Calculation of Parameters of Overhead Power Lines

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ABSTRACT

This paper describes the most common methods used for the calculation of overhead line parameters: Carson's and Complex Depth of Earth Return (CDER) methods for the series impedance and Maxwell's potential coefficients for the parallel capacitance. A program developed in Matlab language was created using the three methods stated above and comparisons were made between the results obtained using this program and those obtained using Line Constants program from ATP/EMTP and indicated very similar results for both programs. This is followed by a study on how the variation of frequency and material conductivity of the line conductors affects the series impedance and the parallel capacitance of the overhead line. An illustrative example of a line is used to observe these variations. For the series impedance it is concluded that results for both methods, Carson's and CDER, are very similar and that the series resistance is the parameter that is most influenced by the frequency and by the conductivity. The series inductance and the parallel capacitance do not vary significantly. Results showed that the CDER method is accurate when comparing to Carson's method and, being easier to use, it can replace Carson's method in most cases.

Key Words

Overhead Power Line Parameters; Series Impedance; Parallel Capacitance; Carson's Method; CDER Method; Line Constants of ATP/EMTP.

1. INTRODUCTION

1.1 Motivation

Electric Energy is nowadays beyond any doubt an essential asset for everyone. Almost every activity, from the industrial field to the domestic one, depends on electric energy. With the development of new technologies, and changes in the way of life of people in general, and generalized use of computers, electric energy became almost indispensable and therefore there is a need to develop and maintain all the structures that allows the creation, propagation and consumption of electric energy.

The transmission of Electric Energy can be done in alternate current or continuous current and can also be done by using underground cables or overhead lines.

Depending on the level of voltage of the transmission of electric energy there can be three kinds of overhead lines: High Voltage or Transmission Overhead Lines, Medium Voltage Overhead Lines and Low Voltage or Distribution Overhead Lines.

The Transmission and Distribution Overhead Lines are very important in Electric Energy Systems, because they are the arteries on witch the Electric Energy flows, from the production centers into the consumption ones. Therefore it is necessary to develop and project overhead lines

that better adapt to the new problems of the distribution systems: evaluation of nominal regime, disturbed and harmonics. In this context there is a need to correctly characterize the overhead lines, about their electric characteristics, resistance, reactance and capacitance.

Some mathematical models have been developed to allow the calculation of these electric parameters. Carson in 1926 published his article and since then it has been the standard model in the calculation of series impedance of overhead lines. Later on, C. Gary developed another model that is as good as Carson's and more simple to implement.

The other electrical characteristic of the overhead line is the parallel capacitance, and this parameter is calculated using Maxwell's potential coefficients.

A program was developed in this project in Matlab language that enables the calculation of the series impedance of three phase overhead lines using both methods, Carson's and CDER, and the parallel capacitance using Maxwell's potential coefficients, for any type of line geometry and type of conductors used.

Another program that is used for the calculation of the parameters of overhead lines is the program Line Constants from ATP/EMTP. This program is not very easy to use because it works with a card system in which the

minimum error in the edition of the input file produces errors in the output file. In the Line Constants Program the Method used for the calculation of the series impedance of the line is Carson's, and the one used for the calculation of the parallel capacitance is the potential coefficient of Maxwell.

The program developed here is easy to use by everyone, even the less experienced user and it also allows one to calculate the series impedance of a line using two different methods, and then compare the results obtained by both.

In this paper, we begin with an explanation about how both methods for the calculation of the series impedance work, Carson's and CDER and then explain how the method for the calculation of the parallel capacitance works.

After that, a practical example of a real overhead line is used to enable the comparison of the results obtained using the three methods and the program Line Constants.

It is also verified how some parameters influence the final results, parameters like frequency and conductivity. This is done in section 4.

1.2   Objective

The objective of this work is to obtain the parameters of three-phase overhead lines, for any geometry using any kind of material and to compare that results to the ones obtained using Line Constants that is a reference program in the calculation of overhead line parameters.

