

PARETO-IMPROVING PRICING FOR TRANSPORTATION NETWORKS

By

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To my parents and my wife

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TABLE OF CONTENTS

	<u>page</u>
ACKNOWLEDGMENTS	4
LIST OF TABLES	7
LIST OF FIGURES	8
ABSTRACT	9
CHAPTER	
1 INTRODUCTION	11
1.1 Background	11
1.2 Objectives	14
1.3 Dissertation Outline	15
2 LITERATURE REVIEW	16
2.1 Background of Road Pricing	16
2.2 Fundamental of Congestion Pricing	17
2.3 Network-Wide Congestion Pricing	19
2.3.1 Fixed-Demand Marginal Cost Pricing	20
2.3.2 Elastic-Demand Marginal Cost Pricing	22
2.3.3 First-Best Toll Set	23
2.3.4 Second-Best Congestion Pricing	24
2.4 Congestion Pricing with Heterogeneous Users	25
2.5 Public Acceptance of Congestion Pricing	27
2.5.1 Weakness of Marginal Cost Pricing	27
2.5.2 Equity Effects of Congestion Pricing	28
2.6 Summary	29
3 PARETO-IMPROVING CONGESTION PRICING APPROACH	32
3.1 The Concept of Pareto-Improving Pricing	32
3.2 Dominating Flow Distribution	34
3.2.1 Feasible Flow Distribution	35
3.2.2 Finding Dominating Flow Distribution	35
3.2.3 Dominating Flow Distribution and Nonnegative Tolls	38
3.3 Nonnegative Pareto-Improving Tolls	42
3.3.1 Existence of Anonymous Pareto-Improving Tolls	43
3.3.2 Existence of Discriminatory Pareto-Improving Tolls	47
3.4 Pareto-Improving Toll Problem	49
3.5 Summary	52

4	PARETO-IMPROVING CONGESTION PRICING SCHEME FOR MULTICLASS NETWORK	59
4.1	Multiclass Pareto-Improving Pricing Scheme	59
4.1.1	Multiclass Network Equilibrium	60
4.1.2	Definition of Multiclass Pareto-Improving Pricing Scheme	61
4.1.3	Finding Multiclass Pareto-Improving Pricing Scheme	63
4.2	Properties and Existence of Multiclass Pareto-Improving Pricing Scheme	65
4.2.1	Multiclass Dominating Flow Distribution	66
4.2.2	Existence of Nonnegative Pareto-Improving Tolls	68
4.3	Manifold Suboptimization Algorithm	70
4.4	Numerical Examples	72
4.5	Summary	74
5	PARETO-IMPROVING HYBRID POLICY FOR TRANSPORTATION NETWORKS	81
5.1	Pareto-Improving Pure Road Space Rationing Policy	82
5.1.1	Problem Setting	83
5.1.2	User Equilibrium under Pure Road Space Rationing Schemes	84
5.1.3	Pareto-Improving Pure Road Space Rationing Problem	86
5.2	Pareto-Improving Hybrid Policy	87
5.2.1	User Equilibrium under Hybrid Policies	88
5.2.2	Pareto-Improving Hybrid Policy Problem	90
5.3	Numerical Examples	92
5.4	Summary	94
6	CONCLUSIONS	99
6.1	Summary of Major Findings	99
6.2	Future Research	101
	REFERENCES	102
	BIOGRAPHICAL SKETCH	108

LIST OF TABLES

<u>Table</u>	<u>page</u>
3-1 Flow distributions under UE and MSC pricing for the five-link network	54
3-2 UE and dominating flow distributions	55
3-3 UE and optimal dominating flow distributions	56
3-4 Optimal dominating flow distribution and tolls	56
3-5 UE and dominating distributions	57
3-6 OD-dependent Pareto-improving tolls	58
4-1 Time-based dominating flow distribution and Pareto-improving scheme	76
4-2 Performance comparisons with UE flow distribution	77
4-3 Cost-based dominating flow distribution and Pareto-improving scheme	77
4-4 Performance comparisons with UE flow distribution	78
4-5 Network attributes	78
4-6 Exact Pareto-improving problem	79
4-7 Approximate Pareto-improving problem	79
5-1 User equilibrium and Pareto-improving pure rationing problems	97
5-2 Pareto-improving hybrid policy problems	98
5-3 System delay reductions under different Pareto-improving strategies	98

LIST OF FIGURES

<u>Figure</u>	<u>page</u>
2-1 Graphical representation of marginal cost pricing	31
3-1 A five-link network	54
3-2 Multi-commodity loop example	55
3-3 Multi-commodity loop example with loop removed	55
3-4 A five-link network with two OD pairs	56
3-5 The longest path tree associated with \bar{v}	57
4-1 A five-link network with multiclass users	76
4-2 Frequencies of ratios for Sioux Falls network	80
4-3 Cumulative distribution of ratios for Sioux Falls network	80
5-1 A multimodal transportation network	96

Abstract of Dissertation Presented to the Graduate School
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Since its inception over 90 years ago, congestion pricing has been recognized by many as an efficient method for alleviating traffic congestion. Despite the successes of pricing projects worldwide and growing government support, congestion pricing remains largely unappealing to the general public, and it is this lack of public support that impedes its further development and implementation. This dissertation focuses on a class of congestion pricing strategies that is Pareto-improving (i.e., a pricing scheme that benefits society while ensuring that no one in the system is worse off). It is believed that such pricing strategies should be able to gain more public acceptance.

This dissertation provides an in-depth investigation of the-state-of-the-art of Pareto-improving pricing strategies for general transportation networks. First, a systematic study of the existence and properties of Pareto-improving pricing schemes is conducted. Second, an anonymous Pareto-improving pricing scheme in a transportation network with a discrete set of value of times (VOTs) for several distinct user classes is proposed and solution algorithms are developed to solve the proposed model efficiently. Last, a Pareto-improving hybrid policy that combines multiple policy instruments is investigated. The proposed hybrid policy takes advantage of the synergistic effects between congestion pricing and free-travel-right assignment. Numerical results demonstrate that the proposed hybrid policy can achieve substantial improvements in transportation system efficiency while maintaining Pareto-improving.

Most importantly, this dissertation tackles a long-standing dilemma for transportation authorities: how to enjoy the efficiency benefits of congestion pricing while keeping the general public happy. The strategies developed demonstrate that Pareto-improving pricing is a viable and promising way to achieve these two seemingly contradictory goals simultaneously. The findings may make congestion pricing no longer a hard sell to decision makers and the general public and lead the nation's transportation system to a more sustainable future.

CHAPTER 1 INTRODUCTION

1.1 Background

Traffic congestion, as an almost inevitable by-product of fast urbanization around the world, is undesirable because it causes deadweight loss to society as a whole. There are two approaches to handling traffic congestion, namely supply-side and demand-side. Transportation authorities used to cope with congestion problems from the supply side by adding more capacity. However, whenever and wherever highway capacity is expanded, latent travel demand soon fills the capacity gains. The problem is especially acute for megacities in developing countries where road infrastructure struggles to keep pace with increasing vehicle ownership and travel demand. [Downs \(1962, 2004\)](#) suggested a “fundamental law of highway congestion” and concluded that once peak-hour congestion has appeared in a region; it is impractical for the region to build its way out of congestion. [Duranton and Turner \(2011\)](#) investigated the relationship between interstate highways and highway vehicle miles traveled (VMT) in US cities and found that increased provision of roads or public transit is unlikely to relieve congestion. From another perspective, traffic congestion can be viewed as an example of the “tragedy of the commons” ([Garrett, 1968](#)) which refers to the fact that people tend to overuse free public goods. The primary reason for such demand-supply mismatches is that road users are selfish decision makers when they are only required to pay their private costs instead of the true social costs.

We contend that sustainable solutions to eliminate or at least reduce traffic congestion are on the demand side. Congestion pricing is one type of travel demand management (TDM) strategies and has been recognized as an efficient method for alleviating traffic congestion. The theoretical foundation of congestion pricing was laid out by [Pigou \(1920\)](#) and [Knight \(1924\)](#). Many have come to recognize congestion pricing as an effective market-based instrument to allocate road space resources to

benefit society as a whole, see, e.g., the review article of [Lindsey \(2006\)](#), [de Palma and Lindsey \(2011\)](#) and references cited therein. The recent advent of electronic tolling makes congestion pricing more practical and there exist several successful implementations worldwide (e.g., San Diego, Singapore, Oslo, and London).

The basic idea of congestion pricing is to let road users pay the true social costs of using the road. In general, road users are selfish when they make travel decisions and are not aware of or not willing to consider the negative congestion externality they impose on other users. The classic marginal social cost (MSC) pricing principle states that users should be charged for the difference between the marginal social costs and private costs. In this way, road users pay the true social costs and subsequently their travel decisions comply with the interests of society. Although MSC pricing makes perfect economic sense, it assumes that there is no constraint on pricing; therefore, it is also called first-best pricing. On the other hand, a second-best pricing scheme removes this strong assumption at the price of achieving less system efficiency. It is worth mentioning that all existing congestion pricing implementations around the world are second-best pricing schemes.

Congestion pricing remains politically difficult in the US. Road users generally view congestion pricing as another tax when they are asked to pay for something they currently receive for free. Hence, most elected officials are reluctant to support congestion pricing knowing road users are voters. Despite strong theoretical arguments and growing government support (e.g., FHWA's Value Pricing Program), getting the public to accept congestion pricing is still a major obstacle. Residents in Hong Kong, Cambridge (England) and Edinburgh voted against the congestion pricing schemes proposed for their cities. More recently, the congestion pricing plan for New York City was "killed" before it reached the State Assembly. [Hau \(2005\)](#) concluded that congestion tolls levied on each road user typically exceed travel time/cost savings they yield, which

implies that road users are usually made worse off unless toll revenues are redistributed to them in certain forms.

It would presumably be easy to gain support for a congestion pricing scheme from the general public if implementing the scheme improves net social benefits without making any stakeholder (road users, society and transportation authorities) worse off. [Lawphongpanich and Yin \(2010\)](#) recently introduced a new class of pricing schemes called “Pareto-improving” congestion pricing, which improves system efficiency while ensuring that, in terms of travel time, no user is made worse off and some users are better off when compared with the situation without any pricing intervention, even before toll revenue redistribution.

In recent years, the concept of Pareto-improving pricing has been picked up by many researchers. They proposed to compensate the unhappy road users who are made worse off by congestion pricing using the toll revenue raised, usually through a revenue refunding scheme. [Liu et al. \(2009\)](#) proposed a Pareto-improving scheme that charges positive tolls on road users and refunds all toll revenue collected to transit users with uniformly distributed users’ value of time (VOT) functions. [Nie and Liu \(2010\)](#) examined how users’ VOT distributions may affect the existence of a Pareto-improving pricing and refunding scheme. [Guo and Yang \(2010\)](#) investigated Pareto-improving pricing and refunding schemes in general transportation networks. They concluded that a congestion pricing scheme is Pareto-improving if it reduces system travel time and all toll revenue is refunded to users. Admittedly toll revenue refunding is an effective way to make a congestion pricing scheme more appealing to the general public. However, if users anticipate toll refunding in some form when they pay, the behavioral implications are much more complicated than what are assumed in models in the literature, which may lead to unexpected outcomes. Therefore, in this dissertation, we focus primarily on nonnegative Pareto-improving congestion pricing schemes.

1.2 Objectives

The main objective of this dissertation is to provide an in-depth investigation of the-state-of-the-art of Pareto-improving pricing strategies for general transportation networks. More specifically, three major components discussed in this dissertation are as follows:

First, for the Pareto-improving pricing problem formulated in [Lawphongpanich and Yin \(2010\)](#), the existence of the pricing scheme is not guaranteed. Why and when such pricing schemes exist are still open questions. This dissertation provides a systematic study of the existence and properties of Pareto-improving schemes in general transportation networks, which is almost an unexplored area in the literature.

Second, one complication that makes practical implementations of congestion pricing challenging is that road users are heterogeneous in many aspects. Many debates on congestion pricing root in user heterogeneity. For instance, road users with different VOTs may have different perceptions of an anonymous (uniform) toll and react differently to the tolling intervention. The Pareto-improving pricing problem formulated by [Lawphongpanich and Yin \(2010\)](#) only considered road users with homogeneous VOT. This dissertation explores the problem of developing Pareto-improving pricing schemes for networks with heterogeneous users, which may have important practical policy implications and is urgently needed.

Last, apart from market-based congestion pricing strategies, regulatory demand management policies, such as road space rationing, can also effectively mitigate traffic congestion (see, e.g., [Wang et al., 2010](#) and [Han et al., 2010](#)). However, in transportation literature, regulatory and market-based demand management policies are usually evaluated separately. The potential of bundling multiple policy instruments are generally overlooked. [Downs \(2004\)](#) concluded that the most effective overall strategy for reducing congestion probably should consist of both market-based and regulatory elements. This dissertation fills this needed gap in the literature by investigating how to design hybrid

policies that take advantage of the synergistic effects between congestion pricing and free-travel-right assignment.

1.3 Dissertation Outline

The remainder of this dissertation is organized as follows. Chapter 2 reviews the background of congestion pricing and highlights two aspects of congestion pricing: user heterogeneity and equity issues. Chapter 3 systematically investigates the Pareto-improving congestion pricing approach. Chapter 4 presents the Pareto-improving congestion pricing problem for network with multiclass users. Chapter 5 proposes a Pareto-improving hybrid policy that combines multiple policy instruments. Chapter 6 discusses future research directions and concludes the dissertation.

CHAPTER 2 LITERATURE REVIEW

2.1 Background of Road Pricing

Broadly speaking, road pricing refers to all charges that users pay for using the road system, which includes fuel taxes, vehicle registration fees, parking fees, road tolls and congestion pricing charges, etc. People may think roads are provided by the government for free, however, they are not aware of the fact that they are actually paying for their travel through various taxes and fees.

Road pricing is not a new concept, toll roads were common in Britain and the United States during most of the 19th century ([Lindsey, 2006](#)). The Highway Trust Fund (HTF) was founded in 1956 to ensure dependable financing for the interstate highway system. The HTF can date back to the 1920s when Oregon first adopted a motor fuel tax that was earmarked for road construction and maintenance. Currently it is derived from two main sources: federal excise taxes on motor fuels (gasoline, diesel, and special fuels) and truck-related taxes (truck and trailer sales, truck tires, and heavy-vehicle use). The HTF is facing shortfall because of declining fuel tax income due to more fuel-efficient vehicles on the road and rising costs of transportation projects. Despite an \$8 billion infusion from the General Fund of the Treasury in September 2008 to replenish the account, HTF needs an additional infusion of funds, about \$15 billion, to remain solvent through the end of fiscal year 2010 ([GAO, 2009](#)). An urgent reform of the existing highway finance system is needed. Interestingly, when it was first proposed in 1920s, fuel taxes were consciously adopted as imperfect substitutes by state legislatures. They believed that direct tolling at the time and place of use is more appropriate, but tolls were expensive and awkward to collect ([Wachs, 2003](#)). As the advent of electronic tolling, we should think of shifting from reliance on fuel tax to direct usage-based charges.

Among various forms of direct usage-based charges, in this chapter, we will focus on congestion pricing, which particularly aims at mitigating congestion using pricing

instruments. For literature review on general road pricing, please refer to [Small et al. \(1989\)](#) and [Gómez-Ibáñez et al. \(1999\)](#).

2.2 Fundamental of Congestion Pricing

The principle of congestion pricing was first proposed by [Pigou \(1920\)](#) and [Knight \(1924\)](#) over 80 years ago. The work has been elaborated and extended by many researchers, see, e.g., [Walters \(1961\)](#), [Mohring and Harwitz \(1962\)](#), [Beckmann et al. \(1956\)](#) and [Vickrey \(1969\)](#). The basic concept of congestion pricing is to charge road users their negative external impacts on others. Consider society of road users, more specifically, the externality is mainly the congestion impact that an individual user imposes on all other road users. [Wardrop \(1952\)](#) stated the user equilibrium (UE) conditions and described road users' selfish routing behavior. Under such behavior assumption, when users enter road network, they either are not aware of or not willing to consider the congestion they impose on others. They tend to only consider their own private cost (marginal private cost). However, the cost (marginal social cost) borne by society is more than the private cost. The discrepancy is the fundamental cause of inefficiency in road resource allocation. Therefore, to achieve the maximum efficiency to society each road user is required to pay a user charge equal to the difference between the marginal social cost and private cost, which ensures that the individual user's decision also reflects the interests of society. Such charge, known as marginal cost pricing or first-best congestion pricing toll, maximizes the net benefit of society and also induces a Pareto-optimal situation, that is, no one can be made better off without making someone else worse off.

The underlying principle of marginal cost pricing can be explained graphically using [Figure 2-1](#) for a single road segment (bottleneck) with homogenous road users. The MC curve represents the marginal social cost of an additional user to traffic flow for the new trip-maker and all existing road users, while the AC curve represents the marginal private cost borne by the new trip-maker alone. Note that when all road users

are assumed to be price-takers, perceived marginal private cost equals to the average private cost. Without any tolling intervention, the equilibrium flow is V_a , where AC and inverse demand curve intersect. From society's point of view, V_a is not optimal because the last user that enters the road network enjoys a benefit of \overline{ab} while imposing a marginal cost of \overline{ac} to society. By charging users the marginal cost toll, the equilibrium flow shifts from V_a to V_o . With this toll scheme, users' interests coincide with society's interests, thereby, the flow V_o is optimal to society. The total social welfare, given in the area \overline{ghqm} , is also maximized. The additional traffic flow beyond V_o generates a cost of the area \overline{ache} and only enjoys a benefit of the area \overline{abhe} . A welfare gain of the area \overline{bch} (the shadow area) is apparent due to the marginal cost pricing toll scheme.

Mathematically, let $c(v)$, v denote road users' average travel cost function or the AC curve and traffic flow on the road respectively. Then the marginal social cost, MC, is defined as

$$MC(v) = \frac{\partial c(v)v}{\partial v} = c(v) + v \frac{\partial c(v)}{\partial v}$$

The marginal cost pricing toll, by definition, is the difference between MC and AC curve. Therefore, the marginal cost pricing toll, $\tau(v)$ is

$$\tau(v) = v \frac{\partial c(v)}{\partial v}$$

The above results also can be derived by solving a social welfare maximization problem. Social welfare (SW) is defined as total social benefit minus total social cost.

$$SW(v) = \int_0^v D^{-1}(\omega) d\omega - c(v)v$$

where $D^{-1}(v)$ is the inverse demand function or benefit function. Maximizing $SW(v)$ with respect to v yields the necessary first-order condition:

$$D^{-1}(v) - c(v) - v \frac{\partial c(v)}{\partial v} = 0$$

By definition of marginal cost pricing toll, the optimal equilibrium flow, v_o , happens when MC curve and inverse demand curve intersect.

$$D^{-1}(v) = c(v) + \tau(v)$$

Thus, the optimal toll is

$$\tau(v) = v \frac{\partial c(v)}{\partial v}$$

Graphically, the social welfare can be seen as the sum of consumer surplus of the area \overline{hqp} and the government surplus of the area \overline{ghpm} .

2.3 Network-Wide Congestion Pricing

In this section, we will review congestion pricing problems in general networks. [Wardrop \(1952\)](#) introduced two important principles in transportation network modeling, namely, Wardrop's first principle and second principle. The first principle depicts users' selfish routing behavior, that is, all users in the network try to minimize their own private travel costs. The resulting equilibrium state is called UE state, where travel costs on all utilized paths are equal or less than those on any unused paths for a specific OD pair. When the network traffic flow distribution is in UE state, no users can further reduce their travel costs by unilaterally changing their paths. The second principle assumes there is a central controller to coordinate the traffic flow so that users work collaboratively to achieve the best system performance. Therefore, such a flow distribution is also called system optimum (SO) state. Note that a SO state is not sustainable because some users may be able to lower their travel costs by switching to other paths. The SO state gives the upper bound of system performance, thus it is an important benchmark and also the target goal for planning and operations. In general networks, a UE flow distribution generally is different from a SO flow distribution due to congestion externality. We will show in this section that the marginal cost pricing principle is still valid in the network context, which can drive a UE flow distribution to a SO one.

Notations that will be used in this section are given as follows. Let N and L denotes the set of nodes and links in a road network, and W be the set of OD pairs. A link $(i, j) \in L$ represents a road segment. For OD pair w , x_{ij}^w denotes the flow on link (i, j) of that OD pair. The vector x is the flow vector for OD pair w with x_{ij}^w as its elements. The aggregate flow on link (i, j) is expressed by $v_{ij} = \sum_w x_{ij}^w$. Due to congestion effects, travel time function $t_{ij}(v_{ij})$ is assumed to be a continuous and monotonically increasing function of the aggregate link flow v_{ij} . Let A be the node-arc incidence matrix for the network and E^w denote a vector in R^m , where m is the number of nodes. Specifically, E^w is an “input-output” vector and has exactly two non-zero components: one has a value 1 corresponding the origin node of OD pair w and the other one has a value -1 in the component for the destination.

2.3.1 Fixed-Demand Marginal Cost Pricing

Consider the case where travel demand for OD pair w , d^w , is given and homogeneous, total system travel time can be set as the system performance measure. Thus SO flow distribution minimizes the total system travel time. Mathematically, the SO flow distribution can be obtained by solving the following mathematical program:

$$\min_{(v,x)} \sum_{(i,j) \in L} t_{ij}(v_{ij}) v_{ij} \quad (2-1)$$

$$\text{s.t. } Ax^w = E^w d^w \quad \forall w \in W \quad (2-2)$$

$$v = \sum_{w \in W} x^w \quad (2-3)$$

$$x \geq 0. \quad (2-4)$$

[Beckmann et al. \(1956\)](#) formulated traffic assignment problem as a mathematical program and proved the equivalence between the optimality conditions of the proposed model and the Wardrop’s first principle. The corresponding mathematical program is as

follows:

$$\min_{(v,x)} \sum_{(i,j) \in L} \int_0^{v_{ij}} t_{ij}(\omega) d\omega \quad (2-5)$$

$$\text{s.t. } Ax^w = E^w d^w \quad \forall w \in W \quad (2-6)$$

$$v = \sum_{w \in W} x^w \quad (2-7)$$

$$x \geq 0. \quad (2-8)$$

Comparing the optimality conditions of the above SO and UE formulations, we note that SO problem is equivalent to a UE problem with the following modified travel cost function

$$\tilde{t}_{ij}(v_{ij}) = t_{ij}(v_{ij}) + v_{ij} \frac{dt_{ij}(v_{ij})}{dv_{ij}}. \quad (2-9)$$

The first term of the modified travel cost function is the user's private travel time function and the second term is the additional travel time imposed on all existing users by an additional user. If all users adopt the modified travel cost function to make their route choice decisions, the resulting flow distribution will be automatically a SO flow distribution. However, in reality users normally only consider their own private cost, $t_{ij}(v_{ij})$. To make users' decision comply with society's interests, the second term of the modified travel cost function should be charged as a congestion toll. The marginal cost pricing toll for each link can be expressed as follows:

$$\tau_{ij} = v_{ij} \left. \frac{dt_{ij}(v_{ij})}{dv_{ij}} \right|_{v_{ij}=v_{ij}^{SO}} \quad (i, j) \in L \quad (2-10)$$

Note that $\tilde{t}_{ij}(v_{ij}) = \frac{dv_{ij} t_{ij}(v_{ij})}{dv_{ij}}$, the modified travel cost function is exactly the marginal cost of adding an additional user to link (i, j) . Furthermore, $t_{ij}(v_{ij})$ is assumed to be separable, therefore we have $\tilde{t}_{ij}(v_{ij}) = \frac{d \sum_{(i,j) \in L} v_{ij} t_{ij}(v_{ij})}{dv_{ij}}$, which is also the marginal social cost of having an additional user to the system. Hence, we can express Wardrop's

second principle in another way: all utilized paths have the same marginal cost for a specific OD pair.

