Calculus Philosophy

Final Report

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6-24-2018

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Final Report

Introduction

The purpose of this instructional improvement grant was to facilitate conversations between mathematics faculty members who regularly teach the calculus sequence. Through twelve regular weekly meetings and one final meeting at the end of the semester, we were able to create a unified philosophy for all calculus courses, review and update the student learning outcomes for all calculus courses, and create teacher notes for Calculus I and Calculus II. We were also to able to create resources for instructors that include examples of *Mathematica* projects, final exam questions for Calculus I, and application problem question banks for Calculus I.

Calculus Philosophy

The main goal of this grant was to discuss our perspectives on calculus education and to create a unified statement. Although the faculty members come from various backgrounds, we were able to agree on the need to present the course material from a multifaceted viewpoint. The students are expected to learn standard calculus mechanics, utilize technology to highlight these ideas, and reinforce this learning through writing.

Through reviewing older extended course outlines, we were able to find previous statements that served as a basis for our new philosophy. However, these statements were listed under sections of the EXO that are no longer present on the current form. The following statement comes from the Fall 2010 MAT221 EXO:

> In this course we develop and apply techniques of limit evaluation and differentiation; definite integrals and integration (topics that will be developed more fully in MAT222) are introduced. Although students will not be responsible for producing formal mathematical proofs it is important that they see a few proofs during the semester, and that they get used to the idea of mathematics being developed from basic definitions. Students begin to develop an awareness of the structures and methods of mathematics through demonstration of theory, directed investigations using technology, and through practice.
>
> Students leaving the course must be comfortable working with limits and derivatives from several points of view: using definitions, numerical approximations, graphical interpretations and algebraic calculations. In addition, students must be able to use graphing calculators and a computer algebra system to investigate and illustrate ideas developed during the semester, and to check their own work. Furthermore, students must be able to articulate the mathematics they use, especially when applied to problems in other areas.

We all agreed that this was an excellent statement, but lacked a focus on writing and needed more clarity on the use of technology. Johanna Halsey was able to distill this statement and work with the group to create a statement that reflects our current views:

In our Calculus sequence, students develop a deeper awareness of the structures and methods of mathematics through demonstration of theory, directed investigations, and practice. While they are not responsible for producing formal mathematical proofs, students repeatedly participate in creating new ideas by working from foundational definitions and core concepts. By engaging in active exploration and incorporating appropriate technology and writing, students gain a deeper understanding of the mathematical process.

The Calculus sequence at DCC affords students the opportunity to:

- **Expand their ability to think from a multifaceted viewpoint, and convey their thought process through writing.** By exploring mathematical ideas from numerical, graphical, algebraic and verbal points of view, students encounter opportunities to develop their mathematical writing skills. Both in class and out of class work involves using mathematical vocabulary to explain concepts as well as a clear decision making process.
- **Utilize technology effectively to support work with complex functions and concepts.** Important course ideas are explored visually, numerically and algebraically. Simpler examples from in class work help students learn important processes which they then apply to more challenging situations on the computer.

This statement contains much of the ideals from the old statement, and solidifies our commitment to using technology and writing. We wanted to keep the emphasis on students developing new idea from definitions and theorems. However, we felt it necessary to stress the importance of students continually reaffirming their understanding through technology and writing.

The unifying philosophy statement encompasses all three courses. We also began work on notes which indicate the types of writing expected in Calculus I and Calculus II. Due to time constraints, we were unable to create similar writing statements about Calculus III. We have plans to make the use of writing standards for Calculus III in the coming academic year.

The philosophy statement will be listed on all calculus course syllabi. The document as a whole, including the philosophy statement as well as notes on the use of writing, can be found by clicking:

Calculus Philosophy Statement.

Updating Student Learning Outcomes and EXOs

Student Learning Outcomes

For our first meeting, we reviewed the current Student Learning Outcomes for the three calculus courses. There were a few reasons for this. First, it is important to constantly review SLOs. Having all the calculus instructors together to discuss the outcomes for the entire sequence of courses does not occur very often and we wanted to take advantage of this opportunity. Second, we wanted to make sure all of us were on the same page with student expectations. Since the meetings were going to involve final exam reviews, having the SLOs as a foundation was essential and beneficial. Third, MCS plans to perform a backwards redesign starting with Precalculus (MAT185) down through the STEM pathway. Therefore, it is important to have accurate SLOs for the calculus sequence when this process begins.

The Calculus I and Calculus II SLOs were relatively unchanged. There was a minor wording change for Calculus I to "linear approximation." For Calculus II, we removed part of an SLO that required the students to model and solve applied problems in physics. As a group, we felt that the majority of students take a physics class where this is covered. We also removed reference to the graphing calculator, as we want to focus on using other forms of technology, such as a computer algebra system.

The biggest changes were applied to the Calculus III Student Learning Outcomes. The most current EXO listed the following outcomes:

- Students will continue to develop and refine their ability to effectively communicate key problem solving processes as well as clearly articulate key mathematical concepts both verbally and in writing.
- Students will continue to develop their ability to recognize essential information in a problem statement, and translate this information into a form which is able to be analyzed using the concepts and ideas developed in the course.
- Students will demonstrate the ability to perform essential computations by hand, as well as use appropriate technology to automate routine calculations in order to investigate more involved problems.
- Students will demonstrate the ability to meaningfully incorporate technology into their work by work on projects given throughout the semester.

Although we all liked these statements and found them valuable, they are not course specific and difficult to measure. Luckily, we found outcomes from an archived EXO that fit perfectly with our expectations. The revised SLOs for Calculus III are:

- Apply vector operations to analyze planes and curves in two or three dimensional space.
- Describe the behavior of multivariable functions graphically and analytically by examining cross sections, contours, graphs, differentials, tangent planes, directional derivatives and gradient vectors.
- Determine the area and volume of regions using double or triple integrals and the appropriate coordinate system.

- Compute line integrals and interpret geometrically the relationship between the path and the vector field.
- Apply the techniques of multivariate calculus both differential and integral to set up and solve contemporary application problems, including constrained optimization.
- Use the language of calculus to explain and interpret the mathematics used in the problem solving process.
- Use a computer algebra system to investigate, illustrate and apply the concepts of multivariate calculus.

A list of the current Student Learning Outcomes for all three courses can be found here: Updated Student Learning Outcomes for the Calculus Sequence.

Course Descriptions and the Graphing Calculator

After a few conversations about the use of technology in all three courses, we updated all of our course descriptions. As a group, we recognized that we no longer use the graphing calculator, though that had been an early and essential tool as we began integration of technology into the courses. We still allow the students to use it on evaluations, but no longer take class time to teach how to use the calculator.

Furthermore, there are many different graphing calculators on the market. The more prominently used calculators are from the TI-83/84 family. Even though these have been used widely for decades, they have never reduced the price, making this an expensive component if we require it for the course. In addition, other newer TI models are available that can do a variety of calculus operations that we expect the students to be able to perform without the aid of technology.

All of the course descriptions have the following reference to the graphing calculator: "A graphing calculator from the TI-83/84 family of calculators is required for this course." We agreed to remove this statement from the course descriptions.

We did want to make sure we referenced the calculator somewhere in the Extended Course Outlines and course syllabi. We wanted to make it clear that the use of certain calculators are prohibited on examinations. The following statement will appear on both the EXOs and syllabi:

> Faculty may demonstrate some course ideas using a TI -84 graphing calculator. Students may use a graphing calculator on tests except for a calculator with a built in CAS. The TI-89, or any calculator with a built in CAS is not allowed on tests.

Other Updates

We reviewed final exams that faculty gave during Fall 2017 for all three courses during our meetings. After reviewing the exams, we realized it was important to emphasize the need for a final exam that covers the core concepts of the entire course. All future documentation will reference the final exam as being comprehensive.

All of these items will be updated in the Extended Courses Outline for the upcoming academic year and presented to the Curriculum Committee.

Course Resources and *Mathematica*

A large amount of our effort went towards creating course resources and discussing *Mathematica* projects. Our focus was to assist full-time and part-time faculty members that were new to teaching the calculus sequence. We emphasized Calculus I as that course has the most sections offered every year.

Calculus I

We began our focus on final exams. For assessment purposes, the full-time faculty created a common portion of the fall 2017 Calculus I final exam. Upon discussion and review during the meetings, we decided to continue having a common portion across all sections. These questions were selected with our philosophy in mind. They cover a combination of mechanical and conceptual questions that require students to take derivatives, interpret graphs, and write about the results. The required questions also provide options to preserve the rigor of the final exam. The list of questions can be found here: Calculus I (MAT221) Final Exam Guideline.

Beyond the common portion of the final, we found it beneficial to create additional guidelines for the Calculus I final exam. We wanted to ensure all final exams are comprehensive and contain vital problems from the text presented in a varied fashion consistent with our philosophy. The guidelines can be found here: Calculus I (MAT221) Final Exam Guideline.

After discussing the final exams, we began going through the textbook section by section. The current textbook is *Calculus: Concepts and Contexts 4th Edition* by James Stewart. We all agreed this is a solid textbook, especially the computer based applications that are embedded within the e-book. Since there are some sections and examples that we do not cover, we created an annotated syllabus for Calculus I that new instructors can use to help navigate the text. The annotated syllabus also contains references to *Mathematica* projects and other helpful resources. It can be found here: Annotated Syllabus.

We decided to go beyond this for two sections in the text, Related Rates and Optimization. One major focus of Calculus I is the inclusion of application problems. The book and online computer assisted instruction do have problems for these two topics. However, the style of problems we find most useful do not have enough exercises in the book. We decided to create problem banks to supplement the text, which can be found here: Related Rates Problem Bank and Optimization Problem Bank.

Calculus II

Although we focused most of our efforts on Calculus I, we were able to create some resources for Calculus II. For final exams, we decided that a common portion is not necessary, but we still wanted to provide similar guidelines as we did for Calculus I. We also created an annotated syllabus for similar reasons. Those items can be found here: Calculus II (MAT222) Final Exam Guideline and Annotated Syllabus.

Mathematica

An integral part of our calculus sequence is writing and the use of technology to support writing. Many of our meetings focused on discussing *Mathematica* projects. From a general standpoint, we discussed the voice used, technical vs instructional, and the use of an introduction or abstract. The group decided that as long as the writing is college level, the instructor can decide on which styles to use.

From a previous grant focused on creating *Mathematica* resources, we had a relatively universal set of beginning and ending projects for Calculus I. All of the instructors require a project using limits to review precalculus ideas and a final project that focuses on describing function behavior using derivatives. There were also individual projects looking at implicit differentiation, linear approximation, and Riemann sums. We were all pleased with how these projects used technology to encourage writing and reinforce materials from lecture. We decided the final project is required, but there is flexibility on the function used.

For Calculus II, we agreed that projects regarding the Fundamental Theorem of Calculus, improper integration, and Taylor polynomials are required for the course. Example projects will be available for these topics. Instructors can adapt these projects as long as they each contain graphical and numerical interpretations. We also discussed the difficulties of finding a project that covers chapters 6 or 7. Projects in these chapters will require further investigation.

We agreed that *Mathematica* is an integral part of Calculus III as it aids the students in visualizing multivariable functions. We did not have time to discuss Calculus III in as much depth as the other two courses, but we did notice similarities in the projects provided. We found projects dealing with vectors and planes, level curves, gradient vectors, triple integrals, and line integrals to be invaluable. The faculty will continue discussing writing in Calculus III in the upcoming academic year.

Using *Mathematica* can be challenging for students. There can be a steep learning curve, particularly for the students who have not had any type of programming course as the language is quite exact and stringent. While we do have some resources available from an earlier grant, we will extend this base by providing templates to students to help them create limit tables. This should help alleviate the difficulty students often have with the multi-step process of creating a table, and allow the students to focus more on the interpretation. A template for Riemann sums was discussed and is currently being created to be shared with the group in the Fall 2018.

Our last meeting was dedicated to reviewing the *Mathematica* manuals created by Johanna Halsey and offering suggestions for updates. It was decided to combine both the Calculus I and Calculus II manuals to create a seamless experience for the students. The faculty will continue to give Johanna Halsey useful edits and suggestions.

All of the course resources, *Mathematica* projects, *Mathematica* templates, and example student work will be made available to all MCS faculty members in the shared Blackboard folder.

Conclusion and Future Work

During this project, we created a philosophy statement, updated course documentation, and created course resources for faculty. These group conversations and collaborations was essential in creating pathways for the courses that we all felt accomplished our departmental goals and standards.

While much work was accomplished, there are still items that we will continue to be work on:

- The course EXOs must be finalize and presented to curriculum committee.

- Jason Gumaer reached out to transfers schools to make sure our calculus sequence is in line with transfer schools. Follow up is required. As Program Chair of LAM, he will continue to email other schools and obtain syllabi.
- Jason Gumaer will work with Sara Taylor to update the shared Blackboard folder with the materials described in this report.
- The faculty will continue to meet during Fall 2018 to discuss Calculus III to create resources, such as an annotated syllabus.
- The faculty will continue to review the material created and discuss the impact on student learning. More *Mathematica* templates may be created depending on student response.
- We will continue to consider and share other technologies we find that help highlight calculus concepts.
- Johanna Halsey will use the information from this project to begin updating the *Mathematica* manuals. The other faculty will assist her with edits.

We are grateful of the opportunities this instructional improvement grant has given us and we look forward to working together in the upcoming academic year.

Grant Award Letter

DUTCHESS
COMMUNITY COLLEGE

December 20, 2017

MEMORANDUM

TO: Sandra
DeGuzman Jason
Gumaer Johanna
Halsey Maryanne
Johnson

FROM: Dr. Holly Molella, Dean of Academic Affairs

SUBJECT: **Approval of DCC Foundation Mini-Grant Awards for 2017-18**

Congratulations! Your application for a DCC Foundation Mini-Grant requesting funds for reviewing the calculus philosophy has been approved for $3120, the total amount of your request. Please contact me if you have any questions or concerns.

A detailed report that describes the project, outcomes, and the benefits of the project should be submitted to the Office of Academic Affairs at the time you request final payment, and should be filed with Academic Affairs by May 18, 2018. This report should also include a one-paragraph summary for publicity purposes.

Please see the enclosed Procedures for Claims and Reimbursement. And once again, congratulations!

cc Sara Taylor, MCS Department Chair
Diana Pollard, Executive Director of the DCC Foundation
Yvonne Flowers, Principal Accounting Clerk

Procedures for Claims and Reimbursement for Mini-Grant Awards

The Business Office and the Office of Academic Affairs have developed the following procedures to administer the Mini-Grant awards. Awards may involve unique situations and much paperwork; it is important that the procedures for reimbursement be clear.

Yvonne Flowers in the Business Office will contact you regarding the specific account number you will need.

If there is more than one recipient of this grant, please specify the **one person** who will act as the contact person with my office and the Business Office on all paperwork.

Submit all completed forms to my office. Depending on the payment requested, you will either submit a paper form to my office, or generate a requisition through the Banner system. I will sign as "the supervising dean" on all payment authorizations and forward them appropriately.

If your award includes equipment that will be ordered, submit a completed requisition via the Banner system. You should follow standard college purchasing procedures (price quotes, bids etc.) contained in the Purchasing Manual available from the Business Office. Questions about purchasing practices should be directed to the Purchasing Department.

