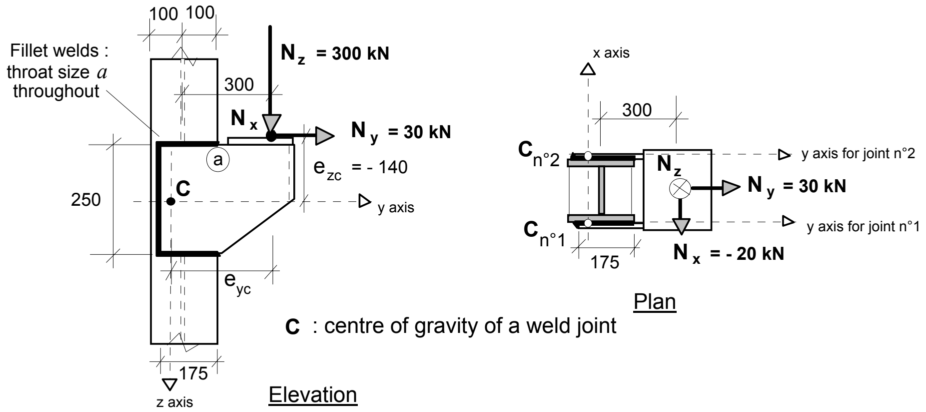
DESIGN EXAMPLE 5 – WELDED JOINT

The joint configuration and its loading are shown in the figure below. Noting that there are two identical plane fillet weld joints of constant throat size sharing the applied loading, the required throat size for the welds shall be determined. Right angle (equal leg) welds will be used throughout.



Material properties

Use material grade 1.4401

0,2% proof stress = 220 N/mm^2 and the tensile strength is = 530 N/mm^2 — Table 3.1

Take f_y as the 0,2% proof stress = 220 N/mm^2 and f_u = 530 N/mm^2 — Section 3.2.4

E = 200 000 N/mm^2 and G = 76 900 N/mm^2 — Section 3.2.4

It is assumed that the yield and ultimate tensile strength of the weld exceed those of the parent metal. — Section 6.4.1

Partial safety factor

Partial safety factor on weld resistance : γ_{M2} = 1,25 — Table 2.1

β_w = 1,0 — Section 6.4.2

The need to include a reduction factor on the weld resistance to account for its length will be examined.

Analysis

An elastic analysis approach is used here for designing the right-angle equal-leg fillet weld for the load case indicated above. An elastic analysis of the welded joint leads to a conservative estimate of the joint resistance. — EN 1993-1-8, clause 2.5

The co-ordinates of a point (x_c , y_c , z_c) on the welded joint are taken with reference to a right hand axis system with an origin at the centre of gravity of the welded joint. (In the present case the joint is taken to be in the y-z plane so that x_c = 0 throughout.)

The main purpose of the elastic analysis is to determine the induced design forces in the weld at the most severely loaded point or points of the welded joint, often referred to as the "critical" points. For the welded joint being examined the critical point can be taken as being the point the farthest removed from the centre of gravity of the joint.

102, Route de Limours
F-78471 St Rémy Lès Chevreuse Cedex
France
Tel : +33 (0)1 30 85 25 00
Fax : +33 (0)1 30 52 75 38

The vectors of the applied force, its eccentricity and the resulting moments acting on a welded joint of general form and centre of gravity C can be expressed as follows :

Applied force

$$\overline{N_{\mathrm{w,Ed}}} = \left[N_{\mathrm{x,Ed}},\ N_{\mathrm{y,Ed}},\ N_{\mathrm{z,Ed}}\right]$$

Eccentricity of the applied force

$\overline{e_{\mathrm{N}}} = \left[e_{\mathrm{xc}},\ e_{\mathrm{yc}},\ e_{\mathrm{zc}}\right]$ which are the co-ordinates of the point of application of the force vector $\overline{N_{\mathrm{w,Ed}}}$

