Calculus Transition: From High School to College

1989 Preface

The importance of the problems identified in the panel's report has been underscored by several recent international and national assessments of mathematics education (e.g., The National Research Council's report *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*).

Since the panel's report was written in 1986, the severity of the transition problems from high school to college calculus has increased both qualitatively and quantitatively. In 1987 there were 59,123 students who took an Advanced Placement Calculus Examination, an increase of 85% from 1982. Although this increase does not necessarily indicate a similar growth rate in the number of students studying calculus in high school, it does document a large increase in the number of students entering college calculus having earned an Advanced Placement Calculus score of three or less. This increase serves to intensify the Report's recommendation that colleges and universities develop special calculus courses for these students.

The development of Computer Algebra Systems designed for classroom teaching introduces a new component into the transition problems students may encounter in going from high school to college calculus. It is vitally important to expand communication between college and high school teachers in regards to the development of this technology, particularly with respect to pedagogical issues.

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Introduction

There is a widespread and growing dissatisfaction with the performance in college calculus courses of many students who had studied calculus in high school. In response to this concern, in the fall of 1983, the Committee on the Undergraduate Program in Mathematics (CUPM) formed a Panel on Calculus Articulation to undertake a three-year study of questions concerning the transition of students from high school calculus to college calculus and submit a report to CUPM detailing the problems encountered and proposals for their solution.

The seriousness of the issues involved in the Panel's study is underscored by the number of students involved and their academic ability. During the ten-year period 1973 to 1982, the number of students in high school calculus courses grew at a rate exceeding 10% annually. Of the 234,000 students who passed a high school calculus course in 1982, 148,600 received a grade of B- or higher [2]. Assuming a continuation of the 10% growth rate and a similar grade distribution there were approximately 200,000 high school students in the spring of 1985 who received a grade of B- or higher in a calculus course. Thus possibly a third or more of the 500,000 college students who began their college calculus program (in Calculus I, Calculus II, or Calculus III) in the fall of 1985 had already received a grade of B- or higher in a high school calculus course.

The students studying calculus in high school constitute a large majority of the more mathematically capable high school students. (In 1982, 55% of high school students attended schools where calculus was taught [2].) Students who score a 4 or 5 on an Advanced Placement (AP) Calculus examination normally do well in maintaining their accelerated mathematics program during the transition from high school to college. However, this is a very small percentage of the students who take calculus in high school. For example, in 1982, of the 32,000 students who took an Advanced Placement calculus examination, just over 12,000 received scores of 4 or 5, which represents only 6% of all high school students who took calculus that year. The primary concern of the Panel was with the transition difficulties associated with the remaining almost 94% of the high school calculus students.

Problem Areas

Past studies and the Panel's surveys of high school teachers, college teachers, and state supervisors suggest that the major problems associated with the transition from high school calculus to college calculus are:

1. High school teacher qualifications and expectations.
2. Student qualifications and expectations.
3. The effect of repeating a course in college after having experienced success in a similar high school course.
4. College placement.
5. Lack of communication between high schools and colleges.

These problems were addressed by first considering accelerated programs in general, high school calculus (successful, unsuccessful), and the responsibilities of the colleges.

Accelerated Programs

Accelerated mathematics programs, usually beginning with algebra in eighth grade, are now well established and accepted in most school systems. The success of these programs in attracting the mathematically capable students was documented in the 1981-82 testing that was done for the "Second International Mathematics Study." The Summary Report [9] states with reference to a comparison between twelfth grade precalculus students and twelfth grade calculus students in the United States:

> We note furthermore that in every content area (sets and relations, number systems, algebra, geometry, elementary functions/calculus, probability and statistics, finite mathematics), the end-of-the-year average achievement of the precalculus classes was less (and in many cases considerably less) than the beginning-of-the-year achievement of the calculus students.

The report continues:

> It is important to observe that the great majority of U.S. senior high school students in fourth and fifth year mathematics classes (that is, those in precalculus classes) had an average performance level that was at or below that of the lower 25% of the countries. The end-of-year performance of the students in the calculus classes was at or near the international means for the various content areas, with the exception of geometry. Here U.S. performance was below the international average.

