fraction symbol.

Start with intermost grouping symbol.

2. Simplify all exponential expressions.
3. Perform all multiplications and divisions in the order they appear.
4. Perform all addition and subtraction in the order they appear.
5. Simplify any unary operations.

Example 2 Order of Operations

Work inside parenthesis

$$\frac{2 \cdot (4+3) - 1 \cdot 3}{7^2 + 1^4}$$

Work above and multiply

$$= \frac{2 \cdot (7) - 1 \cdot 3}{7^2 + 1^4}$$

Work above and subtract

$$= \frac{14 - 3}{7^2 + 1^4}$$

Work below and exponents

$$= \frac{11}{7^2 + 1^4}$$

Work below and add

$$= \frac{11}{49 + 1}$$
$$= \frac{11}{50}$$

TI-Nspire Family Order of Operations

The image shows a TI-Nspire Family Order of Operations interface. At the top, there are navigation buttons for '1.1' and '1.2', and a title bar 'v1s1-examples'. The main display area shows the expression $\frac{2 \cdot (4+3) - 1 \cdot 3}{7^2 + 1^4}$ on the left and the result $\frac{11}{50}$ on the right. Below the expression, there is a vertical cursor bar.

Add a Calculator page. Use the fraction template. Key in the problem as shown followed by **enter**.
Keep My Changes/ctrl S (Save) the file as **v1s1-examples**.

Exercise 2 Order of Operations

Evaluate:

$$4 + 3 \cdot (x - 1)^2 \text{ for } x = 5$$

[Solution](#) >>

The Set of Real Numbers

Real numbers will be the basis for most of this tutorial. Much of our everyday life math experiences can be modeled with real numbers.

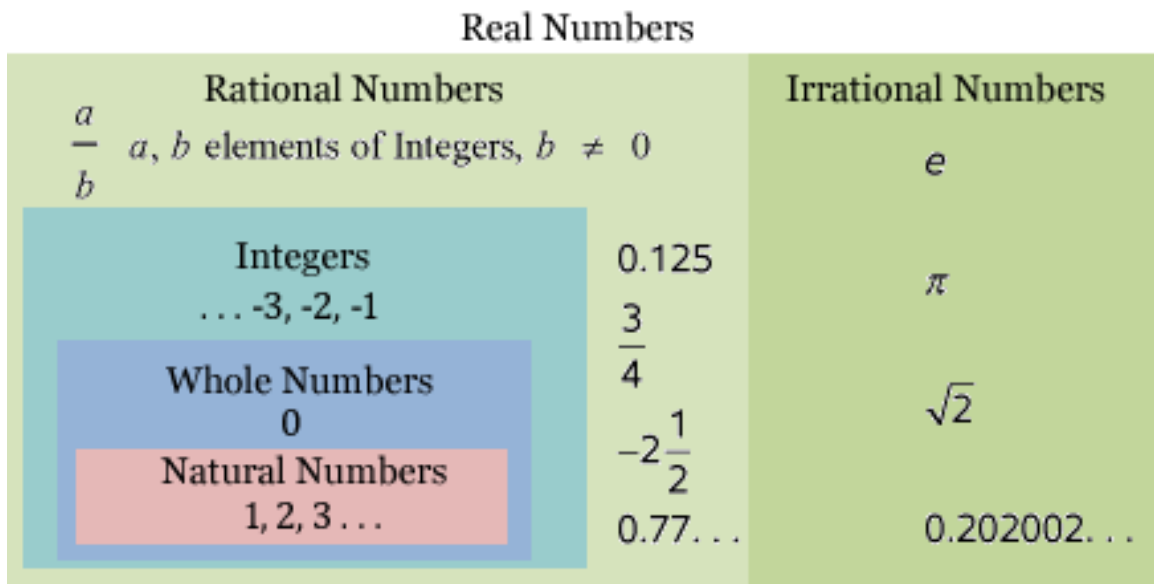
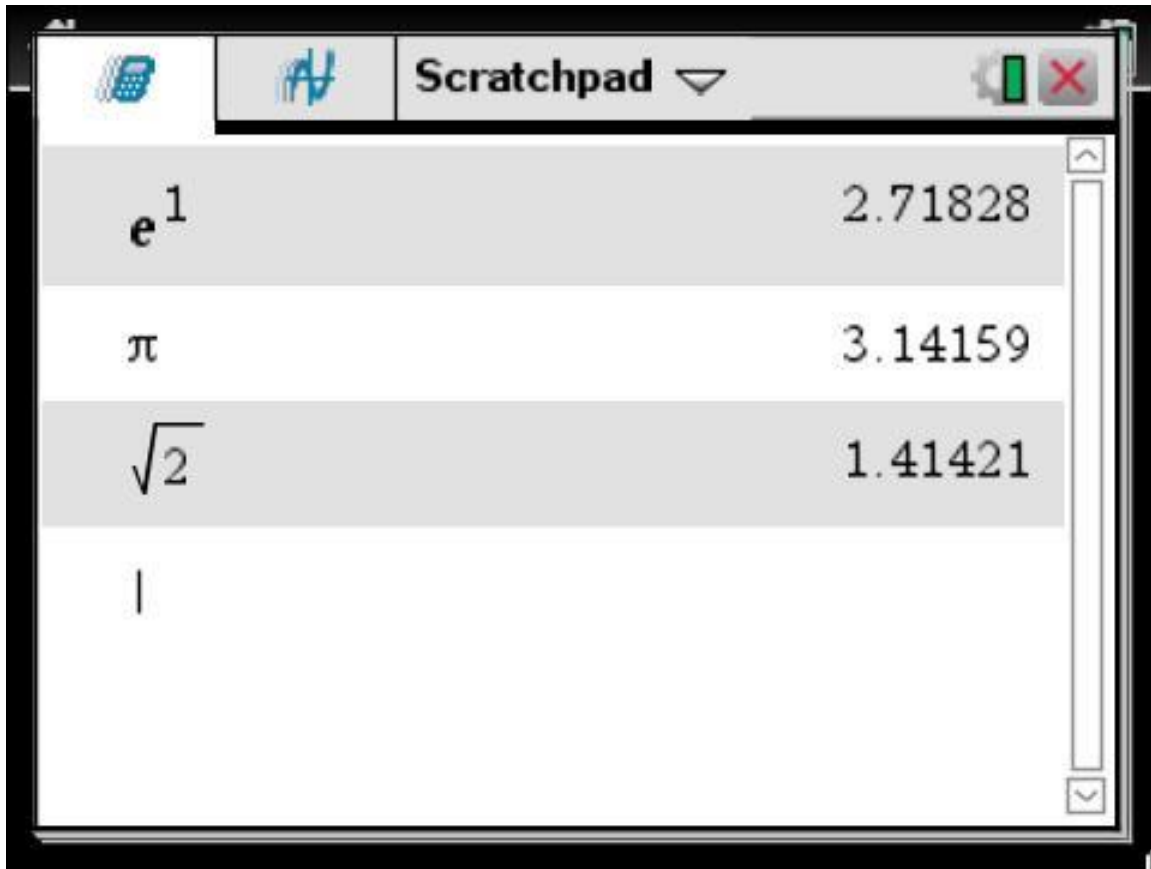


Figure 1 The real number system

TI-Nspire Family Irrational Numbers



Begin by selecting the **Scratchpad** key on the handheld. On the iPad create a document, calculator page, do the entries, and discard when done.

Use the handheld touchpad to activate and move the mouse pointer, select the Down arrow icon to right of word Scratchpad at top of screen and select **Clear Scratchpad** from the Scratchpad's menu or using the **menu** key select **menu > Actions > Clear History** to clear the handheld's calculation workspace.

Key in as shown. This screen was taken from the TI-Nspire CX model. Pressing **enter** gives a decimal approximation. With the TI-Nspire CAS devices we will need to use the approximate key, **ctrl enter** on the handheld and **long press** on the iPad app. Pressing **enter** on CAS devices leaves the answer in exact form. Only the first answer will change by dropping the exponent.

Decimal Representation for the Real Numbers

Every real number can be represented as a decimal. The decimal for the Real number can either be terminating, repeating, nonterminating, or non-repeating.

The set of Real Numbers is made up of the following:

- The Natural numbers are the counting numbers and are represented by terminating decimals.
- The Whole numbers are the Counting numbers plus 0 and are represented by terminating decimals.
- The Integers are the Natural numbers, their negatives plus 0, and are

represented by terminating decimals.

- The Rational numbers are numbers that can be written in the form a/b , where a and b are integers and b is not equal to 0 and are represented by terminating or repeating decimals.
- The Irrational numbers are numbers that cannot be written in the form a/b , where a and b are integers and b is not equal to 0 and are represented by nonterminating, nonrepeating decimals
- The Real numbers are all of the above. They are all types of decimals.

The following are the basic properties of equalities.

Properties of Equality

If $a = b$ and c is any real number, then

1. $a + c = b + c$
2. $a \cdot c = b \cdot c$

Property 1 states that we can add the same number to both sides of an equality. In the next section we introduce the sign rules for real numbers. We show that adding a negative is the equivalent to subtracting. Thus, if c is negative we could think of this as subtracting the same number from both sides.

Property 2 states that we can multiply the same number to both sides of an equality. If the number we multiply is the reciprocal, we can think of this as dividing both sides by the same number.

Since we have real numbers, subtraction and division are not necessary but practically subtraction and division are easier to use as we see in the next example.

Example 3 Rational Representation for repeating decimal

Write $x = 1.2373737\dots$ as a rational number.

Multiply by 10 to get repeating to right of decimal.

$$10x = 12.373737\dots$$

Multiply by 100 because 2 repeating digits

$$1000x = 1237.373737\dots$$

Subtract $10x$ from $1000x$

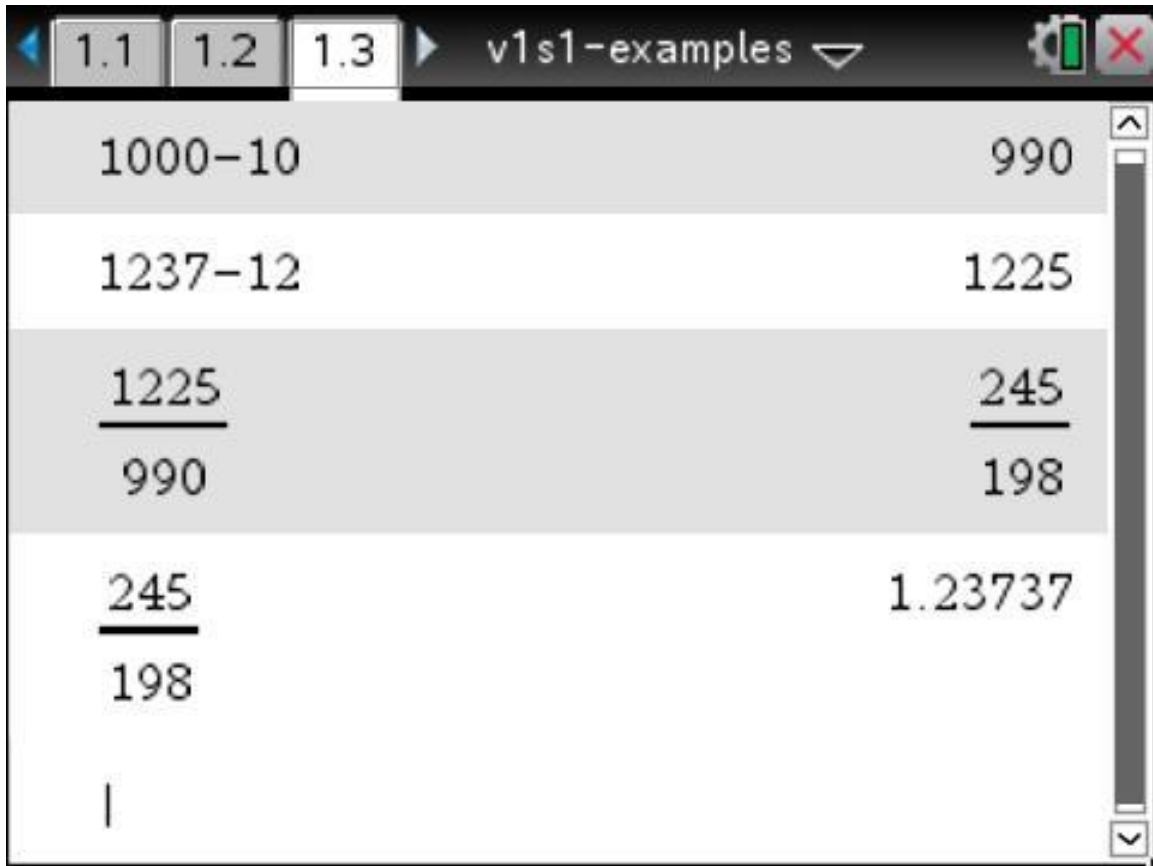
$$\begin{array}{r} 1000x = 1237.373737\dots \\ -10x = -12.373737\dots \\ \hline 990x = 1225 \end{array}$$

Dividing by 990

$$\frac{990x}{990} = \frac{1225}{990} \qquad x = \frac{1225}{990} = \frac{245}{198}$$

To come up with a rational number we used the Property of Equality for multiplication and addition for real numbers. First we used multiplication twice. Subtraction was next. It is easier to think subtraction rather than addition of negative numbers. Then we used division. Likewise, division is easier to do than multiplication of the reciprocal. Final step is to reduce to simplest form.

TI-Nspire Family Repeating Decimal Calculations



Add a Calculator page. Key in all calculation as shown except last followed by **enter**. For the last the calculation **approximate** (\sim) operator is located above the **equal** (=) key. Use **ctrl** on the handheld and **long press** on the iPad for the secondary key. The default mode for an answer for the TI-Nspire is simplified fraction.

Keep My Changes/ctrl S (Save) the file as **v1s1-examples**.

Exercise 3 Repeating Decimal as a Rational Number

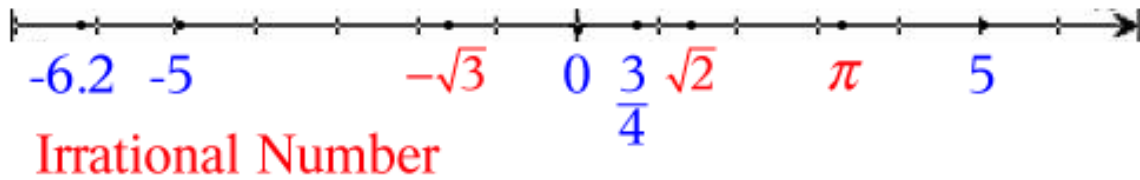
Write $x = 0.666\dots$ as a rational number.

[Solution](#) >>

Representing Real Numbers on a Number Line

We will use the one-to-one correspondence between the set of real numbers and the points on a straight line to create a one-dimensional space.

Rational Number



Irrational Number

Figure 2 The real numbers

Next we use the Cartesian Coordinate system and Pythagorean theorem to locate the square root of 5 on the x-axis. The drawing was made using the [TI Math Nspired](#) activity "Where is Root 2?" (*The Number System: Middle Grades Math: TI Math Nspired*). The drawing was from the Computer Software version of TI-Nspire. We explored in the activity and used the 2.2 tab to display the square root of five shown next. The activity wanted negative root of five, but for our purposes we choose root of five. The resulting screen capture was then edited. For a better understanding of irrational square roots on the number line, we recommend exploring this activity on your TI-Nspire.

TI-Nspire Family Where is Root 5?

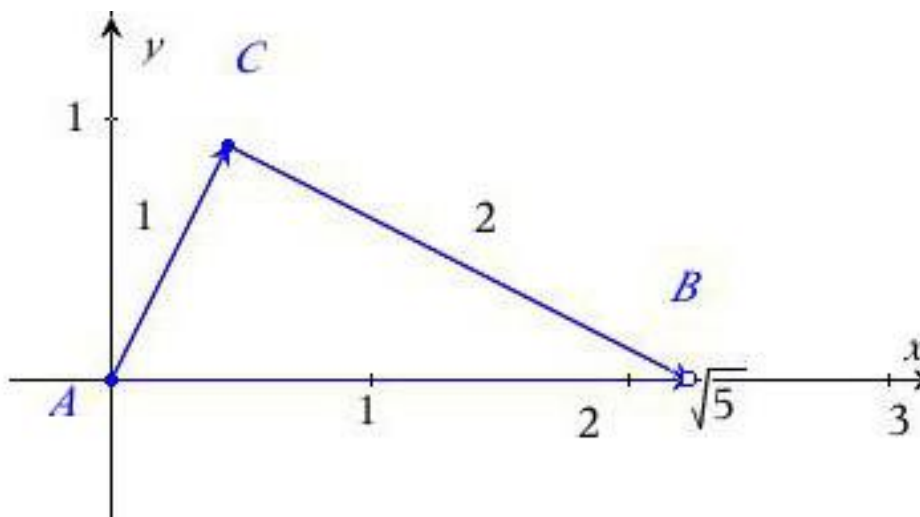


Figure 3 Root 5

Introduction to Absolute Value

We represent a number's distance from zero on a number line with notation $|a|$, read "the absolute value of a ". The distance is always a nonnegative number. To find absolute value, if the number is zero or positive we leave it alone, if the number is negative, we make it positive. See the TI-Nspire Scratchpad entries that follow. The first two entries are the same as the formal definition of absolute value.

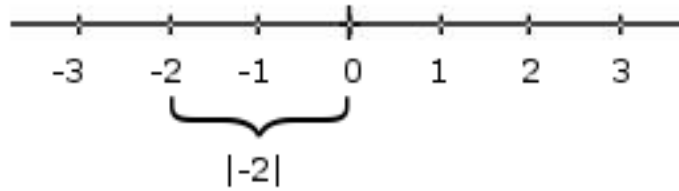


Figure 4 The absolute value of a number is its distance from zero.

Definition of Absolute Value

If a is real number, then the absolute value of a is

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

TI-Nspire Family Absolute Value

The screenshot shows a TI-Nspire Scratchpad window with the following entries:

$ a $ $a \geq 0$	a
$ a $ $a < 0$	$-a$
$ 2 $	2
$ -3 $	3
$-(-3)$	

Begin by selecting the **Scratchpad** key on the handheld. On the iPad create a document, calculator page, do the entries, and discard when done.

