C4: QUESTIONS FROM PAST PAPERS – INTEGRATION

1. Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} \mathrm{e}^{\cos x + 1} \sin x \, \mathrm{d}x \; \mathrm{e}(\mathrm{e} - 1)$$

(Total 6 marks)

2.
$$\mathrm{f}(\theta) = 4\cos^2\theta - 3\sin^2\theta$$

(a) Show that $\mathrm{f}(\theta) = \frac{1}{2} + \frac{7}{2}\cos 2\theta$.

(3)

(b) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta \, \mathrm{f}(\theta) \, \mathrm{d}\theta$

(7)
(Total 10 marks)

3. (a) Find $\int \frac{9x + 6}{x} \, \mathrm{d}x, \; x > 0$.

(2)

(b) Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(9x + 6)y^{\frac{1}{3}}}{x}$$

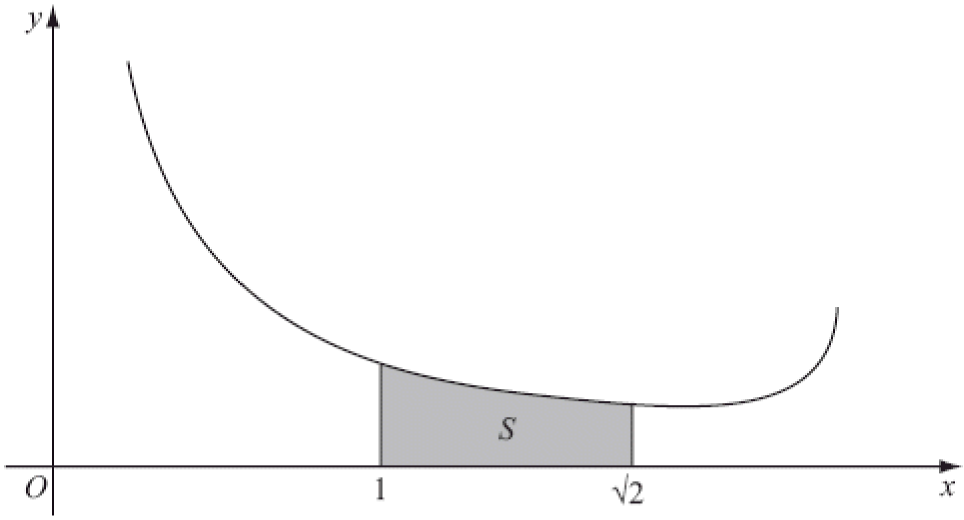
giving your answer in the form $y^2 = \mathrm{g}(x)$.

(6)
(Total 8 marks)

4. (a) Using the substitution $x = 2\cos u$, or otherwise, find the exact value of

$$\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{(4 - x^2)}} \, \mathrm{d}x$$

(7)



The diagram above shows a sketch of part of the curve with equation $y = \dfrac{4}{x(4-x^2)^{\frac{1}{4}}}$,

$0 < x < 2$.

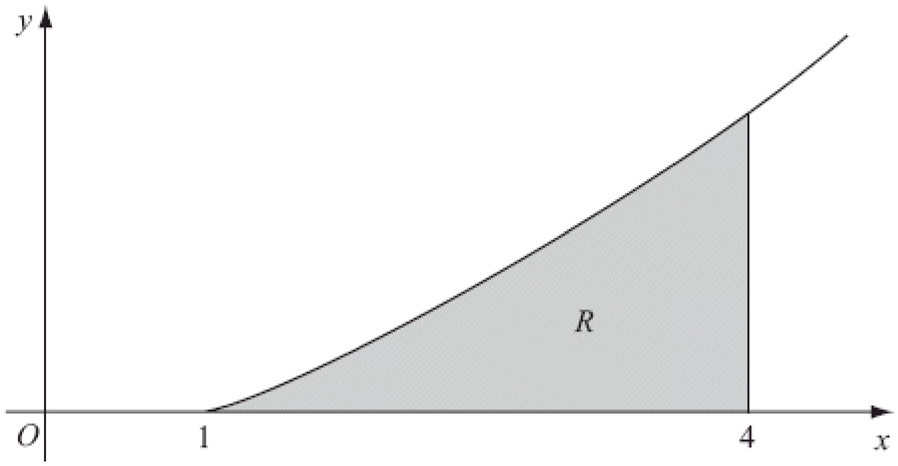
The shaded region S, shown in the diagram above, is bounded by the curve, the x-axis and the lines with equations $x = 1$ and $x = \sqrt{2}$. The shaded region S is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Using your answer to part (a), find the exact volume of the solid of revolution formed.

(3)

(Total 10 marks)

5.



The diagram above shows a sketch of the curve with equation $y = x \ln x$, $x \geq 1$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

(a) Complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places.

(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(4)

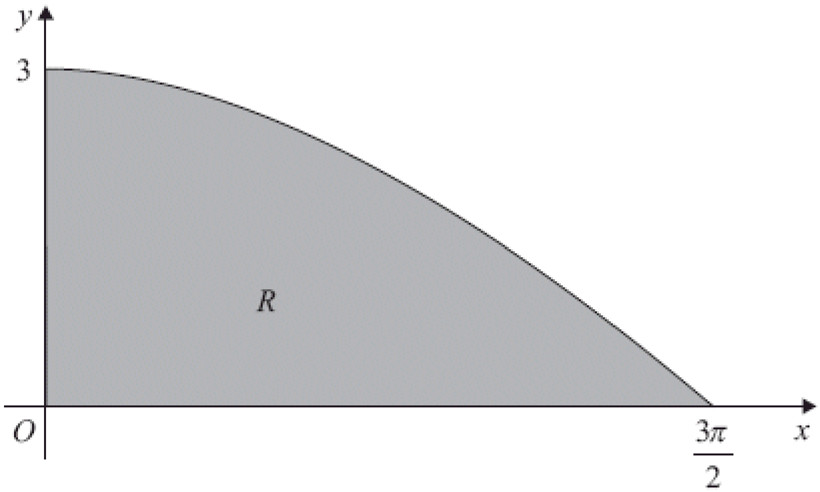
(c) (i) Use integration by parts to find $\int x \ln x \, dx$.

(ii) Hence find the exact area of R, giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where a and b are integers.

(7)

(Total 13 marks)

6.



The diagram above shows the finite region R bounded by the x-axis, the y-axis and the curve with equation $y = 3 \cos\left(\frac{x}{3}\right), 0 \le x \le \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3 \cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

(a) Complete the table above giving the missing value of y to 5 decimal places.

(1)

(b) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R, giving your answer to 3 decimal places.

(4)

(c) Use integration to find the exact area of R.

(3)
(Total 8 marks)

7.

$$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$$

(a) Find the values of the constants A, B and C.

(4)

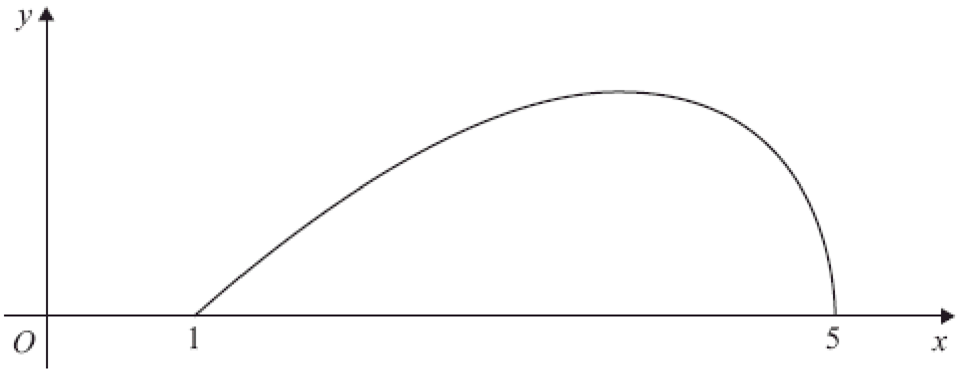
(b) (i) Hence find $\int f(x)\,dx$.

(3)

(ii) Find $\int_0^2 f(x)\,dx$ in the form 1n k, where k is a constant.

(3)
(Total 10 marks)

8. (a) Find $\int \sqrt{(5-x)}\,dx$



(2)

The diagram above shows a sketch of the curve with equation

$$y = (x - 1)\sqrt{(5 - x)}, \qquad 1 \le x \le 5$$

(b) (i) Using integration by parts, or otherwise, find

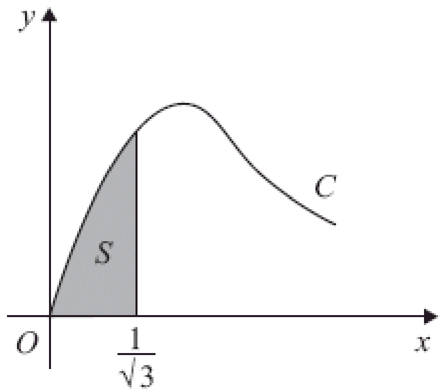
$$\int (x-1)\sqrt{(5-x)}\,dx$$

(4)

(ii) Hence find $\int_1^5 (x-1)\sqrt{(5-x)}\,dx$.

(2)
(Total 8 marks)

9. (a) Using the identity $\cos 2\theta = 1 - 2\sin^2\theta$, find $\int \sin^2\theta\,d\theta$.



(2)

The diagram above shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = 2\sin 2\theta, \qquad 0 \leq \theta < \frac{\pi}{2}$$

The finite shaded region S shown in the diagram is bounded by C, the line $x = \dfrac{1}{\sqrt{3}}$ and the x-axis.

This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Show that the volume of the solid of revolution formed is given by the integral

$$k\int_0^{\frac{\pi}{6}} \sin^2\theta \, \mathrm{d}\theta$$

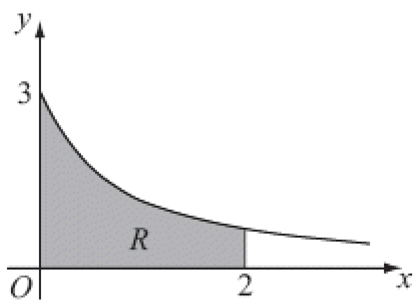
where k is a constant.

(5)

(c) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where p and q are constants.

(3)
(Total 10 marks)

10.



The diagram above shows part of the curve $y = \dfrac{3}{\sqrt{(1+4x)}}$. The region R is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 2$, as shown shaded in the diagram above.

(a) Use integration to find the area of R.

(4)

The region R is rotated 360° about the x-axis.

(b) Use integration to find the exact value of the volume of the solid formed.

(5)
(Total 9 marks)

11. (a) Find $\int \tan^2 x \, dx$.

(2)

(b) Use integration by parts to find $\int \frac{1}{x^3} \ln x \, dx$.

(4)

(c) Use the substitution $u = 1 + e^x$ to show that

$$\int \frac{e^{3x}}{1+e^x} dx = \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k,$$

where k is a constant.

(7)
(Total 13 marks)

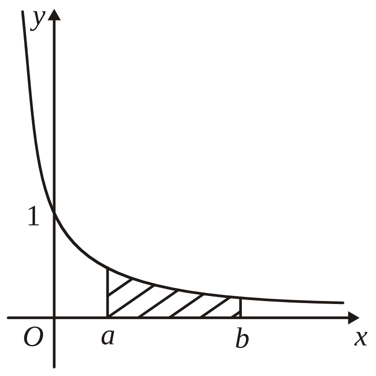
12. (a) Use integration by parts to find $\int xe^x \, dx$.

(3)

(b) Hence find $\int x^2 e^x \, dx$.

(3)
(Total 6 marks)

13.



The curve shown in the diagram above has equation $y = \frac{1}{(2x+1)}$. The finite region bounded by the curve, the x-axis and the lines $x = a$ and $x = b$ is shown shaded in the diagram. This region is rotated through 360° about the x-axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of a and b.

(Total 5 marks)

14. (i) Find $\int 1\text{n}\left(\frac{x}{2}\right)\mathrm{d}x$.

(4)

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 \, \mathrm{d}x$.

(5)
(Total 9 marks)

15. Use the substitution $u = 2^x$ to find the exact value of

$$\int_0^1 \frac{2^x}{\left(2^x+1\right)^2}\,\mathrm{d}x.$$

(Total 6 marks)

16. (a) Find $\int x\cos 2x\,\mathrm{d}x.$.

(4)

(b) Hence, using the identity $\cos 2x = 2\cos^2 x - 1$, deduce $\int x\cos^2 x\,\mathrm{d}x$.

(3)
(Total 7 marks)

17.
$$\frac{2\left(4x^2+1\right)}{(2x+1)(2x-1)} \equiv A + \frac{B}{(2x+1)} + \frac{C}{(2x-1)}$$

(a) Find the values of the constants A, B and C.

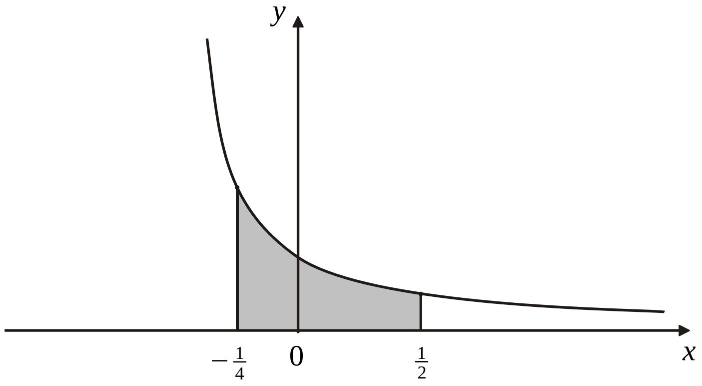
(4)

(b) Hence show that the exact value of $\int_1^2 \frac{2\left(4x^2+1\right)}{(2x+1)(2x-1)}\,\mathrm{d}x$ is 2 + ln k, giving the value of the constant k.

(6)
(Total 10 marks)

18.

Figure 1



The curve with equation $y = \dfrac{1}{3(1+2x)}$, $x > -\dfrac{1}{2}$, is shown in Figure 1.

The region bounded by the lines $x = -\dfrac{1}{4}$, $x = \dfrac{1}{2}$, the x-axis and the curve is shown shaded in Figure 1.

This region is rotated through 360 degrees about the x-axis.

(a) Use calculus to find the exact value of the volume of the solid generated.

(5)

Figure 2

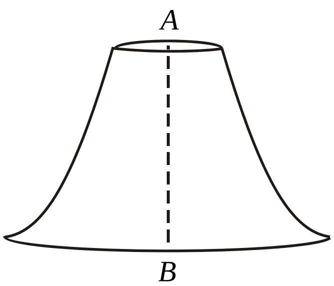


Figure 2 shows a paperweight with axis of symmetry AB where AB = 3 cm. A is a point on the top surface of the paperweight, and B is a point on the base of the paperweight.
The paperweight is geometrically similar to the solid in part (a).

(b) Find the volume of this paperweight.

(2)
(Total 7 marks)

19.

$$I = \int_0^5 e^{\sqrt{(3x+1)}}\,dx.$$

(a) Given that $y = e^{\sqrt{(3x+1)}}$, complete the table with the values of y corresponding to x = 2, 3 and 4.

x	0	1	2	3	4	5
y	e^1	e^2				e^4

(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the original integral I, giving your answer to 4 significant figures.

(3)

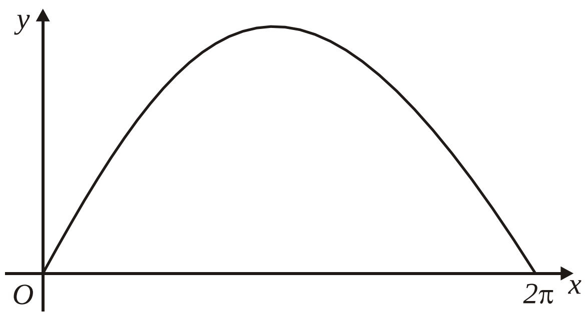
(c) Use the substitution $t = \sqrt{(3x + 1)}$ to show that I may be expressed as $\int_a^b kte^t \, dt$, giving the values of a, b and k.

(5)

(d) Use integration by parts to evaluate this integral, and hence find the value of I correct to 4 significant figures, showing all the steps in your working.

(5)
(Total 15 marks)

20.



The curve with equation, $y = 3\sin\frac{x}{2}, 0 \le x \le 2\pi$, is shown in the figure above. The finite region enclosed by the curve and the x-axis is shaded.

(a) Find, by integration, the area of the shaded region.

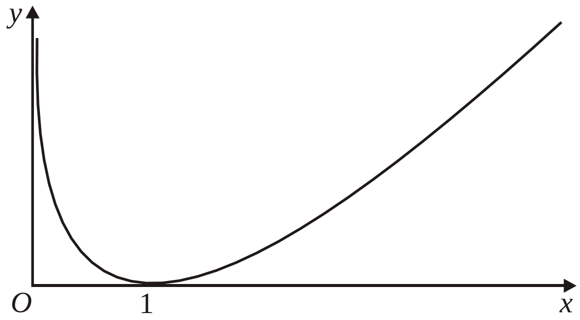
(3)

This region is rotated through 2π radians about the x-axis.

(b) Find the volume of the solid generated.

(6)
(Total 9 marks)

21.



The figure above shows a sketch of the curve with equation $y = (x - 1) \ln x, \quad x > 0$.

(a) Complete the table with the values of y corresponding to $x = 1.5$ and $x = 2.5$.

x	1	1.5	2	2.5	3
y	0		ln 2		2ln 3

(1)

Given that $I = \int_1^3 (x-1) \ln x \, dx$

(b) use the trapezium rule

(i) with values of y at $x = 1$, 2 and 3 to find an approximate value for I to 4 significant figures,

(ii) with values of y at $x = 1$, 1.5, 2, 2.5 and 3 to find another approximate value for I to 4 significant figures.

(5)

(c) Explain, with reference to the figure above, why an increase in the number of values improves the accuracy of the approximation.

(1)

(d) Show, by integration, that the exact value of $\int_1^3 (x-1) \ln x \, dx$ is $\frac{3}{2} \ln 3$.

(6)
(Total 13 marks)

22. $$f(x) = (x^2 + 1) \ln x, \qquad x > 0.$$

(a) Use differentiation to find the value of $f'(x)$ at $x = e$, leaving your answer in terms of e.

(4)

(b) Find the exact value of $\int_1^e f(x) \, dx$

(5)
(Total 9 marks)

23.

$$f(x) = \frac{9+4x^2}{9-4x^2}, \qquad x \neq \pm \tfrac{3}{2}.$$

(a) Find the values of the constants A, B and C such that

$$f(x) = A + \frac{B}{3+2x} + \frac{C}{3-2x}, \qquad x \neq \pm \tfrac{3}{2}.$$

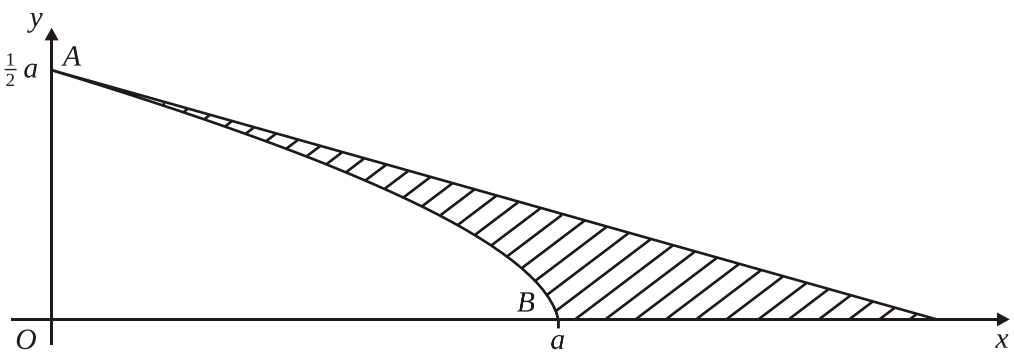
(4)

(b) Hence find the exact value of

$$\int_{-1}^{1} \frac{9+4x^2}{9-4x^2}\,dx$$

(5)
(Total 9 marks)

24.



The curve shown in the figure above has parametric equations

$$x = a\cos 3t,\ y = a\sin t, \quad 0 \leq t \leq \frac{\pi}{6}.$$

The curve meets the axes at points A and B as shown.

The straight line shown is part of the tangent to the curve at the point A.

Find, in terms of a,

(a) an equation of the tangent at A,

(6)

(b) an exact value for the area of the finite region between the curve, the tangent at A and the x-axis, shown shaded in the figure above.

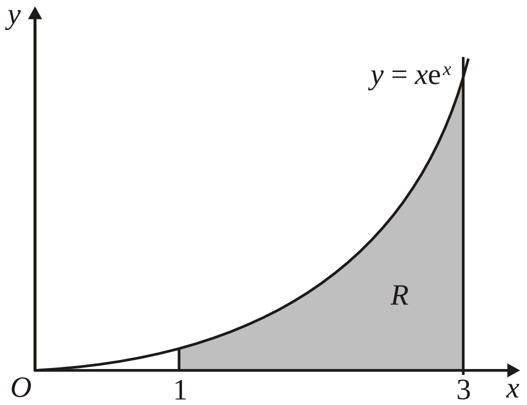
(9)
(Total 15 marks)

25. Using the substitution $u^2 = 2x - 1$, or otherwise, find the exact value of

$$\int_1^5 \frac{3x}{\sqrt{(2x-1)}}\,dx$$

(Total 8 marks)

26.



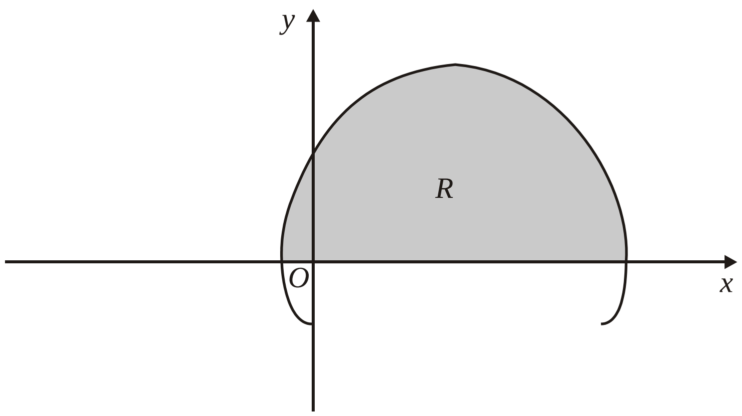
The figure above shows the finite shaded region, R, which is bounded by the curve $y = xe^x$, the line $x = 1$, the line $x = 3$ and the x-axis.

The region R is rotated through 360 degrees about the x-axis.

Use integration by parts to find an exact value for the **volume** of the solid generated.

(Total 8 marks)

27.



The curve shown in the figure above has parametric equations

$$x = t - 2\sin t, \qquad y = 1 - 2\cos t, \qquad 0 \le t \le 2\pi$$

(a) Show that the curve crosses the x-axis where $t = \dfrac{\pi}{3}$ and $t = \dfrac{5\pi}{3}$.

(2)

The finite region R is enclosed by the curve and the x-axis, as shown shaded in the figure above.

(b) Show that the area of R is given by the integral

$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)^2 \, dt.$$

(3)

(c) Use this integral to find the exact value of the shaded area.

