# Calculation of Construction Time for Building Projects - Application of the Monte Carlo Method to Determine the Period Required for Shell Construction Works 

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#### Abstract

Construction time is of crucial importance when it comes to utilizing the production factors in an optimal way. The client determines the available construction period, and the contractor prepares its quotation on the basis of the specification whilst planning the construction process and logistics. Construction times that are too short usually result in higher cost, poorer quality and a larger number of disputes. This paper outlines the calculation of construction time whilst considering key construction management parameters. Beyond a simple, deterministic method, other options for calculation are presented that rely on probability calculus. The deterministic method results in one value per each calculation process (calculation mode 1). In calculation mode 2, probability calculus is applied in a simple fashion. Both range and probability of occurrence can be considered for the relevant input variables. For the third calculation mode (calculation mode 3), the Monte Carlo method is applied using the @RISK software. This method shows a probability distribution for each of the parameters to be determined. Using a high-rise building project, the application of the Monte Carlo method (calculation mode 3) to determine construction time is demonstrated. Weighted triangles are used as distribution functions, which makes it possible to consider minimum and maximum values, as well as expected values. The correlation between probability of occurrence and construction times is reflected by a probability distribution.


Keywords: construction time, production rate, consumption rate, risk, Monte Carlo method, shell construction works

## 1. Introduction

Production quantities and outputs are required to calculate construction time. If the entire construction process is divided into specific phases, additional allocation parameters need to be defined in order to determine construction time. In this case, the shortest possible construction time will result from the critical path. Production quantities result from the dimensions of the structural elements. Production rate parameters are derived from labour (in the case of labour-intensive activities) or equipment outputs (in the case of work steps requiring a high degree of equipment utilization). Depending on the individual project phases, project data and production rate parameters fluctuate to a varying extent. Even after the preparation of the specification, the exact volumes will not be available in most cases.

At the preliminary planning stage, construction time can be estimated using the average output relative to the gross volume of the structure. If, for instance, construction time is calculated for an insitu concrete shell, the amount of reinforced concrete needs to be used as the unit of reference. The amount of labour required for the placement of formwork, reinforcement and concrete is related to this unit.

The various input variables used for the calculation are subject to inaccuracies that result from the prevailing structural, site, management and process conditions, as well as from the construction contract.

## 2. Analysis of situation, objective

The type of work, the conditions under which this work is performed, the amount and quality of the work and the construction time required are factors that determine the cost level, and thus pricing. The client may influence cost and pricing by defining the construction time. Construction times that are too short (i.e. where the maximum values for the productive use of equipment and/or labour are exceeded) result in productivity losses and also higher costs. To a certain extent, the limits for productivity losses are not yet fully understood, or controversial.

In most cases, the client does not sufficiently account for boundary conditions imposed by construction management, or neglects these conditions completely, when determining the construction time that is contractually agreed upon. However, boundary conditions arising from structural or management conditions, for example, have a significant influence on parameters that determine output, such as the maximum number of available workers or the maximum number of available machines or pieces of equipment.

This paper is to systematically present construction time calculation methods. In addition, the calculation procedure should account for limiting construction management factors. Both deterministic and stochastic calculation approaches should be outlined. It should be investigated whether the implementation of probability calculus leads to improved results.

Blecken (1967) already stated that a deterministic assumption made for production would oversimplify the production model. He considers the inclusion of a stochastic approach a way to achieve a significant improvement of results.

## 3. Bases for the calculation of construction time

The following section includes and describes the equations used to calculate duration, production rate and total consumption rate. However, the calculation methods outlined do not in any way replace more detailed analyses for the calculation of construction time.

### 3.1 Duration and production rate of reinforced concrete works

In Eq. 1 , the average values for the concrete quantity $\mathrm{Q}_{\mathrm{C}}\left[\mathrm{m}^{3}\right]$ and daily production rate $\mathrm{PR}_{\mathrm{RCW}}\left[\mathrm{m}^{3} / \mathrm{d}\right]$ are used to calculate the duration $\mathrm{D}_{\mathrm{RCW}}[\mathrm{d}]$.

$$
\begin{equation*}
D_{R C W}=\frac{Q_{C}}{P R_{R C W}} \tag{1}
\end{equation*}
$$

To account for disruptions, the calculation should include a buffer $\mathrm{BU}_{\mathrm{T}, \mathrm{RCW}}$ [\%], which results in the following equation:

$$
\begin{equation*}
D_{R C W \cdot B U}=D_{R C W} *\left(1+\frac{B U_{T, R C W}}{100}\right) \tag{2}
\end{equation*}
$$

The magnitude of the buffer will depend on the complexity of the project and the number of winter construction phases. Experience shows that this buffer should range from 5 to $15 \%$.

