

## C. A Review of Concepts Related to Chapter 3

The fundamental idea behind using the moving reference frame kinematics description is that it is often the case that the motion seen by a moving observer is often easier to describe than that of a fixed observer. Ultimately, we want velocity and acceleration described in terms of a fixed frame observation. The moving reference velocity and acceleration equations provide a means for “correcting” the moving frame observation to produce the fixed frame observation.

The critical components of the moving reference frame kinematics equations can be broken down into two parts:

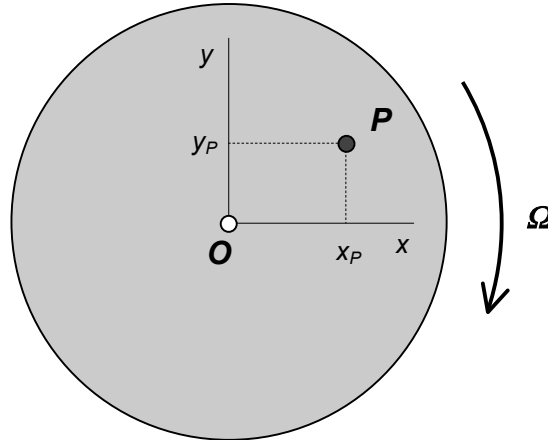
- “How the observer moves”:  $\vec{\omega}$  and  $\vec{\alpha}$  (the angular velocity and acceleration of the observer).
- “What the observer sees”:  $(\vec{v}_{B/A})_{rel}$  and  $(\vec{a}_{B/A})_{rel}$  (the velocity and acceleration of B as seen by the observer).

If you can keep these two sets of terms in the moving frame kinematics equations straight, you are off to a good start on this material.

The following is a set of conceptual/short answer/assessment questions related to the course material covered in Chapter 3. It is expected that you will use these questions to challenge your understanding of the fundamental concepts from the course as you learn the material and prepare for exams. Although these problems are conceptual in nature, you are not expected to be able to answer these questions by inspection. Rather, your answer will come from the use of a short analysis based on the fundamental equations covered. You can expect to see questions such as these on your exams this semester.

### Question C3.1

Particle P moves on the top horizontal surface of a disk that is rotating in a clockwise sense about a vertical axis with a rate of  $\Omega = 5 \text{ rad/s}$ . The position of P is described in terms of a set of Cartesian components  $x_P$  and  $y_P$  measured relative to the disk. When  $(x_P, y_P) = (4, 3) \text{ ft}$ , the velocity of P as seen by a stationary observer is:  $\vec{v}_P = (10\hat{i} - 20\hat{j}) \text{ ft/s}$ . Describe the velocity of P as seen by an observer on the disk.



TOP VIEW

Attach observer to disk

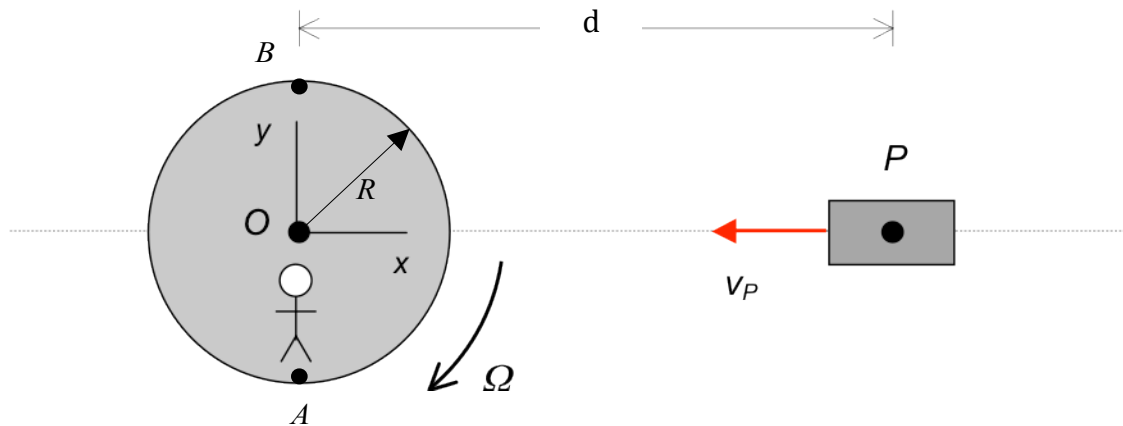
" xyz " "