Applied moments

$$M_{\mathrm{xc,Ed}} = e_{\mathrm{yc}}N_{\mathrm{z,Ed}} - e_{\mathrm{zc}}N_{\mathrm{y,Ed}}$$

$$M_{\mathrm{yc,Ed}} = e_{\mathrm{zc}}N_{\mathrm{x,Ed}} - e_{\mathrm{xc}}N_{\mathrm{z,Ed}}$$

$$M_{\mathrm{zc,Ed}} = e_{\mathrm{xc}}N_{\mathrm{y,Ed}} - e_{\mathrm{yc}}N_{\mathrm{x,Ed}}$$

A linear elastic analysis of the joint for a general load case leads to the following induced force components per unit length of weld at a point with co-ordinates (x_c, y_c, z_c) , where the throat size is denoted by a:

$$F_{\mathrm{wx,Ed}} = a\left[\frac{N_{\mathrm{x,Ed}}}{A_{\mathrm{w}}} + \frac{z_{\mathrm{c}}M_{\mathrm{yc,Ed}}}{I_{\mathrm{yc}}} - \frac{y_{\mathrm{c}}M_{\mathrm{zc,Ed}}}{I_{\mathrm{zc}}}\right]$$

$$F_{\mathrm{wy,Ed}} = a\left[\frac{N_{\mathrm{y,Ed}}}{A_{\mathrm{w}}} + \frac{x_{\mathrm{c}}M_{\mathrm{zc,Ed}}}{I_{\mathrm{zc}}} - \frac{z_{\mathrm{c}}M_{\mathrm{xc,Ed}}}{I_{\mathrm{xc}}}\right]$$

$$F_{\mathrm{wz,Ed}} = a\left[\frac{N_{\mathrm{z,Ed}}}{A_{\mathrm{w}}} + \frac{y_{\mathrm{c}}M_{\mathrm{xc,Ed}}}{I_{\mathrm{xc}}} - \frac{x_{\mathrm{c}}M_{\mathrm{yc,Ed}}}{I_{\mathrm{yc}}}\right]$$

In the above expressions, the resisting sectional throat area and the inertias about the principal axes of the welded joint are:

$A_{\mathrm{w}} = \int a\,dl = \sum a_{\mathrm{i}}l_{\mathrm{i}}$ for a weld of straight segments of length l_i and throat size a_i,

$$I_{\mathrm{xc}} = \int a(y_{\mathrm{c}}^2 + z_{\mathrm{c}}^2)dl$$

$$I_{\mathrm{yc}} = \int a(x_{\mathrm{c}}^2 + z_{\mathrm{c}}^2)dl$$

$$I_{\mathrm{zc}} = \int a(x_{\mathrm{c}}^2 + y_{\mathrm{c}}^2)dl$$

As the throat size, a, is constant throughout the plane joint, one can write :

$$\frac{A_{\mathrm{w}}}{a} = \int dl = \sum l_{\mathrm{i}},$$

Since $x_{\mathrm{c}} = 0$,

Centre Technique Industriel de la Construction Métallique
102, Route de Limours
F-78471 St Rémy Lès Chevreuse Cedex
France
Tel : +33 (0)1 30 85 25 00
Fax : +33 (0)1 30 52 75 38

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$$\frac{I_{zc}}{a} = \int (y_c^2)dl\,, \qquad \frac{I_{yc}}{a} = \int (z_c^2)dl \qquad , \quad \frac{I_{xc}}{a} = \int (y_c^2 + z_c^2)dl = \frac{I_{yc}}{a} + \frac{I_{zc}}{a}$$

Design approaches — Section 6.4.2

Determine the required weld throat size at the critical point.

Two different procedures are allowed for designing fillet welds:

The first procedure is based on the simplified, and more conservative, design shear strength for a fillet weld. The design shear force per unit length of weld at any point of the joint is defined as the vector sum of the induced forces per unit length due to all forces and moments transmitted by the welded joint. This design shear force per unit length should not exceed the design resistance per unit length which is taken as the design shear strength multiplied by the throat size. This approach ignores the throat plane orientation to the direction of resultant weld force per unit length.

The second procedure is based on comparing the basic design strength of the weaker part joined to the applied design weld stress in the weld throat determined by a Von Mises type of formula. This approach is the most precise as it allows for the throat plane orientation to the direction of resultant weld force per unit length.