Thus those students in accelerated programs culminating in a calculus course perform near the international mean level while their classmates in (non-accelerated) programs culminating in a precalculus course perform in the bottom 25% in this international survey. The poor performance in geometry by both the precalculus and calculus students correlates well with the statistic that 38% of the students were never taught the material contained in the geometry section of the test [9, p. 59]. The test data underscores the concern expressed by many college teachers that more emphasis needs to be placed on geometry throughout the high school curriculum. This data does not, however, indicate that accelerated programs emphasize geometry less than non-accelerated programs.

The success of the accelerated programs in completing the "normal" four year high school mathematics program by the end of the eleventh grade presents schools with both an opportunity and a challenge for a "fifth" year program. There are two acceptable options:

1. Offer college-level mathematics courses that would continue the students' accelerated program and thus provide exemption from one or two semesters of college mathematics.
2. Offer high school mathematics courses that would broaden and strengthen a student's background and understanding of precollege mathematics.

Not offering a fifth year course or offering a watered-down college level course with no expectation of students earning advanced placement are not considered to be acceptable options.

A great deal of prestige is associated with offering calculus as a fifth year course. Communities often view the offering of calculus in their high school as an indication of a quality educational program. Parents, school board officials, counselors, and school administrators often demonstrate a competitive pride in their school's offering of calculus. This prestige factor can easily manifest itself in strong political pressure for a school to offer calculus without sufficient regard to the qualifications of teachers or students.

It is important that this political pressure be resisted and that the choice of a fifth year program be made by the mathematics faculty of the local school and be made on the basis of the interest and qualifications of the mathematics faculty and the quality and number of accelerated students. School officials should be encouraged to develop public awareness programs to extend the prestige and support that exists for the calculus to acceleration programs in general. This would help diffuse the political pressure as well as broaden school support within the community.

Schools that elect the first option of offering a college level course should follow a standard college course syllabus (e.g., the Advanced Placement syllabus for calculus). They should use placement test scores along

with the college records of their graduates as primary measures of the validity of their course.

For schools that elect the second option, a variety of courses is possible. The following course descriptions represent four possibilities.

ANALYTICAL GEOMETRY. This course could go well beyond the material normally included in second year algebra and precalculus. It could include Cartesian and vector geometry in two- and three-dimensions with topics such as translation and rotation of axes, characteristics of general quadratic relations, curve sketching, polar coordinates, and lines, planes, and surfaces in three-dimensional space. Such a course would provide specific preparation for calculus and linear algebra, as well as give considerable additional practice in trigonometry and algebraic manipulations.

PROBABILITY AND STATISTICS. This course could be taught at a variety of levels, to be accessible to most students, or to challenge the strongest ones. It could cover counting methods and some topics in discrete probability such as expected values, conditional probability, and binomial distributions. The statistics portion of the course could emphasize exploratory data analysis including random sampling and sampling distributions, experimental design, measurement theory, measures of central tendency and spread, measures of association, confidence intervals, and significance testing. Such an introduction to probability and statistics would be valuable to all students, and for those who do not plan to study mathematics, engineering, or the physical sciences, probably more valuable than a calculus course.

DISCRETE MATHEMATICS. This type of course could include introductions to a number of topics that are either ignored or treated lightly within a standard high school curriculum, but which would be stimulating and widely useful for the college-bound high school student. Suggested topics include permutations, combinations, and other counting techniques: mathematical induction; difference equations; some discrete probability; elementary number theory and modular arithmetic; vector and matrix algebra, perhaps with an introduction to linear or dynamic programming; and graph theory.

MATRIX ALGEBRA. This course could include basic arithmetic operations on matrices, techniques for finding matrix inverses, and solving systems of linear equations and their equivalent matrix equations using Gaussian elimination. In addition, some introduction to linear programming and dynamic programming could be included. This course could also emphasize three-dimensional geometry.

High School Calculus

There are many valid reasons why a fifth year program should include a calculus course. Four major reasons: (1) calculus is generally recognized as the starting point of a college mathematics program, (2) there exists a (nationally accepted) syllabus, (3) the Advanced Placement program offers a nation-wide mechanism for obtaining advanced placement, and (4) there is a large prestige factor associated with offering calculus in high school. Calculus, however, should not be offered unless there is a strong indication that the course will be successful.