Use the handheld touchpad to activate and move the mouse pointer, select the Down arrow icon to right of word Scratchpad at top of screen and select **Clear Scratchpad** from the Scratchpad's menu or using the **menu** key select **menu > Actions > Clear History** to clear the handheld's calculation workspace.

Key in as shown. This screen was taken from the TI-Nspire CX CAS model. Only TI-Nspire CAS devices can do entries one and two. Attempting to do entries one and two on a non-CAS device will result in "Error: Variable is not defined". Continuing on with the first two entries on CAS device. Use the absolute value template. The **with** (|) operator is located above **equal** (=) on the handheld and above the **comma** (,) key on the iPad. The **inequalities** operators are located above **equal** (=) on the handheld and above the **greater than** (>) key on the iPad. Use **ctrl** on the handheld and **long press** on the iPad for the secondary keys.

The first two entries illustrate a nice feature of the CAS devices. Many times we can use unassigned variables to display definitions. This is not possible on non-CAS devices since variables must be assigned a value. With the second entry on CAS models, we need to make one note. " $-a$ " result is not a negative number, but the opposite of the number inside the absolute value notation. Since $a < 0$, the number inside the notation must be negative and the opposite of the negative would be a positive. " $-a$ " result therefore represents a positive number.

With all TI-Nspire devices we will need to use the absolute value template. For the final entry, press **enter** to verify the absolute value technique of making a negative number positive by taking its opposite. The parentheses were added to emphasize that we were taking the opposite of negative 3. With a TI-Nspire device the parentheses are gone when we press **enter** since they are not necessary. We will need to get use to the difference between how we normally write expressions manually and how the TI-Nspire devices like to display those expressions. Both The TI-Nspire device's input and results are often displayed differently then the way we entered them or would manually display the results.

Absolute Value and Distance

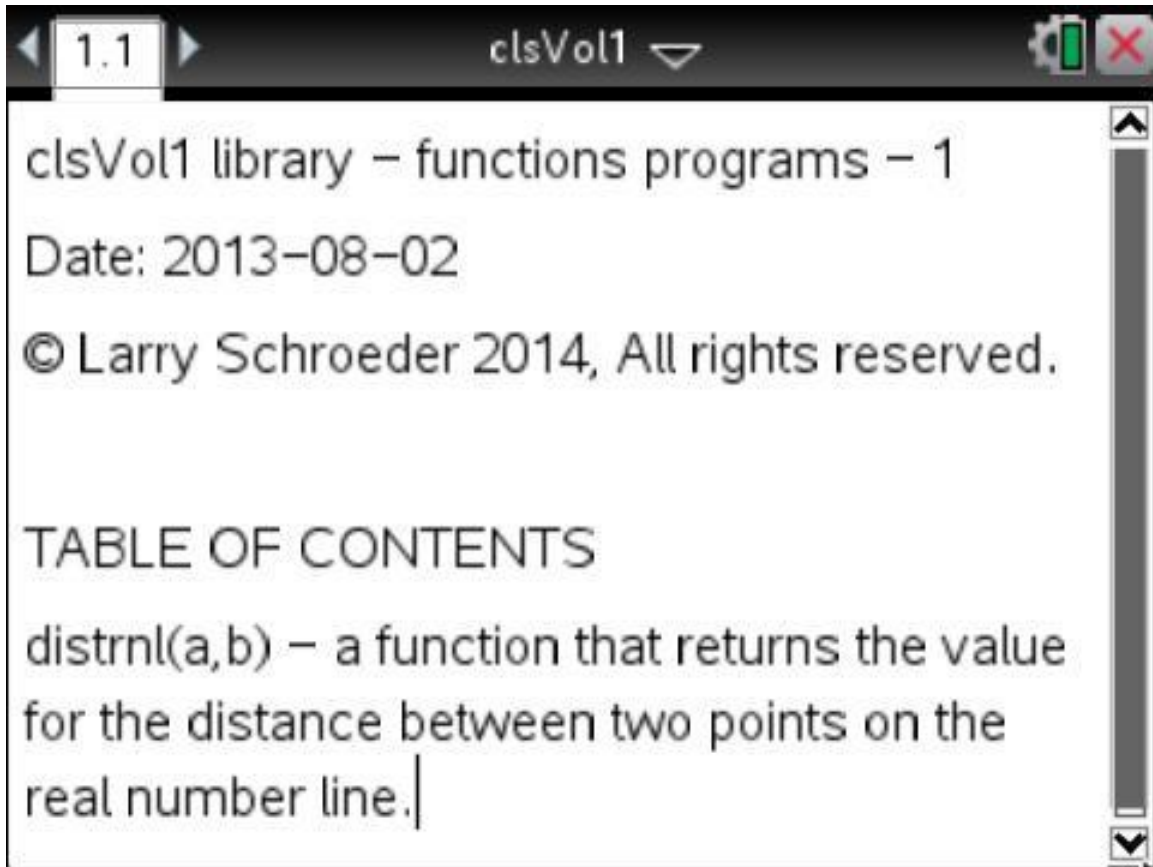
What is distance between two points, -2 and 5, on the real number line? Our intuition say the distance is 7. We can calculate this by finding either $|5 - (-2)| = 7$ or $|(-2) - 5| = 7$. This leads us to the following definition.

Distance between Points on the Real Line

For any real numbers, a and b , then the distance between the points a and b on the real line is

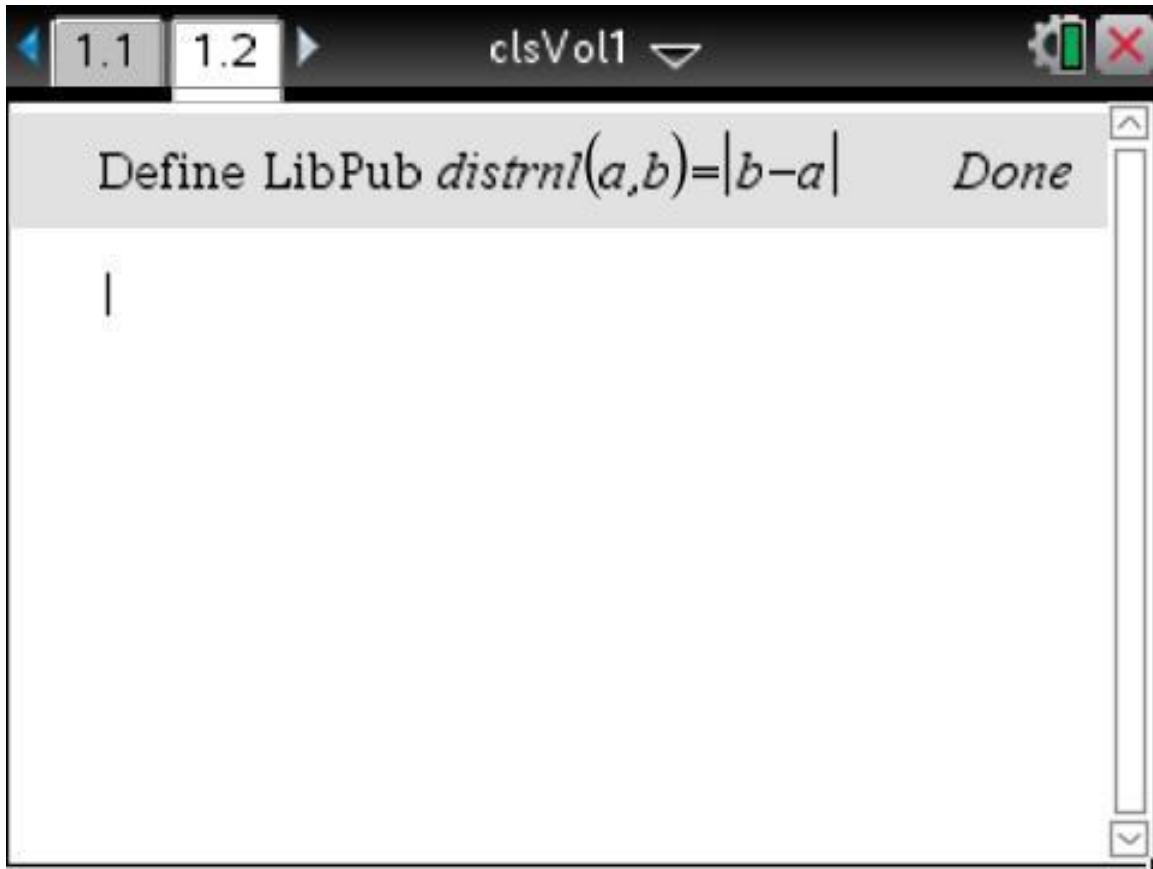
$$d(a,b) = |b - a|$$

TI-Nspire Family Creating the “clsVol1” Function Library - Entering the Function “distrnl” in the Library File and Saving the Library File in the “MyLib” Folder



Begin by creating a new Document and adding a Note page. For our libraries we like to key in the information shown. The 1 in line 1 is how many programs are currently in this library. It will be 1 when we are done here. **ctrl Catalog** icon gave a palette where you will find the copyright symbol. On the iPad press the catalog icon use **Utilities > Symbols** to find the copyright symbol. To keep things simple you could just key in “clsVol1 library” for now and add the other information later. We are giving the file a unique name, **clsVol1**. The “cls” prefix is our company name “Computer Learning Service”. If the library would only work with TI-Nspire CAS we would add the suffix “cas”.

Keep My Changes/ctrl S (Save) the file as **clsVol1**.

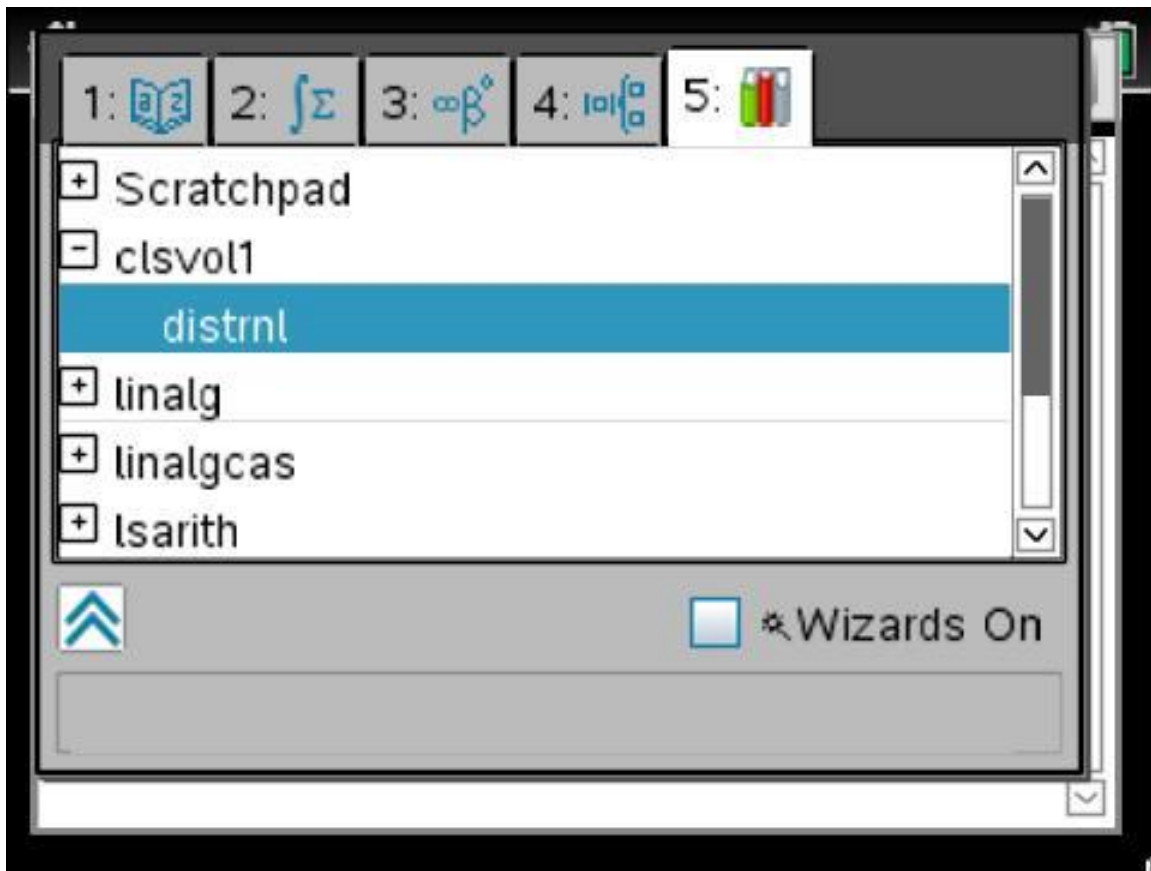


Add a Calculator page. Use **Tools/menu > Actions > Library > Define LibPub (Show in Catalog)**. Key in as shown and press **enter**.

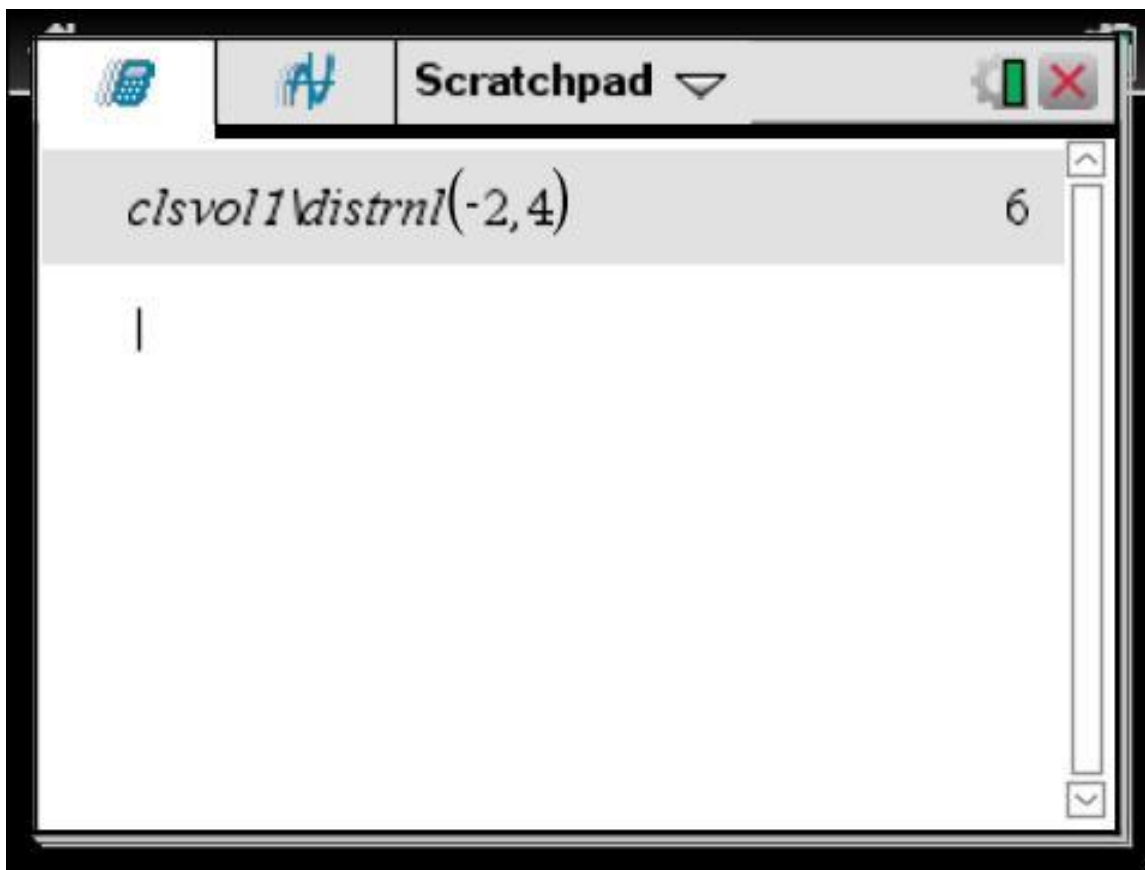
Keep My Changes/ctrl S (Save) the file as **clsVol1**. On the iPad make sure under the Home “clog” icon preferences that **Show MyLib Folder** is turned on. On the iPad move the **clsVol1** file to the MyLib’s folder. On non-iPad file systems, if you did not save the file to MyLib, move it there now.

Close the **clsVol1** file. On the handheld open up the Scratchpad or a new document Calculator page. On the iPad a new document Calculator page. Another option would be to leave the **clsVol1** file open and add a new Calculator page.

Use **Tools/menu > Actions > Library > Refresh Libraries**. On the handheld we can alternately use **doc > Refresh Libraries**. If using the computer software I found the doc refresh option to be the easiest.



Press the Catalog key and choose the last tab. Expand the “clsvol1” library and select “distnl”. The TI-Nspire will be tab 5 and the TI-Nspire CAS will be tab 6. On the iPad press the catalog icon use **Utilities > Libraries > clsvol1 > distnl** and then **Done**.



Key in “-2,4” and press **enter**. An alternative to the Catalog key, last tab sequence, or iPad’s Catalog icon, is to type the library name, back slash, function name. On the iPad key in “(-2,4)” and press **enter**. Remember the iPad has no calculator Scratchpad. On the iPad we need to create a document, calculator page, do the entries, and discard when done. With the iPad’s virtual keyboard it is easy to directly type the path\command.

The function “distrnl”, distance real number line, works equally as well on both the TI-Nspire and TI-Nspire CAS. The variables defined in the function are being assigned the values in the calling statement.

Remark: Adding a Global one-line function is one of the most powerful features of the TI-Nspire family. We can add any command or function that is missing, nest it, and use it in an expression with existing and other new commands or functions. We show how to use these techniques in the [compound fraction section](#) of this tutorial.

As math content is added to this volume and the series, we will utilize Global functions and Global programs to extend the TI-Nspire’s power and facilitate our understanding. We will use Global functions and Global programs to add some very powerful and useful commands, functions, and programs to the built-in commands, functions, and capabilities of the TI-Nspire and TI-Nspire CAS.

To further illustrate Global one-line functions, we take a look at arithmetic sequences. We [add a Library](#) of arithmetic sequence formulas. This Library gives us a model for adding Global one-line functions for formulas. Use this Library’s model

to build and add formulas to your own Libraries. Beside math Libraries build your own Libraries of formulas from any subject area.