1. Simplified design shear strength of the weld

The design resistance check of the fillet weld is as follows:

$$F_{\mathrm{w,Ed}} \;=\; \sqrt{F_{\mathrm{wx,Ed}}^2 + F_{\mathrm{wy,Ed}}^2 + F_{\mathrm{wz,Ed}}^2} \le F_{\mathrm{w,Rd}} \;=\; af_{\mathrm{vw,d}} \;=\; a\left(\frac{f_{\mathrm{u}}/\sqrt{3}}{\beta_{\mathrm{w}}\gamma_{\mathrm{M2}}}\right)$$

EN 1993-1-8, clause 4.5.3.3

Where:

$f_{\mathrm{vw,d}}$ is the design shear strength of the weld

$F_{\mathrm{w,Rd}}$ is the design (shear) resistance per unit length of weld of throat size a.

For stainless steel β_{w} may be take as 1.0 — Section 6.4.2

When the design procedure requires that a suitable throat size be obtained, the design expression becomes :

$$a \;\ge\; \frac{F_{\mathrm{w,Ed}}}{f_{\mathrm{vw,d}}}$$

2. Basic design strength of the weld

In this approach one must check the Von Mises type stress in the weld throat against the basic design strength of the fillet weld material. In general this requires that the stresses in the weld throat, $\sigma_\perp$, $\tau_\perp$ and $\tau_{||}$ be obtained, thus taking account of the orientation of the plane of the throat area to the direction of the resultant induced weld force per unit length.

The design formula is as follows:

$$\sqrt{\sigma_\perp^2 + 3(\tau_\perp^2 + \tau_{||}^2)} \;\le\; \frac{f_{\mathrm{u}}}{\beta_{\mathrm{w}}\gamma_{\mathrm{M2}}}$$

Eq. 6.12a

It is also required to check the normal stress separately:

$$\sigma_\perp \;\le\; \frac{0.9 f_{\mathrm{u}}}{\gamma_{\mathrm{M2}}}$$

Eq 6.12b

102, Route de Limours
F-78471 St Rémy Lès Chevreuse Cedex
France
Tel : +33 (0)1 30 85 25 00
Fax : +33 (0)1 30 52 75 38

For the present case of a plane fillet weld joint with right angle (equal leg) welds this latter check is not critical. However it may be so for partial penetration welds in bevelled joints.

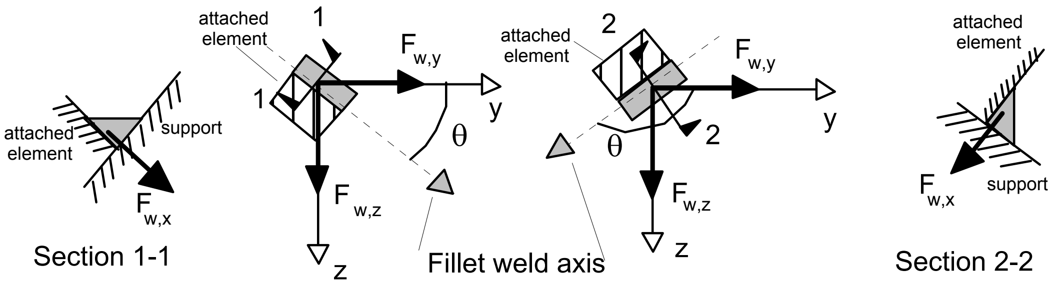
Instead of having to calculate the stresses ($\sigma_{\perp}$, $\tau_{\perp}$ and $\tau_{||}$) in the weld throat the following design check expression may be used for y-z plane joints with right angle (equal leg) welds:

$$2F_{\mathrm{w,x}}^2 + 2F_{\mathrm{w,y}}^2 + 2F_{\mathrm{w,z}}^2 + F_{\mathrm{w,y}}^2 Cos^2\theta + F_{\mathrm{w,z}}^2 Sin^2\theta - 2F_{\mathrm{w,x}}F_{\mathrm{w,y}}Sin\theta + 2F_{\mathrm{w,x}}F_{\mathrm{w,z}}Cos\theta$$

$$+ 2F_{\mathrm{w,y}}F_{\mathrm{w,z}}Sin\theta\, Cos\theta \le \left(a\frac{f_{\mathrm{u}}}{\beta_{\mathrm{w}}\gamma_{\mathrm{M2}}}\right)^2$$

Note : The subscripts have been shortened: $F_{\mathrm{w,x}}$ for $F_{\mathrm{wx,Ed}}$ etc.