If your award involves reimbursement to you for materials and supplies, submit a completed requisition through the Banner system.

If your award includes payments for professional services over and above your normal duties, this will be paid at the rate of $40.00 per hour ($35.00 per hour for someone in their first two semesters), using a Payment Authorization form. Payment Authorizations should be signed by employee and department head and forwarded to my office, where it will be completed and forwarded to Payroll. The form must show an itemized account of your hours. Note that a Fringe Benefit percentage (for retirement, etc.) will be computed as 30% of the amount for labor. The Fringe Benefit does NOT increase the amount of the check you will receive. The Fringe Benefit will be handled by the Business Office; no separate budget transfer is necessary, but do keep track of this amount toward your remaining balance. The amount of the reimbursement including the fringe must not exceed the total amount you were awarded.

If your award includes payments for professional services to persons not employed at the College, use a Payment Voucher used for Honorariums, which should then be sent to my office. Check first with Payroll to see if the person is in our payroll system. [Under IRS guidelines, if they have ever been paid through our payroll, we must continue to pay them this way, but taxes will be withheld.
Payments under the grant may begin anytime. If you need to carry the project over into the next year, please let me know. <u>Please inform me if you will not use all of the allocated funds</u>.

Calculus Philosophy Statement

Calculus Philosophy

In our Calculus sequence, students develop a deeper awareness of the structures and methods of mathematics through demonstration of theory, directed investigations, and practice. While they are not responsible for producing formal mathematical proofs, students repeatedly participate in creating new ideas by working from foundational definitions and core concepts. By engaging in active exploration and incorporating appropriate technology and writing, students gain a deeper understanding of the mathematical process.

The Calculus sequence at DCC affords students the opportunity to:

- **Expand their ability to think from a multifaceted viewpoint, and convey their thought process through writing.** By exploring mathematical ideas from numerical, graphical, algebraic and verbal points of view, students encounter opportunities to develop their mathematical writing skills. Both in class and out of class work involves using mathematical vocabulary to explain concepts as well as a clear decision making process.
- **Utilize technology effectively to support work with complex functions and concepts.** Important course ideas are explored visually, numerically and algebraically. Simpler examples from in class work help students learn important processes which they then apply to more challenging situations on the computer.

For Class Notes - Use of Writing:

In Calculus 1, students begin to learn how to express key ideas in writing by following verbal and written models modeled in class, and then practicing and refining this approach in their graded work. The following areas are emphasized in the course:

- Clearly articulating how to use numerical data to determine if there is a limiting value. Students are expected to weave back and forth between words and symbols.
- Describing whether a derivative will be positive or negative and increasing or decreasing based on the graph of the function.
- Describing function behavior in terms of information gleaned either from a graph or a table of a derivative.
- Explaining clearly what a derivative represents from geometric, numerical and algebraic points of view.
- Interpreting what a derivative represents, and explaining what it tells us about function behavior.
- Explaining what it means for a function to be locally linear.
- Describing what definite integrals represent both conceptually and contextually.

Calculus 2 continues advancing students' ability to express concepts verbally. Students are required to:

- Further develop their ability to explain how to use a graph of a rate of change to examine function behavior. This is done both conceptually and contextually.
- Explain why some integrals are improper, and how they make a decision whether these integrals converge or diverge.

- Be able to clearly define core vocabulary words when working with sequences and series.
- Be able to explain what it means for a sequence or series to converge or diverge.
- Be able to provide written explanations while using tests for convergence/divergence for series.
- Explain how graphs and tables help support the convergence of a power series.
- Explain why certain centers work well for Taylor Series of some functions.
- Explain how to choose the appropriate approach to finding a volume of revolution by washers, disks or shells.

MAT221 – Calculus I Materials

MAT221 Syllabus and Teacher Notes

Department Syllabus

DUTCHESS COMMUNITY COLLEGE
Poughkeepsie, NY 12601
MAT 221 – Calculus I
Departmental Course Information

Course Title: Calculus I

Credits: 4

Prerequisite: MAT 185 with a grade of at least C, OR high school precalculus with a grade of at least a C, OR permission of the department.

Course Description: This course is the first of a three-semester sequence developing calculus for the student majoring in engineering, mathematics, or the sciences. Topics include the derivative, limits, continuity, differentiability, the definite integral, the Fundamental Theorem of Calculus, techniques of differentiation (including for transcendental functions), applications of differentiation, mathematical modeling and computer applications.

Textbooks: *Calculus Concepts & Contexts*, James Stewart, (4e, 2010), Brooks/Cole, Cengage Learning

Mathematica for Calculus 1, Dutchess Community College, Halsey

Calculator: Faculty may demonstrate some course ideas using a TI -84 graphing calculator. Students may use a graphing calculator on tests except for a calculator with a built in CAS. The TI-89,or any calculator with a built in CAS is not allowed on tests.

Chapters Covered: Part of 1,2,3,4, 5.1- 5.4

Philosophy: In our Calculus sequence, students develop a deeper awareness of the structures and methods of mathematics through demonstration of theory, directed investigations, and practice. While they are not responsible for producing formal mathematical proofs, students repeatedly participate in creating new ideas by working from foundational definitions and core concepts. By engaging in active exploration and incorporating appropriate technology and writing, students gain a deeper understanding of the mathematical process.

The Calculus sequence at DCC affords students the opportunity to:

- **Expand their ability to think from a multifaceted viewpoint, and convey their thought process through writing.** By exploring mathematical ideas from numerical, graphical, algebraic and verbal points of view, students encounter opportunities to develop their mathematical writing skills. Both in class and out of class work involves using mathematical vocabulary to explain concepts as well as a clear decision making process.
- **Utilize technology effectively to support work with complex functions and concepts.** Important course ideas are explored visually, numerically and algebraically. Simpler examples from in class work help students learn important processes which they then apply to more challenging situations on the computer.

Learning Outcomes:

- Compute limits of the elementary functions, including limits involving infinity. Distinguish between ordinary limits and limits of indeterminate forms. Choose an appropriate method of either exact evaluation or numerical approximation.
- Compute derivatives of elementary functions by applying, as appropriate, the formal definition of the derivative, numerical approximation or differentiation rules.
- Apply differentiation and limits to solving basic problems including linear approximation, function behavior, and optimization.
- Evaluate simple definite integrals using, as appropriate, Riemann sums or the Fundamental Theorem of Calculus, and interpret the results.
- Use the language of calculus to explain and interpret the mathematics used in the problem solving process.
- Use a graphing calculator and computer algebra system to investigate, illustrate and apply the concepts of calculus.

ASSESSMENT:

- At least 3 full hour tests,
- 3 or 4 substantial projects using Mathematica.
- A comprehensive final exam. The final may include both test and project components, or may be just a test. All finals will include some types of questions chosen by the department.
- Quizzes, turn-in homework, discussions, etc. as specified by your teacher.

TOPICS:

Introduction to semester and Introduction to Mathematica (5 hours): Quick review of select topics from Chapter 1. Begin to use *Mathematica*. Time will be given for preliminary work with *Mathematica* during Lab time over the first few weeks, but students MUST continue work with learning the program as part of their out of class work.

Limits and Continuity (6 hours): 2.2 The Limit of a Function; 2.3 Calculating Limits Using the Limit laws;

2.4 continuity; 2.5 Limits Involving Infinity - Project using Mathematica

The Derivative (10 hours) : 2.1 The Tangent and Veloicty Problems; 2.6 Derivatives and Rates of Change;

2.7 the Derivative as a Function; 2.8: What Does f' Say about f? - Test and/or Project

Differentiation Rules, Linear Approximations and L'Hopital's Rule (15 hours): 3.1Derivatives of Polynomials and Exponential Functions; 3.2 The Product and Quotient Rules; 3.3 Derivatives of Trigonometric Functions; 3.4 The Chain Rule; 3.5 Implicit Differentiation; 3.6 Inverse Trigonometric Functions and Their Derivatives; 3.7 Derivatives of Logarithmic Functions; 3.8 Rates of Change in the Natural and Social Sciences (optional); 3.9 Linear Approximations and Differentials; 4.5 Indeterminate Forms and L'Hopital's Rule - Test and/or Project

Applications of the Derivative (11 hours): 4.1 Related Rates; 4.2 Maximum and Minimum Values;

4.3 Derivatives and the Shapes of Curves; 4.4 Graphing with Calculus and Calculators;

4.6 Optimization Problems - Test and/or Project

Antiderivatives and Integrals (8 hours): 4.8 Antiderivatives; 5.1 Areas and Distances; 5.2 The Definite Integral; 5.3 Evaluating Definite Integrals; 5.4 Fundamental Theroem of Calculus

Review and Final Assessment (5 hours)

Use of Writing:

In Calculus 1, students begin to learn how to express key ideas in writing by following verbal and written models modeled in class, and then practicing and refining this approach in their graded work. The following areas are emphasized in the course:

- Clearly articulating how to use numerical data to determine if there is a limiting value. Students are expected to weave back and forth between words and symbols.
- Describing whether a derivative will be positive or negative and increasing or decreasing based on the graph of the function.
- Describing function behavior in terms of information gleaned either from a graph or a table of a derivative.
- Explaining clearly what a derivative represents from geometric, numerical and algebraic points of view.
- Interpreting what a derivative represents, and explaining what it tells us about function behavior.
- Explaining what it means for a function to be locally linear.
- Describing what definite integrals represent both conceptually and contextually.

WebAssign

WebAssign is used regularly for homework assignments in Calculus 1. At the end of sections there are "just in time" questions that review concepts needed.

Annotated Syllabus

Introduction to semester and Introduction to Mathematica (5 hours): Quick review of select topics from Chapter 1. Begin to use *Mathematica*. Time will be given for preliminary work with *Mathematica* during Lab time over the first few weeks, but students MUST continue work with learning the program as part of their out of class work. Detailed suggestions can be found in a separate document titled *Teacher's Notes on Mathematica.*

Mathematica skills: Key Mathematica skills are:

- Using text and formatting formulas
- Entering functions
- Graphing functions
  - modifying the graphing window as needed
  - using drawing tools to emphasize key features
- Evaluation and solving equations
- Templates for tables should be able to be modified. Students are not expected to memorize these commands.

Writing: Review should include a class exercise where students begin learning to write about a problem. Faculty have found it helpful to pair students to begin the work and then share the paragraphs as the exercise progresses. For many students, writing about the process of doing mathematics is a new skill. Ideas for writing topics may include:

- Compare the long term behavior between a linear function with positive slope and exponential growth.
- Describe the domain of the function __________.

Comments: It is important to get the students using their textbook right from the start. Keep in mind that all students have learned the material in Chapter 1 sections 1 – 6. There is no need to re-teach this material, rather encourage students to use this chapter as their own review guide, and make use of selected exercises from the book to make sure the students are comfortable with technology.

Limits and Continuity (6 hours):

2.2 The Limit of a Function

- Students need to understand limits from both a graphical and numerical point of view.
- Emphasize tables, graphs and the interplay between the numerical approximations and the graph behavior through written descriptions.
- Students should be able to write correct limit notation, including one-sided limits.
- Using piecewise functions is a good way to emphasize the one-sided limit concept.

2.3 Calculating Limits Using the Limit laws;

- Do not emphasize example 10 or the Squeeze Theorem

2.4 Continuity;

- Students need to be able to recognize discontinuities algebraically, numerically and graphically.
- Do emphasize the concepts of removable and non-removable discontinuities.
- Cover the Intermediate Value Theorem.

2.5 Limits Involving Infinity

- Emphasize tables, graphs and the interplay between the numerical approximations and the graph behavior through written descriptions.
- Both horizontal and vertical asymptote definitions should be discussed.

Project using Mathematica -- There is a template available to be given to students that creates short term behavior tables at two points and also creates the tables for long term behavior. A project involving limits from sections 2.2 and 2.5 is in Blackboard. Also for samples of explanations, students should be referred to the Mathematica manual.

Test

The Derivative (10 hours) :

NOTE: Students MUST be able to work with derivatives numerically, graphically and algebraically from the limit definition of derivative before they are allowed to use the short cut methods.

2.1 The Tangent and Veloicty Problems;

- It is important to continually make sure that students understand and can distinguish between the graphic, numeric and algebraic limit definition approach to derivatives.
- The e-text has an applet accessible from p91 that shows a graph and a secant line that can be used to approximate the tangent line. Look for TEC

2.6 Derivatives and Rates of Change (at a point);

- Examples of finding the equation of a tangent line should be emphasized.
- The e-text has an applet that zooms in to show local linearity. Access from p136.
- Algebraic and numerical data should be related to graphical concepts.

- Emphasize example 7 and see the Mathematica Manual for creating derivative limit tables and examples of describing their contents.

  There is a project regarding derivative tables found in Blackboard.

2.7 The Derivative as a Function;

- Students should be able to sketch the derivative function given a graph and identify points where the function is not differentiable.
- The e-text has an applet called "Slope a scope". Given a function the tangent line is shown. Its slope is plotted below as you drag the tangent across the graph. Access from p 147.
- Students should be able to recite the limit definition of the derivative and find the derivate function for a quadratic function, 1/x and $\sqrt{x}$.
- Students should understand the connection of the second derivative to the concavity of the original function.

2.8: What Does f' Say about f? –

- Emphasize the properties here and let students know they will be used again in section 4.2 and 4.3.
- Students will need practice drawing functions with particular properties.

Test and/or Project

Differentiation Rules, Linear Approximations and L'Hopital's Rule (15 hours):

NOTE: As you develop the short cut rules for derivatives, make sure you regularly interrelate the concepts from sections 2.1 and 2.6-2.8, as well as the use of technology. Students need repeated exposure to these ideas in order to fully process them. Students are expected to know the shortcut rules without referencing a fact sheet with the exception of the inverse trigonometric functions. Students should be familiar with the inverse trigonometric derivatives so that they will be recognized in MAT 222.

3.1Derivatives of Polynomials and Exponential Functions;

- The e-text has an applet called "Slope a scope" applied to an exponential function.

3.2 The Product and Quotient Rules;

3.3 Derivatives of Trigonometric Functions;

- Skip the geometric proof of the derivative of sine.
- Use the graphical methods to "see" that the derivative of sine is cosine.
- The e-text has an applet called "Slope a scope" applied to the sine or cosine functions.

3.4 The Chain Rule;

- Do not cover ideas using parametric equations.
- Skip the proof of the chain rule.

3.5 Implicit Differentiation;

- The Mathematica Manual has a section on graphing implicit relations.

3.6 Inverse Trigonometric Functions and Their Derivatives;

3.7 Derivatives of Logarithmic Functions;

3.8 Rates of Change in the Natural and Social Sciences (optional);

3.9 Linear Approximations and Differentials;

- For projects related to this section, see Blackboard.

4.5 Indeterminate Forms and L'Hopital's Rule

- Cover the traditional forms 0/0, inf/inf, inf – inf, 0 * inf

Test and/or Project

Applications of the Derivative (11 hours):

4.1 Related Rates;

- There is a list of problems broken down by shape in Blackboard. Simpler shapes from this list make good test questions.

