SAMPLER

CMP3

CONNECTED MATHEMATICS PROJECT

Lappan
Fey
Fitzgerald
Friel
Phillips

ALWAYS LEARNING

PEARSON

Get connected.

For more information, visit
ConnectedMathematics3.com

Learn more! Insert into a USB port on any computer connected to the Internet.

Welcome

First Look: Connected Mathematics 3 (CMP3)

Engage students in active, personalized learning *CMP3* takes inquiry-based learning to the next level. New digital tools engage students while driving conceptual understanding, procedural skill, and real-world applications.

Teach the Common Core, teach with greater ease *CMP3* aligns to the Common Core State Standards for Mathematics and prepares students for college and careers. Technology applications help you manage your classroom with fidelity, maximize instructional time, and capture needed student data.

Apply a research-proven instructional approach *CMP3* offers the most comprehensive research base of any middle school mathematics program. Download research and evaluation reports at ConnectedMathematics3.com.

Sampler Contents

Overview

CMP3 for the Common Core and digital classroom

Background

The National Science Foundation funded the Connected Mathematics Project (CMP) at Michigan State University between 1991 and 1997. The result was *Connected Mathematics,* a complete mathematics curriculum for Grades 6, 7, and 8. CMP helps students develop an understanding of important concepts, skills, and ways of thinking and reasoning—in number, geometry, measurement, algebra, probability, and statistics. In 2000, the National Science Foundation funded a revision of the *Connected Mathematics* materials, *CMP2,* to take advantage of findings during six years of classroom use.

New Beginnings

In 2012, the same authorship team created the next generation of the Connected Mathematics Project — *CMP3*. This new curriculum aligns the program's existing rigor and emphasis on constructing viable arguments to the Common Core Standards. *CMP3* enhances its problem-based, interactive curriculum with digital instructional tools and content. The trusted authorship team remains the same. The pedagogy and approach to teaching and learning math are preserved and enhanced. And now there are powerful digital tools that allow teachers to manage their classrooms and deliver content in a revolutionary way.

CMP3 Goals

The overarching goal of *Connected Mathematics 3* is to help students develop mathematical knowledge, conceptual understanding, and procedural skills, along with an awareness of the rich connections between math topics—across grades and across content areas. Through the "Launch-Explore-Summarize" model, students investigate and solve problems that develop rigorous higher-order thinking skills and problem-solving strategies.

Curriculum development for *CMP3* has been guided by an important mathematical idea: All students should be able to reason and communicate proficiently in mathematics. They should have knowledge of and skill in the use of the vocabulary, forms of representation, materials, tools, techniques, and intellectual methods of mathematics. This includes the ability to define and solve problems with reason, insight, inventiveness, and technical proficiency.

Common Core

Connected Mathematics 3 embraces the essence of the Common Core State Standards at a deep and organic level. Given its instructional philosophy—the emphasis on inquiry and applications—*CMP3* fully addresses the Common Core State Standards for Mathematical Practice. Throughout the program, students focus on problem-solving strategies, habits of mind, and mathematical proficiency. *CMP3* students learn to communicate their reasoning by constructing viable arguments, offering proofs, and using representations. These approaches, which are aligned with the Standards for Mathematical Practice, are explicitly woven within the content of the curriculum and connected to the Common Core Content Standards.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Technology

Bringing *CMP3* up to digital speed

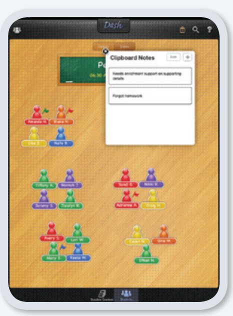
New Teaching, Learning, and Productivity Tools

Technology is part of everyday life. It's with us at home, at work, and when we play and learn. *CMP3* brings the full power of technology into the classroom. *CMP3* uses technology to help you teach with fidelity, thus raising student achievement. It helps you manage your classroom with easy-to-use mobile tools and lets you capture student work on the go. It delivers a full suite of *ExamView®* assessment tools. And *CMP3* technology is the ultimate student motivator. It's engaging and interactive. It makes learning relevant to students' lives. It personalizes learning and practice to meet individual needs and abilities—automatically, with digital speed.

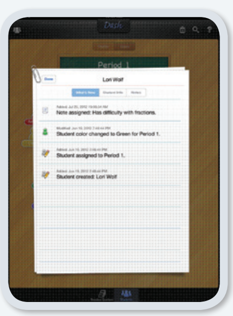
Teacher Place

CMP3 delivers information to your computer or tablet device via the Teacher Place web application. This powerful online hub is your complete source for all *CMP3* content, management tools, assessment, and important program information. Within Teacher Place, *CMP3* uses an interactive dashboard *(Dash)* for the delivery and management of teacher content. Planning and teaching are streamlined, because you now have *CMP3* content available instantly on your tablet device. Every resource you need is provided at point-of-use, eliminating the need to search through multiple booklets to teach a lesson. This enriches the Launch, Explore, and Summarize phases of the lesson, which is critical to student success.

Teacher Place

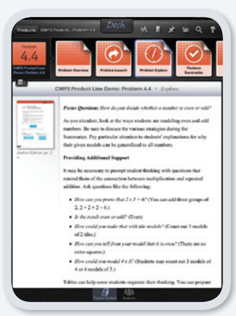


Group Students with drag-and-drop ease.



Instantly access a **Student Profile**. Add notes!

Use a projector to share the **Videos**.



Access the **Teacher Edition** from any computing platform.

Join an **Online Community** of *CMP3* teachers.

Dash works on your Mac or PC computer, as well as your tablet device—including the iPad®—so you're free to use *CMP3* with the device you prefer. And Teacher Place provides an online community to connect *CMP3* teachers with colleagues down the hall and across the country.

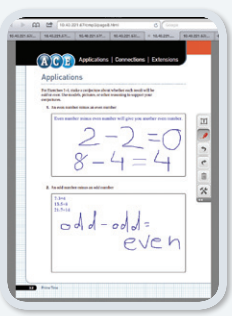
Student Place

Nothing motivates students more than meaningful technology. The digital world is their world. *CMP3* introduces Student Place—a student-centered digital workspace that allows for greater student engagement with *CMP3's* Common Core aligned mathematical content. The centerpiece of Student Place is the *CMP3* digital Student Edition that includes multimedia and interactivity. The digital edition electronically captures students' work while giving them access to an array of virtual math tools. Student Place complements the program's print materials. But if your technology access allows it, *CMP3* can be delivered entirely through digital devices.