More over, a deeper study about these parameters is undertaken here, by varying the frequency and the conductivity of the overhead lines and seeing how this variation affects the final results.

Also a comparison was made between the methods used for the calculation of the series impedance of the overhead line, Carson's method and CDER method.

The results, obtained using the program developed in Matlab, were compared to those obtained using Line Constants.

2. MODELS FOR THE CALCULATION OF THE PARAMETERS OF OVERHEAD LINES

2.1 Series Impedance of the Overhead Line

2.1.1 Carson's Model

Carson's Model [4] although it has been published in 1926 is still today the standard frequency dependent method for the calculation of the series impedance of the line. Carson assumes that the earth is a uniform, plane, solid and infinite surface with a constant resistivity.

Carson's method expresses the frequency dependent series impedance by means of an improper integral that has to be expanded into an infinite series for computation.

Hence if not properly applied it can cause considerable errors at high frequencies [1].

Self Impedance of Conductor i:

The self impedance includes three components: self reactance of the conductor X_{ii} assuming that the line and the earth are both perfect conductors, internal impedance of the line Z_c and ground impedance Z_g [1].

The self impedance is then given by:

$$Z_{ii} = X_{ii} + Z_c + Z_g \qquad (2.1.1)$$

The self reactance of the conductor i is given by

$$X_{ii} = j\omega L_{ii} \qquad (2.1.2)$$

where the self inductance L_{ii} is calculated using this expression:

$$L_{ii} = \frac{\mu_0}{2\pi} \ln \frac{2h_i}{r_i} \qquad (2.1.3)$$

And μ_0 is the magnetic permeability constant in the vacuum and is given by:

$$\mu_0 = 4\pi \times 10^{-7} \text{ [H/m]} . \qquad (2.1.4)$$

By definition r_i is the external radius of the conductor and h_i is the average height of the conductor to the earth.

The conductors are usually tubular, **Fig 1**, with two types of materials. Therefore two radiuses are defined, one that limits the first material, usually steel that supports the cable, q that corresponds to the inner radius and the other that limits the second material, aluminum or copper that conducts the current through the line, r that corresponds to the outer radius.
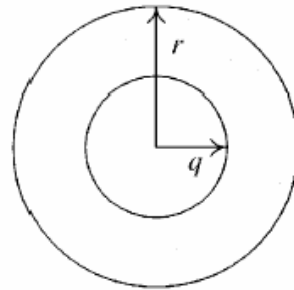
Fig. 1 – Model of the Tubular Conductor

The internal impedance of the conductor Z_c, is calculated using that notion that the conductors are tubular and is given by:

$$Z_c = R_{dc} \frac{j}{2} mr(1 - S^2) \times \frac{(ber\ mr + jbei\ mr) + \varphi(ker\ mr + jkei\ mr)}{(ber'\ mr + jbei'\ mr) + \varphi(ker'\ mr + jkei'\ mr)} \quad (2.1.5)$$

where R_{dc} is the DC resistance of the conductor and it is calculated by:

$$R_{dc} = 1/[\pi\sigma(r^2 - q^2)] \ . \quad (2.1.6)$$

σ is the conductivity of the material of the conductor, aluminum or copper.

The conductivity varies with the temperature, affecting Z_c and consequently Z_{ii}.

The variable S is the relation between the internal and the external radius of the conductor and is given by:

$$S = \frac{q}{r} \quad (2.1.7)$$

and m is a variable that relates the angular frequency ω with the conductivity σ and the permeability of the conductor μ:

$$m = \sqrt{\omega\mu\sigma} \ . \quad (2.1.8)$$

The variable ϕ in equation (2.1.5) is given by:

$$\varphi = -\frac{ber'mq + jbei'mq}{ker'mq + jkei'mq} \quad (2.1.9)$$

and *ber*, *bei*, *ker*, *kei* are Kelvin functions that belong to the family of Bessel functions and *ber'*, *bei'*, *ker'* e *kei'* are their derivatives respectively.