4.2 Maximum and Minimum Values;

- Extreme Value Theorem, Fermat's Theorem, Critical Number

4.3 Derivatives and the Shapes of Curves;

- Mean Value Theorem, Increasing and Decreasing Test, First Derivative Test, Concavity Test
- Be sure to include examples with removable discontinuities, infinite discontinuities, vertical tangents and cusp or corners.
- A project should be done on this section. See Blackboard for project ideas.

4.4 Graphing with Calculus and Calculators;

4.6 Optimization Problems

- Use Mathematica for the messier derivatives so that more examples can be explored.
- There is a list of problems sorted by how they are used in Blackboard. Questions typically given on tests are included and have derivatives that are easier to do without technology.

Test and/or Project

Antiderivatives and Integrals (8 hours):

4.8 Antiderivatives;

5.1 Areas and Distances;

- Students should be able to compute a Riemann Sum by hand for a small number of subdivisions.
- Skip using sums as demonstrated in the solution to example 2 (p334). Ie the formula for the sum of the squares for the first n positive integers.
- There is an applet that allows you to graphically show more rectangles gives a better estimate of the area under a curve.

5.2 The Definite Integral;

- Skip examples using formulas 5-7 on page 346.
- There is an applet that allows you to graphically show more rectangles gives a better estimate of the area under a curve. Selected functions show curves above and below the x-axis.

5.3 Evaluating Definite Integrals;

- Area between curves is part of this section.

5.4 Fundamental Theorem of Calculus

- Make sure sufficient time is spent working with a function defined as an integral.
- Fundamental Theorem of Calculus applet show cumulative area graphs.

Test

NOTE: There often is not time to do a project on integration. A project on integration may be done at the start of calculus 2.

FINAL PROJECT:

A final project that completely analyzes a function should be completed by students using Mathematica to create detailed graphs, to make limit tables, to take derivatives, to evaluate functions, and to solve equations. Example projects are in Blackboard.

Calculus I (MAT221) Final Exam Guideline

The following is a list of topics that are required on a MAT221 final exam. They are broken into two categories, questions which are common to all MAT221 finals and topics which are required but the style of questions is left to the instructor's discretion. The common exam questions may be slightly altered to remove the possibility of cheating between sections.

- **All questions should come from sections listed in the EXO in chapters 2 through 5.**
- **Point values must be included on the exam, as well as overarching instructions.**

Common Exam Questions

- A question using the limit definition of the derivative, one that has the student state the definition of derivative and a follow up that involves them using it on a polynomial (degree 2).
- A question that involves multiple shortcut rules. There needs to be at least one chain rule, product rule, quotient rule problem, and a multiple chain rule (composition of a composition). Those problems need to involve a combination of polynomials, exponential, trigonometric, and inverse functions.
- A question that focuses on local extrema including intervals of increase/decrease and intervals of concavity. This problem may or may not have the student sketch the function after obtaining the results.
- A question involving a Riemann Sum (numerical integration). The problem on the assessment uses values form a table.
- A question using the Fundamental Theorem when given an accumulation function. The function will be given graphically. The student will need to answer a variety of questions based on the graph such as outputs of the accumulation function, intervals of increase/decrease, and concavity.

Required Topics

- Questions or a series of questions that have students finding the values of limits using a variety of techniques. There should be at least one problem that requires the use of L'Hôpital's rule. There should be a mix of limits approached with analytic, numerical, or graphical techniques. The type of approach can be up to the instructor.
- A question on continuity. The approach for these questions can be analytical, numerical, or graphical. This is up to the instructor.
- A question that requires the student to find the equation for a tangent line. The instructor may choose to combine this with another required topic or have the student use the tangent line for the purpose of approximating values of the original function (linear approximation).
- A related rates problem (§4.1). A problem bank of these problems will be provided.
- An optimization problem (§4.6). A problem bank of these problems will be provided.
- Problems that require the student to use the Evaluation Theorem (Fundamental Theorem of Calculus §5.3) to evaluate definite integrals.

The inclusion of additional topics are up to the instructor's discretion. Suggested topics could be differentiability, implicit differentiation, and the extreme value theorem. **All questions should come from sections listed in the EXO in chapters 2 through 5.**

Common Final Exam Question Bank:

In an effort to give all sections of Math 221 a common experience, you are asked to include the following on the final exam. The use of "OR" means that you will choose one of the questions/formulas listed. It is not intended for the student to choose. If you have a similar question you would like to add to this test bank, please forward it to Sandra DeGuzman at deguzman@sunydutchess.edu.

DIFFERENTIATION:

1. a. Write the limit definition of the derivative.
   b. Use the limit definition to find the derivative of the function

$f(x)=5x^2-4x+5$ **OR** $f(x)=3x^2-2x$ **OR** $f(x)=3x^2-7x+1$.

2. Differentiate the following:

a) $f(x)=x^2\ln(x)$ OR $f(x)=x^{3/2}+\ln(x^4)$ OR $f(x)=(x^2)e^{x^2+x}$

b) $h(x)=\dfrac{\cos(x)}{x^2+1}$ OR $h(x)=\dfrac{\cos(x)}{x^3+3x}$ OR $h(x)=\dfrac{\tan(x)+1}{\sin(2x)}$

c) $r(x)=\sqrt{3x^2-7}$ OR $r(x)=\sqrt{e^x-3x^2}$ OR $r(x)=\sqrt{7x^3+6}$

d) $s(x)=\sin^5(x^2)$ OR $t(x)=\sin^3(x^2)$ OR $f(x)=\cos^3(x^2+1)$

3. Suppose $\boldsymbol{f'(x)=(x-6)^2(x+1)}$ **and** $\boldsymbol{f''(x)=(3x-4)(x-6)}$.
The domain of $\boldsymbol{f(x)}$ is given as all real numbers.

Using methods of calculus:

a) Find the location (values of x) of all critical numbers for $\boldsymbol{f(x)}$. For each critical number found, identify each as a local maximum value, a local minimum value, or neither. Justify you answers.

Critical number(s) ______________________

Local maximum (x value(s))______________

Local minimum (x value(s))_______________

b) Over what interval(s) is the function $f(x)$ increasing? ______________________

Over what interval(s) is the function $f(x)$ decreasing? ______________________

Using a complete sentence, explain how these intervals were determined.

c) Find all x coordinate(s) of any inflection points for $f(x)$. Justify your answer.

Inflection point(s) __________________________

d) Over what interval(s) is the function $y = f(x)$ concave up? ______________________

Over what interval(s) is the function $y = f(x)$ concave down? ______________________

Using a complete sentence, explain how these intervals were determined.

e) Using the information gathered from parts (a) – (d) and the fact that $\boldsymbol{f(0) = 0}$, sketch the curve $y = f(x)$ on the axes below.

Be sure to scale the x axis so that the important information can be determined from the graph.

INTEGRATION:

1. Use the data in the table to estimate $\int_0^1 f(x)\,dx$ by taking a left-hand sum, a right hand sum and then averaging the two. Show your work.

x	0	0.2	0.4	0.6	0.8	1.0
$f(x)$	1.8	2.4	3.1	3.9	4.8	5.8

$L_5 =$ ____________ $R_5 =$ ____________ Best Approximation:______________

OR

Using the function $f(x) = x^2 + 1$, complete the table below, and then show clearly how you use the values in the table to compute the left and right hand Riemann Sums to approximate $\int_1^3 (x^2+1)dx$

x	f(x)
1	
1.5	
2	
2	
2.5	
3	

OR

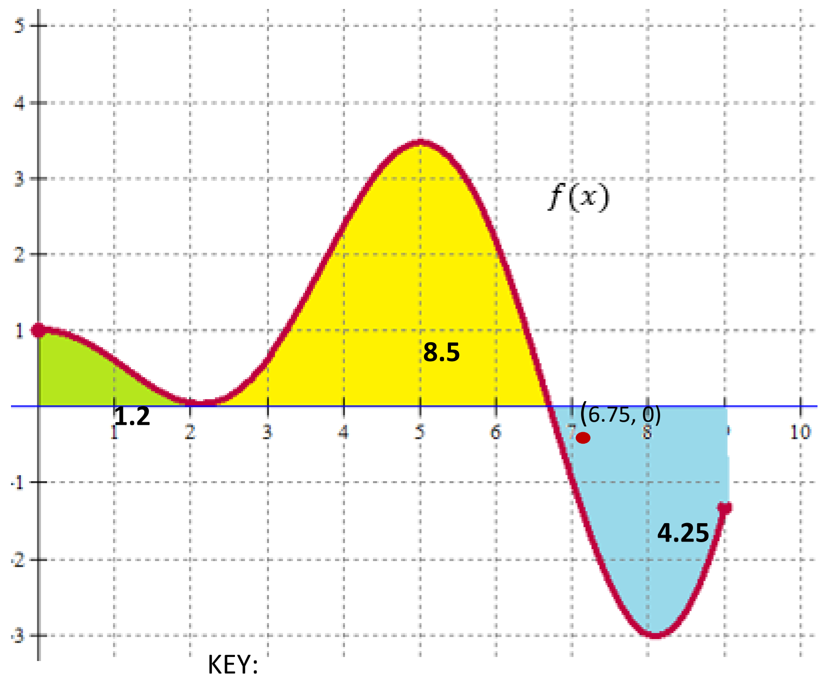
Use the data in the table to estimate $\int_0^{4.5} f(x)dx$ by taking a left-hand sum, a right hand sum and then averaging the two. Show your work.

x	0	0.75	1.50	2.25	3.00	3.75	4.50

$f(x)$	1.8	2.0	3.0	3.9	4.8	5.7	6.0

$L_6 =$ ____________ $R_6 =$ ____________ Best Approximation:______________

2. The graph of $f(x)$ is given at right on the closed interval [0, 9]. The function $g(x)$ is defined $g(x) = \int_0^x f(t)\,dt$.



Find:

a. $g(2) =$ ___________

b. $g(9) =$ ___________

c. $\int_{6.75}^{2} f(x)dx =$ __________

KEY:

Green = 1.2

Yellow = 8.5

d. On the closed interval [0, 9], for what value(s) of *x* does $g(x)$ have local extremes? Using complete sentences, explain your reasoning?

e. On the closed interval [0, 9], at what value of *x* does $g(x)$ attain its global (absolute) maximum value?

Using complete sentences, explain your reasoning?

Comments/Suggestions should be sent to:

Sandra DeGuzman: deguzman@sunydutchess.edu

Related Rates Problem Bank

Below the related rates problems have been grouped by shape. If you have problems to add, please email them to deguzman@sunydutchess.edu.

- Circle:
  1. When a circular plate of metal is heated in an oven, its radius increases at the rate of 0.01 cm/sec. At what rate is the plate's area increasing when the radius is 50 cm?

  2. Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The radius of the spill increases at a rate of 5 m/min. How fast is the area of the spill increasing when the radius is 13 m?
- Square:
  3. P260 #3 Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm^2?

  4. A hypothetical square grows so that the length of its sides are increasing at a rate of 9 m/min. How fast is the area of the square increasing when the sides are 5 m each?

- Rectangle:
  5. (page 260, #4) The length of a rectangle is increasing at a rate of 8 cm/sec and its width is increasing at the rate of 3 cm/sec. At what rate is the area of the rectangle changing when the length is 20 cm and the width is 10cm ?

     Answer : 140 cm^2/sec

  6. The length of a rectangle is increasing at a rate of 4 cm/s and its width is increasing at a rate of 6 cm/s. When the length is 5 cm and the width is 4 cm, how fast is the area of the rectangle increasing?

- Triangle:
  7. P260 #19 The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100cm^2.
  8. A 10-meter ladder is leaning against the wall of a building, and the base of the ladder is sliding away from the building at a rate of 3 meters per second. How fast is the top of the ladder sliding down the wall when the base of the ladder is 6 meters from the wall?

     Answer -9/4 m/ sec. The ladder is sliding down the wall at the rate of 9/4 m/sec.

  9. P260 #15 Two cars start moving from the same point. One travels south at 60 mi/hr and the other travels west at 25 mi/hr. At what rate is the distance between the cars increasing two hours later?
  10. (page 260, #10) At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/hour and ship B is sailing north at 25 km/hour. How fast is the distance etween the two ships changing at 4PM?

Answer : 21.4 km/hr

11. If *a* and *b* are the lengths of two sides of triangle, and ϑ the measure of the included angle, the area A of the triangle is A= (½) *ab sin* ϑ. How is *dA/dt* related to *da/dt*, *db/dt*, and $d\vartheta/dt$?

12. A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 4 ft/s along a straight path. How fast is the tip of his shadow moving when he is 30 ft from the pole?

- Box:

13. If x, y, and z are lengths of the edges of a rectangular box, the common length of the box's diagonals is $s = \sqrt{x^2 + y^2 + z^2}$. How is *ds/dt* related to *dx/dt, dy/dt, and dz/dt*?
14. Suppose the edge lengths x, y, z of a closed rectangular box are change at the following rates: $\frac{dx}{dt} = 1\text{m/sec}$, $\frac{dy}{dt} = -2\text{m/sec}$ and $\frac{dz}{dt} = 1\text{m/sec}$. At the instant when x = 4, y = 3 and z = 2, find the rates at which the box's
    i. volume is changing.
    ii. surface area is changing.
    iii. diagonal length $s = \sqrt{x^2 + y^2 + z^2}$ is changing.

15. A hypothetical cube shrinks at a rate of 27 m^3/min. At what rate are the sides of the cube changing when the sides are 9 m each?

- Sphere: $v = \frac{4\pi}{3} r^3$

1. Example 1 from text: A spherical balloon has its volume increasing at a rate of 100 cm^3/s. How fast is the balloon's radius increasing when the diameter is 50 cm?
2. A spherical snowball is melting. Its radius is decreasing at 0.2 cm per hour when the radius is 15 cm. How fast is its volume decreasing at that time?
3. A spherical balloon is inflated with helium at the rate of 100π ft^3/min.
    i. How fast is the balloons radius increasing at the instant the radius is 5 ft?
    ii. How fast is the surface area increasing at that instant?
4. A hemispherical bowl of radius 10 cm conatins water to a depth of h cm.
    i. Find the radius of the surface of the water as a function of height.
    ii. The water level drops at a rate of 0.1 cm per hour. At what rate is the radius of the water decreasing when the depth is 5 cm.

- Cone:

5. A cone-shaped coffee filter of radius 6cm and depth 10 cm contains water, which drips out through a hole at the bottom at a constant rate of 1.5 cm^3 per second.
    i. If the filter starts out full, how long does it take to empty?
    ii. Find the volume of water in the filter when the depth of water is h cm.

iii. How fast is the water level falling when the depth is 8 cm?

6. (page 325, #36) A paper cup has the shape of a cone with height 10 cm and radius 3 cm. If water is poured into the cup at the rate of 2 cm^3/sec, how fast is the water level rising when the water is 5 cm deep?

   Answer $\frac{8}{9\pi}$ cm/sec.

7. Sand falls from a conveyor belt at the rate of 10 cubic meters per minute onto the top of a conical pile. The height of the pile is always three-eights of the base diameter.
   a) How fast are the height and
   b) how fast are the radius changing when the pile is 4 m high?

- Cylinder:

8. An upright cylindrical tank with radius 7 m is being filled with water at a rate of 4 m^3/min. How fast is the height of the water increasing? (Round the answer to four decimal places.)