Using O as the reference point:

$$\begin{aligned}\vec{v}_P &= \vec{v}_O^0 + (\vec{v}_{P/O})_{rel} + \vec{\omega} \times \vec{r}_{P/O} \\ \hookrightarrow (\vec{v}_{P/O})_{rel} &= \vec{v}_P - \vec{\omega} \times \vec{r}_{P/O} \\ &= (10\hat{i} - 20\hat{j}) - (-5\hat{k}) \times (4\hat{i} + 3\hat{j}) \\ &= [10 - 5(3)]\hat{i} + [-20 + 5(4)]\hat{j} \\ &= (-5\hat{i}) \text{ ft/s}\end{aligned}$$

### Question C3.2

A disk is pinned to ground at its center O with the disk rotating clockwise at a constant rate of  $\Omega = 3 \text{ rad/s}$ . Block P is traveling to the left along a straight path toward O with a constant speed of  $v_P = 20 \text{ ft/s}$ . Determine the acceleration of P as seen by an observer on the disk when P is at a distance of 50 ft from O.



Attach observer to disk

"xyz" " "

$$\vec{V}_P = \vec{V}_O + (\vec{V}_{P/O})_{rel} + \vec{\omega} \times \vec{r}_{P/O}$$

$$\begin{aligned} \hookrightarrow (\vec{V}_{P/O})_{rel} &= \vec{V}_P - \vec{\omega} \times \vec{r}_{P/O} \\ &= (-v_P \hat{i}) - (-\Omega \hat{k}) \times (d \hat{i}) \end{aligned}$$

$$\vec{a}_P = \vec{a}_O + (\vec{a}_{P/O})_{rel} + \cancel{\vec{\omega} \times \vec{r}_{P/O}} + 2\vec{\omega} \times (\vec{V}_{P/O})_{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/O})$$

$$\hookrightarrow (\vec{a}_{P/O})_{rel} = -2\vec{\omega} \times (\vec{V}_{P/O})_{rel} - \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/O})$$

### Question C3.3

Sprinkler arm OA is pinned to a cart at point O. The cart moves to the right with a speed of  $v_{cart}$  with  $\dot{v}_{cart} = 2 \text{ ft/s}^2 = \text{constant}$ . Fluid flows through the sprinkler arm at a rate of  $\dot{d}$  with  $\ddot{d} = -3 \text{ ft/s}^2 = \text{constant}$ . The sprinkler arm is being raised at a constant rate of  $\dot{\theta} = 4 \text{ rad/s}$ . An observer and  $xyz$  coordinate system are attached to the sprinkler arm, as shown in the figure below. The following equation is to be used to find the acceleration of a pellet P that flows with the fluid in the arm:

$$\vec{a}_P = \vec{a}_O + (\vec{a}_{P/O})_{rel} + \vec{\alpha} \times \vec{r}_{P/O} + 2\vec{\omega} \times (\vec{v}_{P/O})_{rel} + \vec{\omega} \times [\vec{\omega} \times \vec{r}_{P/O}]$$

Provide numerical values for the following terms when:  $d = 3 \text{ ft}$ ,  $v_{cart} = 3 \text{ ft/s}$ ,  $\dot{d} = 5 \text{ ft/s}$  and  $\theta = 90^\circ$ .

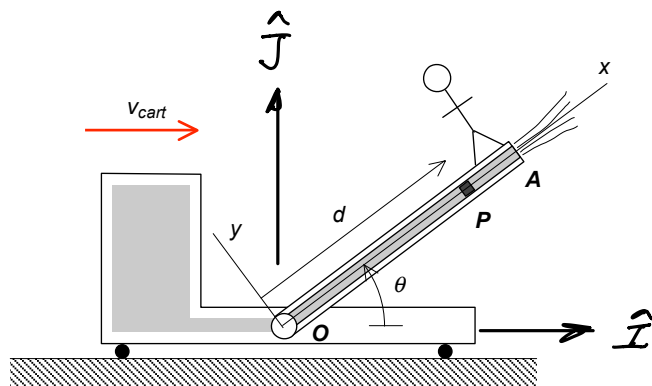
$$\vec{a}_O = 2 \hat{i} = (-2 \hat{j}) \text{ ft/s}^2$$