The average daily production rate for reinforced concrete works $\mathrm{PR}_{\mathrm{RCW}}\left[\mathrm{m}^{3} / \mathrm{d}\right]$ is calculated using Eq. 3. In the numerator, the multiplication of the number of workers $\mathrm{W}_{\mathrm{RCW}}[\mathrm{wh} / \mathrm{hr}$ ] with the daily working time $\mathrm{WT}_{\mathrm{RCW}}[\mathrm{hr} / \mathrm{d}]$ results in the daily hours paid. In the denominator, the total consumption rate $\mathrm{TCR}_{\mathrm{RCW}}\left[\mathrm{wh} / \mathrm{m}^{3}\right]$ for reinforced concrete works is used. The consumption rate (for example as shown in Kenley et al. 2010) expressed as total working hours per unit of quantity [wh/UoQ] for the respective activity.

$$
\begin{equation*}
P R_{R C W}=\frac{W_{R C W} * W T_{R C W}}{T C R_{R C W}} \tag{3}
\end{equation*}
$$

Figure 1 shows major influences on the amount of production rate achieved. Average production rate can be calculated for the entire structure or for individual groups of structural components. The accuracy of the input variables, and thus of the calculation result, will usually increase in line with the degree of detail of the project and project phase.


Figure 1: Influences on the order of magnitude of the required production rate

### 3.2 Number of workers

In building construction, the maximum number of available workers always correlates with the available workspace and number of cranes that can be used. In the literature, approximate values are stated for the number of workers per crane and construction method.

As a rule, the number of workers does not remain constant over the entire construction period. The number of workers required increases to the maximum value as the project ramp-up phase progresses (duration $=\mathrm{D}_{\mathrm{SP}}$ ), and remains at a relatively constant level thereafter. The number of workers decreases again when the final project phase begins (duration $=\mathrm{D}_{\mathrm{FS}}$ ).

Formwork, reinforcement and concrete placement works are the main activities associated with reinforced concrete works. Each of these activities is characterized by the methods, materials and machines or pieces of equipment used. A certain number of workers is required to achieve a defined daily production rate (depending on consumption rate and daily working time). Amongst other factors, the productivity of the workers will also depend on the number of hours worked per day and the available workspace. When planning the construction process, a defined minimum workspace per worker should be ensured at all times.

The minimum workspace (relative to the floor plan of the structure) is a very important parameter for construction process and logistics planning. Provided the minimum workspace requirement is adhered to, it can be assumed that no productivity losses will occur, for instance due to mutual interference of workers or of one or more gangs or teams. In a survey on costing, process planning
and construction work conducted by Graz University of Technology (i.e. an expert survey with 18 respondents from the construction industry), the average minimum workspace was found to be $30 \mathrm{~m}^{2}$ per worker.

If the client specifies a construction time that is too short for any given building project, this construction time can still be adhered to by utilizing existing potentials more effectively. However, the calculated productivity losses should result in higher unit prices to be paid for the work to be performed. Ideally, the client is sufficiently aware of these boundary conditions and considers these key construction management parameters when determining the contractually agreed construction time.


Figure 2: Correlation between number of workers and available workspace for reinforced concrete works

On the one hand, the use of resources (i.e. workers and machines) results from the available construction time and production quantity. On the other, the shortest possible construction time results from the maximum amount of resources available. The number of workers always correlates with the available workspace and the number of machines that can be installed. In building construction, for instance, the maximum number of available workers will depend on the maximum number of cranes that can be used.

Figure 2 shows the correlation between the curve for "usable" workspace and the curve showing the number of workers. The trend in the number of required workers and the development of the workspace are reflected in an idealized model over the construction period (overall duration of reinforced concrete works $=\mathrm{D}_{\text {тот }}[\mathrm{d}]$ ). To simplify the model, a trapezoidal curve was assumed. The "practical feasibility" of this simplification was evaluated on the basis of resource plans prepared for various structures in building construction. A good approximation to the trapezoidal model was found.