- Triangular Prism:

9. (textbook, #27) A trough is 10 feet long and its ends have the shape if isosceles triangles that are 3 feet across at the lop and have a height of 1 foot. If the trough is being filled with water at the rate of 12 ft^3/min, how fast is the water level rising when the water is 6 inches deep?

   Answer $\frac{4}{5}$ ft/min.

- Formula:

10. The Dubois formula relates a person's surface area, s, in meters squared, to weight, w, in kg and height, h, in cm by $s = 0.01\ w^{0.25}h^{0.75}$.
    i. What is the surface area of a person who weighs 60 kg and is 150 cm tall?
    ii. The person in part a stays constant height but increases weight by 0.5kg/year. At what rate is his surface area increasing when his weight is 62 kg?

11. Atmospheric pressure decays exponentially as altitude increases. With pressure, P, in inches of mercury and altitude, h, in feet above sea level, we have $P = 30\ e^{-3.23x10^{-5}h}$.
    i. At what altitude is the atmospheric pressure 25 inches of mercury?
    ii. A Glider measures the pressure to be 25 inches of mercury and experiences a pressure increase of 0.1 inches of mercury per minute. At what rate is the altitude changing?

12. An item costs $500 at time t=0 and costs $P in year t. When inflation is r% per year the price is given by $P = 500e^{rt/100}$.
    i. If r is a constant, at what rate is the price rising in dollars per year?
       1. Initially?
       2. After 2 years?

ii. Now suppose that r is increasing by 0.3 per year when r = 4 and t = 2. At what rate is the price increasing at that time?

Optimization Problem Bank

Below are optimization problems faculty have used on tests, in their classrooms and for homework. If you have examples you would like to share, send them to deguzman@sunydutchess.edu and categorize them according to how you would use them.

The following are used for tests, class examples or homework.

1. Find two numbers whose difference is 10 and whose product is a minimum.

2. What is the smallest perimeter possible for a rectangle whose area is 16^2 in ?

3. SCalcCC4 4.6.005.MI. [1227118]
   Find the dimensions of a rectangle with perimeter 60 m whose area is as large as possible.

4. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single strand electric fence. With 800 meters of wire at your disposal, what is the largest area that you can enclose?

5. You are designing a poster to contain 50 in^2 of printing, with margins of 4 inches each on the top and bottom, and margins of 2 inches on each side. What are the overall dimensions that will minimize the amount of paper used?

6. Two sides of a triangle have lengths a and b (constants), and the angle between them is θ. What value of θ will maximize the area of the triangle? ($Area = .5ab\ Sin(\theta)$).

7. SCalcCC4 4.6.008. [1842054]
   The rate (in mg carbon/m^3/h) at which photosynthesis takes place for a species of phytoplankton is modeled by the function $P = \frac{110\ I}{I^2+I+9}$ where I is the light intensity (measured in thousands of foot-candles). For what light intensity is P a maximum?

8. SCalcCC4 4.6.011.MI. [1222058]
   If 30,000 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

9. SCalcCC4 4.6.012. [1841240]
   A box with a square base and open top must have a volume of 62,500 cm^3. Find the dimensions of the box that minimize the amount of material used.

10. A rectangular pool has an area of 3000 square yards. Zoning regulations require 8 foot clearance at the front and rear of the pool and 3 feet of space on either side of the pool. Find the dimensions of the smallest piece of property on which the pool can be legally constructed to meet the zoning code. Round answers to four decimal places.

The following a recommended for classwork or homework but should only be used on a test if the students have prior experience with similar problems.

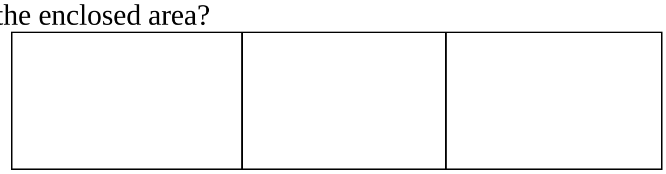
11. You are planning to make an open rectangular box from an 8 inch by 15 inch piece of cardboard by cutting squares from the corners and folding up the sides. What are the dimensions of the box of largest volume that you can make?

12. A box with an open top is to be constructed from a sheet of cardboard which is 3 feet by 3 feet by cutting away squares from each corner and folding up the sides. Find the largest possible volume for this box.

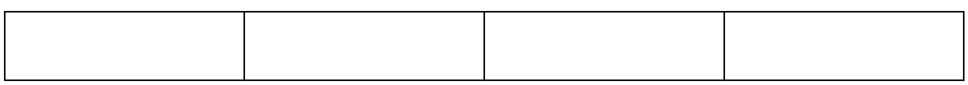
13. What are the dimensions of the lightest (minimal surface area) open top right circular cylindrical can that will hold a volume of 1000 3 cm ?

14. SCalcCC4 4.6.014. [1840827]
    A rectangular storage container with an open top is to have a volume of 10 m^3. The length of this base is twice the width. Material for the base costs \$5 per square meter. Material for the sides costs \$3 per square meter. Find the cost of materials for the cheapest such container. (Round your answer to the nearest cent.)

15. Example: A farmer has 1000 feet of fencing. They want to use this material to fence in three rectangular pens. The pens are all the same size. What dimensions for the three pens maximizes the enclosed area?



16. Consider the following problem: A farmer with 750 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the 4 pens?



17. A can is to be constructed from aluminum. The can should hold 40 in^3 of juice. The can is a cylinder- capped at both ends. What is the minimum amount of material needed ?
    Cylinder: V= $\pi r^2 h$ Surface area S = $2\pi r^2 + 2\pi r h$

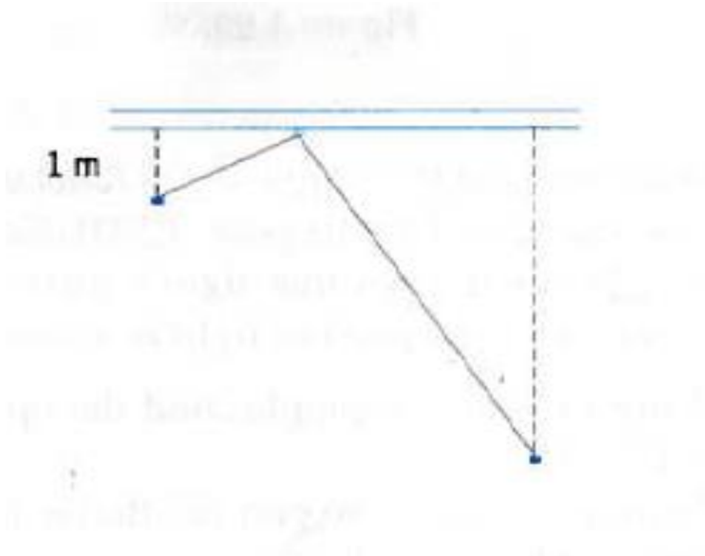
The following a recommended for classwork or homework. The setup is more difficult or the derivative is best taken using Mathematica.

18. A model used for the yield Y of an agricultural crop as a function of the nitrogen level N in the soil (measured in appropriate units) is $Y = \frac{kN}{1+N^2}$ where k is a positive constant. What nitrogen level gives the best yield.

19. A cone shaped paper drinking cup is to hold 27 cm^3 of water. Find the height and radius of the cup that will use the smallest amount of paper.
    Surface area for a cone: Area= $\pi r\sqrt{r^2 + h^2}$ r: radius h: height

20. The manager of a 100 unit apartment complex knows from experience that all units will be occupied if the rent is \$800 a month. A market survey suggests that for each \$10 increase in the rent one additional unit will remain vacant. What rent should the manager charge to maximize the revenue?

21. On the same side of a straight river are two towns and the towns people want to build a pumping station at point S. The pumping station is to be at the edge of the river with pipes extending straight to the two towns. Where should the pumping station be located to minimize the total length of pipe? Assume town B is 4 miles from the river. Also, assume that P1 and P2 are 6



miles apart.

22. A rectangle has one side on the x-axis and two vertices on the curve $\frac{1}{1+x^2}$. Find the vertices of the rectangle with maximum area.

23. A rectangle has one side on the x-axis, one side on the y axis and a vertex (corner) on the curve $y = e^{-2x}$ for x>0. Find the maximum area.

24. A builder wishes to fence in 60,000 square meters of land in a rectangular shape. For security reasons, the fence along the front part of the land will cost \$20 per meter, while the fence for the other three sides will cost \$10 per meter. How much of each type of fence should the builder buy to minimize the cost of the fence? What is the minimum cost?

25. A manufacturer needs to produce a cylindrical container with a capacity of 1000 cubic cm. The top and bottom of the container are made of material that costs \$0.05 per square centimeter while the sides of the container are made of material costing \$0.03 per square centimeter. Find the dimensions that will minimize the company's cost of producing the container.

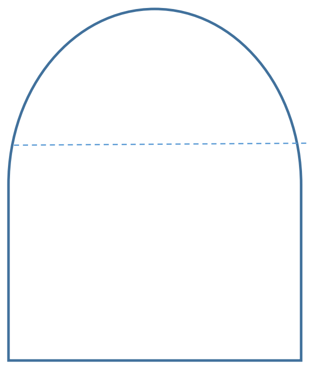
26. A cruise line offers a trip for \$2000 per passenger. If at least 100 passengers sign up, the price is reduced for all passengers by \$10 for every additional passenger (beyond 100) who goes on the trip. For example is 102 passengers go on the trip all 102 receive a \$20 discount and pay only \$1980. The boat can accommodate 250 passengers. What number of passengers maximizes the cruise line's total revenue? What price does each passenger pay then?

The cost to the cruise line for n passengers is 80000+400n. What is the maximum profit that the cruise line can make on one trip? How many passengers must sign up for the maximum to be reached and what price will each pay.

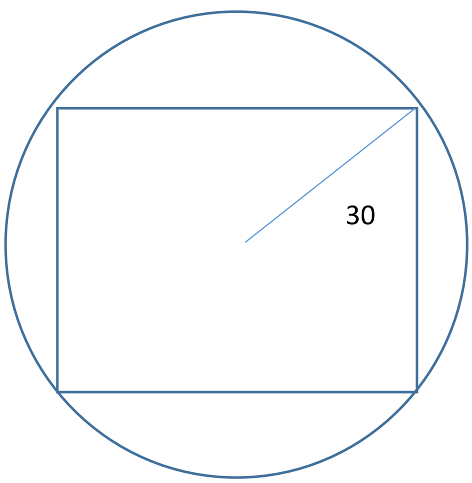
27. The cost of fuel to propel a boat through the water in dollars per hour is proportional to the cube of the speed. A certain ferry boat uses $100 worth of fuel per hour when cruising at 10 miles per hour. Apart from the fuel, the cost of running this ferry (labor, maintenance and so on) is $675 per hour. At what speed should it travel so as to minimize the cost per mile traveled?

    Note that you need to follow the units: First get the cost in terms of speed ($/hr_. Then you will need to get cost per mile but look at (cost($)/hr)/(mi/hr) so you simply divide your cost by the speed and you will get the function to minimize.

28. A Norman window has the shape of a rectangle surmounted by a semicircle of diameter equal to the width of the rectangle. If the perimeter of the window is 10 meters, what dimensions will admit the most light?



29. A rectangular beam is cut from a cylindrical log of radius 30cm. The strength of the beam of width w and height h is proportional to wh^2. Find the width and height of the beam of maximum strength.



MAT222 – Calculus II Materials

MAT222 Syllabus and Teacher Notes

Department Syllabus

DUTCHESS COMMUNITY COLLEGE
Poughkeepsie, NY 12601

MAT 222 – Calculus II
Departmental Course Information

Course Title: Calculus II

Credits: 4

Prerequisite: MAT221 with a grade of C or better, or permission of the department.

Course Description: This course is the second of a three-semester sequence developing calculus for the student majoring in engineering, mathematics, or the sciences. Topics include the Fundamental Theorems of calculus, definite and indefinite integrals, techniques of integration, improper integrals, applications of integration, sequences, series and Taylor Series, differential equations, mathematical modeling and computer applications.

Textbooks: *Calculus Concepts & Contexts*, James Stewart, (4e, 2010), Brooks/Cole, Cengage Learning

Mathematica for Calculus 2, Dutchess Community College, Halsey

Calculator: Faculty may demonstrate some course ideas using a TI -84 graphing calculator. Students may use a graphing calculator on tests except for a calculator with a built in CAS. The TI-89, or any calculator with a built in CAS is not allowed on tests.

Chapters Covered: Parts of 5, 6, 7, and 8.

Philosophy: In our Calculus sequence, students develop a deeper awareness of the structures and methods of mathematics through demonstration of theory, directed investigations, and practice. While they are not responsible for producing formal mathematical proofs, students repeatedly participate in creating new ideas by working from foundational definitions and core concepts. By engaging in active exploration and incorporating appropriate technology and writing, students gain a deeper understanding of the mathematical process.

The Calculus sequence at DCC affords students the opportunity to:

- **Expand their ability to think from a multifaceted viewpoint, and convey their thought process through writing.** By exploring mathematical ideas from numerical, graphical, algebraic and verbal points of view, students encounter opportunities to develop their mathematical writing skills. Both in class and out of class work involves using mathematical vocabulary to explain concepts as well as a clear decision making process.
- **Utilize technology effectively to support work with complex functions and concepts.** Important course ideas are explored visually, numerically and algebraically. Simpler examples from in class work help students learn important processes which they then apply to more challenging situations on the computer.

Learning Outcomes:

- Apply various techniques of integration including integration by substitution, integration by parts, and partial fraction decomposition.
- Recognize and evaluate improper integrals.
- Use approximating techniques to evaluate a definite integral.
- Set up, and evaluate definite integrals for areas, lengths and volumes.
- Apply convergence tests to determine convergence or divergence of an infinite series.
- Model a function with an approximating Taylor polynomial.
- Develop differential equation models for exponential growth and decay.
- Use the language of calculus to explain and interpret the mathematics used in the problem solving process.
- Use a computer algebra system to investigate, illustrate and apply the concepts of calculus

ASSESSMENT:

- At least 3 full hour tests
- At least 3 substantial projects using Mathematica.
- A comprehensive final exam. The final may include both test and project components, or may be just a test. All finals will include some types of questions chosen by the department.
- Quizzes, turn-in homework, discussions, etc. as specified by your teacher.