In the above expression the angle θ is that between the y axis and the axis of the weld as shown in the following figure.



The force components at the critical point of the weld are determined in the Appendix to this design example.

1. Design using the simplified design shear strength approach

The design shear strength for the simplified design approach is:

$$f_{\mathrm{vw,d}} = \frac{f_{\mathrm{u}}}{\beta_w \gamma_{\mathrm{M2}}\sqrt{3}} = \frac{530}{1.0\times 1{,}25\times\sqrt{3}} \approx 245\,\mathrm{N/mm^2}$$

EN 1993-1-8, Eq. 4.4

The value of the resultant induced force per unit length in a weld throat of 1mm is :

$$F_{\mathrm{w,Ed}} = \sqrt{F_{\mathrm{wx,Ed}}^2 + F_{\mathrm{wy,Ed}}^2 + F_{\mathrm{wz,Ed}}^2} = \sqrt{243^2 + 747^2 + 966^2} = 1245\,\mathrm{N/mm}$$

The required throat size is therefore:

$$a \ge \frac{F_{\mathrm{w,Ed}}}{f_{\mathrm{vw,d}}} = \frac{1245}{245} \approx 5{,}0\,\mathrm{mm}$$

102, Route de Limours
F-78471 St Rémy Lès Chevreuse Cedex
France
Tel : +33 (0)1 30 85 25 00
Fax : +33 (0)1 30 52 75 38

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2. Design of the weld using the basic design weld strength approach

The basic design strength of the weld material is taken as follows:

$$\frac{0.9 f_u}{\gamma_{M2}} = \frac{0.9 \times 530}{1{,}25} = 381{,}6 \text{ N/mm}^2$$ Eq. 6.12b

Where f_u is the ultimate tensile strength of the weaker part joined

At the point (a), where the angle θ is 0°, the design check expression becomes:

$$2F_{\text{wx,Ed}}^2 + 3F_{\text{wy,Ed}}^2 + 2F_{\text{wz,Ed}}^2 + 2F_{\text{wx,Ed}}F_{\text{wz,Ed}} \leq \left(a \frac{f_u}{\gamma_{Mw}} \right)^2$$

The required throat size is therefore:

$$a \geq \frac{\sqrt{2 \times (-243)^2 + 3 \times (747)^2 + 2 \times (966)^2 + 2 \times (-243) \times (966)}}{381{,}6} = 4{,}7 \text{ mm}$$

Adopt a 5 mm throat size and assume that the weld is full size over its entire length.

Note :
A reduction factor is required for splice joints when the effective length of fillet weld is greater than 150*a*. The reduction factor would seem to be less relevant for the present type of joint. Nevertheless by considering, safely, the full length of the welded joint and a throat size of 5 mm one obtains:

$$\beta_{\text{LW.1}} = 1{,}2 - 0{,}2L_j/(150a) = 1{,}2 - 0{,}2(600)/(150 \times 5) = 1{,}04 \quad \text{Take } \beta_{\text{LW.1}} = 1{,}0$$ EN 1993-1-8, Eq. 4.9

It is concluded that the use of a reduction factor on the design strength of the weld is not required.

Appendix – Calculation of the force components at the critical point of the weld

Geometric properties of the welded joint

There are two similar joints, one on each side of the column, resisting the applied loads. Only one of the joints needs to be examined. It is placed in the y-z plane.