The Kelvin functions are defined by [3]:

$$ber\ x + jbei\ x = I_0\left(x\sqrt{j}\right)$$
$$ker\ x + jkei\ x = K_0\left(x\sqrt{j}\right) \quad (2.1.10)$$

where I_0 and K_0 are the Modified Bessel functions of order zero, of the first and second type respectively.

Matlab calculates automatically these Kelvin functions using a subprogram.

The derivatives of these functions are given by:

$$ber'\ x + jbei'\ x = \sqrt{j}I_1\left(x\sqrt{j}\right)$$
$$ker'\ x + jkei'\ x = -\sqrt{j}K_1\left(x\sqrt{j}\right) \quad (2.1.11)$$

where I_1 and K_1 are the Modified Bessel functions of first order, of the first and second type respectively.

When q is zero, the line is no longer tubular and in that case ϕ is zero.

The impedance of the earth is expressed by:

$$Z_g = R_g + jX_g \quad (2.1.12)$$

Further on it will be explained how Z_g is calculated.

Mutual Impedance between conductors i and j:

The mutual impedance between the two conductors *i* and *j*, both parallel to the earth with their respective average heights to the ground h_i and h_j, presents two components: The mutual reactance between conductors *i* and *j*, X_{ij}, and the impedance of the earth return path Z_{gm} that is common to the currents in the conductors *i* and *j* .

The mutual impedance is then given by:
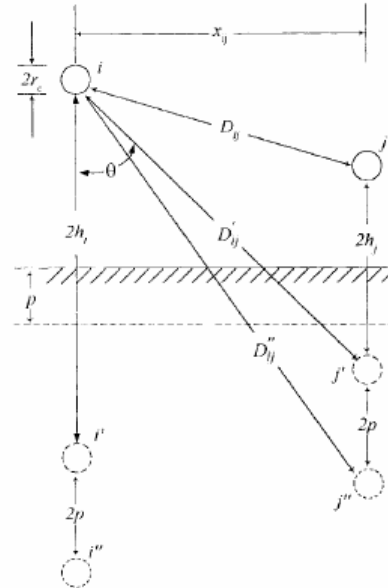
$$Z_{ij} = X_{ij} + Z_{gm} \quad (2.1.13)$$

The mutual reactance between conductors i and j is expressed by:

$$X_{ij} = j\omega L_{ij} \quad (2.1.14)$$

being the mutual inductance L_{ij}:

$$L_{ij} = \frac{\mu_0}{2\pi} ln\frac{D'_{ij}}{D_{ij}} \quad (2.1.15)$$

where D_{ij} is the distance between conductors *i* and *j* and D'_{ij} is the distance between the conductor *i* and the image of conductor *j*, **Fig 2**[1].

Fig. 2 –Allocation of conductors *i* and *j* and their images *i'* and *j'*. [1]

The impedance of the earth return path is given by:

$$Z_{gm} = R_{gm} + jX_{gm} \qquad (2.1.16)$$

Carson's correction terms for the self and mutual impedances due to the earth return path impedances Z_g and Z_{gm}, are given by the following expressions [3]:

$$R_g = 4w \times 10^{-7} \left\{ \begin{array}{l} \dfrac{\pi}{8} - b_1 k + b2 \left[\left(C_2 - lnk \right) k^2 \right] \\ + b_3 k^3 - d_4 k^4 - \ldots \end{array} \right\} \qquad (2.1.17)$$

$$X_g = 4w \times 10^{-7} \left\{ \begin{array}{l} \dfrac{1}{2} \left(0.6159315 - lnk \right) + b_1 k - d_2 k^2 \\ + b_3 k^3 - b_4 \left[\left(C_4 - lnk \right) k^4 \right] + \ldots \end{array} \right\} \qquad (2.1.18)$$