2.3.2 Elastic-Demand Marginal Cost Pricing

In the elastic demand case, travel demand for a specific OD pair w , d^w follows a demand function $D^w(\mu^w)$, where μ^w is the travel cost between that OD pair. Let $D_w^{-1}(d^w)$ denote the inverse demand function for OD pair w . Total system travel time is not appropriate to be used as the system performance measure anymore because of demand elasticity. For the elastic demand case, SO problem can be defined in terms of maximization of social welfare. The mathematical program that corresponding to the SO problem is as follows:

$$\max_{(v,x,d)} \sum_{w \in W} \int_0^{d^w} D_w^{-1}(\omega) d\omega - \sum_{(i,j) \in L} t_{ij}(v_{ij}) v_{ij} \quad (2-11)$$

$$\text{s.t. } Ax^w = E^w d^w \quad \forall w \in W \quad (2-12)$$

$$v = \sum_{w \in W} x^w \quad (2-13)$$

$$x \geq 0. \quad (2-14)$$

The UE problem with elastic demand is formulated as follows:

$$\min_{(v,x,d)} \sum_{(i,j) \in L} \int_0^{v_{ij}} t_{ij}(\omega) d\omega - \sum_{w \in W} \int_0^{d^w} D_w^{-1}(\omega) d\omega \quad (2-15)$$

$$\text{s.t. } Ax^w = E^w d^w \quad \forall w \in W \quad (2-16)$$

$$v = \sum_{w \in W} x^w \quad (2-17)$$

$$x \geq 0. \quad (2-18)$$

Using the exact same procedure as the last section, we can obtain the marginal cost pricing toll for each link as follows:

$$\tau_{ij} = v_{ij} \left. \frac{dt_{ij}(v_{ij})}{dv_{ij}} \right|_{v_{ij}=v_{ij}^{SO}} \quad (i, j) \in L \quad (2-19)$$

Note that although SO state definitions are different for fixed and elastic demand case, the marginal cost pricing toll vectors, (2–10) and (2–19) are identical.

2.3.3 First-Best Toll Set

In most literature, the marginal cost pricing toll vector (2–10 and 2–19) derived in previous two sections has been the only first-best pricing scheme. However, [Hearn and Ramana \(1998\)](#) found that the link-based marginal cost pricing toll scheme does not necessarily have to be unique and proposed a “first-best toll set”, which contains all toll vectors that can drive a UE state to a SO one. The traditional marginal cost pricing toll vector is just one element of this toll set. Consider the following system of equations:

$$x_{ij}^w (t_{ij}(v_{ij}) + \tau_{ij} - \rho_i^w + \rho_j^w) = 0 \quad \forall (i, j) \in L, w \in W \quad (2-20)$$

$$t_{ij}(v_{ij}) + \tau_{ij} \geq \rho_i^w - \rho_j^w \quad \forall (i, j) \in L, w \in W \quad (2-21)$$

$$Ax^w = E^w d^w \quad \forall w \in W \quad (2-22)$$

$$v = \sum_{w \in W} x^w \quad (2-23)$$

$$x \geq 0 \quad (2-24)$$

where ρ_i^w represents the node potential [Ahuja et al. \(1993\)](#) of node i for OD pair w .

These conditions essentially are the Karush-Kuhn-Tucker (KKT) conditions for the tolled UE problem. Recall that the objective of first-best toll is to drive the solution of the tolled UE problem to coincide with the un-tolled SO problem. Therefore, if and only if there exists a ρ vector such that the combined vector (τ, ρ) satisfies the following conditions, the τ vector is a valid first-best toll vector ([Hearn and Ramana, 1998](#)).

$$x_{ij}^w (t_{ij}(v_{ij}^{SO}) + \tau_{ij} - \rho_i^w + \rho_j^w) = 0 \quad \forall (i, j) \in L, w \in W \quad (2-25)$$

$$t_{ij}(v_{ij}^{SO}) + \tau_{ij} \geq \rho_i^w - \rho_j^w \quad \forall (i, j) \in L, w \in W. \quad (2-26)$$

More concisely, the first-best toll set can be expressed in vector form as follows:

$$(t(v^{SO}) + \tau)^T v^{SO} = \sum_{w \in W} (E^w)^T d^w \rho^w \quad (2-27)$$

$$t(v^{SO}) + \tau \geq A^T \rho^w \quad \forall w \in W. \quad (2-28)$$

Since the first-best toll set generally has multiple elements, including the traditional marginal cost pricing toll, it gives more flexibility in setting link tolls to meet secondary objectives, such as minimum number of toll booth, maximum government revenue, etc.

[Yin and Lawphongpanich \(2009\)](#) further investigated the first-best toll set and concluded that all elements of the toll set are actually still in line with the marginal cost pricing principle in path level although they may be different from the traditional link-based marginal cost pricing toll vector.

2.3.4 Second-Best Congestion Pricing

Marginal cost pricing or first-best congestion pricing toll schemes assume that there is no constraint on pricing and reduce congestion to its minimum level when travel demand is fixed or increase the social welfare to its maximum level when travel demand is elastic. On the other hand, for political reasons or otherwise, some roads are not tollable and other restrictions may also apply. Because of these restrictions, generally this type of toll scheme cannot achieve the maximum system efficiency and is called “second-best” congestion pricing scheme. In the literature, many formulated the problem as a bi-level optimization problem (see, e.g., [Bard, 1998](#); [Shimizu et al., 1997](#)) or a mathematical program with equilibrium constraints (see, e.g., [Luo et al., 1996](#)). Generally speaking, the upper-level problems or the objectives of the second-best toll schemes are optimizing certain system performance measures and the lower-level problems or complimentary constraints are used to describe road users’ route choice behaviors, i.e., tolled UE conditions. Other types of second-best pricing schemes include area-based (see, e.g., [Maruyama and Harata, 2006](#); [Maruyama and Sumalee, 2007](#))

and cordon pricing (see, e.g., [Akiyama et al., 2004](#); [Sumalee, 2004](#); [Zhang and Yang, 2004](#)).

2.4 Congestion Pricing with Heterogeneous Users

It is well known that road users consists of heterogeneous users. User heterogeneity can be categorized into two types, namely observable and unobservable heterogeneity. Observable user heterogeneity refers to users' different vehicle types, e.g., truck, bus, car, etc. Unobservable user heterogeneity assumes that all users have the same type of vehicles, while their socioeconomic characteristics vary across different user groups, such as value of times (VOT).

For the observable user heterogeneity, due to the differences of their vehicle types, different user groups may have different congestion externalities imposing on others. Since their physical difference is observable, it is possible to charge differentiated tolls for each vehicle type based on the marginal cost pricing principle. More detailed discussions of congestion pricing on this type of user heterogeneity can be found in [Patriksson \(1994\)](#) and [Nagurney \(1999\)](#).

For unobservable user heterogeneity, all users differ from one another in unobservable ways, it is very difficult if not impossible to introduce differentiated or non-anonymous tolls ([Arnott and Kraus, 1998](#)). Since users have different VOTs, they may react differently to an anonymous (uniform) toll scheme. It is then interesting to investigate whether marginal cost pricing principle is still valid in this case. In transportation literature, this type of user heterogeneity is captured by assuming either users' VOTs follow a continuous distribution or a discrete set of VOTs for several distinct user classes. [Yin and Yang \(2004\)](#) concluded that an anonymous link toll can still be derived based on the marginal cost pricing principle to achieve the cost-based system optimum (total value of system travel time minimization). User externality of travel time is the same for all user groups because they use the same type of vehicle. Thus, the optimal anonymous toll is equal to the user externality of travel time multiplied by the arithmetic

mean of VOTs of all users traversing that link, which is exactly the additional travel cost that a marginal user imposes on others already traveling on that link. On the other hand, the system manager may want to achieve a time-based system optimum (total system travel time minimization). In this case, although the user externality of travel time is still the same across different user groups, no anonymous toll can be derived based on the marginal cost pricing principle. The reason is congestion tolls can only be charged in monetary unit. To internalize the externality, a congestion toll equals to the user externality of travel time multiple the user group's VOT should be charged to each user group, which is no longer anonymous. [Yang and Huang \(2004\)](#) further proved that an anonymous toll scheme that supports a multiclass user equilibrium flow distribution as a time-based system optimum exists and can be obtained by solving a proposed dual linear programming (LP) problem. However, the anonymous toll derived does not reflect the user externality. [Yin and Yang \(2004\)](#) extended the work and proposed a general toll set containing all feasible anonymous toll patterns that support a multiclass user equilibrium flow distribution as a time-based system optimum. The toll set can be expressed as a nonempty polyhedron in a linear inequality and equality system. In a similar way, the general toll set that supports a cost-based system optimum can also be derived. And, the marginal cost link toll pattern is an element of the set. [Engelson and Lindberg \(2006\)](#) pointed out that from a pure economic perspective that cost-based system optimum provides the maximum overall economic efficiency to society with heterogeneous uses.

[Engelson and Lindberg \(2006\)](#) investigated the marginal cost pricing principle in a cost-based framework and concluded that the equilibrium flow distribution is not unique in a cost-based system optimum supported by the marginal cost link toll while the aggregate link flow is indeed unique. Therefore, implementing the marginal cost link toll, the resulting toll equilibrium needs not coincide with the cost-based system optimum flow distribution, which may cause marginal cost pricing principle to fail to work

in real-world implementation. However, they further proved that the objective function value of the cost-based system optimum, i.e., the total value of system travel time is unique for any resulting tolled equilibrium distribution by implementing the marginal cost link toll, even though the objective function in general is non-convex.

2.5 Public Acceptance of Congestion Pricing

Economists have long recognized that congestion pricing is an effective tool to manage and reduce traffic congestion. However, getting the public to accept congestion pricing is still an obstacle. In this section, we will briefly discuss why the marginal cost pricing toll scheme is not appealing to the general public. Equity issues, the major concern preventing congestion pricing gaining public acceptance, will also be investigated.

2.5.1 Weakness of Marginal Cost Pricing

Marginal cost pricing can increase social welfare to the maximum level, which is beneficial to society as a whole. However, it turns out that all road users in the system are worse off compared to the “un-tolled” scenario. The only stakeholder that gains surplus is the government who collects toll revenues. [Hau \(2005\)](#) pointed out that marginal cost pricing is “most likely doomed to be political failures” because users will find themselves worse off. Use [Figure 2-1](#) to illustrate, excess flow beyond tolled equilibrium flow V_o is “priced off” to other times or paths because their willingness to pay is not high enough. These users suffer a loss in consumer surplus of the area \overline{bhk} . For those users remaining on the road, their travel time is reduced at the cost of paying congestion tolls. Their travel time saving (area \overline{mnkg}) is always less than the tolls they pay (area \overline{mphg}). A deficit in consumer surplus of the area \overline{npkh} is expected when marginal cost pricing toll is introduced. If the government does not compensate users directly by revenue refunding or indirectly by improvements in infrastructure and public transit, road users are generally made worse off.

2.5.2 Equity Effects of Congestion Pricing

Equity effects were largely ignored by early researchers on congestion pricing. [Richardson \(1974\)](#) believed that the equity arguments are “murky” and decision should be made in light of efficiency. [Giuliano \(1994\)](#) claimed that congestion pricing is no more regressive than other existing road related charges, such as fuel tax. Indeed, equity effects could be highly subjective and economists have not reached a consensus on the definition of “equity” yet. On the other hand, equity issues play an important role in implementing any public policies. There is no exception for congestion pricing. With the advent of electronic tolling, congestion pricing becomes more practical and many cities around the world are considering implementing it. Therefore, in recent years, researchers have shown an increasing interest in equity effects of congestion pricing. Due to the complexity of equity issues, three dimensions of equity concerns are discussed in this section.

Assuming road users are heterogeneous in their incomes, social (vertical) equity issues arise when an anonymous toll is charged because the uniform charge tends to take up a higher percentage of the budget of a person or family with a lower income. [Foster \(1974\)](#) was the first to argue that congestion pricing is regressive and discriminates against the poor. Since then, many studies have been done on welfare/revenue redistribution to make congestion pricing scheme progressive or less regressive. [Eliasson \(2001\)](#) proposed a toll scheme that reduces aggregate travel time and makes everyone better off by redistributing toll revenues equally to all users. [Yang et al. \(2004\)](#) developed an integrated pricing scheme for a bimodal transportation network with transit subsidy. [Eliasson and Mattsson \(2006\)](#) claimed that with the toll revenue spent on public transportation, which benefits women and those with lower incomes, the proposed Stockholm road pricing scheme is progressive.

Spatial (horizontal) equity emphasizes that the benefits and costs of congestion pricing should be distributed equally over space ([Viegas, 2001](#)). Congestion pricing may

have significantly different impacts on the generalized travel costs between different OD pairs. [Yang and Zhang \(2002\)](#) designed a second-best toll scheme with explicit constraints on social and spatial equity. [Yin and Yang \(2004\)](#) tried to find the most equitable anonymous link toll pattern that can support the multiclass, bi-criterion user equilibrium flow pattern as a system optimum. The total transformed travel disutility was used as the inequality measurement. Social and spatial inequalities were calculated for the purpose of comparison.

Another dimension of equity issue which is often overlooked is intergenerational or temporal equity. Let's consider a broader road pricing concept, toll revenues are usually used to finance new roads and capacity expansions under current practices of transportation infrastructure financing. To what extent the gains and losses are distributed between the present and future generations is the key issue of intergenerational equity. [Szeto and Lo \(2006\)](#) defined a gap function to capture the intergenerational equity and formulated a time-dependent network design problem to obtain the most desirable toll patterns for each planning period (generation).

Since there is no absolute equity, it is more meaningful to discuss the degree of equity using some equity measures. [Ramjerdi \(2006\)](#) examined various equity measures for a proposed pricing scheme in Oslo using two case studies and concluded that it is not appropriate to make judgment about the equity implication of a policy on the basis of a single equity measure, therefore, multiple equity measures should be employed.

2.6 Summary

This chapter briefly reviewed the classic marginal cost pricing principle in the framework of network modeling. Recent developments in first-best and second-best pricing were also covered. Two aspects of congestion pricing were highlighted: user heterogeneity and public acceptance. General speaking, unobservable user heterogeneity is more difficult to deal with because only anonymous toll is feasible to charge in this case and some equity concerns may arise due to the regressiveness

of anonymous tolling. Public acceptance is the key issue of implementing congestion pricing schemes. We explained why marginal cost pricing scheme is not appealing to the general public and how three dimensions of equity concerns play important roles in designing congestion pricing toll schemes.

To address the public acceptance issues, ideally we should directly tackle equity concerns when developing a congestion pricing scheme. However, the equity issue is not to determine whether a pricing scheme is equal or not, but it is a problem about the degree of equity, which relies on some equity measures. Unfortunately, not all equity measures are well defined and suitable to be incorporated in optimization models. Therefore, instead of directly utilizing various equity measures, a Pareto-improving approach is adopted in this dissertation, that is, developing congestion pricing schemes that improve system efficiency while ensuring that everyone is better off and no one is worse off than the “un-tolled” situation, which will be discussed in more details in the next chapter. Although the Pareto-improving approach does not directly address equity concerns, it is easy for the general public to understand. Since no one is worse off in the toll scheme, it would presumably be easier to gain public acceptance and serves as a stepping stone towards more comprehensive road pricing schemes.

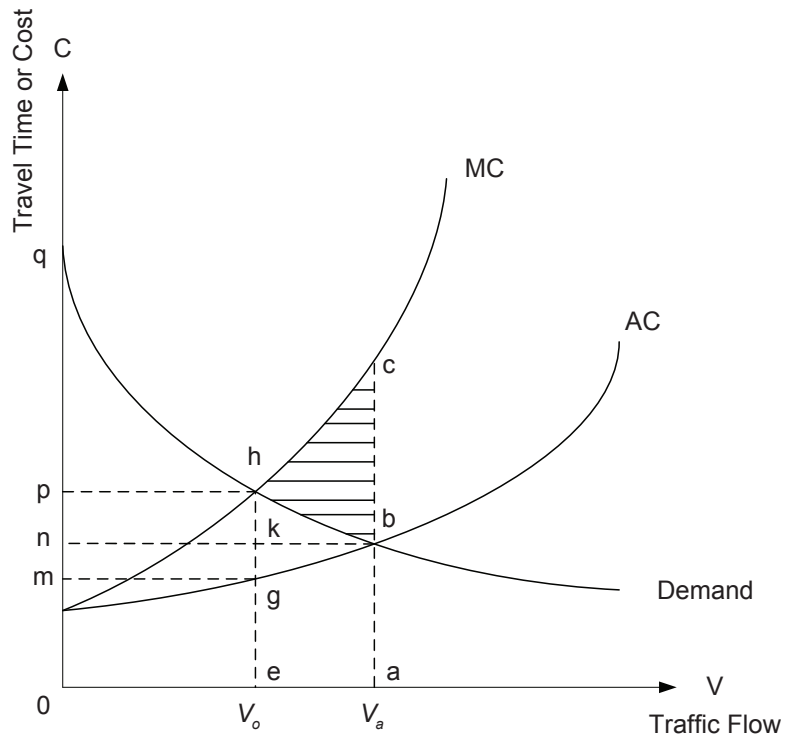


Figure 2-1. Graphical representation of marginal cost pricing

CHAPTER 3 PARETO-IMPROVING CONGESTION PRICING APPROACH

3.1 The Concept of Pareto-Improving Pricing

Pareto efficiency, or Pareto optimality, is a central concept in welfare economics (see, e.g., [Ng, 1979](#)). A change from one economic arrangement to another that can make at least one person better off without making any other person worse off is called a Pareto improvement. A situation is said to be Pareto-efficient or Pareto-optimal when there is no further Pareto improvement can be made. It is named after Vilfredo Pareto (1848-1923), an Italian economist who used the concept in his studies of economic efficiency and income distribution. Almost everyone would agree that society should avoid situations that are not Pareto-optimal. Since a Pareto improvement does not hurt anyone, a Pareto-improving arrangement would presumably have less obstacles of gaining public acceptance. It is well known that if there is perfect competition and no externality, Pareto optimality would hold. However, the statement in general is not true when externality exists.

In transportation literature, users are usually assumed to make their route choices selfishly to minimize their own travel costs. The flow distribution that follows this route choice principle is called a Wardrop's user equilibrium (UE) distribution. Because of the existence of congestion externality, Pareto optimality generally does not hold in a UE state. From the system's perspective, system efficiency is not optimized under a UE flow distribution. System optimum (SO), on the other hand, makes an assumption that all users work cooperatively to improve system efficiency and, furthermore, a SO flow distribution is Pareto-optimal. In general, a Pareto-optimal flow distribution needs not to optimize system efficiency, although a SO flow distribution is always Pareto-optimal. A SO flow distribution is ideal to society; however, it is unstable in the sense that some travelers could unilaterally change routes to reduce their own travel costs. In the literature, a long-established market-based approach to induce a UE flow distribution to

a SO flow distribution is the marginal social cost (MSC) congestion pricing strategy (see, e.g., [Pigou, 1920](#); [Beckmann et al., 1956](#); [Button, 1993](#) and [Arnott and Small, 1994](#)).

Although a SO flow distribution and MSC congestion pricing improve system efficiency to its optimal level, they are not appealing to the general public with good reason ([Hau, 2005](#)). Some users may find themselves suffering longer travel time in a SO flow distribution even before tolling. Moreover, after introducing the MSC tolls, some users may experience higher travel costs (time plus congestion tolls) than those under the original UE condition. To illustrate the concept of MSC congestion pricing, consider the example in [Figure 3-1](#) from [Hagstrom and Abrams \(2001\)](#) in which there is only one OD pair (1, 4) with a demand of 3.6.

[Table 3-1](#) displays the link and path flows under UE and MSC pricing along with the associated costs. Recall that tolls under MSC pricing are of the form $dt_{ij}(v_{ij})/dv_{ij}$ and v_{ij} is the flow on link (i, j) . When implemented, MSC pricing forces 1.54 users to use path 1-2-4 with a total cost of 101.70, of which 75.85 is the travel time and the rest (25.85) is for tolls. Thus, these 1.54 users suffer twice, once for having to use a longer route (75.85 instead of 71.06) and the other for having to pay tolls. Overall, the total cost to the 3.6 users under MSC pricing is 366.13 which is more than the total cost (225.80) under UE, a cost consisting entirely of time or delay. However, MSC pricing yields less total delay (227.11) and generates toll revenue (139.02) for the transportation authority.

In this chapter, instead of considering MSC or similar pricing, we introduce a class of tolling or pricing schemes that improves net social benefits without making any stakeholder (road users, society or the transportation authority) worse off. We term this class of pricing schemes Pareto-improving congestion pricing.

The outline of this chapter is as follows: Section 2 discusses a class of non-equilibrium flow distribution called dominating flow distribution that improves system efficiency without making anyone worse off. Mathematical formulations to find such a flow distribution are given. The conditions when a dominating flow distribution can be

supported by anonymous nonnegative tolls are explored. Section 3 investigates properties and the existence of anonymous and discriminatory Pareto-improving tolls. Section 4 provides mathematical formulations to obtain dominating flow distributions and equilibrium-inducing Pareto-improving toll vectors simultaneously. Section 5 is the summary of this chapter.

3.2 Dominating Flow Distribution

Even though a SO flow distribution can reduce total system delay to its minimum level, some users may suffer longer travel time, and thus are worse off. Such a flow distribution may encounter fierce opposition from the general public, and subsequently some elected officials are reluctant to advocate such a solution. Intuitively, if there exists a flow distribution that improves system efficiency without making any user worse off, the flow distribution would be much easier to gain public support. In this dissertation, such a flow distribution is referred to as a dominating flow distribution. A flow distribution dominates the UE distribution if it strictly improves a measure of system efficiency and allows all users to use routes that are no longer than those under UE. For brevity, we also say that a flow distribution is dominating if it dominates the UE distribution. When compared to the UE distribution, we show later that our definition of domination is consistent with the traditional one, i.e., no user is worse off and some are better off under a dominating flow distribution. Consequently, we say that a pricing or tolling scheme is Pareto-improving if it induces a flow distribution that dominates the UE distribution and generates a generalized travel cost (i.e., time plus congestion tolls) for every user that is no larger than his or her travel time under UE.

In [Hagstrom and Abrams \(2001\)](#) and [Abrams and Hagstrom \(2006\)](#), the authors refer to a dominating flow distribution as a Generalized Braess Paradox. They found non-equilibrium flow distributions may achieve lower travel time for every user than the UE flow distribution does and also subsequently reduce total system delay. The seemingly paradox can be explained by the fact that the UE flow distribution may not be

Pareto-optimal, which implies it is possible to find better alternatives for at least one user without making any other user worse off.

3.2.1 Feasible Flow Distribution

When demands are fixed, V^F denotes a set of all feasible flow distributions, each of which is represented as v . The number of nodes and links in the transportation network are m and n respectively. In particular, $v \in R^n$ and $v^F \in R^n$. The set v^F can be described using either path or link flow variables. Using the former, let f_r^w and d^w denote the amount of flows on path (or route) r and the given demand for OD pair w , respectively. Then,

$$V^F = \{v : v_{ij} = \sum_w \sum_{r \in P^w} \delta_{ijr} f_r^w; \sum_{r \in P^w} f_r^w = d^w, \forall w; f_r^w \geq 0, \forall w, r\},$$

where P^w is the set of paths for OD pair w and δ_{ijr} (equals 0 or 1) indicates whether arc (i, j) is on path r . Alternatively, let A be the node-arc incidence matrix and it is of size $m \times n$. We use w to represent the index for OD pair w and d^w denotes the demand between OD pair w . $E^w \in R^m$ is an “input-output” vector and has exactly two nonzero components, the one corresponding to the origin of OD pair w has a value 1 and the one corresponding to the destination has a value of -1. Let L be the set of links in the network and its element (i, j) represents a link from node i to node j . Travel time function $t_{ij}(v_{ij})$ is assumed to be separable, continuous, strictly monotone and $t_{ij}(v_{ij}) > 0$ for all $v \in V^F$. The feasible flow distribution V^F can be written as follows:

$$V^F = \{v : v = \sum_w x^w, Ax^w = E^w d^w, \forall w\},$$

where $x^w \in R^n$ is a vector whose components are link flows of OD pair w .

3.2.2 Finding Dominating Flow Distribution

To describe a dominating flow distribution mathematically, let f_r^w be the path flow on path r of OD pair w and P^w be the set of paths between OD pair w . A flow distribution $v \in V^F$ dominates a given UE flow distribution, or is dominating if v and its path flow f_r^w

satisfy the flowing conditions:

$$\sum_{(i,j) \in \Phi_r^w} t_{ij}(v_{ij}) \leq c_w^{UE} \quad \forall w, r \in P_+^w \quad (3-1)$$

$$t(v)^T v < t(v^{UE})^T v^{UE} \quad (3-2)$$

where, for OD pair w , Φ_r^w is the set of links on path r and $P_+^w = \{r : r \in P^w \text{ and } f_r^w > 0\}$ is a set of utilized paths associated with the flow distribution v . v^{UE} is the UE flow distribution, i.e., v^{UE} satisfies $t(v^{UE})^T (v - v^{UE}) \geq 0, \forall v \in V^F$. In particular, condition (3-1) ensures that all users are not worse off and condition (3-2) guarantees that some are better off. If v is dominating, then condition (3-2) implies that there must exist at least one utilized path for some OD pair w , such that $\sum_{(i,j) \in \Phi_{\tilde{r}}^w} t_{ij}(v_{ij}) < c_w^{UE}$, i.e., users of path \tilde{r} are better off. Otherwise, $\sum_{(i,j) \in \Phi_r^w} t_{ij}(v_{ij}) = c_w^{UE}$ for every w and $r \in P_+^w$, implying that $t(v)^F = t(v^{UE})^T v^{UE}$. The latter contradicts the fact that v is dominating and satisfies condition (3-2).