Note: An alternative to adding Global one-line functions to a Library using **Define LibPub** (Show in Catalog) is to use the Program Editor. Using the Program Editor is a multi-part process. We need to create and store the program or function. This step is important as saving the document does not store the program or function. Storing and saving are two separate processes. We then need to save the file that it is in to the MyLib folder and refresh the libraries. Using the Program Editor is necessary for multiple line functions and programs. It can be used for Global one-line functions but due to the two separate process of storing and saving and complexity of using its template we recommend the use of the **Define LibPub** (Show in Catalog) command.

TI's Reference Manual list the non-block form as **Define Lib Pub** *Function (Param1, Param2, ...)= Expression*. Rather than using the Program Editor block form use this non-block form. This is one-part process, no need to store, just save in the MyLib folder and refresh the libraries.

We should mention that the TI-Nspire can create Local functions but their scope is limited to the current problem and thus their value is limited. We therefore generally like to create Global functions that are not limited to the current problem.

When adding several related Global [one-line functions](#) to a page or using the Program Editor to add program such as "[snotation](#)" or importing as "[decnotation](#)" using TI-Nspire Computer Software or virtual Keypads of the TI-Nspire iPad app make their entry easier. If using the TI-Nspire Computer Software for entry then use its transfer options to move the file to the handheld. If moving the file from the TI-Nspire iPad app use a combination of Google Drive and TI-Nspire Computer Software to move the files between TI-Nspire devices.

We have the Google Drive software on the iPad and the Computer. On the iPad Google's app is called **Drive**.

Tip: To move a tns file created on our Computer or transferred from a handheld to the Computer to the iPad app, we copy the tns file from the Computer's directory to the desired directory of the Google Drive. Then open the **Drive** app on the iPad and navigate to that directory. Selecting the tns file gives us an option Open in... our iPad's Ti-Nspire CAS or Ti-Nspire app. Open and work with the file. For a Library file, select **HOME** on the iPad's Ti-Nspire CAS or Ti-Nspire app and move the file to the MyLib directory. Refresh the Libraries.

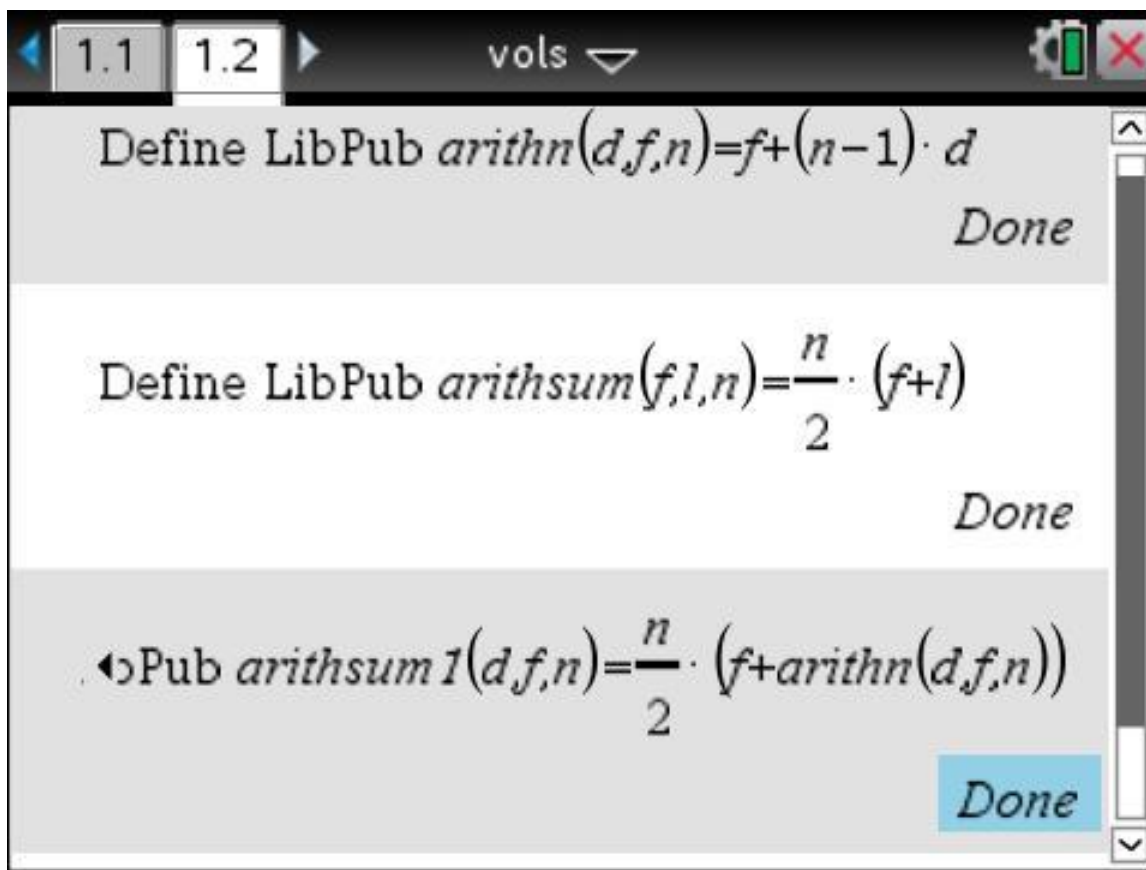
To move a tns file from the iPad's Ti-Nspire CAS or Ti-Nspire app to the Computer, touch the TI-Nspire iPad Toolbar's **Open in** icon, drawing of an up arrow in a box. The Ti-Nspire CAS or TI-Nspire iPad then has an option to Open in **Drive**. Touch the Open in **Drive** icon. The **Upload Item** dialog's window opens. Touch **Upload**, the tns file we are using will be placed in the root directory of the Google Drive. To move to a different directory we use the Computer's Google Drive.

Later in the compound fraction section of this tutorial we will [add a Library](#) dealing with slope of a secant line, equation of a line, equation of a secant line, slope of a tangent line, and equation of a tangent line. Its Library functions demonstrate how Global one-line functions can help with the understanding of complex math concepts. We discuss in the Library's creation how [enhancements](#) such as using Private functions and providing Global function of one Library to another Library can be made.

We then [add another group](#) of functions to this Library. The additional functions include first and second derivative, x -intercepts, minimums and maximums, inflection points, places where a graph is increasing or decreasing, and the concavity of a graph. The additional functions generate a set of tools that can be used to help with graphing and in solving optimization problems.

The series' primary goal is giving us a path to learn math. Revealing the commands and functions of the TI-Nspire and TI-Nspire CAS and learning how to create and use Libraries that extend the TI-Nspire and TI-Nspire CAS's power and usefulness is the other major goal of the series.

TI-Nspire Family Arithmetic Sequences



Here we show the three functions from the MyLib "vols" file. The functions are used for Arithmetic Sequences or also known as Arithmetic Progressions. We probably should have added our prefix

“cls,” Computer Learning Service, to insure the uniqueness of the Library name. A simple Library name shortens the pathname of global one-line functions, which allows to key-in the full pathname\command with fewer keystrokes.

Begin by creating a new Document and adding a Note page (not shown – tab 1.1). Remember that for our libraries on the [note page](#) we like to key in the information such as library name, number of functions and programs, date, copyright notice, and table of contents with a brief description of each function and program. For now, you may only want to key in the first line, “vols library – function programs -3”, and add the other items later.

On the handheld use **ctrl I** to insert a Calculator page. On the iPad **Add** a Calculator page. Use **Tools/menu > Actions > Library > Define LibPub (Show in Catalog)**, key in the remainder of the first entry as shown. Repeat for the second and third entry.

Keep My Changes/ctrl S (Save) the file as **vols**. On the iPad make sure under the Home “clog” icon preferences that **Show MyLib Folder** is turned on. On the iPad move the **vols** file to the MyLib’s folder. On non-iPad file systems, if you did not save the file to MyLib, move it there now.

Close the **vols** file. On the handheld open up the Scratchpad or a new document Calculator page. On the iPad a new document Calculator page. Another option would be to leave the **vols** file open and add a new Calculator page.

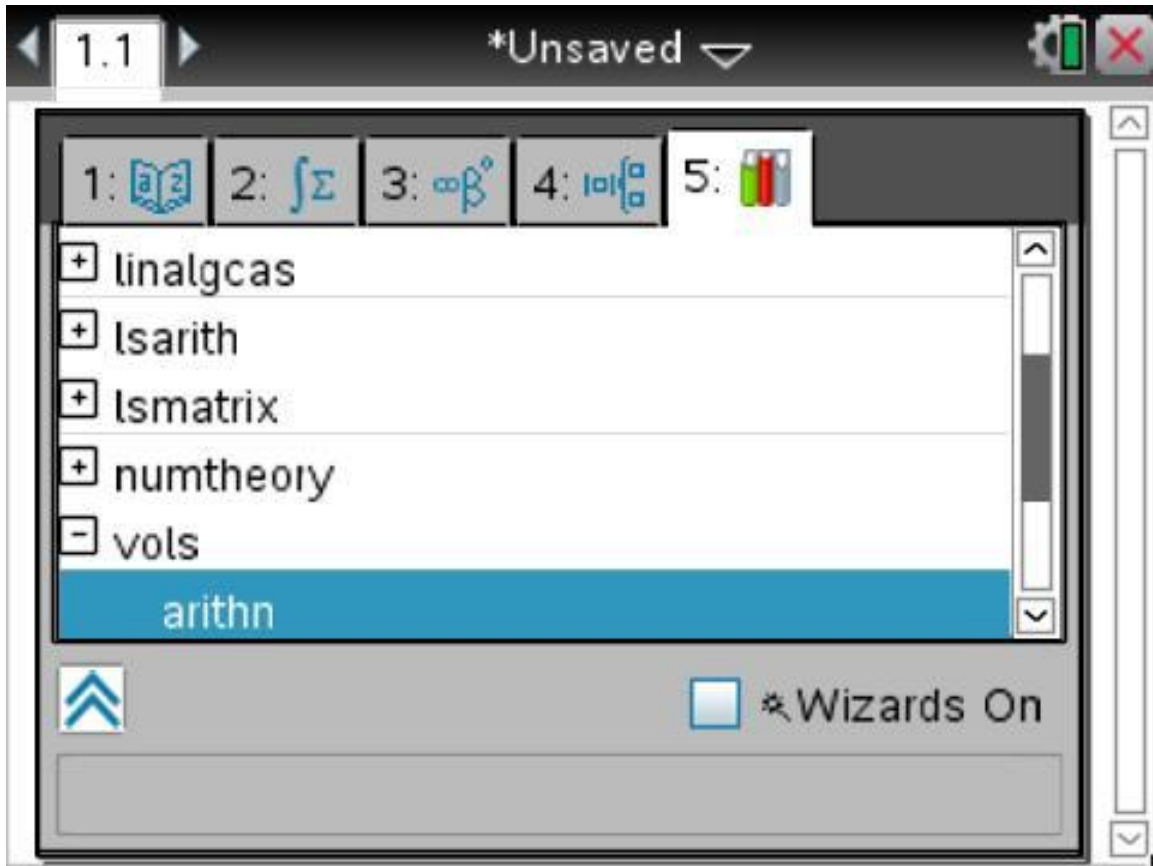
Use **Tools/menu > Actions > Library > Refresh Libraries**. On the handheld we can alternately use **doc > Refresh Libraries**. With the computer software I found the doc refresh option to be the easiest.

The functions in the Library shown above are © Larry Schroeder 2015. All rights reserved.

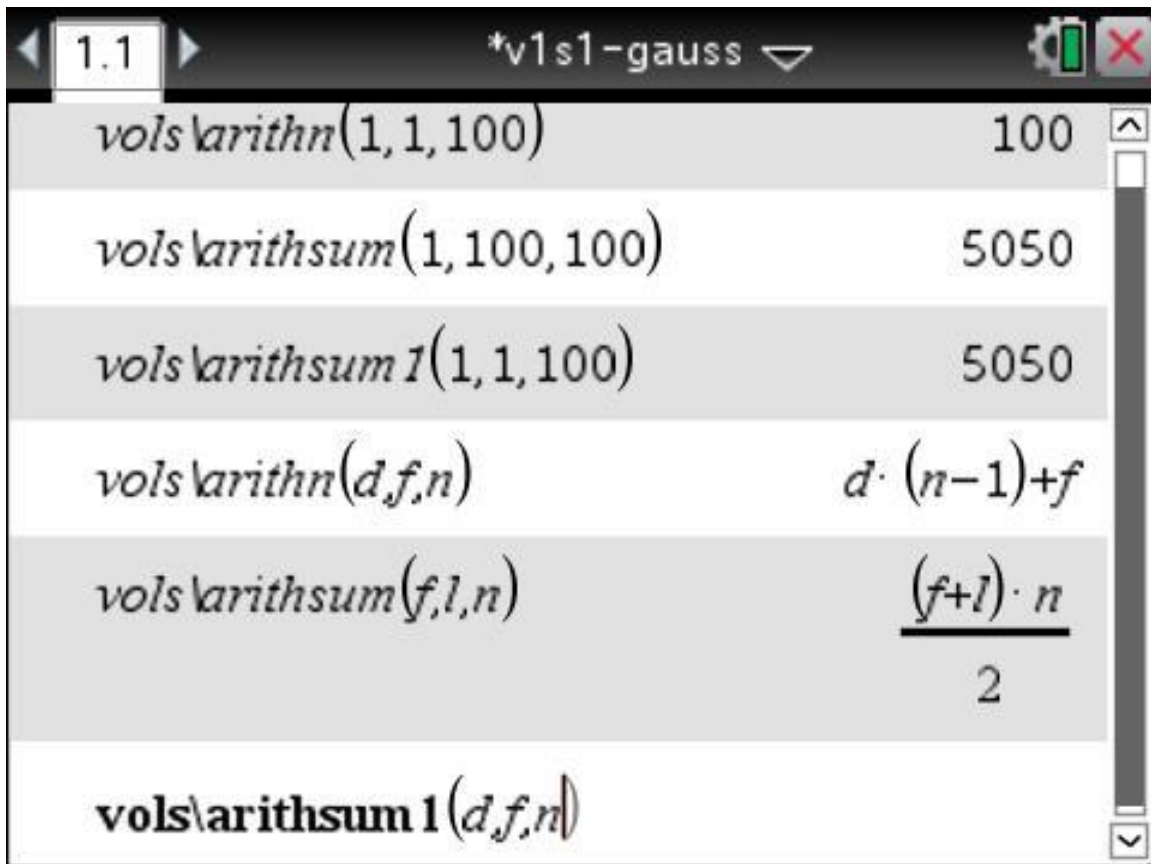
The last entry is an example of how a new function can be added that uses an existing function. We could of used $2 \cdot f + (n - 1) \cdot d$ instead of $f + \mathbf{arithn}(d, f, n)$ expression but we wanted to demonstrate how new functions can be made up of built-in and/or user created functions. If before entering arithsum1, we had previously refreshed the libraries, **arithn** function would show up bold until we completed the entry. Pressing enter would change the bold back to italics as shown in the screenshot.

Note: We choose to list the parameters in alphabetical order, d is the common difference, f is the first term, l is the last term, and n is the number of terms. The order of the parameters is up to us. We will use a logical order if that makes sense for the function or alphabetical order in functions where no logical order exists.

Next we apply this to Gauss’s famous example. Our function “arithsum(f, l, n)” formalizes the technique that Gauss used.



Begin by creating a new Document and adding a Calculator page. On the handheld, use the Catalog key and go to the last tab. Scroll down and select "vols". Expand it and select "arithn". Press **enter**. This screen is from TI-Nspire. The TI-Nspire will be tab 5 and the TI-Nspire CAS will be tab 6. On the iPad press the catalog icon use **Utilities > Libraries > vols > arithn** and then **Done**.



Enter the parameters 1,1,100 as shown in the first entry of the screenshot followed by **enter**. On the iPad we will need to enter the parenthesis as well.

Edit a previous entry or repeat the Catalog procedure for other entries. Alternately we could type entries directly. Do the next two entries.

On the TI-Nspire, use **Keep/Save** the file as **v1s1-gauss** to save the first three entries. We optionally can **Keep/Save** on the TI-Nspire CAS. When we are putting multiple complex statements on a page it is a good idea to save the results periodically.

For TI-Nspire CAS devices do entries four, five, and six. Pressing **enter** on the TI-Nspire CAS to entry six will display the result as the expression, $\{[d(n-1)+2f]n\}/2$.

On the TI-Nspire CAS, use **Keep/Save** the file as **v1s1-gauss** to save all six entries. If we have previously saved use **Keep My Changes/ctrl S**.

Entries one and three show a common difference of 1, a first term of 1, and 100 as the number of terms. Entry two shows a first term of 1, last term of 100, and 100 as the number of terms.

Entries four, five, and six will only work with the TI-Nspire CAS. Entries four and five can be used to display our formulas.

Entry six's result will be the result, $\{[d(n-1)+2f]n\}/2$, of our nested function, not the original expression, $n/2(f+\mathbf{arithn}(d,f,n))$, that we entered.

Gauss Anecdote

From Gauss's [Wikipedia Anecdotes](#), "Another famous story has it that in primary school after the young Gauss misbehaved, his teacher, J.G. Büttner, gave him a task: add a list of integers in arithmetic progression; as the story is most often told, these were the numbers from 1 to 100. The young Gauss reputedly produced the correct answer within seconds, to the astonishment of his teacher and his assistant Martin Bartels.

Gauss's presumed method was to realize that pairwise addition of terms from opposite ends of the list yielded identical intermediate sums: $1 + 100 = 101$, $2 + 99 = 101$, $3 + 98 = 101$, and so on, for a total sum of $50 \times 101 = 5050$. However, the details of the story are at best uncertain (see[30 in the article] for discussion of the original Wolfgang Sartorius von Waltershausen source and the changes in other versions); some authors, such as Joseph Rotman in his book *A first course in Abstract Algebra*, question whether it ever happened (*Carl Friedrich Gauss*).

Investigation

A modern use for the number line would be the measuring wheel. Agriculture uses this tool to measure lengths and widths of fields in determining yields per acre. A commercial model found on the Internet had the specifications listed as a circumference of 5 feet and a diameter of 19 inches. Divide these two numbers. What is this number close to?