(7)
(Total 12 marks)

28.

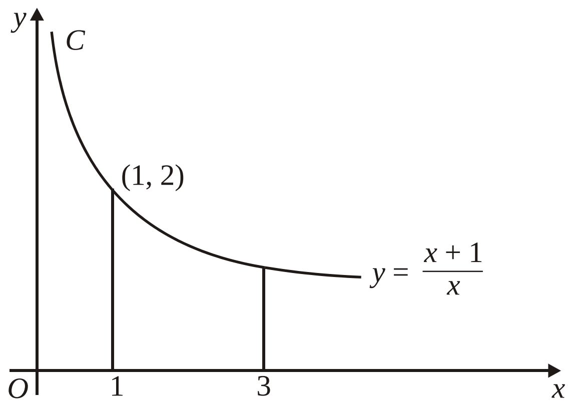


Figure 1

Figure 1 shows part of the curve C with equation $y = \dfrac{x+1}{x}$, $x > 0$.

The finite region enclosed by C, the lines $x = 1$, $x = 3$ and the x-axis is rotated through 360° about the x-axis to generate a solid S.

(a) Using integration, find the exact volume of S.

(7)

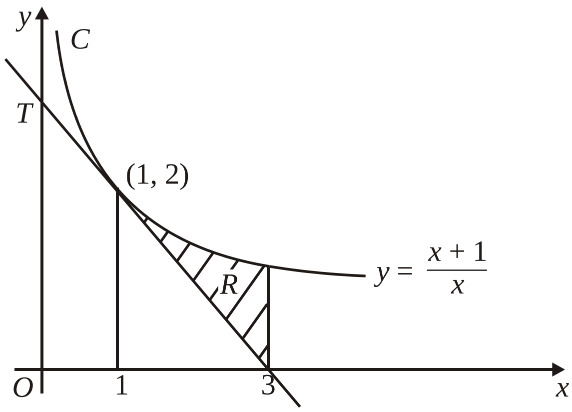


Figure 2

The tangent T to C at the point (1, 2) meets the x-axis at the point (3, 0). The shaded region R is bounded by C, the line $x = 3$ and T, as shown in Figure 2.

(b) Using your answer to part (a), find the exact volume generated by R when it is rotated through 360° about the x-axis.

(3)
(Total 10 marks)

29. (a) Use integration by parts to find

$$\int x \cos 2x \, dx .$$

(4)

(b) Hence, or otherwise, find

$$\int x \cos^2 x \, dx .$$

(3)
(Total 7 marks)

30. (a) Express $\dfrac{5x+3}{(2x-3)(x+2)}$ in partial fractions.

(3)

(b) Hence find the exact value of $\displaystyle\int_2^6 \frac{5x+3}{(2x-3)(x+2)} \, dx$, giving your answer as a single logarithm.

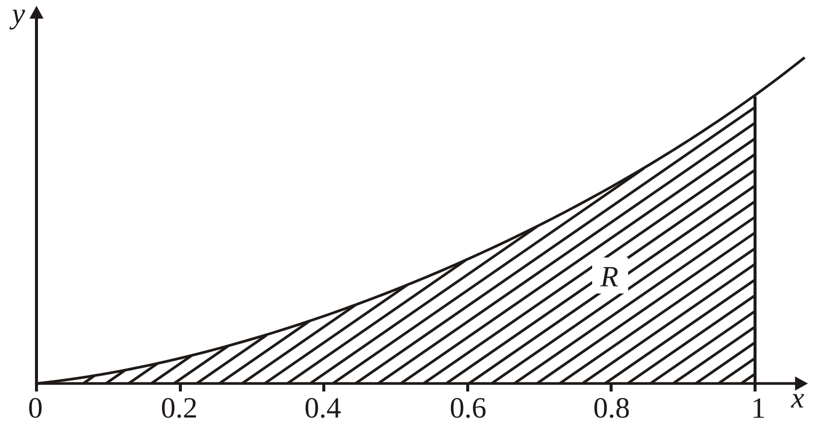
(5)
(Total 8 marks)

31. Use the substitution $x = \sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}} \, dx .$$

(Total 7 marks)

32.



The diagram shows the graph of the curve with equation

$$y = xe^{2x}, \qquad x \geq 0.$$

The finite region R bounded by the lines $x = 1$, the x-axis and the curve is shown shaded in the diagram.

(a) Use integration to find the exact value of the area for R.

(5)

(b) Complete the table with the values of y corresponding to $x = 0.4$ and 0.8.

x	0	0.2	0.4	0.6	0.8	1
$y = xe^{2x}$	0	0.29836		1.99207		7.38906

(1)

(c) Use the trapezium rule with all the values in the table to find an approximate value for this area, giving your answer to 4 significant figures.

(4)

33. (a) Use the formulae for $\sin(A \pm B)$, with $A = 3x$ and $B = x$, to show that $2 \sin x \cos 3x$ can be written as $\sin px - \sin qx$, where p and q are positive integers.

(3)

(b) Hence, or otherwise, find $\int 2 \sin x \cos 3x \, dx$.

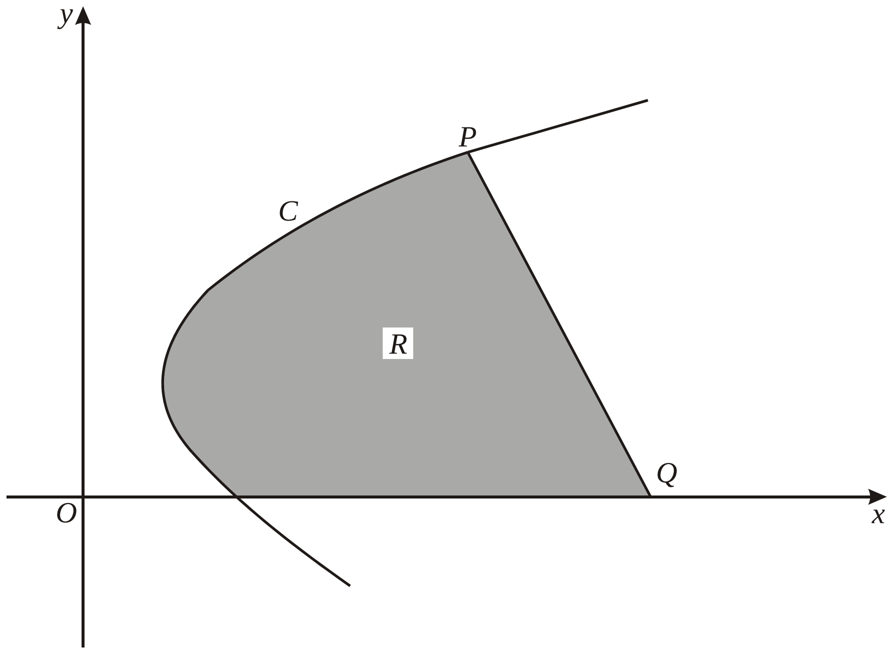
(2)

(c) Hence find the exact value of $\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} 2 \sin x \cos 3x \, dx$

(2)

(Total 7 marks)

34.



The diagram shows a sketch of part of the curve C with parametric equations

$$x = t^2 + 1, \qquad y = 3(1 + t).$$

The normal to C at the point $P(5, 9)$ cuts the x-axis at the point Q, as shown in the diagram.

(a) Find the x-coordinate of Q.

(6)

(b) Find the area of the finite region R bounded by C, the line PQ and the x-axis.

(9)
(Total 15 marks)

35. (a) Use integration by parts to show that

$$\int x \operatorname{cosec}^2\left(x + \frac{\pi}{6}\right) \mathrm{d}x = -x \cot\left(x + \frac{\pi}{6}\right) + \ln\left[\sin\left(x + \frac{\pi}{6}\right)\right] + c, \qquad -\frac{\pi}{6} < x < \frac{\pi}{3}.$$

(3)

(b) Solve the differential equation

$$\sin^2\left(x + \frac{\pi}{6}\right)\frac{\mathrm{d}y}{\mathrm{d}x} = 2xy(y + 1)$$

to show that $\frac{1}{2}\ln\left|\frac{y}{y+1}\right| = -x\cot\left(x + \frac{\pi}{6}\right) + \ln\left[\sin\left(x + \frac{\pi}{6}\right)\right] + c.$

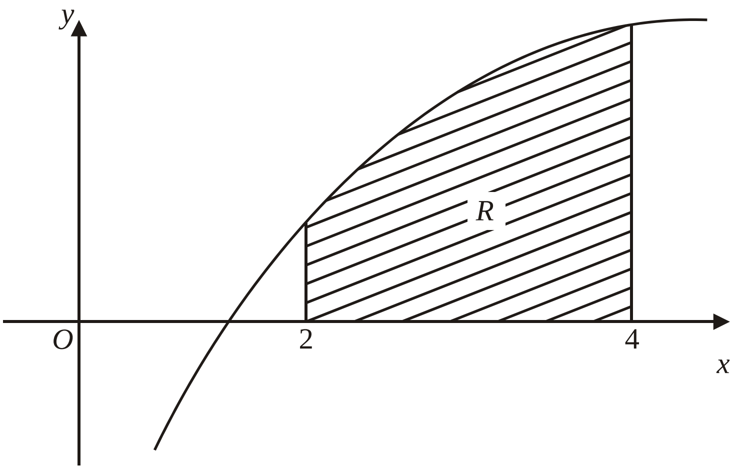
(6)

Given that $y = 1$ when $x = 0$,

(c) find the exact value of y when $x = \dfrac{\pi}{12}$.

(6)
(Total 15 marks)

36.



The diagram above shows part of the curve with equation

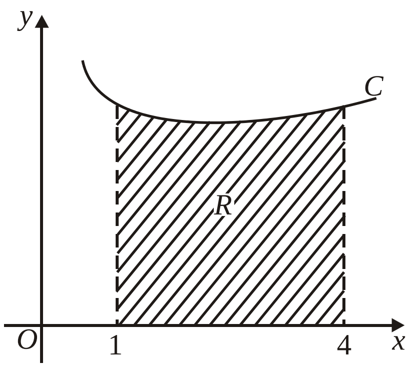
$$y = 4x - \frac{6}{x}, \quad x > 0.$$

The shaded region R is bounded by the curve, the x-axis and the lines with equations $x = 2$ and $x = 4$. This region is rotated through 2π radians about the x-axis.

Find the exact value of the volume of the solid generated.

(Total 8 marks)

37.



The diagram above shows parts of the curve C with equation

$$y = \frac{x+2}{\sqrt{x}}.$$

The shaded region R is bounded by C, the x-axis and the lines $x = 1$ and $x = 4$.

This region is rotated through 360° about the x-axis to form a solid S.

(a) Find, by integration, the exact volume of S.

(7)

The solid S is used to model a wooden support with a circular base and a circular top.

(b) Show that the base and the top have the same radius.

(1)

Given that the actual radius of the base is 6 cm,

(c) show that the volume of the wooden support is approximately 630 cm^3.

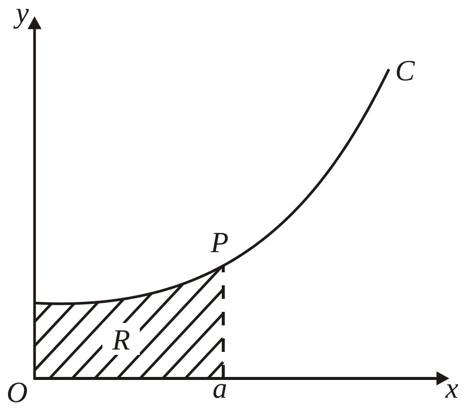
(2)
(Total 10 marks)

38. Use the substitution $u = 1 + \sin x$ and integration to show that

$$\int \sin x \cos x\,(1 + \sin x)^5\,dx = \frac{1}{42}(1 + \sin x)^6\,[6 \sin x - 1] + \text{constant}.$$

(Total 8 marks)

39.



The diagram above shows a sketch of the curve C with parametric equations

$$x = 3t \sin t,\ y = 2 \sec t, \qquad 0 \leq t < \frac{\pi}{2}.$$

The point $P(a, 4)$ lies on C.

(a) Find the exact value of a.

(3)

The region R is enclosed by C, the axes and the line $x = a$ as shown in the diagram above.

(b) Show that the area of R is given by

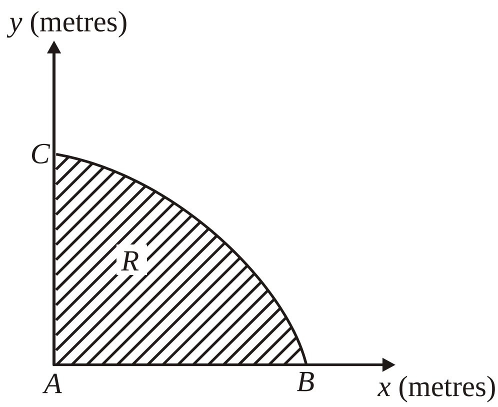
$$6\int_0^{\frac{\pi}{3}} (\tan t + t)\,\mathrm{d}t.$$

(4)

(c) Find the exact value of the area of R.

(4)
(Total 11 marks)

40.



The diagram above shows a cross-section R of a dam. The line AC is the vertical face of the dam, AB is the horizontal base and the curve BC is the profile. Taking x and y to be the horizontal and vertical axes, then A, B and C have coordinates $(0, 0)$, $(3\pi^2, 0)$ and $(0, 30)$ respectively. The area of the cross-section is to be calculated.

Initially the profile BC is approximated by a straight line.

(a) Find an estimate for the area of the cross-section R using this approximation.

(1)

The profile BC is actually described by the parametric equations.

$$x = 16t^2 - \pi^2, \quad y = 30\sin 2t, \quad \frac{\pi}{4} \leq t \leq \frac{\pi}{2}.$$

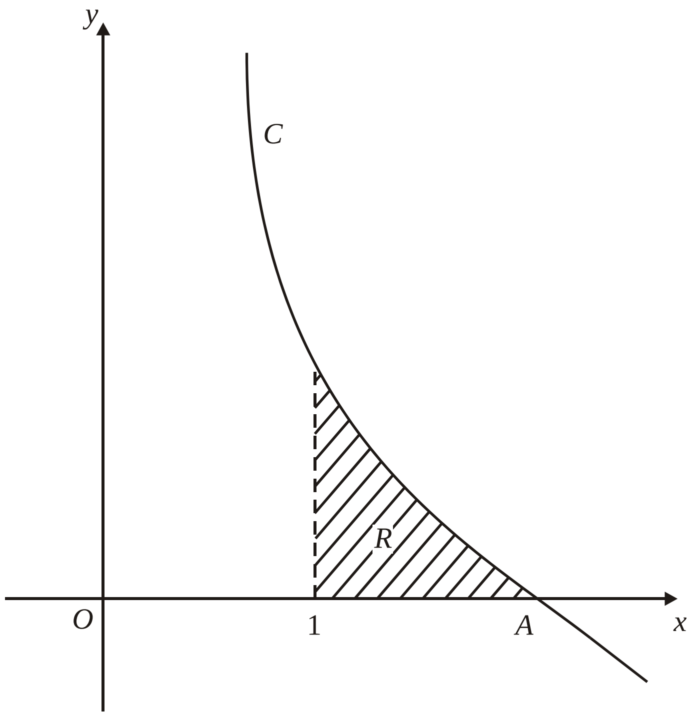
(b) Find the exact area of the cross-section R.

(7)

(c) Calculate the percentage error in the estimate of the area of the cross-section R that you found in part (a).

(2)
(Total 10 marks)

41.



The diagram above shows the curve C with equation $y = f(x)$, where

$$f(x) = \frac{8}{x} - x^2, \quad x > 0.$$

Given that C crosses the x-axis at the point A,

(a) find the coordinates of A.

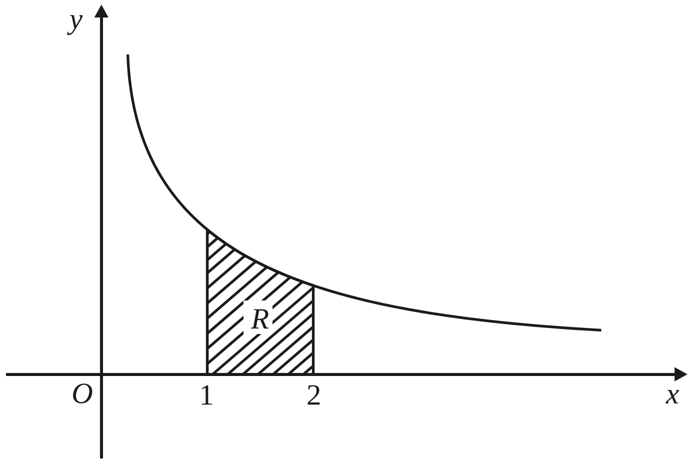
(3)

The finite region R, bounded by C, the x-axis and the line $x = 1$, is rotated through 2π radians about the x-axis.

(b) Use integration to find, in terms of π the volume of the solid generated.

(7)
(Total 10 marks)

42.



The diagram above shows part of the curve with equation $y = 1 + \frac{c}{x}$, where c is a positive constant.

The point P with x-coordinate p lies on the curve. Given that the gradient of the curve at P is -4,

(a) show that $c = 4p^2$.

(2)

Given also that the y-coordinate of P is 5,

(b) prove that $c = 4$.

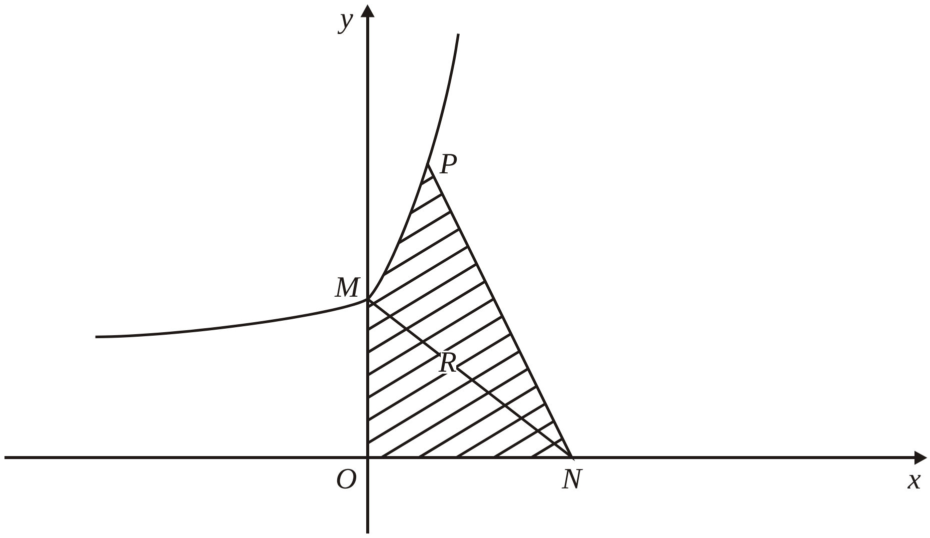
(2)

The region R is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$, as shown in the diagram above. The region R is rotated through 360° about the x-axis.

(c) Show that the volume of the solid generated can be written in the form $\pi(k + q \ln 2)$, where k and q are constants to be found.

(7)
(Total 11 marks)

43.



The curve C with equation $y = 2e^x + 5$ meets the y-axis at the point M, as shown in the diagram above.

(a) Find the equation of the normal to C at M in the form $ax + by = c$, where a, b and c are integers.

(4)

This normal to C at M crosses the x-axis at the point $N(n, 0)$.

(b) Show that $n = 14$.

(1)

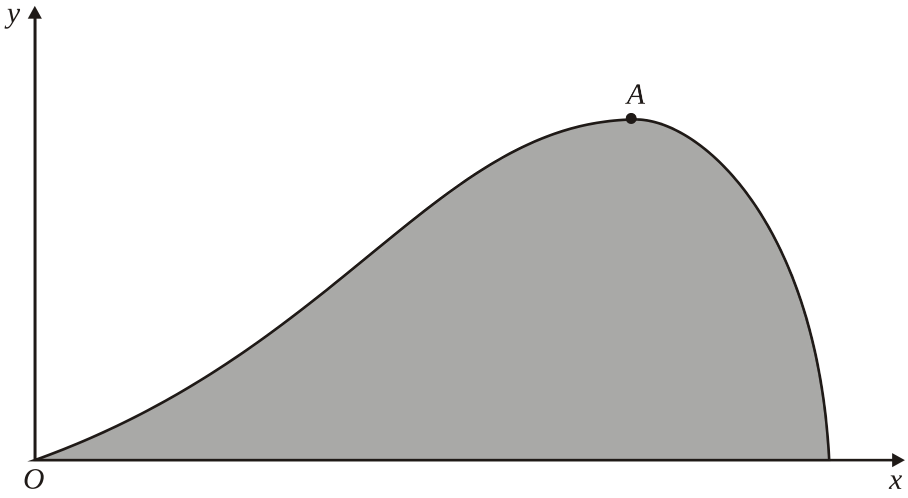
The point $P(\ln 4, 13)$ lies on C. The finite region R is bounded by C, the axes and the line PN, as shown in the diagram above.

(c) Find the area of R, giving your answer in the form $p + q \ln 2$, where p and q are integers to be found.

(7)

(Total 12 marks)

44.



The diagram above shows a graph of $y = x\sqrt{\sin x}$, $0 < x < \pi$. The maximum point on the curve is A.

(a) Show that the x-coordinate of the point A satisfies the equation $2 \tan x + x = 0$.

(4)

The finite region enclosed by the curve and the x-axis is shaded as shown in the diagram above.

A solid body S is generated by rotating this region through 2π radians about the x-axis.

(b) Find the exact value of the volume of S.

(7)
(Total 11 marks)

45. $$\mathrm{f}(x) = \frac{25}{(3+2x)^2(1-x)}, \quad |x| < 1.$$

(a) Express $\mathrm{f}(x)$ as a sum of partial fractions.

(4)

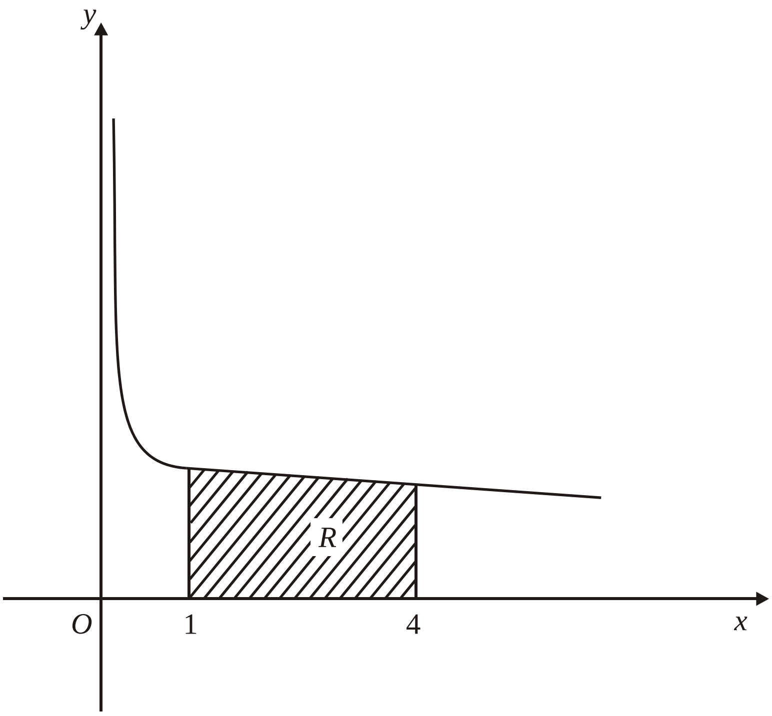
(b) Hence find $\int \mathrm{f}(x)\,\mathrm{d}x$.