### 3.2.1 Calculation of the maximum number of workers

The average available workspace per employee for reinforced concrete works during the main construction period $\mathrm{D}_{\mathrm{MCT}}[\mathrm{d}]$ results from Eq. 4.

$$
\begin{equation*}
W S_{W, R C W}=\frac{T O T_{W S}}{W_{R C W, M A X}} \tag{4}
\end{equation*}
$$

For example, the total workspace $\mathrm{TOT}_{\mathrm{Ws}}\left[\mathrm{m}^{2}\right]$ can be calculated from the floor area of the storey and the number of storeys that can be worked on simultaneously $n_{s t}[-]$ (see Eq. 5).

$$
\begin{equation*}
T O T_{W S}=F A_{S T} * n_{s t} \tag{5}
\end{equation*}
$$

The number of cranes that can be used for construction work is limited by space, construction management and economic constraints. These constraints need to be taken into account when determining the maximum number of available workers. The crane proportionality factor indicates the average number of workers who can be "served" by one crane.

When establishing a relationship between the number of workers and the number of cranes, the maximum number of workers $\mathrm{W}_{\mathrm{RCW}, \mathrm{MAx}}[\mathrm{wh} / \mathrm{hr}$ ] is calculated by multiplying the number of cranes $\mathrm{NUM}_{\mathrm{C}}[-]$ with the crane proportionality factor $\mathrm{PF}_{\mathrm{C}, \mathrm{W}}[\mathrm{W} / 1]$.

$$
\begin{equation*}
W_{R C W, M A X}=N U M_{C} * P F_{C, W} \tag{6}
\end{equation*}
$$

### 3.2.2 Calculation of the average number of workers

The lowest of the maximum values determined is used to proceed with the calculation of the average number of workers. The maximum number of workers can be used only during the main construction phase. The proportion of the average and maximum number of workers results in the workers factor $\mathrm{f}_{\mathrm{W}}[-]$ from Eq. 7.

$$
\begin{equation*}
f_{W}=\frac{W_{R C W, A}}{W_{R C W, M A X}} \tag{7}
\end{equation*}
$$

The main construction phase is usually equivalent to 60 to $80 \%$ of the overall duration of reinforced concrete works. This range can be narrowed down further, for instance on the basis of the experience gained in similar, previously completed projects. The average number of workers can be calculated from the maximum number of workers and the workers factor (see Eq. 8).

$$
\begin{equation*}
W_{R C W, A}=W_{R C W, M A X} * f_{W} \tag{8}
\end{equation*}
$$

Depending on the type of building (e.g. high-rise, power plant), workers factors between 75 and $90 \%$ may be assumed.

### 3.3 Daily working time

The daily working time per employee influences consumption rate. Limits to the maximum daily working time per employee result from applicable labour law and collective agreements entered into with trade unions. In addition, productivity losses should be assumed from a certain daily working time level. Appropriate shift models can be used to increase the productive daily working time by distributing it across several shifts. Shift models are necessary for certain construction processes and methods (such as slip-form construction), or result from a very short construction time.

### 3.4 Total consumption rate for reinforced concrete works

For reinforced concrete works, the total consumption rate $\mathrm{TCR}_{\mathrm{RCw}}\left[\mathrm{wh} / \mathrm{m}^{3}\right]$, which includes formwork, reinforcement and concrete placement works, is calculated using Eq. 9:

$$
\begin{equation*}
T C R_{R C W}=C R_{A, F W} * F R_{A, B D}+C R_{A, R W} * R R_{A, B D}+C R_{A, C W} \tag{9}
\end{equation*}
$$

The first term is the product of the average formwork placement consumption rate $\mathrm{CR}_{\mathrm{A}, \mathrm{FW}}\left[\mathrm{wh} / \mathrm{m}^{2}\right.$ ] and the formwork ratio $\mathrm{FR}_{\mathrm{A}, \mathrm{BD}}\left[\mathrm{m}^{2} / \mathrm{m}^{3}\right]$; the second is the product of the average reinforcement work consumption rate $\mathrm{CR}_{\mathrm{A}, \mathrm{RW}}[\mathrm{wh} / \mathrm{t}]$ and the reinforcement ratio $\mathrm{CR}_{\mathrm{A}, \mathrm{CW}}\left[\mathrm{t} / \mathrm{m}^{3}\right]$; the last term represents the average consumption rate of concrete placement $\mathrm{TCR}_{\mathrm{A}, \mathrm{Cw}}\left[\mathrm{wh} / \mathrm{m}^{3}\right]$. Eq. 9 is used to either estimate or precisely calculate the mean values. A more accurate calculation is carried out as part of a detailed analysis. Depending on the item considered, total consumption rate can be calculated either for the entire structure or for individual groups of structural components. The accuracy of the results usually increases in line with the degree of detail of the analysis.

## 4. Calculation methods

### 4.1 Calculation mode 1 - Deterministic approach

Calculation mode 1 is used to calculate individual values for the duration. In each calculation process, and production rate value and, subsequently, duration are calculated. Parameters can be chosen to ensure that an upper and lower limit value is determined for the duration.