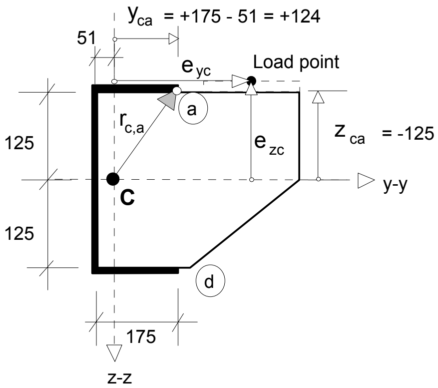
Throat area and positions of the centre of gravity and the critical point

Throat area (resisting section) of each of the joints made up of straight segments of length L_i and constant throat size a is, for each 1mm of throat size:

$$\frac{A_\mathrm{w}}{a} = \frac{a\int ds}{a} = \frac{\sum A_{\mathrm{w},i}}{a} = \frac{\sum aL_{\mathrm{w},i}}{a} = \sum L_i = (2\times175+250) = 600\,\mathrm{mm^2/m}$$

Distance of the centre of gravity from the vertical side (parallel to the z axis) of the joint of constant throat size a:

$$\bar{y} = \frac{\sum \bar{y}_i (A_{\mathrm{w},i}/a)}{\sum A_{\mathrm{w},i}/a} = \frac{\sum \bar{y}_i L_i}{\sum L_i} = \frac{2\times(87{,}5\times175)+(0\times250)}{600} \approx 51\,\mathrm{mm}$$



The co-ordinates of the position of the critical point of the joint, the point (a), relative to the principal axes through centre of gravity (C) are :

$y_\mathrm{ca} = +(175-51) = +124\,\mathrm{mm} \qquad z_\mathrm{ca} = -125\,\mathrm{mm}$

Note: the point (d) might also be chosen as a potential critical point, for which :

$y_\mathrm{cd} = +(175-51) = +124\,\mathrm{mm} \qquad z_\mathrm{cd} = +125\,\mathrm{mm}$

However, for the load case considered it is evident that the point (a) is the most critical.

Inertias of the joint resisting section

For each of the joints, for each 1mm of throat size :

$$\frac{I_\mathrm{yc}}{a} = \int z_\mathrm{c}^2 ds = 2\times175\times125^2 + 250^3/12 = 6{,}77\times10^6\,\mathrm{mm^4/mm}$$

102, Route de Limours
F-78471 St Rémy Lès Chevreuse Cedex
France
Tel : +33 (0)1 30 85 25 00
Fax : +33 (0)1 30 52 75 38

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$$\frac{I_{zc}}{a} = \int y_c^2 ds = 250\times 51^2 + 2\times 175^3/12 + 2\times 175\times(87{,}5-51)^2 = 210 \text{ mm}^4/\text{mm}$$

For the “torsion” moment the relevant inertia, per joint, is :

$$I_{xc} = a\int r_c{}^2 ds = a\int y_c{}^2 ds + a\int z_c{}^2 ds = I_{zc} + I_{yc}$$

So that

$$\frac{I_{xc}}{a} = (6{,}77+2{,}01)\times 10^6 = 8{,}78\times 10^6 \text{ mm}^4/\text{mm}$$

Applied forces and moments

It is assumed that applied loads and moments are shared equally by the two joints.

The applied axial and shear force components per joint are:

$$N_{x,Ed} = -\frac{20}{2} = -10\,\text{kN},\quad N_{y,Ed} = +\frac{30}{2} = +15\,\text{kN},$$

$$N_{z,Ed} = +\frac{300}{2} = +150\,\text{kN}$$

Applied moments are calculated using the applied force components and their eccentricities. The eccentricities, i.e. the co-ordinates of the effective load point, are :

e_{xc} = 0 as the effective load point is taken to be in the y-z plane of the joint,

e_{yc} = (300 – 100 + 175 – 51) = +324 mm,

e_{zc} = –140 mm

The applied moments, per joint, are then;

$$M_{xc,Ed} = e_{yc}N_{z,Ed} - e_{zc}N_{y,Ed} = (+324)\times(+150) - (-140)\times(+15) = +50{,}7\text{ kNm}$$

$$M_{yc,Ed} = e_{zc}N_{x,Ed} - e_{xc}N_{z,Ed} = (-140)\times(-10) - (0)\times(+150) = +1{,}4\text{ kNm}$$

$$M_{zc,Ed} = e_{xc}N_{y,Ed} - e_{yc}N_{x,Ed} = (0)\times(+15) - (+324)\times(-10) = +3{,}24\text{ kNm}$$