(5)

(c) Find the series expansion of $\mathrm{f}(x)$ in ascending powers of x up to and including the term in x^2. Give each coefficient as a simplified fraction.

(7)
(Total 16 marks)

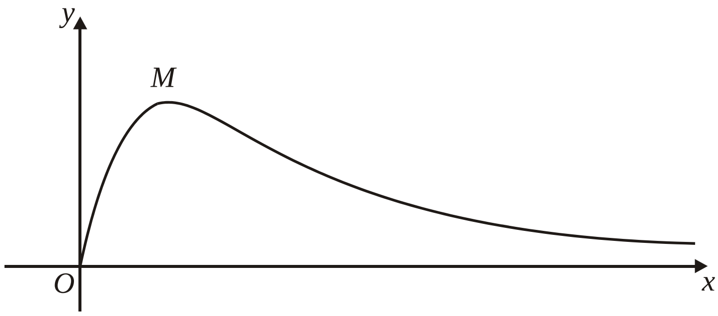
46.



The diagram above shows part of the curve with equation $y = 1 + \dfrac{1}{2\sqrt{x}}$. The shaded region R, bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$, is rotated through 360° about the x-axis. Using integration, show that the volume of the solid generated is $\pi(5 + \frac{1}{2}\ln 2)$.

(Total 8 marks)

47.



The diagram above shows the curve with equation $y = x^{\frac{1}{2}}e^{-2x}$.

(a) Find the x-coordinate of M, the maximum point of the curve.

(5)

The finite region enclosed by the curve, the x-axis and the line $x = 1$ is rotated through 2π about the x-axis.

(b) Find, in terms of π and e, the volume of the solid generated.

(7)
(Total 12 marks)

48. Find the volume generated when the region bounded by the curve with equation $y = 2 + \frac{1}{x}$, the x-axis and the lines $x = \frac{1}{2}$ and $x = 4$ is rotated through 360° about the x-axis.

Give your answer in the form $\pi(a + b \ln 2)$, where a and b are rational constants.

…………………………..……..…………………………………………………………..

(Total 7 marks)

49. Given that $y = 1$ at $x = \pi$, solve the differential equation

$$\frac{dy}{dx} = yx^2 \cos x, \quad y > 0.$$

(Total 9 marks)

50.

$$g(x) = \frac{5x+8}{(1+4x)(2-x)}.$$

(a) Express g(x) in the form $\frac{A}{(1+4x)} + \frac{B}{(2-x)}$, where A and B are constants to be found.

(3)

The finite region R is bounded by the curve with equation $y = g(x)$, the coordinate axes and the line $x = \frac{1}{2}$.

(b) Find the area of R, giving your answer in the form $a \ln 2 + b \ln 3$.

(7)
(Total 10 marks)

51. Use the substitution $u^2 = (x - 1)$ to find

$$\int \frac{x^2}{\sqrt{(x-1)}}\, dx,$$

giving your answer in terms of x.

(Total 10 marks)

52. Use the substitution $u = 4 + 3x^2$ to find the exact value of

$$\int_0^2 \frac{2x}{(4+3x^2)^2}\, dx.$$

(Total 6 marks)

53. $\mathrm{f}(x) = \dfrac{1+14x}{(1-x)(1+2x)}$, $\quad |x| < \dfrac{1}{2}$.

(a) Express f(x) in partial fractions.

(3)

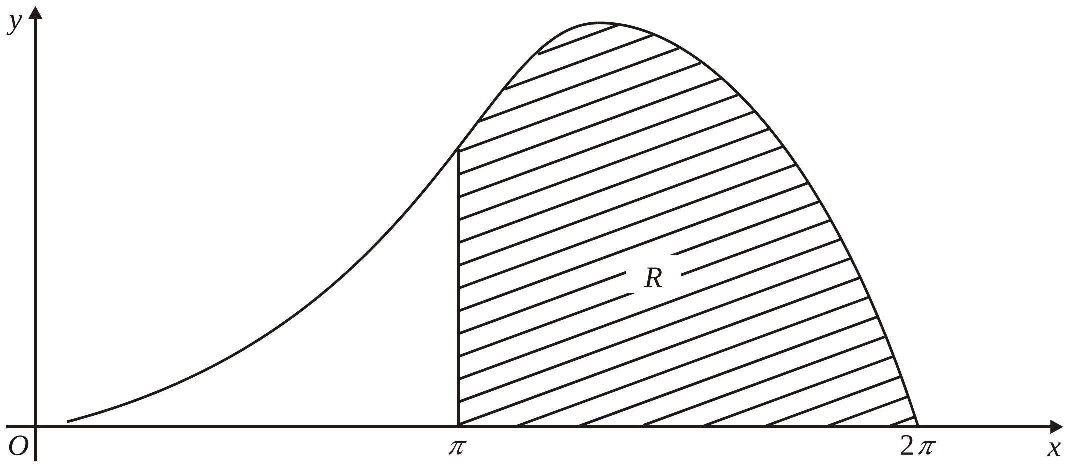
(b) Hence find the exact value of $\displaystyle\int_{\frac{1}{6}}^{\frac{1}{3}} \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in the form ln p, where p is rational.

(5)

(c) Use the binomial theorem to expand f(x) in ascending powers of x, up to and including the term in x^3, simplifying each term.

(5)
(Total 13 marks)

54.



The diagram above shows the curve with equation

$$y = x^2 \sin\left(\tfrac{1}{2}x\right), \qquad 0 < x \leq 2\pi.$$

The finite region R bounded by the line $x = \pi$, the x-axis, and the curve is shown shaded in Fig 1.

(a) Find the exact value of the area of R, by integration. Give your answer in terms of π.

(7)

The table shows corresponding values of x and y.

x	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
y	9.8696	14.247	15.702	G	0

(b) Find the value of G.

(1)

(c) Use the trapezium rule with values of $x^2 \sin\left(\frac{1}{2}x\right)$

(i) at $x = \pi$, $x = \dfrac{3\pi}{2}$ and $x = 2\pi$ to find an approximate value for the area R, giving your answer to 4 significant figures,

(ii) at $x = \pi$, $x = \dfrac{5\pi}{4}$, $x = \dfrac{3\pi}{2}$, $x = \dfrac{7\pi}{4}$ and $x = 2\pi$ to find an improved approximation for the area R, giving your answer to 4 significant figures.

(5)
(Total 13 marks)

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MARK SCHEME

1. $\dfrac{du}{dx} = -\sin x$ B1

$\int \sin x\, e^{\cos x+1} dx = -\int e^u du$ M1 A1

$= -e^u$ ft sign error A1 ft

$= -e^{\cos x+1}$

$\left[-e^{\cos x+1}\right]_0^{\frac{\pi}{2}} = -e^1 - \left(-e^2\right)$ or equivalent with u M1

$= e(e-1)$ ∗ cso A16

[6]

2. (a) $f(\theta) = 4\cos^2\theta - 3\sin^2\theta$

$= 4\left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right) - 3\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)$ M1 M1

$= \frac{1}{2} + \frac{7}{2}\cos 2\theta$ ∗ cso A13

(b) $\int \theta \cos 2\theta\, d\theta = \frac{1}{2}\theta \sin 2\theta - \frac{1}{2}\int \sin 2\theta\, d\theta$ M1 A1

$= \frac{1}{2}\theta \sin 2\theta + \frac{1}{4}\cos 2\theta$ A1

$$\int \theta \operatorname{f}(\theta)\,\mathrm{d}\theta = \frac{1}{4}\theta^2 + \frac{7}{4}\theta\sin 2\theta + \frac{7}{8}\cos 2\theta$$ M1 A1

$$\left[\ \ldots\ \right]_0^{\frac{\pi}{2}} = \left[\frac{\pi^2}{16} + 0 - \frac{7}{8}\right] - \left[0 + 0 + \frac{7}{8}\right]$$ M1

$$= \frac{\pi^2}{16} - \frac{7}{4}$$ A17

[10]

3. (a) $\int \frac{9x+6}{x}\,\mathrm{d}x = \int \left(9 + \frac{6}{x}\right)\mathrm{d}x$ M1

$=9x + 6\ln x\ (+C)$ A12

(b) $\int \frac{1}{y^{\frac{1}{3}}}\,\mathrm{d}y = \int \frac{9x+6}{x}\,\mathrm{d}x$ Integral signs not necessary B1

$$\int y^{-\frac{1}{3}}\,\mathrm{d}y = \int \frac{9x+6}{x}\,\mathrm{d}x$$

$$\frac{y^{\frac{2}{3}}}{\frac{2}{3}} = 9x + 6\ln x(+C)$$ $\pm ky^{\frac{2}{3}} =$ their (a) M1

$$\frac{3}{2}y^{\frac{2}{3}} = 9x + 6\ln x(+C)$$ ft their (a) A1ft

$y = 8$, $x = 1$

$$\frac{3}{2}8^{\frac{2}{3}} = 9 + 6\ln 1 + C$$ M1

$C = -3$ A1

$$y^{\frac{2}{3}} = \frac{2}{3}(9x + 6\ln x - 3)$$

$y2 = (6x + 4\ln x - 2)^3$ $\left(= 8(3x + 2\ln x - 1)^3\right)$ A16

[8]

4. (a) $\frac{\mathrm{d}x}{\mathrm{d}u} = -2\sin u$ B1

$$\int \frac{1}{x^2\sqrt{4-x^2}}\,\mathrm{d}x = \int \frac{1}{(2\cos u)^2\sqrt{4-(2\cos u)^2}} \times -2\sin u\,\mathrm{d}u$$ M1

		$= \int \dfrac{-2\sin u}{4\cos^2 u\sqrt{4\sin^2 u}}\mathrm{d}u$	Use of $1-\cos^2 u = \sin^2 u$	M1
		$= -\dfrac{1}{4}\int \dfrac{1}{\cos^2 u}\mathrm{d}u$	$\pm k\int \dfrac{1}{\cos^2 u}\mathrm{d}u$	M1
		$= -\dfrac{1}{4}\tan u(+C)$	$\pm k \tan u$	M1
		$x = \sqrt{2} \Rightarrow \sqrt{2} = 2\cos u \Rightarrow u = \dfrac{\pi}{4}$		
		$x = 1 \Rightarrow 1 = 2\cos u \Rightarrow u = \dfrac{\pi}{3}$		M1
		$\left[-\dfrac{1}{4}\tan u\right]_{\frac{\pi}{4}}^{\frac{\pi}{4}} = -\dfrac{1}{4}\left(\tan\dfrac{\pi}{4} - \tan\dfrac{\pi}{3}\right)$		
		$= -\dfrac{1}{4}\left(1-\sqrt{3}\right)\left(= \dfrac{\sqrt{3}-1}{4}\right)$		A17
	(b)	$V = \pi\int_1^{\sqrt{2}}\left(\dfrac{4}{x\left(4-x^2\right)^{\frac{1}{4}}}\right)^2\mathrm{d}x$		M1
		$= 16\pi\int_1^{\sqrt{2}}\dfrac{1}{x^2\sqrt{4-x^2}}\mathrm{d}x$	$16\pi \times$ integral in (a)	M1
		$= 16\pi\left(\dfrac{\sqrt{3}-1}{4}\right)$	$16\pi \times$ their answer to part (a)	A1ft3
				[10]
5.	(a)	1.386, 2.291	awrt 1.386, 2.291	B1 B12
	(b)	$A \approx \dfrac{1}{2} \times 0.5(\ldots)$		B1
		= ... (0+2(0.608+1.386+2.291+3.296 +4.385)+5.545)		M1
		= 0.25(0+2(0.608+1.386+2.291+3.296 +4.385)+5.545)	ft their (a)	A1ft
		=0.25×29.477 ... ≈7.37	cao	A14
	(c)	(i) $\int x\ln x\,\mathrm{d}x = \dfrac{x^2}{2}\ln x - \int \dfrac{x^2}{2}\times\dfrac{1}{x}\mathrm{d}x$		M1 A1

$$= \frac{x^2}{2}\ln x - \int \frac{x}{2}\,\mathrm{d}x$$

$$= \frac{x^2}{2}\ln x - \frac{x^2}{4}(+C)$$ M1 A1

(ii) $$\left[\frac{x^2}{2}\ln x - \frac{x^2}{4}\right]_1^4 = (8\ln 4 - 4) - \left(-\frac{1}{4}\right)$$ M1

$=8\ln 4 - \dfrac{15}{4}$

$=8(2\ln 2) - \dfrac{15}{4}$ ln 4 = 2ln 2 seen or implied M1

$= \dfrac{1}{4}(64\ln 2 - 15)$ a = 64, b = –15 A17

[13]

6. (a) 1.14805 awrt 1.14805 B11

(b) $A \approx \dfrac{1}{2} \times \dfrac{3\pi}{8}(\ldots)$ B1

= ... (3 + 2(2.77164 + 2.12132 + 1.14805) + 0) 0 can be implied M1

= $\dfrac{3\pi}{16}$ (3 + 2(2.77164 + 2.12132 + 1.14805)) ft their (a) A1ft

$\dfrac{3\pi}{16}$ × 15.08202 ... = 8.884 cao A14

(c) $$\int 3\cos\left(\frac{x}{3}\right)\mathrm{d}x = \frac{3\sin\left(\frac{x}{3}\right)}{\frac{1}{3}}$$ M1 A1

$$= 9\sin\left(\frac{x}{3}\right)$$

$$A = \left[9\sin\left(\frac{x}{3}\right)\right]_0^{\frac{3\pi}{2}} = 9 - 0 = 9$$ cao A13

[8]

7. (a) f(x) = $\dfrac{4-2x}{(2x+1)(x+1)(x+3)} = \dfrac{A}{2x+1} + \dfrac{B}{x+1} + \dfrac{C}{x+3}$

$4 - 2x = A(x + 1)(x + 3) + B(2x + 1)(x + 3) + C(2x + 1)(x + 1)$ M1

A method for evaluating one constant M1

$x \to -\frac{1}{2}$, $5 = A\left(\frac{1}{2}\right)\left(\frac{5}{2}\right) \Rightarrow A = 4$ any one correct constant A1

$x \to -1$, $6 = B(-1)(2) \Rightarrow B = -3$

$x \to -3$ $10 = C(-5)(-2) \Rightarrow C = 1$ all three constants correct A14

(b) (i) $\displaystyle\int \left(\frac{4}{2x+1} - \frac{3}{x+1} + \frac{1}{x+3}\right) dx$

$= \dfrac{4}{2}\ln(2x+1) - 3\ln(x+1) + \ln(x+3) + C$ A1 two

ln terms correct M1 A1ft

All three ln terms correct and "+C" ; ft constants A1ft3

(ii) $\left[2\ln(2x+1) - 3\ln(x+1) + \ln(x+3)\right]_0^2$

$= (2\ln 5 - 3\ln 3 + \ln 5) - (2\ln 1 - 3\ln 1 + \ln 3)$ M1

$= 3\ln 5 - 4\ln 3$

$= \ln\left(\dfrac{5^3}{3^4}\right)$ M1

$= \ln\left(\dfrac{125}{81}\right)$ A13

[10]

8. (a) $\displaystyle\int \sqrt{(5-x)}\, dx = \int (5-x)^{\frac{1}{2}}\, dx = \frac{(5-x)^{\frac{1}{2}}}{-\frac{3}{2}} (+C)$ M1 A12

$\left(= -\dfrac{2}{3}(5-x)^{\frac{3}{2}} + C\right)$

(b) (i) $\displaystyle\int (x-1)\sqrt{(5-x)}\,dx = -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} + \frac{2}{3}\int (5-x)^{\frac{3}{2}}\, dx$ M1 A1ft

$= \quad \ldots \quad + \dfrac{2}{3} \times \dfrac{(5-x)^{\frac{5}{2}}}{-\frac{5}{2}} (+C)$ M1

$-\dfrac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \dfrac{4}{15}(5-x)^{\frac{5}{2}} (+C)$ A14

(ii) $\left[-\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}}-\frac{4}{15}(5-x)^{\frac{5}{2}}\right]_1^5=(0-0)-\left(0-\frac{4}{15}\times 4^{\frac{5}{2}}\right)$

$=\frac{128}{15}\left(=8\frac{8}{15}\approx 8.53\right)$ awrt 8.53 M1 A12

Alternatives for (b)

$u^2=5-x\Rightarrow 2u\frac{du}{dx}=-1\left(\Rightarrow\frac{dx}{du}=-2u\right)$

$\int(x-1)\sqrt{(5-x)}dx=\int(4-u^2)u\frac{dx}{du}du=\int(4-u^2)u(-2u)du$ M1 A1

$=\int(2u^4-8u^2)du=\frac{2}{5}u^5-\frac{8}{3}u^3(+C)$ M1

$=\frac{2}{5}(5-x)^{\frac{5}{2}}-\frac{8}{3}(5-x)^{\frac{3}{2}}\ (+C)$ A1

Alternatives for (c)

$x=1\ \Rightarrow\ u=2,\ x=5\ \Rightarrow\ u=0$

$\left[\frac{2}{5}u^5-\frac{8}{3}u^3\right]_2^0=(0-0)-\left(\frac{64}{5}-\frac{64}{3}\right)$ M1

$=\frac{128}{15}\left(=8\frac{8}{15}\approx 8.53\right)$ awrt 8.53 A12

[8]

9. (a) $\int\sin^2\theta\,d\theta=\frac{1}{2}\int(1-\cos 2\theta)\,d\theta=\frac{1}{2}\theta-\frac{1}{4}\sin 2\theta(+C)$ M1 A12

(b) $x=\tan\theta\Rightarrow\frac{dx}{d\theta}=\sec^2\theta$

$\pi\int y^2\,dx=\pi\int y^2\,\frac{dx}{d\theta}d\theta=\pi\int(2\sin 2\theta)^2\sec^2\theta\,d\theta$ M1 A1

$=\pi\int\frac{(2\times 2\sin\theta\cos\theta)^2}{\cos^2\theta}d\theta$ M1

$16\pi\int\sin^2\theta\,d\theta$ $k=16\pi$ A1

$x = 0 \Rightarrow \tan\theta = 0 \Rightarrow \theta = 0$, $x = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$ B15

$$\left(V = 16\pi \int_0^{\frac{\pi}{6}} \sin^2\theta \, d\theta\right)$$

(c) $V = 16\pi\left[\frac{1}{2}\theta - \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{6}}$ M1

$= 16\pi\left[\left(\frac{\pi}{12} - \frac{1}{4}\sin\frac{\pi}{3}\right) - (0 - 0)\right]$ Use of correct limits *M1

$= 16\pi\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) = \frac{4}{3}\pi^2 - 2\pi\sqrt{3}$ $p = \frac{4}{3}, q = -2$ A13

[10]

10. (a) Area(R) = $\int_0^2 \frac{3}{\sqrt{(1+4x)}} \, dx = \int_0^2 3(1+4x)^{-\frac{1}{2}} dx$

Integrating $3(1 + 4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$. M1

$= \left[\frac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2}.4}\right]_0^2$ Correct integration. Ignore limits. A1

$= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}}\right]_0^2$

$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$ Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round. M1

$= \frac{9}{2} - \frac{3}{2} = 3(\text{units})^2$ 3 A14

(Answer of 3 with no working scores M0A0M0A0.)

(b) Volume = $\pi \int_0^2 \left(\frac{3}{\sqrt{(1+4x)}}\right)^2 dx$ Use of $V = \pi \int y^2 \, dx$ ..

Can be implied. Ignore limits and dx. B1

$= (\pi)\int_0^2 \frac{9}{1+4x} dx$

$= (\pi)\left[\frac{9}{4}\ln|1+4x|\right]_0^2$ $\pm k\ln|1+4x|$ M1

$\frac{9}{4}\ln|1+4x|$ A1

$= (\pi)\left[\left(\tfrac{9}{4}\ln 9\right)-\left(\tfrac{9}{4}\ln 1\right)\right]$ Substitutes limits of 2 and 0 and subtracts the correct way round. dM1

Note that ln1 can be implied as equal to 0.

So Volume = $\underline{\tfrac{9}{4}\pi \ln 9}$ $\underline{\tfrac{9}{4}\pi \ln 9}$ or $\underline{\tfrac{9}{2}\pi \ln 3}$ or $\underline{\tfrac{18}{4}\pi \ln 3}$ A1 oe isw5

Note the answer must be a one term exact value. Note that $\tfrac{9}{4}\pi \ln 9 + c$ (oe.) would be awarded the final A0.

Note, also you can ignore subsequent working here.

[9]

11. (a) $\int \tan^2 x \, dx$

[*NB* : $\underline{\sec^2 A = 1 + \tan^2 A}$ gives $\underline{\tan^2 A = \sec^2 A - 1}$] The correct underlined identity. M1 oe

$$= \int \sec^2 x - 1 \, dx$$

= $\underline{\tan x - x (+c)}$ Correct integration with/without + c A12

(b) $\int \frac{1}{x^3} \ln x \, dx$

$$\left\{ \begin{array}{l} u = \ln x \quad \Rightarrow \quad \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^{-3} \Rightarrow v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{array} \right\}$$

$$= -\frac{1}{2x^2}\ln x - \int -\frac{1}{2x^2} \cdot \frac{1}{x} \, dx$$

Use of 'integration by parts' formula in the correct direction. M1

Correct direction means that $u = \ln x$.

Correct expression. A1

$$= -\frac{1}{2x^2}\ln x + \frac{1}{2}\int \frac{1}{x^3} \, dx$$

An attempt to multiply through $\frac{k}{x^n}$, $n \in$ □ $n \ldots 2$ by $\frac{1}{x}$ and an attempt to ...

$$= \underline{-\frac{1}{2x^2}\ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)} (+c)$$

... "integrate"(process the result); M1

correct solution with/without + c A1 oe4

(c) $\int \frac{e^{3x}}{1+e^x} \, dx$

$$\left\{ u = 1 + e^x \Rightarrow \underline{\frac{du}{dx} = e^x}, \underline{\frac{dx}{du} = \frac{1}{e^x}}, \underline{\frac{dx}{du} = \frac{1}{u-1}} \right\}$$

Differentiating to find

any one of the <u>three underlined</u> B1

$\int \frac{e^{2x}.e^{x}}{1+e^{x}} dx = \int \frac{(u-1)^2.e^{x}}{u}.\frac{1}{e^{x}} du$ Attempt to substitute for

e^{2x} = f(u), their $\frac{dx}{du} = \frac{1}{e^{x}}$ and $u = 1 + e^{x}$

or = $\int \frac{(u-1)^3}{u}.\frac{1}{(u-1)} du$ or e^{3x} =f(u), their $\frac{dx}{du} = \frac{1}{u-1}$ M1 *

and u =1 + e^{x}.