### 4.2 Calculation mode 2 - Simplified Stochastic approach

Three values are used for the respective input variables of the equations: a minimum value, an expected value and a maximum value. These values are multiplied with the specific probability of occurrence, which was defined on the basis of a subjective assessment. We arrive at the value with the greatest subjective probability by adding up the three products.

### 4.3 Calculation Mode 3 - Application of Monte Carlo method

For the stochastic method, a distribution function is allocated to selected parameters (see Figure 3 for flow chart). The values for the range are defined on the basis of construction management and specific structural boundary conditions. The bases for these values can be, for instance, internal logs or data taken from the literature (such as approximate working times for building construction).


Figure 3: Calculation Mode 3 - construction time: Flow chart

Results can be improved significantly by including probability considerations in the calculations. On the basis of the ranges and distribution functions established, probability distributions are shown for the required values depending on the number of simulations to be chosen. The Monte Carlo method makes it possible to calculate the probability distribution for the total consumption rate of reinforced concrete works. In a freely selectable number of iterative steps, a software program (in this case, @RISK) generates random numbers for the input parameters. These random numbers are each allocated to the predefined distribution functions and combined according to a specified computation rule (i.e. the equations used for the deterministic method). The input parameters are ranges and distribution functions.

As regards distribution functions, Raaber (2003) states that triangular, parabolic or, less commonly, rectangular distributions should be used for calculations in the field of construction, where limits are almost always identifiable.

## 5. Application of the Monte Carlo method

### 5.1 High-rise building project - key details

The building comprises two basements, one ground floor, ten upper storeys and one attic floor (see Figure 4). The maximum area of the building at basement level amounts to approx. $3,025 \mathrm{~m}^{2}$. The two basements, the ground floor, the standard floors and the attic storey have four different ground plans. However, some of the components or members are almost identical across several storeys. The highrise building comprises a total of 14 floors, with the standard floor having the following dimensions: $50 \mathrm{~m} * 47 \mathrm{~m}$. Figure 5 shows the layout of a standard floor.


Figure 4: Section (Doka, 2009)


Figure 5: Floor plan - standard floor (Doka, 2009)

Table 1 contains information on the quantities of formwork, reinforcement and concrete. Quantities are derived from the determination of quantities on the basis of available plans and drawings. The formwork ratio of approx. $3.9 \mathrm{~m}^{2} / \mathrm{m}^{3}$ for the entire building is derived from a formwork surface of approx. $49,300 \mathrm{~m}^{2}$ and a concrete volume of about $12,600 \mathrm{~m}^{3}$. The reinforcement ratio of approx. $147 \mathrm{~kg} / \mathrm{m}^{3}$ is calculated from the amount of reinforcement and concrete.

Table 1: Quantities of formwork, reinforcement and concrete

| Components | Formwork area |  | Reinforcement quantity |  | Concrete volume |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | [ $\mathrm{m}^{2}$ ] | [\%] | [t] | [\%] | [ $\mathrm{m}^{3}$ ] | [\%] |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Foundation plate | 225 | 0.46 | 431 | 23.28 | 3,080 | 24.39 |
| Walls | 5,553 | 11.26 | 103 | 5.56 | 739 | 5.85 |
| Columns | 3,598 | 7.30 | 89 | 4.81 | 270 | 2.14 |
| Shafts and cores | 15,554 | 31.54 | 266 | 14.37 | 1,902 | 15.06 |
| Slabs | 24,386 | 49.45 | 962 | 51.97 | 6,638 | 52.56 |
| Amount: | 49,316 | 100 | 1,851 | 100 | 12,629 | 100 |

### 5.2 Solution for calculation mode 3 - Calculation using the Monte Carlo method

The analysis of the ground plan and site conditions establishes that a maximum of two cranes can be used. The maximum number of workers (40) is derived from two cranes and a maximum of 20 workers per crane. For an average floor area of $2,000 \mathrm{~m}^{2}$, the assumption is made that 0.65 floors can be worked on simultaneously during the main construction phase. The maximum available workspace amounts to approx. $1,300 \mathrm{~m}^{2}$ (insert in Eq. 5). Assuming that the minimum workspace amounts to 30 $\mathrm{m}^{2}$ per worker, we use Eq. 4 to arrive at a maximum of about 43 workers during the main construction phase. The lower of the two maximum values calculated is to be used for the calculation of the average number of workers. The duration of reinforced concrete works is calculated using the
mode shown in Figure 3. The values from Table 2 are used for the input variables to calculate total consumption rate, daily production rate and duration (weighted triangular distribution).