$$R_{gm} = 4w \times 10^{-7} \left\{ \begin{array}{l} \dfrac{\pi}{8} - b_1 k_m cos\theta \\ + b2 \left[\begin{array}{l} \left(C_2 - ln\,k_m \right) k_m^{\,2} cos2\theta \\ + \theta k_m^{\,2} sin2\theta \end{array} \right] \\ + b_3 k_m^{\,3} cos3\theta - d_4 k_m^{\,4} cos4\theta - \\ - \ldots \end{array} \right\} \qquad (2.1.19)$$

$$X_{gm} = 4w \times 10^{-7} \left\{ \begin{array}{l} \dfrac{1}{2} \left(0.6159315 - lnk_m \right) + b_1 k_m cos\theta \\ - d_2 k_m^{\,2} cos2\theta + b_3 k_m^{\,3} cos3\theta \\ - b_4 \left[\begin{array}{l} \left(C_4 - lnk_m \right) k_m^{\,4} cos4\theta + \\ \theta k_m^{\,4} sin4\theta \end{array} \right] + \\ + \ldots \end{array} \right\} \qquad (2.1.20)$$

where

$$b_1 = \frac{\sqrt{2}}{6} \qquad (2.1.21)$$

$$b_2 = \frac{1}{16} \qquad (2.1.22)$$

$$b_i = b_{i-2} \frac{sign}{i(i+2)} \qquad (2.1.23)$$

$$C_i = C_{i-2} + \frac{1}{i} + \frac{1}{i+2} \qquad (2.1.24)$$

$$C_2 = 1.3659315 \qquad (2.1.25)$$

$$d_i = \frac{\pi}{4} b_i \qquad (2.1.26)$$

In equation (2.1.23) the *sign* of coefficient b_i changes every four terms, that is *sign* = +1 for i = 1,2,3,4 and then *sign* = -1 for i = 5,6,7,8 and so on.

The variables k and k_m from equations (2.1.17) to (2.1.20) are frequency related and are given by:

$$k = 4\pi\sqrt{5} \times 10^{-4} \left(2h_i \right) \sqrt{f/\rho} \,, \qquad (2.1.27)$$

$$k_m = 4\pi\sqrt{5} \times 10^{-4} D'_{ij} \sqrt{f/\rho} \,, \qquad (2.1.28)$$

Where f is the frequency and ρ is the resistivity of the earth. The angle θ is indicated in **Fig.2** and is the angle between i-i' e i-j' and is given by:

$$\theta = sin^{-1} \left(x_{ij} / D'_{ij} \right), \qquad (2.1.29)$$

and the result is expressed in radians.

2.1.2 The Complex Depth of Earth Return Model (CDER)

In 1976, 50 years after the publication of Carson's method, C. Gary [5] a French investigator proposed an alternative method to Carson's in which the earth could be replaced by a set of earth return conductors located directly under the overhead lines at a complex depth. That is, the distances between the imagined earth return conductors and the overhead line conductors are complex numbers. This model presented very good results for the whole frequency range and being easier to implement than Carson's method it is expected that in the near future it can replace Carson's method for the calculation of overhead line parameters [1].

The CDER model assumes that the current that flows through conductor i returns through an imagined earth path located directly bellow the original conductor, under the ground at a depth of $\left(h_i + 2p \right)$ as shown in **Fig.2**. In this figure i'' refers to the imagined earth return conductor of conductor i and p to the skin depth of the ground.