$$\vec{\omega} = (4 \hat{k}) \frac{\text{rad}}{\text{s}}$$

$$\vec{\alpha} = \vec{0}$$

$$(\vec{v}_{P/O})_{rel} = \dot{d} \hat{u} = (5 \hat{u}) \text{ ft/s}$$

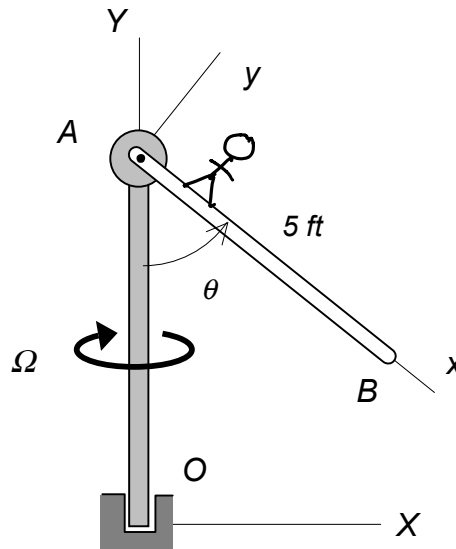
$$(\vec{a}_{P/O})_{rel} = \ddot{d} \hat{u} = \vec{0}$$



$$\hat{I} = \cos\theta \hat{u} - \sin\theta \hat{j} = -\hat{j}$$

### Question C3.4

The vertical shaft OA rotates about a fixed axis with a constant rate of  $\Omega = 8 \text{ rad/s}$ . The arm AB is pinned to OA and is being raised at a constant rate of  $\dot{\theta} = 10 \text{ rad/s}$ . An observer and  $xyz$  axes are attached to AB. The  $XYZ$  axes are stationary. What is the angular acceleration vector for arm AB when  $\theta = 90^\circ$ ?



Attach observer to AB  
 " " " " " " " "

$$\begin{aligned} \vec{\omega} &= -\Omega \hat{j} + \dot{\theta} \hat{k} \quad ; \quad \hat{j} = \hat{j} @ \theta = 90^\circ \\ \vec{\alpha} &= \frac{d\vec{\omega}}{dt} = \underbrace{-\dot{j}}_{\downarrow 0} \hat{j} - \Omega \hat{j} + \underbrace{\dot{\theta}}_{\downarrow 0} \hat{k} + \dot{\theta} \hat{k} = \vec{0} \\ &= \dot{\theta} (\vec{\omega} \times \hat{k}) = \dot{\theta} [-\Omega \hat{j} + \dot{\theta} \hat{k}] \times \hat{k} \\ &= -\dot{\theta} \Omega (\hat{j} \times \hat{k}) = -\dot{\theta} \Omega (\hat{i} \times \hat{k}) \\ &= -\dot{\theta} \Omega \hat{i} \end{aligned}$$

### Question C3.5

Arm AB rotates about a fixed vertical axis with a constant rate of  $\omega_1$ . A ring, with its center at O and of radius  $r$ , rotates about arm AB with a constant rate of  $\omega_2$ . A particle P moves along the ring with  $\dot{\theta} = \text{constant}$ . Let the  $XYZ$  axes be fixed, and the  $xyz$  axes be attached to the ring. At the position shown,  $\theta = 90^\circ$  and the  $xyz$  axes are aligned with the  $XYZ$  axes. It is desired to use the following equation to determine the acceleration of P for the position shown:

$$\vec{a}_P = \vec{a}_O + (\vec{a}_{P/O})_{rel} + \vec{\alpha} \times \vec{r}_{P/O} + 2\vec{\omega} \times (\vec{v}_{P/O})_{rel} + \vec{\omega} \times [\vec{\omega} \times \vec{r}_{P/O}]$$

Provide expressions for the following terms appearing in this equation.