Any feasible flow pattern that satisfies the above two inequalities is a dominating flow distribution. When such a distribution exists, however, it may not be unique. To illustrate, Table 3-2 lists three dominating flow distributions for the five-link network in Figure 3-1. They are in columns labeled DF-F-1, DF-F-2, and DF-F-3. As displayed in the penultimate row in the table, the lengths of the longest utilized path for the three dominating flow distributions are no greater than the one for UE. Among the three, the longest path for DF-F-1 is the shortest while DF-F-3 yields the smallest total delay. In addition, a convex combination of DF-F-2 and DF-F-3 is also dominating. Thus, the number of dominating flow distributions for the five-link network is infinite.

Among all dominating flow distributions, we define the one with the least total system delay as the *optimal* dominating flow distribution, which not only dominates the UE distribution but also dominates all other dominating flow distributions. To find the optimal dominating flow distribution, we formulate the following path-based optimization

problem. The optimization problem is equivalent to the formulation in [Abrams and Hagstrom \(2006\)](#) of finding a Generalized Braess Paradox.

$$\text{P1: } \min_v t(v)^T v \quad (3-3)$$

$$\text{s.t. } v \in V^F \quad (3-4)$$

$$f_r^w \left(\sum_{(i,j) \in \Phi_r^w} t_{ij}(v_{ij}) - c_w^{UE} \right) \leq 0 \quad \forall w, r \in P^w \quad (3-5)$$

In the above, constraint (3-4) requires v to be a feasible flow distribution. Constraint (3-5) ensures that when path r is utilized, its travel time should be less than or equal to that under the UE flow distribution. The objective function is to minimize total system delay. Thus, if the optimal solution to P1 is v^* and $t(v^*)^T v^* < t(v^{UE})^T v^{UE}$, then condition (3-2) holds. The complementarity constraint implies for $f_r^w > 0$, $\sum_{(i,j) \in \Phi_r^w} t_{ij}(v_{ij}^*) - c_w^{UE} \leq 0$ and hence, condition (3-1) also holds. Furthermore, if there exists another dominating flow distribution other than v^* that has less total system delay, v^* cannot be the optimal solution to problem P1. Thus, if dominating flow distributions exist, v^* must be the optimal dominating flow distribution. When $t(v^*)^T v^* = t(v^{UE})^T v^{UE}$, there is no $v \in V^F$ that satisfies both (3-1) and (3-2), i.e., no dominating distribution exists. Note that for the five-link network, DF-F-3 in Table 2 solves the P1 problem and the other two do not.

To make the problem more computationally tractable, we formulate the following link-based optimization problem:

$$\text{P2: } \min_{(v,\rho)} t(v)^T v \quad (3-6)$$

$$\text{s.t. } v \in V^F \quad (3-7)$$

$$x_{ij}^w (\rho_i^w - \rho_j^w - t_{ij}(v_{ij})) \geq 0 \quad \forall w, (i, j) \in L \quad (3-8)$$

$$\rho_{o(w)}^w - \rho_{d(w)}^w \leq c_w^{UE} \quad \forall w \quad (3-9)$$

When a link (i, j) is utilized, i.e., $x_{ij}^w > 0$, constraint (3-8) reduces to $(\rho_i^w - \rho_j^w - t_{ij}(v_{ij})) \geq 0$. Adding this expression up for each link along a utilized path r of OD pair w yields the

following:

$$\sum_{(i,j) \in \Phi_r^w} t_{ij}(v_{ij}) \leq \sum_{(i,j) \in \Phi_r^w} (\rho_i^w - \rho_j^w) = \rho_{o(w)}^w - \rho_{d(w)}^w,$$

where, for OD pair w , Φ_r^w is a set containing links on path r and $o(w)$ and $d(w)$ represent the origin and destination nodes. Thus, along with constraint (3–9), it implies that condition (3–1) is satisfied for every utilized path and OD pair. With a similar argument to problem P1, we can conclude that the optimal solution to P2 is the optimal dominating flow distribution if there exists a dominating flow distribution.

As formulated above, mathematical program P1 and P2 are optimization problems with complementarity constraints (MPCC), a difficult class of problem to solve optimally (see, e.g., [Scheel and Scholtes, 2000](#)), and [Abrams and Hagstrom \(2006\)](#) propose a practical methodology for finding a local optimal solution to problem P1 by allowing only routes utilized in the UE distribution to have positive flows.

3.2.3 Dominating Flow Distribution and Nonnegative Tolls

Although a dominating flow distribution can improve traffic condition without making anyone worse off, the flow distribution is not in equilibrium state and hence is unstable. In real-world implementation, the dominating flow distribution has to be realized through either regulatory or market-based approach. In this chapter, road congestion pricing is used as an instrument to induce such a dominating flow distribution. In this section, we would like to explore the relationship between a dominating flow distribution and nonnegative tolls. More specifically, we will discuss when a dominating flow distribution can be supported by an anonymous nonnegative toll vector as a Wardrop equilibrium flow distribution.

Before we start answering this question, the concept of Gallager loop, or loop in Gallager's sense ([Gallager, 1977](#)) needs to be introduced first.

Definition of Gallager loop: An aggregate feasible flow v that satisfies demand d contains loops if it can be expressed as $v = \bar{v} + \tilde{v}$, where \bar{v} is a feasible flow distribution for d and $\tilde{v} > 0$ is a circulation flow, such that $A\tilde{v} = 0$.

In other words, for a flow distribution with Gallager loop, if the circulation flow \tilde{v} is removed, \bar{v} is still a feasible flow distribution for demand d . Hereafter we will use the term *commodity* to refer to users between a particular OD pair. By the definition of Gallager loop, a flow distribution is loop-free if there exists no decomposition into commodity link flows that use all links of a directed cycle. [Gallager \(1977\)](#) also provided the following necessary and sufficient condition for a flow distribution to contain a Gallager loop.

Lemma 1: An aggregate feasible flow v contains loops if and only if \bar{v} is another feasible aggregate flow for the same demand with $\bar{v} < v$.

A circulation flow of a single commodity along a directed cycle is an obvious example of a Gallager loop. We call such a circulation flow a single-commodity loop. It is generally easy to identify a single-commodity loop by visual inspection. However, a Gallager loop also exists when an aggregate circulation flow is formed by flows of different OD pairs, which we refer to as a multi-commodity loop. It is generally not a straightforward task to detect a multi-commodity loop through visual inspection. The examples in [Figure 3-2](#) and [3-3](#) extracted from [Gallager \(1977\)](#) and [Hagstrom and Abrams \(2009\)](#) are used to illustrate.

In the examples, three OD pairs and their corresponding demands are shown in the figures. Different dotted links are used to distinguish paths taken by users of different OD pairs. For example, in [Figure 3-2](#), travel demand from the origin node 2 to the destination node 5 follows path 2-4-3-5. By inspection, there is no single-commodity loop in [Figure 3-2](#). However, a multi-commodity loop can be found, which consists of flows of three different OD pairs along directed cycle 1-2-4. The existence of a Gallager loop can also be verified using Lemma 1. Let v denote aggregate flow vector in [Figure 3-2](#). [Figure 3-3](#) shows another feasible flow distribution with an aggregate flow vector \bar{v} by removing traffic flows from the directed cycle 1-2-4. By Lemma 1, for the same travel

demand, we have another aggregate flow \bar{v} and $\bar{v} < v$, therefore, the flow distribution in Figure 3-2 must contains a Gallager loop.

Given a feasible flow distribution \bar{v} with travel demand d , we formulate the following optimization problem to explore the existence of a Gallager loop mathematically.

$$\text{P3: } \min_v \sum_{(i,j) \in L} (v_{ij} - \bar{v}_{ij}) \quad (3-10)$$

$$\text{s.t. } v \in V^F \quad (3-11)$$

$$0 \leq v_{ij} \leq \bar{v}_{ij} \quad \forall (i, j) \in L \quad (3-12)$$

where, constraint (3-11) is to make sure the flow distribution v is feasible. Constraint (3-12) requires aggregate link flow v should be no greater than the original feasible flow distribution. The objective function is to maximize the difference between v and \bar{v} , and the value is always less than or equal to 0. The above optimization problem always has at least one feasible solution when $v_{ij} = \bar{v}_{ij}, \forall (i, j) \in L$. If the optimal value of the objective function is 0, we can conclude that the given feasible flow distribution \bar{v} is loop-free. When the optimal objective function value is strictly less than 0, the given feasible flow distribution \bar{v} contains at least one loop, and the optimization problem also generates a loop-free feasible flow distribution \bar{v} for travel demand d .

Hagstrom and Abrams (2009) investigated loops and the existence of equilibrium-inducing tolls. They established necessary and sufficient conditions for a particular traffic flow to be supported by various classes of tolls. The following theorem is from Hagstrom and Abrams (2009). We restate it here for convenience.

Theorem 1: The feasible flow \bar{v} is realizable as a Wardrop equilibrium with positive anonymous cost vector $c(\bar{v})$ if and only if \bar{v} does not contain a loop.

To relate a given feasible flow distribution to nonnegative tolls, we have the following lemma as a direct extension of the above theorem.

Lemma 2: The absence of loops (single- and multi-commodity) is necessary and sufficient for the existence of anonymous nonnegative tolls to support a given feasible flow distribution as a Wardrop equilibrium.

Proof: We prove the necessary part first. If a feasible flow, \bar{v} , contains no loop, according to Theorem 1, it is realizable with positive cost $c(\bar{v})$. The cost function can be written as follows: $c_{ij}(\bar{v}_{ij}) = t_{ij}(\bar{v}_{ij}) + \tau_{ij}$. The toll for link (i, j) can be expressed as $\tau_{ij} = c_{ij}(\bar{v}_{ij}) - t_{ij}(\bar{v}_{ij})$. In a tolled UE, the following system of equations holds:

$$\sum_{(i,j) \in \Phi_r^w} c_{ij}(\bar{v}_{ij}) = \sum_{(i,j) \in \Phi_r^w} (t_{ij}(\bar{v}_{ij}) + \tau_{ij}) = \lambda^w, \quad \forall w \text{ and } r \in \bar{P}_+^w \quad (3-13)$$

$$\sum_{(i,j) \in \Phi_r^w} c_{ij}(\bar{v}_{ij}) = \sum_{(i,j) \in \Phi_r^w} (t_{ij}(\bar{v}_{ij}) + \tau_{ij}) \geq \lambda^w, \quad \forall w \text{ and } r \in \bar{P}_0^w \quad (3-14)$$

where $\bar{P}_+^w = \{r : r \in P^w \text{ and } \bar{f}_r^w > 0\}$ and $\bar{P}_0^w = \{r : r \in P^w \text{ and } \bar{f}_r^w = 0\}$. We further know that the UE flow distribution does not change when the travel cost function of each link is multiplied by a positive scale, i.e., if link travel cost becomes $\alpha c_{ij}(\bar{v}_{ij})$ for every link $(i, j) \in L$, the UE flow distribution still holds. Therefore, we know that $\tau_{ij} = \alpha c_{ij}(\bar{v}_{ij}) - t_{ij}(\bar{v}_{ij})$ is also a valid toll to support the UE flow. By changing the scale α , the existence of a nonnegative toll vector is guaranteed.

To prove the sufficient part, generally every link is assumed to have a positive free flow travel time in a transportation network, which implies that $t_{ij}(\bar{v}_{ij}) > 0$. Therefore, when there is a nonnegative toll vector to support a feasible flow distribution \bar{v} , we have $c_{ij}(\bar{v}_{ij}) = t_{ij}(\bar{v}_{ij}) + \tau_{ij} > 0$. By Theorem 1, we know that the feasible flow distribution \bar{v} does not contain any loop (single- and multi-commodity).

Corollary 1: The absence of loops (single- and multi-commodity) is necessary and sufficient for the existence of anonymous nonnegative tolls to support a dominating flow distribution as a Wardrop equilibrium.

Essentially, a dominating flow distribution is just a special class of feasible flow distribution. As a result, by Lemma 2, we have the above corollary.

For a transportation network with a single OD pair, the relationship between the *optimal* dominating flow distribution and nonnegative tolls can be further strengthened.

Theorem 2: When a transportation network has a single OD pair, there always exists a nonnegative toll vector to support the optimal dominating flow distribution.

Proof: To obtain a contradiction, let (v^*, ρ^*) be the solution to the optimization problem P2. If v^* contains a Gallager's loop, it can be expressed as $v^* = \bar{v} + \tilde{v}$, where \bar{v} is another feasible flow, $\bar{v} \in V^F$, and $\bar{v} < v^*$. $t_{ij}(v_{ij})$ is strictly monotone, thus $t_{ij}(\bar{v}_{ij}) \leq t_{ij}(v_{ij}^*)$ for all $(i, j) \in L$. Since the transportation network has a single OD pair, the second constraint of P2 can be reduced to $v_{ij}(\rho_i - \rho_j - t_{ij}(v_{ij})) \geq 0$. When $(\rho_i^* - \rho_j^* - t_{ij}(v_{ij}^*)) \geq 0$, for the feasible flow \bar{v} , we have $(\rho_i^* - \rho_j^* - t_{ij}(\bar{v}_{ij})) \geq 0$ because $t_{ij}(\bar{v}_{ij}) \leq t_{ij}(v_{ij}^*)$ for all $(i, j) \in L$. When $(\rho_i^* - \rho_j^* - t_{ij}(v_{ij}^*)) \leq 0$, in order to make the second constraint hold, $v_{ij}^* = 0$. Since $\bar{v}_{ij} \leq v_{ij}^*$ for all $(i, j) \in L$, we have $\bar{v}_{ij} = 0$. Therefore, $\bar{v}_{ij}(\rho_i^* - \rho_j^* - t_{ij}(\bar{v}_{ij})) \geq 0$ also holds. We can conclude that if we keep ρ^* unchanged, the feasible flow \bar{v} satisfies all three constraints of problem P2. In other words, \bar{v} is also a feasible solution to problem P2. Since $\bar{v} < v^*$ and $t_{ij}(\bar{v}_{ij}) \leq t_{ij}(v_{ij}^*)$ for $(i, j) \in L$, $\sum_{(i,j) \in L} t_{ij}(\bar{v}_{ij})\bar{v}_{ij} < \sum_{(i,j) \in L} t_{ij}(v_{ij}^*)v_{ij}^*$. We have a contradiction that a feasible solution to a minimization problem cannot lead to a lower value in the objective function than the optimal solution does.

3.3 Nonnegative Pareto-Improving Tolls

In previous sections, we investigated the relationship between dominating flow distributions, and nonnegative congestion tolls and concluded that the absence of loops (single- and multi-commodity) is necessary and sufficient for the existence of anonymous nonnegative tolls to support a given dominating flow distribution as a Wardrop equilibrium. A further question we are interested in is: when a given dominating flow distribution can be supported by anonymous nonnegative tolls, does there always exist a Pareto-improving toll vector to induce the dominating flow distribution? The

following discussions will provide an answer to this question. In this section, we assume that a dominating distribution is given.

3.3.1 Existence of Anonymous Pareto-Improving Tolls

Let \bar{v} denote a given dominating distribution. With respect to the travel costs under UE, a nonnegative toll vector, τ , is Pareto-improving if it satisfies the following conditions:

$$\sum_{(i,j) \in \Phi_r^w} (t_{ij}(\bar{v}_{ij}) + \tau_{ij}) = \lambda^w \quad \forall w, r \in \bar{P}_+^w \quad (3-15)$$

$$\sum_{(i,j) \in \Phi_r^w} (t_{ij}(\bar{v}_{ij}) + \tau_{ij}) \geq \lambda^w \quad \forall w, r \in \bar{P}_0^w \quad (3-16)$$

$$\tau > 0 \quad (3-17)$$

$$\lambda^w \leq c_{UE}^w \quad \forall w \quad (3-18)$$

Similar to before, $\bar{P}_+^w = \{r : r \in P^w \text{ and } \bar{f}_r^w > 0\}$ and $\bar{P}_0^w = \{r : r \in P^w \text{ and } \bar{f}_r^w = 0\}$. For each OD pair, conditions (3-15) and (3-16) ensure that τ is a valid toll vector (see, e.g., [Hearn and Ramana, 1998](#)), in that they force the utilized paths associated with \bar{v} to have the same generalized travel cost that is no greater than those for the non-utilized paths. Condition (3-18) ensures that the generalized cost for each OD pair is no greater than the user equilibrium travel time. Without the nonnegativity requirement in (3-17), setting $\tau = -t(\bar{v})$ trivially satisfies conditions (3-15) – (3-18) and the resulting generalized travel cost is zero for all paths. Thus, $\tau = -t(\bar{v})$ is Pareto improving. When τ must be nonnegative, a Pareto-improving toll vector may not exist. As discussed in the previous section, if a dominating flow distribution contains a single- or multi-commodity loop, no nonnegative tolls can be found to support the dominating flow distribution. However, even if a dominating flow distribution can be supported by a nonnegative toll vector, does that indicate that we can always find a nonnegative Pareto-improving toll vector to induce the flow distribution? We will illustrate the problem using the following numerical

example. The five-link network in Figure 3-4 has two OD pairs (1, 4) and (3, 2) with demands of 3.6 and 1.0 respectively.

To obtain a dominating flow distribution of the example in Figure 3-4, we can solve the optimization problem P2. The UE flow distribution and optimal dominating flow distribution are given in Table 3-3.

We can observe from Table 3-3 that users between OD pair (3, 2) are better off in terms of lower travel time, and total system delay is also reduced. Let a feasible flow distribution \bar{v} be a dominating flow distribution. The following linear program is constructed to examine the existence of a nonnegative Pareto-improving toll vector to support the dominating flow distribution \bar{v} .

$$\text{P4: } \min_{(y, \rho, \tau)} y \quad (3-19)$$

$$\text{s.t. } \bar{x}_{ij}^w (t_{ij}(\bar{v}_{ij}) + \tau_{ij} - \rho_i^w + \rho_j^w) = 0 \quad \forall w, (i, j) \in L \quad (3-20)$$

$$t_{ij}(\bar{v}_{ij}) + \tau_{ij} \geq \rho_i^w - \rho_j^w \quad \forall w, (i, j) \in L \quad (3-21)$$

$$\rho_{o(w)}^w - \rho_{d(w)}^w - c_w^{UE} \leq y \quad \forall w \quad (3-22)$$

$$\tau, y \geq 0 \quad (3-23)$$

Constraints (3-20) and (3-21) ensure that the toll vector τ supports the flow distribution \bar{v} as a user equilibrium, i.e., $(t(\bar{v}) + \tau)^T (v - \bar{v}) \geq 0, \forall v \in V^F$. When the dominating flow distribution contains no loop, it can be supported by a nonnegative toll vector.

By setting y equals to a large number, constraint (3-22) is not binding anymore. The problem reduces to solving a system of equations to get a nonnegative toll vector that supports the dominating flow distribution. Since the non-tolled UE flow distribution is always feasible to the system of equations, the optimization problem always has a solution. If the optimal objective function value is less than or equal to 0 and $\tau > 0$, we find a Pareto-improving toll vector τ to support the dominating flow distribution. If the optimal objection value is strictly greater than 0, we can conclude that there is no

Pareto-improving toll vector to support the dominating flow distribution. In the meantime, the program also generates a nonnegative toll vector that can induce the dominating flow distribution to a user equilibrium flow distribution.

The following example demonstrates that a nonnegative Pareto-improving toll vector may not exist to support a dominating flow distribution even though the flow distribution can be supported by nonnegative tolls.

The UE and dominating flow distributions are the same as the previous example in Figure 3-2. By solving problem P4, the optimal objective function value is 12.16, which indicates that there is no Pareto-improving toll vector to support the given dominating flow distribution. The solutions to problem P4 also demonstrate that the given dominating flow distribution actually can be supported by a nonnegative toll vector. The toll vector is listed in the last column of Table 3-4. The equilibrium travel cost of OD pair (1, 4) increases from 71.25 to 82.69 after implementing the nonnegative tolls, and hence the equilibrium-inducing toll vector is not Pareto-improving.

Generally, there may not exist a set of Pareto-improving tolls that induces a given dominating flow distribution. The theorem below provides a necessary and sufficient condition under which such tolls exist.

Theorem 3: Let \bar{v} be a dominating flow distribution such that $\bar{v} = \sum_w \bar{x}^w$, where $A\bar{x}^w = E^w d^w$ and $\bar{x}^w \geq 0$ for all w . Then, there exists a Pareto-improving toll vector that induces \bar{v} if and only if the vector $(\bar{x}^w, 0)$ solves the following linear program:

$$\min_{(y,z)} t(\bar{v})^T \left(\sum_w y^w \right) + \sum_w c_w^{UE} z^w \quad (3-24)$$

$$\text{s.t. } Ay^w + E^w z^w = E^w d^w \quad \forall w \quad (3-25)$$

$$\sum_w y_{ij}^w \leq \bar{v}_{ij} \quad \forall (i, j) \in L \quad (3-26)$$

$$y^w, z^w \geq 0 \quad \forall w \quad (3-27)$$

Proof: Because the Karush-Kuhn-Tucker (KKT) conditions are both necessary and sufficient for linear programs, $(\bar{x}^w, 0)$ solves the above linear program if and only if there

exists multipliers ρ^w , τ , σ^w and γ^w such that the following hold:

$$t_{ij}(\bar{v}) + \tau_{ij} - (\rho_i^w - \rho_j^w) - \sigma_{ij}^w = 0, \quad \forall w, (i, j) \in L \quad (3-28)$$

$$c_w^{UE} - E_w^T \rho^w - \gamma^w = 0, \quad \forall w \quad (3-29)$$

$$(\bar{x}^w)^T \sigma^w = 0, \quad \forall w \quad (3-30)$$

$$\tau, \gamma^w, \sigma^w \geq 0 \quad (3-31)$$

Because $\bar{x}^w > 0$ when link (i, j) is on a utilized path, (3-30) implies that $\sigma_{ij}^w = 0$ for all (i, j) on a utilized path. By adding equations in (3-28) associated with arcs on the same path together and using that the fact that $\sigma_{ij}^w = 0$ for links on utilized path, the following must hold for every OD pair $w = (o, d)$:

$$\sum_{(i,j) \in \Phi^w} (t_{ij}(\bar{v}) + \tau_{ij}) = (\rho_o^w - \rho_d^w), \quad \forall r \in \bar{P}_0^w \quad (3-32)$$

$$\sum_{(i,j) \in \Phi^w} (t_{ij}(\bar{v}) + \tau_{ij}) \geq (\rho_o^w - \rho_d^w), \quad \forall r \in \bar{P}_0^w \quad (3-33)$$

where, similar to before, $\bar{P}_+^w = \{r : r \in P^w \text{ and } \bar{f}_r^w > 0\}$ is the set of utilized paths associated with \bar{v} and $\bar{P}_0^w = \{r : r \in P^w \text{ and } \bar{f}_r^w = 0\}$ is the set of unutilized paths. Next, observe that (3-29) implies that $c_w^{UE} - E_w^T \rho^w = \gamma^w \geq 0$ or $c_w^{UE} \geq \rho_o^w - \rho_d^w$ for all w . Thus, letting $\lambda^w = \rho_o^w - \rho_d^w$ yields a triplet (\bar{v}, τ, λ) that satisfies (3-15) – (3-18). Because \bar{v} is dominating, it follows that $t(\bar{v})^T \bar{v} < t(v^{UE})^T v^{UE}$ and τ satisfies (3-15) – (3-18), i.e., τ is Pareto-improving.

When the network has only one origin and $t_{ij}(v_{ij})$ is an increasing function for all (i, j) , the above theorem can be strengthened.

Theorem 4: Assume that there is only one origin and $t_{ij}(v_{ij})$ is an increasing function for all (i, j) . Let \bar{v} be a given dominating flow distribution without any single-commodity loop. Then, there always exists a Pareto-improving toll vector that induces \bar{v} .

Proof: Because $t_{ij}(v_{ij})$ is an increasing function and \bar{v} contains no single-commodity loop, the subnetwork consisting of arcs such that $\bar{v}_{ij} > 0$ also contains no directed cycle.