= $\int \frac{(u-1)^2}{u} du$ $\underline{\int \frac{(u-1)^2}{u} du}$ A1

= $\int \frac{u^2 - 2u + 1}{u} du$ An attempt to

multiply out their numerator
to give at least three terms

= $\int u - 2 + \frac{1}{u} du$ and divide through each term by u dM1 *

= $\frac{u^2}{2} - 2u + 1\text{n}u\ (+c)$ Correct integration

with/without +c A1

= $\frac{(1+e^{x})^2}{2} - 2(1+e^{x}) + 1\text{n}(1+e^{x}) + c$ Substitutes u = 1 + e^{x} back

into their integrated expression with at least two terms. dM1 *

= $\frac{1}{2} + e^{x} + \frac{1}{2}e^{2x} - 2 - 2e^{x} + 1\text{n}\ (1 + e^{x}) + c$

= $\frac{1}{2} + e^{x} + \frac{1}{2}e^{2x} - 2 - 2e^{x} + \ln(1 + e^{x}) + c$

= $\frac{1}{2}e^{2x} - e^{x} + \ln(1 + e^{x}) - \frac{3}{2} + c$

= $\frac{1}{2}e^{2x} - e^{x} + \ln(1 + e^{x}) + k$ **AG** $\underline{\frac{1}{2}e^{2x} - e^{x} + 1\text{n}(1+e^{x}) + k}$

must use a + c + and "$-\frac{3}{2}$ "

combined. A1 cso7

[13]

12. (a) $\left\{ \begin{array}{l} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^{x} \Rightarrow v = e^{x} \end{array} \right\}$

$\int xe^{x} dx = xe^{x} - \int e^{x}.1dx$

$= xe^{x} - \int e^{x} dx$

= $x\,e^{x} - e^{x}$ (+ c)

Use of 'integration by parts' formula in the **correct direction**.
(See note.) M1

Correct expression. (Ignore dx) A1

Correct integration with/without + c A1 3

Note integration by parts in the **correct direction** means that u and $\frac{dv}{dx}$ must be assigned/used as $u = x$ and $\frac{dv}{dx} = e^x$ in this part for example.

+ c is not required in this part.

(b) $$\left\{\begin{array}{l} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array}\right\}$$

$$\int x^2 e^x dx = x^2 e^x - \int e^x .2x dx$$

$$= x^2 e^x - 2\int x e^x dx$$

$$= x^2 e^x - 2(xe^x - e^x) + c$$

$$\left\{\begin{array}{l} = x^2 e^x - 2xe^x + 2e^x + c \\ = e^x (x^2 - 2x + 2) + c \end{array}\right\}$$

Use of 'integration by parts' formula in the **correct direction**. M1

Correct expression. (Ignore dx) A1

Correct expression **including + c**. **(seen at any stage!)**
You can ignore subsequent working. A1 **ISW** 3

Ignore subsequent working

+ c is required in this part.

[6]

13. Volume = $\underline{\pi\int_a^b \left(\frac{1}{2x+1}\right)^2 dx} = \pi\int_a^b \frac{1}{(2x+1)^2} dx$

$$= \pi\int_a^b (2x+1)^{-2} dx$$

$$= (\pi)\left[\frac{(2x+1)^{-1}}{(-1)(2)}\right]_a^b$$

$$= (\pi)\left[\underline{-\frac{1}{2}(2x+1)^{-1}}\right]_a^b$$

$$= (\pi)\left[\left(\frac{-1}{2(2b+1)}\right) - \left(\frac{-1}{2(2a+1)}\right)\right]$$

$$= \frac{\pi}{2}\left[\frac{-2a-1+2b+1}{(2a+1)(2b+1)}\right]$$

$$= \frac{\pi}{2}\left[\frac{2(b-a)}{(2a+1)(2b+1)}\right]$$

$$= \frac{\pi(b-a)}{(2a+1)(2b+1)}$$

Use of $V = \underline{\pi\int y^2 \mathrm{d}x}$.

Can be implied. Ignore limits. B1

Integrating to give $\underline{\pm p(2x+1)^{-1}}$ M1

$\underline{-\frac{1}{2}(2x+1)^{-1}}$ A1

Substitutes limits of b and a and subtracts the correct way round. dM1

$\underline{\frac{\pi(b-a)}{(2a+1)(2b+1)}}$ (*) A1 aef 5

(*) Allow other equivalent forms such as

$$\frac{\pi b-\pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b-\pi a}{4ab+2a+2b+1}.$$

Note that n is not required for the middle three marks of this question.

Aliter
Way 2

Volume = $\underline{\pi\int_a^b \left(\frac{1}{2x+1}\right)^2 \mathrm{d}x} = \pi\int_a^b \frac{1}{(2x+1)^2}\mathrm{d}x$

$$= \pi\int_a^b (2x+1)^{-2}\mathrm{d}x$$

Use of $V = \underline{\pi\int y^2 \mathrm{d}x}$.

Can be implied. Ignore limits. B1

Applying substitution $u = 2x + 1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2$ and changing limits $x \to u$ so that $a \to 2a + 1$ and $b \to 2b + 1$, gives

$$= (\pi)\int_{2a+1}^{2b+1} \frac{u^{-2}}{2} \mathrm{d}u$$

$$= (\pi)\left[\frac{u^{-1}}{(-1)(2)}\right]_{2a+1}^{2b+1}$$

$$= (\pi)\left[-\frac{1}{2}u^{-1}\right]_{2a+1}^{2b+1}$$

$$= (\pi)\left[\left(\frac{-1}{2(2b+1)}\right) - \left(\frac{-1}{2(2a+1)}\right)\right]$$

$$= \frac{\pi}{2}\left[\frac{-2a-1+2b+1}{(2a+1)(2b+1)}\right]$$

$$= \frac{\pi}{2}\left[\frac{2(b-a)}{(2a+1)(2b+1)}\right]$$

$$= \frac{\pi(b-a)}{(2a+1)(2b+1)}$$

Integrating to give $\underline{\pm pu^{-1}}$ M1

$\underline{-\frac{1}{2}u^{-1}}$ A1

Substitutes limits of $2b + 1$ and $2a + 1$ and subtracts the correct way round. dM1

$\underline{\frac{\pi(b-a)}{(2a+1)(2b+1)}}$ (*) A1 aef 5

(*) Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b - \pi a}{4ab+2a+2b+1}.$$

Note that π is not required for the middle three marks of this question.

[5]

14. (i) $\int \ln\left(\frac{x}{2}\right)\mathrm{d}x = \int 1.\ln\left(\frac{x}{2}\right)\mathrm{d}x \Rightarrow \left\{\begin{matrix} u = \ln\left(\frac{x}{2}\right) \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{\frac{1}{2}}{\frac{x}{2}} = \frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = 1 \qquad \Rightarrow v = x \end{matrix}\right\}$

$$\int \ln\left(\frac{x}{2}\right)\mathrm{d}x = x\ln\left(\frac{x}{2}\right) - \int x.\frac{1}{x}\mathrm{d}x$$

$$= x\ln\left(\frac{x}{2}\right) - \int \underline{1}\ \mathrm{d}x$$

$$= x\ln\left(\frac{x}{2}\right) - x + c$$

Use of 'integration by parts' formula in the correct direction. M1

Correct expression. A1

Note: $\int \ln\left(\frac{x}{2}\right) dx = (\text{their}v)\ln\left(\frac{x}{2}\right) - \int (\text{their}v).(\text{their}\frac{du}{dx})dx$ for M1 in part (i).

An attempt to multiply x by a candidate's $\frac{a}{x}$ or $\frac{1}{bx}$ or $\frac{1}{x}$. dM1

Correct integration with + c A1 aef 4

Aliter
Way 2

$$\int \ln\left(\frac{x}{2}\right) dx = \int (\ln x - \ln 2)dx = \int \ln x\, dx - \int \ln 2\, dx$$

$$\int \ln x\, dx = \int 1.\ln x\, dx \Rightarrow \begin{Bmatrix} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = 1 \quad \Rightarrow v = x \end{Bmatrix}$$

$$\int \ln x\, dx = x \ln x - \int x.\frac{1}{x} dx$$

$= x \ln x - x + c$

$$\int \ln 2 dx = x \ln 2 + c$$

Hence, $\int \ln\left(\frac{x}{2}\right) dx = x \ln x - x - x \ln 2 + c$

Use of 'integration by parts' formula in the correct direction. M1

Note: $\int \ln x\ dx = (\text{their}v)\ln x - \int (\text{their}v).(\text{their}\frac{du}{dx})dx$ for M1 in part (i).

Correct integration of ln x with or without + c. A1

Correct integration of ln 2 with or without + c M1

Correct integration with + c A1 aef 4

Aliter
Way 3

$$\int \ln\left(\frac{x}{2}\right) dx$$

$$u = \frac{x}{2} \Rightarrow \frac{du}{dx} = \frac{1}{2}$$

$$\int \ln\left(\frac{x}{2}\right) dx = 2\int \ln u\, du$$

$$\int \ln u\, dx = \int 1.\ln u\, du$$

$$\int \ln u\, dx = u \ln u - \int u.\frac{1}{u} du$$

$= u \ln u - u + c$

$$\int \ln\left(\frac{x}{2}\right) \mathrm{d}x = 2(u \ln u - u) + c$$

$$\text{Hence, } \int \ln\left(\frac{x}{2}\right) \mathrm{d}x = x \ln\left(\frac{x}{2}\right) - x + c$$

Use of 'integration by parts' formula in the correct direction.	M1
Correct integration of ln u with or without + c.	A1
Decide to award 2nd M1 here!	M1
Correct integration with + c	A1 aef 4

(ii) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, \mathrm{d}x$

$$\left[\text{NB}: \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \text{ or } \underline{\sin^2 x = \frac{1}{2}(\pm 1 \pm \cos 2x)}\right]$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} \mathrm{d}x = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\underline{1 - \cos 2x}) \mathrm{d}x$$

$$= \frac{1}{2}\left[\underline{x - \frac{1}{2}\sin 2x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin(\pi)}{2}\right) - \left(\frac{\pi}{4} - \frac{\sin\left(\frac{\pi}{2}\right)}{2}\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{2} - 0\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)\right]$$

$$= \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi}{8} + \frac{1}{4}$$

Consideration of double angle formula for cos $2x$	M1
<u>Integrating to give ± ax ± b sin $2x$</u>; $a, b \neq 0$	
Correct result of anything equivalent to $\frac{1}{2}x - \frac{1}{4}\sin 2x$	A1
Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
$\underline{\frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)}$ or $\underline{\frac{\pi}{8} + \frac{1}{4}}$ or $\underline{\frac{\pi}{8} + \frac{2}{8}}$	A1 **aef, cso**

Candidate must collect their π term and constant term together for A1
No fluked answers, hence **cso**.

Aliter
Way 2

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x . \sin x \, dx \text{ and } I = \int \sin^2 x \, dx$$

$$\left\{\begin{array}{l} u = \sin x \quad \Rightarrow \dfrac{du}{dx} = \cos x \\ \dfrac{dv}{dx} = \sin x \Rightarrow v = -\cos x \end{array}\right\}$$

$$\therefore I = \underline{\left\{-\sin x \cos x + \int \cos^2 x dx\right\}}$$

$$\therefore I = \left\{-\sin x \cos x + \int (1 - \sin^2 x) dx\right\}$$

$$\int \sin^2 x \, dx = \left\{-\sin x \cos x + \int 1 \, dx - \int \sin^2 x \, dx\right\}$$

$$2\int \sin^2 x \, dx = \left\{-\sin x \cos x + \int 1 \, dx\right\}$$

$$2\int \sin^2 x \, dx = \left\{-\sin x \cos x + x\right\}$$

$$\int \sin^2 x \, dx = \underline{\left\{-\frac{1}{2}\sin x \cos x + \frac{x}{2}\right\}}$$

$$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \left[\left(-\frac{1}{2}\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right) + \frac{\left(\frac{\pi}{2}\right)}{2}\right) - \left(-\frac{1}{2}\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) + \frac{\left(\frac{\pi}{4}\right)}{2}\right)\right]$$

$$= \left[(0 + \frac{\pi}{4}) - (-\frac{1}{4} + \frac{\pi}{8})\right]$$

$$= \frac{\pi}{8} + \frac{1}{4}$$

An attempt to use the correct by parts formula.	M1
For the LHS becoming $2I$	dM1
<u>Correct integration</u>	A1
Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
$\underline{\frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)}$ or $\underline{\frac{\pi}{8} + \frac{1}{4}}$ or $\underline{\frac{\pi}{8} + \frac{2}{8}}$	A1 **aef, cso**

Candidate must collect their π term and constant term together for A1
No fluked answers, hence **cso**.

Note: $\frac{\pi}{8} + \frac{1}{4}$ = 0.64269...

[9]

15. $\int_0^1 \frac{2^x}{(2^x + 1)^2} \, dx$, with substitution $u = 2^x$

$$\frac{du}{dx} = 2^x.\ln 2 \Rightarrow \frac{dx}{du} = \frac{1}{2^x.\ln 2}$$

$$\int \frac{2^x}{(2^x+1)^2}\,dx = \left(\frac{1}{\ln 2}\right)\int \frac{1}{(u+1)^2}\,du$$ 6

$$= \left(\frac{1}{\ln 2}\right)\left(\frac{-1}{(u+1)}\right) + c$$

B1 $\frac{du}{dx} = 2^x.\ln 2$ or $\frac{du}{dx} = u.\ln 2$ or $\left(\frac{1}{u}\right)\frac{du}{dx} = \ln 2$

M1* $k\int \frac{1}{(u+1)^2}\,du$ where k is constant

M1 $(u + 1)^{-2} \rightarrow a(u + 1)^{-1}$ (*)

A1 $(u + 1)^{-2} \rightarrow -1.(u + 1)^{-1}$ (*)

(*) If you see this integration applied anywhere in a candidate's working then you can award M1, A1

change limits: when $x = 0$ & $x = 1$ then $u = 1$ & $u = 2$

$$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx = \frac{1}{\ln 2}\left[\frac{-1}{(u+1)}\right]_1^2$$

$$= \frac{1}{\ln 2}\left[\left(-\frac{1}{3}\right) - \left(-\frac{1}{2}\right)\right]$$

$$= \frac{1}{6\ln 2}$$

depM1 Correct use of limits $u = 1$ and $u = 2$

A1aef $\underline{\frac{1}{6\ln 2}}$ or $\underline{\frac{1}{\ln 4} - \frac{1}{\ln 8}}$ or $\underline{\frac{1}{2\ln 2} - \frac{1}{3\ln 2}}$ (*)

Exact value only!

(*) There are other acceptable answers for A1. eg: $\underline{\frac{1}{2\ln 8}}$ or $\underline{\frac{1}{\ln 64}}$

NB: Use your calculator to check eg. 0.240449...

Alternatively candidate can revert back to x...

$$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx = \frac{1}{\ln 2}\left[\frac{-1}{(2^x+1)}\right]_0^1$$

$$= \frac{1}{\ln 2}\left[\left(-\frac{1}{3}\right) - \left(-\frac{1}{2}\right)\right]$$

$$= \underline{\frac{1}{6\ln 2}}$$

depM1* Correct use of limits $x = 0$ and $x = 1$

A1aef $\dfrac{1}{6\ln 2}$ or $\dfrac{1}{\ln 4}-\dfrac{1}{\ln 8}$ or $\dfrac{1}{2\ln 2}-\dfrac{1}{3\ln 2}$ (*)

Exact value only!

(*) There are other acceptable answers for A1. eg: $\dfrac{1}{2\ln 8}$ or $\dfrac{1}{\ln 64}$

NB: Use your calculator to check eg. 0.240449...

[6]

16. (a)
$$\left\{\begin{array}{ll} u = x & \Rightarrow \dfrac{du}{dx} = 1 \\ \dfrac{dv}{dx} = \cos 2x \Rightarrow v = \dfrac{1}{2}\sin 2x & \end{array}\right\}$$

$$\text{Int} = \int x\cos 2x\,dx = \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x.1\,dx$$

$$= \frac{1}{2}x\sin 2x - \frac{1}{2}\left(-\frac{1}{2}\cos 2x\right) + c$$ 4

$$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$$

M1 **(*see note below*)**
Use of 'integration by parts' formula in the correct direction.

A1 Correct expression.

dM1 $\sin 2x \to -\dfrac{1}{2}\cos 2x$

or $\sin kx \to -\dfrac{1}{k}\cos kx$

with $k \neq 1$, $k > 0$

A1 Correct expression with + c

(b)
$$\int x\cos^2 x\,dx = \int x\left(\frac{\cos 2x + 1}{2}\right)dx$$

$$= \frac{1}{2}\int x\cos 2x\,dx + \frac{1}{2}\int x\,dx$$

$$= \underline{\frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right)}; + \frac{1}{2}\int x\,dx$$ 3

$$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$$

M1 Substitutes correctly for $\cos^2 x$ in the given integral

A1ft $\dfrac{1}{2}$ (their answer to (a)); or <u>underlined expression</u>

A1 Completely correct expression with/without $+c$

Notes:

Int = $\int x\cos 2x \,\mathrm{d}x = \frac{1}{2}x\sin 2x \pm \int \frac{1}{2}\sin 2x.1\mathrm{d}x$

M1 This is acceptable for M1

$$\left\{\begin{array}{ll} u = x & \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1 \\ \frac{\mathrm{d}v}{\mathrm{d}x} = \cos 2x \Rightarrow v = \lambda \sin 2x \end{array}\right\}$$

Int = $\int x\cos 2x \,\mathrm{d}x = \lambda x\sin 2x \pm \int \lambda \sin 2x.1\mathrm{d}x$

M1 This is also acceptable for M1

Aliter (b) Way 2

$$\int x\cos^2 x\,\mathrm{d}x = \int x\left(\frac{\cos 2x + 1}{2}\right)\mathrm{d}x$$

$$\left\{\begin{array}{ll} u = x & \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1 \\ \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1}{2}\cos 2x + \frac{1}{2} & \Rightarrow v = \frac{1}{4}\sin 2x + \frac{1}{2}x \end{array}\right\}$$

$$= \frac{1}{4}x\sin 2x + \frac{1}{2}x^2 - \int\left(\frac{1}{4}\sin 2x + \frac{1}{2}x\right)\mathrm{d}x$$

$$= \underline{\frac{1}{4}x\sin 2x} + \frac{1}{2}x^2 + \underline{\frac{1}{8}\cos 2x} - \frac{1}{4}x^2 + c$$

$$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$$

M1 Substitutes **<u>correctly</u>** for $\cos^2 x$ in the given integral...
...or
$u = x$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1}{2}\cos 2x + \frac{1}{2}$

A1ft $\frac{1}{2}$ (their answer to (a)); or <u>underlined expression</u>

A1 Completely correct expression with/without + c

Aliter (b) Way 3

$$\int x\cos 2x\,\mathrm{d}x = \int x(2\cos^2 x - 1)\mathrm{d}x$$

$$\Rightarrow 2\int x\cos^2 x\,\mathrm{d}x - \int x\mathrm{d}x = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$$

$$\Rightarrow \int x\cos^2 x\,\mathrm{d}x = \underline{\frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right)};+\frac{1}{2}\int x\mathrm{d}x$$

$$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$$

M1 Substitutes **correctly** for cos $2x$ in $\int x\cos 2x\,dx$

A1ft $\frac{1}{2}$ (their answer to (a)); or underlined expression

A1 Completely correct expression with/without + c

[7]

17. (a) **Way 1**

A method of long division gives,

$$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv 2 + \frac{4}{(2x+1)(2x-1)}$$

$$\frac{4}{(2x+1)(2x-1)} \equiv \frac{B}{(2x+1)} + \frac{C}{(2x-1)}$$

$4 \equiv B(2x-1) + C(2x+1)$

or their remainder, $Dx + E \equiv B(2x-1) + C(2x+1)$

Let $x = -\frac{1}{2}$, $4 = -2B \Rightarrow B = -2$

Let $x = \frac{1}{2}$, $4 = 2C \Rightarrow C = 2$ 4

B1 $A = 2$

M1 Forming any one of these two identities. Can be implied.

See note below

A1 either one of $B = -2$ or $C = 2$

A1 both B and C correct

Aliter (a) Way 2

$$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \frac{B}{(2x+1)} + \frac{C}{(2x-1)}$$

See below for the award of B1

$2(4x^2+1) \equiv A(2x+1)(2x-1) + B(2x-1) + C(2x+1)$

Equate x^2, $8 = 4A \Rightarrow A = 2$

Let $x = -\frac{1}{2}$, $4 = -2B \Rightarrow B = -2$

Let $x = \frac{1}{2}$, $4 = 2C \Rightarrow C = 2$

B1 ***decide to award B1 here!! ...***
... for $A = 2$

M1 Forming this identity. Can be implied.
If a candidate states one of either B or C correctly then the method mark M1 can be implied.

See note below

A1 either one of $B = -2$ or $C = 2$

A1 both B and C correct

(b) $\int \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = \int 2 - \frac{2}{(2x+1)} + \frac{2}{(2x-1)}\,dx$

$= 2x - \frac{2}{2}\ln 2(2x+1) + \frac{2}{2}\ln(2x-1)(+c)$

6

M1* Either $p\ln(2x+1)$ or $q\ln(2x-1)$
or either $p\ln 2x+1$ or $q\ln 2x-1$
Some candidates may find rational values for B and C.
They may combine the denominator of their B or C with
$(2x+1)$ or $(2x-1)$. Hence:

Either $\frac{a}{b(2x-1)} \to k\ln(b(2x-1))$ or $\frac{a}{b(2x+1)} \to k\ln(b(2x+1))$

is okay for M1.
Candidates are not allowed to fluke $-\ln(2x+1)+\ln(2x-1)$ for A1. Hence **cso**. If they do fluke this, however, they can gain the final A1 mark for this part of the question.

B1ft $A \to Ax$

A1cso&aef $-\frac{2}{2}\ln(2x+1) + \frac{2}{2}\ln(2x-1)$ or $-\ln(2x+1)+\ln(2x-1)$

See note below.

$\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = [2x - \ln(2x+1) + \ln(2x-1)]_1^2$

$= (4 - \ln 5 + \ln 3) - (2 - \ln 3 + \ln 1)$

$= 2 + \ln 3 + \ln 3 - \ln 5$

$= 2 + \ln\left(\frac{3(3)}{5}\right)$

$= 2 + \ln\left(\frac{9}{5}\right)$

depM1* Substitutes limits of 2 and 1 and subtracts the correct way round. (Invisible brackets okay).

M1 Use of correct product (or power) and/or quotient laws for logarithms to obtain a single logarithmic term for their numerical expression.
To award this M1 mark, the candidate must use the appropriate law(s) of logarithms for their ln terms to give a **one single** logarithmic term. Any error in ***applying the laws of logarithms*** would then earn M0.
Note: This is not a dependent method mark.

A1 $2 + \ln\left(\frac{9}{5}\right)$ or $2 - \ln\left(\frac{5}{9}\right)$ **and** k stated as $\frac{9}{5}$.