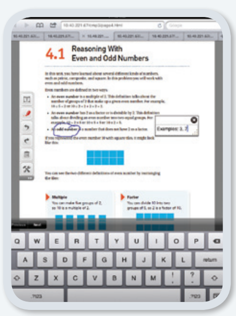
Connecting MathXL® to CMP3

In need of extra skills practice? *CMP3* incorporates interactive skills practice with *MathXL®* software on Student Place. Because this additional math practice is completely adaptive, based on pre-assessment results, *CMP3* students receive a fully personalized learning experience. *CMP3* skills practice with *MathXL®* features algorithmically generated exercises, supplemental learning aids, and auto-grading. The problems are correlated to standards; thus, regardless of how students complete the practice items, teachers can compare student results against Common Core Standards. *CMP3* skills practice with *MathXL®* is also a great tool to use with students who are ahead and could benefit from higher-level skills practice.

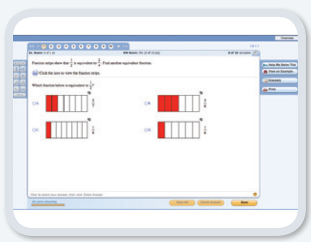
Student Place



Practice and apply with interactive **Student Pages**.



Add notes, highlight words, use **Virtual Math Tools**.



Learn at your pace with adaptive **MathXL® Assignments**.

Comparing CMP2 with CMP3

Good Gets Better

What are the biggest differences between *CMP2* and *CMP3?* For starters, the lesson content is completely updated from the ground up, and the table of contents is newly reorganized and enhanced to meet the full requirements of the Common Core State Standards. The CMP authors reviewed all grade-level content to ensure Common Core alignment and to actively implement the Standards for Mathematical Practice. And, *CMP3* has an accelerated version of content so schools can now introduce Algebra 1 in the 8th grade.

More Digital, More Interactive

Technology permeates our lives. School should be the place that harnesses the full power of technology in a meaningful way. *CMP3* is reengineered with digital resources that motivate students and make you more productive.

- **Teacher Place** was created to make planning, teaching, and classroom management a whole lot easier. The online teacher dashboard, powered by *Dash,* allows users to interact with instructional content in a flexible, personalized way.
- **Teachability** is a new community on Teacher Place. It lets *CMP3* teachers collaborate with other *CMP3* instructors. Teachers can share ideas, ask questions, offer tips, and share helpful strategies to improve student achievement.
- **Student Place** is where *CMP3* students access their online math content, graded homework, and teacher comments. It gives students (and their parents) anytime-anywhere access for a personalized learning experience.
- **MathXL®** is now part of *CMP3.* This online powerhouse provides personalized skills practice. A Pre-Test automatically determines the level of instruction.

Instructional Features	CMP2	CMP3
Renowned authorship team	✓	✓
Problem-centered curriculum that fosters reflective learners through discussion of solution methods, embedded mathematics, and appropriate generalizations	✓	✓
Three-part lesson structure: Launch, Explore, Summarize	✓	✓
ACE (Applications, Connections, Extensions) homework questions	✓	✓
Mathematical Practices serve as the pedagogical fabric of the program	✓	✓
TOC and Unit Investigation Booklets completely aligned by grade level to Common Core domains, clusters, and standards		✓
Interactive Class and Student Management features		✓
Personalized Skills Practice powered by *MathXL®*		✓

Components	CMP2	CMP3
Investigations: print Student Editions (booklets, single bind, English, Spanish)	✓	✓
Investigations: print Teacher's Guides	✓	✓
Interactive digital Student Edition		✓
Interactive digital Teacher's Guide with additional point-of-use teaching resources		✓
Program Implementation and Overview Guide	✓	✓
Digital QuickStart Guide		✓
Program Resources package (print, digital)	✓	✓
Online Teacher Community		✓
Assessments CD with ExamView®	✓	✓
Teacher Resource DVD		✓
Manipulatives Kits	✓	✓

Authors

A trusted team of experts

Glenda Lappan is a University Distinguished Professor in the Program in Mathematics Education (PRIME) and the Department of Mathematics at Michigan State University. Her research and development interests are in the connected areas of students' learning of mathematics and mathematics teachers' professional growth and change related to the development and enactment of K–12 curriculum materials.

James T. Fey is a Professor Emeritus at the University of Maryland. His consistent professional interest has been development and research focused on curriculum materials that engage middle and high school students in problem-based collaborative investigations of mathematical ideas and their applications.

William M. Fitzgerald *(Deceased)* was a Professor in the Department of Mathematics at Michigan State University. His early research was on the use of concrete materials in supporting student learning and led to the development of teaching materials for laboratory environments. Later he helped develop a teaching model to support student experimentation with mathematics.

Susan N. Friel is a Professor of Mathematics Education in the School of Education at the University of North Carolina at Chapel Hill. Her research interests focus on statistics education for middle-grade students and, more broadly, on teachers' professional development and growth in teaching mathematics K–8.

Elizabeth Difanis Phillips is a Senior Academic Specialist in the Program in Mathematics Education (PRIME) and the Department of Mathematics at Michigan State University. She is interested in teaching and learning mathematics for both teachers and students. These interests have led to curriculum and professional development projects at the middle school and high school levels, as well as projects related to the teaching and learning of algebra across the grades.

SAMPLER

CMP3®

CONNECTED MATHEMATICS PROJECT

Prepublication Sample

- Course Content: Grades 6, 7, 8
- Sample Investigation (Grade 6)

Contents

Grade 6

Contents

Grade 7

Contents

Grade 8

Prime Time

Factors and Multiples

Grade 6 Unit Contents

"Prime Time": Investigation 4 Follows

Investigation 4

Linking Multiplication and Addition

You can write a product using a "times sign," as in $12 = 2 \times 6$. You can also show a product with a "raised dot" or with parentheses.

$$12 = 2 \times 6 = 2 \bullet 6 = 2(6)$$

The parentheses mean that the amount inside the parentheses is multiplied by the amount outside the parentheses.

In this Unit, you have studied factors and multiples. For example, 12 can be written as the product of two or more numbers, called factors of 12. 12 is also a multiple of its factors.