Using the algorithm in Dial (1999) for the one origin problem, it is possible to construct a toll vector $\bar{\tau} \geq 0$ such that, for each OD pair, the generalized costs of all utilized paths are the same and at least one of which is toll-free. The latter implies that, for every OD pair w , $\sum_{(i,j) \in \Phi_r^w} \bar{\tau}_{ij} = 0$ for some $\tilde{r} \in \bar{P}_+^w$ and it follows from (3-15) that, for every $r \in \bar{P}_+^w$,

$$\bar{\lambda}^w = \sum_{(i,j) \in \Phi_r^w} (t_{ij}(\bar{v}) + \tau_{ij}) = \sum_{(i,j) \in \Phi_{\tilde{r}}^w} t_{ij}(\bar{v}) \leq c_w^{UE}, \quad \forall r \in \bar{P}_+^w$$

From the above, $\bar{\tau}$ satisfies (3-15) – (3-18), and \bar{v} is dominating. Thus, $\bar{\tau}$ is Pareto-improving.

To illustrate, consider the five link example in Figure 3-1. Table 3-5 provides the UE and a dominating distribution. The latter solves problem P2 and is labeled as ‘DF-F-3’ in Table 3-5.

The algorithm in Dial (1999) constructs the longest path tree in Figure 3-5 using the link cost $t_{ij}(\bar{v})$ in Table 3-5. Based on this longest path tree, Dial’s algorithm would set the tolls on link (3, 2) and (3, 4) to be $\tau_{32} = (51.36 - 22.4) - 10.61 = 18.35$ and $\tau_{34} = (71.06 - 22.4) - 42.74 = 5.91$. Doing so produces the toll vector $\tau^1 = [0.0, 0.0, 18.35, 5.91, 0.0]^T$ and makes the generalized cost (time plus tolls) of every path equal 71.06. However, other Pareto-improving toll vectors exist. For example, $\tau^2 = [5.91, 0.0, 12.44, 0.0, 0.0]^T$ and $\tau^3 = [2.26, 0.0, 16.09, 3.65, 0.0]^T$ are both Pareto-improving. When considering a secondary objective, τ^1 and τ^2 may be more attractive because, e.g., they require a smaller number of toll collection facilities than τ^3 . With all three toll vectors, $\lambda^w = c_w^{UE}$ because the longest paths under \bar{v} and UE have the same length.

3.3.2 Existence of Discriminatory Pareto-Improving Tolls

So far in this chapter, all pricing schemes we have discussed are anonymous pricing schemes, i.e., road users are charged the same amount of money on links that are subject to congestion pricing regardless of their individual differences. However, as the technology of vehicle positioning system and electric tolling advances, discriminatory congestion pricing schemes are becoming more readily available to real-world

implementations. One form of discriminatory pricing is to charge users differently based on their origins and destinations, which is referred to as commodity-dependent (OD-dependent) pricing. In this section, we will investigate the relationship between OD-dependent tolls and dominating flow distributions.

Let \bar{v} denote a given dominating distribution. With respect to the travel costs under UE, a OD-dependent nonnegative toll vector, τ , is Pareto-improving if it satisfies the following conditions:

$$\sum_{(i,j) \in \Phi_r^w} (t_{ij}(\bar{v}_{ij}) + \tau_{ij}^w) = \lambda^w \quad \forall w, r \in \bar{P}_+^w \quad (3-34)$$

$$\sum_{(i,j) \in \Phi_r^w} (t_{ij}(\bar{v}_{ij}) + \tau_{ij}^w) \geq \lambda^w \quad \forall w, r \in \bar{P}_0^w \quad (3-35)$$

$$\tau \geq 0 \quad (3-36)$$

$$\lambda^w \leq c_{UE}^w \quad (3-37)$$

where $\bar{P}_+^w = \{r: r \in P^w \text{ and } \bar{f}_r^w > 0\}$ and $\bar{P}_0^w = \{r: r \in P^w \text{ and } \bar{f}_r^w = 0\}$. Similar to its anonymous pricing counterpart, a mathematical program is constructed to examine the existence of a Pareto-improving toll vector to support the dominating flow distribution \bar{v} .

$$\text{P5: } \min_{(y, \rho, \tau)} y \quad (3-38)$$

$$\text{s.t. } \bar{x}_{ij}^w (t_{ij}(\bar{v}_{ij}) + \tau_{ij}^w - \rho_i^w + \rho_j^w) = 0 \quad \forall w, (i, j) \in L \quad (3-39)$$

$$t_{ij}(\bar{v}_{ij}) + \tau_{ij}^w \geq \rho_i^w - \rho_j^w \quad \forall w, (i, j) \in L \quad (3-40)$$

$$\rho_{o(w)}^w - \rho_{d(w)}^w - c_w^{UE} \leq y \quad \forall w \quad (3-41)$$

$$\tau, y \geq 0 \quad (3-42)$$

Using the same demand and network settings as the previous example in Figure 3-4, we solve problem P5 and present the results in Table 3-5.

OD-dependent tolls are listed in the last two columns of Table 3-6. We can observe that road users between OD pair (3, 2) are better off and total system delay is also

reduced, therefore, the resulting OD-dependent tolling scheme is Pareto-improving. The above numerical examples in Table 3-4 and 3-6 demonstrate that a dominating flow distribution that cannot be supported by anonymous Pareto-improving tolls may be supported as a tolled UE by an OD-dependent Pareto-improving toll vector.

Hagstrom and Abrams (2009) concluded that any feasible flow distribution with no single-commodity loop can be realized as a Wardrop equilibrium using commodity-dependent nonnegative tolls even if the flow contains a multi-commodity loop. The following Lemma is a direct result of their conclusion.

Lemma 3: If a dominating flow distribution does not contain a single-commodity loop, then there always exists a nonnegative OD-dependent toll vector to support it.

To explore the relationship between a given dominating flow distribution and OD-dependent Pareto-improving toll vectors in general, we have the following Theorem.

Theorem 5: Given a dominating flow distribution without a single-commodity loop, we can always find a nonnegative OD-dependent Pareto-improving toll vector to support it as a Wardrop equilibrium.

Proof: The system of equations (3-34) to (3-37) can be decomposed into separate sub-problems for each OD pair. Each sub-problem is equivalent to finding an anonymous Pareto-improving nonnegative toll vector for a given dominating flow distribution with single OD pair. Using the algorithm in Dial (1999), it is possible to construct a toll vector $\bar{\tau}^w \geq 0$ for the sub-problem of OD pair w , such that all utilized paths of OD pair w have the same cost and at least one of them is toll-free. By running the algorithm for each OD pair, we can construct a nonnegative OD-dependent Pareto-improving toll vector to induce any dominating flow distribution without a single-commodity loop.

3.4 Pareto-Improving Toll Problem

The previous two sections investigated properties and existence of dominating flow distributions and Pareto-improving tolls. To find a Pareto-improving toll vector,

a two-step procedure can be followed. For a given network configuration, find a dominating flow distribution first (problem P1 or P2), and if it exists then try to find corresponding Pareto-improving toll vectors (problem P4 or P5) to induce the dominating flow distribution to a Wardrop equilibrium flow distribution. Nevertheless, the two-step procedure may not be particularly efficient in finding a Pareto-improving toll vector. In this section, we introduce a new mathematical problem called Pareto-improving toll problem to find a dominating flow distribution and equilibrium-inducing Pareto-improving tolls simultaneously. In general, Pareto-improving pricing schemes may be classified as second-best pricing schemes, which do not necessarily reduce congestion to the minimum level possible.

To find an anonymous Pareto-improving toll vector, the following mathematical program is formulated:

$$\text{P6: } \min_{(v, x, \rho, \tau)} t(v)^\top v \quad (3-43)$$

$$\text{s.t. } v \in V^F \quad (3-44)$$

$$x_{ij}^w (t_{ij}(v_{ij}) + \tau_{ij} - \rho_i^w + \rho_j^w) = 0 \quad \forall w, (i, j) \in L \quad (3-45)$$

$$t_{ij}(v_{ij}) + \tau_{ij} \geq \rho_i^w - \rho_j^w \quad \forall w, (i, j) \in L \quad (3-46)$$

$$\rho_{o(w)}^w - \rho_{d(w)}^w \leq c_w^{UE} \quad \forall w \quad (3-47)$$

$$\tau \geq 0 \quad (3-48)$$

The objective function of the above problem is to minimize the total travel time or delay in the system. When combined together, constraints (3-45) and (3-46) are the KKT conditions associated with $(t(v) + \tau)^\top (u - v) \geq 0, \forall u \in V^F$. In other words, these two constraints ensure flow distribution v is a tolled equilibrium flow distribution. When link (i, j) is utilized, i.e., $x_{ij}^w > 0$, constraint (3-45) forces the equation $t_{ij}(v_{ij}) + \tau_{ij} = \rho_i^w - \rho_j^w$ to hold. Combing together this equation for each link on a utilized path yields the following:

$$\sum_{(i,j) \in \Phi_r^w} (t_{ij}(v_{ij}) + \tau_{ij}) = \sum_{(i,j) \in \Phi_r^w} (\rho_i^w - \rho_j^w) = \rho_{o(w)}^w - \rho_{d(w)}^w$$

where, for OD pair w , Φ_r^w is a set containing links on path r and $o(w)$ and $d(w)$ represent the origin and destination nodes. Thus, constraint (3–44) implies that the generalized cost (time plus tolls) of every utilized path equals $(\rho_{o(w)}^w - \rho_{d(w)}^w)$ for OD pair w . Consequently, constraint (3–47) guarantees that no utilized path costs more than c_w^{UE} , the cost under UE, and doing so makes no one worse off. Finally, constraint (3–48) requires the link tolls to be nonnegative.

As formulated, problem P6 is always feasible. In particular, let v^{UE} denote a UE distribution, i.e., v^{UE} satisfies $t(v^{UE})^T(u - v^{UE}) \geq 0, \forall u \in V^F$. Then, $(v, \rho, \tau) = (v^{UE}, \rho^{UE}, 0)$ is feasible to problem P6, where ρ^{UE} is the KKT multipliers associated with the preceding variational inequality. On the other hand, a dominating flow distribution or Pareto-improving tolls may not exist. Let (v^*, ρ^*, τ^*) denote an optimal solution to problem P6. If $\sum_{(i,j) \in L} t_{ij}(v^*_{ij})v^*_{ij} < \sum_{(i,j) \in L} t_{ij}(v^{UE}_{ij})v^{UE}_{ij}$, then v^* is a dominating flow distribution and τ^* is Pareto-improving. When $\sum_{(i,j) \in L} t_{ij}(v^*_{ij})v^*_{ij} = \sum_{(i,j) \in L} t_{ij}(v^{UE}_{ij})v^{UE}_{ij}$, then no dominating flow distribution and Pareto-improving toll exists.

Similar to the above problem of developing an anonymous pricing scheme, the following mathematical program can be formulated to find an OD-dependent Pareto-improving toll vector.

$$\text{P7: } \min_{(v, \rho, \tau)} t(v)^T v \quad (3-49)$$

$$\text{s.t. } v \in V^F \quad (3-50)$$

$$x_{ij}^w (t_{ij}(v_{ij}) + \tau_{ij}^w - \rho_i^w + \rho_j^w) = 0 \quad \forall w, (i, j) \in L \quad (3-51)$$

$$t_{ij}(v_{ij}) + \tau_{ij} \geq \rho_i^w - \rho_j^w \quad \forall w, (i, j) \in L \quad (3-52)$$

$$\rho_{o(w)}^w - \rho_{d(w)}^w \leq c_w^{UE} \quad \forall w \quad (3-53)$$

$$\tau \geq 0 \quad (3-54)$$

Constraints (3–51) and (3–52) in the above optimization problem are KKT conditions that ensure flow distribution v is induced by OD-dependent tolls as a tolled UE

distribution. Constraint (3–53) confines equilibrium travel cost of each OD pair to no more than that under original UE condition. As formulated, by setting $\tau = 0$, v^{UE} is always a feasible solution to problem P7. However, an OD-dependent Pareto-improving toll vector only exists if the optimal value of the objective function of problem P7 is strictly less than $t(v^{UE})^T v^{UE}$.

The above Pareto-improving toll problems are formulated as MPCC, a class of optimization problems difficult to solve (see, e.g., [Scheel and Scholtes, 2000](#)). Standard stationarity conditions such as the KKT conditions do not hold for MPCC and many (see, e.g., [Luo et al., 1996](#) and references cited therein) have proposed special algorithms to solve them. The manifold suboptimization algorithm developed by [Lawphongpanich and Yin \(2010\)](#) is worth particular mentioning because it can solve large-scale MPCC problems effectively. Their algorithm focuses on finding a strongly stationary solution by solving a sequence of relaxed problems. More details about the algorithm can be found in [Lawphongpanich and Yin \(2010\)](#).

3.5 Summary

This chapter systematically investigated Pareto-improving congestion pricing tolls. In the beginning of the chapter, we discussed why a SO flow distribution is not appealing to the general public. The fundamental reason that a Pareto-improving congestion pricing scheme is achievable is that a UE flow distribution may not be Pareto-optimal. A Pareto improvement over the UE flow distribution termed as a dominating flow distribution was introduced in Section 2. Mathematical formulations to find dominating flow distributions were presented. Since a dominating flow distribution is not in an equilibrium state, how to induce a dominating flow distribution using nonnegative tolls was discussed. The absence of loops (single- and multi-commodity) was identified as the necessary and sufficient condition for the existence of anonymous nonnegative tolls to support a given dominating flow distribution. For a transportation network with one OD pair, we further concluded that there always exists a nonnegative toll vector

to support the optimal dominating flow distribution. In Section 3, we investigated the existence of nonnegative Pareto-improving toll vectors that can support a given dominating flow distribution as an equilibrium flow distribution in both anonymous and discriminatory congestion pricing settings. Two examples were used to show that an OD-dependent Pareto-improving toll vector may exist to support a dominating flow distribution that cannot be supported by an anonymous Pareto-improving toll vector. These examples were particularly enlightening because they demonstrated the potential benefits of introducing discriminatory pricing schemes. We then proved that a nonnegative OD-dependent Pareto-improving toll vector can always be found to induce any dominating flow distribution without a single-commodity loop as a Wardrop equilibrium flow distribution. In Section 4, instead of following a two-step procedure to find a dominating flow distribution and its corresponding Pareto-improving toll vector, we formulated a MPCC that can solve both problems simultaneously.

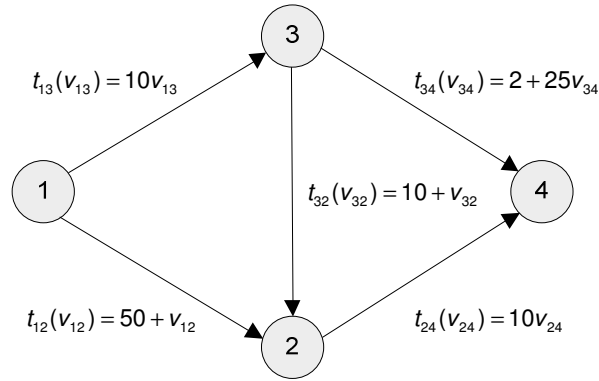


Figure 3-1. A five-link network

Table 3-1. Flow distributions under UE and MSC pricing for the five-link network

Link	UE		Flow (SO)	MSC pricing		
	Flow	Time		Time	Toll	Gen. cost
(1, 3)	3.60	36.00	2.06	20.64	20.64	41.28
(1, 2)	0.00	50.00	1.54	51.54	1.54	53.07
(3, 2)	2.28	12.28	0.90	10.90	0.90	11.79
(3, 4)	1.32	35.06	1.17	31.21	29.21	60.43
(2, 4)	2.28	22.78	2.43	24.32	24.32	48.63
Path						
1-3-4	1.32	71.06	1.17	51.85	49.85	101.70
1-3-2-4	2.28	71.06	0.90	55.85	45.85	101.70
1-2-4	0.00	72.78	1.54	75.85	25.85	101.70
Costs to users		255.80		227.11	139.02	366.13

Table 3-2. UE and dominating flow distributions

Link	UE	Link flows		
		DF-F-1	DF-F-2	DF-F-3
(1, 3)	3.60	1.90	2.24	2.24
(1, 2)	0.00	1.70	1.36	1.36
(3, 2)	2.28	0.00	0.45	0.61
(3, 4)	1.32	1.90	1.79	1.63
(2, 5)	2.28	1.70	1.81	1.97
Cost of the longest utilized path	71.06	68.70	69.40	71.06
System cost or total delay	255.78	247.10	241.17	234.99

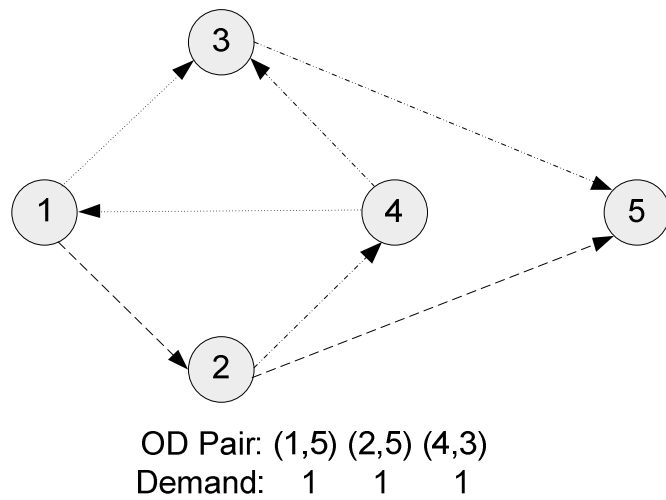


Figure 3-2. Multi-commodity loop example

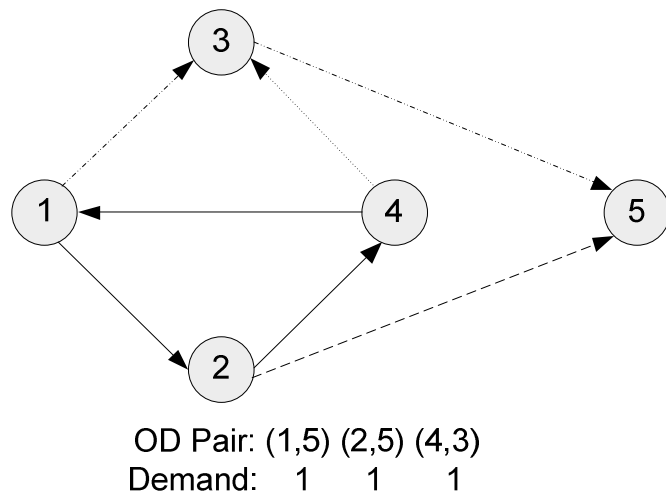


Figure 3-3. Multi-commodity loop example with loop removed

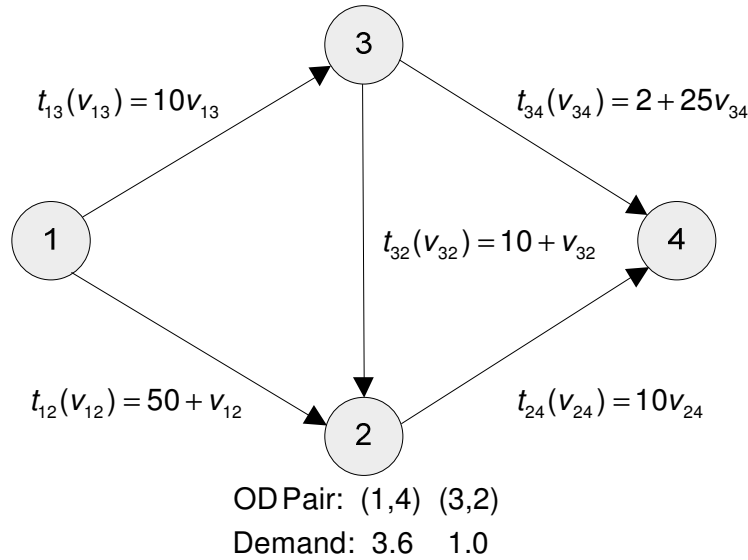


Figure 3-4. A five-link network with two OD pairs

Table 3-3. UE and optimal dominating flow distributions

Link	UE		Optimal dominating flow	
	OD 1-4 flow	OD 3-2 flow	OD 1-4 flow	OD 3-2 flow
(1, 3)	3.60	0.00	2.13	0.00
(1, 2)	0.00	0.00	1.47	0.00
(3, 2)	2.25	1.00	0.55	1.00
(3, 4)	1.35	0.00	1.57	0.00
(2, 4)	2.25	0.00	2.03	0.00
Longest utilized path	71.75	13.25	71.75	11.55
Total delay		271.55		245.09

Table 3-4. Optimal dominating flow distribution and tolls

Link	UE			Dominating flow and toll			
	OD 1-4 flow	OD 3-2 flow	Time	OD 1-4 flow	OD 3-2 flow	Time	Toll
(1, 3)	3.60	0.00	36.00	2.13	0.00	21.27	18.65
(1, 2)	0.00	0.00	50.00	1.47	0.00	51.47	0.00
(3, 2)	2.25	1.00	13.25	0.55	1.00	11.55	0.00
(3, 4)	1.35	0.00	35.75	1.57	0.00	41.31	1.46
(2, 4)	2.25	0.00	22.50	2.03	0.00	20.28	10.94
Equilibrium cost of OD 1-4			71.75				82.69
Equilibrium cost of OD 3-2			13.25				11.55
Total delay			271.55				245.09

Table 3-5. UE and dominating distributions

Link flow	v^{UE}	\bar{v} or DF-F-3	$t_{ij}(\bar{v})$
(1, 3)	3.60	2.24	22.40
(1, 2)	0.00	1.36	51.36
(3, 2)	2.28	0.61	10.61
(3, 4)	1.32	1.63	42.75
(2, 5)	2.28	1.97	19.70
Cost of the longest utilized path	71.06	71.06	
System cost or total delay	255.78	234.99	

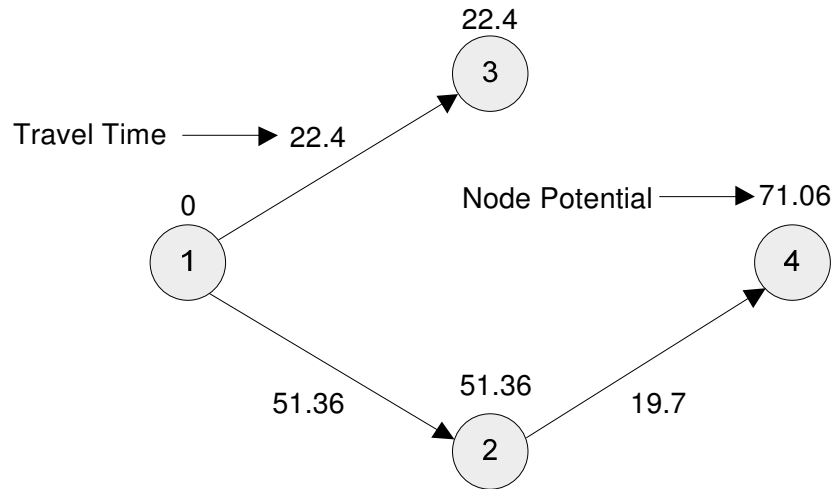


Figure 3-5. The longest path tree associated with \bar{v}

Table 3-6. OD-dependent Pareto-improving tolls

Link	UE			Pareto-improving toll				
	OD 1-4 flow	OD 3-2 flow	Time	OD 1-4 flow	OD 3-2 flow	Time	OD 1-4 toll	OD 3-2 toll
(1, 3)	3.60	0.00	36.00	2.13	0.00	21.27	6.14	0.00
(1, 2)	0.00	0.00	50.00	1.47	0.00	51.47	0.00	0.00
(3, 2)	2.25	1.00	13.25	0.55	1.00	11.55	12.51	1.44
(3, 4)	1.35	0.00	35.75	1.57	0.00	41.31	3.04	0.00
(2, 4)	2.25	0.00	22.50	2.03	0.00	20.28	0.00	0.00
Equilibrium cost OD 1-4			71.75					71.75
Equilibrium cost OD 3-2			13.25					12.99
Total delay			271.55					245.09

CHAPTER 4 PARETO-IMPROVING CONGESTION PRICING SCHEME FOR MULTICLASS NETWORK

[Lawphongpanich and Yin \(2010\)](#) introduced and proposed an algorithm for finding Pareto-improving tolls on general networks assuming that users are homogenous. Because there is substantial heterogeneity in value of travel time (VOT) across the driving population (e.g., between \$20 and \$40 per hour observed by [Brownstone and Small, 2005](#)) and the heterogeneity may have important implications in designing congestion pricing schemes ([Small et al., 2005](#); [Small and Yan, 2001](#)), this chapter extends the work of [Lawphongpanich and Yin \(2010\)](#) to the case with user heterogeneity. In particular, this chapter presents optimization models that determine an anonymous Pareto-improving toll vector or indicate that one does not exist. In the literature, tolls are “anonymous” or “uniform” if they are the same for all user classes. Such tolls are of interest because it is generally difficult or impractical to determine the VOT of a user arriving at a toll facility (e.g., [Yang and Huang, 2004](#)).