[10]

18. (a) Volume $= \underline{\pi\int_{-\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{3(1+2x)}\right)^2 dx} = \frac{\pi}{9}\int_{-\frac{1}{4}}^{\frac{1}{2}} \frac{1}{(1+2x)^2}\,dx$

$$= \left(\frac{\pi}{9}\right)\int_{-\frac{1}{4}}^{\frac{1}{2}} (1+2x)^{-2}\,dx$$

$$= \left(\frac{\pi}{9}\right)\left[\frac{(1+2x)^{-1}}{(-1)(2)}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$$

$$= \left(\frac{\pi}{9}\right)\left[-\frac{1}{2}(1+2x)^{-1}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$$

$$= \left(\frac{\pi}{9}\right)\left[\left(\frac{-1}{2(2)}\right)-\left(\frac{-1}{2\left(\frac{1}{2}\right)}\right)\right]$$

$$= \left(\frac{\pi}{9}\right)\left[-\frac{1}{4}-(-1)\right]$$

$$= \frac{\pi}{12}$$

Use of $V = \underline{\pi\int y^2\,dx}$

Can be implied. Ignore limits. B1

Moving their power to the top.
(Do not allow power of –1.)

Can be implied. Ignore limits and $\frac{\pi}{9}$ M1

Integrating to give $\underline{\pm p(1 + 2x)^{-1}}$ M1

$\underline{-\frac{1}{2}(1+2x)^{-1}}$ A1

Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef A1aef5

Note: $\frac{\pi}{9}$ (or implied) is not needed for the middle three marks.

Aliter
Way 2

Volume = $\underline{\pi\int_{-\frac{1}{4}}^{\frac{1}{2}}\left(\frac{1}{3(1+2x)}\right)^2 dx} = \pi\int_{-\frac{1}{4}}^{\frac{1}{2}}\frac{1}{(3+6x)^2}dx$

$$= (\pi)\int_{-\frac{1}{4}}^{\frac{1}{2}} (3+6x)^{-2}\,\mathrm{d}x$$

$$= (\pi)\left[\frac{(3+6x)^{-1}}{(-1)(6)}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$$

$$= (\pi)\left[\underline{-\frac{1}{6}(3+6x)^{-1}}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$$

$$= (\pi)\left[\left(\frac{-1}{6(6)}\right) - \left(\frac{-1}{6\left(\frac{3}{2}\right)}\right)\right]$$

$$= (\pi)\left[-\frac{1}{36} - \left(-\frac{1}{9}\right)\right]$$

$$= \frac{\pi}{12}$$

Use of $V = \underline{\pi\int y^2\,\mathrm{d}x}$

Can be implied. Ignore limits. B1

Moving their power to the top.
(Do not allow power of –1.)
Can be implied. Ignore limits and π M1

Integrating to give $\underline{\pm p(3 + 6x)^{-1}}$ M1

$\underline{-\frac{1}{6}(3+6x)^{-1}}$ A1

Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef A1aef5

Note: π (or implied) is not needed for the middle three marks.

(b) From Fig.1, AB = $\frac{1}{2} - \left(-\frac{1}{4}\right) = \frac{3}{4}$ units

As $\frac{3}{4}$ _units ≡ 3cm

then scale factor k = $\frac{3}{\left(\frac{3}{4}\right)}$ = 4.

Hence Volume of paperweight = $(4)^3\left(\frac{\pi}{12}\right)$

V = $\underline{\frac{16\pi}{3}}$ cm^3 =16.75516... cm^3

$(4)^3$ × (their answer to part (a)) M1

$\frac{16\pi}{3}$ or awrt 16.8 or $\frac{64\pi}{12}$ or aef A12

[7]

19. (a)

x	0	1	2	3	4	5
y	e^1	e^2	$e^{\sqrt{7}}$	$e^{\sqrt{10}}$	$e^{\sqrt{13}}$	e^4
or y	2.71828…	7.38906…	14.09403…	23.62434…	36.80197…	54.59815…

Either $e^{\sqrt{7}}, e^{\sqrt{10}}$ and $e^{\sqrt{13}}$ or awrt 14.1, 23.6 and 36.8 or e to the power awrt 2.65, 3.16, 3.61 (or mixture of decimals and e's)

At least two correct B1

All three correct B12

(b) $I \approx \frac{1}{2} \times 1; \times \underline{\left\{e^1 + 2(e^2 + e^{\sqrt{7}} + e^{\sqrt{10}} + e^{\sqrt{13}}) + e^4\right\}}$

$= \frac{1}{2} \times 221.1352227... = 110.5676113..$ = $\underline{110.6}$ (4sf)

Outside brackets $\frac{1}{2} \times 1$ B1;

<u>For structure of trapezium rule {...............}.</u> M1ft

<u>110.6</u> A1 **cao**3

Beware: Candidates can add up the individual trapezia:

$$I \approx \frac{1}{2}.1(\underline{e^1 + e^2}) + \frac{1}{2}.1(\underline{e^2 + e^{\sqrt{7}}}) + \frac{1}{2}.1(\underline{e^{\sqrt{7}} + e^{\sqrt{10}}}) + \frac{1}{2}.1(\underline{e^{\sqrt{10}} + e^{\sqrt{13}}}) + \frac{1}{2}.1(\underline{e^{\sqrt{13}} + e^4})$$

(c) $t = (3x+1)^{\frac{1}{2}} \Rightarrow \frac{dt}{dx} = \frac{1}{2}.3.(3x+1)^{-\frac{1}{2}}$

... or $t^2 = 3x + 1 \Rightarrow$ $\underline{2t\frac{dt}{dx} = 3}$

so $\frac{dt}{dx} = \frac{3}{2.(3x+1)^{\frac{1}{2}}} = \frac{3}{2t} \Rightarrow \frac{dx}{dt} = \frac{2t}{3}$

$\therefore I = \int e^{\sqrt{(3x+1)}} dx = \int e^t \frac{dx}{dt}.dt = \int e^t . \frac{2t}{3}.dt$

$\therefore I = \underline{\int \frac{2}{3} te^t dt}$

change limits:

When $x = 0$, $t = 1$ & when $x = 5$, $t = 4$

Hence $I = \int_1^4 \frac{2}{3} te^t dt$; where $a = 1$, $b = 4$, $k = \frac{2}{3}$

$A(3x+1)^{-\frac{1}{2}}$ or $t\frac{dt}{dx} = A$ M1

$\underline{\frac{3}{2}(3x+1)^{\frac{1}{2}}}$ or $\underline{2t\frac{dt}{dx}=3}$ A1

Candidate obtains either $\frac{dt}{dx}$ or $\frac{dx}{dt}$ in terms of t …

… and moves on to substitute this into I to convert an integral wrt x to an integral wrt t. dM1

$\underline{\int \frac{2}{3}te^{t}}$ A1

changes limits $x \to t$ so that $0 \to 1$ and $5 \to 4$ B15

(d) $\left\{ \begin{array}{l} u = t \Rightarrow \frac{du}{dt} = 1 \\ \frac{dv}{dt} = e^{t} \Rightarrow v = e^{t} \end{array} \right\}$

$k\int te^{t}\,dt = k(te^{t} - \int e^{t}.1dt)$

$= k(\underline{te^{t} - e^{t}}) + c$

$\therefore \int_1^4 \frac{2}{3}te^{t}\,dt = \frac{2}{3}\{(4e^{4} - e^{4}) - (e^{1} - e^{1})\}$

$= \frac{2}{3}(3e^{4}) = \underline{2e^{4}} = 109.1963...$

Let k be any constant for the first three marks of this part.

Use of 'integration by parts' formula in the correct direction. M1

Correct expression with a constant factor k. A1

Correct integration with/without a constant factor k A1

Substitutes their changed limits into the integrand and subtracts oe. dM1 oe.

either $2e^{4}$ or awrt 109.2 A15

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.

[15]

20. (a) Area Shaded = $\int_0^{2\pi} 3\sin\left(\frac{x}{2}\right)dx$

$= \left[\frac{-3\cos\left(\frac{x}{2}\right)}{\frac{1}{2}}\right]_0^{-2\pi}$

Integrating 3sin($\frac{x}{2}$) to give k cos ($\frac{x}{2}$) with k ≠ 1. M1
Ignore limits.

$= \left[-6\cos\left(\frac{x}{2}\right)\right]_0^{2\pi}$

$-6\cos\left(\frac{x}{2}\right)$ or $\frac{-3}{\frac{1}{2}}\cos\left(\frac{x}{2}\right)$ A1 oe

$= [-6(-1)] - [-6(1)] = 6 + 6 = \underline{12}$ A1 cao 3

(Answer of 12 with no working scores M0A0A0.)

(b) Volume = $\underline{\pi \int_0^{2\pi} \left(3\sin\left(\frac{x}{2}\right)\right)^2} \, dx = 9\pi \int_0^{2\pi} \sin^2\left(\frac{x}{2}\right) dx$

Use of $V = \pi \int y^2 \, dx$.
Can be implied. Ignore limits. M1

[NB: $\cos 2x = \pm 1 \pm 2\sin^2 x$ gives $\sin^2 x = \frac{1 - \cos 2x}{2}$]

[NB: $\cos x = \pm 1 \pm 2\sin^2\left(\frac{x}{2}\right)$ gives $\sin^2\left(\frac{x}{2}\right) = \frac{1 - \cos x}{2}$] M1

Consideration of the Half Angle Formula for $\sin^2\left(\frac{x}{2}\right)$ *or the Double Angle Formula for* $\sin^2 x$

$\therefore \text{Volume} = 9(\pi) \int_0^{2\pi} \left(\frac{1 - \cos x}{2}\right) dx$ A1

Correct expression for Volume
Ignore limits and π.

$= \frac{9(\pi)}{2} \int_0^{2\pi} (\underline{1 - \cos x}) \, dx$

$= \frac{9(\pi)}{2} [x - \sin x]_0^{2\pi}$

Integrating to give ± ax ± bsinx ; depM1;
Correct integration
k − k cos x → kx − k sin x A1

$= \frac{9\pi}{2}[(2\pi - 0) - (0 - 0)]$

$= \frac{9\pi}{2}(2\pi) = \underline{9\pi^2}$ or $\underline{88.8264.}$ A1 cso 3

Use of limits to give either $9\pi^2$ *or awrt 88.8*
Solution must be completely correct. No flukes allowed.

[6]

21. (a)

x	1	1.5	2	2.5	3

y	0	0.5 ln 1.5	ln 2	1.5 ln 2.5	2 ln 3
or y	0	0.2027325541 …	ln2	1.374436098 …	2 ln 3

Either 0.5 ln 1.5 and 1.5 ln 2.5 B1
or awrt 0.20 and 1.37
(or mixture of decimals and ln's)

(b) (ii) $l_1 \approx \frac{1}{2} \times 1 \times \{\underline{0 + 2(\ln 2) + 2\ln 3}\}$ M1;

For structure of trapezium rule {…………};

$\frac{1}{2} \times$ 3.583518938... = 1.791759... = 1.792 (4sf) A1 cao

(ii) $l_2 \approx \frac{1}{2} \times 0.5; \times \{\underline{0 + 2(0.5\ln 1.5 + \ln 2 + 1.5\ln 2.5) + 2\ln 3}\}$

Outside brackets $\frac{1}{2}$ *× 0.5* B1

For structure of trapezium rule {…………..}; M1 ft

$= \frac{1}{4} \times 6.737856242... = 1.684464$ A1 5

awrt 1.684

(c) With increasing ordinates, the line segments at the top of the trapezia are closer to the curve. B1 1

Reason or an appropriate diagram elaborating the correct reason.

(d) $\left\{ \begin{array}{ll} u = \ln x & \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x - 1 & \Rightarrow v = \frac{x^2}{2} - x \end{array} \right\}$ M1

Use of 'integration by parts' formula in the correct direction

$$\mathrm{I} = \left(\frac{x^2}{2} - x\right)\ln x - \int \frac{1}{x}\left(\frac{x^2}{2} - x\right) dx$$ A1

Correct expression

$$= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\int \left(\frac{x}{2} - x\right) dx}$$

An attempt to multiply at least one term through by $\frac{1}{x}$ *and an attempt to ...*

$$= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\left(\frac{x^2}{4} - x\right)} (+c)$$

... integrate; M1
correct integration A1

$$\therefore \mathrm{I} = \left[\left(\frac{x^2}{2} - x\right)\ln x - \frac{x^2}{4} + x\right]_1^3$$

$= \left(\frac{3}{2}\ln 3 - \frac{9}{4} + 3\right) - \left(-\frac{1}{2}\ln 1 - \frac{1}{4} + 1\right)$ ddM1

Substitutes limits of 3 and 1 and subtracts.

$$=\tfrac{3}{2}\ln 3+\tfrac{3}{4}+0-\tfrac{3}{4}=\underline{\tfrac{3}{2}\ln 3}$$ **AG** A1 cso 6

Aliter Way 2

(d) $$\int (x-1)\ln x\,dx=\int x\ln x\,dx-\int \ln x\,dx$$

$$\int x\ln x=\frac{x^2}{2}\ln x-\int \frac{x^2}{2}.\left(\frac{1}{x}\right)dx$$ J M1

Correct application of 'by parts'

$$=\frac{x^2}{2}\ln x-\frac{x^2}{4}(+c)$$ A1

Correct integration

$$\int \ln x\,dx=x\ln x-\int x.\left(\frac{1}{x}\right)dx$$ M1

Correct application of 'by parts''

$$=x\ln x-x(+c)$$ A1

Correct integration

$$\therefore \int_1^3 (x-1)\ln x\,dx=\left(\tfrac{9}{2}\ln 3-2\right)-\left(3\ln 3-2\right)=\tfrac{3}{2}\ln 3$$ **AG**

Substitutes limits of 3 and 1 into both integrands and subtracts. dd M1

$\tfrac{3}{2}\ln 3$ A1 cso 6

Aliter Way 3

(d) $$\left\{\begin{array}{ll} u=\ln x & \Rightarrow \frac{du}{dx}=\frac{1}{x} \\ \frac{dv}{dx}=x-1 & \Rightarrow v=\frac{(x-1)^2}{2} \end{array}\right\}$$ M1

Use of 'integration by parts' formula in the correct direction

$$\mathrm{I}=\frac{(x-1)^2}{2}\ln x-\int \frac{(x-1)^2}{2x}dx$$ A1

Correct expression

$$=\frac{(x-1)^2}{2}\ln x-\int \frac{x^2-2x+1}{2x}dx$$

$$=\frac{(x-1)^2}{2}\ln x-\int\left(\frac{1}{2}x-1+\frac{1}{2x}\right)dx$$

Candidate multiplies out numerator to obtain three terms...

... multiplies at least one term through by $\frac{1}{x}$ and then attempts to ...

... integrate the result; M1

<u>correct</u> integration A1

$$= \frac{(x-1)^2}{2}\ln x - \left(\frac{x^2}{4} - x + \frac{1}{2}\ln x\right)(+c)$$

$$\therefore \text{I} = \left[\frac{(x-1)^2}{2}\ln x - \frac{x^2}{4} + x - \frac{1}{2}\ln x\right]_1^3$$

$$= (2\ln 3 - \tfrac{9}{4} + 3 - \tfrac{1}{2}\ln 3) - (0 - \tfrac{1}{4} + 1 - 0)$$ ddM1

Substitutes limits of 3 and 1 and subtracts.

$$= 2\ln 3 - \tfrac{1}{2}\ln 3 + \tfrac{3}{4} + \tfrac{1}{4} - 1 \;= \underline{\tfrac{3}{2}\ln 3}\ \textbf{AG}$$ A1 cso 6

Aliter Way 4

(d) By substitution

$$u = \ln x \qquad \Rightarrow \tfrac{du}{dx} = \tfrac{1}{x}$$

$$\text{I} - \int (e^u - 1).ue^u \, du$$

Correct expression

$$= \int u\,(e^{2u} - e)\,du$$ M1

Use of 'integration by parts' formula in the correct direction

$$= u\left(\frac{1}{2}e^{2u} - e^u\right) - \int \underline{\left(\frac{1}{2}e^{2u} - e^u\right)} dx$$ A1

Correct expression

$$= u\left(\frac{1}{2}e^{2u} - e^u\right) - \underline{\left(\frac{1}{4}e^{2u} - e^u\right)}(+c)$$

Attempt to *__integrate__*; M1;
__correct integration__ A1

$$\therefore l = \left[\frac{1}{2}ue^{2u} - ue^u - \frac{1}{4}e^{2u} + e^u\right]_{\ln 1}^{\ln 3}$$

$$= \left(\tfrac{9}{2}\ln 3 - 3\ln 3 - \tfrac{9}{4} + 3\right) - (0 - 0 - \tfrac{1}{4} + 1)$$ ddM1

Substitutes limits of ln3 and ln1 and subtracts.

$$= \tfrac{3}{2}\ln 3 + \tfrac{3}{4} + \tfrac{1}{4} - 1 \;= \underline{\tfrac{3}{2}\ln 3}\ \textbf{AG}$$ A1 cso 6

$$\tfrac{3}{2}\ln 3$$

[13]

22. (a) $f'(x) = (x^2 + 1) \times \dfrac{1}{x} + \ln x \times 2x$ M1 A1

		$f'(e) = (e + 1) \times \frac{1}{e} + 2e = 3e + \frac{1}{e}$	M1 A1	4
	(b)	$(\frac{x^3}{3} + x)\ln x - \int (\frac{x^3}{3} + x)\frac{1}{x}dx$	M1 A1	
		$= (\frac{x^3}{3} + x)\ln x - \int (\frac{x^3}{3} + 1)dx$		
		$= \left[(\frac{x^3}{3} + x)\ln x - (\frac{x^3}{9} + x) \right]_1^e$	A1	
		$= \frac{2}{9}e^3 + \frac{10}{9}$	M1 A1	5

[9]

23.	(a)	$\frac{9+4x^2}{9-4x^2} = -1 + \frac{18}{(3+2x)(3-2x)}$, so $A = -1$	B1	
		Uses $18 = B(3 - 2x) + C(3 + 2x)$ and attempts to find B and C	M1	
		$B = 3$ and $C = 3$	A1 A1	4
		Or		
		Uses $9 + 4x^2 = A(9 - 4x^2) + B(3 - 2x) + C(3 + 2x)$ and attempts to find A, B and C	M1	
		$A = -1$, $B = 3$ and $C = 3$	A1, A1, A1	4
	(b)	Obtains $Ax + \frac{B}{2}\ln(3 + 2x) - \frac{C}{2}\ln(3 - 2x)$	M1 A1	
		Substitutes limits and subtracts to give $2A + \frac{B}{2}\ln(5) - \frac{C}{2}\ln(\frac{1}{5})$	M1 A1ft	
		$= -2 + 3\ln 5$ or $-2 + \ln 125$	A1	5

[9]

24.	(a)	$\frac{dx}{dt} = -3a\sin 3t$, $\frac{dy}{dt} = a\cos t$ therefore $\frac{dy}{dx} = \frac{\cos t}{-3\sin 3t}$	M1 A1	
		When $x = 0$, $t = \frac{\pi}{6}$	B1	
		Gradient is $-\frac{\sqrt{3}}{6}$	M1	
		Line equation is $(y - \frac{1}{2}a) = -\frac{\sqrt{3}}{6}(x - 0)$	M1 A1	6
	(b)	Area beneath curve is $\int a\sin t\,(-3a\sin 3t)dt$	M1	
		$= -\frac{3a^2}{2}\int(\cos 2t - \cos 4t)dt$	M1	
		$\frac{3a^2}{2}[\frac{1}{2}\sin 2t - \frac{1}{4}\sin 4t]$	M1 A1	
		Uses limits 0 and $\frac{\pi}{6}$ to give $\frac{3\sqrt{3}a^2}{16}$	A1	

Area of triangle beneath tangent is $\frac{1}{2} \times \frac{a}{2} \times \sqrt{3}a = \frac{\sqrt{3}a^2}{4}$ M1 A1

Thus required area is $\frac{\sqrt{3}a^2}{4} - \frac{3\sqrt{3}a^2}{16} = \frac{\sqrt{3}a^2}{16}$ A1 9

N.B. The integration of the product of two sines is worth 3 marks
(lines 2 and 3 of to part (b))
If they use parts

$\int \sin t \sin 3t dt$ $= -\cos t \sin 3t + \int 3\cos 3t \cos t dt$ M1

$= -\cos t \sin 3t + 3\cos 3t \sin t + \int 9 \sin 3t \sin t dt$

$8I = \cos t \sin 3t - 3\cos 3t \sin t$ M1 A1

[15]

25. Uses substitution to obtain $x = f(u)$ $\left[\frac{u^2+1}{2}\right]$, M1

and to obtain $u\frac{du}{dx}$ = const. or equiv. M1

Reaches $\int \frac{3(u^2+1)}{2u} u du$ or equivalent A1

Simplifies integrand to $\int \left(3u^2 + \frac{3}{2}\right)$ or equiv. M1

Integrates to $\frac{1}{2}u^2 + \frac{3}{2}u$

A1ft dependent on all previous Ms

Uses new limits 3 and 1 substituting and subtracting

(or returning to function of x with old limits) M1

To give 16 cso A1 8

[8]

"By Parts"

Attempt at " right direction" by parts M1

[$3x\ (2x-1)^{\frac{1}{2}} - \{ \int 3(2x-1)^{\frac{3}{2}}\ dx\}$] M1{M1A1}

.... – $(2x-1)^{\frac{3}{2}}$ M1A1ft

Uses limits 5 and 1 correctly; [42 – 26] 16 M1A1

26. Attempts $V = \pi \int x^2 e^{2x} dx$ M1

$= \pi \left[\frac{x^2 e^{2x}}{2} - \int x e^{2x} dx\right]$ (M1 needs parts in the correct direction) M1 A1

$= \pi \left[\frac{x^2 e^{2x}}{2} - \int x e^{2x} dx\right]$ (M1 needs second application of parts) M1 A1ft

M1A1ft refers to candidates $\int xe^{2x}\mathrm{d}x$, but dependent on prev. M1

$= \pi\left[\frac{x^2e^{2x}}{2} - \left(\frac{xe^{2x}}{2} - \int\frac{e^{2x}}{4}\right)\right]$ A1 cao

Substitutes limits 3 and 1 and subtracts to give... dM1
[dep. on second and third Ms]

$= \pi \left[\frac{13}{4}e^6 - \frac{1}{4}e^2\right]$ or any correct exact equivalent. A1 8

[Omission of π loses first and last marks only]

[8]

27. (a) Solves y = 0 $\Rightarrow$ cos$t$ = $\frac{1}{2}$ to obtain $t = \frac{\pi}{3}$ *or* $\frac{5\pi}{3}$

(need both for A1) M1 A1 2

Or substitutes **both** values of t and shows that $y = 0$

(b) $\frac{dx}{dt}$ = 1 – 2 cos$t$ M1 A1

Area = $\int y dx = \int_{\pi/3}^{5\pi/3}(1 - 2\cos t)\ (1 - 2\cos t)dt$

$= \int_{\pi/3}^{5\pi/3}(1 - 2\cos t)^2\ dt$ AG B1 3

(c) Area $= \int 1 - 4\cos t + 4\cos^2 t\ dt$ 3 terms M1

$\int 1 - 4\cos t + 2(\cos 2t + 1)dt$ (use of correct double angle formula) M1

$= \int 3 - 4\cos t + 2\cos 2t dt$ M1 A1

= [3t – 4 sin t + sin 2t] M1 A1

Substitutes the two correct limits $t = \frac{5\pi}{3}$ and $\frac{\pi}{3}$ and subtracts. M1

= 4π + 3 $\sqrt{3}$ A1A1 7

[12]

28. (a) $\left(\frac{x+1}{x}\right)^2 = 1 + \frac{2}{x} + \frac{1}{x^2}$ anywhere B1

$V = \pi\int\left(\frac{x+1}{x}\right)^2 dx$ M1

$\int\left(\frac{x+1}{x}\right)^2 dx = x - \frac{1}{x}, +2\ln x$ M1A1,A1

M1 attempt to $\int$

Using limits correctly in their integral:

$(\pi) = \{[x + 2\ln x - \frac{1}{x}]^3 - [x + 2\ln x - \frac{1}{x}]_1\}$ M1

$V = \pi[2^2/_3 + 2\ln 3]$ A1 7

Must be exact

(b) Volume of cone (or vol. generated by line) = $\frac{1}{3}\pi \times 2^2 \times 2$ B1

$V_R = V_S$ – volume of cone = $V_S - {}^1/_3\pi \times 2^2 \times 2$ M1

$= 2\pi\ln 3$ or $\pi\ln 9$ A1 3

[10]

29. (a) Attempt at integration by parts, i.e. $kx \sin 2x \pm \int k \sin 2x\,dx$, with $k = 2$ or ½ M1

$= \frac{1}{2}x \sin 2x - \int \frac{1}{2}\sin 2x\,dx$ A1

Integrates sin 2x correctly, to obtain $\frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + c$ M1, A1 4

(penalise lack of constant of integration first time only)

(b) Hence method: Uses $\cos 2x = 2\cos^2 x - 1$ to connect integrals B1

Obtains

$$\int x\cos^2 x\,dx = \frac{1}{2}\{\frac{x^2}{2} + \text{answer to part}(a)\} = \frac{x^2}{4} + \frac{x}{4}\sin 2x + \frac{1}{8}\cos 2x + k$$ M1A1 3

Otherwise method

$$\int x\cos^2 x\,dx = x\left(\frac{1}{4}\sin 2x + \frac{x}{2}\right) - \int \frac{1}{4}\sin 2x + \frac{x}{2}\,dx$$ B1, M1

B1 for $(\frac{1}{4}\sin 2x + \frac{x}{2})$

$$= \frac{x^2}{4} + \frac{x}{4}\sin 2x + \frac{1}{8}\cos 2x + k$$ A1 3

[10]

30. (a) $$\frac{5x+3}{(2x-3)(x+2)} = \frac{A}{2x-3} + \frac{B}{x+2}$$

$5x + 3 = A(x + 2) + B(2x - 3)$

Substituting $x = -2$ or $x = \frac{3}{2}$ and obtaining A or B; or equating coefficients and solving a pair of simultaneous equations to obtain A or B. M1

$A = 3$, $B = 1$ A1, A1 3

If the cover-up rule is used, give M1 A1 for the first of A or B found, A1 for the second.