Sometimes, you can use an expression as a factor. A **numerical expression** combines numbers using one or more mathematical operations.

$$12 = 2(5 + 1)$$

This example shows the product of 2 and the expression $5 + 1$.

You can also write numbers as the sum of two or more numbers.

$$12 = 10 + 2 \quad \text{or} \quad 12 = 5 + 7 \quad \text{or} \quad 12 = 3 + 4 + 5$$

The expressions 12, $2 \bullet 6$, $10 + 2$, $5 + 7$, $3 + 4 + 5$, and $2(5 + 1)$ are **equivalent expressions**. They have the same numerical value.

You will learn how multiplication and addition are related.

Common Core State Standards

6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.

4.1 Reasoning With Even and Odd Numbers

In this Problem you will work with odd and even numbers. An **odd number** is a number that does *not* have 2 as a factor. *Even numbers* are defined in two ways:

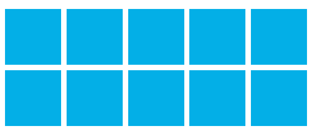
- An **even number** is a *multiple of 2*. This definition talks about the number of groups of 2 that make up a given even number.

$$10 = \overbrace{2 + 2 + 2 + 2 + 2}^{\text{five groups of 2}} \quad \text{or} \quad 10 = 5 \times 2$$

- An **even number** has *2 as a factor*. It is *divisible by 2*. This definition talks about dividing an even number into two equal groups.

$$10 \div 2 = 5 \quad \text{or} \quad \overbrace{10 = 2 \times 5}^{\text{2 is a factor of 10.}} \quad \text{or} \quad 10 = 5 + 5$$

If you represented the even number 10 with square tiles, it might look like this:



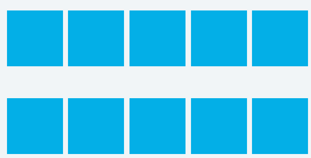
You can see the two different definitions of *even number* by rearranging the tiles:

Multiple

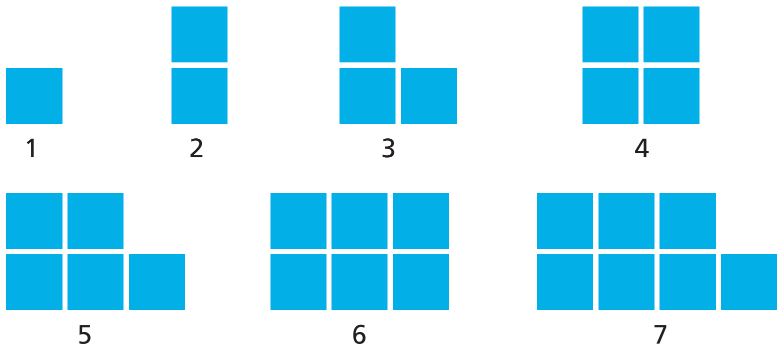
You can make five groups of 2, so 10 is a multiple of 2.

Factor

You can divide 10 into two groups of 5, so 2 is a factor of 10.



Lila wanted to know more about even and odd numbers. She arranged square tiles in a pattern to make models for whole numbers. Lila's tile models for the numbers from 1 to 7 look like this:



- Which numbers are even? Which numbers are odd?
- How are the models of even numbers different from the models of odd numbers?
- Describe what the models for 50 and 99 would look like.

When you predict what you think will happen in a mathematical situation, you are making a *conjecture*. A **conjecture** is your best guess about an observed pattern or relationship. You can use models, drawings, or other kinds of evidence to support your conjectures.

- Make a conjecture about what happens when you add two even numbers. Do you get an even number or an odd number? Explain.

Problem 4.1 asks you to make several conjectures about even and odd numbers.

Problem 4.1 Reasoning With Even and Odd Numbers

A Make conjectures about whether the results below will be *even* or *odd*. Then use tile models or some other method to support your conjectures.

1. the sum of two even numbers
2. the sum of two odd numbers
3. the sum of an even number and an odd number
4. the product of two even numbers
5. the product of two odd numbers
6. the product of an even number and an odd number

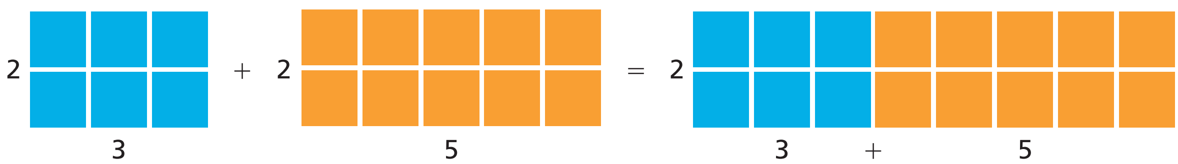
B How can you determine whether a sum of numbers, such as 127 + 38, is even or odd without building a tile model or computing the sum?

C Is 0 an even number or an odd number? How do you know?

ACE Homework starts on page 72.

4.2 Using the Distributive Property

Lila made the conjecture that the sum of two even numbers is an even number. She used square tiles to show why the sum of two even numbers is always even.



$2(3) + 2(5) = 16$ and $2(3+5) = 16$

Alex wondered if this means that $2(3) + 2(5) = 2(3 + 5)$.

What do you think? Are Lila and Alex correct? Explain why or why not.

Lila's picture represents an important property of numbers called the **Distributive Property**. The Distributive Property connects the operations of addition and multiplication.

$$\underline{2(3)} + \underline{2(5)} = \underline{2}(\underline{3 + 5}) = 16$$

sum of two terms — product of two factors

You can write the number 16 as a sum of two quantities, 6 and 10. You can also write the number 16 as the sum of two quantities, 2(3) and 2(5).

$$16 = 6 + 10 = 2(3) + 2(5)$$

You can write the number 16 as a product of two factors, 2 and 8. You can also write 16 as the product of two factors, 2 and (3 + 5).

$$16 = 2(8) = 2(3 + 5)$$

The expressions 2(3) + 2(5) and 2(3 + 5) are equivalent expressions.

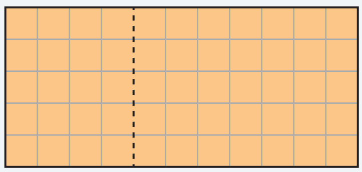
$$2(3) + 2(5) = 2(3 + 5)$$

Lila thinks that the Distributive Property explains how the area of a rectangle can be found in two different ways. Is she correct? Explain.