For the remainder of this chapter, Section 2 introduces the definition of Multiclass Pareto-improving congestion pricing scheme and presents two mathematical programming models for finding a Multiclass Pareto-improving scheme. Section 3 analyzes the problem and establishes the existence conditions of Multiclass Pareto-improving schemes through a two-step procedure. Section 4 proposes an algorithm that can solve the problem using manifold suboptimization procedure. Section 5 presents numerical examples, followed by concluding remarks in Section 6.

4.1 Multiclass Pareto-Improving Pricing Scheme

This section defines a Multiclass Pareto-improving pricing scheme mathematically and formulates the problem of finding such a scheme as a mathematical program with complementarity constraints (MPCC). To highlight key ideas, we assume that the travel demand for every OD pair is fixed.

4.1.1 Multiclass Network Equilibrium

Similar to many previous studies in the literature (e.g., [Dafermos, 1972](#); [Engelson and Lindberg, 2006](#); [Yang and Huang, 2004](#)), we represent user heterogeneity by a discrete set of VOTs, and classify accordingly the users into multiple classes. For each user class k , let $d^{w,k}$ and β^k denote the travel demand between OD pair w and the corresponding VOT, respectively.

To describe feasible flow distributions, let n be the number of links in the network and K , the number of user classes. Then, a feasible multiclass flow distribution is represented as a K -tuple of vectors in R^n , i.e. (v^1, \dots, v^K) , where $v^i \in R^n, i = 1, \dots, K$, is a vector of link flows for class k with v_a^k as its element. Mathematically, the set of all feasible multiclass flow distributions, denoted as V^F , can be described using either path or link flow variables. Using the former, let $f_r^{w,k}$ be the flows of user class k on path r for OD pair w . Then,

$$V^F = \left\{ (v^1, \dots, v^K) : v_a^k = \sum_w \sum_{r \in P^w} \delta_{ar} f_r^{w,k}; \right. \\ \left. \sum_{r \in P^w} f_r^{w,k} = d^{w,k}, \forall w, k; f_r^{w,k} \geq 0, \forall w, k, r \right\} \quad (4-1)$$

where P^w is the set of paths for OD pair w and $\delta_{ar} = 1$ if link a is on path r , 0 otherwise. To describe the set in terms of link flows, let A be the node-arc incidence matrix for the network and E^w denote a vector in R^m , where m is the number of nodes. Specifically, E^w is an “input-output” vector and has exactly two non-zero components: one has a value 1 corresponding the origin node of OD pair and the other one has a value -1 in the component for the destination. Then, V^F can be written as

$$V^F = \{ (v^1, \dots, v^K) : v^k = \sum_w x^{w,k}, Ax^{w,k} = E^w d^{w,k}, x^{w,k} \geq 0, \forall w, k \}.$$

In the above, $x^{w,k} \in R^n$ is a vector whose components are link flows of user class k for OD pair w .

As an alternative to (v^1, \dots, v^K) , we also let (u^1, \dots, u^K) denote an element of V^F . In addition, the letters u and v without any super or subscript denote the aggregation

of class-specific link flow vectors, i.e., $u = \sum_k u^k$ and $v = \sum_k v^k$. Generally, u^{UE} and u^{SO} denote the aggregate multiclass user equilibrium (UE) and system optimum (SO) distribution.

For each link a , $t_a(u_a)$ denotes the travel time function that is convex and monotonically increasing with respect to the aggregate flow u_a . The multiclass UE flow distribution u^{UE} can be obtained by solving the following variational inequality (e.g., Engelson and Lindberg, 2006):

$$\sum_k t(u^{UE})^T (v^k - u^{k,UE}) \geq 0, \quad \forall v \in V^F$$

Under the above assumptions concerning the travel time functions, the aggregate vector u^{UE} is unique. However, the class-specific link flows $u^{k,UE}$ may not be unique. There may be another K -tuple (v^1, \dots, v^K) such that $u^{UE} = \sum_{k=1}^K v^k$.

4.1.2 Definition of Multiclass Pareto-Improving Pricing Scheme

Considering a network with an anonymous pricing scheme $\tau \in R^n$, we assume that users perceive their travel costs (disutility) as a sum of travel-time cost and toll fare, although other criteria may apply (e.g., Nagurney, 2000; Yin and Yang, 2004). We further assume that the travel costs are additive and thus a generalized route travel cost is the sum of generalized costs associated with links that the route comprises. Consequently, the tolled multiclass UE distribution, \tilde{v} , can be obtained by solving the following variational inequality:

$$\sum_k (\beta^k t(\tilde{u}) + \tau)^T (v^k - \tilde{u}^k) \geq 0, \quad \forall v \in V^F$$

A nonnegative toll vector, τ , is “Pareto-improving” if the following conditions are satisfied:

$$\sum_a \delta_{ar} (\beta^k t_a(\tilde{u}_a) + \tau_a) = \lambda^{w,k} \quad \forall w, k, r \in \tilde{P}_+^{w,k} \quad (4-2)$$

$$\sum_a \delta_{ar} (\beta^k t_a(\tilde{u}_a) + \tau_a) \geq \lambda^{w,k} \quad \forall w, k, r \in \tilde{P}_0^{w,k} \quad (4-3)$$

$$\tau \geq 0 \quad (4-4)$$

$$\lambda^{w,k} \leq c_{w,k}^{UE} \quad \forall w, k \quad (4-5)$$

$$t(\tilde{u})^T \tilde{u} < t(u^{UE})^T u^{UE} \quad (4-6)$$

or

$$\sum_k \beta^k t(\tilde{u})^T \tilde{u}^k < \sum_k \beta^k t(u^{UE})^T u^{k,UE} \quad (4-7)$$

where $\tilde{P}_+^{w,k} = \{r : r \in P^w \text{ and } \tilde{f}_r^{w,k} > 0\}$, $\tilde{P}_0^{w,k} = \{r : r \in P^w \text{ and } \tilde{f}_r^{w,k} = 0\}$, and $c_{w,k}^{UE}$ denotes the travel cost of user class k for OD pair w under the UE condition without tolling, i.e., $c_{w,k}^{UE} = \beta_k \sum_a \delta_{ar} t_a(u_a^{UE})$ for all paths r utilized by users in class k under UE.

For each OD pair and user class, the first two conditions ensure that, with a toll pattern τ , the tolled UE holds and the equilibrium cost is $\lambda^{w,k}$. Condition (4-4) requires all tolls to be nonnegative because it is more sensible to penalize or tax negative externalities such as traffic congestion, instead of subsidizing or rewarding. Moreover, pricing schemes with subsidies are generally more complex to implement in practice. Condition (4-5) requires that the generalized cost for each OD pair and user class with tolls is no greater than the situation without, i.e., no user is made worse off. Condition (4-6) or (4-7) guarantees a strict improvement in system performance. The former ensures that the total system travel time will be strictly reduced while the latter is concerned with the total system travel cost. These two objectives have different policy implications. In the former, travel times among all users are weighted equally while the latter weighs travel times by users' VOT. Because VOT is positively correlated with income, the former seems more progressive (Mayet and Hansen, 2000). To differentiate, the former is called as time-based Pareto-improving scheme while the latter cost-based.

As stated above, condition (4-5) applies to all OD pairs and user classes. However, this is unnecessary because it is sufficient to require (4-5) to hold only for the user class with the lowest VOT, i.e., (4-5) can be replaced by

$$\lambda^{w,L} \leq c_{w,L}^{UE}, \quad \forall k \quad (4-8)$$

where L is a class index such that $\beta_L \leq \beta_k, \forall k$. Assume that $(\tilde{u}, \tau, \lambda)$ satisfies (4–2) – (4–4), (4–6), (4–7), and (4–8). Let $r \in \tilde{P}_+^{w,L}$ be a path utilized by users in class L under \tilde{u} . Then, the following must hold from the fact that $\beta_L \leq \beta_k, \forall k$:

$$\sum_a \delta_{ar} (t_a(\tilde{u}_a) + \frac{\tau_a}{\beta^k}) \leq \sum_a \delta_{ar} (t_a(\tilde{u}_a) + \frac{\tau_a}{\beta^L}) = \frac{\lambda^{w,L}}{\beta^L} = \frac{c_{w,L}^{UE}}{\beta^L} = t_w^{UE}$$

When multiplied by β^k , the above implies that $\sum_a \delta_{ar} (\beta^k t_a(\tilde{u}_a) + \tau_a) \leq \beta^k t_w^{UE} = c_{w,k}^{UE}$.

Thus, (4–8) implies (4–5) for all $r \in \tilde{P}_+^{w,L}$. For paths not utilized by users in class L under \tilde{u} , i.e., $r' \in \tilde{P}_0^{w,L}$, assume there exists a class k such that r' is utilized under \tilde{u} , i.e., $r' \in \tilde{P}_+^{w,k}$, and $\sum_a \delta_{ar'} (\beta^k t_a(\tilde{u}_a) + \tau_a) = \lambda^{w,k} > c_{w,k}^{UE}$. However, the latter implies that path r' costs more than $r \in \tilde{P}_+^{w,L}$ which contradicts the fact that r' is utilized by users in class k and \tilde{u} is in user equilibrium.

4.1.3 Finding Multiclass Pareto-Improving Pricing Scheme

Conditions (4–2) – (4–6) or (4–7) define a Pareto-improving pricing scheme that makes no user worse off while improving system performance and generating toll revenue. The scheme may not always exist in a general road network. The section presents mathematical formulations for finding such a Pareto-improving pricing scheme, if it exists.

A multiclass time-based Pareto-improving (MTPI) scheme can be obtained by solving the following mathematical program:

$$\text{MTPI-P: } \min \quad t(u)^T u \quad (4-9)$$

$$\text{s.t. } u \in V^F \quad (4-10)$$

$$f_r^{w,k} \left(\sum_a \delta_{ar} (\beta^k t_a(u_a) + \tau_a) - \lambda^{w,k} \right) = 0 \quad \forall w, k, r \in P_k^w \quad (4-11)$$

$$\sum_a \delta_{ar} (\beta^k t_a(u_a) + \tau_a) \geq \lambda^{w,k} \quad \forall w, k, r \in P_k^w \quad (4-12)$$

$$\lambda^{w,k} \leq c_{w,k}^{UE} \quad \forall w, k \quad (4-13)$$

$$\tau \geq 0 \quad (4-14)$$

When the optimal objective function value of MTPI-P is strictly less than $t(u^{UE})^T u^{UE}$, the solution τ is a Pareto-improving pricing scheme. As formulated, MTPI-P is an MPCC, a class of optimization problems difficult to solve. Standard stationarity conditions such as the Karush-Kuhn-Tucker conditions do not hold for MPCC and many have proposed special algorithms to solve them (e.g., [Scheel and Scholtes, 2000](#)). [Lawphongpanich and Yin \(2010\)](#) applied concepts from manifold suboptimization and proposed a new algorithm that converges to a strongly stationary solution in a finite number of iterations. The algorithm will be applied to solve the above MTPI-P problem for strongly stationary solutions later in this chapter.

Although intuitive, the above formulation is not computationally convenient because it requires knowing or generating all paths between each OD pair. An alternative formulation using link flow variables and node potentials (see [Ahuja et al., 1993](#)) can be formulated below:

$$\text{MTPI-L: } \min \quad t(u)^T u \quad (4-15)$$

$$\text{s.t. } u \in V^F \quad (4-16)$$

$$x_a^{w,k} (\beta^k t_a(u_a) + \tau_a - \rho_i^{w,k} + \rho_j^{w,k}) = 0 \quad \forall a, w, k, a = (i, j) \quad (4-17)$$

$$\beta^k t_a(u_a) + \tau_a \geq \rho_i^{w,k} - \rho_j^{w,k} \quad \forall a, w, k, a = (i, j) \quad (4-18)$$

$$\rho_{o(w)}^{w,k} - \rho_{d(w)}^{w,k} \leq c_{w,k}^{UE} \quad \forall w, k \quad (4-19)$$

$$\tau \geq 0 \quad (4-20)$$

In the above, $\rho^{w,k}$ are node potentials. Moreover, nodes i and j are the starting and ending nodes of link a , and $o(w)$ and $d(w)$ represent the origin and destination nodes of OD pair w . As stated above, $\rho^{w,k}$ are unrestricted. However, it is possible to set $\rho_{d(w)}^{w,k} = 0$ and doing so forces $\rho_i^{w,k}$ to be nonnegative for all $i = 1, \dots, m$.

Since Pareto-improving toll scheme may not always exist, when it does not exist, we contend that a congestion pricing scheme would still be appealing if users are not worse

off by a significant amount. This leads to the concept of approximate Pareto-improving toll schemes, which can be determined by solving the following MPCC:

$$\text{MTPI-L: } \min \quad t(u)^T u \quad (4-21)$$

$$\text{s.t. } u \in V^F \quad (4-22)$$

$$x_a^{w,k} (\beta^k t_a(u_a) + \tau_a - \rho_i^{w,k} + \rho_j^{w,k}) = 0 \quad \forall a, w, k, a = (i, j) \quad (4-23)$$

$$\beta^k t_a(u_a) + \tau_a \geq \rho_i^{w,k} - \rho_j^{w,k} \quad \forall a, w, k, a = (i, j) \quad (4-24)$$

$$\rho_{o(w)}^{w,k} - \rho_{d(w)}^{w,k} \leq (1 + \alpha) c_{w,k}^{UE} \quad \forall w, k \quad (4-25)$$

$$\tau \geq 0 \quad (4-26)$$

Where α is called relaxation factor, which indicates users are allowed to be worse off by at most α comparing with their travel costs under UE condition. The above formulation is conceptually similar as the bi-level programming formulation proposed by [Yang and Zhang \(2002\)](#). [Jahn et al. \(2005\)](#) propose a similar formulation that uses routing instead of tolling to achieve approximate Pareto improvement.

Similarly, a path or link-based MPCC can be formulated for finding a cost-based Pareto-improving scheme by replacing the objective function of the above formulations with $\min \sum_k \beta^k t(u)^T v^k$.

4.2 Properties and Existence of Multiclass Pareto-Improving Pricing Scheme

The section discusses the properties of a Pareto-improving scheme and establishes the existence conditions.

The fundamental reason for the existence of a Pareto-improving pricing scheme is that the original Wardropian user equilibrium may not be Pareto optimal. In game theory ([Fisk, 1984](#)), a Wardropian user equilibrium corresponds to a Nash equilibrium, where a player corresponds to a traveler or user. As the prisoner's dilemma ([Poundstone, 1992](#)) often used to demonstrate, a Nash equilibrium needs not be strongly Pareto optimal. There may be another situation that is as good for everyone and strictly preferred

by some. In network flow modeling, such a property is characterized by (Hagstrom and Abrams, 2001) as generalized Braess's paradox. Given a UE flow distribution in a general road network, a generalized Braess's paradox occurs if there exists some other feasible flow distribution under which the total system travel time is smaller and all utilized paths are no longer than those under UE. Such a flow distribution may be more desirable since it dominates the UE distribution (Lawphongpanich and Yin, 2010). Because the dominating flow distribution is not in equilibrium and thus non-sustainable, we use tolls as a mechanism to evolve the traffic flow to the dominating distribution. In other words, we seek a tolling scheme such that the tolled UE flow distribution is the dominating flow distribution. If after paying the tolls, no users have larger travel costs than before, the tolling scheme is Pareto improving.

Therefore, finding Pareto-improving congestion pricing schemes can be decomposed into two steps: the first step is to detect whether a generalized Braess paradox occurs, i.e., finding a dominating flow. Given a dominating distribution, we then investigate whether there exists a nonnegative tolling scheme that induces the dominating flow distribution without making any user worse off.

4.2.1 Multiclass Dominating Flow Distribution

A flow distribution $v \in V^F$ dominates a given multiclass UE distribution or is dominating if it strictly reduces the total system travel time or travel cost and allows all users to use routes that are no more expensive than those under UE. Mathematically, v and its path flows $f_r^{w,k}$ satisfy the following conditions:

$$\sum_a \delta_{ar} \beta^k t_a(u_a) \leq c_{w,k}^{UE}, \quad \forall w, k, r \in P_+^{w,k} \quad (4-27)$$

$$t(u)^T u < t(u^{UE})^T u^{UE} \quad (4-28)$$

or

$$\sum_k \beta^k t(u)^T v^k < \sum_k \beta^k t(u^{UE})^T v^{k,UE} \quad (4-29)$$

where $P_+^{w,k} = \{r : r \in P^w \text{ and } f_r^{w,k} > 0\}$. It follows immediately from condition (4-27) that a dominating flow distribution makes no user experience longer travel time when compared to the UE distribution. Condition (4-28) or (4-29) ensures that the system performance is strictly improved under a dominating flow distribution, which implies that some users will be better off.

To find a time-based multiclass dominating flow distribution, we formulate the following path-based optimization problem:

$$\text{TBDF-P: } \min \quad t(u)^T u \quad (4-30)$$

$$\text{s.t. } v \in V^F \quad (4-31)$$

$$f_r^{w,k} \left(\sum_a \delta_{ar} \beta^k t_a(u_a) - c_{w,k}^{UE} \right) \leq 0 \quad \forall w, k, r \in P^w \quad (4-32)$$

In the above, the first constraint requires v to be a feasible flow distribution and the second ensures that, if path r is utilized, its travel cost for user k , $\sum_a \delta_{ar} \beta^k t_a(u_a)$, must be no greater than the corresponding UE travel cost. The objective of the problem is to minimize total system travel time. Thus, if v^{DF} solves the TBDF-P problem and $t(u^{DF})^T u^{DF} < t(u^{UE})^T u^{UE}$, then v^{DF} is a dominating flow distribution. Otherwise, no dominating flow distribution exists.

An equivalent but more computationally-convenient link-based formulation for finding dominating flow distribution is as follows:

$$\text{TBDF-L: } \min \quad t(u)^T u \quad (4-33)$$

$$\text{s.t. } v \in V^F \quad (4-34)$$

$$x_a^{w,k} (\rho_i^{w,k} - \rho_j^{w,k} - \beta^k t_a(u_a)) \geq 0 \quad \forall a, w, k, a = (i, j) \quad (4-35)$$

$$\rho_{o(w)}^{w,k} - \rho_{d(w)}^{w,k} \leq c_{w,k}^{UE} \quad \forall w, k \quad (4-36)$$

In the above, when link a is utilized by user class k , i.e., $x_a^{w,k} > 0$, the second constraint reduces to $\rho_i^{w,k} - \rho_j^{w,k} - \beta^k t_a(u_a) \geq 0$. Collectively, this expression for each link on a utilized path yields the following

$$\sum_{a \in \Phi_r^w} \beta^k t_a(u_a) \leq \sum_{a \in \Phi_r^w, a=(i,j)} (\rho_i^{w,k} - \rho_j^{w,k}) = \rho_{o(w)}^{w,k} - \rho_{d(w)}^{w,k},$$

where Φ_r^w is a set containing links on path r between OD pair w . Thus, the second constraint implicitly ensures that the length of every utilized path for user class k and OD pair w is no greater than $(\rho_{o(w)}^{w,k} - \rho_{d(w)}^{w,k})$. Consequently, the last constraint guarantees that no utilized path has more expensive travel cost than the UE cost. Note that TBDF-P and TBDF-L are similar in structure to MTPI-P and MTPI-L and can be transformed into MPCC by introducing auxiliary variables. And if we replace the objective functions with $\min \sum_k \beta^k t(u)^T v^k$, the problem of finding the cost-based dominating flow distributions can be formulated.

4.2.2 Existence of Nonnegative Pareto-Improving Tolls

For a given dominating flow distribution, this section investigates whether there exists a Pareto-improving toll vector that can induce such a flow distribution.

Let u^{DF} denotes a given aggregate dominating flow distribution. The set of nonnegative Pareto-improving toll patterns consists of the τ component of the solution (τ, λ, \hat{v}) to the following system of equations and inequalities:

$$\sum_k \beta_k \sum_a (t_a(u_a^{DF}) + \tau_a) \hat{v}_a^k = \sum_k \sum_w \lambda^{w,k} d^{w,k} \quad (4-37)$$

$$\sum_a \delta_{ar} (\beta^k t_a(u_a^{DF}) + \tau_a) \geq \lambda^{w,k} \quad \forall w, k, r \in P_k^w \quad (4-38)$$

$$u^{DF} = \sum_k \hat{v}^k \quad (4-39)$$

$$\hat{v} \in V^F \quad (4-40)$$

$$\lambda^{w,k} \leq c_{w,k}^{UE} \quad \forall w, k \quad (4-41)$$

In the above, for each OD pair and each user class, the first four conditions ensure that after adding toll τ , the tolled multiclass UE holds and the equilibrium cost for user class k and OD pair w is $\lambda^{w,k}$. Note that \hat{v} is not necessarily v^{DF} , which is the class specific

flow distribution associated with u^{DF} obtained by solving TBDF-L. Condition (4-41) ensures that the generalized cost for each OD pair and each user class under the tolled UE is no greater than the original UE travel cost.

The above toll set could be empty. To establish an existence condition, consider the following problem for a given dominating flow u^{DF} :

$$\text{MPIT: min } t(u^{DF})^T \left(\sum_w \sum_k \beta_k y^{w,k} \right) + \sum_w \sum_k c_{w,k}^{UE} z^{w,k} \quad (4-42)$$

$$\text{s.t. } Ay^{w,k} + E^w z^{w,k} = E^w d^{w,k} \quad \forall w, k \quad (4-43)$$

$$\sum_w \sum_k y_a^{w,k} \leq u_a^{DF} \quad \forall a \quad (4-44)$$

$$y^{w,k}, z^{w,k} \geq 0 \quad \forall w, k \quad (4-45)$$

We refer to the above problem as the multiclass Pareto-improving toll (MPIT) problem because the theorem below shows that the dual variables or multipliers associated with the second constraint becomes Pareto-improving tolls. Observe that the MPIT problem is linear with respect to its decision variables, $y_a^{w,k}$ and $z^{w,k}$, because u^{DF} and $c_{w,k}^{UE}$ are given. The following theorem gives a necessary and sufficient condition for the existence of a Pareto-improving toll vector.

Theorem: Let u^{DF} be a multiclass aggregate dominating flow distribution. A Pareto-improving toll vector exists if and only if $(\hat{x}^{w,k}, 0)$ solves the MPIT problem and the inequality constraints are binding, i.e., $\sum_w \sum_k \hat{x}^{w,k} = u^{DF}$.

Proof: Since KKT conditions are both necessary and sufficient for linear programs, $(\hat{x}^{w,k}, 0)$ solves the MPIT problem if and only if there exists multipliers $\rho_i^{w,k}, \tau_a, \sigma^{w,k}$ and $\gamma_a^{w,k}$ such that the following hold:

$$\beta^k t_a(u_a^{DF}) - (\rho_i^{w,k} - \rho_j^{w,k}) + \tau_a - \gamma_a^{w,k} = 0 \quad \forall w, k \text{ and } a = (i, j) \quad (4-46)$$

$$c_{w,k}^{UE} - (E^w)^T \rho^{w,k} - \sigma^{w,k} = 0 \quad \forall w, k \quad (4-47)$$

$$(\hat{x}^{w,k})^T \gamma^{w,k} = 0 \quad \forall w, k \quad (4-48)$$

$$\tau, \sigma, \gamma \geq 0 \quad (4-49)$$

$$A\hat{x}^{w,k} = E^w d^{w,k} \quad (4-50)$$

$$\sum_w \sum_k \hat{x}^{w,k} = u^{DF} \quad (4-51)$$

Because $\hat{x}_a^{w,k} > 0$ when link a is on a utilized path, the complementary slackness condition implies that $\gamma_a^{w,k} = 0$ for all a on a utilized path. By adding the first equation associated with links on the same path together and using the fact that $\gamma_a^{w,k} = 0$ for links on utilized paths, the following must hold for every user class k and OD pair $w = (o, d)$:

$$\sum_a \delta_{ar} (\beta^k t_a(u_a^{DF}) + \tau_a) = \rho_o^{w,k} - \rho_d^{w,k} \quad \forall w, k \text{ and } r \in \hat{P}_+^{w,k} \quad (4-52)$$

$$\sum_a \delta_{ar} (\beta^k t_a(u_a^{DF}) + \tau_a) \geq \rho_o^{w,k} - \rho_d^{w,k} \quad \forall w, k \text{ and } r \in \hat{P}_0^{w,k} \quad (4-53)$$

where $\hat{P}_+^{w,k} = \{r : r \in P^w \text{ and } \hat{f}_r^{w,k} > 0\}$, and $\hat{P}_0^{w,k} = \{r : r \in P^w \text{ and } \hat{f}_r^{w,k} = 0\}$. Let $\rho_o^{w,k} - \rho_d^{w,k} = \lambda^{w,k}$. Together with (4-50) and $\hat{v}^k = \sum_w \hat{x}^{w,k}$, the above two conditions imply that conditions (4-37) and (4-38) hold (Heran and Ramana, 1998).