(b) $$\int \frac{5x+3}{(2x-3)(x+2)}dx = \frac{3}{2}\ln(2x-3) + \ln(x+2)$$ M1 A1ft

$[\ldots]_2^6 = \frac{3}{2}\ln 9 + \ln 2$ M1 A1

$= \ln 54$ cao A1 5

[8]

31. $\int \frac{1}{(x-x^2)^{\frac{1}{2}}}\,\mathrm{d}x = \int \frac{1}{(1-\sin^2\theta)^{\frac{3}{2}}}\cos\theta\,\mathrm{d}\theta$ M1

Use of $x = \sin\theta$ and $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \cos\theta$

$= \int \frac{1}{\cos^2\theta}\,\mathrm{d}\theta$ M1 A1

$= \int \sec^2\theta\,\mathrm{d}\theta = \tan\theta$ M1 A1

Using the limits 0 and $\frac{\pi}{6}$ to evaluate integral M1

$[\tan\theta]_0^{\frac{\pi}{6}} = \frac{1}{\sqrt{3}}\left(=\frac{\sqrt{3}}{3}\right)$ cao A1

Alternative for final M1 A1

Returning to the variable x and using the limits 0 and $\frac{1}{2}$ to evaluate integral M1

$\left[\frac{x}{\sqrt{(1-x^2)}}\right]_0^{\frac{1}{2}} = \frac{1}{\sqrt{3}}\left(=\frac{\sqrt{3}}{3}\right)$ cao A1

[7]

32. (a) $\int x\mathrm{e}^{2x}\,\mathrm{d}x = \frac{1}{2}x\mathrm{e}^{2x} - \frac{1}{2}\int \mathrm{e}^{2x}\,\mathrm{d}x$ M1 A1

Attempting parts in the right direction

$= \frac{1}{2}x\mathrm{e}^{2x} - \frac{1}{4}\mathrm{e}^{2x}$ A1

$\left[\frac{1}{2}x\mathrm{e}^{2x} - \frac{1}{4}\mathrm{e}^{2x}\right]_0^1 = \frac{1}{4} + \frac{1}{4}\mathrm{e}^2$ M1 A1 5

(b) $x = 0.4 \Rightarrow y \approx 0.89022$
$x = 0.8 \Rightarrow y \approx 3.96243$ B1 1

Both are required to 5.d.p.

(c) $I \approx \frac{1}{2} \times 0.2 \times [\ldots]$ B1

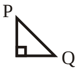
$\approx \ldots \times [0 + 7.38906 + 2(0.29836 + .89022 + 1.99207 + 3.96243)]$ M1 A1ft

ft their answers to (b)

$\approx 0.1 \times 21.67522$
≈ 2.168 cao A1 4

Note: $\frac{1}{4} + \frac{1}{4}e^2 \approx 2.097\ldots$

[10]

33.	(a)	$\sin(3x + x) = \sin 3x \cos x + \cos 3x \sin x$	M1	
		$\sin(3x - x) = \sin 3x \cos x - \cos 3x \sin x$	A1	
		(subtract) $\Rightarrow$ $\underline{\sin 4x - \sin 2a = 2\sin x \cos 3x}$	A1c.s.o.	3
	(b)	$\int 2\sin x \cos 3x \, dx = \int(\sin 4x - \sin 2x)dx$	M1	
		$= \underline{-\dfrac{\cos 4x}{4} + \dfrac{\cos 2x}{2} + c}$	A1ft	2
		ft their p, q		
	(c)	$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} 2\sin x \cos 3x dx = \left(-\frac{1}{4}\cos\frac{10\pi}{3} + \frac{1}{2}\cos\frac{5\pi}{3}\right) - \left(-\frac{1}{4}\cos 2\pi + \frac{1}{2}\cos\pi\right)$	M1	
		$= \underline{\dfrac{9}{8}}$	A1	2
			[7]	
34.	(a)	$\dfrac{dy}{dx} = \dfrac{\dot{y}}{\dot{x}} = \dfrac{3}{2t}$	M1	
		Gradient of normal is $-\dfrac{2t}{3}$	M1	
		At P $t = 2$	B1	
		$\therefore$ Gradient of normal @ P is $-\dfrac{4}{3}$	A1	
		Equation of normal @ P is $y - 9 = -\dfrac{4}{3}(x - 5)$	M1	
		Q is where $y = 0 \therefore x = \dfrac{27}{4} + 5 = \underline{\dfrac{47}{4}}$ (o.e.)	A1	6
	(b)	Curved area = $\int y dx = \int y \dfrac{dx}{dt} dt$	M1	
		$= \int 3(1 + b).2t \, dt$	A1	
		$= [3t^2 + 2t^3]$	M1A1	
		Curve cuts x-axis when $t = -1$	B1	
		Curved area = $[3t^2 + 2t^3]_{-1}^{2} = (12 + 16) - (3 - 2)$ (= 27)	M1	
		Area of  triangle = $\dfrac{1}{2}((a) - 5) \times 9$ (= 30.375)	M1	
		Total area of R = curved area + Δ	M1	
		= 57.375 or AWRT $\underline{57.4}$	A1	9
			[15]	
35.	(a)	$I = \int x \operatorname{cosec}^2(x + \dfrac{\pi}{6})dx = \int x d(-\cot(x + \dfrac{\pi}{6}))$	M1	
		$= -x \cot(x + \dfrac{\pi}{6}) + \int \cot(x + \dfrac{\pi}{6})dx$	A1	

$= \underline{-x\cot(x+\frac{\pi}{6}) + \ln(\sin(x+\frac{\pi}{6}) / c}$ (*) A1c.s.o. 3

(b) $\int \frac{1}{y(1+y)}\mathrm{d}x = \int 2x\mathrm{cosec}^2(x+\frac{\pi}{6})\mathrm{d}x$ M1

LHS = $\int\left(\frac{1}{y} - \frac{1}{1+y}\right)\mathrm{d}y$ M1 A1

$\therefore \ln y - \ln|1+y|$ or $\ln\left|\frac{y}{1+y}\right|$ = 2(a) M1 M1

$\therefore \underline{\frac{1}{2}\ln\left|\frac{y}{1+y}\right| = -x\cot\left(x+\frac{\pi}{6}\right) + \ln\left|\sin\left(x+\frac{\pi}{6}\right)\right| + c}$ (*) A1c.s.o. 6

(c) $y = 1, x = 0 \Rightarrow \frac{1}{2}\ln\frac{1}{2} = \ln\left(\sin\frac{\pi}{6}\right) + c$ M1

$\therefore c = -\frac{1}{2}\ln\frac{1}{2}$ A1 M1

$x = \frac{\pi}{12} \Rightarrow \frac{1}{2}\ln\left|\frac{y}{1+y}\right| = -\frac{\pi}{12}.1 + \ln\frac{1}{\sqrt{2}} - \frac{1}{2}\ln\frac{1}{2}$ A1

$\sqrt{\text{"}c\text{"}}$

(i.e. $\ln\left|\frac{y}{1+y}\right| = -\frac{\pi}{6}$)

$\frac{y+1}{y} = e^{\frac{\pi}{6}}$ M1

(o.e.)

$1 = y(e^{\frac{\pi}{6}} - 1)$

$\therefore y = \underline{\frac{1}{e^{\frac{\pi}{6}} - 1}}$ A1 6

(o.e.)

[15]

36. Use of $V = \pi\int y^2\mathrm{d}x$ M1

$y^2 = 16x^2 + \frac{36}{x^2}$; –48

Integrating to obtain $(\pi)\left[\frac{16x^3}{3} - \frac{36}{x}; -48x\right]$ ft constants only M1 A1ft; A1ft

$(\pi)\left[\frac{16x^3}{3} - \frac{36}{x} - 48x\right]_2^4 = (\pi)\left[140\frac{1}{3} - (-71\frac{1}{3})\right]$ correct use of limits M1

$V = 211\frac{2}{3}\pi$ (units^3) A1 8

[8]

37. (a) $y^2 = \left(\frac{x+2}{\sqrt{x}}\right)^2 = \frac{x^2+4x+4}{x} = x + 4 + \frac{4}{x}$ M1A1

$\pi\int y^2 \mathrm{d}x$ [dependent on attempt at squaring y] B1

$\int y^2 \mathrm{d}x = \int (\frac{x^2+4x+4}{x})\mathrm{d}x; = \frac{x^2}{2} + 4x + 4\ln x$ M1;A1 ft

[A1√ must have ln x term]

Correct use of limits: $[\ ]_1^4 = [\ ]^4 - [\ ]_1$ M1

[M dependent on prev. M1]

Volume = $\left(\frac{39}{2} + 4\ln 4\right)\pi$ or equivalent exact A1 7

(b) Showing that $y = 3$ at $x = 1$ and $x = 4$ B1 1

(c) Volume = 2^3 × answer to (a) ; = 629.5 cm^3 ≈ 630 cm^3 (*) M1;A1 2

[allow 629 – 630]

[10]

38. $u = 1 + \sin x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \cos x$ or $\mathrm{d}u = \cos x\,\mathrm{d}x$ or $\mathrm{d}x = \frac{\mathrm{d}u}{\cos x}$ M1

$I = \int (u-1)u^5 \mathrm{d}u$ M1, A1

Full sub. to I in terms of u, correct

$= \int (u^6 - u^5)\,\mathrm{d}u$ M1

Correct split

$= \frac{u^7}{7} - \frac{u^6}{6}$ $(+ c)$ M1, A1

M1 for $u^n \rightarrow u^{n+1}$

$= \frac{u^6}{42}(6u - 7)$ $(+ c)$ M1

Attempt to factorise

$= \frac{(1+\sin x)^6}{42}(6\sin x + 6 - 7) + c \rightarrow \underline{\frac{(1+\sin x)^6}{42}}$ <u>(6 sin x – 1) (+ c)</u> (*) A1 cso

[8]

Alt: <u>Integration by parts</u> M1

		$I = (u-1)\frac{u^6}{6} - \frac{1}{6}\int u^6\,\mathrm{d}u$ Attempt first stage	M1	
		$= (u-1)\frac{u^6}{6} - \frac{u^7}{42}$ Full integration	A1	
		$(= \frac{u^7}{6} - \frac{u^6}{6} - \frac{u^7}{42}$ <u>or</u> $\frac{6u^7-7u^6}{42}$) rest as scheme		
39.	(a)	$4 = 2\sec t \Rightarrow \cos t = \frac{1}{2}, \Rightarrow t = \frac{\pi}{3}$	M1, A1	
		$\therefore a = 3\times\frac{\pi}{3}\times\sin\frac{\pi}{3} = \underline{\frac{\pi\sqrt{3}}{2}}$	B1	3
	(b)	$A = \int_0^a y\,\mathrm{d}x = \int y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t$ *Change of variable*	M1	
		$= \int 2\sec t \times [3\sin t + 3t\cos t]\,\mathrm{d}t$ *Attempt* $\frac{\mathrm{d}x}{\mathrm{d}t}$	M1	
		$= \int_0^{\frac{\pi}{3}} (6\tan t, + 6t)\,\mathrm{d}t$ (*) *Final A1 requires limit <u>stated</u>*	A1, A1cso	
	(c)	$A = [6\ln\sec t + 3t^2]_0^{\frac{\pi}{3}}$ *Some integration (M1) both correct (A1) ignore lim.*	M1, A1	
		$= (6\ln 2 + 3\times\frac{\pi^2}{9}) - (0)$ Use of $\frac{\pi}{3}$	M1	
		$= \underline{6\ln 2 + \frac{\pi^2}{3}}$	A1	4
			[11]	
40.	(a)	Area of triangle $= \frac{1}{2}\times 30\times 3\pi^2$ (= 444.132) *Accept 440 or 450*	B1	1
	(b)	**Either** Area shaded $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 30\sin 2t.32t\mathrm{d}t$	M1 A1	
		$= [-480t\cos 2t + \int 480\cos 2t]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	M1 A1	

$= [-480t\cos 2t + 240\sin 2t]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ A1 ft

$= 240(\pi - 1)$ M1 A1 7

or $\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} 60\cos 2t.(16t^2 - \pi^2)\mathrm{d}t$ M1 A1

$= [(30\sin 2t\,(\pi^2 - 16t^2) - 480t\cos 2t + \int 480\cos 2t]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ M1 A1

$= [-480t\cos 2t + 240\sin 2t]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ A1 ft

$= 240(\pi - 1)$ M1 A1 7

(c) Percentage error = $\dfrac{240(\pi-1) - \text{estimate}}{240(\pi-1)} \times 100 = 13.6\%$ M1 A1 2

(Accept answers in the range 12.4% to 14.4%)

[10]

41. (a) $\dfrac{8}{x} - x^2 = 0 \Rightarrow x^3 = 8 \Rightarrow x = 2$ M1 A1 2

(b) $\left(\dfrac{8}{x} - x^2\right)^2 = x^4 - 16x + \dfrac{64}{x^2}$ M1

$\int (x^4 - 16x + 64x^{-2})\mathrm{d}x = \dfrac{x^5}{5} - 8x^2 - \dfrac{64}{x}$ M1 A1

$\left[\dfrac{x^5}{5} - 8x^2 - \dfrac{64}{x}\right]_1^2 = \left(\dfrac{32}{5} - 32 - 32\right) - \left(\dfrac{1}{5} - 8 - 64\right)$ M1 A1 ft

Volume is $\dfrac{71}{5}\pi$ (units^3) A1 7

[9]

42. (a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{c}{x^2}$ M1

Attempt $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

When $x = p \Rightarrow -4 = -\dfrac{c}{p^2}$ $\therefore$ $\underline{c = 4p^2}$ (*) A1 cso 2

(b) $5 = 1 + \dfrac{c}{p}$ and solve with $c = 4p^2$ M1

$5 = 1 + 4p \Rightarrow p = 1$ $\therefore$ $\underline{c = 4}$ (*) A1 cso 2

(c) $y^2 = 1 + \dfrac{8}{x} + \dfrac{16}{x^2}$ M1

y^2 = ; ≥ 2 terms correct

$$\int y^2 \mathrm{d}x = \left[x + 8\ln x - \frac{16}{x} \right]$$

some correct ∫ M1
all correct A1

$$\int_1^2 y^2 \mathrm{d}x = (2 + 8\ln 2 - \frac{16}{2}) - (1 + 8\ln 1 - 16)$$ M1

Use of correct limits

= 9 + 8ln2

$V = \pi \int_1^2 y^2 \mathrm{d}x$; ∴ $\underline{V = \pi(9 + 8\ln 2)}$

$V = \pi\int y^2 dx$ B1
k = 9; A1
q = 8 A1 7

[11]

43. (a) M is (0, 7) B1

$\frac{\mathrm{d}y}{\mathrm{d}x} = 2e^x$ M1

Attempt $\frac{\mathrm{d}y}{\mathrm{d}x}$

∴ gradient of normal is $-\frac{1}{2}$ M1

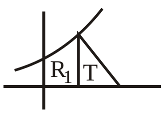
ft their y′(0) or = $-\frac{1}{2}$

(Must be a number)

∴ equation of normal is $y - 7 = -\frac{1}{2}(x - 0)$ or $\underline{x + 2y - 14 = 0}$
$\underline{x + 2y = 14}$ o.e. A1 4

(b) $y = 0$, $x = 14$ ∴ $\underline{N \text{ is } (14, 0)}$ (*) B1 cso 1

(c)



$\int (2e^x + 5)\, \mathrm{d}x = [2e^x + 5x]$ M1
some correct ∫

$R_1 = \int_0^{\ln 4} (2e^x + 5)\, \mathrm{d}x = (2 \times 4 + 5 \ln 4) - (2 + 0)$ M1

limits used

= 6 + 5 ln 4 A1

		$T = \frac{1}{2} \times 13 \times (14 - \ln 4)$ *Area of T*	B1	
		$T = 13(7 - \ln 2)$; $R_1 = 6 + 10 \ln 2$ *Use of ln 4 = 2ln 2*	B1	
		$R = T + R_1$, $\underline{R = 97 - 3 \ln 2}$	M1, A1	7
			[12]	
44.	(a)	$\frac{dy}{dx} = \sqrt{\sin x} + \frac{x}{2}(\sin x)^{-\frac{1}{2}} \cos x$	M1, A1	
		At A $\sqrt{\sin x} + \frac{x}{2}(\sin x)^{-\frac{1}{2}} \cos x = 0$	dM1	
		$\therefore \sin x + \frac{x}{2} \cos x = 0$ (essential to see intermediate line before given answer)		
		$\therefore 2\tan x + x = 0$ (*)	A1	4
	(b)	$V = \pi \int y^2 dx = \pi \int x^2 \sin x dx$	M1	
		$= \pi \left[-x^2 \cos x + \int 2x \cos x dx\right]_0^\pi$	M1 A1	
		$= \pi \left[-x^2 \cos x + 2x \sin x - \int 2 \sin x dx\right]_0^\pi$	M1	
		$= \pi \left[-x^2 \cos x + 2x \sin x + 2 \cos x\right]_0^\pi$	A1	
		$= \pi [\pi^2 - 2 - 2]$	M1	
		$= \pi [\pi^2 - 4]$	A1	7
			[11]	
45.	(a)	Method using either $\frac{A}{(1-x)} + \frac{B}{(2x+3)} + \frac{C}{(2x+3)^2}$ **or** $\frac{A}{1-x} + \frac{Dx+E}{(2x+3)^2}$	M1	
		A = 1 C = 10 , B = 2 **or** D = 4 and E = 16	B1, A1, A1	4
	(b)	$\int [\frac{1}{1-x} + \frac{2}{2x+3} + 10(2x+3)^{-2}] dx$ **or** $\int \frac{A}{1-x} + \frac{Dx+E}{(2x+3)^2} dx$	M1	
		$-\ln\|1 - x\| + \ln\|2x + 3\| - 5(2x + 3)^{-1}$ $(+c)$ **or**		
		$-\ln\|1 - x\| + \ln\|2x + 3\| - (2x + 8)(2x + 3)^{-1}$ $(+c)$	M1 A1 ftA1 ftA1 ft	
	(c)	Either $(1 - x)^{-1} + 2(3 + 2x)^{-1} + 10(3 + 2x)^{-2} =$	M1	
		$1 + x + x^2 + \ldots$	A1 ft	
		$+\frac{2}{3}(1 - \frac{2x}{3} + \frac{4x^2}{9} \ldots)$	M1 A1 ft	
		$+\frac{10}{9}(1 + (-2)(\frac{2x}{3}) + \frac{(-2)(-3)}{2}(\frac{2x}{3})^2 + \ldots)$	A1 ft	

$$=\frac{25}{9}-\frac{25}{27}x+\frac{25}{9}x^2\ldots$$ M1 A1 7

Or

$25[(9 + 12x + 4x^2)(1 - x)]^{-1} = 25[(9 + 3x - 8x^2 - 4x^3)]^{-1}$ M1 A1

$$\frac{25}{9}\left[1+\frac{3x}{9}-\frac{8x^2}{9}-\frac{4x^3}{9}\right]^{-1}=\frac{25}{9}\left[1-\left(\frac{3x}{9}-\frac{8x^2}{9}-\frac{4x^3}{9}\right)+\left(\frac{x^2}{9}..\right).\right]$$ M1 A1 A1

$$=\frac{25}{9}-\frac{25}{27}x+\frac{25}{9}x^2$$ M1, A1 7

[16]

46. Volume = $\pi\int_1^4\left(1+\frac{1}{2\sqrt{x}}\right)^2\mathrm{d}x$ M1

$$\int\left(1+\frac{1}{2\sqrt{x}}\right)^2\mathrm{d}x=\int\left(1+\frac{1}{\sqrt{x}}+\frac{1}{4x}\right)\mathrm{d}x$$ B1

$$=\left[x+2\sqrt{x}+\frac{1}{4}\ln x\right]$$ M1 A1 A1 ft

Using limits correctly M1

Volume = $\pi\left[\left(8+\frac{1}{4}\ln 4\right)+3\right]$ A1

$$=\pi\left[5+\frac{1}{2}\ln 2\right]$$ A1 8

[8]

47. (a) $\frac{\mathrm{d}y}{\mathrm{d}x}=2e^{-2x}\sqrt{x}+\frac{e^{-2x}}{2\sqrt{x}}$ M1 A1 A1