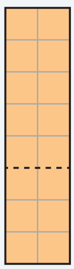
Problem 4.2 Using the Distributive Property

A In each diagram below, a large rectangle has been made from two smaller rectangles. In each case, show two different ways to calculate the area of the large rectangle.

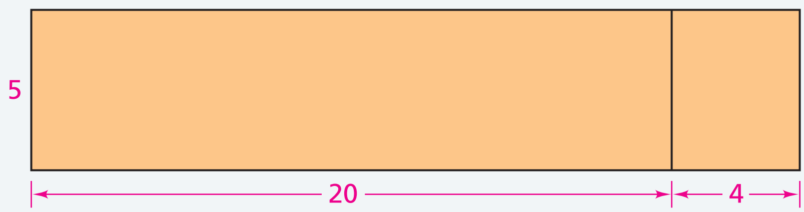
1.



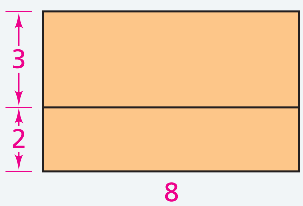
2.



3.



4.



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Problem 4.2 continued

B A large rectangle has an area of 28 square units. It has been divided into two smaller rectangles. One of the smaller rectangles has an area of 4 square units. What are possible whole-number dimensions of the large rectangle? Justify your reasoning.

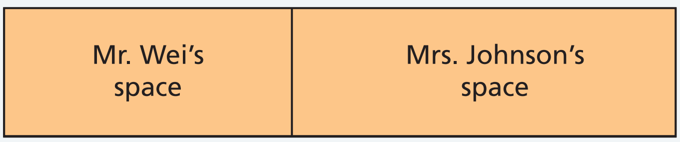
C Each of the following numerical expressions represents the area of a rectangle that has been divided into two smaller rectangular pieces. For each expression

- sketch a rectangle whose area can be represented by the expression.
- write an equivalent expression for the area of the original rectangle.

1. $6(5 + 9)$

2. $4(7) + 4(3)$

D Ms. Johnson's and Mr. Wei's classes are participating in the Meridian School Arts and Crafts exhibit. Their spaces will be next to each other. Ms. Johnson's space will be longer, but it will have the same width as Mr. Wei's space. Ms. Johnson has 48 carpet squares. Mr. Wei has 36 carpet squares.



1. Use the Distributive Property to find the possible whole-number dimension of the classes' total exhibition space. Explain what each number means in your expressions.

2. What is the greatest width that their exhibition space can have? What is the corresponding depth/length? Which expression represents these dimensions?

3. What is the length of each class's space? Do these lengths have any common factors? Explain.

4. Mr. Casey writes the equation $40 = 16 + 24 = a(b + c)$. What whole numbers can he choose for a, b, and c if he wants b and c to have no common factors greater than 1?

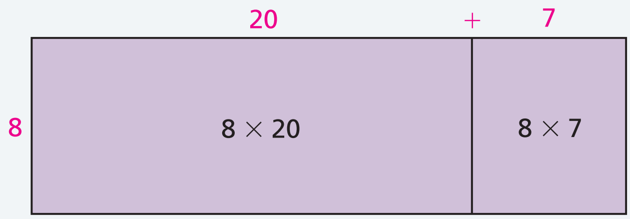
continued on the next page >

Problem 4.2 *continued*

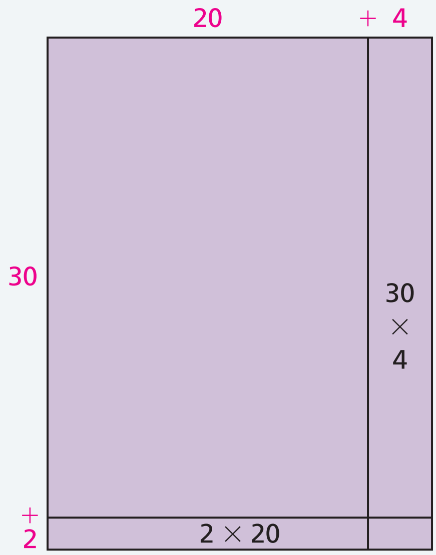
E You can use the area of a rectangle, the Distributive Property, and what you know about place value to find products.

1. Explain how you can use the diagram to find the product 8×27. How does this process represent the Distributive Property?

$$8 \times 27 = 8\,(20 + 7) = (8 \times 20) + (8 \times 7)$$



2. Explain how the diagram and the Distributive Property show a way to find the product 32×24.



   a. Find the missing areas.

   b. Write two expressions to find the product 32×24. Simplify the expressions.

   c. Explain how finding the area of the rectangle in two different ways is related to the algorithm for finding the product 32×24.

3. How is finding the area of a rectangle related to the Distributive Property?

ACE Homework starts on page 72.

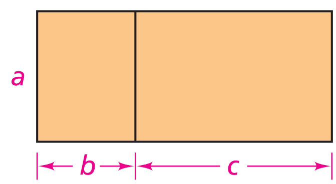
4.3 Ordering Operations

When you multiply a number by a letter variable, you can leave out the multiplication sign or parentheses. So, $3n$ means $3 \times n$ or $3(n)$. This is also true for a product with more than one letter variable. So, ab means $a \times b$ or $a(b)$.

The Distributive Property can be useful when solving problems. The Distributive Property states that if a, b, and c are any numbers, then

$$a(b + c) = a(b) + a(c)$$

- A number can be expressed as both a product and a sum.
- The area of a rectangle can be found in two different ways.



The expression $a(b + c)$ is in **factored form**. The expression $a(b) + a(c)$ is in **expanded form**. The two expressions $a(b + c)$ and $a(b) + a(c)$ are equivalent expressions.

$$\underline{a}(\underline{b + c}) = a(b) + a(c) = \underline{ab} + \underline{ac}$$

factored form (product of two factors) — expanded form (sum of two terms)

In addition to using the Distributive Property, there needs to be an agreement as to which operation should be done first in an expression. In evaluating the expression $3 + 4 \times 6$, Mary thinks you get 42 and Hank thinks you get 27.

Who is correct?