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In other words, the earth can be replaced by a set of earth return conductors. The distance between a conductor and its imagined earth return conductor is $2(h_i + p)$. This distance is a complex number because p is a complex number given by:

$$p = \sqrt{\frac{\rho}{j\omega\mu_0}} \, . \qquad (2.1.30)$$

Therefore the self and mutual impedances are calculated using the following expressions [3]:

Self Impedance:

$$Z_{ii} = j\omega \frac{\mu_0}{2\pi} ln \frac{2(h_i + p)}{r_i} + Z_c \qquad (2.1.31)$$

Mutual Impedance:

$$Z_{ij} = j\omega \frac{\mu_0}{2\pi} ln \frac{\sqrt{\left(h_i + h_j + 2p^2\right) + x_{ij}^2}}{\sqrt{\left(h_i - h_j\right)^2 + x_{ij}^2}} =$$

$$= j\omega \frac{\mu_0}{2\pi} ln \frac{D_{ij}''}{D_{ij}} \qquad (2.1.32)$$

and h_i, h_j, x_{ij}, p, D_{ij} e D_{ij}'' are shown in **Fig. 2**.

2.2 Parallel Capacitance of the Overhead Line

The ideal line is formed of perfect conductors and dielectrics. Electromagnetic field does not penetrate perfect conductors so their frontiers are coincidental with the surfaces of the conductors and the earth. Therefore, the geometry of the conductors alongside with the parameters of the dielectric ε and μ, are the only parameters needed for the calculation of the capacitance coefficients of the overhead line.

The calculation of these coefficients is done through Maxwell potential coefficients P.

$$[C] = [P]^{-1} \, . \qquad (2.2.1)$$

The matrix **C** represents the parallel capacitance matrix by unit of length of the line and has dimension ($n \times n$), being n the number of conductors of the line.

The distances between the conductors when compared with the radiuses of the aerial conductors are much bigger and then, the matrix **P** can be calculated using the following expressions [1]:

$$P_{ii} = \frac{1}{2\pi\varepsilon_0} ln \frac{2h_i}{r_i} \qquad (2.2.2)$$

$$P_{ij} = \frac{1}{2\pi\varepsilon_0} ln \frac{D_{ij}'}{D_{ij}} \qquad (2.2.3)$$

In which P_{ii} corresponds to the terms in the main diagonal and P_{ij} corresponds to the terms off the main diagonal, and D_{ij} e D_{ij}' are the distances shown in **Fig.2.1.2**.

So using the previous expressions one can obtain the matrix **P** and inverting this matrix calculates the matrix **C**, whose elements can be separated into two components, one that can be named as self capacitance and the other the mutual capacitance.

The self capacitance C_{ii} corresponds to the main diagonal elements of the matrix **C** and is given by the sum of the parallel capacitances by unit of length from conductor i to all the other conductors as for the earth also.

The mutual capacitance C_{ij}, corresponds to the elements off the main diagonal of the matrix **C** and is given by the negative value of the parallel capacitance by unit of length from the conductor i to k.

Again due to the symmetry of the line, C_{ij} equals C_{ji}.

Because the line is not ideal, as one supposed before, and the conductors aren't really circular and cylindrical and because they are suspended in poles and they are not really parallel to the earth, there is the need to introduce the average height concept for the calculation of the potential coefficients.

3. COMPARISON OF RESULTS BETWEEN THE TWO METHODS AND ATP/EMTP

In this section one will present the results obtained using the program developed here and the results obtained using the program Line Constants of ATP/EMTP [3] for overhead lines with and without bundling and with different geometries.

Results will be presented and compared between the three methods used (Carson, CDER and Maxwell potential coefficient), so one can verify which one of both methods for the calculation of the series impedance can approximate more to the results obtained with the reference program Line Constants.

The comparisons will be made using the positive-sequence element obtained in all the matrices, the series impedance matrix and also the parallel capacitance matrix.

All the conductors are considered as being made of aluminum and steal at a temperature of 40ºC.

The conductivity of the conductors is a characteristic that has a major influence in the calculation of the overhead line parameters, especially in the series impedance, and it depends on the material that the conductors are made of.