TOPICS:

Review (5 hours)

Mathematica, derivatives, antidifferentiation, Riemann sums, definite integrals, the Evaluation Theorem

Unit 1: Chapter 5 Antiderivatives, and Integration Techniques
(15 hours)

5.4 The Fundamental Theorem of Calculus; 5.5 Substitution; 5.6 Integration by Parts;
5.7 Additional Integration Techniques (Powers of Sine and Cosine, Partial Fractions, Trig Substitutions);
5.9 Approximate Integration; 5.10 Improper Integrals
Review/ Test

Unit 2: Chapter 6 Applications of Integration
(9 hours)

6.1 Areas; 6.2 Volumes; 6.3 Volumes with Cylindrical Shells; 6.4 Arc Length; 6.5 Average Value
Review Test

Unit 3: Chapter 7 Differential Equations
(7 hours)
7.1 Modeling with Differential Equations; 7.2 Direction Fields and Euler's Method;
7.3 Separable Equations; 7.4 Exponential Growth and Decay; 7.5 (Optional) The Logistic Equation
Review / Test
Unit 4: Chapter 8 Infinite Sequences and Series
(18 hours)
8.1 Sequences; 8.2 Series; 8.3 The Integral and Comparison Tests; 8.4 Other Tests(Alternating Series Test, Ratio Test); 8.5 Power Series; 8.6 Functions as Power Series; 8.7 Taylor and Maclaurin Series; 8.8 (Optional) Applications
Review /Test

Review and Final Assessment (6 hours)

Use of Writing:

Calculus 2 continues advancing students' ability to express concepts verbally. Students are required to:

- Further develop their ability to explain how to use a graph of a rate of change to examine function behavior. This is done both conceptually and contextually.
- Explain why some integrals are improper, and how they make a decision whether these integrals converge or diverge.
- Clearly define core vocabulary words when working with sequences and series.
- Explain what it means for a sequence or series to converge or diverge.
- Provide written explanations while using tests for convergence/divergence for series.
- Explain how graphs and tables help support the convergence of a power series.
- Explain why certain centers work well for Taylor Series of some functions.
- Explain how to choose the appropriate approach to finding a volume of revolution by washers, disks or shells.

WebAssign

WebAssign is used regularly for homework assignments in Calculus 2. At the end of sections there are "just in time" questions that review concepts needed.

Annotated Syllabus

The ordering of these units are up to preference. An alternative sequence is Unit 1 (5.4-5.10), Unit 4 (8.1-8.9), Unit 2 (6.1-6.5), and then Unit 3 (7.1-7.5).

Introduction to semester and review of Calculus I and Mathematica (5 hours): Review topics from Calculus 1 with an emphasis on differentiation, Riemann sums, and definite integrals. Review of Mathematica can be done through an early project. The focus should be on writing, text formatting, plotting, derivatives, and integration. Section 5.4, The Fundamental Theorem of Calculus, is an excellent section for a first project.

Unit 1: Chapter 5 Antiderivatives, and Integration Techniques (15 hours)

The focus of this unit is for students to learn the integration techniques. The largest focus should be on Substitution and Integration by Parts. Stewart has an excellent Integration Techniques PDF that works really well as a review after you finish section 5.7. It gives an example of each technique, a question bank, and worked out solutions for odd problems. Section 5.8 is not covered as it is expected that *Mathematica* is used throughout the course.

Section 5.4 is a great section for a *Mathematica* project. Page 400 of the book also has a good basis for a project on integration techniques. A project on section 5.10, Improper Integration, is highly recommend as it can be used to review limit tables.

5.4 The Fundamental Theorem of Calculus

- Students need to understand the connection between the derivative and the integral.
- They should be familiar with half of the FTC, the book calls this the "Evaluation Theorem," so the focus should be on the first part: *If* $g(x) = \int_a^x f(t)dt$, *then* $g'(x) = f(x)$.
- Accumulation functions are a major focus of this section and should be used often to highlight connections between Calculus I and II.
- There is an example *Mathematica* project in the department Blackboard folder.

5.5 The Substitution Rule

- This section and the next are the most important integration techniques. As such, a good amount of time should be dedicated to this section.
- Be sure to show examples using powers/radicals, trigonometric functions, and exponential functions.
- The integral for $\tan(x)$ should be shown in this section.
- Although indefinite integral is the highlight for these techniques, be sure to show how to use substitution with a definite integral by changing the bounds.
- Symmetry is introduced in this section, but should not be the main focus. It could be helpful in section 5.10

5.6 Integration by Parts

- This is the other major section for this unit. More time should be dedicated to this section than 5.5 as it is a more difficult topic.
- The main focus should be identifying when to use Parts and deciding which expressions should be u and dv.
- Give examples that require Parts twice. You may talk about using parts more than twice on some examples. However, using Parts more than twice takes time and should not be used on an examination.
- Show the major inverse function antiderivatives, $\ln(x)$ and $\arctan(x)$. Explain how the others can be derived as well, but showing them is not necessary.
- Example 4 in this section and problems like it make for great guided exercises or homework problems.

5.7 Additional Techniques of Integration

- There are three techniques in this section: Powers of Sine and Cosine, Trigonometric Substitution, and Partial Fractions.
- This section is not as important as 5.5 and 5.6, so it is not recommended to do every example.
- Partial Fractions with two linear factors must be approached, as it is required for the final exam.
- The other partial fractions techniques can be shown, but takes a lot of time and nuance. They can be glossed over.
- An example of an odd power of sine or cosine and an even power of sine or cosine should be shown. They use two different trigonometric identities. For testing, it is recommended to use odd powers as the Pythagorean Identities are more familiar and it highlights the use of the Substitution Rule.
- Trigonometric Substitution should not be used on an exam. One example of it should be shown in class. This technique is used in other courses that the students may be taking, such as Physics and Engineering courses.

5.9 Approximate Integration

- This section does a good job highlighting what happens when you cannot find an antiderivative. It is also a solid review of Riemann Sums. However, with the use of *Mathematica*, it should not take much focus.
- One example of Midpoint, Trapezoidal, and Simpson's Rule each should be shown.
- Error formulas should not be emphasized.

5.10 Improper Integrals

- This is another focus of this unit. This section connects with limits learned in Calculus I.
- Both types of improper integrals need to be shown.
- Be sure to discuss the comparison test as it will be used in section 8.3.

Unit 2: Chapter 6 Applications of Integration
(9 hours)
This is the second shortest unit covered in this course. For testing purposes, you may combine this unit with sections from another unit to create a full exam. The focus is on Area and Volume. Be sure to highlight the ability to integrate with respect to x and y. There are wooden models in Jason Gumaer's office (W113) that can be used for Disks and Shells.

6.1 More About Area

- Students should already have a good grasp on integration being connected to area beneath a curve.
- Focus on area between two or more curves and how to properly set up those integrals.
- Be sure to include problems that involve integration with respect to x and y. Problems that are ambiguous and require the students to choose are recommended.

6.2 Volumes

- This section and the next are where the bulk time of this unit are spent.
- At the start, focus on connecting Riemann Sums with volume of revolution with simple curves.
- Highlight that Disks and Washers are essentially the same method.
- Avoid using cross sections that are not circular.
- Examples and problems should be given with rotating about lines that are not the axes.

6.3 Volumes by Cylindrical Shells

- As with the previous sections, be sure to set up integrals with respect to x and y.
- Be sure to describe when to use each method, Washers vs Shells.
- Examples and problems should be given with rotating about lines that are not the axes.

6.4 Arc Length

- Depending on time, this section can be done relatively quickly by just giving the result for functions with respect to x and y.
- If you are deriving the parametric formula, this may be the first time students see parametric equations.
- The Arc Length will appear in Calculus 3.

6.5 Average Value of a Function

- The formula is simple and intuitive for the students, so it should be quick to derive.
- Be sure to go over the Mean Value Theorem as it connects with Calculus I.
- Minimum time is required for this section.

Unit 3: Chapter 7 Differential Equations

(7 hours)

This is the quickest unit to cover. It is a visual chapter. Although it is useful for students to know how a slope field is generated, be sure to use technology to make all slope fields. Desmos has a simple slope field generator: https://www.desmos.com/calculator/p7vd3cdmei

7.1 Modeling with Differential Equations

- This is the first exposure the students have to differential equations.
- Be sure to highlight how differential equations are used to model observations.
- Focus on first-order differential equations, solutions being a family of functions, initial value problems, and how to confirm solutions.

7.2 Direction Fields and Euler's Method

- Use technology to create slope fields.
- The first example in the book, $y' = x + y$, is a good starting problem as it has one obvious linear solution.
- Emphasize the connection between Euler's Method and Linearization from Calculus 1.
- The ebook has a lot of videos and links to demonstrations in this section, including this cdf: http://demonstrations.wolfram.com/EulersMethodForTheExponentialFunction/

7.3 Separable Equations

- This is the only technique to solve a differential equation students are responsible to know for Calculus 2.
- Be sure to remind the students about their family constant after integrating, $+C$.
- After solving for the general solution, it is recommended that students plot a few specific solutions on top of a slope field to confirm the ideas from the previous sections.
- Reinforce the difference between a general solution and an initial value solution from section 7.1.
- Do not cover orthogonal trajectories.

7.4 Exponential Growth and Decay

- This section is relatively quick to cover, as the students should be familiar with exponential models.
- The only tricky example that will probably be new for students is Newton's Law of Cooling.

7.5 (Optional) The Logistic Equation

- Time pending, this is a very interesting model with two equilibrium solutions.
- Emphasize that this is a more realistic model for population growth with limited resources.

Unit 4: Chapter 8 Infinite Sequences and Series

(18 hours)

This is by far the longest chapter and the students generally struggle with this chapter. On the surface, the content is very different from everything else in the course and students may need reassurance that there is a connection to integration near the end of the unit, with sections 8.6 and 8.7. When going through the early sections, make sure to emphasize the difference between a sequence and a series.

A list of series convergence tests must be provided to students to be used on all evaluations, including the final exam. A copy of the list is available in the Blackboard folder or can be found here: http://www.bates.edu/math-stat-workshop/files/2010/05/convergence_tests.pdf . Some instructors also provide a flowchart that can be found here: http://www.math.hawaii.edu/~ralph/Classes/242/SeriesConvTests.pdf

There should be at least one project from this unit, specifically on Taylor polynomial approximations. It is a large unit, so a second project is more than possible, preferably on the interval of convergence of a power series.

8.1 Sequences

- This section lays the groundwork for series convergence tests in the next three sections.
- Be sure to emphasize a sequence is a list of values or outputs from a function with a domain of nonnegative integers.
- Provide a few examples of sequences, especially those that have a kth term. Get students to start working between the expanded form (list) and kth term as this will be useful in the coming sections.
- Recursive sequences are interesting, but not useful for this unit.
- Focus heavily on the limit of a sequence. This is a good section to review limits from Calculus 1.
- The students may or may not have seen the squeeze theorem in Calculus 1. This is useful, but should not be as large a focus as matching a sequence to a function.
- Monotonic Sequence Theorem should not be emphasized.

8.2 Series

- Be sure to emphasize the difference between a sequence and a series. This is also very tricky when discussing a series, the sequence of partial sums, and the limit of the sequence of partial sums.
- Students should be familiar with the terms converges and diverges from Calculus 1 and section 5.10. This is a good section to remind them of those connections.
- The geometric series is extremely useful and should be a focus. It is the only series in this unit where the students can calculate the sum.

- Highlight the harmonic series. It is important for students to see a series that has a sequence of the terms going to 0 yet the series does not coverage.
- Emphasize that the test for divergence does not say anything about convergence.

8.3 The Integral and Comparison Tests; Estimating Sums

- The three major tests in this section are the Integral Test, p-series Test, and Comparison Test.
- The integral test is an excellent place to review improper integration from section 5.10.
- The p-series test is extremely important in this section.
- The Comparison Test needs extra focus as it is difficult for students to determine if the terms need to be greater than or less than the terms of the compared series. It is vital that students can quickly identify whether a geometric or p-series converges or diverges so they can compare.
- Be sure to talk about the situations where the Limit Comparison Test needs to be used.
- Do not cover error and estimation.

8.4 Other Convergence Tests

- The most important test out of this section is the Ratio Test. It is consistently used in the next sections.
- The Alternating Series test is also important, especially for end point test in 8.5.
- Absolute Convergence should be defined.

NOTE: The list of convergence tests pdf provided to students has the Root Test listed. The Root Test is not covered in this course. Also, the sheet calls the Test for Divergence the "nth term test."

8.5 Power Series

- Be sure the students have a strong foundation on the convergence tests before they begin this section, as all the tests may need to be used.
- Make sure to emphasize the center of a power series, the radius, and the interval of convergence.
- Make sure to show examples of all three types of intervals.

8.6 Representations of Functions as Power Series

- Emphasize writing functions as power series.
- Continue to find intervals of convergence to help reinforce section 8.5.
- Be sure to derive the series expansions for $\ln(1+x)$ and $\arctan(x)$.

8.7 Taylor and Maclaurin Series

- This section is the culmination of all the work of this unit.
- Be sure to derive the expansion for e^x, $\sin(x)$, and $\cos(x)$.
- Use examples and problems where the series is centered somewhere other than 0.

- Use expansions to approximate values and integrals.
- There is an example *Mathematica* project in the department Blackboard folder.

8.8 (Optional) Applications of Taylor Polynomials

- Even though this section is listed as optional, many of the important uses of Taylor Polynomials that are show in 8.7 are expanded here.
- This section is perfect for the *Mathematica* project.

Calculus II (MAT222) Final Exam Guideline

The following is a list of topics that are required on a MAT222 final exam. It is highly recommended that there should be a combination of conceptual problems as well as standard mechanical problems. The final may include both test and project components, or may be just a test.

- **All questions should come from sections listed in the EXO in chapters 5 through 8.**
- **Point values must be included on the exam, as well as overarching instructions.**

Required Topics

- Integration techniques. A major focus should be indefinite integration for the majority of the problems, but definite integrals are suitable for applications.
    - Integrals that require substitution and integrals that require integration by parts. These techniques should appear multiple times on the exam.
    - An integral using partial fraction with linear factors in denominator, non-repeatable factors.
    - There should be a mix of the two approaches. It is vital that students who are going on in engineering be able to determine the correct technique for integration. Therefore, there should be at least one problem for each technique that does not give direction on which technique to use. From there, directions may be given to assess individual techniques.
- Improper integrals. Students should be able to explain why an integral is improper. They should also be able to use the proper limit notation. It is the instructor's preference to have the student complete the integral.
- Questions that require a student to set up an integral with respect to both *x* and *y*. This can be done through area between two curves or solids of revolution.
- Solving a separable differential equation for the general solution or with an initial value.
- Euler's Method. There should be an opportunity for students to work with Euler's on the final. This could be a mechanical, conceptual, or geometric problem.
- Questions that use the series convergence tests.
    - Students will have access to the series convergence tests sheet.
    - It is optional whether we tell them which tests to use.
    - There should be an emphasis on comparison and the ratio test.
- Find the interval of convergence for a power series. Optional whether you ask them to check the endpoints. Either way, you should make sure they know that it would be part of the process.
- Taylor Series.
    - Can have them create a Taylor Series from scratch. Can be at a center other than 0.
    - Can have them use a given series to create an expansion for a modified function.
    - Can use the series to approximate a definite integral.

MAT223 – Calculus III Course Materials

MAT223 Department Syllabus

DUTCHESS COMMUNITY COLLEGE
Poughkeepsie, NY 12601

MAT 223 – Multivariable Calculus
Departmental Course Information

Course Title: Calculus III

Credit Hours: 4

Prerequisites: Prerequisite: MAT 222 with a grade of C or better or advanced placement with the permission of the department.

Course Objectives: A continuation of MAT 222. Topics include vectors in the plane, solid analytic geometry, functions of several variables, partial differentiation, multiple integration, line integrals and vector fields, and Green's Theorem. Use of appropriate technology is required.