Mathematicians know that some numerical situations might be interpreted in more than one way. Therefore, they agreed on an order for simplifying expressions called the **Order of Operations**. When an expression includes more than one operation, you simplify it by following these steps.

1. Work within **parentheses**.
2. Write numbers written with **exponents** in standard form.
3. Do all **multiplication and division** in order from left to right.
4. Do all **addition and subtraction** in order from left to right.

? Using the Order of Operations, what is 3 + 2(5 + 4)?

Problem 4.3 Ordering Operations

A 1. Jenn bought 12 pens for $2 each and 6 pads of paper for $3 each. She was in a hurry and forgot to include the operations. She wrote

12 2 6 3

Place parentheses and operations signs to write an expression for the total cost of Jenn's purchase. How much did she spend?

2. Nic bought 12 pens for $2 each and 12 pads of paper for $3 each. Write two expressions for how Nic could calculate his total. Write one expression in expanded form and one in factored form.

3. Can Jenn write two expressions for her calculation? Explain.

B Without changing the order of the numbers, how many different numbers can you find by inserting parentheses and/or addition signs between the numbers below? For example, 2(5) + 1 + 3 = 14.

2 5 1 3

continued on the next page >

Problem 4.3 continued

C Simplify each expression below. Compare your answers with your classmates' answers.

1. $3 + 5 \times 2 + 4$
2. $3 + 5(2 + 4)$
3. $4 + (3 + 7) \div 2 - 2(4)$
4. $3^2 + 5(2 + 3) - 25$
5. $2 + 5^3 \times 10$
6. $4 \div 4 + 7^2$

D When simplifying the expression $3 + 5(2 + 4)$, Kalia applied the Distributive Property first, and then performed the Order of Operations. She wrote,

$$\begin{aligned} 3 + 5(2 + 4) &= 3 + 5(2) + 5(4) \\ &= 3 + 10 + 20 \\ &= 33 \end{aligned}$$

Do you agree with Kalia? Explain.

ACE Homework starts on page 72.

4.4 Choosing an Operation

In Problem 4.3, you used the Distributive Property and the Order of Operations to solve problems. Before you can apply the Distributive Property or the Order of Operations, you need to identify which operations are needed to solve a problem. In Problem 4.4, you will first identify which operations you need. Then you can use what you have learned about the Distributive Property and the Order of Operations to answer the questions.

Problem 4.4 Choosing an Operation

In each of the following situations

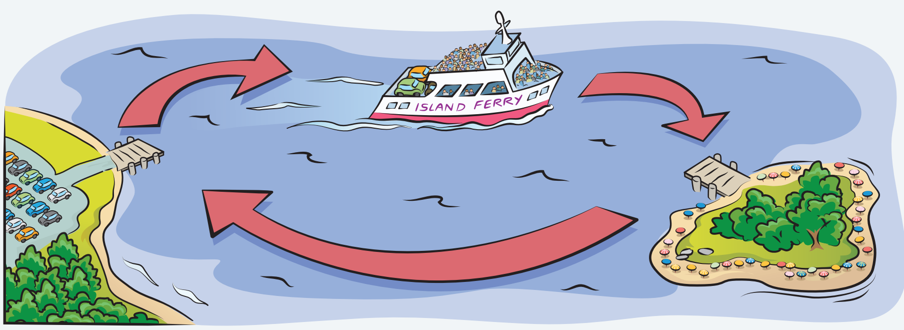
- decide which operations are needed to solve the problem.
- write one or two expressions to represent each problem.
- use the Order of Operations to simplify your expression.
- explain your reasoning.

A Dan is selling fudge to raise money for the school band. Each box of fudge costs \$8. One week he sells 15 boxes, and the next week he sells 17 boxes. How much money has Dan made at the end of the two weeks?

B An American football team is on their 35-yard line. Suppose they lose 5 yards on each of the next three plays. At what yard line will they be after the three plays?

C Leslie is in charge of packing snacks for her class. She has 30 cookies and 20 apples. She wants to put the same number of cookies and the same number of apples in each pack. What are the possible numbers of packs she can make? How many items are in each bag?

D The ferry is the only connection between an island and the mainland. The ferry makes one round trip a day. On Monday the ferry carried 83 people to the island and returned 114 people to the mainland. Compare the population of the island at the beginning of the day to the population after the ferry returned to the mainland.



E Two student clubs plan to share a bus on a trip to the capital. Transportation and lunch for one day costs \$12 per student. One club has 25 members and the other club has 18 members. What is the total cost of the trip for the two clubs?

ACE Homework starts on page 72.

Applications

For Exercises 1–4, make a conjecture about whether each result will be *odd* or *even*. Use models, pictures, or other reasoning to support your conjectures.

1. an even number minus an even number

2. an odd number minus an odd number

3. an even number minus an odd number

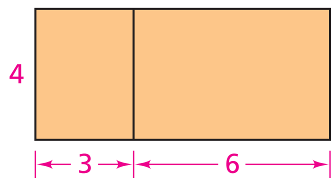
4. an odd number minus an even number

5. How can you tell whether a number is even or odd? Explain or illustrate your answer in at least two different ways.

6. How can you determine whether the sum of several numbers, such as 13 + 45 + 24 + 17, is even or odd without actually calculating the sum?

For Exercises 7–9, write expressions for the area of each rectangle in two different ways. Then find the area using both expressions.

7.



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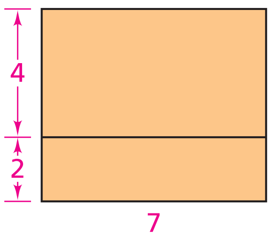
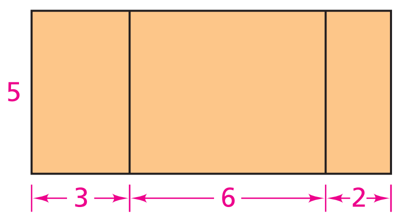


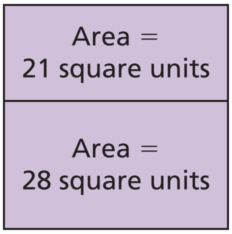
9.



In Exercises 10–12, the dimensions of the rectangles are whole numbers. Find the area of the rectangle composed of the two smaller rectangles. Then find the dimensions of all three rectangles.

10.

11.



12.