From equation (4-47), we have $c_{w,k}^{UE} - (E^w)^T \rho_k^{w,k} = \sigma^{w,k} \geq 0$, i.e., $c_{w,k}^{UE} \geq \rho_o^{w,k} - \rho_d^{w,k} = \lambda^{w,k}$ for all w, k . Thus, the above equations (4-47) – (4-51) yield a solution (τ, λ, \hat{v}) satisfying conditions (4-37) – (4-41), and τ is Pareto-improving.

Notice that imposing the above toll vector τ will not necessarily replicate \hat{v}^k , because the multiclass tolled UE problem may admit multiple network equilibrium. However, the corresponding aggregate link flow pattern, the path travel times and the generalized path travel costs are the same under these equilibrium (Proposition 2, Engelson and Lindberg, 2006). In other words, the resulting aggregate link flow will be u_a^{DF} for each link and the generalized travel cost $\lambda^{w,k} \leq c_{w,k}^{UE}$ for each OD pair and user class, thereby ensuring that τ is Pareto improving.

4.3 Manifold Suboptimization Algorithm

The problems we formulated in Sections 2 and 3, namely MTPI-P, MTPI-L, TBDF-P, and TBDF-L, are MPCC problem, which is a class of optimization problems

difficult to solve. The reasons are two folds. One is the fact that MPCC violates the Magasarian-Fromovitz constraint qualification (MFCQ) and the other is that its feasible region is non-convex (see, e.g., [Chen and Florian, 1995](#); [Scheel and Scholtes, 2000](#)). Standard algorithms for nonlinear programs become ineffective for MPCC. [Lawphongpanich and Yin \(2010\)](#) developed a Manifold Suboptimization algorithm for finding a “strongly stationary” solution to Pareto-improving problem by solving a sequence of relaxed problems. More details about the algorithm can be found in [Lawphongpanich and Yin \(2010\)](#).

Below is a version of the manifold suboptimization algorithm for solving MTPI-L problem.

Manifold Suboptimization Algorithm

Step 0 Let v be a UE distribution and $x_{ij}^{w,k}$ denotes the corresponding link flow for OD pair w of user class k . Set $n=1$, $\Omega_{x,w,k}^1 = \{(i, j) : x_{ij}^{w,k} = 0\}$, and $\Omega_{z,w,k}^1 = \{(i, j) : x_{ij}^{w,k} > 0\}$.

Step 1 Let $(v^n, \rho^n, \beta^n, x^n, z^n)$ solve the following problem:

$$\text{R-MTPI: } \min t(u)^T v \quad (4-54)$$

$$\text{s.t. } u = \sum_w \sum_k x^{w,k} \quad (4-55)$$

$$Ax^{w,k} = E^{w,k} d^{w,k} \quad \forall w, k \quad (4-56)$$

$$z_{ij}^{w,k} = \beta^k t_{ij}(u_{ij}) + \tau_{ij} - \rho_i^{w,k} + \rho_j^{w,k} \quad \forall w, k \text{ and } (i, j) \in L \quad (4-57)$$

$$\rho_{o(w)}^{w,k} - \rho_{d(w)}^{w,k} \leq c_{w,k}^{UE} \quad \forall w, k \quad (4-58)$$

$$x_{ij}^{w,k} = 0 \quad \forall w, k \text{ and } (i, j) \in \Omega_{x,w,k}^n \quad (4-59)$$

$$z_{ij}^{w,k} = 0 \quad \forall w, k \text{ and } (i, j) \in \Omega_{z,w,k}^n \quad (4-60)$$

$$x_{ij}^{w,k} \geq 0 \quad \forall w, k \text{ and } (i, j) \notin \Omega_{x,w,k}^n \quad (4-61)$$

$$z_{ij}^{w,k} \geq 0 \quad \forall w, k \text{ and } (i, j) \notin \Omega_{z,w,k}^n \quad (4-62)$$

$$\tau_{ij} \geq 0 \quad \forall (i, j) \in L \quad (4-63)$$

Step 2 Let $\Gamma_{w,k}^n = \{(i, j) \in \Omega_{x,w,k}^n : \delta_{ij,w,k}^n < 0 \text{ and } z_{ij,w,k}^n = 0\}$, where $\delta_{ij,w,k}^n$ is the multiplier associated with the constraint $x_{ij}^{w,k} = 0$. If $\Gamma_{w,k}^n$ is empty for all w, k , stop and $(v^n, \rho^n, \beta^n, x^n, z^n)$ is strongly stationary. Otherwise, do the following and go to Step 1:

- a) Set $\Omega_{x,w,k}^{n+1} = \Omega_{x,w,k}^n - \Gamma_{w,k}^n$,
- b) Set $\Omega_{z,w,k}^{n+1} = \{(i, j) : z_{ij,w,k}^n = 0\}$,
- c) Set $n=n+1$.

One of the advantages of this algorithm is its fast computation time, which is very desirable in large-scale applications. We will use this algorithm to solve MTPI problems in three test networks in the next section.

4.4 Numerical Examples

Consider the network in Figure 4-1 in which there is only one OD pair (1, 4) and two user classes, whose VOTs are 0.6 and 1.4 respectively. The total demand is 3.6 and each user class has a demand of 1.8.

Instead of directly solving the Pareto-improving toll problems proposed in Section 2, we adopt the two-step procedure to gain more insights. We first solve the TBDF-F problem for a time-based dominating flow and then obtain a Pareto-improving scheme by solving MPIT and examining the multipliers. Table 4-1 presents the results and comparisons with the multiclass UE flow distribution are reported in Table 4-2.

For the case with Pareto-improving tolls, the results in Table 4-2 indicate that the travel cost for user class 1 is the same as before while the cost for class 2 decreases from 99.48 to 94.76. In the presence of anonymous tolls, the users with lower VOTs are more likely to be made worse off than those with higher VOTs (Yin and Yang, 2004). If the users with the lowest VOT are no worse off, then all the other users will be better off. At the same time, the Pareto-improving tolls reduce the total system travel time (and cost), and generate an amount of toll revenue of 18.57. Therefore, the pricing scheme is beneficial to society, the government and users of class 2 without hurting

users of class 1. The total system travel time is reduced from 255.80 to 234.99, which is very close to the total system travel time under SO condition, 227.11. The reduction is approximately 72.53% of the maximum possible reduction. It is also interesting to note that the time-based class-specific flow distribution from solving the TBDF-L problem, $v^{DF,1} = \{0.99, 0.81, 0.30, 0.69, 1.11\}^T$ and $v^{DF,2} = \{1.25, 0.55, 0.30, 0.94, 0.86\}^T$, does not solve the MPIT problem while the one reported in Table 1 does. However, since their corresponding aggregate flows are the same (both are u^{DF} reported in the table), the resulting system travel time is the same.

We also solve for a cost-based dominating flow and the resulting flow distribution along with the tolls are shown in Table 4-3. Similarly, Table 4-4 compares the resulting performance measures with the multiclass UE. By making similar comparisons, the results in Table 4-4 show that the Pareto-improving scheme benefits every other stakeholder without making users of class 1 worse off. As expected, the cost-based Pareto-improving scheme will lead to smaller total cost while the time-based scheme will result in a smaller total system time.

To explore the existences and properties of multiclass Pareto-improving toll schemes in general networks, we further solve three networks in the literature: nine-node (see, e.g., [Hearn and Ramana, 1998](#)), Sioux Falls (see, e.g., [Bai et al., 2004](#)), and Hull (see, e.g., [Florian et al., 1987](#)). We keep the original network-related settings but introduce two classes of users, whose value of time is 0.6 and 1.4 respectively. We further assume a uniform 60 – 40% split between the low and high VOT users for each OD pair. Some general attributes of these three networks are listed in Table 4-5. Table 4-6 shows the total system travel time under multiclass system optimum (MSO), multiclass user equilibrium (MUE) and multiclass time-based Pareto-improving toll scheme (MTPI). The last column of the table reports the reduction in travel time from MUE delay to MTPI delay as a fraction of the difference between MUE delay and SO delay, which is the maximum possible reduction. Observe from Table 4-6 that no

Pareto-improving toll scheme exists for nine-node and Sioux Falls network and the resulting MTPI delay of Hull network reduces the travel delay by no more than 1% of the maximum possible amount, i.e., Pareto-improving toll schemes do not lead to a significant improvement in travel delay. Table 4-7 reports the results from solving the approximate Pareto-improving toll problem or AMITPI with $\alpha = 0.05, 0.1, 0.15$ and 0.20 . These results illustrate that allowing users to be slightly worse off may lead to substantially improvement in the system performance.

Although marginal cost pricing can achieve the SO flow distribution and reduce the total system travel time to its minimum level, it may cause inequality among users, such as spatial inequality (e.g., Yang and Zhang, 2002). Figure 4-2 and 4-3 shows the frequencies and cumulative distributions of two ratios, C_w^{SO}/C_w^{UE} and C_w^{AMTPI}/C_w^{UE} , where C_w^{SO} is the travel cost (time plus toll) between OD pair w under the marginal cost pricing; C_w^{UE} and C_w^{AMTPI} are the equilibrium travel costs under MUE and an approximate MTPI scheme with $\alpha = 15\%$ respectively. The data for the figure are from Sioux Falls. Figure 4-2 shows that marginal cost pricing makes all users worst off than the MUE condition. Even without including the tolls, around 12% of the OD pairs under MSO will have more than 15% higher travel times than MUE. From Figure 4-3, as expected, equilibrium travel costs of all OD pairs are no more than 15% of those under MUE. The Gini coefficients associated with the ratio of C_w^{SO}/C_w^{UE} and C_w^{AMTPI}/C_w^{UE} are 0.122 and 0.016 respectively, suggesting that compared with marginal cost pricing, Pareto-improving scheme may induce less spatial inequality with respect to the *status quo*. It is also clear from Figure 4-3 that the high VOT users tend to be better off than the low VOT users. Unfortunately, such social inequality may be inevitable with anonymous tolling and can only be addressed by other subsidy schemes (e.g., Yin and Yang, 2004).

4.5 Summary

This chapter discusses Pareto-improving pricing schemes for a general network when users belong to a discrete set of classes, each one with a different VOT. We

determine whether an anonymous Pareto-improving pricing scheme exists by solving an optimization and observing its optimal objective value. We provide formulations for finding a Pareto-improving scheme as well as a dominating flow distribution. Given a dominating flow distribution, we provide necessary and sufficient conditions for the existence of a nonnegative Pareto-improving toll vector. The numerical results from three networks illustrate that multiclass Pareto-improving pricing schemes are less prevalent when compared to its single-class counterpart. The existence and effectiveness Pareto-improving pricing schemes not only depend on the network configurations but also on demographic features (e.g., VOT values) of the population.

To facilitate the presentation of the key ideas, this chapter assumes that travel demand is deterministic. The theory for elastic demand can be similarly established, e.g., see [Lawphongpanich and Yin \(2010\)](#) for the case with one user class.

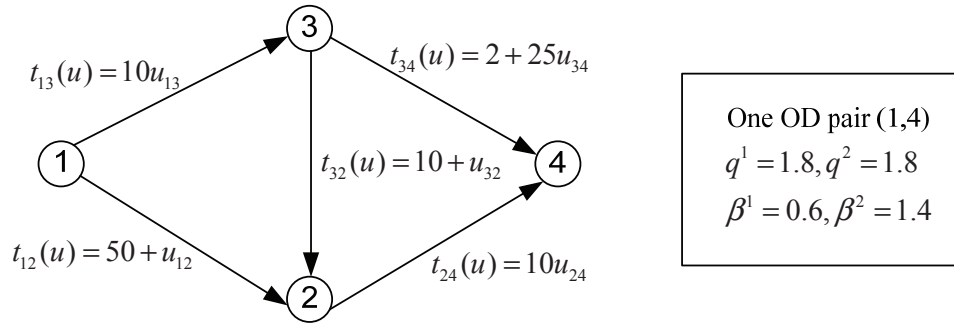


Figure 4-1. A five-link network with multiclass users

Table 4-1. Time-based dominating flow distribution and Pareto-improving scheme

Link	UE		Pareto-improving				τ
	u^{UE}	$t(u^{UE})$	u^{DF}	\hat{v}^1	\hat{v}^2	$t(u^{DF})$	
(1, 3)	3.60	36.00	2.24	0.44	1.80	22.40	0.00
(1, 2)	0.00	50.00	1.36	1.36	0.00	51.36	0.00
(3, 2)	2.28	12.28	0.61	0.00	0.61	10.61	20.96
(3, 4)	1.32	35.06	1.63	0.44	1.19	42.75	3.55
(2, 4)	2.28	22.78	1.97	1.36	0.61	19.70	0.00

Table 4-2. Performance comparisons with UE flow distribution

	User equilibrium		Pareto-improving	
	Class 1	Class 2	Class 1	Class 2
Path travel time				
1-3-4	71.06	71.06	65.15	65.15
1-3-2-4	71.06	71.06	52.71	52.71
1-2-4	72.78	72.78	71.06	71.06
Longest utilized path	71.06	71.06	71.06	65.15
Path travel cost				
1-3-4	42.64	99.48	42.64	94.76
1-3-2-4	42.64	99.48	52.59	94.76
1-2-4	43.67	101.89	42.64	99.48
Equilibrium cost	42.64	99.48	42.64	94.76
Total system travel time		255.80		234.99
Total system travel cost		255.80		228.74
Toll revenue		0.00		18.57

Table 4-3. Cost-based dominating flow distribution and Pareto-improving scheme

Link	UE			Pareto-improving			
	u^{UE}	$t(u^{UE})$	u^{DF}	\hat{v}^1	\hat{v}^2	$t(u^{DF})$	τ
(1, 3)	3.60	36.00	2.15	0.35	1.80	21.49	0.00
(1, 2)	0.00	50.00	1.45	1.45	0.00	51.45	0.00
(3, 2)	2.28	12.28	0.51	0.00	0.51	10.51	21.96
(3, 4)	1.32	35.06	1.64	0.35	1.29	42.98	3.96
(2, 4)	2.28	22.78	1.96	1.45	0.51	19.61	0.00

Table 4-4. Performance comparisons with UE flow distribution

	User equilibrium		Pareto-improving	
	Class 1	Class 2	Class 1	Class 2
Path travel time				
1-3-4	71.06	71.06	64.47	64.47
1-3-2-4	71.06	71.06	51.61	51.61
1-2-4	72.78	72.78	71.06	71.06
Longest utilized path	71.06	71.06	71.06	64.47
Path travel cost				
1-3-4	42.64	99.48	42.64	94.21
1-3-2-4	42.64	99.48	52.92	94.21
1-2-4	43.67	101.89	42.64	99.49
Equilibrium cost	42.64	99.48	42.64	94.21
Total system travel time		255.80		235.09
Total system travel cost		255.80		228.64
Toll revenue		0		17.68

Table 4-5. Network attributes

Network	Links	Nodes	OD pairs
Nine-node	9	18	4
Sioux Falls	76	24	528
Hull	798	501	158

Table 4-6. Exact Pareto-improving problem

Network	MSO delay	MUE delay	MTPI delay	Reduction (% of Max)
Nine-node	2253.92	2455.87	2455.87	0.00%
Sioux Falls	3514.39	3654.46	3654.46	0.00%
Hull	50542.64	51350.74	51347.14	0.44%

Table 4-7. Approximate Pareto-improving problem

Relaxation factor (α)	Nine-node		Sioux Falls		Hull	
	AMTPI delay	Reduction (% of Max)	AMTPI delay	Reduction (% of Max)	AMTPI delay	Reduction (% of Max)
5%	2455.87	0%	3650.73	2.67%	51279.49	8.82%
10%	2392.27	31.50%	3649.88	3.27%	51274.31	9.46%
15%	2385.81	34.70%	3614.85	28.28%	51274.31	9.46%
20%	2385.81	34.70%	3607.94	33.21%	51208.48	17.60%

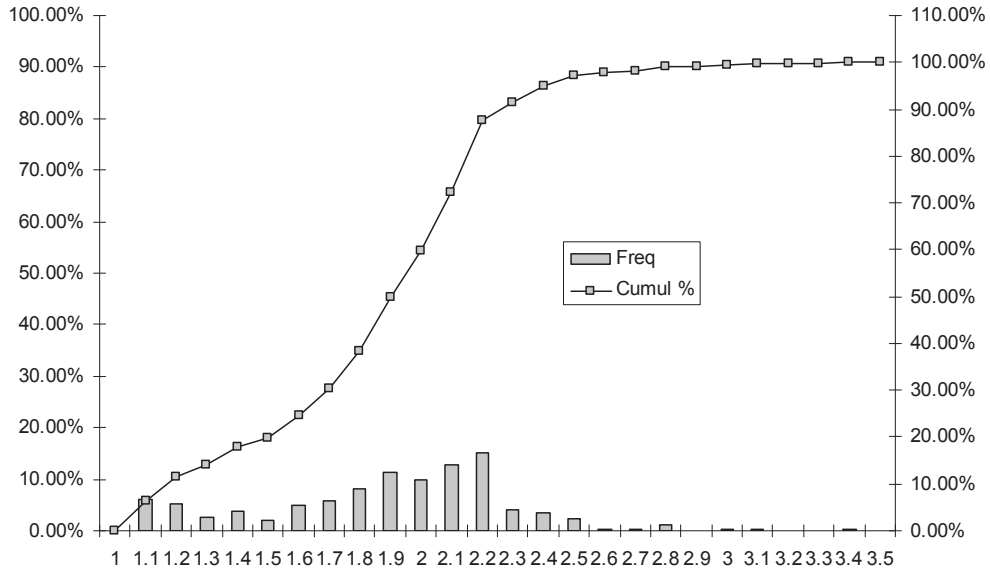


Figure 4-2. Frequencies of ratios for Sioux Falls network

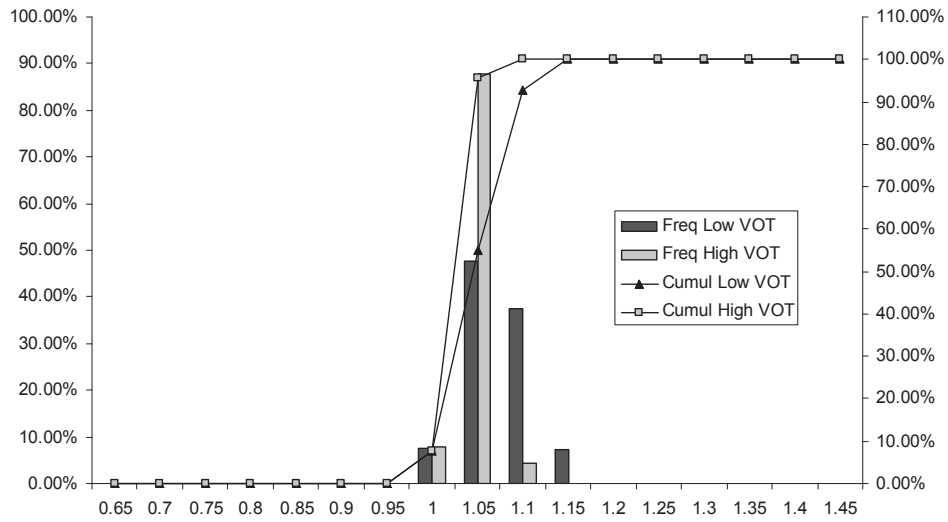


Figure 4-3. Cumulative distribution of ratios for Sioux Falls network

CHAPTER 5 PARETO-IMPROVING HYBRID POLICY FOR TRANSPORTATION NETWORKS

[Lawphongpanich and Yin \(2010\)](#) proposed a Pareto-improving congestion pricing scheme that leads a transportation system to a Pareto improvement over *status quo* even before toll revenue redistribution. The fundamental reason for a nonnegative Pareto-improving toll vector to exist is the user equilibrium (UE) flow distribution may not be Pareto-optimal, which indicates there may exist another flow distribution that makes at least one user better off without making any other user worse off. [Song et al. \(2009\)](#) and [Wu et al. \(2011\)](#) extended the Pareto-improving congestion pricing model to networks with multiclass and modes. Although it is relatively easy to implement a nonnegative Pareto-improving pricing scheme, the existence of the pricing scheme is not guaranteed. Even it does exist; we found the reduction in total system delay may not be substantial.

This chapter explores the potential of combining multiple policy instruments to achieve Pareto improvements. We consider a hybrid policy that combines travel-right assignment and congestion pricing. In the presence of congestion tolling, the policy allocates road users the rights of toll exemption on designated days instead of paying tolls on every single day. On days that users are exempted (free days), they are allowed to navigate through the road network without being charged while the others are subject to congestion tolls. Users may experience different travel times from day to day depending on whether they are exempted from congestion tolls on a particular day. To make meaningful comparisons with the *status quo*, average travel cost is defined as the weighted average of travel costs on free days and restricted days. Subsequently, users are considered better off if their average travel costs are less than those before the implementation of the hybrid policy.

From another perspective, the allocation of rights for traveling for free can be viewed as a form of road space rationing. As a regulatory travel demand management

strategy, road space rationing has been used to mitigate traffic congestion for decades. Cities such as Athens, Santiago, Mexico City, Beijing and Guangzhou have resorted to using plate-number-based road space rationing to reduce congestion and air pollution (see, e.g., [Wang et al., 2010](#) and [Han et al., 2010](#)). [Downs \(2004\)](#) concluded that the most effective overall strategy for reducing traffic congestion probably should consist of both market-based and regulatory elements. [Daganzo \(1995\)](#) proposed a scheme that can be viewed as hybrid between pricing and rationing to control flow through a bottleneck. Users are allowed to choose the form of the penalty they must pay for using the bottleneck based on their personal needs and differences. The hybrid strategy could achieve a Pareto improvement in theory.

In a similar spirit to the one by [Daganzo \(1995\)](#), the hybrid policy proposed in this chapter attempts to make use of the synergistic effects of multiple policy instruments to achieve Pareto improvements in a general network. The aim of this chapter is to establish a mathematical framework of designing an optimal road space rationing policy and an optimal hybrid policy for a general network and compares the performance of these two policies. The remainder of this chapter is organized as follows. Section 2 discusses road space rationing schemes and formulates a mathematical program to design optimal Pareto-improving road space rationing scheme. Section 3 formulates a mathematical program to design an optimal Pareto-improving hybrid policy. Section 4 compares these two policies on a toy network. The last section offers concluding remarks.

5.1 Pareto-Improving Pure Road Space Rationing Policy

Road space rationing has been used by many cities for years to mitigate traffic congestion and/or air pollution. However [Wang et al. \(2010\)](#) is the first to demonstrate that road space rationing schemes could lead to Pareto improvements. Furthermore, in practice, rationing ratio, which indicates the percentage of users that are prohibited from using the road network, is determined primarily based on engineering judgment.

This section follows an optimization approach and provides the formulation of a Pareto-improving pure road space rationing problem, which we believe may offer more insights into the pure road space rationing problem.

5.1.1 Problem Setting

Let $G = (N, L)$ be a directed transportation network, where N is the set of nodes and L is the set of directed links. A is the node-arc incidence matrix of a multi-modal network, which consists of two sub-networks, namely road network and transit network. \bar{A} is the node-arc incidence matrix of the transit network only. An element of L is denoted as (i, j) and $t_{ij}(v_{ij})$ represents the travel time function for link (i, j) , which is assumed to be separable, continuous and monotonically increasing respect to the aggregate link flow v_{ij} . Let L_n and L_t denote the set of links in the road and transit network, respectively. It is assumed that each origin-destination (OD) pair is connected by a transit line whose travel time is constant. To be realistic, the transit travel time is assumed to be longer than the travel time on road for the same OD pair. Let W be the set of OD pairs and d_w be the deterministic number of trips made between OD pair $w \in W$. Users can be divided into two groups ($k = 1, 2 \in K$), namely, general users and restricted users. For OD pair w , $x_{ij}^{w,k}$ represents the link flow of user group k on link (i, j) and $v_{ij} = \sum_w \sum_{k=1,2} x_{ij}^{w,k}$.

In a pure road space rationing scheme, generally rationing is enforced on all road links, L_n . The percentage of restricted road users who are forced to use alternative modes, e.g., transit service, is termed as rationing ratio α . Each day a fraction of road users (user group $k = 1$), $100(1 - \alpha)\%$ of the total demand, are allowed to access the entire transportation network freely. The rest (user group $k = 2$), $100\alpha\%$ of the total demand, are prohibited from using the road network.