Successful Calculus Courses

The primary characteristics of a successful high school calculus course are:
1. A qualified and motivated instructor with a mathematics degree that included at least one semester of a junior-senior real analysis course involving a rigorous treatment of limits, continuity, etc.
2. Administrative support, including provision of additional preparation time for the instructor (e.g., as recommended by the North Central Accreditation Association).
3. A full year program based on the Advanced Placement syllabus.
4. A college text should be used (not a watered-down high school version).
5. Advanced placement for students (rather than mere preparation for repeating calculus in college) is a major goal.
6. Course evaluation based primarily on college placement and the performance of its graduates in the next higher level calculus course.
7. Restriction of course enrollment to only qualified and interested students.
8. The existence of an alternative fifth year course that students may select who are not qualified for or interested in continuing in an accelerated program.

The bottom line of what makes a high school calculus course successful is no surprise to anyone. A qualified teacher with high but realistic expectations, using somewhat standard course objectives, and students who are willing and able to learn result in a successful transition at any level of our educational process. Problems appear when any of the above ingredients are missing.

Unsuccessful Calculus Courses

Two types of high school calculus courses have an

undesirable impact on students who later take calculus in college.

One type is a one semester or partial year course that presents the highlights of calculus, including an intuitive look at the main concepts and a few applications, and makes no pretense about being a complete course in the subject. The motivation for offering a course of this kind is the misguided idea that it prepares students for a *real* course in college.

However, such a preview covers only the glory and thus takes the excitement of calculus away from the college course without adequately preparing students for the hard work and occasional drudgery needed to understand concepts and master technical skills. Professor Sherbert has commented: "It is like showing a ten minute highlights film of a baseball game, including the final score, and then forcing the viewer to watch the entire game from the beginning—with a quiz after each inning."

The second type of course is a year-long, semi-serious, but watered-down treatment of calculus that does not deal in depth with the concepts, covers no proofs or rigorous derivations, and mostly stresses mechanics. The lack of both high standards and emphasis on understanding dangerously misleads students into thinking they know more than they really do.

In this case, not only is the excitement taken away, but an unfounded feeling of subject mastery is fostered that can lead to serious problems in college calculus courses. Students can receive respectable grades in a course of this type, yet have only a slight chance of passing an examination. Those who place into second term calculus in college will find themselves in heavy competition with better prepared classmates. Those who elect (or are selected) to repeat first term calculus believe they know more than they do, and the motivation and willingness to learn the subject are lacking.

College Programs

Several studies ([1], [3], [5], [6], [7]) have been conducted on the performance in later courses by students who have received advanced placement (and possibly college credit) by virtue of their scores on Advanced Placement Calculus examinations. The studies show that, overall, students earning a score of 4 or 5 on either the AB or BC Advanced Placement Calculus examination do as well or better in subsequent calculus courses than the students who have taken all their calculus in college. It is therefore strongly recommended that colleges recognize the validity of the Advanced Placement

Calculus program by the granting of one semester advanced placement with credit in calculus for students with a 4 or 5 score on the AB examination, and two semesters of advanced placement with credit in calculus for students with a 4 or 5 score on the BC examination.

The studies reviewed by the Panel do not indicate any clear conclusions concerning performance in subsequent calculus courses by students who have scored a 3 on an Advanced Placement Calculus examination. The treatment of these students is a very important transition problem since approximately one-third of all students who take an Advanced Placement Calculus examination are in this group and many of them are quite mathematically capable.

It is therefore recommended that these students be treated on a special basis in a manner that is appropriate for the institution involved. For example, several colleges offer a student who has earned a 3 on an Advanced Placement Calculus examination the opportunity to upgrade this score to an "equivalent 4" by doing sufficiently well on a Department of Mathematics placement examination. Another option is to give such students one semester of advanced placement with credit for Calculus I upon successful completion of Calculus II. A third option is to give one semester of advanced placement with credit for Calculus I and provide a special section of Calculus II for such students.

Other important transition problems are associated with students who have studied calculus in high school, but have not attained advanced placement either through the Advanced Placement Calculus program or effective college procedures. These students pose an important and difficult challenge to college mathematics departments, namely: How should these students be dealt with so that they can benefit from their accelerated high school program and not succumb to the negative and (academically) destructive attitude problems that often result when a student repeats a course in which success has already been experienced? There are three major factors to consider with respect to these students.

1. The lack of uniformity of high school calculus courses. The wide diversity in the backgrounds of the students necessitates that a large review component be included in their first college calculus course to guarantee the necessary foundation for future courses.

2. The mistaken belief of most of these students that they really know the calculus when, in fact, they do not. Thus they fail to study enough at the beginning of the course. When they realize their mistake (if they do), it is often too late. These students often

become discouraged and resentful as a result of their poor performance in college calculus, and believe that it is the college course that must be at fault.