For Exercises 13–16, draw a rectangle with the given width and length. Then use the Distributive Property to write each area as a product and as a sum.

13. 3 and $(4 + 6)$

14. 3 and $(5 + 1 + 3)$

15. N and $(2 + 6)$

16. 5 and $(N + 2)$

For Exercises 17–18, use the area of a rectangle and the Distributive Property to find each product.

17. 9×34

18. 35×18

19. **a.** Write 60 as the sum of two numbers.

b. Write 60 as the product of two numbers.

c. Write 60 as the product of two factors. In your expression, write one of the factors as a sum of two numbers. Find an equivalent way to write this expression.

90 = 20 + 70. Use the Distributive Property and the GCF of 20 and 70 to write another related expression for 90. Could you write another expression with a different common factor?

b. 90 = 36 + 54. Use the Distributive Property and the GCF of 36 and 54 to write another related expression for 90. Could you write another expression with a different common factor?

21. Consider a 3-by-3 grid.

- Choose four numbers, such as 2, 8, 6, and 3. Write the numbers along the border of the grid as shown.

	6	3	Sum
2			
8			
Sum			

- Enter the product of the border numbers into the corresponding cells.

	6	3	Sum
2	12	6	
8	48	24	
Sum			

- Add across the rows and columns.

	6	3	Sum
2	12	6	18
8	48	24	72
Sum	60	30	90

a. What is the relationship between the black number in the lower right-hand cell and the red numbers along the edges of the grid?

b. Start a new 3-by-3 grid. Pick another set of numbers for the border. Does the same relationship hold for the lower right-hand cell and the numbers along the edges? Explain.

c. Shalala claims she used the Distributive Property to show that the sum of the numbers in the bottom row was the same as the sum of numbers in the last column. Do you agree? Explain.

For Exercises 22–23, use rectangles to show that each statement is true.

22. $3(7 + 2) = 3(7) + 3(2)$

23. $5(6) + 5(2) = 5(6 + 2)$

For Exercises 24–27, replace *m* with a whole number to make each statement true.

24. $7(4 + m) = 49$

25. $8(m - 3) = 56$

26. $m \bullet 10 - m \bullet 2 = 8$

27. $m \bullet 10 + m \bullet 13 = 138$

For Exercises 28–31, identify which expression has the greater value.

28. $3 + 4 \times 2$ $(3 + 4) \times 2$

29. $12 \div 6 \times 2$ $12 \div (6 \times 2)$

30. $11 \times 2 + 1$ $16 - 5 \times 3$

31. 4×3^2 $3^2 \times 3^2$

For Exercises 32–35, insert operation signs to make each equation true.

32. 2 ■ 5 ■ 3 = 17

33. 2 ■ 5 ■ 3 = 13

34. 2 ■ 5 ■ 3 = 30

35. 2 ■ 5 ■ 3 = 7

For Exercises 36–40, insert parentheses and/or addition signs to make each equation true. Remember that parentheses can indicate multiplication.

36. 3 2 4 1 = 9

37. 3 2 4 1 = 13

38. 3 2 4 1 = 21

39. 3 2 4 1 = 12

40. 3 2 4 1 = 10

41. Without changing the order of the numbers below, insert parentheses and/or addition signs so that the computation results in the number described below.

4 3 6 1

a. The number is a multiple of 5.

b. The number is a factor of 36.

42. Andrea thought about how she could rewrite numbers as products and sums in different ways. She came up with the following method.

$36 = 3 \times 12$	First, I find a factor pair of a number.
	3 and 12 are factors of 36.
$= 3 \times (5 + 7)$	Next, I rewrite one factor as a sum. 12 is the sum of 5 + 7.
$= (3 \times 5) + (3 \times 7)$	Next, I use the Distributive Property.
$= 15 + 21$	Last, I find the product in each pair of parentheses.

Use Andrea's method to rewrite each number in different ways.

a. 21 **b.** 24 **c.** 55 **d.** 48

Exercises 43–46 show Devin's work. Devin made some mistakes. Identify where he made each mistake. Then correct his work.

43.

$3^2 \times 2^2 - 3^3$
$= 6 \times 4 - 9$
$= 24 - 9$
$= 15$

44.

$8 + 2 \times 3^2$
$= 8 + 6^2$
$= 8 + 36$
$= 44$

45.

$18 - 6 + 2 \times 3$
$= 18 - 6 + 6$
$= 18 - 12$
$= 6$

46.

$24 \div 6 \times (5 - 1)$
$= 24 \div 6 \times 4$
$= 24 \div 24$
$= 1$

Exercises 47–49 list the steps of an arithmetic trick. Explain why each trick works.

47. **Step 1:** Think of a whole number.

Step 2: Add 15 to the number.

Step 3: Multiply the result by 2.

Step 4: Subtract 30.

The result is double the original number.

48. **Step 1:** Think of a number.

Step 2: Double it.

Step 3: Add 6.

Step 4: Divide by 2.

Step 5: Subtract 3.

The result is the original number.

49. **Step 1:** Think of a number.

Step 2: Add 4.

Step 3: Multiply by 2.

Step 4: Subtract 6.

Step 5: Divide by 2.

Step 6: Subtract the original number.

The result is 1.

For Exercises 50–51, find a number to make each statement true.

50. $12 \times (6 + 4) = (12 \times \blacksquare) + (12 \times 4)$

51. $2 \times (n + 4) = (\blacksquare \times n) + (\blacksquare \times 4)$

For Exercises 52–57, determine whether the number sentence is true. In each case explain how you could answer without calculating. Check your answers by doing the indicated calculations.

52. $50 \times 432 = (50 \times 400) + (50 \times 32)$

53. $50 \times 368 = (50 \times 400) - (50 \times 32)$

54. $50 \times 800 = (50 \times 1000) + (50 \times 200)$

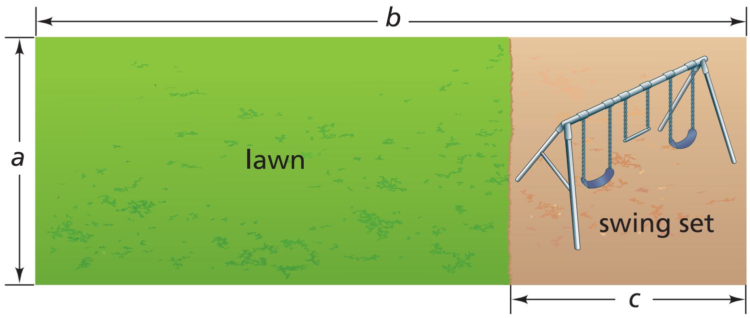
55. $(70 \times 20) + (50 \times 20) = 90 \times 70$

56. $50 + (400 \times 32) = (50 + 400) \times (50 + 32)$

57. $6 \times 17 = 6 \times 20 - 6 \times 3$

58. Sophia used the Order of Operations to simplify $4(5 - 2)$ to 4×3. Jose used the Distributive Property to simplify $4(5 - 2)$ to $20 - 8$. Are they both correct? Explain.