5.1.2 User Equilibrium under Pure Road Space Rationing Schemes

The user equilibrium problem under a pure road space rationing policy can be formulated as follows:

$$\text{P1: } \min_{(x,v)} \sum_{(i,j) \in L_n} \int_0^{v_{ij}} t_{ij}(\omega) d\omega + \sum_{(i,j) \in L_t} t_{ij} v_{ij} \quad (5-1)$$

$$\text{s.t. } Ax^{w,1} = (1 - \alpha) E^w d^w \quad \forall w \quad (5-2)$$

$$\bar{A}x^{w,2} = \alpha E^w d^w \quad \forall w \quad (5-3)$$

$$v_{ij} = \sum_w \sum_k x_{ij}^{w,k} \quad (5-4)$$

$$x \geq 0 \quad (5-5)$$

where constraints (5-2) and (5-3) describe flow balance constraints for general and restricted users respectively. E^w is an input-output vector, i.e., a vector with exactly two non-zero components, to specify the origin and destination of OD pair w . The component of E^w corresponding the origin has a value 1 and the one corresponding the destination has a value -1. Constraints (5-2) to (5-5) essentially are the feasible flow region of a multiclass multimodal network, which is convex. The objective function is strictly convex with respect to the aggregate link flow v_{ij} . Therefore, problem P1 is a strictly convex optimization problem and there is a unique solution of the user equilibrium link flow distribution. For a strictly convex mathematical program, the Karush–Kuhn–Tucker (KKT) conditions are both necessary and sufficient for optimality. To show the above mathematical program is equivalent to the user equilibrium under a pure road space rationing policy, the KKT conditions of the above optimization problem can be stated as follows,

$$t_{ij}(v_{ij}) + \rho_i^{w,1} - \rho_j^{w,1} \geq 0 \quad \forall (i,j) \in L_n, w \quad (5-6)$$

$$x_{ij}^{w,1} [t_{ij}(v_{ij}) + \rho_i^{w,1} - \rho_j^{w,1}] = 0 \quad \forall (i,j) \in L_n, w \quad (5-7)$$

$$t_{ij} + \rho_i^{w,k} - \rho_j^{w,k} \geq 0 \quad \forall (i,j) \in L_t, w, k = 1, 2 \quad (5-8)$$

$$x_{ij}^{w,k} [t_{ij} + \rho_i^{w,k} - \rho_j^{w,k}] = 0 \quad \forall (i, j) \in L_t, w, k = 1, 2 \quad (5-9)$$

$$Ax^{w,1} = (1 - \alpha) E^w d^w \quad \forall w \quad (5-10)$$

$$\bar{A}x^{w,2} = \alpha E^w d^w \quad \forall w \quad (5-11)$$

$$v_{ij} = \sum_w \sum_k x_{ij}^{w,k} \quad (5-12)$$

$$x \geq 0 \quad (5-13)$$

where ρ^w is a vector of Lagrange multipliers associated with the flow balance constraints (5-2) and (5-3), which are also known as node potentials (Ahuja et al., 1993) for OD pair w . Let's derive the equivalence conditions for restricted users first. Restricted users (user group $k = 2$) are allowed to access transit links only, therefore, only one pair of complementarity constraints (5-8) and (5-9) applies to restricted users. When a link (i, j) is utilized, i.e., $x_{ij}^{w,2} > 0$, constraint (5-9) forces the equation $t_{ij} + \rho_i^{w,2} - \rho_j^{w,2} = 0$ to hold. Combing together this equation for each link on a utilized path yields the following:

$$\sum_{(i,j) \in \Phi_r^{w,2}} t_{ij} = \sum_{(i,j) \in \Phi_r^{w,2}} \rho_j^{w,2} - \rho_i^{w,2} = \rho_{d(w)}^{w,2} - \rho_{o(w)}^{w,2}$$

where, for OD pair w , $\Phi_r^{w,2}$ is a set containing transit links that are available to restricted users on path r and $d(w)$ and $o(w)$ denote the destination and origin nodes of OD pair w . Thus, constraints (5-9) implies that the generalized travel time of every utilized path equals $\rho_{d(w)}^{w,2} - \rho_{o(w)}^{w,2}$ for general users between OD pair w . When a link (i, j) is not utilized, i.e., $x_{ij}^{w,2} = 0$, constraints (5-8) to (5-9) imply the inequality $t_{ij} + \rho_i^{w,2} - \rho_j^{w,2} \geq 0$ holds. Adding this inequality along a non-utilized path yields $\sum_{(i,j) \in \Phi_r^{w,2}} t_{ij} \geq \rho_{d(w)}^{w,2} - \rho_{o(w)}^{w,2}$. Therefore, for restricted users, we have all utilized paths have the same travel time $\rho_{d(w)}^{w,2} - \rho_{o(w)}^{w,2}$ and it is no longer than those of all non-utilized paths. For general users, there are two pairs of complementarity constraints (5-6) to (5-9) involved because they are allowed to use all links in the transportation network freely. Similar procedures can be applied to construct the user equilibrium conditions for general users (user group

$k = 1$). Let $C_{w,k}^{UE} = \rho_{d(w)}^{w,k} - \rho_{o(w)}^{w,k}$, then $C_{w,k}^{UE}$ is the equilibrium travel time (cost) for users of group k between OD pair w . Hence, we prove that the KKT conditions of the above optimization problem are equivalent to the user equilibrium conditions under pure road space rationing schemes.

5.1.3 Pareto-Improving Pure Road Space Rationing Problem

Wang et al. (2010) demonstrated that under certain rationing ratio and network configurations, it is possible to achieve Pareto improvements using pure road space rationing. The following mathematical program is formulated to find the optimal rationing ratio and its corresponding flow pattern to minimize the total system delay while ensures that the rationing scheme can lead to a Pareto-improvement. The Pareto-improving pure road space rationing problem can be formulated as a mathematical program with complementarity constraints (MPCC) as follows,

$$\text{P2: } \min_{(x,v,\rho,\alpha)} \sum_{(i,j) \in L} t_{ij}(v_{ij}) v_{ij} \quad (5-14)$$

$$\text{s.t. } Ax^{w,1} = (1 - \alpha) E^w d^w \quad \forall w \quad (5-15)$$

$$\bar{A}x^{w,2} = \alpha E^w d^w \quad \forall w \quad (5-16)$$

$$v_{ij} = \sum_w \sum_k x_{ij}^{w,k} \quad (5-17)$$

$$x \geq 0 \quad (5-18)$$

$$t_{ij}(v_{ij}) + \rho_i^{w,1} - \rho_j^{w,1} \geq 0 \quad \forall (i,j) \in L_n, w \quad (5-19)$$

$$x_{ij}^{w,1} [t_{ij}(v_{ij}) + \rho_i^{w,1} - \rho_j^{w,1}] = 0 \quad \forall (i,j) \in L_n, w \quad (5-20)$$

$$t_{ij} + \rho_i^{w,k} - \rho_j^{w,k} \geq 0 \quad \forall (i,j) \in L_t, w, k = 1, 2 \quad (5-21)$$

$$x_{ij}^{w,k} [t_{ij} + \rho_i^{w,k} - \rho_j^{w,k}] = 0 \quad \forall (i,j) \in L_t, w, k = 1, 2 \quad (5-22)$$

$$(1 - \alpha) [\rho_{d(w)}^{w,1} - \rho_{o(w)}^{w,1}] + \alpha [\rho_{d(w)}^{w,2} - \rho_{o(w)}^{w,2}] \leq C_w^{UE} \quad \forall w \quad (5-23)$$

where C_w^{UE} is the equilibrium travel time of OD pair w before the policy implementation, i.e., *status quo*. The objective function minimizes the total system delay. Constraints

(5–15) to (5–22) are the user equilibrium flow conditions under a pure road space rationing policy. Constraint (5–23) guarantees that when averaged across restricted and free days, no user is made worse off compared with the *status quo*. As formulated, the above problem is a MPCC, which is a class of problem that is difficult to solve for mainly two reasons. One is because MPCC violates certain constraint qualification and the other is due to the fact that the feasible region is non-convex.

5.2 Pareto-Improving Hybrid Policy

The major drawback of a pure road space rationing policy is that as a regulatory demand management strategy it requires users to behave according to certain compulsory rules (taking transit when they are restricted) that apply to everyone in the same manner. Downs (2004) pointed out it may not be the most effective overall strategy. The hybrid policy we proposed integrates regulatory and market-based demand management strategies.

In the presence of a hybrid policy, road link set L_n can be further divided into two mutually exclusive sets: a general link set L_g and a restricted link set L_r . We thus have $L_n = \{L_g \cup L_r\}$. Each day a fraction of road users (user group $k = 1$), $100(1 - \alpha)\%$ of the total demand, are entitled to travel the entire road network for free, where α is called restriction ratio. The remaining restricted users (user group $k = 2$), however, are not forced to take transit service. They are provided with the opportunity to pay tolls to access the restricted links, L_r and also travel on the general links for free. Nonnegative congestion tolls, β_{ij} , are only charged on restricted links, $(i, j) \in L_r$, for users on their restricted days. To design a hybrid policy, transportation agencies need to specify three crucial components: restriction ratio α , locations to set up restricted links and their corresponding toll rates β_{ij} .

5.2.1 User Equilibrium under Hybrid Policies

The user equilibrium flow distribution in the presence of a hybrid policy can be estimated by solving the following mathematical program.

$$\text{P3: } \min_{(x,v)} \sum_{(i,j) \in L_n} \int_0^{v_{ij}} t_{ij}(\omega) d\omega + \sum_{(i,j) \in L_t} v_{ij} t_{ij} + \sum_{(i,j) \in L_r} \beta_{ij} v_{ij}^2 \quad (5-24)$$

$$\text{s.t. } Ax^{w,1} = (1 - \alpha) E^w d^w \quad \forall w \quad (5-25)$$

$$Ax^{w,2} = \alpha E^w d^w \quad \forall w \quad (5-26)$$

$$v_{ij} = \sum_w \sum_k x_{ij}^{w,k} \quad (5-27)$$

$$x \geq 0 \quad (5-28)$$

where constraints (5-25) to (5-26) describe flow balance constraints for general and restricted users. For restricted users, however, they have to pay β_{ij} if they choose to use restricted links $(i, j) \in L_r$. Constraints (5-25) to (5-28) essentially are the feasible flow region of a multiclass multimodal network, which is convex. The objective function is strictly convex with respect to the aggregate link flow v_{ij} . Therefore, problem P3 is a strictly convex optimization problem and there is a unique solution of the user equilibrium link flow distribution. To show the above mathematical program is equivalent to the user equilibrium under a hybrid policy, the KKT conditions of the above optimization problem can be stated as follows. For a strictly convex mathematical program, the KKT conditions are both necessary and sufficient for optimality.

$$t_{ij}(v_{ij}) + \rho_i^{w,k} - \rho_j^{w,k} \geq 0 \quad \forall (i, j) \in L_g, w, k = 1, 2 \quad (5-29)$$

$$x_{ij}^{w,k} [t_{ij}(v_{ij}) + \rho_i^{w,k} - \rho_j^{w,k}] = 0 \quad \forall (i, j) \in L_g, w, k = 1, 2 \quad (5-30)$$

$$t_{ij} + \rho_i^{w,k} - \rho_j^{w,k} \geq 0 \quad \forall (i, j) \in L_t, w, k = 1, 2 \quad (5-31)$$

$$x_{ij}^{w,k} [t_{ij} + \rho_i^{w,k} - \rho_j^{w,k}] = 0 \quad \forall (i, j) \in L_t, w, k = 1, 2 \quad (5-32)$$

$$t_{ij}(v_{ij}) + \rho_i^{w,1} - \rho_j^{w,1} \geq 0 \quad \forall (i, j) \in L_r, w \quad (5-33)$$

$$x_{ij}^{w,1} [t_{ij}(v_{ij}) + \rho_i^{w,1} - \rho_j^{w,1}] = 0 \quad \forall (i, j) \in L_r, w \quad (5-34)$$

$$t_{ij}(v_{ij}) + \beta_{ij} + \rho_i^{w,2} - \rho_j^{w,2} \geq 0 \quad \forall (i, j) \in L_r, w \quad (5-35)$$

$$x_{ij}^{w,2} [t_{ij}(v_{ij}) + \beta_{ij} + \rho_i^{w,2} - \rho_j^{w,2}] = 0 \quad \forall (i, j) \in L_r, w \quad (5-36)$$

$$Ax^{w,1} = (1 - \alpha) E^w d^w \quad \forall w \quad (5-37)$$

$$Ax^{w,2} = \alpha E^w d^w \quad \forall w \quad (5-38)$$

$$v_{ij} = \sum_w \sum_k x_{ij}^{w,k} \quad (5-39)$$

$$x \geq 0 \quad (5-40)$$

where ρ^w is a vector of Lagrange multipliers associated with the flow balance constraints (5-25) and (5-26), which are also known as node potentials (Ahuja et al., 1993) for OD pair w . There are four pairs of complementarity constraints (5-29) to (5-36). Let's derive the equivalence conditions for general users first. General users (user group $k = 1$) are allowed to access all links, i.e., $\{L_g \cup L_t \cup L_r\}$ in the transportation network for free. When a link (i, j) is utilized, i.e., $x_{ij}^{w,1} > 0$, constraints (5-30), (5-32) and (5-34) force the equation $t_{ij}(v_{ij}) + \rho_i^{w,1} - \rho_j^{w,1} = 0$ to hold. Combing together this equation for each link on a utilized path yields the following:

$$\sum_{(i,j) \in \Phi_r^{w,1}} t_{ij}(v_{ij}) = \sum_{(i,j) \in \Phi_r^{w,1}} \rho_j^{w,1} - \rho_i^{w,1} = \rho_{d(w)}^{w,1} - \rho_{o(w)}^{w,1}$$

where, for OD pair w , $\Phi_r^{w,1}$ is a set containing links that are available to general users on path r and $d(w)$ and $o(w)$ denote the destination and origin nodes of OD pair w . Thus, constraints (5-30), (5-32) and (5-34) imply that the generalized travel time of every utilized path equals $\rho_{d(w)}^{w,1} - \rho_{o(w)}^{w,1}$ for general users between OD pair w . When a link (i, j) is not utilized, i.e., $x_{ij}^{w,1} = 0$, constraints (5-29) to (5-34) imply the inequality $t_{ij}(v_{ij}) + \rho_i^{w,1} - \rho_j^{w,1} \geq 0$ holds. Adding this inequality along a non-utilized path yields $\sum_{(i,j) \in \Phi_r^{w,1}} t_{ij}(v_{ij}) \geq \rho_{d(w)}^{w,1} - \rho_{o(w)}^{w,1}$. Therefore, we have the user equilibrium conditions for general users. Similar procedures can be applied to complementarity constraints (5-29) to (5-33), (5-35) and (5-36) to construct the tolled user equilibrium conditions for

restricted users (user group $k = 2$). Hence, the KKT conditions of optimization problem P1 are equivalent to the user equilibrium conditions when a hybrid policy is in place.

The above formulation also provides a way to achieve the UE solution under a pure road space rationing scheme. Instead of solving problem P1 directly, we can put an arbitrary large toll on all road links for restricted users to prevent them from using road links in problem P3. Under such settings, by solving problem P3, the resulting flow distribution should coincide with the solution to problem P1.

5.2.2 Pareto-Improving Hybrid Policy Problem

Similar to congestion pricing and other demand management instruments, the utmost goal of a hybrid policy is to improve transportation system efficiency. While for a Pareto-improving policy, we also have to ensure the policy lead to a Pareto-improvement over the *status quo*. To design a hybrid policy, the restriction ratio α , locations of restricted links, i.e., partitioning of L_g and L_r , and their corresponding toll rates β for restricted users need to be specified to minimize total system delay. A MPCC is formulated as follow,

$$\text{P4: } \min_{(x,v,\rho,\alpha,\beta)} \sum_{(i,j) \in L_n} t_{ij}(v_{ij})v_{ij} + \sum_{(i,j) \in L_t} t_{ij}v_{ij} \quad (5-41)$$

$$\text{s.t. } Ax^{w,1} = (1 - \alpha) E^w d^w \quad \forall w \quad (5-42)$$

$$Ax^{w,2} = \alpha E^w d^w \quad \forall w \quad (5-43)$$

$$v_{ij} = \sum_w \sum_k x_{ij}^{w,k} \quad (5-44)$$

$$x \geq 0 \quad (5-45)$$

$$t_{ij}(v_{ij}) + \rho_i^{w,1} - \rho_j^{w,1} \geq 0 \quad \forall (i,j) \in L_n, w \quad (5-46)$$

$$x_{ij}^{w,1} [t_{ij}(v_{ij}) + \rho_i^{w,1} - \rho_j^{w,1}] = 0 \quad \forall (i,j) \in L_n, w \quad (5-47)$$

$$t_{ij}(v_{ij}) + \beta_{ij} + \rho_i^{w,2} - \rho_j^{w,2} \geq 0 \quad \forall (i,j) \in L_n, w \quad (5-48)$$

$$x_{ij}^{w,2} [t_{ij}(v_{ij}) + \beta_{ij} + \rho_i^{w,2} - \rho_j^{w,2}] = 0 \quad \forall (i,j) \in L_n, w \quad (5-49)$$

$$t_{ij} + \rho_i^{w,k} - \rho_j^{w,k} \geq 0 \quad \forall (i,j) \in L_t, w, k = 1, 2 \quad (5-50)$$

$$x_{ij}^{w,k} [t_{ij} + \rho_i^{w,k} - \rho_j^{w,k}] = 0 \quad \forall (i, j) \in L_t, w, k = 1, 2 \quad (5-51)$$

$$(1 - \alpha) [\rho_{d(w)}^{w,1} - \rho_{o(w)}^{w,1}] + \alpha [\rho_{d(w)}^{w,2} - \rho_{o(w)}^{w,2}] \leq C_w^{UE} \quad \forall w \quad (5-52)$$

$$\beta_{ij} \geq 0 \quad \forall (i, j) \in L_n \quad (5-53)$$

Constraints (5-42) to (5-51) are equilibrium flow conditions assuming all road links are potential restricted links. Constraint (5-52) guarantees that when averaged across restricted and free days, no user is made worse off compared with the *status quo*.

Constraint (5-53) requires the toll vector to be nonnegative. In the above formulation, the restriction ratio and toll rates are decision variables and all road links are treated as potential restricted links. The designation of restricted links can be determined by referring to the optimal toll rates. If the resulting toll rate for restricted users is strictly positive on link (i, j) , the link belongs to L_r , otherwise, $(i, j) \in L_g$.

Note that the above formulation can also be used to obtain the solution to a Pareto-improving pure rationing problem (P2) by setting an arbitrary large toll on all road links for restricted users so that they choose not to use the road network anyhow. Since the optimal solution to the Pareto-improving pure rationing problem (P2) is always a feasible solution to the above formulation, we can conclude that the solution to problem P2 should be dominated by the optimal solution to the problem P4 in theory.

It is also worth mentioning that Pareto improvements in above example are attained before redistribution of toll revenues. In other words, we could achieve even better improvements if certain revenue redistribution plan is integrated in the model. As an example, let's consider a transit subsidy plan, which directly transfers toll revenues to subsidize transit fares. The hybrid policy with transit subsidy problem can be formulated as a MPCC as follows:

$$\text{P5: } \min_{(x, v, \rho, \alpha, \beta)} \sum_{(i, j) \in L_n} t_{ij}(v_{ij}) v_{ij} + \sum_{(i, j) \in L_t} t_{ij} v_{ij} \quad (5-54)$$

$$\text{s.t. } Ax^{w,1} = (1 - \alpha) E^w d^w \quad \forall w \quad (5-55)$$

$$Ax^{w,2} = \alpha E^w d^w \quad \forall w \quad (5-56)$$

$$v_{ij} = \sum_w \sum_k x_{ij}^{w,k} \quad (5-57)$$

$$x \geq 0 \quad (5-58)$$

$$t_{ij}(v_{ij}) + \rho_i^{w,1} - \rho_j^{w,1} \geq 0 \quad \forall (i, j) \in L_n, w \quad (5-59)$$

$$x_{ij}^{w,1} [t_{ij}(v_{ij}) + \rho_i^{w,1} - \rho_j^{w,1}] = 0 \quad \forall (i, j) \in L_n, w \quad (5-60)$$

$$t_{ij}(v_{ij}) + \beta_{ij} + \rho_i^{w,2} - \rho_j^{w,2} \geq 0 \quad \forall (i, j) \in L_n, w \quad (5-61)$$

$$x_{ij}^{w,2} [t_{ij}(v_{ij}) + \beta_{ij} + \rho_i^{w,2} - \rho_j^{w,2}] = 0 \quad \forall (i, j) \in L_n, w \quad (5-62)$$

$$(t_{ij} - s_{ij}) + \rho_i^{w,k} - \rho_j^{w,k} \geq 0 \quad \forall (i, j) \in L_t, w, k = 1, 2 \quad (5-63)$$

$$x_{ij}^{w,k} [(t_{ij} - s_{ij}) + \rho_i^{w,k} - \rho_j^{w,k}] = 0 \quad \forall (i, j) \in L_t, w, k = 1, 2 \quad (5-64)$$

$$(1 - \alpha) [\rho_{d(w)}^{w,1} - \rho_{o(w)}^{w,1}] + \alpha [\rho_{d(w)}^{w,2} - \rho_{o(w)}^{w,2}] \leq C_w^{UE} \quad \forall w \quad (5-65)$$

$$\sum_{(i,j) \in L_t} s_{ij} v_{ij} \leq \sum_{(i,j) \in L_n} \sum_{w \in W} \beta_{ij} x_{ij}^{w,2} \quad (5-66)$$

$$s_{ij}, \beta_{ij} \geq 0 \quad \forall (i, j) \in L_n \quad (5-67)$$

where s_{ij} is an anonymous nonnegative subsidy to transit users using link (i, j) .

Nonnegative tolls, β_{ij} , are imposed on restricted users who choose to use restricted links and subsidies, s_{ij} , are provided to users using the transit links. Constraint (5-66) makes sure the total subsidy to transit users should be less than total toll revenues collected. Constraint (5-67) requires both tolls and subsidies are nonnegative.

5.3 Numerical Examples

Consider a multimodal network in Figure 5-1, which contains four OD pairs: $(1, 3)$, $(1, 4)$, $(2, 3)$ and $(2, 4)$. It consists of 18 auto links and 4 transit links. The link performance functions associated with auto links are assumed to follow the BPR function: $t_{ij}(v_{ij}) = t_{ij}^0 [1 + 0.15(v_{ij}/b_{ij})^4]$, where t_{ij}^0 is the free-flow travel time and b_{ij} is the capacity of that link. The parenthesis near auto link (i, j) in the figure denotes

(t_{ij}^0, b_{ij}) . Four transit links directly connect the origin and destination node of each OD pair. Aggregate demand of each OD pair is also shown in the Figure 5-1.

Mathematical programs formulated (P2, P4 and P5) are MPCC problems, which is a class of problems that is difficult to solve using commercial software. In this chapter, we revised the manifold suboptimization algorithm proposed by Lawphongpanich and Yin (2010) to find strongly stationary solutions. The algorithm is implemented using GAMS (Brooke et al., 2003) and non-linear sub-problems involved are solved by a commercial nonlinear programming solver called CONOPT. As a note on the solution procedure, it is generally not effective to use the manifold suboptimization algorithm to solve the Pareto-improving hybrid policy problem (P4) directly. The resulting flow distribution of the pure rationing scheme (P2) is used as an initial solution to the hybrid policy problem because it is always a feasible solution to the later.

For the first scenario, we assume that travel times on all transit links are 150% of their corresponding travel times under the original UE condition. By solving problem P2, P4 and P5, these two Pareto-improving strategies are compared, and results are summarized in Table 5-1 and 5-2.

When transit travel time is 150% the UE equilibrium travel cost, total system delay is 4904.0961 and 5883.8004 respectively under SO and UE conditions. For comparison purposes, results for transit links are listed in the first four rows of both tables. As shown in the tables, both the pure rationing and hybrid policies generate Pareto improvements. We can observe that although the average equilibrium travel cost under the hybrid policy is slightly higher than that under pure rationing scheme, the hybrid policy provides substantially better system performance. More importantly, we can observe that users that are forced to use transit links, e.g., link_(2, 3) under pure rationing scheme choose to pay tolls to access road networks under the hybrid policy, which indicates that users are enjoying the flexibility provided by the hybrid policy. If toll revenues are used to subsidize transit users, we observe that system efficiency can be improved even further.

In Table 5-2, transit subsidy on transit link (2, 4) is 4.06 and system delay reduction is 87.82% of the maximum possible reduction. Also, as a note to the Pareto-improving pricing problem proposed by Lawphongpanich and Yin (2010), no Pareto improvement is observed under pure Pareto-improving pricing schemes.

