# Direct Variation

By

Samuel Chukwuemeka
(Samdom For Peace)

# Objectives

- At the end of this presentation, we shall

- Understand the meaning of variation
- Be familiar with the vocabulary terms
- Know the types of variation
- Demonstrate the concepts of direct variation
- Solve problems involving direct variation

# What do we mean by Variation?

- Have you ever used these words, "vary, varies, variable, variability?" These are the related terms. What do we mean when we say that something varies? It is evident that something that varies, always changes. It is not fixed.

- When we talk of Variation as it relates to mathematics, we are thinking of how a variable relates to another variable or several other variables. Variables are things that changes or varies. They are not fixed. Examples are weight, age, economy, and so on.

# So, what is Variation?

- Variation is simply a relationship between one variable and another variable or between one variable and several other variables.
- So, we are looking at a
- One variable to one variable relationship (One-to-One) OR a
- One variable to many variables relationship (One-to-Many)

- Lets say compare the economy of Greece with the economy of the United States OR
- Compare the economy of Greece with the economies of other Euro zone nations like Germany, France, Italy and so on.

- Is there any relationship? If yes, what kind of relationship?

# That brings us to

- The types of Variation: these are the different kinds of relationship that exists between variables; either one variable to another variable OR one variable to several other variables.
- The types of variation are:
- Direct Variation
- Inverse Variation
- Joint Variation and
- Partial Variation
- We shall not discuss Partial Variation since it is not in your textbook.
- We shall only discuss Direct Variation in this presentation.

# Direct Variation
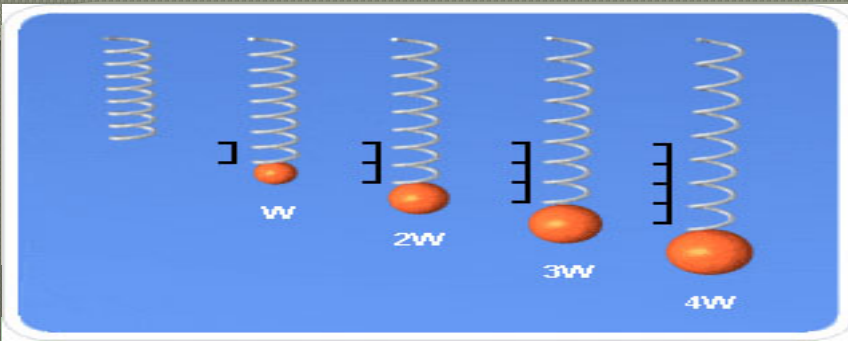
- This is the type of variation in which a variable is directly proportional to another variable.
- An increase in one variable leads to an increase in the other variable.
- Similarly, a decrease in one variable leads to a decrease in the other variable.

# Examples of Direct Variation

- A four-cylinder car engine has a lower gas consumption that a six-cylinder engine; and a six-cylinder engine consume less gas than an eight-cylinder engine.
- Based on this, we can say that the smaller the number of cylinders in a car engine, the lower the gas consumption.
- Also, the bigger the number of cylinders in a car engine, the higher the gas consumption.
- So, we say that the number of cylinders in a car engine is directly related to the amount of gas consumption.

# Another Example

- A biker travelling at a higher speed will cover a greater distance than a biker travelling at a lower speed.
- In other words, the higher the speed, the greater the distance covered; and the lower the speed, the lesser the distance covered.
- So, we say that speed varies directly with distance.

# Solved Example: Page 150, No. 41 (Property Tax)

- The property tax, t, on a home is directly proportional to the assessed value, v, of the home. If the property tax on a home with an assessed value of $140,000 is $2,100, what is the property tax on a home with an assessed value of $180,000?
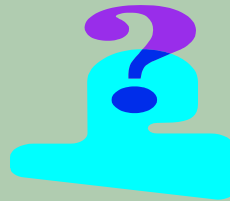
- **<u>Solution</u>**

- What will the property tax be? Is it going to higher or lower?
- It is going to be higher because $180,000 > $140,000
- (a.) $t \alpha v$
- This implies that $t = k * v$  »  $t = kv$  where k is the constant of proportionality

# Solved Example: Page 150, No. 41 (Property Tax)

- (b.) If the property tax on a home with an assessed value of $140,000 is $2,100, here; t = 2100; v = 140000; k =?

- 2100 = k * 140000  implies that k * 140000 = 2100

- Dividing both sides by 140000 gives
- $\underline{k * 140000} = \underline{2100}$
  140000     140000

- k = $\underline{2100}$ = $\underline{3}$
  140000   200

- what is the property tax on a home with an assessed value of $180,000? Here, k = 3/200; v = 180000; t = ?
-  t = k*v implies that t = <u>3</u>   *   180000  »
-                                                 200

- t = <u>3 * 180000</u>  »
-           200


- t = <u>540000</u>  » t = $2,700
-          200

# As you can see

- The property tax had to increase from
- $ 2,100 to $ 2,700 because
- The assessed value increased from
- $140,000 to $ 180,000
- Also note:
- Put your **units** in your final answer.
- The final answer is **$** 2,700

- The loudness of a stereo speaker, L, measured in decibels (dB), is inversely proportional to the square of the distance, d, of the listener from the speaker. If the loudness is 20dB when the listener is 6ft from the speaker, what is the loudness when the listener is 3ft from the speaker?