Textbook: *Calculus Concepts & Contexts*, James Stewart, (4e, 2010), Brooks/Cole, Cengage Learning

Mathematica for Calculus 3, Johanna Halsey

Chapters Covered: 9-13

Philosophy: In our Calculus sequence, students develop a deeper awareness of the structures and methods of mathematics through demonstration of theory, directed investigations, and practice. While they are not responsible for producing formal mathematical proofs, students repeatedly participate in creating new ideas by working from foundational definitions and core concepts. By engaging in active exploration and incorporating appropriate technology and writing, students gain a deeper understanding of the mathematical process.

The Calculus sequence at DCC affords students the opportunity to:

- **Expand their ability to think from a multifaceted viewpoint, and convey their thought process through writing.** By exploring mathematical ideas from numerical, graphical, algebraic and verbal points of view, students encounter opportunities to develop their mathematical writing skills. Both in class and out of class work involves using mathematical vocabulary to explain concepts as well as a clear decision making process.
- **Utilize technology effectively to support work with complex functions and concepts.** Important course ideas are explored visually, numerically and algebraically. Simpler examples from in class work help students learn important processes which they then apply to more challenging situations on the computer.

Learning Outcomes:

- Apply vector operations to analyze planes and curves in two or three dimensional space.

- Describe the behavior of multivariable functions graphically and analytically by examining cross sections, contours, graphs, differentials, tangent planes, directional derivatives and gradient vectors.
- Determine the area and volume of regions using double or triple integrals and the appropriate coordinate system.
- Compute line integrals and interpret geometrically the relationship between the path and the vector field.
- Apply the techniques of multivariate calculus both differential and integral to set up and solve contemporary application problems, including constrained optimization.
- Use the language of calculus to explain and interpret the mathematics used in the problem solving process.
- Use a computer algebra system to investigate, illustrate and apply the concepts of multivariate calculus.

See other side of this sheet for specific sections covered.

MAT 223
Multivariable Calculus

ASSESSMENT:

- At least 2 in class full hour tests.
- Several (3+) substantial projects using Mathematica.
- A cumulative final exam. The final may include both test and project components, or may be just a test.
- Quizzes, turn-in homework, discussions, etc. as specified by your teacher.
- Your instructor will inform you as to dates and topics covered for each assessment.

TOPICS: The following sections represent the typical content coverage for the course. Time spent on each topic will vary.

Unit 1: **Vectors and the Geometry of Space (≈ 13 hours)**

9.1 Three-Dimensional Coordinate Systems, 9.2 Vectors, 9.3 The Dot Product,

9.4 The Cross Product, 9.5 Equations of Lines and Planes, 10.1 Vector Functions and Space Curves, 10.2 Derivatives and Integrals of Vector Functions, 10.3 Arc Length and curvature

Unit 2: **Partial Derivatives and Optimization (≈ 15 hours)**

9.6 Functions and Surfaces, 11.1 Functions of Several Variables, 11.2 Limits and Continuity, 11.3 Partial Derivatives, 11.4 Tangent Planes and Linear Approximations, 11.5 The Chain Rule, 11.6 Directional Derivatives and the Gradient Vector, 11.7 Maximum and Minimum Values, 11.8 Lagrange Multipliers (Optional)

Unit 3: **Multiple Integrals (≈ 20 hours)**

12.1 Double Integrals over Rectangles, 12.2 Iterated Integrals, 12.3 Double Integrals over General Regions, 12.4 Double Integrals in Polar Coordinates, 12.5 Applications of Double Integrals (Optional), 12.6 Surface Area (Optional), 9.7 Cylindrical and Spherical Coordinates, 12.7 Triple Integrals 12.8 Integrals in Cylindrical and Spherical Coordinates

Unit 4: **Vector Functions and Vector Calculus (≈ 7 hours)**

13.1 Vector Fields, 13.2 Line Integrals, 13.3 The Fundamental Theorem for Line Integrals, 13.4 Green's Theorem

Review and Final Assessment (5 hours)

MJ/2018

Additional Documents

SUNY Seamless Transfer Calculus Course Descriptions

Calculus I

The first semester of a multi-semester sequence of differential and integral calculus. Topics include limits, derivatives, considered algebraically, symbolically and graphically; differentials and their use as approximations, the indefinite and definite integrals with applications to areas, volumes, surface area, arc length, moments and center of mass.

Appropriate for math majors and students in partner disciplines requiring understanding of fundamental principles of calculus, with emphasis on deductive reasoning and proof.

Prerequisite: mathematical knowledge at the level of trigonometry and college pre-calculus (successful performance in three years of Regents-level mathematics).

Note: (Similar note as Calculus II, removed to save space.)

Calculus II

A continuation of Calculus I. Includes applications of the definite integral, inverse functions, logarithmic and exponential functions, separable differential equations and their applications, symbolic and numeric methods of integration; area accumulation functions; volume; applications such as work and probability; improper integrals and l'hospital's rule; complex numbers; sequences; series; Taylor series; differential equations; parametric equations, polar coordinates and modeling.

Prerequisites: Calculus I or Equivalent (e.g. Honors Calculus.)

Note: some topics in this description of Calculus II are covered in Calculus I on some campuses and vice versa. Students should take both Calculus I and Calculus II before transferring to be guaranteed acceptance for the mathematics major as well as for majors in engineering and sciences that require two semesters of Calculus such as chemistry and physics. Calculus I does transfer as a single course for majors that only require one semester of calculus.

Calculus III

Multivariable calculus. Geometry of three dimensional space, vector functions in three space, partial differentiation, multiple integrals, functions of several variables, partial differentiation, multiple integration, line integral. Green's theorem, and Stokes' theorem. Applications studied through algorithmic techniques and/or computer usage.

Prerequisites: Calculus I and II.

Note: Some campuses divide the topics described above for Calculus I, II and III into four semesters. For these courses to be guaranteed to transfer for a mathematics major, the student should take all four semesters to have covered the topics of Calculus I, II and III.

Updated Student Learning Outcomes for the Calculus Sequence

MAT221 – Calculus 1

Course Student Learning Outcomes

- Compute limits of the elementary functions, including limits involving infinity. Distinguish between ordinary limits and limits of indeterminate forms. Choose an appropriate method of either exact evaluation or numerical approximation.
- Compute derivatives of elementary functions by applying, as appropriate, the formal definition of the derivative, numerical approximation or differentiation rules.
- Apply differentiation and limits to solving basic problems including linear approximation, function behavior, and optimization.
- Evaluate simple definite integrals using, as appropriate, Riemann sums or the Fundamental Theorem of Calculus, and interpret the results.
- Use the language of calculus to explain and interpret the mathematics used in the problem solving process.
- Use a graphing calculator and computer algebra system to investigate, illustrate and apply the concepts of calculus.

MAT222 – Calculus 2

Course Student Learning Outcomes

Students who successfully complete the course will be able to:

- Apply various techniques of integration including integration by substitution, integration by parts, and partial fraction decomposition.
- Recognize and evaluate improper integrals.
- Use approximating techniques to evaluate a definite integral.
- Set up, and evaluate definite integrals for areas, lengths and volumes.
- Apply convergence tests to determine convergence or divergence of an infinite series.
- Model a function with an approximating Taylor polynomial.
- Develop differential equation models for exponential growth and decay.
- Use the language of calculus to explain and interpret the mathematics used in the problem solving process.
- Use a computer algebra system to investigate, illustrate and apply the concepts of calculus

MAT223 – Calculus 3

Student Learning Outcomes

- Apply vector operations to analyze planes and curves in two or three dimensional space.
- Describe the behavior of multivariable functions graphically and analytically by examining cross sections, contours, graphs, differentials, tangent planes, directional derivatives and gradient vectors.
- Determine the area and volume of regions using double or triple integrals and the appropriate coordinate system.
- Compute line integrals and interpret geometrically the relationship between the path and the vector field.
- Apply the techniques of multivariate calculus both differential and integral to set up and solve contemporary application problems, including constrained optimization.
- Use the language of calculus to explain and interpret the mathematics used in the problem solving process.
- Use a computer algebra system to investigate, illustrate and apply the concepts of multivariate calculus.

Items to be Placed in Department Blackboard Shell

1. Calculus I Folder
   - 1.1. Philosophy Statement
   - 1.2. Department Course Information and Annotated Syllabus
   - 1.3. Final Exam Guidelines
   - 1.4. Common Final Exam Questions
   - 1.5. Example Final Exams Folder
   - 1.6. Instructor Resources Folder
     - 1.6.1. Problem Banks Folder
       - 1.6.1.1. Related Rates Problem Bank
       - 1.6.1.2. Optimization Problem Bank
     - 1.6.2. Current Resources Already in Shared Folder
   - 1.7. Mathematica Resources Folder
     - 1.7.1. Sample Projects Folder
     - 1.7.2. Examples of Student Work Folder
     - 1.7.3. Templates Folder
     - 1.7.4. Current Resources Already in Shared Folder
2. Calculus II Folder
   - 2.1. Philosophy Statement
   - 2.2. Department Course Information and Annotated Syllabus
   - 2.3. Final Exam Guidelines
   - 2.4. Example Final Exams Folder
   - 2.5. Instructor Resources Folder
     - 2.5.1. Series Test and Flowchart
     - 2.5.2. Current Resources Already in Shared Folder
   - 2.6. Mathematica Resources Folder
     - 2.6.1. Sample Projects Folder
     - 2.6.2. Templates Folder
     - 2.6.3. Current Resources Already in Shared Folder
3. Calculus III Folder
   - 3.1. Philosophy Statement
   - 3.2. Department Course Information
   - 3.3. Example Final Exams Folder
   - 3.4. Instructional Resources Folder
     - 3.4.1. Worksheets from Hughes Hallet
   - 3.5. Mathematica Resources Folder
     - 3.5.1. Sample Projects Folder
     - 3.5.2. Templates Folder

Discussion Minutes

Meeting 1 (1/25/18)

Meeting for Calculus Group – 1/25/18 – 11 AM – 12:25 PM
Sandra DeGuzman, Jason Gumaer, Johanna Halsey, Maryanne Johnson

Summary: Discussion revolved around:

A. The order of topics we cover in our Calculus 2 and 3 classes, and how that fits in with the Physics and Engineering courses that students take. An example is when we cover information on vectors, and how that relates to when they see that material in other courses. We will look at the seamless transfer information for these courses, and will consider potentially rearranging the order or depth of coverage for some topics.
B. Student Learning Outcomes: We reviewed the SLOs for Calculus 1 and 2, and agreed that we will make some minor adjustments to those.
C. Use of Technology, in particular the Graphing Calculator. We questioned whether this should continue to be required for the course. We will be investigating other web based programs, as well as Mathematica demonstrations. We are also discussing creating some Mathematica "worksheets" that allow our students to explore ideas without having to worry about the coding. Creating tables is one example of a particularly cumbersome process in Mathematica.

A. Discussion on Order of Topics:

- Consider seamless transfer and see if there is anything that we can move out of Calc 2 so that we could potentially move vectors into Calc 2.
  - Vectors come up in Physics 151, and students who have not taken Calc 3 are at a disadvantage.
  - Seeing vectors is also important for Linear Algebra. Since students can now take Linear Algebra without completing Calc 3, seeing vectors in Calc 2 would be beneficial.
  - Look at some of the schools where our students transfer the most. Need to be very cognizant of what the transfer schools are expecting.
- It would be good to find out more specifically where the material is used in the different curriculums – i.e. engineering, mathematics, computer science. Series and Sequences in particular.

Action over next two weeks:

- Maryanne will look to see if she can get us printouts of the seamless transfer materials.
- Jason will reach out to department chairs from some of the 4 year schools to see about getting syllabi from the calculus courses.
- Maryanne will talk to Tammy McBrian. She remembers her having some information on the articulation agreements.

B. Discussion on Learning Outcomes:

- Realized that sometime when we were making the change of taking the DCC Objectives out of the EXO's that our objectives for 223 got used for the SLO's and the outcomes disappeared.

  - We will work to update the SLOs to reflect the learning outcomes from the Spring 2008 EXO. We will update these. They will then be in line with the Calc 1 and 2 SLOs.
  - We believe we should take the graphing calculator out of the course learning outcomes and syllabi, and make use of computer algebra system and appropriate applets, or other web based programs.
- Maybe we should just say that students can use certain calculators on tests or quizzes – but not require that students buy a TI calculator for the course. Limit the calculators that are acceptable for tests or quizzes.
- Question about what programs we would have on the classroom sets of calculators to potentially use on tests or quizzes. This will be further discussed as we delve into each course separately.
- For Calculus 2, take out the "Also use definite integrals to model (and solve) applied problems in physics."
  - Overarching philosophy: We are teaching them the calculus – we should mention where ideas are used. We do not have to assess on specific applications for other disciplines. Those are assessed in other courses.

Action for the future:

- Over the course of this project, we will update each of the EXOs for Calc 1, 2, 3.
  - Get rid of references to DCC Objectives
  - Modify Calc 1 outcome 3 to say: "including linear approximation".

- Consider finding web based applications that show the concepts of Riemann sums as the interval decreases. (Throughout the semester.)
  - WinPlot – Maryanne will look at this
  - Applets- We can all search as we come across material where we might use them.
  - Look at the applications in the e book. Some of these are Mathematica demonstrations. We will look at these more carefully as we each teach the courses.
  - Sandra has been playing with creating a Mathematica template that will create the tables to consider limits for two points for a function. She has shared with Maryanne, and they will continue to refine that.
  - Jason has a template for looking at RSums if we increase the number of subdivisions. He will be sharing it with the rest of the group.

- Create a list of calculators that will be acceptable on tests or quizzes.

- Calculus 2: Write a new SLO for differential equations. Consider what other schools are doing.

- Try to trim down series and sequences – We will talk more about this when we are discussing that specific course curriculum.

- Ask for input from Physics and Engineering teachers, as well as some of our math faculty who are closer to graduate school to help develop "fact sheets" about where students in different curriculums will need to use the concepts we cover in one course, but don't necessarily use in our math sequence at DCC. For example: Where do students use series and sequences? This is developed in Calc 2, but not used in the rest of our math sequence at DCC.

Next Meeting:

- Start by looking at the 221 finals and further discuss major components of that course.
- Bring projects for 221 for next week, but may not get to them until week after next.

Meeting 2 (2/1/18)

Meeting 2 - Calculus Group Meeting - February 1st, 2018 – 11 AM – 12:20 PM
DeGuzman, Gumaer, Halsey, Johnson

Reviewed what actions we have accomplished or worked towards

- Maryanne
  - Sent info to team members on seamless transfer
  - Sent information on applets for Riemann Sum
  - Sent information from Tammy McBrian
- Sandra looked up the calculators that are approved for the AP Exam.

Discussion: Student Learning Outcomes for 223

These look good. Some discussion on including some information in Teacher Notes to help people understand that some of the learning outcomes will take the bulk of time in a course.

Discussion: Review of 221 Final Exams

We had copies of 4 Final Exams from Fall 2017

All FT faculty had 5 common questions. The adjunct faculty final did not contain these. We will be including 5 common questions in the Spring for assessment purposes.

Discussion on Exams A, B and C. We did not have time to review Exam D.