Figure 5 Measuring wheel

$\pi = \text{Circumference} \div \text{diameter} \approx 3.14159$

$$\pi \approx 5 \cdot 12 \div 19 = 60 \div 19 = 60 \cdot \frac{1}{19} = \frac{60}{19} = 3 \frac{3}{19} \approx 3.15789$$

Unitary Analysis $\frac{5 \text{ ft}}{1} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{1}{19 \text{ in}} \approx 3.15789$

Identity for multiplication $a \cdot 1 = a$ $5 \text{ ft} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 60 \text{ in}$

Invert and multiply $a \div b = a \cdot \frac{1}{b}$ $60 \div 19 = 60 \cdot \frac{1}{19}$

What does it mean to divide these two numbers?

We are looking at a mathematical constant π that is the ratio of a circle's circumference to its diameter, which is approximately equal to 3.14159. When comparing numbers in the ratio, we recognize that they must be the same units. Thus, we change the circumference to inches, since it is the easier unit to change. Remember π is an irrational number, which means we are unable to express it actually as the ratio of two integers. The more common approximate ratio that people use is $22/7$ where the specification for our measuring wheel turns out to be $60/19$.

To mechanically work this problem we take advantage of unitary analysis, the identity property for multiplication, and that division is the same as multiplying by the reciprocal.

Unitary analysis is where we setup the units to get the desired unit(s) or number needed in the outcome. We do not want to directly divide 5 by 19. But change the 5 feet to 60 inches and then divide by 19 inches.

In the next section we discuss the Operations and Properties of Real numbers. We will use two of these properties now.

First we use the [identity property](#) for multiplication, $a \times 1 = a$. Here we multiply 5 feet by 1 to get 60 inches, the identical amount but using a different unit of measure. The 1 is $12\text{in}/1\text{ft}$. To understand why this is 1, consider the number and unit of measure as a whole. Thus both 12in and 1ft are the same, and when we have the

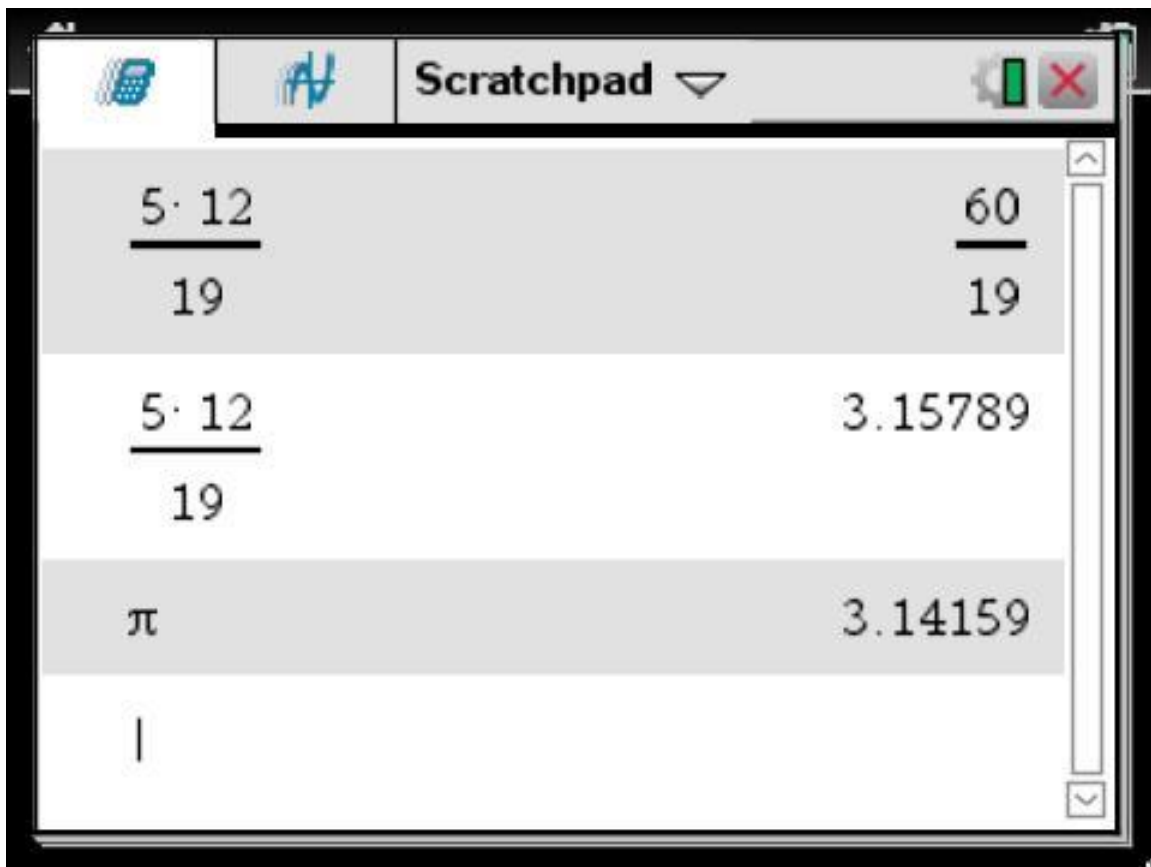
same over the same the result is 1. Keeping track of the units and canceling the units helps to perform the correct conversion. Common mistakes in conversions like this would be to divide by 12 rather than multiply by 12. Using units in the calculation prevents this kind of error.

Next, to divide we used the rule that we can [multiply by the reciprocal](#). Commonly we think of this with fractions, “Invert and multiply”. Historically math education has varied between various philosophies of how to accomplish skills and understanding. Hopefully rules such as “Invert and multiply” are preceded by a proper discussion that teaches learners the mathematics behind the rule not just a mechanical way to do it.

In learning mathematics, we need to understand that writing $60/19$ as a fraction is just a way to specify division of 60 by 19. Pressing TI-Nspire divide key displays the problem as a fraction. Thus, the understanding between fraction notation and division is essential to working with the TI-Nspire.

Understanding and mechanical techniques both play essential roles in learning and using mathematics.

TI-Nspire Family Pi Approximation



Begin by selecting the **Scratchpad** key on the handheld. On the iPad create a document, calculator

page, do the entries, and discard when done.

Use the handheld touchpad to activate and move the mouse pointer, select the Down arrow icon to right of word Scratchpad at top of screen and select **Clear Scratchpad** from the Scratchpad's menu or using the **menu** key select **menu > Actions > Clear History** to clear the handheld's calculation workspace.

Key in as shown. This screen was taken from the TI-Nspire CX model. Pressing **enter** gives the answer as a fraction. With all TI-Nspire devices we will need to use the approximate key, **ctrl enter** on the handheld and **long press** on the iPad app to see the fraction as a decimal. Pressing **enter** switches the entry from its original form of $5 \times 12 / 19$ to a fractional form. Note the use of left to right precedence for multiplication and division. π is shown for comparison purposes. CAS devices will need to use the approximate key.

The Great Pyramid appears to have π associated with a lot of its measurements and calculations. Use a search engine to investigate π in general and the occurrence of π in The Great Pyramid's measurements and calculations. It is theorized that using a wheel for measurements was what caused π to be in many of the pyramid's results. It is generally considered that the existence of π was unknown to the Egyptians at this time.



Astronaut photograph ISS007-E-12915 was taken August 18, 2003 by Cynthia A Evans, Great Pyramids as seen from the International Space Station.

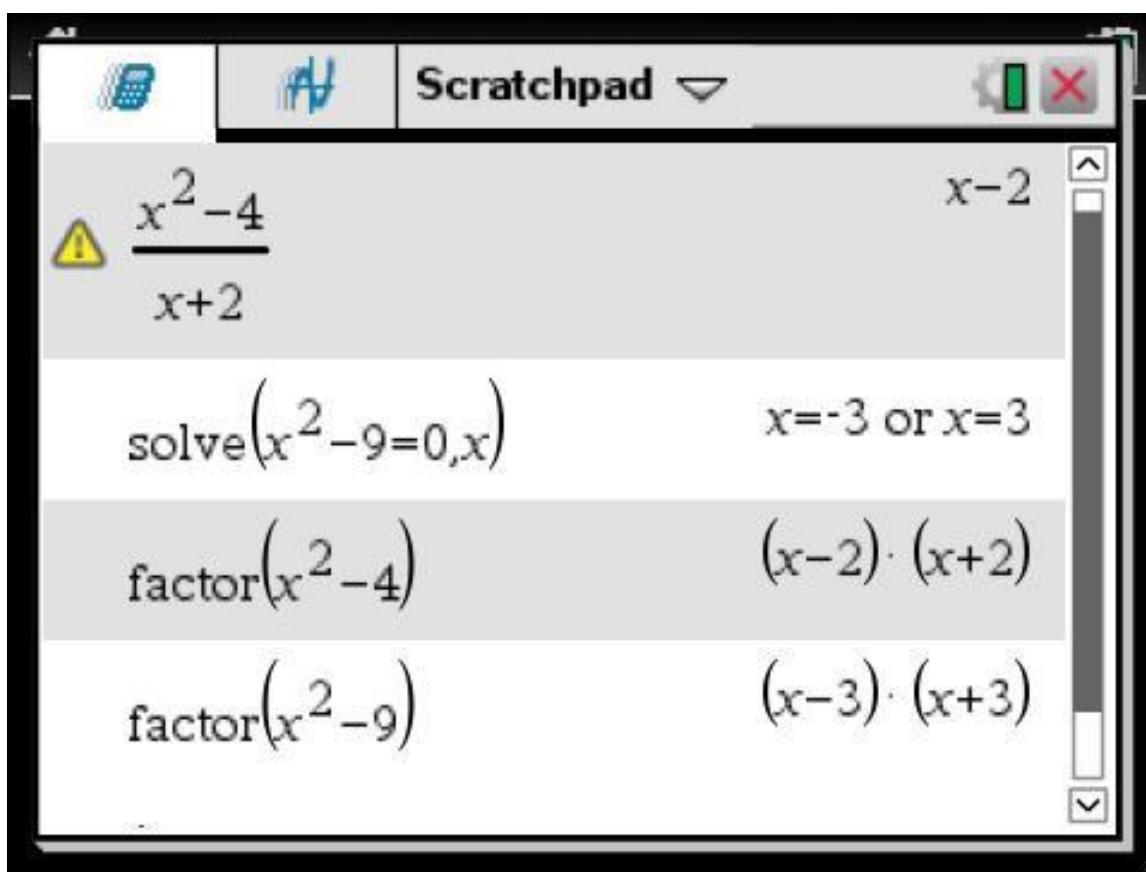
Explanation 1.6 - Factoring

In this section we discuss **factoring**. We will go over common factors, factoring a trinomial, factor by grouping, and special factoring formulas.

Factoring is where we express the sum of terms as a product. Remember expand and factor are the opposite directions of the distributive property. Factoring is the reverse of expand.

Factoring is used to reduce complicated expressions to several simpler expressions or a single expression, which can aid us in simplifying expressions and finding solutions to algebraic equations.

TI-Nspire CAS Simplifying Expressions – Solving Algebraic Equations



The screenshot shows the TI-Nspire CAS Scratchpad interface. At the top, there are icons for a calculator, a handheld device, and the word "Scratchpad" with a dropdown arrow. To the right are window control icons (green, red, and a close button). The main area contains four rows of mathematical entries:

$\frac{x^2-4}{x+2}$	$x-2$
$\text{solve}(x^2-9=0,x)$	$x=-3$ or $x=3$
$\text{factor}(x^2-4)$	$(x-2) \cdot (x+2)$
$\text{factor}(x^2-9)$	$(x-3) \cdot (x+3)$

Begin by selecting the **Scratchpad** key on the handheld. On the iPad create a document, calculator page, do the entries, and discard when done.

Use the handheld touchpad to activate and move the mouse pointer, select the Down arrow icon to right of word Scratchpad at top of screen and select **Clear Scratchpad** from the Scratchpad's menu or using the **menu** key select **menu > Actions > Clear History** to clear the handheld's calculation workspace.

Key in as shown. Notice the warning in entry one since -2 is not allowed in the original expression but is allowed in the result. This use of factoring in expression simplification will be covered in the next section.

Use **Tools/menu > Algebra > Solve** to show the solution of an algebraic equation containing a polynomial. We will be covering the use of factoring to solve algebraic equations in the next tutorial. For now, we will say if the product $ab = 0$ then either $a = 0$ or $b = 0$ or both a and b equal 0, thus the need to use factoring to change the algebraic expression to products of simpler expressions.

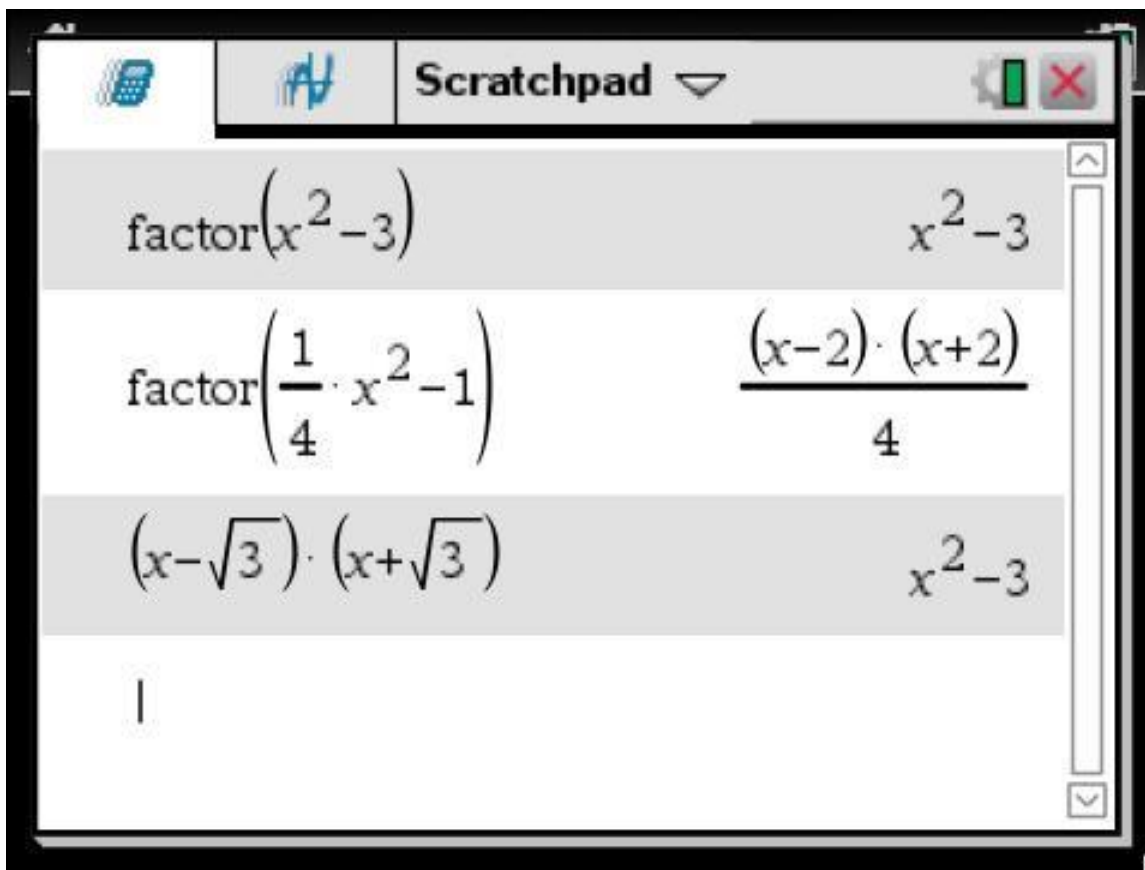
In entries three and four we use **Tools/menu > Algebra > Factor** to show the sum of terms as products.

Note: For the TI-Nspire CAS simplifications and algebraic solutions there is no need to use factor. We displayed entries three and four to help us see the processes that we use in manually simplify and solving algebraic equations.

Each term of a polynomial can be thought of as $a_n x^n$ with n a non-negative integer. The a_n is considered the **coefficient** of that term. Factoring requires us to specify the number system that the coefficients will be using. Unless otherwise stated, we will restrict our manual factoring to factoring with integer coefficients. Polynomials that cannot be factored using integer coefficients are **prime**, or **irreducible** over the integers. When we talk about prime or irreducible, we need to talk about a set of numbers. Factoring is the process where we express a polynomial as the product of irreducible polynomials.

In our next screenshot we show a polynomial that is irreducible over the rational numbers but not over real. The TI-Nspire CAS when using the **factor** command only allows factoring with rational results. Thus $x^2 - 3$ is irreducible over the rational for the TI-Nspire CAS. Entry two shows us that the TI-Nspire CAS is able to factor rational coefficients. Entry three shows that $x^2 - 3$ can be obtained by multiplying two real factors even though the TI-Nspire CAS cannot factor over real coefficients.

TI-Nspire CAS Factor Command



Begin by selecting the **Scratchpad** key on the handheld. On the iPad create a document, calculator page, do the entries, and discard when done.

Use the handheld touchpad to activate and move the mouse pointer, select the Down arrow icon to right of word Scratchpad at top of screen and select **Clear Scratchpad** from the Scratchpad's menu or using the **menu** key select **menu > Actions > Clear History** to clear the handheld's calculation workspace.

Key in as shown. Use the **Tools/menu > Algebra > Factor** command.

Notice in entry one that the TI-Nspire CAS gives back our original expression because the original expression is irreducible over rational numbers. The TI-Nspire CAS uses the rational coefficients for the **factor** command.

We see that entry two factors out one-fourth but writes it in the denominator. As we previously mentioned we will not manually factor rational values unless factoring rational coefficients is stated in the directions or there is a need to do so. Earlier we talked about [graphing a parabola](#). This would be a case where there is a need to factor out rational numbers if possible and then factor the remaining equivalent expression into rational coefficients.

Before we present our factoring rules let us take a look at how we could manually factor entry one and two in the previous screenshot. We will use [Difference of Squares](#) and [Common Factor](#).

(a) Factor the polynomial $x^2 - 3$ over reals

Radical rules $x^2 - 3$

Difference of squares $= x^2 - \sqrt{3}^2$
 $= (x - \sqrt{3})(x + \sqrt{3})$

(b) Factor the polynomial $\frac{1}{4}x^2 - 1$ over rationals

Multiply by 1 $\frac{1}{4}x^2 - 1$

Distributive - factor $\frac{1}{4} = \frac{1}{4}x^2 - 1 \cdot \frac{4}{4}$

Difference of squares $= \frac{1}{4}(x^2 - 4)$
 $= \frac{1}{4}(x - 2)(x + 2)$

Part (a) looks at manually factoring using real numbers. We make the second term a perfect square by using the rules of radicals. The second term would be the square root of 3 raised to the second power. After making it a perfect square we use the special factor formula for difference of two squares to change the expression to factored form. The special factor formulas are discussed at the end of this section.