3. The "Pecking Order" syndrome. The better the student, the more upsetting are the understandable feelings of uncertainty about his or her position relative to the others in the class. Although this is a common problem for all college freshmen, it is compounded when the student appears to be repeating a course in which success had been achieved the preceding year. This promotes feelings of anxiety and produces an accompanying set of excuses if the student does not do at least as well as in the previous year.

The uncertainty of one's position relative to the rest of the class often manifests itself in the student not asking questions or discussing in (or out of) class for fear of appearing *dumb*. This is in marked contrast to the highly confident high school senior whose questions and discussions were major components in his or her learning process.

The unpleasant fact is that the majority of students who have taken calculus in high school and have not clearly earned advanced placement do not *fit* in either the standard Calculus I or Calculus II course. The students do not have the level of mastery of Calculus I topics to be successful if placed in Calculus II and are often doomed by attitude problems if placed in Calculus I. In modern parlance, this is the *rock and hard place.*

An additional factor to consider is the negative effect that a group of students who are repeating most of the content of Calculus I has on the rest of the class as well as on the level of the instructor's presentations.

What is needed are courses designed especially for students who have taken calculus in high school and have not clearly earned advanced placement. These courses need to be designed so that they:

1. Acknowledge and build on the high school experiences of the students;
2. Provide necessary review opportunities to ensure an acceptable level of understanding of Calculus I topics;
3. Are *clearly different* from high school calculus courses (in order that students do not feel that they are essentially just repeating their high school course);
4. Result in an equivalent of one semester advanced placement.

Altering the traditional lecture format or rearranging and supplementing content seem to be two promising approaches to developing courses that will satisfy

the above criteria. For example, Colby College has successfully developed a two semester calculus course that fulfills the four conditions. The course integrates multivariable with single variable calculus, and thereby covers the traditional three semester program in two semesters [10].

Of course, the introduction of a new course entails an accompanying modification of college placement programs. However, providing new or alternative courses should have the effect of simplifying placement issues and easing transition difficulties that now exist.

Recommendations

1. School administrators should develop public awareness programs with the objective of extending the support that exists for fifth year calculus courses to accelerated programs including all of the fifth year options.
2. A fifth year program should offer a student a choice of courses (not just calculus).
3. The choice of fifth year options should be made by the high school mathematics faculty on the basis of their interest and qualifications and the quality and number of the accelerated students.
4. If a fifth year course is intended as a college level course, then it should be treated as a college level course (text, syllabus, rigor).
5. A fifth year college level course should be taught with the expectation that successful graduates (B- or better) would not repeat the course in college.
6. A fifth year program should provide an alternative option for the student who is not qualified to continue in an accelerated program.
7. A mathematics degree that includes at least one semester of a junior-senior real analysis course involving a rigorous treatment of limit, continuity, etc., is strongly recommended for anyone teaching calculus.
8. A high school calculus course should be a full year course based on the Advanced Placement syllabus.
9. The instructor of a high school calculus course should be provided with additional preparation time for this course.
10. High school calculus students should take either the AB or BC Advanced Placement calculus examination.
11. The evaluation of a high school calculus course should be based primarily on college placement and the performance of its graduates in the next level calculus course.

12. Only interested students who have successfully completed the standard four year college preparatory program in mathematics should be permitted to take a high school calculus course.

13. Colleges should grant credit and advanced placement out of Calculus I for students with a 4 or 5 score on the AB Advanced Placement calculus examination, and credit and advanced placement out of Calculus II for students with a 4 or 5 score on the BC Advanced Placement calculus examination. Colleges should develop procedures for providing special treatment for students who have earned a score of 3 on an Advanced Placement calculus examination.

14. Colleges should individualize as much as possible the advising and placement of students who have taken calculus in high school. Placement test scores and personal interviews should be used in determining the placement of these students.

15. Colleges should develop special courses in calculus for students who have been successful in accelerated programs, but have clearly not earned advanced placement.

Colleges have an opportunity and responsibility to develop and foster communication with high schools. In particular:

16. Colleges should establish periodic meetings where high school and college teachers can discuss expectations, requirements, and student performance.

17. Colleges should coordinate the development of enrichment programs (courses, workshops, institutes) for high school teachers in conjunction with school districts and state mathematics coordinators.

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