The above numerical example demonstrates that the hybrid policy can lead to Pareto improvements and achieve much better system efficiency than pure road space rationing schemes. To further investigate and compare three Pareto-improving strategies mentioned in this chapter, three problems (P2, P4 and P5) are solved for another scenario with higher transit travel time (200% of equilibrium travel times under UE). The results are shown in Table 5-3.

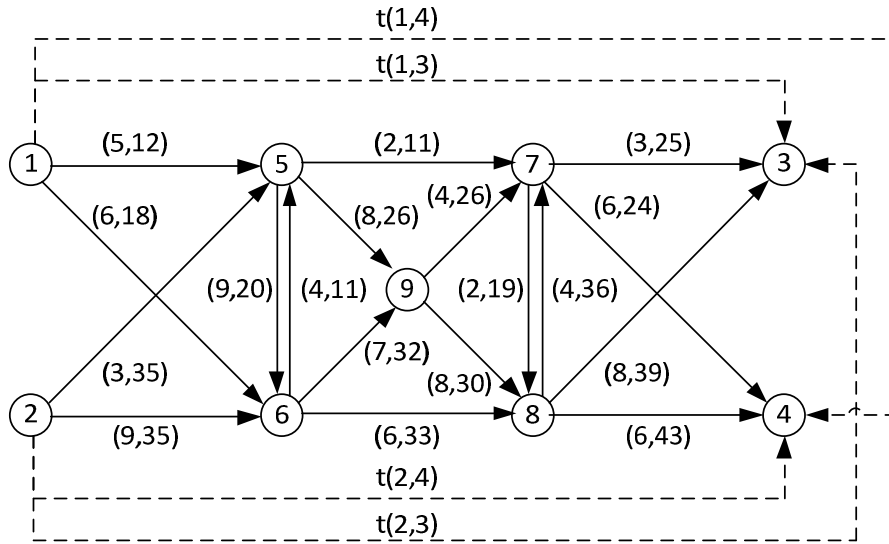
When travel times on transit links are 150% of equilibrium travel times under UE, as shown in the first scenario, the hybrid policy can provide better system efficiency. And, if toll revenues are used to subsidize transit users, system efficiency can be further improved as expected. When travel times on transit links are higher, which could happen in reality, we find that pure rationing scheme fails to generate a Pareto improvement. On the other hand, the hybrid policy is still able to achieve a Pareto improvement and attain 62.68% reduction of the maximum possible reduction in system performance. In this scenario, putting money back to subsidize transit users does not improve system efficiency further because travel times of transit service are so long than the transit mode is not attractive even if subsidy is provided to users. The above example demonstrates that the proposed hybrid policy is more robust than pure rationing policy in balancing system efficiency and Pareto improvements, which is an important feature required for real-world implementations.

5.4 Summary

This chapter proposes a new hybrid Pareto-improving policy that combines multiple policy instruments. The proposed hybrid policy provides greater flexibility than pure road space rationing. Generally rationing is enacted on the whole network and rationing

ratio is uniform for all OD pairs. Travel demands are diverted to transit links at the same ratio regardless of the network traffic conditions, which may cause inefficient allocation of road resources. In the proposed hybrid policy, although restriction ratio is still the same for all OD pairs, tolls are introduced to adjust roadway demand levels of different OD pairs. For some OD pairs, users may be better off paying tolls to enter the road network on restricted days instead of taking transit service. Furthermore, although tolls are charged on restricted users only, their route choices may influence route choices of non-restricted users and achieve better system performance.

The reasons that the proposed policy is more prominent in leading to a Pareto improvement are two folds. First, only a certain portion of users have to pay tolls to use restricted links while all users are required to pay anonymous tolls if they choose to use tolled links in pure congestion pricing. Essentially, the hybrid policy can be viewed as a realization of differentiated pricing schemes, although tolls being charged are still anonymous. Second, by introducing time dimension, a Pareto-improving solution is more likely to exist. To achieve an overall Pareto improvement, users do not necessarily have to be better off every single day, as long as the average travel time of rationing and non-rationing days is better than the original equilibrium travel time, i.e., the *status quo*.



OD Pair:	(1,3)	(1,4)	(2,3)	(2,4)
Demand:	30	30	40	50

Figure 5-1. A multimodal transportation network

Table 5-1. User equilibrium and Pareto-improving pure rationing problems

Link	UE	Pure rationing policy	
	User flow	General user flow	Restricted user flow
(1, 3)	0.00	0.00	9.00
(1, 4)	0.00	0.00	9.00
(2, 3)	0.00	0.00	12.00
(2, 4)	0.00	0.00	15.00
(1, 5)	22.94	14.50	0.00
(1, 6)	37.06	27.50	0.00
(2, 5)	48.11	43.76	0.00
(2, 6)	41.90	19.24	0.00
(5, 6)	0.00	0.00	0.00
(5, 7)	31.65	28.31	0.00
(5, 9)	39.39	29.95	0.00
(6, 5)	0.00	0.00	0.00
(6, 8)	59.12	46.74	0.00
(6, 9)	19.83	0.00	0.00
(7, 3)	45.75	42.50	0.00
(7, 4)	28.32	15.76	0.00
(7, 8)	0.00	0.00	0.00
(8, 3)	24.25	6.50	0.00
(8, 4)	51.68	40.24	0.00
(8, 7)	0.00	0.00	0.00
(9, 7)	42.42	29.95	0.00
(9, 8)	16.80	0.00	0.00
OD pairs	Equilibrium cost	Average equilibrium cost	
(1, 3)	45.63	40.50	
(1, 4)	45.33	39.45	
(2, 3)	35.22	34.07	
(2, 4)	34.92	33.52	
Optimal ratio	0.00	0.30	
Total delay	5883.80	5437.17	
System delay reduction	0%	46.00%	

Table 5-2. Pareto-improving hybrid policy problems

Link	Hybrid policy			Hybrid policy with subsidy		
	General Flow	Restricted user Flow	Toll	General Flow	Restricted user Flow	Toll
(1,3)	0.00	10.02	0.00	0.00	11.46	0.00
(1,4)	0.00	5.48	0.00	0.00	1.84	0.00
(2,3)	0.00	0.00	0.00	0.00	0.00	0.00
(2,4)	0.00	10.56	0.00	0.00	9.13	-4.06
(1,5)	12.32	3.60	22.72	8.32	9.62	26.08
(1,6)	27.64	0.94	14.32	28.76	0.00	18.96
(2,5)	43.92	0.00	10.31	41.50	0.00	11.27
(2,6)	16.01	19.50	0.00	14.12	25.25	0.00
(5,6)	0.00	0.00	0.00	0.00	0.00	0.00
(5,7)	28.55	0.00	15.90	28.51	0.00	10.41
(5,9)	27.69	3.60	0.00	21.31	9.62	0.00
(6,5)	0.00	0.00	0.00	0.00	0.00	0.00
(6,8)	43.65	0.00	22.75	42.88	0.00	17.37
(6,9)	0.00	20.45	7.34	0.00	25.25	7.00
(7,3)	38.95	0.00	1.25	33.04	0.00	7.36
(7,4)	17.29	2.77	0.00	16.78	9.12	0.00
(7,8)	0.00	0.00	0.00	0.00	0.00	0.00
(8,3)	7.66	13.36	2.80	10.22	15.28	7.52
(8,4)	35.99	7.92	3.46	32.66	10.47	4.31
(8,7)	0.00	0.24	0.00	0.00	0.00	0.00
(9,7)	27.69	2.52	15.90	21.31	9.12	10.41
(9,8)	0.00	21.52	8.68	0.00	25.75	2.89
OD pairs	Average equilibrium cost			Average equilibrium cost		
(1,3)	41.90			43.85		
(1,4)	41.00			42.87		
(2,3)	34.55			34.89		
(2,4)	34.92			34.92		
Optimal ratio	0.33			0.38		
Total delay	5078.11			5023.47		
System delay reduction	80.48%			87.82%		

Table 5-3. System delay reductions under different Pareto-improving strategies

Transit time	Pure rationing	Hybrid policy	Hybrid policy with transit subsidy
150% UE	46.00%	80.48%	87.82%
200% UE	0%	62.68%	62.68%

CHAPTER 6 CONCLUSIONS

6.1 Summary of Major Findings

Despite the successes of pricing projects worldwide and growing government support, congestion pricing remains largely unappealing to the general public. A long-standing dilemma for transportation authorities is how to enjoy the efficiency benefits of congestion pricing while keeping the general public happy. In this dissertation, we proposed that using a Pareto-improving congestion pricing approach will bridge the gap between these two seemingly contradictory goals. This dissertation provides an in-depth investigation of the-state-of-the-art of Pareto-improving pricing strategies for general transportation networks.

The fundamental reason that a Pareto-improving congestion pricing scheme is achievable is that a UE flow distribution may not be Pareto-optimal. If a flow distribution is not Pareto-optimal, there may exist a dominating flow distribution that improves system efficiency without making any user worse off. Since a dominating flow distribution is not in equilibrium, congestion pricing can be used as an instrument for inducing such a flow distribution. As shown Chapter 3, the absence of loops (single- and multi-commodity) is the necessary and sufficient condition for the existence of anonymous nonnegative tolls to support a given dominating flow distribution. Also, the relationship between a dominating flow distribution and anonymous nonnegative Pareto-improving toll vectors was examined. We further proved that a nonnegative OD-dependent Pareto-improving toll vector can always be found to induce any dominating flow distribution without a single-commodity loop as an equilibrium flow distribution.

Chapter 4 discussed Pareto-improving pricing schemes for a general network when users belong to a discrete set of VOT classes. Mathematical formulations for finding a Pareto-improving scheme as well as a dominating flow distribution were developed.

Given a dominating flow distribution with heterogeneous users, we also provided necessary and sufficient conditions for the existence of an anonymous nonnegative Pareto-improving toll vector to induce the distribution. The numerical results from three networks illustrate that multiclass Pareto-improving pricing schemes are less prevalent when compared to its single-class counterpart.

A new hybrid Pareto-improving policy that combines congestion pricing and free-travel-right assignment was proposed in Chapter 5. Mathematical formulations for developing Pareto-improving pure road space rationing schemes and hybrid policies were given. The hybrid policy proposed offers greater flexibility than pure road space rationing. Numerical examples demonstrated that the proposed hybrid policy is more prominent in leading to a Pareto improvement than both pure congestion pricing and road space rationing schemes.

Three major contributions of this dissertation to the literature can be summarized as follows:

- This dissertation provided a systematic study of the existence and properties of Pareto-improving pricing schemes in general transportation networks.
- It explored the problem of Pareto-improving pricing schemes for networks with heterogeneous users.
- It proposed a hybrid policy that takes advantage of the synergistic effects between congestion pricing and free-travel-right assignment.

The congestion pricing and hybrid strategies developed in this dissertation demonstrate that the Pareto-improving approach is a viable and promising way of designing efficient demand management strategies that are also appealing to the general public. The findings may make congestion pricing no longer a hard sell to decision makers and the general public, which may eventually lead the nation's transportation system to a more sustainable future.

6.2 Future Research

In Chapter 4, we assumed that road users are categorized into a discrete set of VOT classes. Users who belong to the same user class are assumed to have the same VOT, which is a rough approximation of the actual continuous VOT distribution. [Nie and Liu \(2010\)](#) examined the existence of a Pareto-improving pricing scheme in a bottleneck network (one OD pair) with heterogeneous users characterized by continuous VOT distributions. It would be a valuable addition to the literature of the Pareto-improving congestion pricing approach, if we can extend the model to general road networks.

The hybrid policy proposed in Chapter 5 combines regulatory and market-based demand management policies. Future research could introduce heterogeneous users into the proposed formulation. By doing so, we may obtain more insights into the policy implications of the hybrid model. Also, it might be interesting to investigate the long-term effects of the hybrid policy. Road users may purchase additional vehicles to bypass restrictions. However, whether they choose to do so in the long term depends on toll rates on restricted links and the restriction ratio. Because users are allowed to pay tolls to use restricted links on restricted days, it may not be beneficial for them to maintain a second vehicle to bypass the restriction. On the other hand, if the restriction ratio and toll rates are high, users may find themselves better off purchasing a second car in the long run.

REFERENCES

- Abrams, R. A., Hagstrom, J. N., 2006. Improving traffic flows at no cost. In: S. Lawphongpanich, D. W. Hearn, M. J. S. (Ed.), *Mathematical and Computational Models for Congestion Pricing*. Springer.
- Ahuja, R. K., Magnanti, T. L., Orlin, J. B., 1993. *Network Flows: Theory, Algorithms, and Applications*. Prentice Hall, Englewood Cliffs, NJ.
- Akiyama, T., Mun, S., Okushima, M., 2004. Second-best congestion pricing in urban space: cordon pricing and its alternatives. *The Review of Network Economics* 3 (4), 401–414.
- Arnott, R., Kraus, M., 1998. When are anonymous congestion charges consistent with marginal cost pricing? *Journal of Public Economics* 67 (1), 45–64.
- Arnott, R., Small, K., 1994. The economics of traffic congestion. *American Scientist* 82, 446–455.
- Bai, L., Hearn, D., Lawphongpanich, S., 2004. Decomposition techniques for the minimum toll revenue problem. *Networks* 44 (2), 142–150.
- Bard, J. F., 1998. *Practical Bilevel Optimization: Algorithms and Applications*. Kluwer Academic Publishers, Dordrecht, The Netherlands.
- Beckmann, M., McGuire, C., Winsten, C., 1956. *Studies in the Economics of Transportation*. Yale University Press, New Haven, CT.
- Brooke, A., Kendrick, D., Meeraus, A., Raman, R., 2003. *GAMS: A User's Guide*. GAMS Development Corporation, Washington, DC.
- Brownstone, D., Small, K., 2005. Valuing time and reliability: Assessing the evidence from road pricing demonstrations. *Transportation Research Part A: Policy and Practice* 39 (4), 279–293.
- Button, K., 1993. *Transport economics*. Edward Elgar, Northampton, MA.
- Chen, Y., Florian, M., 1995. The nonlinear bilevel programming problem: Formulations, regularity and optimality conditions. *Optimization* 32 (3), 193–209.
- Dafermos, S., 1972. The traffic assignment problem for multiclass-user transportation networks. *Transportation Science* 6 (1), 73–87.
- Daganzo, C. F., 1995. A pareto optimum congestion reduction scheme. *Transportation Research Part B: Methodological* 29 (2), 139–154.
- de Palma, A., Lindsey, R., 2011. Traffic congestion pricing methodologies and technologies. *Transportation Research Part C: Emerging Technologies* 19 (6), 1377–1399.

- Downs, A., 1962. The law of peak-hour expressway congestion. *Traffic Quarterly* 16 (3), 393–409.
- Downs, A., 2004. *Still Stuck in Traffic: Coping with Peak-Hour Traffic Congestion*. Brookings Institution Press, Washington, D.C.
- Duranton, G., Turner, M. A., 2011. The fundamental law of road congestion: Evidence from us cities. *American Economic Review*, 101 (6), 2616–2652.
- Eliasson, J., 2001. Road pricing with limited information and heterogeneous users: A successful case. *The Annals of Regional Science* 35 (4), 595–604.
- Eliasson, J., Mattsson, L., 2006. Equity effects of congestion pricing quantitative methodology and a case study for Stockholm. *Transportation Research Part A: Policy and Practice* 40 (7), 602–620.
- Engelson, L., Lindberg, P. O., 2006. Congestion pricing of road networks with users having different time values. In: Lawphongpanich, S., Hearn, D. W., Smith, M. J., Pardalos, P. M., Hearn, D. (Eds.), *Mathematical and Computational Models for Congestion Charging*. Vol. 101 of *Applied Optimization*. Springer US, pp. 81–104.
- Fisk, C. S., 1984. Game theory and transportation systems modelling. *Transportation Research Part B: Methodological* 18 (4-5), 301–313.
- Florian, M., Guelat, J., Spiess, H., 1987. An efficient implementation of the Partan variant of the linear approximation method for the network equilibrium problem. *Networks* 17 (3), 319–339.
- Foster, C. D., 1974. The regressiveness of road pricing. *International Journal of Transport Economics* 1, 273–275.
- Gallager, R. G., dec. 1977. Loops in multicommodity flows. In: *Decision and Control including the 16th Symposium on Adaptive Processes and A Special Symposium on Fuzzy Set Theory and Applications, 1977 IEEE Conference on*. Vol. 16. pp. 819 –825.
- GAO, February 2009. *Highway trust fund: Improved solvency mechanisms and communication needed to help avoid shortfalls in the highway account*. Tech. Rep. GAO-09-316, United States Government Accountability Office.
- Garrett, H., 1968. The Tragedy of The Commons. *Science* 162 (1968), 1243–1248.
- Giuliano, G., 1994. Equity and fairness considerations of congestion pricing. In: *Curbing Gridlock: Peak-Period Fees to Relieve Traffic Congestion*. Vol. 2. Transportation Research Board, pp. 250–279.
- Gómez-Ibáñez, J., Tye, W., Winston, C., 1999. *Essays in Transportation Economics and Policy: A Handbook in Honor of John R. Meyer*. Brookings Institution Press, Washington, D.C.

- Guo, X., Yang, H., 2010. Pareto-improving congestion pricing and revenue refunding with multiple user classes. *Transportation Research Part B: Methodological* 44 (8-9), 972–982.
- Hagstrom, J., Abrams, R., 2001. Characterizing braess's paradox for traffic networks. In: *Intelligent Transportation Systems, 2001. Proceedings. 2001 IEEE*. pp. 836 –841.
- Hagstrom, J. N., Abrams, R. A., 2009. Loops and the existence of equilibrium-inducing tolls. Working paper, Department of Information and Decision Sciences, University of Illinois at Chicago.
- Han, D., Yang, H., Wang, X., 2010. Efficiency of the plate-number-based traffic rationing in general networks. *Transportation Research Part E: Logistics and Transportation Review* 46 (6), 1095–1110.
- Hau, T., 2005. Economic fundamentals of road pricing: A diagrammatic analysis, part I-fundamentals. *Transportmetrica* 1 (2), 81–117.
- Hearn, D., Ramana, M., 1998. Solving congestion toll pricing models. In: *Equilibrium and Advanced Transportation Modeling*. Kluwer Academic Publishers, Dordrecht, The Netherlands, pp. 109–124.
- Jahn, O., Mohring, R., Schulz, A., Stier-Moses, N., 2005. System-optimal routing of traffic flows with user constraints in networks with congestion. *Operations Research* 53 (4), 600–616.
- Knight, F., 1924. Some fallacies in the interpretation of social cost. *The Quarterly Journal of Economics* 38, 582–606.
- Lawphongpanich, S., Yin, Y., 2010. Solving the pareto-improving toll problem via manifold suboptimization. *Transportation Research Part C: Emerging Technologies* 18 (2), 234–246.
- Lindsey, R., 2006. Do economists reach a conclusion on road pricing? *Scholarly Comments on Academic Economics* 3 (2), 292–379.
- Liu, Y., Guo, X., Yang, H., 2009. Pareto-improving and revenue-neutral congestion pricing schemes in two-mode traffic networks. *NETNOMICS* 10, 123–140.
- Luo, Z., Pang, J., Ralph, D., 1996. *Mathematical programs with equilibrium constraints*. Cambridge University Press, Cambridge, England.
- Maruyama, T., Harata, N., 2006. Difference between area-based and cordon-based congestion pricing: investigation by trip-chain-based network equilibrium model with nonadditive path costs. *Transportation Research Record: Journal of the Transportation Research Board* 1964, 1–8.

- Maruyama, T., Sumalee, A., 2007. Efficiency and equity comparison of cordon-and area-based road pricing schemes using a trip-chain equilibrium model. *Transportation Research Part A: Policy and Practice* 41 (7), 655–671.
- Mayet, J., Hansen, M., 2000. Congestion pricing with continuously distributed values of time. *Journal of Transport Economics and Policy* 34, 359–369.
- Mohring, H., Harwitz, M., 1962. *Highway Benefits: An Analytical Framework*. Northwestern University Press, Evanston, IL.
- Nagurney, A., 1999. *Network Economics: A Variational Inequality Approach*. Kluwer Academic Publishers, Dordrecht, The Netherlands.
- Nagurney, A., 2000. A multiclass, multicriteria traffic network equilibrium model. *Mathematical and Computer Modelling* 32, 393–411.
- Ng, Y. K., 1979. *Welfare economics*. Macmillan, London.
- Nie, Y., Liu, Y., 2010. Existence of self-financing and pareto-improving congestion pricing: Impact of value of time distribution. *Transportation Research Part A: Policy and Practice* 44 (1), 39–51.
- Patriksson, M., 1994. *The Traffic Assignment Problem: Models and Methods*. V.S.P. Intl Science, Leiden, Netherlands.
- Pigou, A., 1920. *The Economics of Welfare*. Macmillan, London, UK.
- Poundstone, W., 1992. *Prisoner's Dilemma: John Von Neumann, Game Theory and the Puzzle of the Bomb*. Doubleday, New York, NY.
- Ramjerdi, F., 2006. Equity measures and their performance in transportation. *Transportation Research Record: Journal of the Transportation Research Board* 1983, 67–74.
- Richardson, H., 1974. A note on the distributional effects of road pricing. *Journal of Transport Economics and Policy* 8 (1), 82–85.
- Scheel, H., Scholtes, S., 2000. Mathematical programs with complementarity constraints: Stationarity, optimality, and sensitivity. *Mathematics of Operations Research* 25, 1–22.
- Shimizu, K., Ishizuka, Y., Bard, J., 1997. *Nondifferentiable and Two-Level Mathematical Programming*. Kluwer Academic Publishers, Dordrecht, The Netherlands.
- Small, K., Winston, C., Evans, C., 1989. *Road Work: A New Highway Pricing and Investment Policy*. Brookings Institution Press, Washington, D.C.
- Small, K., Winston, C., Yan, J., 2005. Uncovering the distribution of motorists' preferences for travel time and reliability. *Econometrica* 73 (4), 1367–1382.

- Small, K., Yan, J., 2001. The value of value pricing of roads: Second-best pricing and product differentiation. *Journal of Urban Economics* 49 (2), 310–336.
- Song, Z., Yin, Y., Lawphongpanich, S., 2009. Nonnegative pareto-improving tolls with multiclass network equilibria. *Transportation Research Record: Journal of the Transportation Research Board* 2091, 70–78.
- Sumalee, A., 2004. Optimal road user charging cordon design: A heuristic optimization approach. *Computer-Aided Civil and Infrastructure Engineering* 19 (5), 377–392.
- Szeto, W., Lo, H., 2006. Transportation network improvement and tolling strategies: The issue of intergeneration equity. *Transportation Research Part A: Policy and Practice* 40 (3), 227–243.
- Vickrey, W., 1969. Congestion theory and transport investment. *The American Economic Review* 59 (2), 251–260.
- Viegas, J., 2001. Making urban road pricing acceptable and effective: searching for quality and equity in urban mobility. *Transport Policy* 8 (4), 289–294.
- Wachs, M., 2003. Then and now: the evolution of congestion pricing in transportation and where we stand today. In: *International Perspectives on Road Pricing*. Transportation Research Board, pp. 63–72.
- Walters, A., 1961. The theory and measurement of private and social cost of highway congestion. *Econometrica* 29 (4), 676–699.
- Wang, X., Yang, H., Han, D., 2010. Traffic rationing and short-term and long-term equilibrium. *Transportation Research Record: Journal of the Transportation Research Board* 2196, 131–141.
- Wardrop, J., 1952. Some theoretical aspects of road traffic research. *Proceedings of Institution of Civil Engineers* 1 (3), 325–362.
- Wu, D., Yin, Y., Lawphongpanich, S., 2011. Pareto-improving congestion pricing on multimodal transportation networks. *European Journal of Operational Research* 210 (3), 660–669.
- Yang, H., Huang, H., 2004. The multi-class, multi-criteria traffic network equilibrium and systems optimum problem. *Transportation Research Part B: Methodological* 38 (1), 1–15.
- Yang, H., Meng, Q., Hau, T., 2004. Optimal integrated pricing in a bi-modal transportation network. In: Lee, D. (Ed.), *Urban and Regional Transportation Modeling: Essays in Honor of David Boyce*. Edward Elgar Publishing, Northampton, MA, pp. 134–156.
- Yang, H., Zhang, X., 2002. Multiclass network toll design problem with social and spatial equity constraints. *Journal of Transportation Engineering* 128, 420–428.

- Yin, Y., Lawphongpanich, S., 2009. Alternative marginal-cost pricing for road networks. *NETNOMICS* 10 (1), 77–83.
- Yin, Y., Yang, H., 2004. Optimal tolls with a multiclass, bicriterion traffic network equilibrium. *Transportation Research Record: Journal of the Transportation Research Board* 1882, 45–52.
- Zhang, X., Yang, H., 2004. The optimal cordon-based network congestion pricing problem. *Transportation Research Part B: Methodological* 38 (6), 517–537.

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