Part (b) looks at manually factoring using rational numbers. If the manual directions for the expression stated that we were to factor with rational numbers we would need to factor out one-fourth from both terms. To manually do this we would need to multiply the -1 term by $\frac{4}{4}$ so we could factor out one-fourth from both the first and second terms. Multiplying by $\frac{4}{4}$ is an example of using the identity for multiplication, $a \times 1 = a$, with the 1 being $\frac{4}{4}$. The identity properties of multiplying by 1 and adding 0 are often used to change expressions to an equivalent expression in a form that allows us to complete the solution. After factoring out the one-fourth we again use the special factor formula for difference of two squares to change the expression to factored form.

Entry three used the expand component of the distributive property. Since each factor has radicals it is not necessary to use the TI-Nspire CAS **expand** command.

Reading this entry in reverse shows us that under the real numbers $x^2 - 3$ is factorable by using radicals.

Common Factors

When factoring polynomials it is advisable to factor common factors first. We look at each term of the polynomial to see if there are factors common to all the terms. We factored out the smallest non-zero exponent found in any term. The product of these is the **greatest common factor (gcf)**.

Greatest Common Factor

The **greatest common factor** is the product of the factors that appear in all terms. The smallest non-zero exponent found in any term is factored out.

When factoring polynomials with two or more terms, it is advantageous for leading coefficient to be positive. To accomplish this factor a -1 out of every term.

Example 1 Common Factors

(a) Factor the polynomial $3x^2 - 6$

$$\begin{aligned} \text{3 is the gcf} \quad & 3x^2 - 6 \\ \text{Distributive - factor} \quad & = 3x^2 - 3 \cdot 2 \\ & = 3(x^2 - 2) \end{aligned}$$

(b) Factor the polynomial $-3x^3 + 9x^2 - 18x$

$$\begin{aligned} \text{Factor -1} \quad & -3x^3 + 9x^2 - 18x \\ \text{3x is the gcf} \quad & = -1(3x^3 - 9x^2 + 18x) \\ \text{Distributive - factor} \quad & = -1(3x \cdot x^2 - 3x \cdot 3x + 3x \cdot 6) \\ & = -3x(x^2 - 3x + 6) \end{aligned}$$

When factoring we need to factor completely. In part (a) we ended up with a constant times a prime polynomial. It is prime since unless stated otherwise, we are factoring over integers coefficients and there is no integer that squared gives us 2.

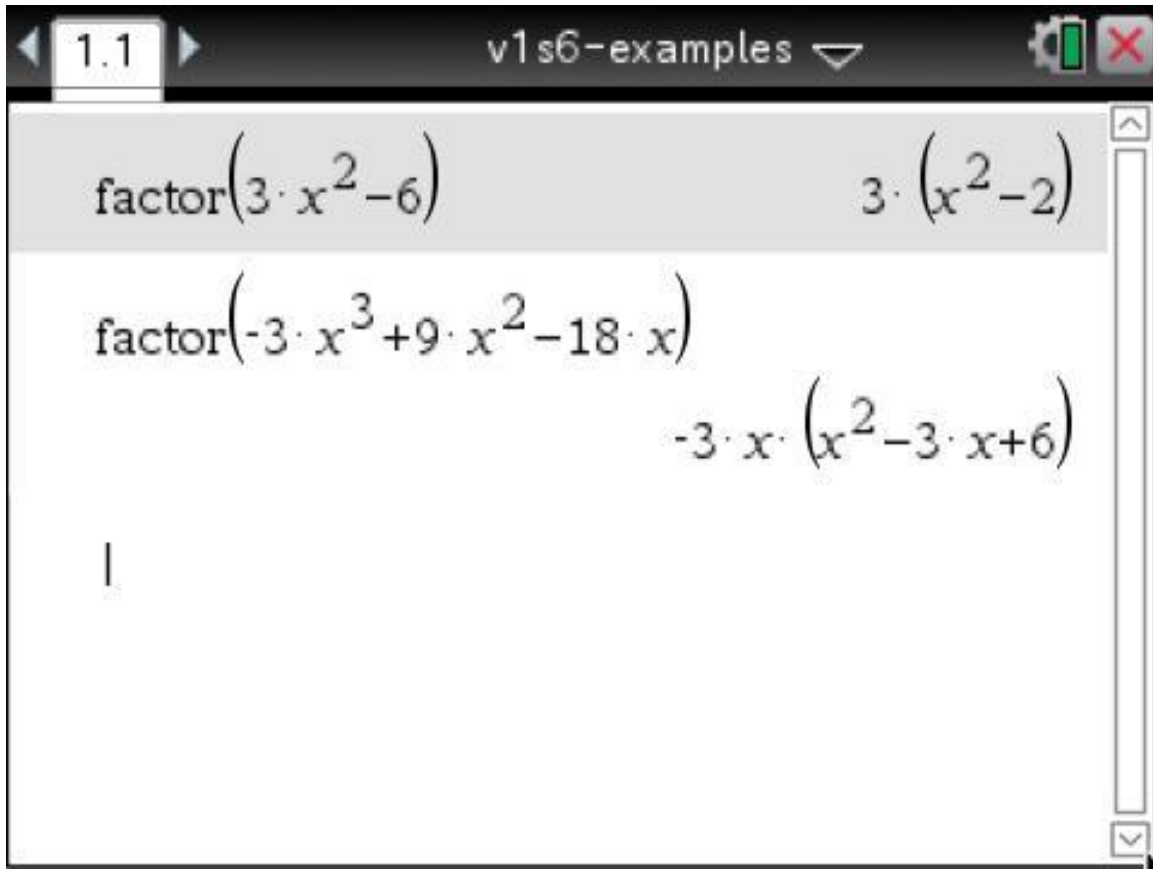
In part (b) we have a monomial times an irreducible trinomial. Trinomials of the form $x^2 + bx + c$ are prime, that is irreducible, when no factors using the FOIL technique produce it. Demonstrating that a trinomial is irreducible over integer coefficients will be discussed next.

Common Factors

1. Factor out the gcf.
2. Write as the product of gcf and a blank:
gcf · ()
3. Supply value for missing factor. Use the distributive expand mnemonic.

For common factors we work the problem mentally by placing the gcf in front of an empty set of parenthesis and think what we need to multiply the gcf by to get our original polynomial. We then use the full distributive property expand or our [distributive expand mnemonic](#) to aid us in supplying the terms of the missing factor. See exercise 1 for an example of this procedure.

TI-Nspire CAS Common Factors - Create the Example File



Begin by creating a new Document and adding a Calculator page. Key in the entries as shown followed by **enter**. Use the **Tools/menu > Algebra > Factor** command.

Keep My Changes/ctrl S (Save) the file as **v1s6-examples**.

Exercise 1 Common Factors

(a) Factor the polynomial $-4x - 12$

(b) Factor the polynomial $8x^3y - 16x^2y^2$

[Solution](#) >>

Factoring Trinomials

When dealing with two or more terms we commented earlier that it is advantageous for the leading coefficient to be positive. We find it advantageous to have trinomials in descending order. Some tutorials or textbooks will keep trinomials in ascending order if the x^2 coefficient is negative. We prefer to always keep ours in descending order.

Before using the **Factoring Trinomials** procedure shown next, remember to factor out common factors first. We like to break our trinomials $ax^2 + bx + c$, into the case

where $a = 1$ and case where $a > 1$. If a is negative we will factor out a -1 so that we end up with a trinomial that has an a being 1 or an a greater than 1.

Factoring Trinomials

ac Test

$$ax^2 + bx + c, a > 0$$

find $|a| \cdot |c|$

list positive pairs of factors of $|a| \cdot |c|$

i) If c positive, add the pair of factors

If b positive, each factor of the pair is positive

If b negative, each factor of the pair is negative

ii) If c negative, subtract the pair of factors

If b positive, largest positive, smallest negative

If b negative, largest negative, smallest positive

If no pairs work for i) or ii) then trinomial is prime.

If $a=1$ then we have the factors. If $a>1$ then we regroup bx using these values.

We will next look at the above type of trinomials with the leading coefficient being 1. There are four cases that factor as described by the ac test above. Everything else not having a b of the four cases will be prime.

Trinomials of this type with $a > 1$ can also be factored using this technique or they can be factored by trial and error. To use this technique with $a > 1$ we will split bx into the pair that allows us to factor by grouping.

We will cover the $a > 1$ type of trinomials plus other polynomials that factor using [grouping](#) in the next section. For the $a > 1$ type of trinomials, there are four cases that factor as described by the ac test above. Everything else not having a b of the

four cases will be prime.

If you wish to use trial and error for the $a > 1$ type of trinomials see online tutorials or a textbook that demonstrates how trial and error works.

Example 2 Factoring Trinomials

(a) Factor the polynomial $x^2 + 6x + 8$

$$|a| \cdot |c| = 1 \cdot 8 = 8 \quad \text{pairs: } 1, 8; 2, 4$$

$$c > 0, b > 0 \quad \text{add } 1 + 8 = 9; 2 + 4 = 6$$

$$\begin{aligned} \text{pair is positive} \quad x^2 + 6x + 8 \\ = (x + 2)(x + 4) \end{aligned}$$

(b) Factor the polynomial $36 - 12x - 3x^2$

$$\text{Factor } -3 \quad 36 - 12x - 3x^2$$

$$\begin{aligned} \text{Decending order} \quad &= -3(-12 + 4x + x^2) \\ &= -3(x^2 + 4x - 12) \end{aligned}$$

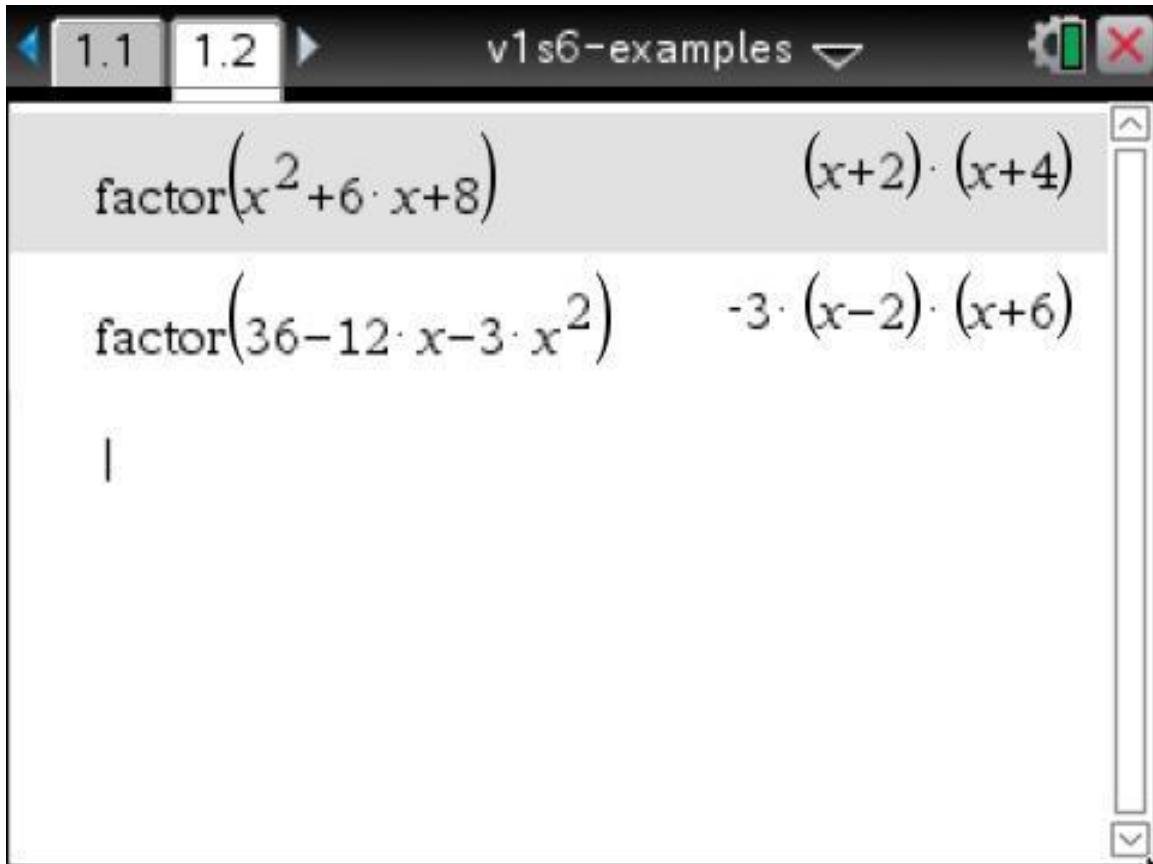
$$|a| \cdot |c| = 1 \cdot 12 = 12 \quad \text{pairs: } 1, 12; 2, 6; 3, 4$$

$$c < 0, b > 0 \quad \text{subtract } 12 - 1 = 11; 6 - 2 = 4; 4 - 3 = 1$$

$$\begin{aligned} \text{largest, positive} \quad &-3(x^2 + 4x - 12) \\ &= -3(x + 6)(x - 2) \end{aligned}$$

In part (b) by factoring out a -3, we perform both common factor of the terms and negative removal of the x^2 term in a single step. We complete the common factor mentally by filling in the blank parenthesis with $x^2 + 4x - 12$. We then use our Factoring Trinomial procedure on $x^2 + 4x - 12$.

TI-Nspire CAS Factoring Trinomials



Add a Calculator page. Key in the entries as shown followed by **enter**. Use the **Tools/menu > Algebra > Factor** command.

Keep My Changes/ctrl S (Save) the file as **v1s6-examples**.

Exercise 2 Factoring Trinomials

(a) Factor the polynomial $x^2 - 7x + 12$

(b) Factor the polynomial $2x^2 - 6x - 8$

[Solution](#) >>

Before considering factor by grouping, lets us look at algebraic expressions with fractional exponents. We will need to factor algebraic expressions with fractional exponents in later tutorials.

Factor the expression $x^{5/3} - 16x^{2/3} + 48x^{-1/3}$

Factor $x^{-1/3}$ $x^{5/3} - 16x^{2/3} + 48x^{-1/3}$
 $= x^{-1/3}(x^2 - 16x + 48)$

$|a| \cdot |c| = 1 \cdot 48 = 48$ pairs: 1,48; 2,24; 3,16; 4,12; 6,8

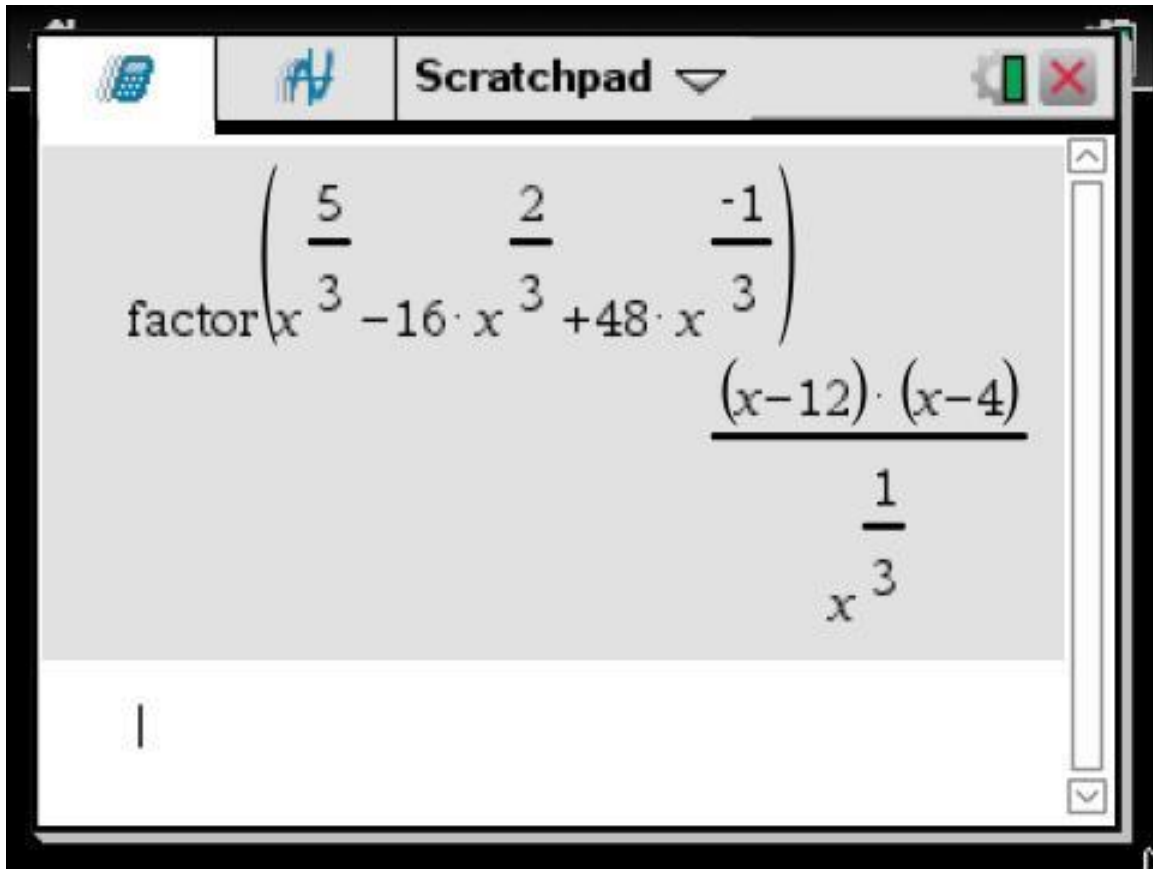
$c > 0, b < 0$ add $1 + 48 = 49$; $2 + 24 = 26$;
 $3 + 16 = 19$; $4 + 12 = 16$; $6 + 8 = 14$

pair is negative $x^{-1/3}(x^2 - 16x + 48)$
 $= x^{-1/3}(x - 4)(x - 12)$

We factored out the smallest power of x , $x^{-1/3}$. Factor using our standard procedure, use the original exponent subtracting the factored out exponent to find the exponent to use inside the parenthesis.

The first term's exponent of the second factor would be $5/3 - (-1/3) = 6/3 = 2$, the second term exponent would be $2/3 - (-1/3) = 3/3 = 1$ and the last term would have the whole $x^{-1/3}$ factored out. Use the Factor Trinomial procedure to finish the problem.