The calculation of the conductivity of the conductors depends on the temperature that these are subjected and is given by the inverse of the resistivity. The resistivity is given by the following expression:

$$\rho = \rho_0 (1 + \alpha \Delta T) \qquad (3.1)$$

in which α is the temperature coefficient and therefore the conductivity will be given by:

$$\sigma = \frac{1}{\rho} . \qquad (3.2)$$

Being the conductors of overhead lines made of aluminum and steal, but being the aluminum the material responsible for the electrical conduction, one will consider for the calculation of the parameters of overhead lines that the conductivity of the conductors is the conductivity of aluminum.

For aluminum conductors $\alpha = 0.00429 \, ^{\circ}C^{-1}$ and for this temperature $\sigma = 3.22 * 1E + 7 [Sm^{-1}]$.

Overhead Line 1:

Overhead three phase line with two conductors per phase and two guard cables.

In this example the line is at 50Hz.

All the conductors including the guard cables are made of aluminum and steal and the conductivity of the conductors was calculated as being aluminum conductors.

The conductors of the phases are at the same height and the guard cables are at a higher position.

In the next tables, results of the calculation of the parameters are presented, using both methods and the program Line Constants:

Table 1 – Results of the calculation of the Series Impedance

	R (ohm/m)	L (H/m)
M.Carson	3,45E-5	9,63E-7
M.CDER	3,45E-5	9,63E-7
ATP/EMTP	3,45E-5	9,63E-7

Table 2 – Results of the calculation of the Parallel Capacitance

	C (F/m)
Maxwell Potential Coeff.	1,182E-11
ATP/EMTP	1,19E-11

All the three ways for the calculation of the series impedance, Carson, CDER and Line Constants, gave approximated results. Also for the parallel capacitance, the results obtained using Line Constants and the program developed here was much approximated. So, the three methods are good for the calculation of the parameters of overhead lines, because when comparing to the reference program, Line Constants, the results were very similar.

4. ANALYSIS OF THE INFLUENCE OF THE INITIAL VARIABLES IN THE FINAL RESULTS

Here an analysis is made of how varying the initial parameters the final results get affected.

Some of the initial parameters have a major influence in the final results, especially the conductivity of the conductor which is usually aluminum or copper and the frequency. Other initial parameters affect the results but here in this paper only these two are taken into account.

The overhead line used here in this section is the Overhead Line 1 presented in the previous section.

4.1 Analysis of the final results with the variation of the frequency

In this sub section the frequency of the line is varied from 50 to 2000Hz and one can see how the final parameters are affected.

Also a comparison was made between the results obtained through Carson's method, CDER method and Line Constants of ATP/EMTP.

The following figures show us how the final parameters (R, L and C) of the direct component vary with the frequency.