- **<u>Solution</u>**

- $L \propto \dfrac{1}{d^2}$ » $L = \dfrac{k}{d^2}$

- If the loudness is 20dB when the listener is 6ft from the speaker; this means that L = 20dB; d = 6ft; k =?

- $20 = \dfrac{k}{6^2}$ » $\dfrac{k}{6^2} = 20$ » $\dfrac{k}{36} = 20$ » $k = 20*36 = 720$

- what is the loudness when the listener is 3ft from the speaker? this means that d = 3ft; k = 720; L =?

- $L = \dfrac{k}{d^2}$ » $L = \dfrac{720}{3^2}$ » $L = \dfrac{720}{9}$ » $L = 80dB$

- As usual, ensure that you put the appropriate unit on the final answer.

# Joint and Combined Variations

- This is the type of variation in which a variable is jointly related to several other variables. This type of relation can be direct or inverse or a combination of both. But with reference to your textbook, the relation is direct, if it is Joint Variation; and a combination of both direct and inverse, if it is a Combined Variation.
- In my earlier slide, I mentioned that Combined Variation is the same as Joint Variation in the sense that a variable is proportional (either directly or inversely or both) to several other variables. The only difference is that in Combined Variation, the proportion is both direct and inverse whereas in Joint Variation, the proportion is direct. Also, it can be said that Joint variation is associated with the product of the several variables while Combined Variation is associated with both the product and quotient of the several variables.
- Let's study both cases.

# Inverse Variation

- This is the type of variation in which a variable is inversely proportional to another variable. An increase in one variable leads to a decrease in the other variable. Similarly, a decrease in one variable leads to an increase in the other variable. For instance,

- In kick boxing (though I am not a kick boxer), a greater force is needed to break a short board, while a lesser force is needed to break a long board of the same thickness. So, we say that the force required to break a board is inversely proportional to the length of the board.

- Also in Economics, based on the Law of Demand, the lower the price, the more products people will buy; and the higher the price, the less products people will buy if nothing changes. So, we note that price is inversely proportional to demand.

- From the latter, say that price = P and demand = D,
- Then P α $\frac{1}{D}$    »    P = $\frac{k}{D}$    where k = proportionality constant

# Joint Variation

- Say that earthly happiness, E varies jointly as the health, h and wealth, w of the individual, then it can be written as
- E α hw » E = khw where k is the proportionality constant
- **Solved Example:  Page 150; Number 40.**
- S varies jointly as I and the square of T. If S = 4 when I = 10 and T = 4, determine S when I = 4 and T = 6.
- **Solution**
- S α I * T$^2$  » S = kIT$^2$ where k is the constant of proportionality
- If S = 4 when I = 10 and T = 4;
- 4 = k * 10 * 4$^2$ » k * 10 * 4$^2$ = 4 » k * 10 * 16 = 4 » k * 160 = 4
- Dividing both sides by 4 gives:
- $\frac{k * 160}{160}$ = $\frac{4}{160}$ » k = $\frac{4}{160}$ » k = $\frac{1}{40}$
- determine S when I = 4 and T = 6
- S = k * I * T$^2$ » S = $\frac{1}{40}$ * 4 * 6$^2$ » S = $\frac{4 * 36}{40}$ » S = $\frac{144}{40}$ » S = 3.6

# Combined Variation

- Say that earthly happiness, E varies jointly as the health, h and wealth, w of the individual, and inversely as the greed, g of the individual, then it can be written as
- E α $\frac{hw}{g}$  »  E = $\frac{khw}{g}$ where k is the proportionality constant
- 
- **Solved Challenge Problems/Group Activities: Page 151, Number 53.**
- Water Cost: In a specific region of the country, the amount of a customer's water bill, W, is directly proportional to the average daily temperature for the month, T, the lawn area, A, and the square root of F, where F is the family size, and inversely proportional to the number of inches of rain, R. In one month, the average daily temperature is 78°F and the number of inches of rain is 5.6. If the average family of four who has a thousand square feet of lawn pays $72.00 for water for that month, estimate the water bill in the same month for the average family of six who has 1500ft$^2$ of lawn
- **Solution**
- W α $\frac{TAF}{R}$  »  W = $\frac{kTAF}{R}$  where k is the constant of proportionality

- In one month, the average daily temperature is 78°F and the number of inches of rain is 5.6. If the average family of four who has a thousand square feet of lawn pays \$72.00 for water for that month,: this means that T = 78; R = 5.6; F = 4; A = 1000; W = 72; k = ?

- $72 = \dfrac{k * 78 * 1000 * \sqrt{4}}{5.6}$ » $\dfrac{k * 78000 * 2}{5.6} = 72$ » $\dfrac{k * 156000}{5.6} = 72$

- » k * 156000 = 72 * 5.6 » k * 156000 = 403.2

- Dividing both sides by 156000 gives

- $\dfrac{k * 156000}{156000} = \dfrac{403.2}{156000}$ » $k = \dfrac{403.2}{156000}$ » k = 0.00258

- estimate the water bill in the same month for the average family of six who has 1500ft$^2$ of lawn. A = 1500; F = 6; T = 78; R = 5.6; W = ?; k = 0.00258

- $W = \dfrac{0.00258 * 78 * 1500 * \sqrt{6}}{5.6}$ » $W = \dfrac{302.4 * 2.44949}{5.6}$ » $W = \dfrac{740.7257}{5.6}$

- » W = \$132.27

- **<span style="color:red">www.samuelchukwuemeka.com</span>**