TI-Nspire CAS Expression with Fractional Exponents



Begin by selecting the **Scratchpad** key on the handheld. On the iPad create a document, calculator page, do the entries, and discard when done.

Use the handheld touchpad to activate and move the mouse pointer, select the Down arrow icon to right of word Scratchpad at top of screen and select **Clear Scratchpad** from the Scratchpad's menu or using the **menu** key select **menu > Actions > Clear History** to clear the handheld's calculation workspace.

Key in as shown. Use the **Tools/menu > Algebra > Factor** command. Notice the TI-Nspire CAS switches the negative exponent to the denominator.

The next type of problem is the result that occurs in calculus from using the Product Rule to take the derivative of the cube root of $4 + x$ times x , $(4 + x)^{1/3}x$. We then use factoring to simplify this result.

Factor the expression $(4+x)^{1/3} + \frac{1}{3} \cdot (4+x)^{-2/3} \cdot x$

Multiply by 1 $(4+x)^{1/3} + \frac{1}{3} \cdot (4+x)^{-2/3} \cdot x$

Factor $\frac{1}{3}(4+x)^{-2/3} = \frac{3}{3}(4+x)^{1/3} + \frac{1}{3} \cdot (4+x)^{-2/3} \cdot x$

Simplify $= \frac{1}{3}(4+x)^{-2/3} [3(4+x)^1 + x]$

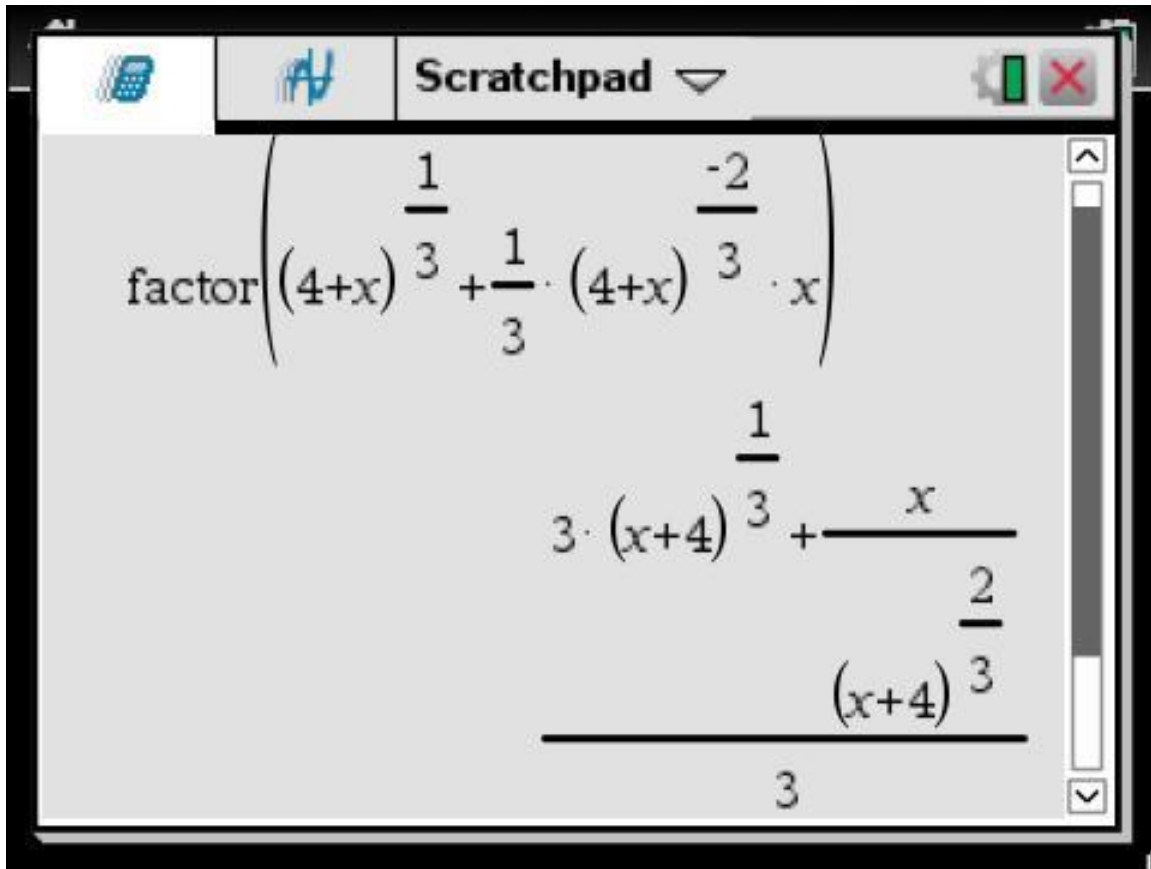
Factor 4 $= \frac{1}{3}(4+x)^{-2/3} (12+4x)$

$$= \frac{4}{3}(4+x)^{-2/3} (3+x)$$

We factored out the smallest power of $4+x$, $(4+x)^{-2/3}$. Use the original exponent subtracting the factored out exponent to find the exponent to use inside the parenthesis.

The first term's exponent of the second factor would be $1/3 - (-2/3) = 3/3 = 1$ and the last term would have the whole $1/3(4+x)^{-2/3}$ factored out leaving x . Simplify and factor out 4 to finish the problem.

TI-Nspire CAS Algebraic Expressions with Fractional Exponents



Begin by selecting the **Scratchpad** key on the handheld. On the iPad create a document, calculator page, do the entries, and discard when done.

Use the handheld touchpad to activate and move the mouse pointer, select the Down arrow icon to right of word Scratchpad at top of screen and select **Clear Scratchpad** from the Scratchpad's menu or using the **menu** key select **menu > Actions > Clear History** to clear the handheld's calculation workspace.

Key in as shown. Use the **Tools/menu > Algebra > Factor** command. As we see this answer slightly changed the form but did not help us with factoring. Although **Factor** worked will for the previous fractional exponent problem, we will shortly see that this kind of complex fraction multiple term problem needs a different TI-Nspire command.

In the next section we will cover [Compound Rational Expressions](#). Compound Rational Expressions are also known as compound fractions or complex fractions. Various forms exist, usually fractions inside of fractions or multiple complex terms.

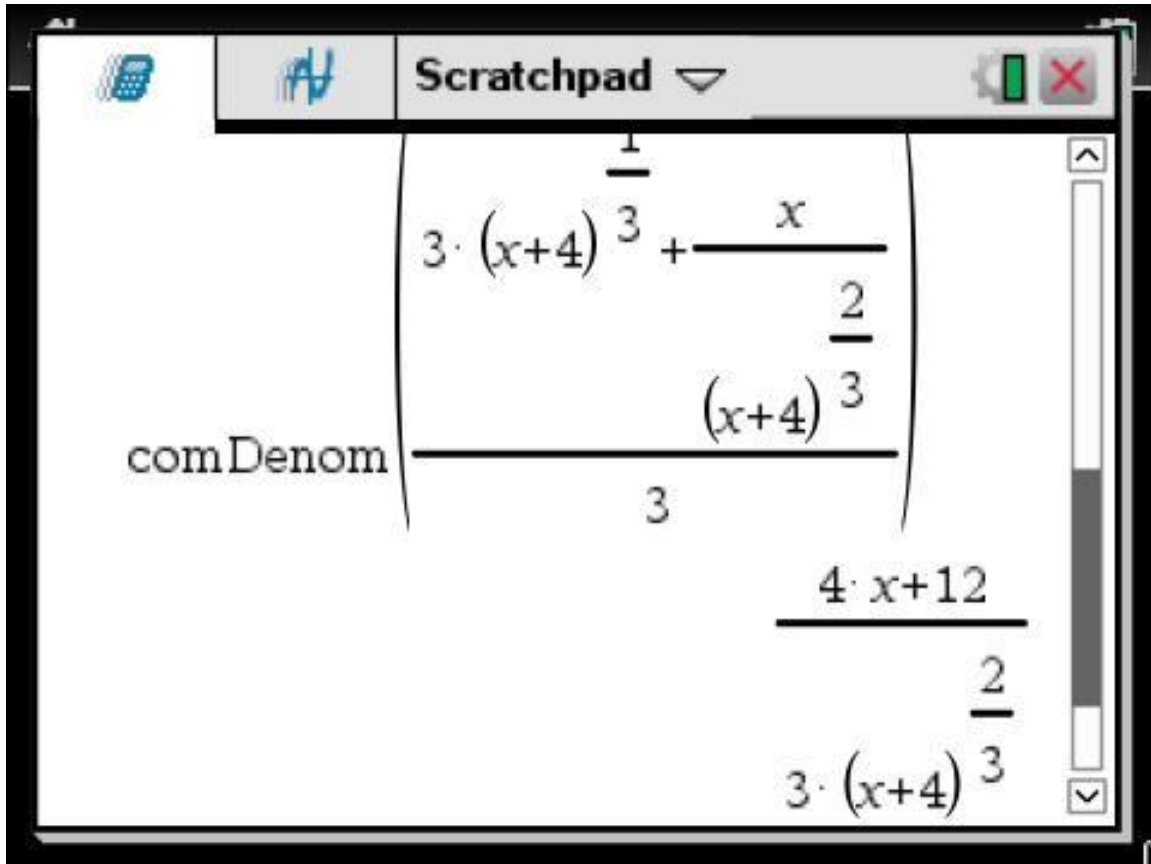
The problem in our screenshot is a multiple complex term problem.

There are three manual ways to deal with Compound Rational Expressions. Method one is to factor out negative, which is the manual technique that we used.

Method two and three normally deal with fractions inside of fractions. Method two and three use the least common denominator (lcd) and/or inverting and multiplying. Method two can be used with multiple complex terms problems as well.

To work our problem we need to use the TI-Nspire's **ComDenom** command. The TI-Nspire's **ComDenom** produces our manual factored result. The TI-Nspire CAS uses this method to solve Compound Rational Expressions. The TI-Nspire's **ComDenom** command can also be applied to a factored result that failed to factor. Either using it first or after **Factor** produces the same result.

We will cover manually working with [method 2 and 3](#) in the next section. For now we will just use TI-Nspire's **ComDenom** command.



Use the **Tools/menu > Algebra > Fraction Tools > Common Denominator** command. Use the **up arrow enter** combination to transfer the previous factored results to the command. Press **enter**. Notice that it places the negative exponent in the denominator and does not factor out the 4.

Factor by Grouping

We first take a general look at how to factor common binomial terms by grouping and then apply it to our Factoring Trinomial procedure for $a > 1$.

When a polynomial contains four or more terms we may be able to factor common binomial terms by grouping.

We will need to find a common binomial term by grouping the polynomial into two groups of two terms. Sometimes to factor we will have to rearrange the four terms.

Remember that the polynomial can turn out to be prime as well.

Example 3 Factor by Grouping

(a) Factor the polynomial $2x^3 - 2x^2 + 3x - 3$

$$\begin{aligned} \text{Group} & \quad 2x^3 - 2x^2 + 3x - 3 \\ \text{Factor } 2x^2, 3 & \quad = (2x^3 - 2x^2) + (3x - 3) \\ \text{Factor } x - 1 & \quad = 2x^2(x - 1) + 3(x - 1) \\ & \quad = (2x^2 + 3)(x - 1) \end{aligned}$$

(b) Factor the polynomial $9x^2 - 15x + 4$

$$\begin{aligned} |a| \cdot |c| &= 9 \cdot 4 = 36 \text{ pairs: } 1, 36; 2, 18; 3, 12; 4, 9; 6, 6 \\ c > 0, b < 0 & \text{ add } 36 + 1 = 37; 2 + 18 = 20; 3 + 12 = 15; \\ & 4 + 9 = 13; 6 + 6 = 12 \end{aligned}$$

$$\begin{aligned} \text{pair is negative} & \quad 9x^2 - 15x + 4 \\ \text{Factor } 3x, -4 & \quad = 9x^2 - 3x - 12x + 4 \\ \text{Factor } 3x - 1 & \quad = 3x(3x - 1) - 4(3x - 1) \\ & \quad = (3x - 4)(3x - 1) \end{aligned}$$

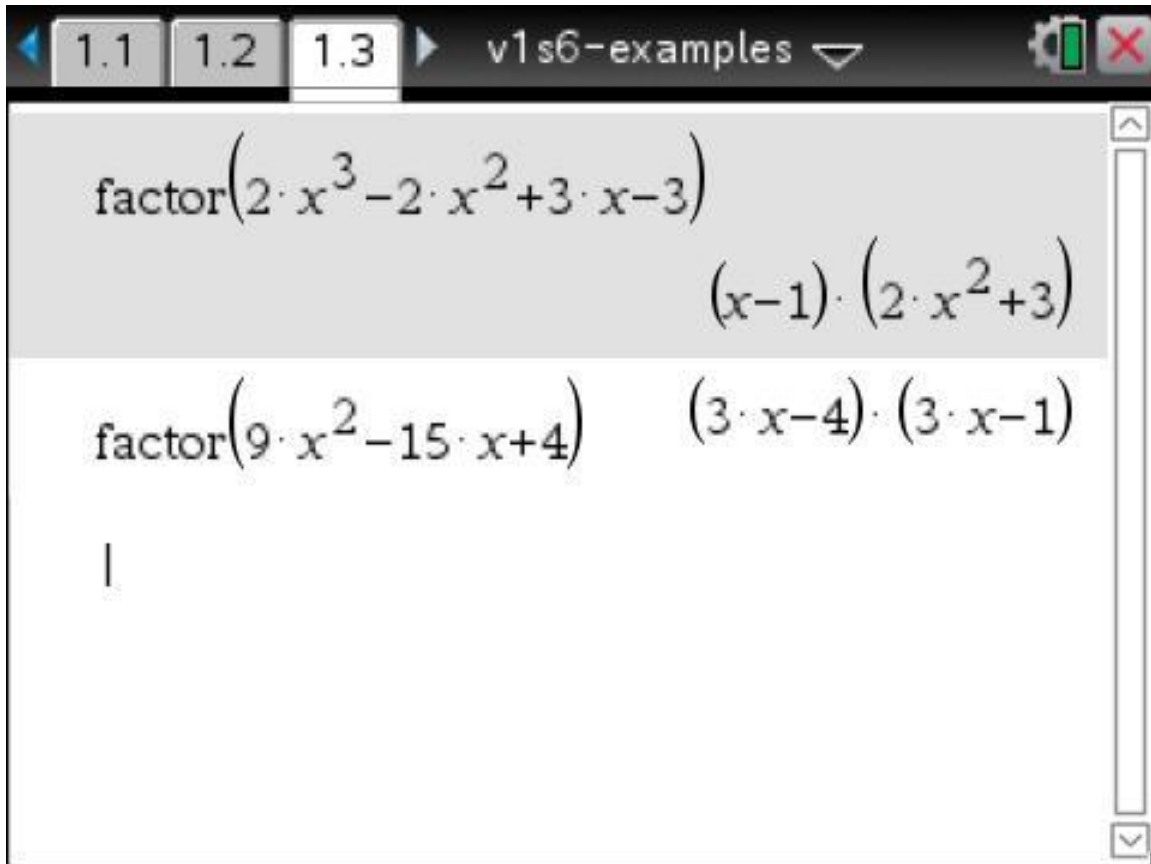
In part (a) we use the associative property to group the first two and last two terms. Then we use the left distributive property to factor each group. We then use the right distributive property to factor out the $x - 1$.

In part (b) we show how to factor trinomials with $a > 1$ by splitting the middle into two terms. In the next section we cover Special Factor Formulas. If the middle term of part (b) trinomial were $-12x$ this would be an example of a perfect square. Because the middle is 15, it is not a perfect square. Thus to factor, we need to use grouping or trial and error. If the middle were not any of the sum values, then the trinomial would be prime under integer coefficients.

Unlike part (a) we did not show the associative property's grouping, but extended our mental common factor approach to factor each of the parts. Since the first part common factor produced $3x - 1$ it was necessary to factor out a -4 from the second

part to produce $3x - 1$. Factor by grouping needs the same expression for both parts. We then use the right distributive property to factor out the $3x - 1$.

TI-Nspire CAS Factor by Grouping



Add a Calculator page. Key in the entries as shown followed by **enter**. Use the **Tools/menu > Algebra > Factor** command.

Keep My Changes/ctrl S (Save) the file as **v1s6-examples**.

Exercise 3 Factor by Grouping

(a) Factor the polynomial $2bx - 2cx + cy - by$

(b) Factor the polynomial $8x^2 + 10x - 3$

[Solution](#) >>

Special Factoring Formulas

In this section we make use of the first two groups, Square of a Binomial and Sum and Difference of a Binomial, of the [Special Products Formulas](#). We will use these in reverse. The third group, Cubes of Binomial, is generally not reversed and designated as a Special Factoring Formulas. However another third group involving

the Difference of Cubes and Sum of Cubes is added. We need to be careful when considering cubes because the product and factoring special formulas are not reverses of each other where all of the other special product and factoring formulas are.

To summarize, in this section we will look at factoring the two Perfect Square Trinomials, the Difference of Squares, the Difference of Cubes, and Sum of Cubes Binomials.

Special Factoring Formulas

Let u and v be real numbers or algebraic expressions.