For the series resistance and inductance, **Fig.3** and **Fig.4**, Carson method is represented in blue, CDER in red and the Program Line Constants in green, and for the parallel capacitance, **Fig.5** the Maxwell potential coefficient is represented in red and Line Constants in green:
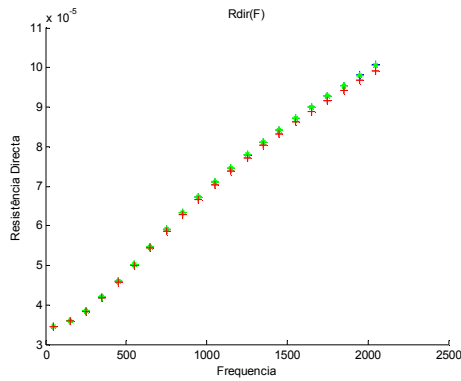
Fig.3 – Variation of the positive-sequence resistance of the line with the frequency.
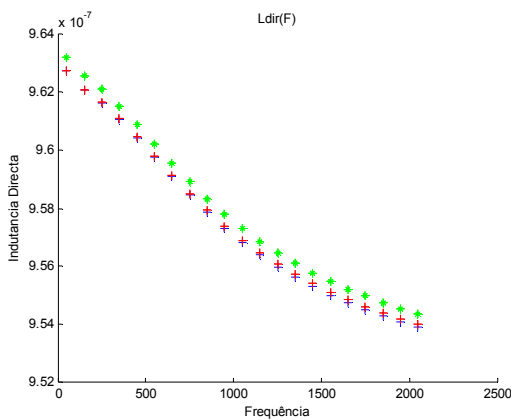
Fig.4– Variation of the positive-sequence inductance of the line with the frequency.
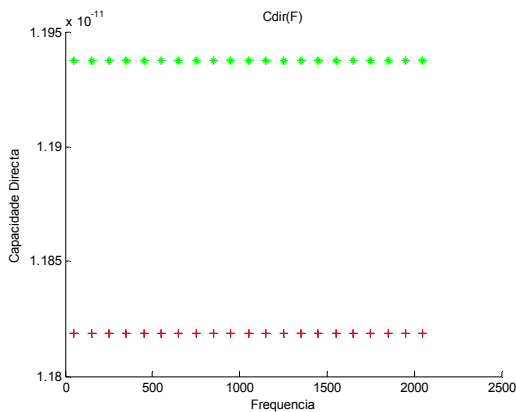
Fig.5– Variation of the positive sequence-capacitance of the line with the frequency.

In **Fig.3** one can see that the positive-sequence resistance increases with the frequency almost proportionally, something that was expected due to the skin effect, that with the increase of the frequency it increases the resistance on the conductors.

Also in **Fig.3** one can observe that with the increasing of frequency the results obtained with both methods and between both methods and Line Constants present some differences, but these are small differences, and the errors are not that significant.

The positive-sequence inductance almost doesn't vary with the increasing of the frequency. It decreases slightly as one can see in **Fig.4**. Still, from 750Hz approximately the results for the Carson and CDER methods start to get different, but this difference is still not significant.

The positive-sequence parallel capacitance doesn't vary with the frequency and present only a small difference between the values obtained with the program developed here and Line Constants that leads into a small error.

Both methods present very similar values for the positive-sequence resistance and also for the inductance, and therefore one can say that the results obtained from the program developed in Matlab Language, using Carson or CDER are very good and accurate, because when compared to Line Constants program the differences are almost non existent.

4.2 Analysis of the final results with the variation of the conductivity of the conductors of the line

Another initial parameter that affects the final parameters is the conductivity of the conductors of the line. So, a variation of this parameter was done, to observe how it influenced (R, L and C).

The conductors are made of aluminum and steal, but the conductivity of the conductors is considered to be the conductivity of aluminum, because the steel only supports the weight of the conductor and the aluminum conducts the current.

The conductivity is a variable that varies with temperature and its variation is shown in (3.1) and (3.2). At 20ºC the conductivity of the aluminum is:

$$\sigma = 3.77E7 \ [S \ m^{-1}] \qquad (4.2.1)$$

and through (3.1) and (3.2) one can calculate the conductivity of the conductors for any other temperature.

Here the line used for this study was the overhead Line 1 at 50 Hz.

The next table shows the results of the line parameters when varying the temperature:

Table 3 – Results for the calculation of the positive sequence parameters when varying the conductivity of the conductors

T(ºC)	Método de Carson			Método CDER		
	R (Ω/m)	L (H/m)	C (F/m)	R (Ω/m)	L (H/m)	C (F/m)
20	2,96E-5	9,63E-7	1,18E-11	2,96E-5	9,63E-7	1,18E-11
30	3,08E-5	9,63E-7	1,18E-11	3,08E-5	9,63E-7	1,18E-11
40	3,20E-5	9,63E-7	1,18E-11	3,20E-5	9,63E-7	1,18E-11
50	3,33E-5	9,63E-7	1,18E-11	3,33E-5	9,63E-7	1,18E-11
60	3,45E-5	9,63E-7	1,18E-11	3,45E-5	9,63E-7	1,18E-11
70	3,57E-5	9,63E-7	1,18E-11	3,57E-5	9,63E-7	1,18E-11
80	3,70E-5	9,63E-7	1,18E-11	3,70E-5	9,63E-7	1,18E-11

As the temperature increases the conductivity decreases and as a consequence of that the positive sequence series resistance increases for both, the Carson and CDER methods.