Force components at the critical point of the weld

For the y-z plane joint, the force components per unit length of weld at the point (a) are:

$$F_{wx,Ed} = \frac{N_{x,Ed}}{A_w/a} + \frac{z_{ca}M_{yc,Ed}}{I_{yc}/a} - \frac{y_{ca}M_{zc,Ed}}{I_{zc}/a}$$

$$F_{wy,Ed} = \frac{N_{y,Ed}}{A_w/a} - \frac{z_{ca}M_{xc,Ed}}{I_{xc}/a}$$

$$F_{wz,Ed} = \frac{N_{zc,Ed}}{A_w/a} + \frac{y_{ca}M_{xc,Ed}}{I_{xc}/a}$$

The contributions to the weld force components (at all points of the welded joint) from the applied force components are :

$$F_{\mathrm{w,x}}^{\mathrm{N_x}} = \frac{N_{\mathrm{x,Ed}}}{A_{\mathrm{w}}\ /\ a} = \frac{-10}{600} = -0{,}017\ \mathrm{kN/mm}$$

$$F_{\mathrm{w,y}}^{\mathrm{N_y}} = \frac{N_{\mathrm{y,Ed}}}{A_{\mathrm{w}}\ /\ a} = \frac{+15}{600} = +0{,}025\ \mathrm{kN/mm}$$

$$F_{\mathrm{w,z}}^{\mathrm{N_z}} = \frac{N_{\mathrm{z,Ed}}}{A_{\mathrm{w}}\ /\ a} = \frac{+150}{600} = +0{,}25\ \mathrm{kN/mm}$$

The various contributions to the weld force components per unit length of weld at the point (a) from the applied moment components are :

$$F_{\mathrm{w,y}}^{\mathrm{M_{xc}}} = -M_{\mathrm{xc,Ed}}\frac{z_{\mathrm{c,a}}}{(I_{\mathrm{xc}}\ /\ a)} = -50{,}7\times10^{6}\times\frac{(-125)}{8{,}78\times10^{6}} = +722\ \mathrm{N/mm}$$

$$F_{\mathrm{w,z}}^{\mathrm{M_{xc}}} = +M_{\mathrm{xc,Ed}}\frac{y_{\mathrm{c,a}}}{(I_{\mathrm{xc}}\ /\ a)} = +50{,}7\times10^{6}\times\frac{(+124)}{8{,}78\times10^{6}} = +716\ \mathrm{N/mm}$$

$$F_{\mathrm{w,x}}^{\mathrm{M_{yc}}} = +M_{\mathrm{yc,Ed}}\frac{z_{\mathrm{c,a}}}{(I_{\mathrm{yc}}\ /\ a)} = +1{,}41\times10^{6}\times\frac{(-125)}{6{,}77\times10^{6}} = -26\ \mathrm{N/mm}$$

$$F_{\mathrm{w,x}}^{\mathrm{M_{zc}}} = -M_{\mathrm{zc,Ed}}\frac{y_{\mathrm{c,a}}}{(I_{\mathrm{zc}}\ /\ a)} = -3{,}24\times10^{6}\times\frac{(+124)}{2{,}01\times10^{6}} = -200\ \mathrm{N/mm}$$

Combining the contributions at the point (a) from the forces and the moments one obtains :

$$F_{\mathrm{wx,Ed}} = F_{\mathrm{w,x}}^{\mathrm{N_x}} + F_{\mathrm{w,x}}^{\mathrm{M_{yc}}} + F_{\mathrm{w,x}}^{\mathrm{M_{zc}}} = -17 - 26 - 200 = -243\ \mathrm{N/mm}$$

$$F_{\mathrm{wy,Ed}} = F_{\mathrm{w,y}}^{\mathrm{N_y}} + F_{\mathrm{w,y}}^{\mathrm{M_{xc}}} = +25 + 722 = +747\ \mathrm{N/mm}$$

$$F_{\mathrm{wz,Ed}} = F_{\mathrm{w,z}}^{\mathrm{N_z}} + F_{\mathrm{w,z}}^{\mathrm{M_{xc}}} = +250 + 716 = +966\ \mathrm{N/mm}$$

These resultant induced force components per unit length are for a welded joint with a weld throat size of 1mm throughout its entire effective length.