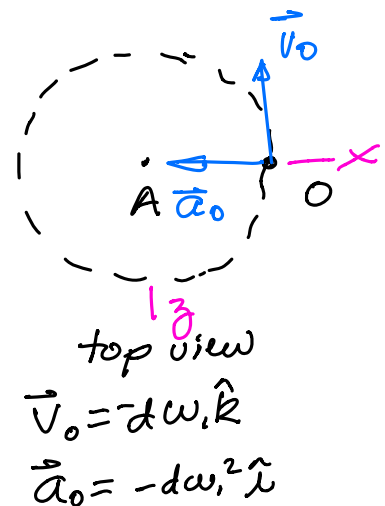
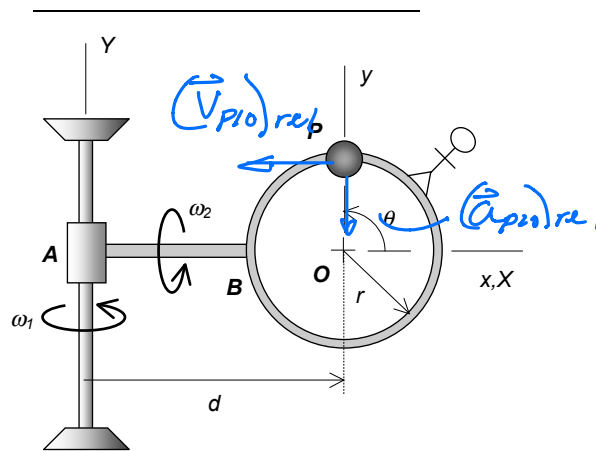
$$\vec{a}_O = -d\omega_1^2 \hat{i}$$

$$\vec{\omega} = \omega_1 \hat{j} + \omega_2 \hat{i}$$

$$\begin{aligned} \vec{\alpha} &= \dot{\omega}_1 \hat{j} + \omega_1 \dot{\hat{j}} + \dot{\omega}_2 \hat{i} + \omega_2 \dot{\hat{i}} = \omega_2 (\vec{\omega} \times \hat{i}) = \omega_2 (\omega_1 \hat{j} + \omega_2 \hat{i}) \times \hat{i} \\ &= \omega_1 \omega_2 (\hat{j} \times \hat{i}) = \omega_1 \omega_2 (\hat{j} \times \hat{i}) = -\omega_1 \omega_2 \hat{k} \end{aligned}$$

$$(\vec{v}_{P/O})_{rel} = -r\dot{\theta} \hat{i}$$

$$(\vec{a}_{P/O})_{rel} = -r\dot{\theta}^2 \hat{j}$$



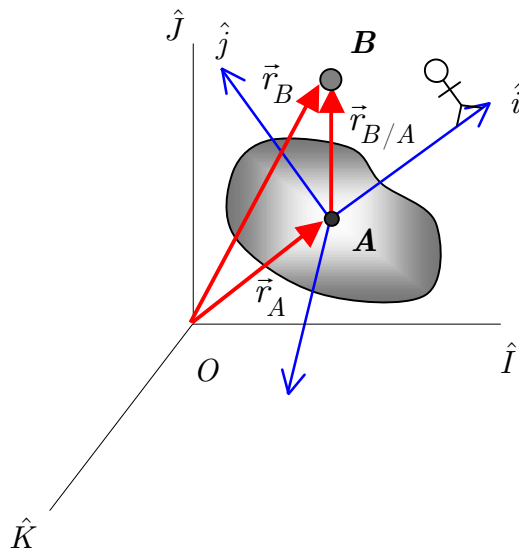
### Question C3.6

Consider an observer who is riding along on a moving (translating and rotating) rigid body. We wish to use the observation of this person in describing the motion of some point B, which is not fixed to the body, in the following moving reference frame acceleration equation.

$$\begin{cases} \vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times (\vec{v}_{B/A})_{rel} + \vec{\omega} \times [\vec{\omega} \times \vec{r}_{B/A}] \\ = \vec{a}_A + \vec{a}_{B/A} \end{cases}$$

Answer the following questions in words:

- What is the meaning of  $\vec{\omega}$ ? *ang. vel. of observer*
- What is the meaning of  $\vec{\alpha}$ ? *ang. acc. of observer*
- What is the meaning of  $(\vec{v}_{B/A})_{rel}$ ? *vel. of B as seen by observer*
- What is the meaning of  $(\vec{a}_{B/A})_{rel}$ ? *acc. of B as seen by observer*
- What restrictions, if any, are on the choice of point A? *must be on same rigid body as observer*
- What is the difference in meaning between  $\vec{a}_{B/A}$  and  $(\vec{a}_{B/A})_{rel}$ ?



$$\begin{aligned} \vec{a}_{B/A} &= \underbrace{(\vec{a}_{B/A})_{rel}}_{\text{acc. as seen by observer}} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times (\vec{v}_{B/A})_{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A}) \\ &= \vec{a}_B - \vec{a}_A \end{aligned}$$