Table 2: Input values to calculate the construction time

|  | MIN | EXP | MAX |
| :---: | :---: | :---: | :---: |
| Average labor consumption rate - formwork works | $0.65 \mathrm{wh} / \mathrm{m}^{2}$ | $0.70 \mathrm{wh} / \mathrm{m}^{2}$ | $0.85 \mathrm{wh} / \mathrm{m}^{2}$ |
| Average formwork ratio for the entire building | $3.60 \mathrm{~m}^{2} / \mathrm{m}^{3}$ | $3.90 \mathrm{~m}^{2} / \mathrm{m}^{3}$ | $4.00 \mathrm{~m}^{2} / \mathrm{m}^{3}$ |
| Average labor consumption rate - reinforcement works | 8.00 wh/t | $9.00 \mathrm{wh} / \mathrm{t}$ | $10.50 \mathrm{wh} / \mathrm{t}$ |
| Average reinforcement ratio for the entire building | $140.00 \mathrm{~kg} / \mathrm{m}^{3}$ | $150.00 \mathrm{~kg} / \mathrm{m}^{3}$ | $160.00 \mathrm{~kg} / \mathrm{m}^{3}$ |
| Average labor consumption rate - concrete works | $0.45 \mathrm{wh} / \mathrm{m}^{3}$ | $0.55 \mathrm{wh} / \mathrm{m}^{3}$ | $0.70 \mathrm{wh} / \mathrm{m}^{3}$ |
| Maximum number of workers | $37.00 \mathrm{wh} / \mathrm{hr}$ | $38.00 \mathrm{wh} / \mathrm{hr}$ | $40.00 \mathrm{wh} / \mathrm{hr}$ |
| Proportion of the average number of workers | 75.00 \% | 80.00 \% | 90.00 \% |
| Daily working time | $8.00 \mathrm{hr} / \mathrm{d}$ | $8.50 \mathrm{hr} / \mathrm{d}$ | $9.00 \mathrm{hr} / \mathrm{d}$ |
| Concrete Quanity | 12,000 m ${ }^{3}$ | 12,629 m ${ }^{3}$ | 13,000 m ${ }^{3}$ |
| Buffer | 5.00 \% | 8.00 \% | 10.00 \% |

50,000 iterative steps were performed in the @RISK program to calculate construction time. Figure 6 shows the calculation results for the duration (including buffer) as a probability distribution. This probability distribution can be used to determine the probability of occurrence for selected values. If, for example, it was specified internally that the probability of occurrence must at least be equal to $40 \%$ for the duration, the corresponding value is easy to determine.


Figure 6: Probability distribution for construction time, including contingencies

The duration of the project will be between 216 and 269 days with a $90 \%$ probability, which translates into a spread of about 53 days. The probability for the duration to exceed 269 days amounts to $5 \%\left(\mathrm{X}_{95}\right)$. On the other hand, the duration will be shorter than 216 days at a $5 \%$ probability $\left(\mathrm{X}_{5}\right)$. The expected value (mean) amounts to approx. 242 days, with a standard deviation of 16 days.

## 6. Conclusion

The specification of a reasonable construction time makes a substantial contribution towards the economical use of elementary and dispositive production factors, or at least creates the preconditions to do so. Workers, equipment and materials can be used cost-efficiently.

A realistic construction time can be calculated once construction management boundary conditions have been considered. Normal construction times still involve a certain potential for being shortened without necessarily running the risk of immediate productivity losses. The application of probability calculus (calculation mode 3) does not result in a single correct value. Rather, the correlation between the probability of occurrence and the magnitude of the value is reflected (probability distribution). The decision-making process can be facilitated by showing the results as a distribution curve. If the ranges can be reduced further by appropriate measures, the distance between the quantiles is also reduced ( $\mathrm{X}_{5}, \mathrm{X}_{95}$ ).

As far as reasonably possible, clients should specify a normal construction time for their projects. In this regard, it is crucial to consider the construction management constraints in terms of the number of resources and logistics-related boundary conditions. Construction time should at least be calculated using calculation mode 2 but ideally also by including a probability calculus approach (such as the Monte Carlo method). However, the correct interpretation of probability calculus results will always require expert knowledge with regard to construction management and related economic aspects.

The author is currently conducting further research into the interdependencies of the risks and the influence of the type of distribution function used (not referred to in the paper). In particular, the research focuses on the possible implications on the construction time probability distributions. The calculation methods outlined above can be applied to a wide range of projects. They can be used by all parties involved in the project.

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