59. a. Mr. and Mrs. Lee are adding a swing set to their backyard. Their yard measures $a \times b$ feet. The space for the swing set measures $a \times c$ feet. They want to figure out how much of their yard will be lawn after they add the swing set.



Mrs. Lee said, "The area of the whole yard is $a \times b$. The area for the swing set is $a \times c$. Therefore, the area of the lawn is $a \times b - a \times c$." Is she correct? Explain.

b. Mr. Lee said, "The length of the lawn is $b - c$. The width is a. I know that area = length × width. Therefore, the area of the lawn is $a \times (b - c)$." Is he correct? Explain.

60. The Distributive Property also applies to subtraction.

$$a(b - c) = ab - ac$$

Draw a rectangle to represent $7(10 - 1)$. Use the Distributive Property to write the expression in expanded form. Show that the two expressions are equivalent.

For Exercises 61–65, decide on the operation(s) needed to solve the problem. Then write a mathematical sentence, solve the problem, and explain your reasoning.

61. A pack of baseball cards has 12 trading cards and 2 stickers. In a box of 36 packs, how many stickers and how many trading cards are there?

62. Swanson's Theatre sells a combo pack of popcorn, a soft drink, and a movie ticket for $12. For groups of 5 or greater, the theater gives a group discount of $3 a person. What is the total cost for a class of 30 students?

63. Monday's high temperature was 5 degrees warmer than Sunday's high temperature. Tuesday's high temperature was 8 degrees colder than Monday's high temperature. How does Sunday's high temperature compare to Tuesday's?

64. Elijah is selling coupon books for a school fundraiser. The coupon books sell for $11. $8 goes to the school and $3 goes to Elijah's homeroom. If Elijah sold 24 coupon books, how much money did he collect? How much went to his homeroom? How much went to the school?

65. Samantha charges $15 to mow a lawn. Each week she mows 1 lawn on Tuesday, 3 on Wednesday, and 2 on Thursday. How much money does she earn in 4 weeks?

Connections

66. Multiple Choice What is my number?

Clue 1: My number has two digits, and both are even.

Clue 2: The sum of my number's digits is 10.

Clue 3: The difference of the two digits of my number is 6.

Clue 4: My number has 4 as a factor.

A. 28 **B.** 46 **C.** 64 **D.** 82

For Exercises 67–74, find the sum, difference, product, or quotient.

67. 50×70

68. 25×70

69. $2{,}200 \div 22$

70. 50×120

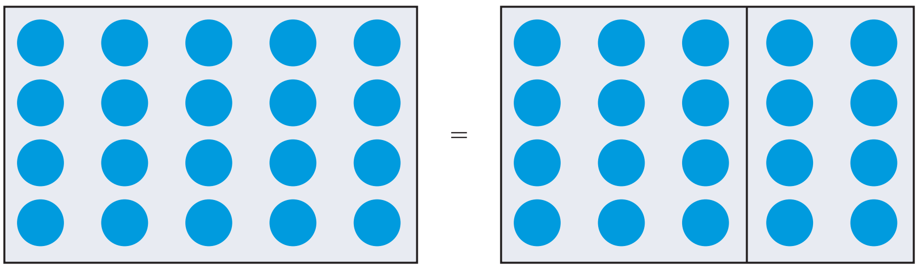
71. $39 + 899$

72. $4{,}400 - 1{,}200$

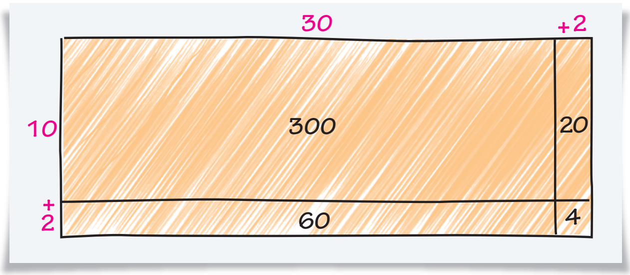
73. $9{,}900 \div 99$

74. $580 + 320$

75. Dot patterns can illustrate the Distributive Property. Write a number sentence suggested by the dot patterns below.



76. To multiply 32 × 12, Jim used the area of a rectangle.



a. How does the diagram relate to finding the product?

$$\begin{array}{r} 32 \\ \underline{\times 12} \\ 64 \\ \underline{+320} \\ 384 \end{array}$$

b. Basilio computed 32 × 8 as shown.

$$32 \times 8 = 16 + 240$$
$$= 256$$

Draw a rectangle to model his thinking.

c. Use Jim's area-of-a-rectangle method and Basilio's method to find the product 45 × 36.

For Exercises 77–79, find the whole number values that *n* can have for each statement to be true.

77. $3(n + 2)$ is a multiple of 5.

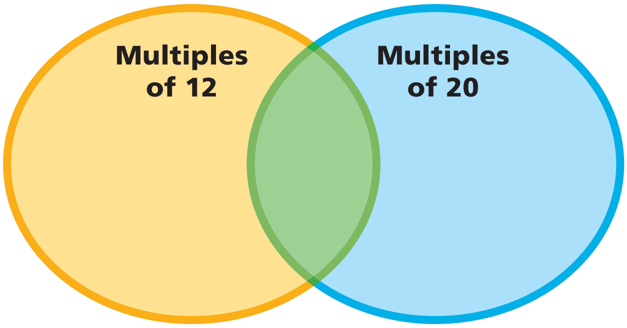
78. $3(n + 2)$ is a factor of 24.

79. $4n + 6$ is a factor of 20.

Extensions

80. **a.** Find at least five numbers that belong in each region of the Venn diagram below.

b. What do the numbers in the intersection have in common?