Putting $\frac{\mathrm{d}y}{\mathrm{d}x}$ = 0 and attempting to solve dM1

$x = \frac{1}{4}$ A1 5

(b) Volume = $\pi\int_0^1(\sqrt{x}e^{-2x})^2\,\mathrm{d}x=\pi\int_0^1 xe^{-4x}\mathrm{d}x$ M1 A1

$$\int xe^{-4x}\mathrm{d}x=-\frac{1}{4}xe^{-4x}+\int\frac{1}{4}e^{-4x}\mathrm{d}x$$ M1 A1

$$=-\frac{1}{4}xe^{-4x}-\frac{1}{16}e^{-4x}$$ A1 ft

Volume = $\pi\left[-\frac{1}{4}e^{-4}-\frac{1}{16}e^{-4}\right]-\left[-\frac{1}{16}\right]=\frac{\pi}{16}[1-5e^{-4}]$ M1 A1 7

[12]

48. Volume = $\pi\int_{\frac{1}{2}}^{4}\left(2+\frac{1}{x}\right)^2 \mathrm{d}x$, = $\pi\int_{\frac{1}{2}}^{4}\left(4+\frac{4}{x}+\frac{1}{x^2}\right)\mathrm{d}x$ M1, M1

$= \pi\left[4x+4\ln x-x^{-1}\right]_{\frac{1}{2}}^{4}$ A1 A1

$= \pi\left[16+4\ln 4-\frac{1}{4}-2-4\ln\frac{1}{2}+2\right]$ M1 A1

$= \pi[15.75 + 12 \ln 2]$ A1

[7]

49. Separating the variables

$\int\frac{\mathrm{d}y}{y}=\int x^2\cos x\,dx$ M1

$\text{LHS}=\ln y$ B1

$\text{RHS}=x^2\sin x-\int 2x\sin x\,\mathrm{d}x$ M1 A1 A1

$= x^2\sin x - 2[x(-\cos x) - \int 1(-\cos x)\,\mathrm{d}x]$ A1

$= x^2\sin x + 2x\cos x - 2\sin x + c$ A1

y =1 at $x = \pi$ gives

$0 = 0 + 2\pi(-1) - 0 + c$

$\Rightarrow c = 2\pi$

$\ln y = x^2\sin x + 2x\cos x - 2\sin x + 2\pi$ M1 A1

[9]

50. (a) $A(2-x) + B(1+4x) = 5x + 8$ M1

$x = 2$ $9B = 18$ $\Rightarrow B = 2$ A1

$x = -\frac{1}{4}$ $\frac{9A}{4}=\frac{27}{4}$ $\Rightarrow A = 3$ A13

(b) Area = $\int_0^{\frac{1}{2}}\mathrm{g}(x)\,\mathrm{d}x$ + (attempt to integrate)

$= 3\int \frac{\mathrm{d}x}{(1+4x)} + 2\int \frac{\mathrm{d}x}{(2-x)}$

$= \frac{3}{4}\left[\ln(1+4x)\right]_0^{\frac{1}{2}} - 2\left[\ln(2-x)\right]_0^{\frac{1}{2}}$ A1 A1

$= \frac{3}{4}$ ln 3 – 2 ln ($\frac{3}{2}$) + 2 ln 2 M1 A1 A1

$= \frac{3}{4}$ ln 3 – 2 ln 3 + 2 ln 2 + 2 ln 2

= 4 ln 2 – $\frac{5}{4}$ ln 3 A17

[10]

51. $u^2 = x - 1$

$2u \frac{\mathrm{d}u}{\mathrm{d}x} = 1$ $x = u^2 + 1$ M1 A1 A1

$I = \int \frac{(u^2+1)^2}{u} 2u \,\mathrm{d}u$

$= 2\int (u^4 + 2u^2 + 1)\,\mathrm{d}u$ A1

$= 2\left[\frac{u^5}{5} + \frac{2u^3}{3} + u\right] + c$ M1 A1

$= \frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{4}{3}(x-1)^{\frac{3}{2}} + 2(x-1)^{\frac{1}{2}} + c$ M1 A110

[10]

52. Uses $\frac{\mathrm{d}u}{\mathrm{d}x} = 6x$ M1

To give $\int \frac{1}{u^2} \frac{\mathrm{d}u}{3}$ A1

Integrates to give $-\frac{1}{3u}$ M1, A1

Uses correct limits 16 and 4 (or 2 and 0 for x) M1

To obtain $-\frac{1}{48} + \frac{1}{12} = \frac{1}{16}$ A16

[6]

53. (a) $\frac{1+14x}{(1-x)(1+2x)} \equiv \frac{A}{1-x} + \frac{B}{1+2x}$ and attempt A and or B M1

$A = 5, B = -4$ A1, A13

(b) $\int \frac{5}{1-x} - \frac{4}{1+2x}\, dx = [-5\ln|1-x| - 2\ln|1+2x|]$ M1 A1

$= (-5\ln\frac{2}{3} - 2\ln\frac{5}{3}) - (-5\ln\frac{5}{6} - 2\ln\frac{4}{3})$ M1

$= 5\ln\frac{5}{4} + 2\ln\frac{4}{5}$

$= 3\ln\frac{5}{4} = \ln\frac{125}{64}$ M1 A15

(c) $5(1-x)^{-1} - 4(1+2x)^{-1}$ B1 ft

$= 5(1 + x + x^2 + x^3) - 4$

$(1 - 2x + \frac{(-1)(-2)(2x)^2}{2} + \frac{(-1)(-2)(-3)(2x)^3}{6} + \ldots)$ M1 A1

$= 1 + 13x - 11x^2 + 37x^3 \ldots.$ M1 A15

[13]

54. (a) $R = \int_{\pi}^{2\pi} x^2 \sin\left(\frac{1}{2}x\right) dx = -2x^2\cos\left(\frac{1}{2}x\right) + \int 4x\cos\left(\frac{1}{2}x\right) dx$ M1 A1

$= -2x^2\cos\left(\frac{1}{2}x\right) + 8x\sin\left(\frac{1}{2}x\right) - \int 8\sin\left(\frac{1}{2}x\right)$ M1 A1

$= -2x^2\cos\left(\frac{1}{2}x\right) + 8x\sin\left(\frac{1}{2}x\right) + 16\cos\left(\frac{1}{2}x\right)$ A1

Use limits to obtain $[8\pi^2 - 16] - [8\pi]$ M1 A17

(b) Requires 11.567 B11

(c) (i) Area = $\frac{\pi}{4}$, [9.8696 + 0 + 2 × 15.702]

(B1 for $\frac{\pi}{4}$ in (i) or $\frac{\pi}{8}$ in (ii)) B1, M1

= 32.42 A1

(ii) Area = $\frac{\pi}{8}$ [9.8696 + 0 + 2(14.247 + 15.702 + 11.567)] M1

= 36.48A1 5

[13]

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EXAMINERS' REPORTS

1. This question was generally well done and, helped by the printed answer, many produced fully correct answers. The commonest error was to omit the negative sign when differentiating

$\cos x + 1$. The order of the limits gave some difficulty. Instead of the correct $-\int_1^2 e^u \, du$, an incorrect version $-\int_1^2 e^u \, du$ was produced and the resulting expressions manipulated to the printed result and working like $-(e^2 - e^1) = -e^2 + e^1 = e(e - 1)$ was not uncommon.

Some candidates got into serious difficulties when, through incorrect algebraic manipulation, they obtained $-\int e^u \sin^2 x \, du$ instead of $-\int e^u \, du$. This led to expressions such as $\int e^u (u^2 - 2u) \, du$ and the efforts to integrate this, either by parts twice or a further substitution, often ran to several supplementary sheets. The time lost here inevitably led to difficulties in finishing the paper. Candidates need to have some idea of the amount of work and time appropriate to a 6 mark question and, if they find themselves exceeding this, realise that they have probably made a mistake and that they would be well advised to go on to another question.

2. Candidates tended either to get part (a) fully correct or make no progress at all. Of those who were successful, most replaced the $\cos^2 \theta$ and $\sin^2 \theta$ directly with the appropriate double angle formula. However many good answers were seen which worked successfully via $7\cos^2 \theta - 3$ or $4 - 7\sin^2 \theta$.

Part (b) proved demanding and there were candidates who did not understand the notation $\theta f(\theta)$. Some just integrated $f(\theta)$ and others thought that $\theta f(\theta)$ meant that the argument 2θ in $\cos 2\theta$ should be replaced by θ and integrated $\frac{1}{2}\theta + \frac{7}{2}\cos\theta$. A few candidates started by writing $\int \theta f(\theta) d\theta = \theta \int f(\theta) d\theta$, treating θ as a constant. Another error seen several times was

$$\int \theta \, f(\theta) d\theta = \int \left(\frac{1}{2}\theta + \frac{7}{2}\cos 2\theta^2 \right) d\theta .$$

Many candidates correctly identified that integration by parts was necessary and most of these were able to demonstrate a complete method of solving the problem. However there were many errors of detail, the correct manipulation of the negative signs that occur in both integrating by parts and in integrating trigonometric functions proving particularly difficult. Only about 15% of candidates completed the question correctly.

3. Part (a) of this question proved awkward for many. The integral can be carried out simply by decomposition, using techniques available in module C1. It was not unusual to see integration by parts attempted. This method will work if it is known how to integrate $\ln x$, but this requires a further integration by parts and complicates the question unnecessarily. In part (b), most could separate the variables correctly but the integration of $\frac{1}{y^{\frac{1}{3}}}$, again a C1 topic, was frequently incorrect.

Weakness in algebra sometimes caused those who could otherwise complete the question to lose the last mark as they could not proceed from $y^{\frac{2}{3}} = 6x + 4 1n x - 2$ to $y^2 = (6x + 4 1n x - 2)^3$. Incorrect answers, such as $y^2 = 216x^3 + 64 \ln x^3 - 8$, were common in otherwise correct solutions.

4. Answers to part (a) were mixed, although most candidates gained some method marks. A surprisingly large number of candidates failed to deal with $\sqrt{4 - 4\cos^2 u}$ correctly and many did not recognise that

$\int \frac{1}{\cos^2 x} dx = \int \sec^2 x \, dx = \tan x (+C)$ in this context. Nearly all converted the limits correctly. Answers to part (b) were also mixed. Some could not get beyond stating the formula for the volume of revolution while others gained the first mark, by substituting the equation given in part (b) into this formula, but could not see the connection with part (a). Candidates could recover here and gain full follow through marks in part (b) after an incorrect attempt at part (a).

5. Nearly all candidates gained both marks in part (a). As is usual, the main error seen in part (b) was finding the width of the trapezium incorrectly. There were fewer errors in bracketing than had been noted in some recent examinations and nearly all candidates gave the answer to the specified accuracy. The integration by parts in part (c) was well done and the majority of candidates had been well prepared for this topic.

Some failed to simplify $\int \frac{x^2}{2} \times \frac{1}{x} dx$ to $\int \frac{x}{2} dx$ and either gave up or produced $\frac{\frac{1}{3}x^3}{x^2}$.

In evaluating the definite integral some either overlooked the requirement to give the answer in the form $\frac{1}{4}(a 1n 2 + b)$ or were unable to use the appropriate rule of logarithms correctly.

6. Most candidates could gain the mark in part (a) although 2.99937, which arises from an incorrect angle mode, was seen occasionally. The main error seen in part (b) was finding the width of the trapezium incorrectly, $\frac{3\pi}{10}$ being commonly seen instead of $\frac{3\pi}{8}$.

This resulted from confusing the number of values of the ordinate, 5, with the number of strips, 4. Nearly all candidates gave the answer to the specified accuracy. In part (c), the great majority of candidates recognised that they needed to find $\int 3\cos\left(\frac{x}{3}\right)\mathrm{d}x$ and most could integrate correctly. However sin x, 9sin x, 3sin $\left(\frac{x}{3}\right), -9\sin\left(\frac{x}{3}\right), -\sin\left(\frac{x}{3}\right) \text{and} -3\sin\left(\frac{x}{3}\right)$ were all seen from time to time. Candidates did not seem concerned if their answers to part (b) and part (c) were quite different, possibly not connecting the parts of the question. Despite these difficulties, full marks were common and, generally, the work on these topics was sound.

7. Part (a) was well done with the majority choosing to substitute values of x into an appropriate identity and obtaining the values of A, B and C correctly. The only error commonly seen was failing to solve $5 = \frac{5}{4}A$ for A correctly. Those who formed simultaneous equations in three unknowns tended to be less successful. Any incorrect constants obtained in part (a) were followed through for full marks in part (b)(i). Most candidates obtained logs in part (b)(i). The commonest error was, predictably, giving $\int \frac{4}{2x+1}dx = 4\ln(2x+1)$, although this error was seen less frequently than in some previous examinations. In indefinite integrals, candidates are expected to give a constant of integration but its omission is not penalised repeatedly throughout the paper. In part (b)(ii) most applied the limits correctly although a minority just ignored the lower limit 0. The application of log rules in simplifying the answer was less successful. Many otherwise completely correct solutions gave 3 ln 3 as ln 9 and some "simplified" 3 ln 5 – 4 ln 3 to $\frac{3}{4}\ln\left(\frac{5}{3}\right)$.

8. Throughout this question sign errors were particularly common. In part (a), nearly all recognised that $(5-x)^{\frac{3}{2}}$ formed part of the answer, and this gained the method mark, but $\frac{3}{2}(5-x)^{\frac{3}{2}}, -\frac{3}{2}(5-x)^{\frac{3}{2}} \text{and} \frac{2}{3}(5-x)^{\frac{3}{2}}$, instead of the correct $-\frac{2}{3}(5-x)^{\frac{3}{2}}$, were all frequently seen. Candidates who made these errors could still gain 3 out of the 4 marks in part (b)(i) if they proceeded correctly. Most candidates integrated by parts the "right way round" and were able to complete the question. Further sign errors were, however, common.

9. The responses to this question were very variable and many lost marks through errors in manipulation or notation, possibly through mental tiredness. For examples, many made errors in manipulation and could not proceed correctly from the printed cos 2θ = 1 – 2sin$^2\theta$ to sin^2 $\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$ and the answer $\frac{x}{2} - \frac{1}{4}\sin 2\theta$ was often seen, instead of $\frac{\theta}{2} - \frac{1}{4}\sin 2\theta$. In part (b), many never found $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ or realised that the appropriate form for the volume was $\pi\int y^2\frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta.$

However the majority did find a correct integral in terms of θ although some were unable to use the identity sin 2θ = 2sinθ cosθ to simplify their integral. The incorrect value $k = 8\pi$ was very common, resulting from a failure to square the factor 2 in sin 2θ = 2sinθ cosθ. Candidates were expected to demonstrate the correct change of limits. Minimally a reference to the result tan $\frac{\pi}{6} = \frac{1}{\sqrt{3}}$, or an equivalent, was required. Those who had complete solutions usually gained the two method marks in part (c) but earlier errors often led to incorrect answers.

10. Q2 was generally well answered with many successful attempts seen in both parts. There were few very poor or non-attempts at this question.

In part (a), a significant minority of candidates tried to integrate $3(1+4x)^{\frac{1}{2}}$. Many candidates, however, correctly realised that they needed to integrate $3(1+4x)^{-\frac{1}{2}}$. The majority of these candidates were able to complete the integration correctly or at least achieve an integrated expression of the form $k(1+4x)^{\frac{1}{2}}$. Few candidates applied incorrect limits to their integrated expression. A noticeable number of candidates, however, incorrectly assumed a subtraction of zero when substituting for $x = 0$ and so lost the final two marks for this part. A minority of candidates attempted to integrate the expression in part (a) by using a substitution. Of these candidates, most were successful.

In part (b), the vast majority of candidates attempted to apply the formula $\pi\int y^2\mathrm{d}x$, but a few of them were not successful in simplifying y^2. The majority of candidates were able to integrate $\frac{9}{1+4x}$ to give $\frac{9}{4}\ln|1+4x|$. The most common error at this stage was for candidates to omit dividing by 4. Again, more candidates were successful in this part in substituting the limits correctly to arrive at the exact answer of $\frac{9}{4}\pi\ln 9$. Few candidates gave a decimal answer with no exact term seen and lost the final mark.

11. In part (a), a surprisingly large number of candidates did not know how to integrate $\tan^2 x$. Examiners were confronted with some strange attempts involving either double angle formulae or logarithmic answers such as $\ln(\sec^2 x)$ or $\ln(\sec^4 x)$. Those candidates who realised that the needed the identity $\sec^2 x = 1 + \tan^2 x$ sometimes wrote it down incorrectly.

Part (b) was probably the best attempted of the three parts in the question. This was a tricky integration by parts question owing to the term of $\frac{1}{x^3}$, meaning that candidates had to be especially careful when using negative powers. Many candidates applied the integration by parts formula correctly and then went on to integrate an expression of the form $\frac{k}{x^3}$ to gain 3 out of the 4 marks available. A significant number of candidates failed to gain the final accuracy mark owing to sign errors or errors with the constants α and β in $\frac{\alpha}{x^2}\ln x + \frac{\beta}{x^2} + c$. A minority of candidates applied the by parts formula in the 'wrong direction' and incorrectly stated that $\frac{\mathrm{d}v}{\mathrm{d}h} = \ln x$ implied $v = \frac{1}{x}$.

In part (c), most candidates correctly differentiated the substitution to gain the first mark. A significant proportion of candidates found the substitution to obtain an integral in terms of u more demanding. Some candidates did not realise that e^{2x} and e^{3x} are $(e^x)^2$ and $(e^x)^3$ respectively and hence $u^2 - 1$, rather than $(u-1)^2$ was a frequently encountered error seen in the numerator of the substituted expression. Fewer than half of the candidates simplified their substituted expression to arrive at the correct result of $\int\frac{(u-1)^2}{u}\mathrm{d}u$.
Some candidates could not proceed further at this point but the majority of the candidates who achieved this result were able to multiply out the numerator, divide by u, integrate and substitute back for u. At this point some candidates struggled to achieve the expression required. The most common misconception was that the constant of integration was a fixed constant to be determined, and so many candidates concluded that $k = -\frac{3}{2}$. Many candidates did not realise that $-\frac{3}{2}$ when added to c combined to make another arbitrary constant k.

12. In part (a), many candidates were able to use integration by parts in the right direction to produce a correct solution. Common errors included integrating e incorrectly to give ln x or applying the by parts formula in the wrong direction by assigning u as e^x to be differentiated and $\frac{\mathrm{d}v}{\mathrm{d}x}$ as x to be integrated.

Many candidates were able to make a good start to part (b), by assigning u as x^2 and $\frac{\mathrm{d}v}{\mathrm{d}x}$ and e^x again correctly applying the integration by parts formula. At this point, when faced with integrating $2xe^x$, some candidates did not make the connection with their answer to part (a) and made little progress, whilst others independently applied the by parts formula again. A significant proportion of candidates made a bracketing error and usually gave an incorrect answer of $e^x(x^2 - 2x - 2) + c$.

In part (b), a few candidates proceeded by assigning u as x and $\frac{\mathrm{d}v}{\mathrm{d}x}$ as xe^x and then used their answer to part (a) to obtain v. These candidates were usually produced a correct solution.

13. Most candidates used the correct volume formula to obtain an expression in terms of x for integration. At this stage errors included candidates using either incorrect formulae of $\pi\int y\mathrm{d}x, 2\pi\int y^2\mathrm{d}x$ or $\int y^2\mathrm{d}x$. Many candidates realised that they needed to integrate an expression of the form $(2x + 1)^{-2}$ (or equivalent). The majority of these candidates were able to complete the integration correctly or at least achieve an integrated expression of the form
$p(1 + 2x)^{-1}$. A few candidates, however, integrated to give an expression in terms of natural logarithms. A significant minority of candidates substituted the limits of $b$and a into their integrand the wrong way round. Only a minority of candidates were able to combine together their rational fractions to give an answer as a single simplified fraction as required by the question.

14. It was clear to examiners that a significant proportion of candidates found part (i) unfamiliar and thereby struggled to answer this part. Weaker candidates confused the integral of ln x with the differential of ln x. It was therefore common for these candidates to write down the integral of ln x as $\frac{1}{x}$, or the integral of ln $\left(\frac{x}{2}\right)$ as either $\frac{2}{x}$ or $\frac{4}{x}$. A significant proportion of those candidates, who proceeded with the expected by parts strategy, differentiated ln $\left(\frac{x}{2}\right)$ incorrectly to give either or 2 $x$and usually lost half the marks available in this part. Some candidates decided from the outset to rewrite ln $\left(\frac{x}{2}\right)$ as 'ln x – ln 2', and proceeded to integrate each term and were usually more successful with integrating ln x than ln 2. It is pleasing to report that a few determined candidates were able to produce correct solutions by using a method of integration by substitution. They proceeded by either using the substitution as

$u = \frac{x}{2}$ or $u = \ln\left(\frac{x}{2}\right)$.

A significant minority of candidates omitted the constant of integration in their answer to part (i) and were penalised by losing the final accuracy mark in this part.

In part (ii), the majority of candidates realised that they needed to consider the identity
$\cos 2x \equiv 1 - 2\sin^2 x$ and so gained the first method mark. Some candidates misquoted this formula or incorrectly rearranged it. A majority of candidates were then able to integrate
$\frac{1}{2}(1 - \cos 2x)$, substitute the limits correctly and arrive at the correct exact answer.

There were, however, a few candidates who used the method of integration by parts in this part, but these candidates were usually not successful in their attempts.

15. Many candidates had difficulties with the differentiation of the function $u = 2^x$, despite the same problem being posed in the January 2007 paper, with incorrect derivatives of $\frac{\mathrm{d}u}{\mathrm{d}x} = 2^x$ and $\frac{\mathrm{d}u}{\mathrm{d}x} = x\,2^{x-1}$ being common. Those candidates who differentiated u with respect to x to obtain either $2^x \ln 2$ or 2^x often failed to replace 2^x with u; or if they did this, they failed to cancel the variable u from the numerator and the denominator of their algebraic fraction. Therefore, at this point candidates proceeded to do some "very complicated" integration, always with no chance of a correct solution.

Those candidates who attempted to integrate $k(u + 1)^{-2}$ usually did this correctly, but there were a significant number of candidates who either integrated this incorrectly to give $k(u + 1)^{-3}$ or ln f(u).

There were a significant proportion of candidates who proceeded to integrate $u(u + 1)^{-2}$ with respect to x and did so by either treating the leading u as a constant or using integration by parts.

Many candidates correctly changed the limits from 0 and 1 to 1 and 2 to obtain their final answer. Some candidates instead substituted u for 2^x and used limits of 0 and 1.

16. In part (a), many candidates recognised that the correct way to integrate $x \cos 2x$ was to use integration by parts, and many correct solutions were seen. Common errors included the incorrect integration of $\cos 2x$ and $\sin 2x$; the incorrect application of the 'by parts' formula even when the candidate had quoted the correct formula as part of their solution; and applying the by parts formula in the wrong direction by assigning $\frac{\mathrm{d}v}{\mathrm{d}x}$ as x to be integrated.

In part (b), fewer than half of the candidates deduced the connection with part (a) and proceeded by using "Way 1" as detailed in the mark scheme. A significant number of candidates used integration by parts on $\int x\left(\frac{\cos 2x + 1}{2}\right)$ and proceeded by using "Way 2" as detailed in the mark scheme.