- Important for students to be able to choose/know the input values that would be chosen near a point to numerically determine if a limit exists.
  - If we make a template for tables in Mathematica, we won't really be assessing this any longer in a project.
  - Talked about how we might address a question so that we could see that students knew what point we were approaching, what direction we were coming from, and then have them create the appropriate limit definition without them having to put a complicated function into the calculator.
- Got side tracked a bit on calculator talk – should be required or not? No. Should we limit the calculators they could have access to? Pulled back on track.
- Talked about limits and difference in language (algebraically and analytically and symbolically)
- Word problems – need to make sure that students have been exposed to certain types of problems
  - Talk about homework or testing. Is there a difference in the type of word problems we would use for homework as opposed to those we would use on a test?
- We want all Final Exams to have an optimization problem as well as a related rates problem.

- Teachers can select from a common pool.
- Talked about importance of definition of derivative and why we should reinforce it in Calc 1 final.
- Side conversation on academic freedom. There was a question raised as to whether requiring the same final was an infringement on academic freedom. We do want to set the standard for the course. We want to let individual faculty decide how to teach the material. Reminder that when we had block finals, we all gave the exact same final for a course. While we do not need to do that now, we would like there to be topics that occur at a similar level on all Calc 1 finals. We can create a template of what type of problems should be included, and give faculty the ability to choose particular problems that they want to use. Since this is a course in a sequence, it is very important that students all leave with a core set of material that they had been exposed to and tested on at a similar level no matter who they have for the course.

Action:

- Sandra will send an email out to all current faculty teaching Calc 1 as well as all other Calculus Faculty indicating that we will be giving 5 common questions on the Final Exam. She will send out the questions from last year. We will be creating a similar experience of 5 common questions for this semester's Final Exam for assessment purposes.

- Sandra will also share that we are creating a list of question types that should be on each Calc 1 final, and explain that we will be creating a question bank for some of the types.

- We need to create a packet of problems that we all agree are appropriate for this course.
    - Sandra will send us the word problems that she uses and has students write up solutions for.
    - We will hone in on what type of problems we think should be focused on.

- Jason will be creating an assessment grant that will look at the data collected from Fall 2017 and Spring 2018.

For next time:

- Finish looking at Final Exams. Need to focus on Exam D in entirety.
- Create a list of required concepts on any Calc 1 final.
- Start talking about projects

Meeting 3 (2/8/18)

Meeting 3 - Calculus Group Meeting - February 8st, 2018 – 11 AM – 12:15 PM
DeGuzman, Gumaer, Johnson

Discussion: Review of 221 Finals

The group reviewed exams A, B, and C in the previous meeting.

This meeting we focused on exam D.

- A discussion on multiple choice questions occurred. The instructors in the group that use multiple choice questions also require the student to defend the answer. All agreed allowing supporting work is preferred.
- A discussion on the inclusion of Chapter 1 material (Precalculus review) ensued.
  - Even though many of the instructors include Precalculus review in their course, the group felt it should not be on a final exam.
  - It is more than appropriate to include the reviewed material on the first unit exam.
  - This will need to be emphasized when creating the Teacher Notes
- Talk of derivative shortcut rules took place. We felt the common assessment questions used last semester provide a decent starting point for our expectations on the final.
- There were further discussion on certain topics being optional or required on a final. That conversation was paused until reviewing exam D was finished.
- While reviewing exam D, there were questions the group liked and other questions we felt do not reflect what our values in the course. The Teacher Notes should help with this.

Discussion: Topics for 221 Final Exams

The discussion of the four final exams led into discussing which topics should be required on a MAT221 final exam and which should be suggested.

- The group agreed that questions similar to those included in the common questions used in Fall 2017 were all good questions to require on a final exam.
- The group agreed that Chapter 1 material should not be included on a final exam.
- The group reviewed the topics covered in the course by chapter.
  - A comprehensive list of topics for a MAT221 final exam will be provided.
  - There was discussion on how some questions have multiple approaches (analytical, numerical, and graphical). For certain topics, the approach may be up to the instructor.
  - A discussion between the differences of linear approximation and the creation of tangent lines ensued. All agreed having a student create a tangent line on the final is important. It is the instructors' choice as to whether the student would then use that tangent line to approximate values of the original function.
  - There was conversation on which type of extrema questions to ask on the final exam. For the assessment project, the question focused on local extrema (§4.3). Global extrema (§4.2) is also important for the course, however, including a question on global extrema should be left to the instructor's discretion.

Action

- Sandra will compile a problem bank for both Related Rates (§4.1) problems and Optimization (§4.6) problems. Please send Sandra those types of problems and note the expected level of difficulty for the problems (i.e. Homework, Exam, Final).
- Be ready to discuss Calculus I projects.

Meeting 4 (2/15/18)

Meeting 4 - Calculus Group Meeting - February 15th, 2018 – 11 AM – 12:15 PM
DeGuzman, Gumaer, Halsey, Johnson

Review and Reminders:

- Reviewed the minutes, and the list of Calc 1 final exam topics from Meeting 3.

- Discussed that we will be making the List of Calculus Final Exam Topics available in Blackboard so all teachers in future semesters can access this.

- Sandra reminded us to send her the problems that we use for optimization and related rates so that she can make up a list of problems.

D**iscussion of Calculus 1 Projects:**

- Shared projects with each other.
- Began looking at project 1.
    - Discussed the formatting of the projects. We agreed that all table commands,and long plot commands should not appear in the presentation work. They should be shown at the end of the project.
    - We can either allow them to include the solve command in the presentation, or write that they have solved a particular equation.
- **Discussion on writing**
    - What style do we each use? Technical or Instructional? Does it matter? We agreed that we will disagree with each other.
    - J. Halsey asked for us to consider working again with each other and with other STEM faculty to continue to develop the ideas of writing an introduction and a summary. Perhaps writing an abstract makes the most sense. Maybe we should write an improvement of instruction grant to meet with other faculty in other disciplines in the future.
- **Discussion on Mathematica**
    - Agreed that we should make available a template based on Sandra's trial template this semester for creating limit tables
- **Discussion of Reading projects.**
    - Should we try to have them read in advance, or after. Or both?
    - We DO want to try to encourage them to be able to make some headway with material on their own, and then create questions that help pinpoint where they have some issues.
    - We will continue to find the areas where pre-reading and/or reading quizzes/homework's can most easily be incorporated into the course. For example, continuity, linear approximation.
    - In some ways, having students pre-read can help speed things up.

Actions for the Future:

- We should set up a list of what we give projects on – between first project and end project. Send the list to Jason before our next meeting.

- Start the philosophy file for Calc 1. Jo will do this and have something after Spring Break.

- Jason will begin setting up the Blackboard site for Calc 1. It will include:

  Departmental Syllabus

  Need to edit the EXO

  Philosophy

  Teacher Notes – Philosophy and then suggestions for each chapter and section covered.

- Jo will look for old teacher notes.

Meeting 5 (3/1/18)

Meeting 5 - Calculus Group Meeting – March 1st, 2018 – 11 AM – 12:15 PM
DeGuzman, Gumaer, Halsey, Johnson

Initial Discussion:

- We spent some time reviewing the list of Calculus courses for Fall 2018, and discussed possible coverage for these courses.
- Will make a suggestion that one of the Calculus 1 courses have a time switched so that we have more flexibility in terms of who can cover that course.

Review and Decisions for Calculus 1

- Final Exam Multiple choice should be defended, or limit the number.
- Point values should be on the exam
- Overarching directions should be included.
- Nothing should be included from Chapt 1
- Don't give limit definition, expect students to produce it.
- Summation notation for Riemann sums should be expected. If a Riemann sum is calculated by hand, use a finite number of subdivisions, rather than using summation properties.
- While u substitution method is not required in Calculus 1, it can be introduced if there is time.
- Reviewed questions 28 and # 29 on exam D and agreed they are not appropriate for a Calculus 1 Exam.

Actions Needed:

- Need to include the point structure on all final exams – add that in to the list of Calculus Final Exam Topics and Guidelines. Jo will update this and pull out this info from minutes to a separate file.
- Teacher notes – Sandra and Jo will look for the past version to use as a start.
- Jo will have draft philosophy by the end of spring break
- Reviewed that In MCS folder, we will have notes on final, philosophy, teacher notes, word problem question banks, folder for projects.
- We will all send Jason our most recent projects.
- Sandra will take all of our questions from our Calc 1 exams and create a question bank for in class tests.
- All of us will send her out Calc 1 tests.
- She will Include a justification for some of the unique problems

Suggested Outline for Caluclus 1 Folder in MCS Blackboard:

- One folder for general info – EXO,
- Project folder
- Final Exam question folder

- Question bank for regular tests/quizzes.

Discussion on Calculus 2 Finals:

- Started process of reviewing Calc 2 exams. 3 exams A, B, C

- Does the EXO say cumulative final? We want cumulative final exam since this is a pre-requisite course.

- Solids of integration
  - 2 skills – 1) sketch and determine which technique to use, and then 2) set up from a sketch and given plan. Won't really ask them to integrate.
- Improper Integral – both types – ask why and ask for set up at least. May have them finish integration, not necessary.

Actions After Meeting:

- Jason will work on Teacher Notes for Calc 2

- Need to pull out EXO's and update them for all 3 courses.

Meeting 6 (3/8/18)

Combined Meetings 6 & 7 - Calculus Group Meeting – March 8th and March 22nd, 2018 – 11 AM – 12:15 PM
DeGuzman, Gumaer, Halsey, Johnson

March 8th – Meeting 6, March 22nd – Meeting 7

- Reviewed minutes from Meeting 4 and 5
- Sandra handed out a Linear Approximation Worksheet

Actions Assigned:

- Jason will include a sub folder in the Project folder that includes Mathematica Templates.
- Sandra will send us the Mathematica worksheets on creating Tables, as well as the Linear Approximation Worksheet.
- Sandra will also share the Reading Assignments with the group.
- Sandra will be responsible for updating the MAT 221 EXO and will include the departmental syllabus.
    - We are removing the DCC Objectives, moving the philosophy to section 4.
    - Calculator change: Faculty may demonstrate some course ideas using a TI -84 graphing calculator. Students may use a graphing calculator on tests except for a calculator with a built in CAS (For example, no TI 89, or TI Inspire without a TI 84 face plate.)
- Johanna will send the Calc 1 departmental syllabus to the group.
- She will add the date to the Calc 1 Final Exam Topics.

Return to reviewing the Calculus 2 Exams

- Average Value of the Function – not required
- Arc length is optional. You should only have them do the set up, not the full integration
- Differential Equations –
    - We should have a separable DE that students have to find a specific solution for.

  - We can ask them to verify a solution
  - We can include a slope field question.
- Find the interval of convergence for a power series. Optional whether you ask them to check the endpoints. Either way, you should make sure they know that it would be part of the process.
- There should be some question that goes after the concept of interval of convergence from a graphical point of view.
- Area between two curves – optional. If asked, can include the graph with points of intersection.
- We need to make sure that students are asked to set up integrals with respect to both x and y, though this can be done in the section of solids of revolution.
- Taylor Series
  - Can have them create a Taylor Series from scratch. Can be at a center other than 0.
  - Can have them use a given series to find the series expansion for a modified function
  - Can use the series to approximate a definite integral.
- Integration
  - Need to have an emphasis on substitution, and parts.
  - Should include a partial fraction with linear factors in denominator, non-repeatable factors.
- Series –
  - We should give them the sheet with the tests on it during class exams and the final exam.
  - It's optional whether we tell them what test they use.
  - We want an emphasis on comparison test and ratio test.
  - Can use integral test.
  - We can leave it optional whether we tell them what test to use or not. Some of this is dependent upon the length of the final and the strength of the group.
- Approximation Techniques – This is optional. We want them to know there are other techniques, but it is not necessary to cover this in any depth. Do NOT focus on error testing at all!
- Euler's method – Discussed that doing the algorithmic approach was just time consuming and easy to make a mistake with. Could ask a conceptual question or a geometric based question instead. There should be some opportunity for students to work with Euler's on the final.
- Improper Integrals – there should be a question asking why it is improper. You should make sure they have to set up the proper notation. You may have them complete the integration.

We will come back one last time to recap ideas on the final.

Meeting 7 (3/22/18)

March 22nd – Added some detail to minutes above as we recapped ideas on the final. Continued discussions below.

Brief discussion on the Assessment for 221 and writing an Assessment grant. Discussed the time needed for each activity.

Finishing Ideas about Calc 2 Final and Discussion on Philosophy of the Final Exam:

- Talked about the idea of having a conceptual part of the final as well as the typical mechanical part of the course.
- This fits well with them going on into Linear Algebra. Since Calc 2 is the only pre-req for Linear, it is important to get the students ready for dealing with Linear Algebra which is more theoretical, but also contains some computational components.
- We do have them do the thorough explanation on projects but need to make sure they are carrying some of that explanation onto the final.
- Discussion of whether they should be told what technique to use for integration and series tests on the final or not.
    - They should have to make the decisions on the in-class tests.
    - There is an advantage for making the students choose the right technique on the final also particularly with integration. Other areas may not be as important.
    - There should be a mix of the two approaches. It is vital that students who are going on in engineering be able to determine the correct technique for integration.
    - Can tell them disks, washers, shells.
    - It is vital that students show that they have retained the core skills from earlier in the course.
    - Big discussion of the philosophy of the final exam.
- If we ask them to do the set up and integration for a volume of revolution, then we have to weight the question more and it is much harder to grade potentially.
- They will see trig sub in their physics 2 and engineering statics course, and maybe the dynamics course. So we should at least show them trig sub. Won't necessarily put this on the final, or even a test, but should at least over it. Maybe integrate into a homework.

Actions Required:

- Don't necessarily need to develop a full-fledged question bank like Calc 1, but maybe some examples and some sample finals.
- Definitely need to work up teacher notes.
- Again, have to really delve into what we want our philosophy to be for the course

We will start with reviewing the projects next week.

Meeting 8 (3/29/18)

Combined Meetings 8, 9 & 10 - Calculus Group Meeting –
DeGuzman, Gumaer, Halsey, Johnson

Meeting 8 – March 29 – 11AM – 12:15 PM

Meeting 9 – April 5 – 11AM – 12:15 PM

Meeting 10 – April 12 – 11AM – 12:15 PM

Reviewed last minutes.

- Agreed that we will work on the EXO for 222 when we review the Teacher Notes that Jason will be working on.

Discussion of Calculus 2:

- Timing of last part of Calc 1 – students are working on final project, learning techniques of integration, and working to understand integration conceptually. Some will not have completed much work that uses the type of writing about FTC problems like J. Halsey's Graded Work 2 from Fall 2017.

- Jason gave idea of using a question like this as a starting point where teacher writes with the students through the beginning part of project to model what type of writing is expected. Then give a similar project that would be graded.

- Concept of making the first work with Mathematica something that teacher works on with the student. This helps them get up and running with the expectations of the course.

- Maryanne says that it really helps at the beginning of Calc 2 to determine the pairs and then tell them to work together, rather than have them self select. Pairs a strong with a weak.

Discussion of Mathematica:

- Talked about Mathematica and using : = as opposed to =. := makes Mathematica to always go back to re-evaluate the function, while the = will just store the result in cache.