The results were always the same for both methods for the same temperature, something that was observed before. The variation of temperature doesn't affect the series inductance and the parallel capacitance as one expected.

The only parameter influenced by the variation of conductivity is the series resistance which makes sense, because if there is a decreasing of the conductivity it happens because the resistance is increasing.

5. CONCLUSIONS

Being the objective of the work to create an easy way of calculating the parameters of overhead lines with good and accurate results for a variety of geometries, sections and materials one can say that the objective was reached.

A program developed in Matlab language was created for the calculation of parameters of overhead three phase lines and the results obtained with this program were compared to those obtained with the reference program Line Constants of ATP/EMTP.

For the correct structure of the program developed, a study of other programs that are used for the calculation of the parameters of overhead lines was undertaken, especially the Line Constants program of ATP/EMTP. One studied the methodology and functioning of Line Constants program.

In this work two different methods for the calculation of the series impedance (series resistance and inductance) per unit of length were studied and implemented and compared with each other to find which one of both was the most accurate and easy to implement.

The methods implemented in this program were Carson's Method from 1926 [4] and CDER (Complex Depth of Earth Return) Method introduced by C. Gary in 1976 [5]. Afterwards these methods were compared with Line Constants for three phase overhead lines with different geometries and materials in Section 4.

Carson's Method is the classic method for the calculation of overhead lines parameters and the only method that gives an analytical complete solution for this calculation. However, this solution is expressed in function of improper integrals that have to be expanded into infinite series to allow calculation. The series converge slowly at high frequencies, but nowadays computers allow the calculation of these series easily, something that in the past was difficult to do [1].

CDER method is indeed an approximation to Carson's method. This is a much simpler method to implement and as accurate as Carson's. It can replace Carson's method in almost all cases of calculation of series impedance of overhead lines [1].

After explained the methods, it is important to take into account the historical path that lead into them. Over the last years many efforts have been made in the way to find new solutions for the calculation of overhead line parameters, and new methods like the Finite Element method, have been developed. What one can conclude on this is that it is always possible to find a better and newer solution for an old problem.

Also along with this work the method for the calculation of the parallel capacitance was studied and implemented. This method does the calculation of the parallel capacitance using Maxwell's potential coefficients [2].

In this work an overhead line was studied and the calculation of the parameters of the line was done and the results were compared for the series impedance, between the two methods presented in the work, Carson and CDER, and between Line Constants program. Also comparison of results was made between Line Constants and Maxwell potential coefficients method used here. After observing the results one can conclude that the program developed here presented very good results for all three methods, Carson, CDER and Maxwell potential coefficients, when comparing the results with Line Constants [3].

Using the example of a real overhead line, a variation of the frequency and the temperature was taken into account to see how these variations influenced the final parameters of the overhead line.

After observing the results one can say that as the frequency increases the series resistance of the overhead line also increases almost proportionally, the series inductance decreases insignificantly and the parallel capacitance does not vary.

As for the conductivity, with the increasing of the temperature the conductivity decreases and the only parameter that changes its value is the series resistance of the overhead line that increases with the temperature.

The program developed here presents two advantages relatively to Line Constants, one is that it is easier to use, because of the system of cards that Line Constants still uses, and the other one is that this program allows the calculation of the series impedance overhead line using

two different methods, Carson and CDER, and Line Constants uses Carson's method only.

Here in this project, an easier and also very accurate way of calculating overhead line parameters was developed and as for the objectives one proposed initially, one can say they were accomplished.

6. REFERENCES

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