Consecutive numbers are whole numbers in sequence, such as 31, 32, 33, or 52, 53, 54. For Exercises 81–84, think of different consecutive numbers.

81. For any three consecutive numbers, what can you say about odd numbers and even numbers? Explain.

82. **a.** Mirari conjectures that, in any three consecutive numbers, one number would be divisible by 3. Do you think Mirari is correct? Explain.

b. Gia claims that the sum of any three consecutive whole numbers is divisible by 6. Is this true? Explain.

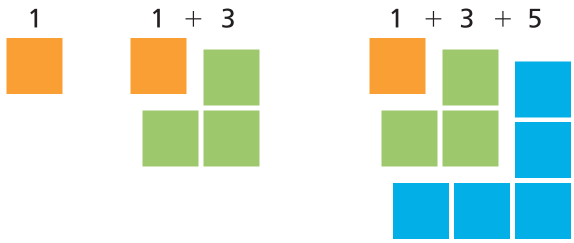
c. Kim claims that the product of any three consecutive whole numbers is divisible by 6. Is this true? Explain.

d. Does the product of any four consecutive whole numbers have any interesting properties? Explain.

83. How many consecutive numbers do you need to guarantee that one of the numbers is divisible by 5?

84. How many consecutive numbers do you need to guarantee that one of the numbers is divisible by 6?

85. Examine the number pattern below.



Stage 1: 1 = 1

Stage 2: 1 + 3 = 4

Stage 3: 1 + 3 + 5 = 9

Stage 4: 1 + 3 + 5 + 7 = 16

a. Find the next four stages and their sums.

b. What is the sum in Stage 20?

c. In what stage will the sum be 576? What is the greatest number added in the sum of this pattern? Explain.

86. Goldbach's Conjecture is a famous conjecture that has never been proven true or false. The conjecture states that every even number, except 2, can be written as the sum of two prime numbers. For example, 16 can be written as 5 + 11.

a. Write the first six even numbers greater than 2 as the sum of two prime numbers.

b. Write 100 as the sum of two primes.

c. The number 2 is a prime number. Can an even number greater than 4 be written as the sum of two prime numbers if you use 2 as one of the primes? Explain why or why not.

87. The chart below shows the factor counts for the numbers from 975 to 1,000. Each star stands for one factor. For example, the four stars after 989 indicate that 989 has four factors.

975	☆☆☆☆☆☆☆☆☆☆☆☆
976	☆☆☆☆☆☆☆☆☆☆
977	☆☆
978	☆☆☆☆☆☆☆☆
979	☆☆☆☆
980	☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆
981	☆☆☆☆☆☆
982	☆☆☆☆
983	☆☆
984	☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆
985	☆☆☆☆
986	☆☆☆☆☆☆☆☆
987	☆☆☆☆☆☆☆☆
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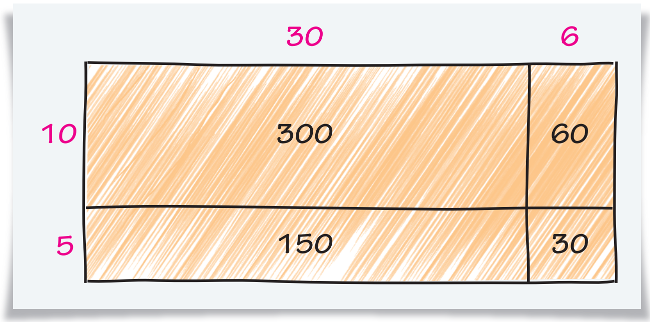
997	☆☆
998	☆☆☆☆
999	☆☆☆☆☆☆☆☆
1000	☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆

a. Boris thinks that numbers that have many factors, such as 975 and 996, must be abundant numbers. (Recall that an *abundant number* is a number whose proper factors have a sum greater than the number.) Is Boris correct? Explain.

b. Doris thinks that there is at least one square number on the list. Is Doris correct? Explain.

88. Evan found a way to find the product of 36 × 15. He drew this diagram and wrote these computations.



$36 \times 15 = (30 + 6)\,(10 + 5) = 300 + 150 + 60 + 30 = 540$

a. Does Evan's method work for finding 36 × 15? Explain.

b. Use Evan's method to find $(2 + n)(3 + 5)$.

c. Use Evan's method to find $(n + 2)(a + 3)$.

d. Use Evan's method to find $(a + b)(c + d)$.

89. Use the Distributive Property to prove each statement. *Hint:* You can write an even number as $2n$ and an odd number as $2n + 1$, where n represents any whole number.

a. The sum of two even numbers is even.

b. The sum of two odd numbers is even.

c. The sum of an odd number and an even number is odd.

90. Use a rectangular model to show the equations below are true.

a. $3(3 + 1 + 7) = 3(3) + 3(1) + 3(7)$

b. $a(b + c + d) = a(b) + a(c) + a(d)$ for any four whole numbers a, b, c, and d.

91. Use the numbers, 1, 2, 3, and 4 exactly once each in any order. Insert operation signs and parentheses to make as many different numbers as you can.

Mathematical Reflections 4

In this Investigation you have studied an important relationship between multiplication and addition, called the Distributive Property. The following questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary in your notebook.

1. a. **Explain** what the Distributive Property means for multiplication, addition, and subtraction. Use the area of a rectangle to illustrate your answer.

   b. **Explain** how you can use the Distributive Property to write a number as two equivalent expressions. Give two examples.

2. a. **What** rules for ordering computations with numbers does the Order of Operations convention provide? Why is it important?

   b. **How** do you decide what operation, addition, subtraction, multiplication, or division, is needed to solve a problem?

WHAT'S NEXT

Unit Project

Don't forget your special number! Can you use the Distributive Property to express it in another form? What about the Order of Operations?

Mathematical Practices

As you worked on each Problem in this Investigation, you used prior knowledge to make sense of the Problem and applied mathematical practices used by mathematicians. Think back over your work, the ways you thought about the Problems, and how you used mathematical practices in solving the Problems.

"The Order of Operations convention helps me to make sense of an arithmetic statement. It also helped me make exact calculations when needed in Problem 4.3."

MP2. Reason abstractly and quantitatively

Describe one other instance of the Mathematical Practices that you and your classmates used in this Investigation to solve a Problem.

Notes

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