Perfect Square Trinomials

$$u^2 + 2uv + v^2 = (u + v)^2$$

$$u^2 - 2uv + v^2 = (u - v)^2$$

Difference of Squares Binomial

$$u^2 - v^2 = (u + v)(u - v)$$

Difference of Cubes Binomial

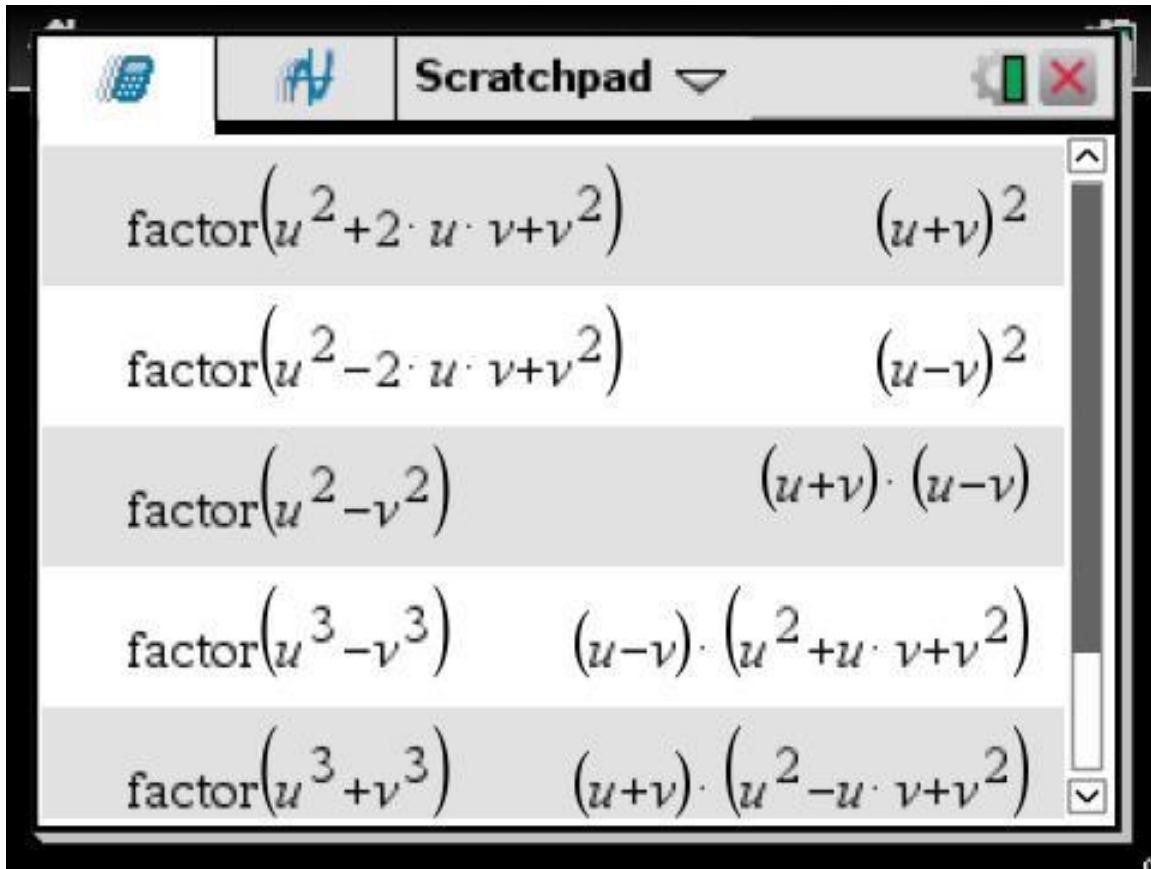
$$u^3 - v^3 = (u - v)(u^2 + uv + v^2)$$

Sum of Cubes Binomial

$$u^3 + v^3 = (u + v)(u^2 - uv + v^2)$$

We can use the TI-Nspire CAS to display them as well.

TI-Nspire CAS Special Factoring Formulas



Begin by selecting the **Scratchpad** key on the handheld. On the iPad create a document, calculator page, do the entries, and discard when done.

Use the handheld touchpad to activate and move the mouse pointer, select the Down arrow icon to right of word Scratchpad at top of screen and select **Clear Scratchpad** from the Scratchpad's menu or using the **menu** key select **menu > Actions > Clear History** to clear the handheld's calculation workspace.

Key in as shown. Use **Tools/menu > Algebra > Factor** to show the entries as terms. Press **enter** for all entries.

We used the **up arrow enter** combination to transfer a previous entry to the edit line in the creation of each of the above entries

Example 4 Special Factoring Formulas

(a) Factor the polynomial $x^2 - 12x + 36$

The trinomial takes the form $u^2 - 2uv + v^2 = (u - v)^2$

Replace u and v $x^2 - 12x + 36$

Change form
$$= x^2 - 2(x)(6) + 6^2$$
$$= (x - 6)^2$$

(b) Factor the polynomial $x^6 - y^6$

The binomial takes the form $u^2 - v^2 = (u - v)(u + v)$

Replace u and v $x^6 - y^6$

Change form
$$= (x^3)^2 - (y^3)^2$$
$$= (x^3 - y^3)(x^3 + y^3)$$

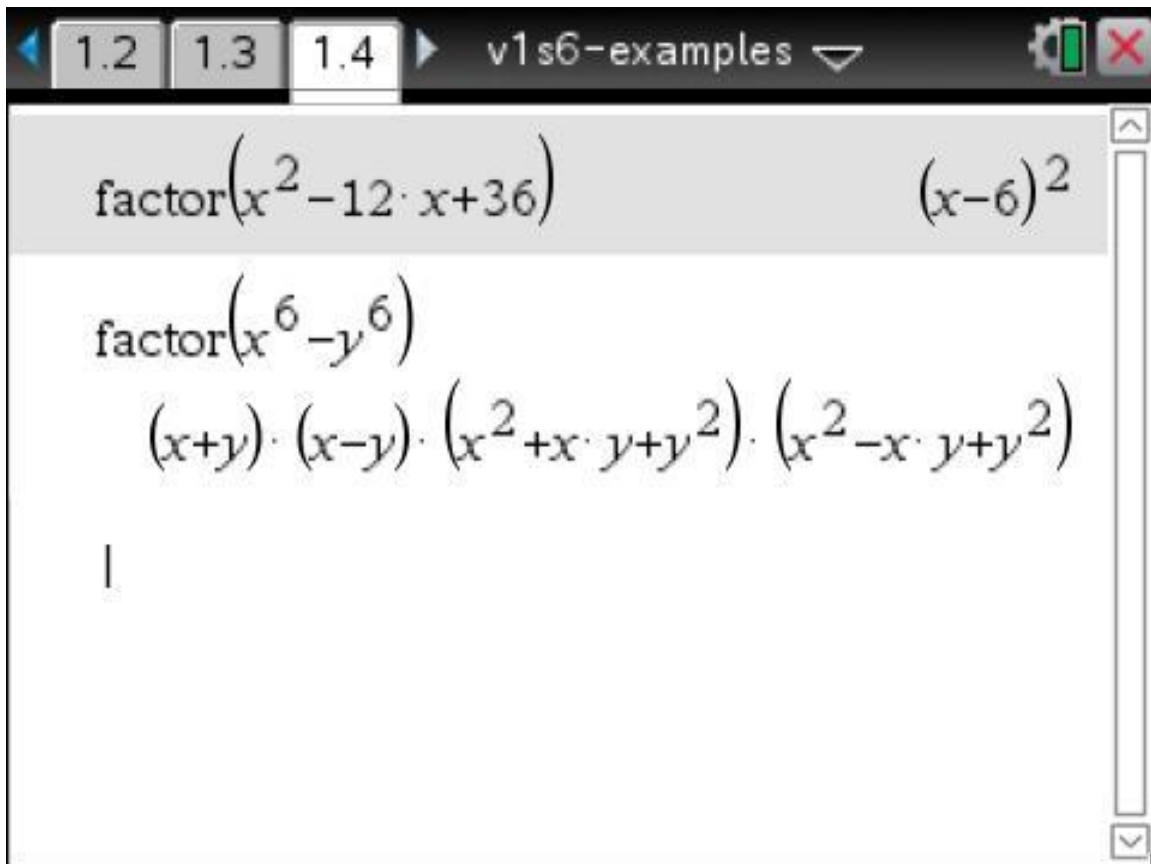
The binomial takes the form

$$u^3 \pm v^3 = (u \pm v)(u^2 \mp uv + v^2)$$

Change form
$$= (x - y)(x^2 + xy + y^2)(x + y)$$
$$(x^2 - xy + y^2)$$

In part (b) we used the Difference of Squares Binomial followed by Difference of Cubes Binomial and Sum of Cubes Binomial. To manually factor we need to work the problem in this order.

TI-Nspire CAS Special Factoring Formulas



Add a Calculator page. Key in the entries as shown followed by **enter** Use the **Tools/menu > Algebra > Factor** command. Notice in entry two that our TI-Nspire CAS is able to handle multiples of the exponents without us doing any extra work.

Keep My Changes/ctrl S (Save) the file as **v1s6-examples**.

Exercise 4 Special Factoring Formulas

(a) Factor the polynomial $4x^2 + 20x + 25$

(b) Factor the polynomial $x^6 + 125$

[Solution](#) >>

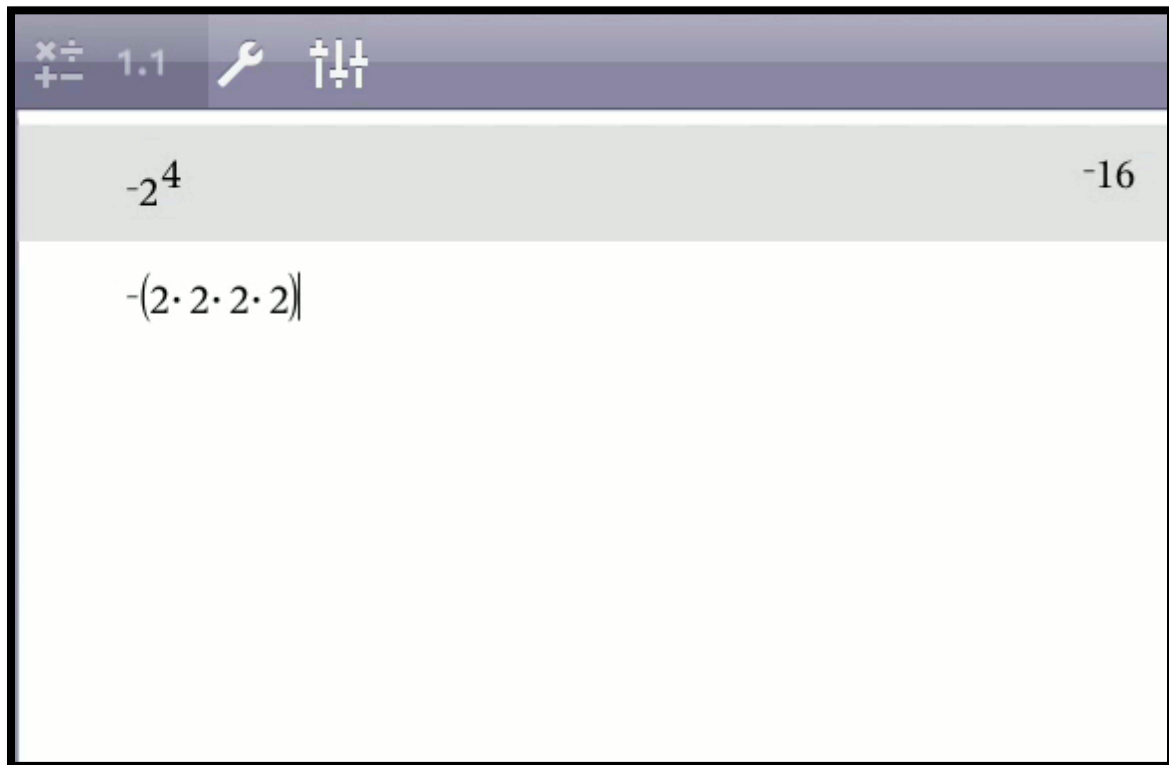
Solutions 1.1 - Real Numbers

Exercise 1 Exponential Notation

$$-2^4 = -(2^4) = -16 \text{ opposite of (2 to the 4th power)}$$

This leads us to our next topic, order of operations. Exponential expressions are done before the unary operation of opposite. This is not the same as (negative 2) to the fourth power. That answer would be 16. Order makes a difference.

TI-Nspire Family Using Exponents - Creating the Problem File



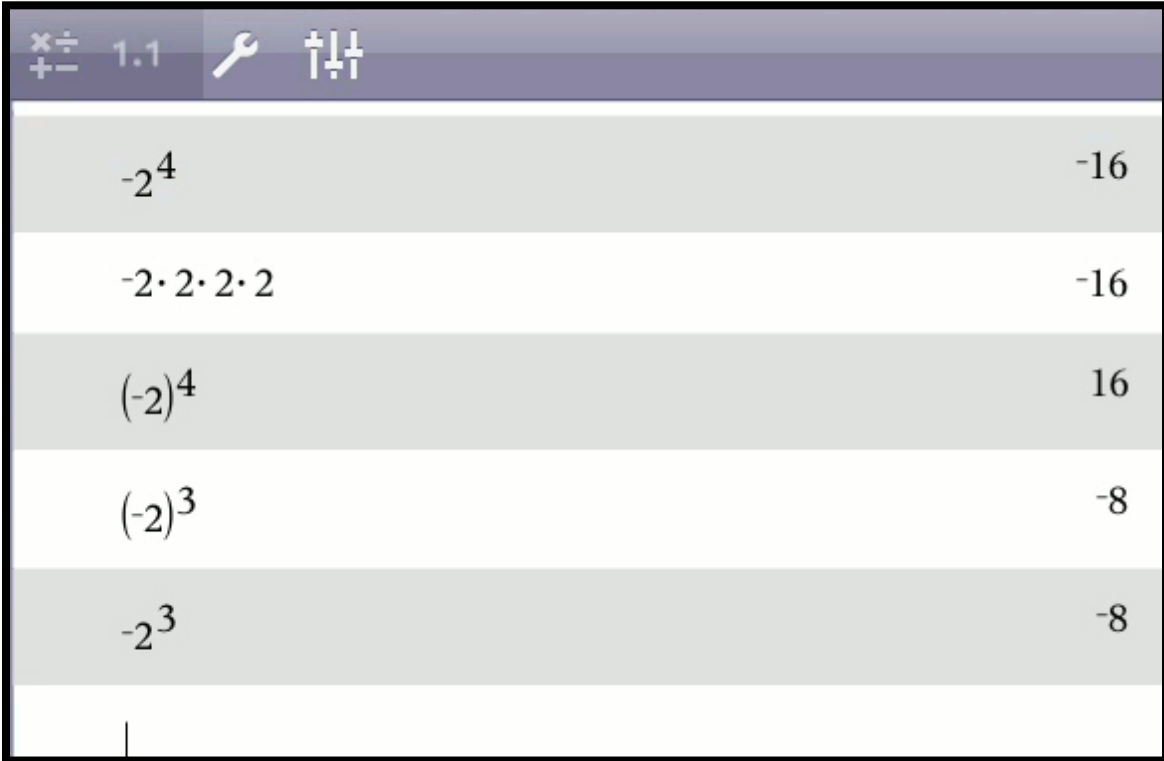
Begin by creating a new Document, and adding a Calculator page. Use the negative (-) key and type as shown. Here because of precedence the key means **opposite** of (2 to the fourth power) not (**negative** 2) to the fourth power.

The second entry with parenthesis is shown to emphasis that we should do the products first. However, when you press **enter** the TI-Nspire will drop the parenthesis and place the negative with the first number. Try it for yourself. The TI-Nspire is moving the opposite to the first entry of the products. The Ti-Nspire is using the Properties of Negatives.

You could also type (-2) raised to the fourth to see that its answer is 16. As we will see next, precedence can and often does make a difference. in our answer. If this exercise had an odd power would doing the wrong order give a wrong answer?

Keep/Save as **v1s1-exercises.tns**.

TI-Nspire Family Additional Explorations



-2^4	-16
$-2 \cdot 2 \cdot 2 \cdot 2$	-16
$(-2)^4$	16
$(-2)^3$	-8
-2^3	-8

This screen illustrates the questions posed in the previous discussion. We must be careful. The last two entries give the same answer but for different reasons. Entry four illustrates that an odd number of negatives when multiplied results in a negative. The last entry is illustrating the **opposite** of (2 to the any power) is a negative.

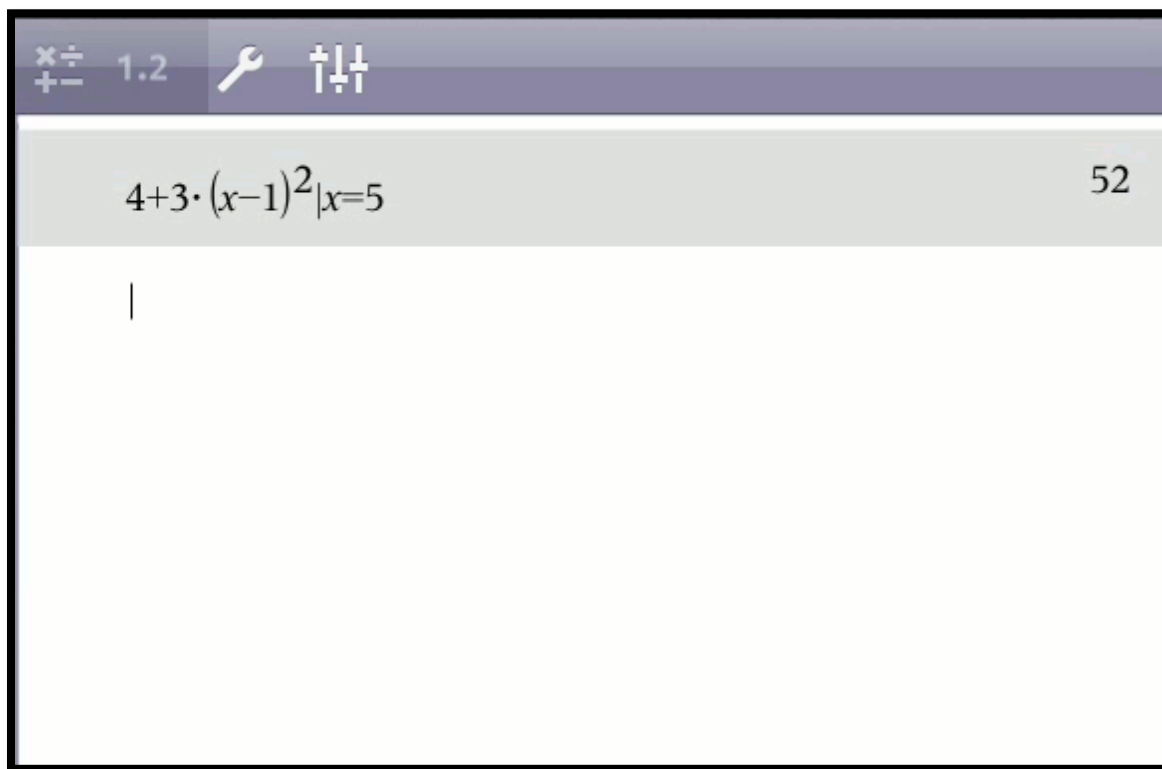
As we can see with the screenshot, in the absence of parenthesis, the precedence rule makes a difference. The negative is applied after the number is raised to the exponent.