- Discussed the idea that when I revise the Mathematica manuals that I will combine Calc 1 and 2.

Discussion of Projects:

- Project 1: Agreed that it is good to have a writing project/ homework reviewing integration in a contextual problem (FTC). The writing could be done in Mathematica or not.

- Project 2: Using Mathematica.
  - J. Halsey's project goes after the FTC, using Mathematica essential skills.
  - J. Gumaer goes over pattern recognition using integration.

  - We discussed that it is a good idea to work with the students in creating certain parts of the material in Mathematica – in particular tables. Gives us the opportunity to reiterate some key ideas of using Mathematica efficiently by copying and pasting and then making appropriate changes.

General Discussion Wrap up:

- Further discussion of what is reasonable to get done by the end of this semester in terms of this Improvement of instruction project.

- Some discussion on the philosophy of why and how we use Mathematica.

Meeting 9 (4/5/18)

Seamlessly picked up on April 5th.

Use of Mathematica

- Agreed that all should be able to fluently use Mathematica to do the following.

  Graphing

  Integration

  Derivatives

  Solving

  Evaluating

- Tables produced by a template, or we give them. We don't expect them to make tables on their own.

- Want them to be able to create a table that shows discrete points and then has them create the first and second derivative and then use that to determine if there are CP

- Use Mathematica as a utility. The idea is to create the "stuff" so that you can get the students to focus on the explanation and the work of thinking about coming at ideas from numerical, graphical and algebraic points of view. Want the evidence. Then want to focus on discussion.

Use of Technology in High School

- Maryanne was talking to high school teacher – why can't they use the TI Inspire? They grab evidence from the screen of the emulator on their computer, and then bring it into word and then type up the explanations. Hard to justify to the students learning a new tool for something that they have a tool for already.
- How important is it for students to learn a computer system? Chances are that they will be exposed to some CAS in their school work.
- Students who come here for Calc 2 or 3 won't have exposure to Mathematica, while other students will be proficient.
- Sandra suggested that the tutor center offer some workshops on Mathematica in the first weeks of the semester. We would need to design the material that they would use with the students.
- Sandra brought up that Dover didn't have a connection for internet that would allow them to tap into the VM to get NSFLAB image. Are we open to schools making their own solutions.
- What is it we need? Need to have the integration of the graphs, solutions, tables right in with the writing and explanations in one well presented document.

- **We will take this question to the department for discussion.**

Back to Mathematica Discussion at DCC:

- So…. Can students use Mathematica and use snippet to use Word?

- What if students go on to a school that doesn't use Mathematica? They can't access the materials they created.

- We agreed that it is the department's preference to create one complete document using Mathematica, however, it is the ultimately the instructor's decision whether to use Mathematica or Word to create for students to create their final documents. What we are looking for is the professional presentation, with the Mathematical evidence from Mathematica.

General End Discussion:

- Discussion of how to have them talk about function behavior. Have to include the output value of a point based on how the question is asked.

- Next week we will finish Calc 2 projects, and then start Calc 3 final exams.

Meeting 10 (4/12/18)

Pick up from here on April 12th

General Opening Discussion:

- Discussed a bit about helping the students learn to write. It is beneficial for them to see each other's writing. J. Halsey talked a bit about Yamir's class, where each student wrote and results were projected on the screen.

- J. Halsey handed out her first draft of the philosophy and asked people to look at it before next week. We will pick up with that at the beginning of our next meeting.

- S. DeGuzman said next time we have to decide what pre-req will be for our Calc 1 exo. She is working on that. Need to discuss AP scores.

Projects for Calc 2:

- Highly Recommended: Writing assignment that covers contextual integration problem
- Potential:
  - For reviewing the basics of Mathematica: Possible review project using FTC. Review graphing, solving, integration. Integrate the writing with the explanations
  - Talked about pairing students up. Put a student who had no Mathematica experience with a student who felt they were strong.
  - Maryanne's approach: Need to show them some good writing from other students so they start a project in the last part of the class. Then put it in the Dropbox. The next day show some of the writing on the screen and talk about it.
  - OR looking for pattern for integration formulas.
- Required: For first use of Mathematica, the improper integral projects gets them using Mathematica and also explaining difficult concepts clearly.
- Can do a project on Probability if you want to have a project for Chapter 6.
  - J. Halsey holds off Chapter 6 until the end so that their last project is on TP.
  - That helps them ease in to the end of the semester.
  - J. Gumaer is trying a project for Chapter 6 which has them use previous concepts and apply them to probability.
  - Look up Revolution plot 3D. Jason will experiment with it for problems from chapter 6 solids of revolution and share.
- Power Series – Can focus on intervals of convergence and have them consider tables and graphs and talk about center. May have the students create some of the partial sums. Focus is on explaining.
- Taylor Series – Can consider many possible variations. We will put some examples in the drop box.
- Next time – start with philosophy and then review Calc 3 exams.

Meeting 11 (4/19/18)

Combined Minutes for Meetings 11 & 12 - Calculus Group Meeting
DeGuzman, Gumaer, Halsey, Johnson

Thursday – April 19th 11AM – 12:15 PM

Thursday – April 26th 11 AM – 12:15 PM

Calculus Philosophy

- Briefly looked at the Philosophy that J. Halsey had written up.
- Also looked at the statement that S. DeGuzman had found on a previous EXO.

Statement from previous EXO

In this course we develop and apply techniques of limit evaluation and differentiation; definite integrals and integration (topics that will be developed more fully in MAT222) are introduced. Although students will not be responsible for producing formal mathematical proofs it is important that they see a few proofs during the semester, and that they get used to the idea of mathematics being developed from basic definitions. Students begin to develop an awareness of the structures and methods of mathematics through demonstration of theory, directed investigations using technology, and through practice.

Students leaving the course must be comfortable working with limits and derivatives from several points of view: using definitions, numerical approximations, graphical interpretations and algebraic calculations. In addition, students must be able to use graphing calculators and a computer algebra system to investigate and illustrate ideas developed during the semester, and to check their own work. Furthermore, students must be able to articulate the mathematics they use, especially when applied to problems in other areas.

- We agreed on the following principles:
  - The rote problems allow students to develop technical and mechanical fluency with core ideas.
  - The writing promotes a deeper understanding and deep learning.
  - Mathematica is used as a tool to facilitate messier computational problems , as well as a tool that enables clear visualization of concepts.

- We agree that we want a philosophy that is no more than 2 paragraphs long. Much of the detail for how we accomplish the progressive writing ability will be in the teacher notes.

- J. H. Will take the statement and suggestions above and weave them into some stunning statement.

Reviewed what we will get done by end of this grant:

- Philosophy Statement for our Calculus Sequence
- List of topics for Mat 221 and 222 (maybe 223)
- Teacher notes for 221 by end of summer
- Teacher notes for 222
- Related Rates question bank ranked.
- Optimization question bank by end of summer.
- Materials for Calc 1 and Calc 2 in MCS Blackboard by end of May.
- Updated EXO and syllabus for Calc 1, 2 and 3 by end of semester. We want to present the updated EXOs at the last department meeting.
- For 221 Collect sample work which exemplifies good writing and presentation for one of the projects as well as writing that needs refinements.
- Review Calc 1 and 2 manuals.

Still out there to be done:

- A deeper look at 223, similar to what we have done with 221 and 222. Maybe do this with an assessment grant later next year.
- Refinement of Calc 3 Mathematica manuals

Discussed Pre-req statement

- Just say MAT 185 or High School Precalculus with a grade of C or better.
- This allows us to not worry about the regents tests.
- It seems that level 5 only pertains to 221.
- If they have Precalculus in high school, they are ready for 221, even if it's been more than 2 years.
- Issue comes up at times in 222, since there is a fair amount of algebraic nuance in this course. Do we need to create an algebra review for 222? WebAssign has Just In Time Review for these types of questions. These appear at the end of the problem sets. Can assign these as an extra WebAssign problem.
- Some of the more nuanced algebra just doesn't come up in Precalculus or 221.
- Maybe come up with a review book that students can reference between Precalculus and Calculus.
- If they have had precalculus anywhere and have gotten a C or better, they should go into Calculus.

Tasks List: Goal is to have EXO's done by department meeting and rest done by end of semester.

- Johanna – Calculus Philosophy
- Sandra is completing the 221 EXO and syllabus
- Jason – EXO for Calc 2 and 3, and syllabus for Calc 2
- Maryanne will do syllabus for Calc 3

Review feedback from Sara on our previous minutes:

- We like points on our test questions. (Rachel had found that it can distract lower level students.)

- Should we share our full minutes with all Calc teachers? No. We will share all relevant decisions and information through thorough Teacher Notes. These notes will include some specifics about what areas of writing are included in each course. (Started on Calculus Philosophy draft by J. Halsey for Calc 1 and 2.)

- We will be putting sample finals in black board.

For next week:

- Hopefully review Calc 3 finals
- A brief description for topics for projects.

Meeting 12 (4/26/18)

Thursday, 4/26/2018 – Another incredibly seamless extension of our vibrant discussions!

Calculus Philosophy:

- We reviewed the Calculus Philosophy. Suggestions for some minor tweaks, but almost there.

- We will put these right before the SLO on the syllabi for Calc 1, 2 and 3.

Discussion of how to use the traditional "Departmental Syllabus".

- Contract uses the idea of syllabus differently than what we have in the past. We always saw the syllabus as the document that we have created and maintained for many, many years.
- Need to be DUE compliant
- One document allows for just one update as things change.
- We will name Departmental Syllabus as Departmental Course Information.
- We will add an example syllabus that meets DUE requirements to the Dropbox in Blackboard once someone has one ready for the Fall.

Moved to reviewing the list of Topics on the Departmental Course Information for Calc 3

Projects – Use of Mathematica:

- Vectors – Not heavy writing, more process, and then visualization and mechanical computations
- Functions of Several Variables: - Project that involves cross sections and average rates of change in comparison to contours and average rates of change
- Gradient Vector and Directional Derivative
- Multiple Integrals – more use it to check that when they change coordinate systems, or set up an integration when they are given the region of integration.
- Vector Functions – project on a vector field with a parametric curve, and study a line integral.

Rubric for STEM Writing:

- Some discussion of concept of introduction and conclusion and how it is morphing into more of a final statement about what the project was about.

General Discussion:

- Discussion of how the work in Calculus 3 inter-relates with other courses they are taking – and how to fit the work load in in sustainable and reasonable ways.
- It is difficult to cover the entire syllabus at the level we have been working at.

- Some schools have 3 credit multi variable and then a one credit vector calculus. Since we have a 4 credit course, it is essential that we cover the vector calculus. This is important for transfer. We have not traditionally gotten all the way through Green's Theorem on a regular basis. It is particular difficult to cover the vector calculus when we have snow days. This is the material that gets cut out.

- We will think carefully about the amount of time we spend on each unit, and see where we can possibly "create more time" for the vector calculus.

And so it ends….. a wonderfully productive semester of conversations and collegiality! We rock!!!!!

Faculty Time Logs

Sandra DeGuzman

Improvement of Instruction - Calculus Project Spring 2018

Date	Time	Description
1/25/2018	1.25	Meeting W110
2/1/2018	1.25	Meeting W110
2/8/2018	1.25	Meeting W110
2/15/2018	1.25	Meeting W110
3/1/2018	1.25	Meeting W110
3/8/2018	1.25	Meeting W110
3/15/2018	1	Related Rates Word Problem Draft
3/22/2018	1.25	Meeting W110
3/29/2018	1.25	Meeting W110
4/5/2018	1.25	Meeting W110
4/11/2018	1	MAT221 EXO and Syllabus
4/12/2018	1.25	Meeting W110
4/19/2018	1.25	Meeting W110
4/26/2018	1.25	Meeting W110
5/2/2018	1	MAT221 EXO and Syllabus
5/10/2018	2	Optimization Word Problem Draft and Related Rates Problems Updates
5/11/2018	1.5	Meeting W110
5/14/2018	1	Annotated Syllabus for 221
5/15/2018	1	Annotated Syllabus for 221
5/16/2018	1	Annotated Syllabus for 221
5/23/2018	1	Annotated Syllabus for 221
Total	25.5	

Jason Gumaer

Improvement of Instruction - Calculus Project Spring 2018

Date	Time	Description
1/25/2018	1.25	Meeting W110
2/1/2018	1.25	Meeting W110
2/8/2018	1.25	Meeting W110
2/10/2018	0.75	Created MAT221 Final Exam Guidelines
2/15/2018	1.25	Meeting W110
3/1/2018	1.25	Meeting W110
3/8/2018	1.25	Meeting W110
3/22/2018	1.25	Meeting W110
3/29/2018	1.25	Meeting W110
4/5/2018	1.25	Meeting W110
4/12/2018	1.25	Meeting W110
4/19/2018	1.25	Meeting W110
4/26/2018	1.25	Meeting W110
5/11/2018	1.5	Meeting W110
5/29/2018	1.5	Created MAT222 Department Information and Final Exam Guidelines
6/15/2018	3.5	Created MAT222 Annotated Syllabus
6/18/2018	1.5	Worked on Final Report
6/20/2018	2.5	Worked on Final Report
6/22/2018	2	Worked on Final Report
6/23/2018	2.5	Finished Final Report
Total	30.75	

Johanna Halsey

Improvement of Instruction - Calculus Project Spring 2018

Date	Time	Description
1/25/18	1.25	Meeting W120
1/27/18	0.5	Edited and formatted minutes
2/1/18	1.25	Meeting W120
2/6/18	0.5	Edited and formatted minutes
2/8/18	1.25	Meeting W120
2/15/18	1.25	Meeting W120
2/26/18	0.5	Edited and formatted minutes
3/1/18	1.25	Meeting W120
3/8/18	1.25	Meeting W120
3/22/18	1.25	Meeting W120
4/3/18	1	Edited and formatted minutes
4/5/18	1.25	Meeting W120
4/12/18	1.25	Meeting W120
4/17/18	0.5	Edited and formatted minutes
4/19/18	1.25	Meeting W120
4/26/18	1.25	Meeting W120
5/1/18	0.75	Edited and formatted minutes
5/11/18	1.5	Meeting W120
Total	19	

Maryanne Johnson

Improvement of Instruction - Calculus Project Spring 2018

Date	Time	Description
1/20/2018	0.5	Complied MAT 221/223 final exams and projects
1/25/2018	1.25	Meeting W110
1/27/2018	0.5	Compliled applet sites for use in MAT 221/222
2/1/2018	1.25	Meeting W110
2/8/2018	1.25	Meeting W110
2/15/2018	1.25	Meeting W110
3/1/2018	1.25	Meeting W110
3/1/2018	0.5	Compiled and sent RR and Opt questions to SD
3/8/2018	1.25	Meeting W110
3/22/2018	1.25	Meeting W110
3/29/2018	1.25	Meeting W110
4/5/2018	1.25	Meeting W110
4/1/2018	1	Updated MAT 221 Common Final
4/12/2018	1.25	Meeting W110
4/19/2018	1.25	Meeting W110
4/25/2018	0.75	Compliled and sent student work MAT 221
4/26/2018	1.25	Meeting W110
5/1/2018	1	Updated MAT 223 "Syllabus"
5/11/2018	1.5	Meeting W110
Total	20.75	