In part (b), the biggest source of error was in the rearranging and substituting of the identity into the given integral. Some candidates incorrectly rearranged the cos $2x$ identity to give $\cos^2 x = \frac{\cos 2x - 1}{2}$. Other candidates used brackets incorrectly and wrote

$\int x\cos^2 x\,\mathrm{d}x$ as either $\int\left(\frac{x}{2}\cos 2x + 1\right)\mathrm{d}x$ or $\int\left(\frac{x}{2}\cos 2x + \frac{1}{2}\right)\mathrm{d}x$.

A significant number of candidates omitted the constant of integration in their answers to parts (a) and (b). Such candidates were penalised once for this omission in part (a).

17. This question was well done with many candidates scoring at least eight of the ten marks available.

In part (a), the most popular and successful method was for candidates to multiply both sides of the given identity by $(2x + 1)(2x – 1)$ to form a new identity and proceed with "Way 2" as detailed in the mark scheme. A significant proportion of candidates proceeded by using a method ("Way 1") of long division to find the constant A. Common errors with this way included algebraic and arithmetic errors in applying long division leading to incorrect remainders; using the quotient instead of the remainder in order to form an identity to find the constants B and C; and using incorrect identities such as $2(4x^2 + 1) \equiv B(2x – 1) + C(2x + 1)$.

In part (b), the majority of candidates were able to integrate their expression to give an expression of the form $Ax + p \ln(2x + 1) + q \ln(2x – 1)$. Some candidates, however, incorrectly integrated $\dfrac{B}{(2x+1)}$ and $\dfrac{C}{(2x-1)}$ to give either $B \ln(2x + 1)$ and $C \ln(2x – 1)$ or $2B \ln(2x + 1)$ and $2C \ln(2x – 1)$. A majority of candidates were able to substitute their limits and use the laws of logarithms to find the given answer. Common errors at this point included either candidates writing $–\ln(2x + 1) + \ln(2x – 1)$ as $\ln(4x^2 – 1)$; or candidates writing $–\ln 5 + \ln 3$ as either $\pm\ln 15$ or $\pm\ln 8$.

18. In part (a), most candidates used the correct volume formula to obtain an expression in terms of x for integration. At this stage errors included candidates using either incorrect formulae of $\pi\int y\mathrm{d}x$ or $\int y^2\mathrm{d}x$. Many candidates realised that they needed to integrate an expression of the form $k(1 + 2x)^{-2}$ (or equivalent). The majority of these candidates were able to complete the integration correctly or at least achieve an integrated expression of the form $p(1 + 2x)^{-1}$. At this stage, however, a common error was for candidates to integrate to give an expression in terms of natural logarithms. A significant number of candidates were unable to cope with substituting the rational limits to achieve the correct answer of $\dfrac{\pi}{12}$.

The vast majority of candidates were unable to gain any marks in part (b). Some candidates understood how the two diagrams were related to each other and were able to find the linear scale factor of 4. Few candidates then recognised that this scale factor needed to be cubed in order for them to go onto find the volume of the paperweight. Instead, a significant number of candidates applied the volume formula they used in part (a) with new limits of 0 and 3.

19. Part (a) was invariably well answered as was part (b). In part (b), some candidates incorrectly stated the width of each of the trapezia as $\dfrac{5}{6}$ whilst a few candidates did not give their answer to 4 significant figures.

The most successful approach in part (c) was for candidates to rearrange the given substitution to make x the subject. The expression for x was differentiated to give $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{2t}{3}$ and then substituted into the original integral to give the required integral in terms of t. Weaker candidates, who instead found $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{3}{2}(3x+1)^{-\frac{1}{2}}$, then struggled to achieve the required integral in terms of t. Most candidates were able to correctly find the changed limits although a sizeable number of candidates obtained the incorrect limits of $t = 2$ and $t = 4$.

Those candidates, who had written down a form of the required integral in part (c), were usually able to apply the method of integration by parts and integrate kte^t with respect to t and use their correct changed limits to find the correct answer of 109.2. Some candidates incorrectly used 'unchanged' limits of $t= 0$ and $t = 5$.

20. In part (a), most candidates realised that to find the shaded area they needed to integrate $3\sin(\frac{x}{2})$ with respect to x, and the majority of them produced an expression involving $\cos(\frac{x}{2})$; so gaining the first method mark. Surprisingly a significant number of candidates were unable to obtain the correct coefficient of -6, so thereby denying themselves of the final two accuracy marks. Most candidates were able to use limits correctly, though some assumed that cos0 is zero.
In part (b), whilst most candidates knew the correct formula for the volume required, there were numerous errors in subsequent work, revealing insufficient care in the use or understanding of trigonometry. The most common wrong starting point was for candidates to write y^2 as $3\sin^2\left(\frac{x^2}{4}\right)$, $9\sin^2(\frac{x^2}{4})$or $3\sin^2(\frac{x}{2})$. Although some candidates thought that they could integrate $\sin^2(\frac{x}{2})$directly to give them an incorrect expression involving $\sin^3(\frac{x}{2})$, many realised that they needed to consider the identity $\cos 2A \equiv 1 – 2\sin^2 A$ and so gained a method mark. At this stage, a significant number of candidates found difficultly with rearranging this identity and using the substitution $A= \frac{x}{2}$ to give the identity $\sin^2\left(\frac{x}{2}\right) \equiv \dfrac{1-\cos x}{2}$. Almost all of those candidates who were able to substitute this identity into their volume expression proceeded to correct integration and a full and correct solution.
There were, however, a significant minority of candidates who used the method of integration by parts in part (b), but these candidates were usually not very successful in their attempts.

21. In part (a), the first mark of the question was usually gratefully received, although for x 1.5 it was not uncommon to see $\frac{1}{2}$ ln($\frac{1}{2}$).

In part (b), it was not unusual to see completely correct solutions but common errors included candidates either stating the wrong width of the trapezia or candidates not stating their final answer correct to four significant figures.
Answers to part (c) were variable and often the mark in this part was not gained.
In part (d) all four most popular ways detailed in the mark scheme were seen. For weaker candidates this proved a testing part. For many candidates the method of integration by parts provided the way forward although some candidates applied this formula in the 'wrong direction' and incorrectly stated that $\frac{dv}{dx}$ =ln x implied v=$\frac{1}{x}$. Sign errors were common in this part, eg: the incorrect statement of $\int\left(\frac{x}{2}-1\right)dx=-\frac{x^2}{4}-x$, and as usual, where final answers have to be derived, the last few steps of the solution were often not convincing.
In summary, this question proved to be a good source of marks for stronger candidates, with 12 or 13 marks quite common for such candidates; a loss of one mark was likely to have been in part (c).

22. The product rule was well understood and many candidates correctly differentiated f(x) in part (a). However, a significant number lost marks by failing to use ln e = 1 and fully simplify their answer.

Although candidates knew that integration by parts was required for part (b), the method was not well understood with common wrong answers involving candidates mistakenly suggesting that $\int \ln x \, \mathrm{d}x=\frac{1}{x}$ and attempting to use $u = x^2 + 1$ and $\frac{\mathrm{d}v}{\mathrm{d}x}=\ln x$ in the formula $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x=uv-\int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$.

Candidates who correctly gave the intermediate result $\left[\left(\frac{x^3}{3}+x\right)\ln x\right]_1^{\mathrm{e}}-\int_1^{\mathrm{e}}\left(\frac{x^3}{3}+x\right)\frac{1}{x}\mathrm{d}x$ often failed to use a bracket for the second part of the expression when they integrated and went on to make a sign error by giving $-\frac{x^3}{9}+x$ rather than $-\frac{x^3}{9}-x$.

23. Many correct answers were seen to part (a). Candidates who used long division were generally less successful often leading to the misuse of $1+\frac{8x^2}{9-4x^2}$ to attempt partial fractions. The few candidates who ignored the 'hence' in part (b) made no progress in their integration, but would not have gained any marks anyway. Although candidates knew $\int\frac{1}{ax+b}dx=k\ln(ax+b)$, they were less accurate with the value of k. This question provided another challenge for candidates who were careless in their use of brackets whose answers often led to giving –[–(–1)] as +1. A few candidates ignored the requirement for an exact value.

24. This question proved a significant test for many candidates with fully correct solutions being rare. Many candidates were able to find $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$, although confusing differentiation with integration often led to inaccuracies. Some candidates attempted to find the equation of the tangent but many were unsuccessful because they failed to use $t=\frac{\pi}{6}$ in order to find the gradient as $-\frac{\sqrt{3}}{6}$.

Those candidates who attempted part (b) rarely progressed beyond stating an expression for the area under the curve. Some attempts were made at integration by parts, although very few candidates went further than the first line. It was obvious that most candidates were not familiar with integrating expressions of the kind $\int \sin at \sin bt \, \mathrm{d}t$. Even those who were often spent time deriving results rather than using the relevant formula in the formulae book.

Those candidates who were successful in part (a) frequently went on to find the area of a triangle and so were able to gain at least two marks in part (b).

25. Most candidates chose to use the given substitution but the answers to this question were quite variable. There were many candidates who gave succinct, neat, totally correct solutions, and generally if the first two method marks in the scheme were gained good solutions usually followed.

The biggest problem, as usual, was in the treatment of "dx": those who differentiated

$u^2 = 2x - 1$ implicitly were usually more successful, in their subsequent manipulation, than those who chose to write $u = \sqrt{(2x-1)}$ and then find $\frac{du}{dx}$; many candidates ignored the dx altogether, or effectively treated it as du, and weaker candidates often "integrated" an expression involving both x and u terms. Some candidates spoilt otherwise good solutions by applying the wrong limits.

26. There were many excellent solutions to this question but also too many who did not know the formula for finding the volume of the solid. Candidates who successfully evaluated

$\int_1^3 x^2 e^{2x}\, dx$ were able to gain 6 of the 8 marks, even if the formula used was

$k\int y^2 dx$ with $k \neq \pi$, but there were many candidates who made errors in the integration, ranging from the slips like sign errors and numerical errors to integrating by parts "in the wrong direction". An error with serious consequences for most who made it was to write $(x\,e^x)^2$ as

$x^2\ e^{x^2}$; for some it was merely a notational problem and something could be salvaged but for most it presented a tricky problem !

27. The majority of candidates gained the marks in part (a) and a good proportion managed to produce the given result in part (b). Some candidates suggested that the area of R was $\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} y^2 dt$, which made the question rather trivial; although that happened to be true here as $y = \frac{dx}{dt}$, working was needed to produce that statement.

The integration in part (c), although well done by good candidates, proved a challenge for many; weaker candidates integrating $(1\text{-}2\cos t)^2$ as $\frac{(1-2\cos t)^3}{3}$ or something similar. It may have been that some candidates were pressed for time at this point but even those who knew that a cosine double-angle formula was needed often made a sign error, forgot to multiply their expression for $\cos^2 t$ by 4, or even forgot to integrate that expression. It has to be mentioned again that the limits were sometimes used as though π = 180,

so that $\left[3t - 4\sin t + \sin 2t\right]_{\frac{\pi}{3}}^{\frac{5\pi}{3}}$ became [900 – …….] – [180 – ….].

28. For many candidates this proved to be the most testing question on the paper.

(a) This was attempted with some degree of success by most candidates and many scored full marks. Most errors occurred in squaring and simplifying y. Some of the integration was not good – candidates attempting to integrate a fraction by integrating numerator and denominator separately, a product by integrating separately and then finding the product, or a squared function by squaring the integral of the original function. In some cases candidates failed to recognise that 2/x integrated to a *log* function.

(b) Very few candidates knew the formula for the volume of a cone, so a further volume of revolution was often found. Candidates working with the incorrect equation for the line expended considerable time calculating an incorrect value for the volume. Many candidates produced answers that were clearly not dimensionally correct in (b), and hence lost all 3 marks. It was common to see expressions of the form "final volume = volume – area" or "final volume = volume – area^2"

29. (a) This was a straightforward integration by parts, which was recognised as such and done well in general. The most common error was the omission of the constant of integration, but some confused signs and others ignored the factors of two.

(b) This was done well by those students who recognised that $\cos^2 x = (1 + \cos 2x)/2$ but there was a surprisingly high proportion who were unable to begin this part. Lack of care with brackets often led to errors so full marks were rare. There was also a large proportion of candidates who preferred to do the integration by parts again rather than using their answer to (a).

30. Almost all candidates knew how to do this question and it was rare to see an incorrect solution to part (a) and nearly all could make a substantial attempt at part (b).

However the error $\int \frac{3}{2x-3}\,\mathrm{d}x = 3\ln(2x-3)$ was common. The standard of logarithmic work seen in simplifying the final answer was good and this, and the manipulation of exponentials elsewhere in the paper, is another area in which the standard of work has improved in recent examinations.

31. This was a question which candidates tended to either get completely correct or score very few marks. If the $\mathrm{d}x$ is ignored when substituting, an integral is obtained which is extremely difficult to integrate at this level and little progress can be made. Those who knew how to deal with the $\mathrm{d}x$ often completed correctly. A few, on obtaining tan θ, substituted 0 and $\frac{1}{2}$ instead of 0 and $\frac{\pi}{6}$. Very few attempted to return to the original variable and those who did were rarely successful.

32. Those who recognised that integration by parts was needed in part (a), and these were the great majority, usually made excellent attempts at this part and, in most cases, the indefinite integral was carried out correctly. Many had difficulty with the evaluating the definite integral. There were many errors of sign and the error $e^0 = 0$ was common. The trapezium rule was well known, although the error of thinking that 6 ordinates gave rise to 6 strips, rather than 5, was often seen and some candidates lost the final mark by not giving the answer to the specified accuracy.

33. Candidates commonly misunderstood that both the formulae for sin(A+B) and sin(A–B) were required by the rubric. The vast majority chose only one, prohibiting progress and usually abandoned the question at this stage. Many who did not complete part (a) attempted to integrate by parts, usually twice, in part (b) before leaving unfinished working. The more successful, or those with initiative, continued with both parts (b) and (c), either with their p and q values, the letters p and q, or hopefully guessed p and q values.

34. Many good attempts at part (a) were seen by those who appreciated t = 2 at P, or by those using the Cartesian equation of the curve. Algebraic errors due to careless writing led to a loss of accuracy throughout the question, most commonly

$$\frac{3}{2t} \to \frac{3}{2}t \to \frac{3t}{2}$$

Part (b) undoubtedly caused candidates extreme difficulty in deciding which section of the shaded area R was involved with integration. The majority set up some indefinite integration and carried this out well. Only the very able sorted out the limits satisfactorily. Most also evaluated the area of either a triangle or a trapezium and combined, in some way, this with their integrand, demonstrating to examiners their overall understanding of this situation.

35. Candidates found this question challenging; however those who read all the demands of the question carefully were able to score some marks, whilst quite an appreciable minority scored them all. In part (a), the crucial step involved keeping signs under control. Seeing

$$-x\cot\left(x+\frac{x}{6}\right) - \int -\cot\left(x+\frac{x}{6}\right)\mathrm{d}x$$

or a correct equivalent, demonstrated to examiners a clear method. Sign confusions sometimes led to a solution differing from the printed answer.

Part (b) was the main source for the loss of marks in this question. It was disappointing that so many candidates rushed through with barely more than three lines of working between separating the variables and quoting the printed answer, losing the opportunity to demonstrate their skills in methods of integration. The majority separated the variables correctly. Very few made any attempt to include the critical partial fractions step, merely stating

$$\frac{1}{2}\int \frac{1}{y(y+1)}\,\mathrm{d}y = \frac{1}{2}\ln\left(\frac{y}{1+y}\right)$$

as printed. Some did not recognise the right hand side of their integral related to part (a), producing copious amounts of working leading to nowhere.

In part (c), the working to evaluate the constant c was often untidy and careless. Those who persevered to a stage of the form ln P = Q + R generally were unable to move on to $P = e^{Q+R}$ in a satisfactory manner, often writing $P = e^Q + e^R$.

36. Although there were a minority who thought that the volume required was $2\pi\int y^2\,\mathrm{d}x$, this is a question for which most candidates knew the correct procedure and it is disappointing to record that less that half were able to give completely correct solutions. Many were unable to square $\left(4x-\frac{6}{x}\right)$ correctly and the integral of $\frac{36}{x^2}$ gave difficulty, both $36\ln x^2$ and $\frac{36}{\frac{1}{3}x^3}$ being seen.

Calculator errors were also frequently noted when simplifying the final fractions. These arose mainly from the incorrect use of brackets. A few candidates gave the final answer as an approximate decimal, failing to note that the question asked for an exact value of the volume.

37. In general, the attempts at part (a) were good and there was a large number of candidates who scored 6 or 7 marks. Even with poor squaring of $\left(\frac{x+2}{\sqrt{x}}\right)$ it was possible for candidates to gain 5 marks, which helped many, but some attempts at integration were not so kindly looked upon; a maximum of three marks was available for candidates who integrated the numerator and denominator separately or who produced $\left(\frac{x^3}{3} + 2x^2 + 4x\right)\ln x$. The majority of candidates gained the mark in part (b), but, surprisingly, correct reasoning in part (c) was uncommon. The most common approach was to approximate the support to a cylinder of radius 6cm and height 6cm, so $\pi(6)^2 \times 6 = 678.6 \approx 630$ was seen frequently

38. Most candidates were able to make a start here but a number did not progress much beyond $\frac{\mathrm{d}u}{\mathrm{d}x} = \cos x$. Some failed to realize that sin$x$ = (1 – u) and others tried integrating an expression with a mixture of terms in x and u. Those who reached $\int (u-1)u^5\ \mathrm{d}u$ usually went on to score the first 6 marks, but the final two marks were only scored by the most able who were able to complete the factorization and simplification clearly and accurately.

39. Part (a) was often answered well but some candidates who worked in degrees gave the final answer as 90 $\sqrt{3}$. Part (b) proved more challenging for many; some did not know how to change the variable and others failed to realize that $\frac{\mathrm{d}x}{\mathrm{d}t}$ required the chain rule. Most candidates made some progress in part (c) although a surprising number thought that $\int \tan t\ \mathrm{d}t$ was $\sec^2 t$. The examiners were encouraged to see most candidates trying to give an exact answer (as required) rather than reaching for their calculators.

40. This was found to be the most difficult question on the paper. Some excellent candidates did not appear to have learned how to find the area using parametric coordinates and could not even write down the first integral. A few candidates used the formula $\frac{1}{2}\int x\frac{dy}{dt} - \frac{1}{2}\int y\frac{dx}{dt}$ from the formula book. The integration by parts was tackled successfully, by those who got to that stage and there were few errors seen. The percentage at the end of the question was usually answered well by the few who completed the question.

41. In part (a) errors in indices were seen and in part (b) many found expanding $\left(\frac{8}{x} - x^2\right)^2$ a stumbling block. In the integration, as expected, integrating $\frac{64}{x^2}$ was the major source of error. However, for the majority, the methods and formulae needed for this question were well known and there were many completely correct solutions.

42. The key to success in part (a) was to realize the need to differentiate the curve. Many weaker candidates did not appreciate this but there were many good solutions to this part. In part (b) many candidates did not realize that they needed to combine the result in part (a) with $y = 1 + \frac{c}{p}$ and circular arguments that started by assuming c = 4 and only used one of these statements were seen. Part (c) was answered very well and many fully correct solutions were seen. The volume formula was well known and working exactly caused few problems. There were a few errors in squaring, where the $\frac{8}{x}$ term was missing, and some thought that the integral of $\frac{16}{x^2}$ was 16 ln x^2.

43. Whilst the majority of answers to part (a) were fully correct, some candidates found difficulties here. A small number failed to find the coordinates of M correctly with (0, 5) being a common mistake. Others knew the rule for perpendicular gradients but did not appreciate that the gradient of a normal must be numerical. A few students did not show clearly that the gradient of the curve at x = 0 was found from the derivative, they seemed to treat $y = 2e^x + 5$ and assumed the gradient was always 2. Some candidates failed to obtain the final mark in this section because they did not observe the instruction that a, b and c must be integers.

For most candidates part (b) followed directly from their normal equation. It was disappointing that those who had made errors in part (a) did not use the absence of n = 14 here as a pointer to check their working in the previous part. Most preferred to invent all sorts of spurious reasons to justify the statement.

Many candidates set out a correct strategy for finding the area in part (c). The integration of the curve was usually correct but some simply ignored the lower limit of 0. Those who used the simple "half base times height" formula for the area of the triangle, and resisted the lure of their calculator, were usually able to complete the question. Some tried to find the equation of *PN* and integrate this but they usually made no further progress. The demand for exact answers proved more of a challenge here than in 6(c) but many candidates saw clearly how to simplify $2e^{\ln 4}$ and convert ln 4 into 2 ln 2 on their way to presenting a fully correct solution.

44. Most candidates made some attempt to differentiate $x\sqrt{\sin x}$, with varying degrees of success. $\sqrt{\sin x} + x\sqrt{\cos x}$ was the most common wrong answer. Having struggled with the differentiation, several went no further with this part. It was surprising to see many candidates with a correct equation who were not able to tidy up the √ terms to reach the required result.

Most candidates went on to make an attempt at $\int \pi y^2 \, dx$. The integration by parts was generally well done, but there were many of the predictable sign errors, and several candidates were clearly not expecting to have to apply the method twice in order to reach the answer. A lot of quite good candidates did not get to the correct final answer, as there were a number of errors when substituting the limits.

45. Most candidates chose an appropriate form for the partial fractions, and they demonstrated knowledge of how to find values for their constants. Where things did go wrong, it was often because of errors in arithmetic, but also in forming the correct numerator for the combined partial fractions – an extra factor of $2x + 3$ was popular.

Those who chose constant numerators for the fractions had the easier time in part (b), although there were frequent errors involving wrong signs and wrong constants in the integrals. Many candidates could not integrate $(3 + 2x)^{-2}$, and erroneously used a log. Few candidates with the two-fraction option could see how to make progress with the integration of the fraction with the quadratic denominator, though there were a number of possible methods.

In part (c) very few candidates achieved full marks for this part as many made errors with signs when expanding using the binomial. The most common mistake however was to take out the 3 from the brackets incorrectly. Not as $\frac{1}{3}$ and $\frac{1}{9}$, but as 3 and 9. However many candidates did manage to get some marks, for showing they were trying to use negative powers and also for the $1 + x + x^2$, and for trying to collect all their terms together at the end.

46. It was pleasing to see most candidates applying the volume of revolution formula correctly. However, although the question was well attempted, with the majority of candidates scoring at least 5 marks despite errors listed below, fully correct solutions were usually only seen from the better candidates.

Algebraic errors arose in expanding $\left(1 + \frac{1}{2\sqrt{x}}\right)^2$; the most common wrong attempts being

$1 + \frac{1}{4x}$, $1 + 4x^{-\frac{1}{2}} + 4x^{-1}$, $1 + \frac{1}{4}x + \frac{1}{4}x$ and $1 + \frac{1}{2}x^{-1} + \frac{1}{4}x^{-\frac{1}{4}}$.

Common errors in integration were $\int \frac{1}{4x} dx =$ ln 4*x* and $\int \frac{1}{\sqrt{x}} dx = \ln \sqrt{x}$.

47. This question involved differentiation using the product rule in part (a) and integration using parts in part (b). It was answered well, with most of the difficulties being caused by the use of indices and the associated algebra. Some candidates wasted time in part (a) by finding the *y*-coordinate which was not requested. A sizeable proportion of the candidates misquoted the formula for volume of revolution.

48-54. No Reports available for these questions.