<< [Return](#)

Exercise 2 Order of Operation

Substituting 5 for x	$4 + 3 \cdot (x - 1)^2$
Work inside parenthesis	$= 4 + 3 \cdot (5 - 1)^2$
Simplify exponent	$= 4 + 3 \cdot (4)^2$
Multiplying	$= 4 + 3 \cdot 16$
Adding	$= 4 + 48$
	$= 52$

TI-Nspire Family Order of Operations



Add a new calculator page. Key in the problem as shown followed by the **enter** key. The **width** (|) operator is located above **equal** (=) on the handheld and above the **comma** (,) key on the iPad. Use **ctrl** on the handheld and **long press** on the iPad for the secondary keys.

Keep My Changes/ctrl S (Save) the file as **v1s1-exercises.tns**.

<< [Return](#)

Exercise 3 Repeating Decimal as a Rational Number

Write $x = 0.666\dots$ as a rational number.

Multiply by 10 because 1 repeating digits

$$10x = 6.666\dots$$

Subtract x from $10x$

$$\begin{array}{r} 10x = 6.666\dots \\ -x = -0.666\dots \\ \hline 9x = 6 \end{array}$$

Dividing by 9

$$\frac{9x}{9} = \frac{6}{9} \qquad x = \frac{6}{9} = \frac{2}{3}$$

TI-Nspire Family Repeating Decimal Calculations

The image shows a TI-Nspire calculator interface. At the top, there are icons for multiplication/division, addition/subtraction, a wrench, and a fraction template. Below the icons, the number 1.3 is displayed. The main display area shows a series of calculations: 10-1, 6-0, 6/9, and 2/3. The result 0.666667 is shown to the right of the fraction 2/3.

$10-1$	9
$6-0$	6
$\frac{6}{9}$	$\frac{2}{3}$
$\frac{2}{3}$	0.666667

Add a Calculator page. Key in all calculation as shown except last followed by **enter**. For the last the calculation **approximate** (~) operator is located above the **equal** (=) key. Use **ctrl** on the handheld and **long press** on the iPad for the secondary key.

Keep My Changes/ctrl S (Save) the file as **v1s1-exercises**.

<< [Return](#)

Solutions 1.6 - Factoring

Exercise 1 Common Factors

(a) Factor the polynomial $-4x - 12$

$$\begin{aligned}\text{Factor } -4 & & -4x - 12 \\ \text{Missing factor - Supply value} & = -4(\quad) \\ & = -4(x + 3)\end{aligned}$$

(b) Factor the polynomial $8x^3y - 16x^2y^2$

$$\begin{aligned}\text{Factor } 8x^2y & & 8x^3y - 16x^2y^2 \\ \text{Missing factor - Supply value} & = 8x^2y(\quad) \\ & = 8x^2y(x - 2y)\end{aligned}$$

For part (a) place the -4 in front of an empty set of parenthesis and think what we need to multiply the -4 by to get our original polynomial, $-4x - 12$. We then use the full distributive property expand or our [distributive expand mnemonic](#) to see that $x + 3$ when multiplied by -4 will produce the original polynomial.

For part (b) factor out $8x^2y$ and follow the same procedure. We see that a missing factor of $x - 2y$ is needed.

TI-Nspire CAS Common Factors

The image shows a TI-Nspire calculator interface. At the top, there is a toolbar with icons for multiplication/division, addition/subtraction, a wrench (Tools menu), and three vertical sliders. Below the toolbar, the calculator screen displays two rows of mathematical expressions. The first row shows the command $\text{factor}(-4 \cdot x - 12)$ on the left and the result $-4 \cdot (x + 3)$ on the right. The second row shows the command $\text{factor}(8 \cdot x^3 \cdot y - 16 \cdot x^2 \cdot y^2)$ on the left and the result $8 \cdot x^2 \cdot (x - 2 \cdot y) \cdot y$ on the right. A vertical cursor is visible on the left side of the screen below the second row.

Begin by creating a new Document, and adding a Calculator page. Key in the entries as shown followed by **enter**. Use the **Tools/menu > Algebra > Factor** command. Notice that the TI-Nspire displays the solution with the gcf split into the front and back.

Keep My Changes/ctrl S (Save) the file as **v1s6-exercises.tns**.

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Exercise 2 Factoring Trinomials

(a) Factor the polynomial $x^2 - 7x + 12$

$$|a| \cdot |c| = 1 \cdot 12 = 12 \quad \text{pairs: } 1, 12; 2, 6; 3, 4$$

$$c > 0, b < 0 \quad \text{add} \quad 1 + 12 = 13; 2 + 6 = 8; 3 + 4 = 7$$

$$\begin{aligned} \text{pair is negative} \quad x^2 - 7x + 12 \\ = (x - 3)(x - 4) \end{aligned}$$

(b) Factor the polynomial $2x^2 - 6x - 8$

$$\begin{aligned} \text{Factor 2} \quad 2x^2 - 6x - 8 \\ = 2(x^2 - 3x - 4) \end{aligned}$$

$$|a| \cdot |c| = 1 \cdot 4 = 4 \quad \text{pairs: } 1, 4; 2, 2$$

$$c < 0, b < 0 \quad \text{subtract} \quad 4 - 1 = 3; 2 - 2 = 0$$

$$\begin{aligned} \text{largest, negative} \quad 2(x^2 - 3x - 4) \\ = 2(x - 4)(x + 1) \end{aligned}$$

In part (b) we factor out the common factor by factoring a 2. We complete the common factor mentally by filling in the blank parenthesis with $x^2 - 3x - 4$. We then use the Factoring Trinomial procedure on $x^2 - 3x - 4$.

TI-Nspire CAS Factoring Trinomials

The screenshot shows a TI-84 Plus calculator interface. At the top, the mode is set to '1.2' (Real mode). The calculator displays two rows of algebraic results:

$\text{factor}(x^2 - 7 \cdot x + 12)$	$(x - 4) \cdot (x - 3)$
$\text{factor}(2 \cdot x^2 - 6 \cdot x - 8)$	$2 \cdot (x - 4) \cdot (x + 1)$

Add a Calculator page. Key in the expressions followed by **enter**. Key in the entries as shown followed by **enter**. Use the **Tools/menu > Algebra > Factor** command.

Keep My Changes/ctrl S (Save) the file as **v1s6-exercises.tns**.

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Exercise 3 Factor by Grouping

(a) Factor the polynomial $2bx - 2cx + cy - by$

$$\begin{aligned} \text{Factor } 2x, -y & \quad 2bx - 2cx + cy - by \\ \text{Commutative add} & \quad = 2x(b-c) - y(-c+b) \\ \text{Factor } b-c & \quad = 2x(b-c) - y(b-c) \\ & \quad = (2x - y)(b-c) \end{aligned}$$

(b) Factor the polynomial $8x^2 + 10x - 3$

$$|a| \cdot |c| = 8 \cdot 3 = 24 \text{ pairs: } 1, 24; 2, 12; 3, 8; 4, 6$$

$$\begin{aligned} c < 0, b > 0 \text{ subtract } & \quad 24 - 1 = 23; 12 - 2 = 10; 8 - 3 = 5 \\ & \quad 6 - 4 = 2 \end{aligned}$$

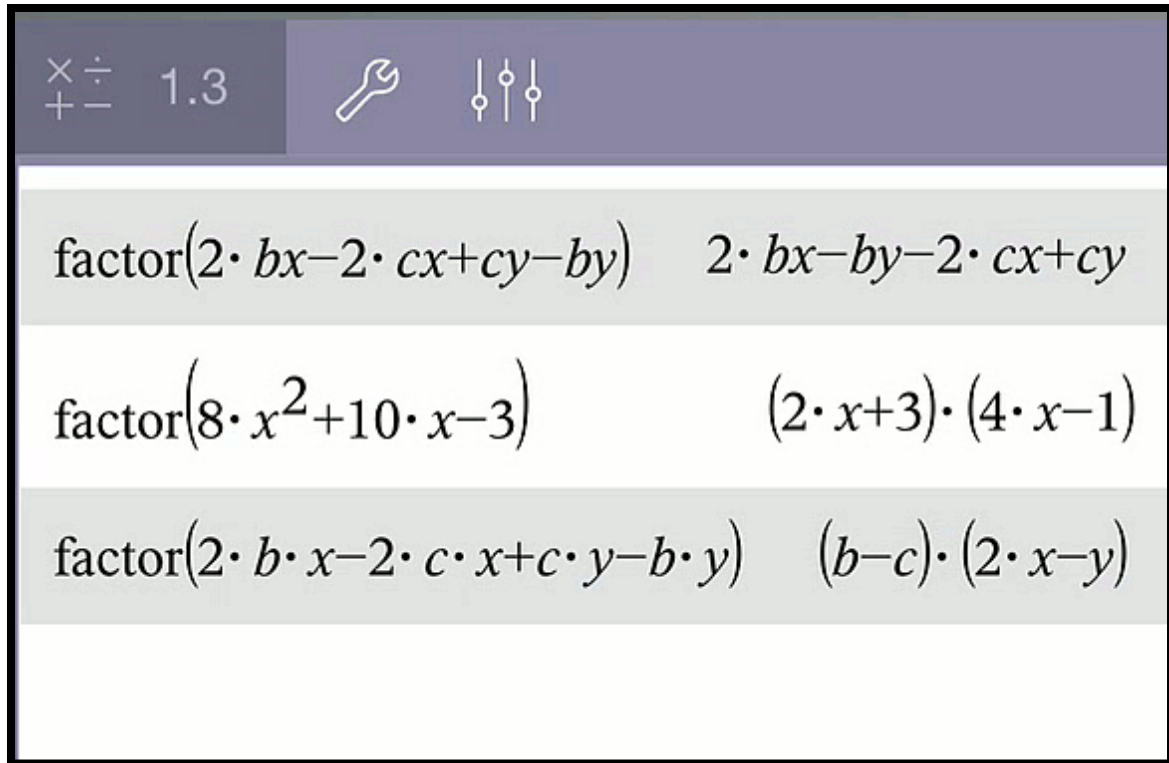
$$\begin{aligned} \text{largest positive} & \quad 8x^2 + 10x - 3 \\ \text{Factor } 4x, -1 & \quad = 8x^2 + 12x - 2x - 3 \\ \text{Factor } 2x+3 & \quad = 4x(2x+3) - 1(2x+3) \\ & \quad = (4x-1)(2x+3) \end{aligned}$$

In part (a) we use the left distributive property to factor each group. We next use the commutative property to get the terms of the $-y$ factor in the right order. What we are using is that if we keep the sign we can move the terms around. This is a shortcut for the more formal approach of changing “subtraction to addition of the opposite.” An alternate approach would have been to move the third and fourth terms first. We then use the right distributive property to factor out the $b - c$.

In part (b) we factor trinomials with $a > 1$ by splitting the middle into two terms. Thus to factor, we need to use grouping or trial and error. If the middle were not any of the difference values, then the trinomial would be prime under integer coefficients.

We extended our mental common factor approach to factor each of the parts. Since the first part common factor produced $2x + 3$ it was necessary to factor out a -1 from the second part to produce $2x + 3$. As we saw in part (a), factor by grouping needs the same expression for both parts. We then use the right distributive property to factor out the $2x + 3$.

TI-Nspire CAS Factor by Grouping



Add a Calculator page. Key in the expressions followed by **enter**. Use the **Tools/menu > Algebra > Factor** command.

Entry one's result only rearranged the terms. Entry one appears not to factor. If we look closely we forgot to place times between the letters, thus our TI-Nspire CAS found the expression we input to be irreducible. When we forget the times the variable is both letters thus resulting in a 4-term expression that is not factorable.

Use **up arrow** to select the first entry, then **Paste History** and key in using times between the variables followed by **enter**. On the handheld use **up arrow enter**.

Keep My Changes/ctrl S (Save) the file as **v1s6-exercises.tns**.

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Exercise 4 Special Factor Formulas

(a) Factor the polynomial $4x^2 + 20x + 25$

The trinomial takes the form $u^2 + 2uv + v^2 = (u + v)^2$

Replace u and v $4x^2 + 20x + 25$

Change form $= (2x)^2 + 2(2x)(5) + 5^2$

$$= (2x + 5)^2$$

(b) Factor the polynomial $x^6 + 125$

The binomial takes the form

$$u^3 + v^3 = (u + v)(u^2 - uv + v^2)$$

Replace u and v $x^6 + 125$

Change form $= (x^2)^3 + 5^3$

Simplify $= (x^2 + 5)[(x^2)^2 - (x^2)5 + 5^2]$

$$= (x^2 + 5)(x^4 - 5x^2 + 25)$$

The trinomial in part (b) solution looks like it could be a Perfect Square Trinomial with $u = x^2$. However the middle would have to be $-10x^2$ instead of $-5x^2$. Using our Factor Trinomial procedure the only other value for the middle that would factor would be $-26x^2$. Thus, the trinomial in the second factor of part (b) result is prime.

TI-Nspire CAS Special Factor Formulas

The screenshot shows a TI-84 Plus calculator interface. At the top, the display shows the number 1.4. Below the display, there are two rows of algebraic expressions. The first row shows the command `factor(4 * x^2 + 20 * x + 25)` followed by the result $(2 \cdot x + 5)^2$. The second row shows the command `factor(x^6 + 125)` followed by the result $(x^2 + 5) \cdot (x^4 - 5 \cdot x^2 + 25)$. A vertical cursor is visible on the left side of the screen.

Add a Calculator page. Key in the expressions followed by **enter**. Use the **Tools/menu > Algebra > Factor** command.

Keep My Changes/ctrl S (Save) the file as **v1s6-exercises.tns**.

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Appendix A – TI-Nspire CAS and Ti-Nspire Techniques – Activities

[TI-Nspire CAS Techniques – Activities](#) / [TI-Nspire Techniques Activities](#)

The links in the Explanation sections takes us to a discussion of the topic or activity.

From the discussion we can find TI-Nspire CAS and TI-Nspire Family screenshots and/or a manual example with TI-Nspire CAS or TI-Nspire Family solutions with a self-checking related exercise with manual, TI-Nspire CAS, and TI-Nspire Family solutions.

If it is TI Math Nspire Activity we will find a link to TI's Math Nspire Activity website.

Use the screenshots, example, and exercise for the topic to see the TI-Nspire CAS and TI-Nspire features listed after the link.

TI-Nspire CAS

Explanation 1.1 – Real Numbers

[Using Exponents](#) (Operations with exponents, precedence order negative and exponent such as -2^4)

[Order of Operations](#) (Default precedence order $+ - x /$, using fraction template, parenthesis, exponents)

[Irrational Numbers](#) (Using approximation to display decimal approximation)

[Rational Representation for Repeating Decimal](#) (Technique to represent a decimal number as a rational number, arithmetic operations)

[Root on a Number Line](#) (TI Math Nspire Activity link and computer software screenshot)

[Absolute Value](#) (Using absolute value with a variable to display definition, using the with operator, inequalities operations, absolute value with positive and negative numbers)

[Absolute Value and Distance on a Number Line](#) (Creating the “clsVol1” Function Library – Entering the Function “distrnl” in the Library File, Saving the Library File in the “MyLib” Folder, Refreshing the Libraries, Using the Catalog to type the Function, evaluating the function results by entering the parameters for the function)

[Arithmetic Sequences](#) (Adding to the “clsVol1” Function Library – Entering the Global one-line Functions “arithn”, “arithsum”, “arithsum1” in the Library File, Saving the Library File in the “MyLib” Folder, Refreshing the Libraries, Using the Catalog to input the Function, evaluating the function results by entering the parameters for the function, enter the parameters as variables to recall the

arithmetic nth term and arithmetic sum formula)

[Pi Approximation](#) (Using a measuring wheel to approximate pi, arithmetic operations)

Explanation 1.6 – Factoring

[Simplifying Expressions – Solving Algebraic Equations](#) (Relationship between simplifying complicated equations and factoring, finding solutions to algebraic equations by using the solve command and factoring)

[Factoring Over a Set of Numbers](#) – (Manual factor – set of integers, TI-Nspire CAS – set of rational numbers, using multiplication of factors to show factoring over real numbers)

[Common Factors](#) – (Factoring out the gcd, the order of TI-Nspire CAS factors vary from manual factoring)

[Factoring Trinomials](#) – (Looking at leading coefficient or is not necessary for TI-Nspire CAS, negative cases also done directly)

[Factoring Expressions with Fractional Exponents](#) – (Factor out smallest power of variable – this can be a negative power, displays common factor of negative power as a positive power by displaying in the denominator)

[Algebra from Calculus – Product Rule](#) – (Algebraic expression involving factoring fractions and negative exponents, need to use comDenom command)

[Factor by Grouping](#) – (Factor by grouping – direct for 4-term expressions, direct for trinomials with $a > 1$, with multiple variables in 4-term expressions – TI-Nspire CAS needs times signs between variables otherwise appears not to factor)

[Special Factoring Formulas](#) – (Use the TI-Nspire CAS to display the formulas, work special formulas and nested special formulas direct, interpret where the perfect square special factoring formulas do not work)

TI-Nspire

Explanation 1.1 – Real Numbers

[Using Exponents](#) (Operations with exponents, precedence order negative and exponent such as -2^4)

[Order of Operations](#) (Default precedence order $+x/$, using fraction template, parenthesis, exponents)

[Irrational Numbers](#) (Using approximation to display decimal approximation)

[Rational Representation for Repeating Decimal](#) (Technique to represent a decimal number as a rational number, arithmetic operations)

[Root on a Number Line](#) (TI Math Nspire Activity link and computer software screenshot)

[Absolute Value](#) (Absolute value with positive and negative numbers, error when variable is not defined)

[Absolute Value and Distance on a Number Line](#) (Creating the “clsVol1” Function Library – Entering the Global one-line Function “distrn1” in the Library File, Saving the Library File in the “MyLib” Folder, Refreshing the Libraries, Using the Catalog to type the Function, evaluating the function results by entering the parameters for the function)

[Arithmetic Sequences](#) (Adding to the “clsVol1” Function Library – Entering the Global one-line Functions “arithn”, “arithsum”, “arithsum1” in the Library File, Saving the Library File in the “MyLib” Folder, Refreshing the Libraries, Using the Catalog to input the Function, evaluating the function results by entering the parameters for the function)

[Pi Approximation](#) (Using a measuring wheel to approximate pi, arithmetic operations)

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