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Calibration and Uncertainties of Pipe Roughness Height

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Outline

1 Introduction

The average roughness height *k* of pipe is a key parameter in hydraulic calculation of pipe systems. For any type of pipes, as soon as *k* is known, the capacity of water conveyance of urban water-supply and drainage systems can be predicted in the design. Therefore, for a new type of pipes, the first thing from the hydraulics view of point is to calibrate this value in hydraulic labs precisely.

If flows are in fully rough region, *k* can be obtained by the Nikuradse equation (1933). Otherwise, it can be estimated by the Colebrook equation (1939), which covers not only the transition region but also the fully developed smooth and rough pipes.

1 Introduction

Recently, several authors carried out the further investigation for estimation of the roughness coefficient.

Author	Contents
Shockling, Allen and Smits (2006)	the roughness coefficient for honed surfaces follows the Nikuradse (1933) form with dips and bellies rather than the monotonic relations seen in the Moody diagram based on the Colebrook equation
Yang and Joseph (2009)	derived an accurate composite friction factor versus Reynolds number correlation formula for laminar, transition and turbulent flow in smooth and rough pipes.
……	

We did the experiment to calibrate the values of k for the three types of ductile cast iron pipes lined with cement mortar, epoxy resin and polyethylene.

The results shown that the values of k found by the Colebrook equation varied significantly with the change in Re(Reynolds number).

Photos of three types of pipes

Cement mortar lining pipes

Epoxy resin lining pipes

Polyethylene lining pipes

1 Introduction

Author	Contents
Yang Kailin et al.	The values of k were between 0.024mm and 0.040mm for the pipes lined with cement mortar, between 0.015mm and 0.078mm for the pipes lined with epoxy resin,and between 0.0039mm and 0.0106mm for the pipes lined with polyethylene.

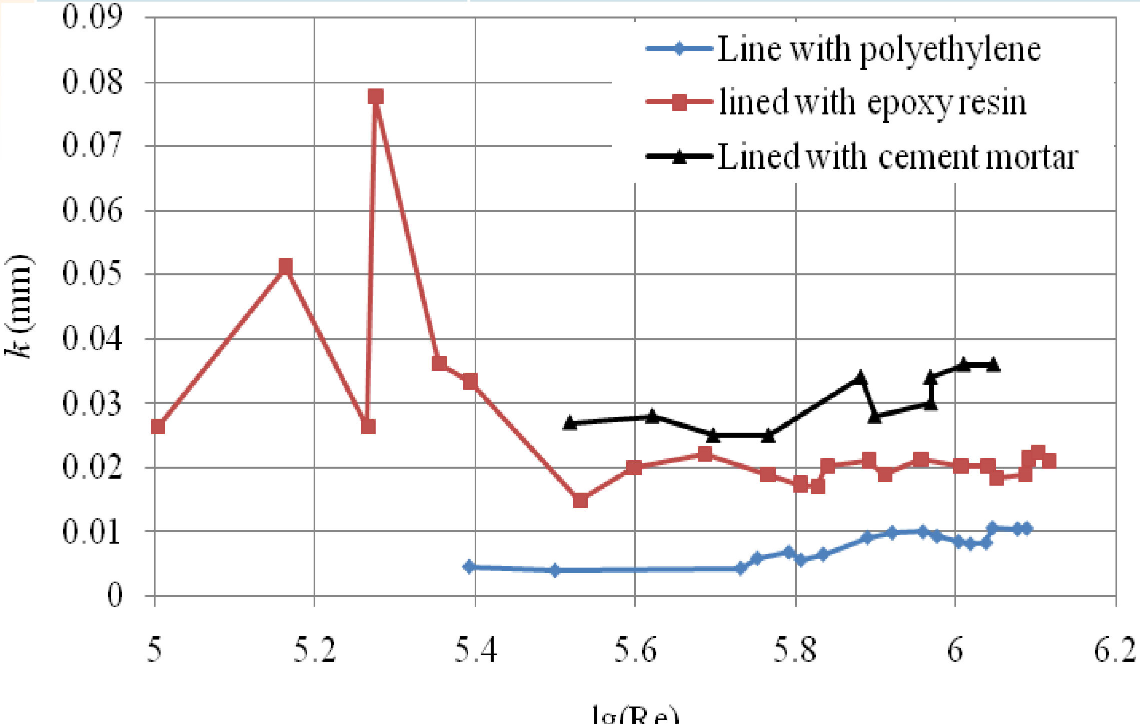

Figure 1. Found by the Colebrook equation

As well known, however, its value should be constant for the given pipe within a short time. Why does the contradiction occur? One of the reasons is that there are the uncertainties in the measured data.

1 Introduction

Our Work

It is mainly to demonstrate how to determine the value of k reasonably by the systematical analysis of the uncertainties for the measured parameters, such as pipe diameter, length, flow rate and headloss as well as width, height and head above crest level of weir for flow rate.

2 Estimation of roughness height and uncertainty

2.1 Estimation of roughness coefficient

The head loss may be estimated using the Darcy-Weisbach equation

$$h_f = \frac{\lambda L Q^2}{2gDA^2}$$

h_f depends on the piezometer heads at the inlet and the outlet of a pipe

The dimensionless friction number λ depends on both the pipe roughness and the Reynolds number

$$\frac{1}{\sqrt{\lambda}} = -2\lg\left(\frac{k}{3.71D} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$$

" k " is roughness height

Thus

$$\begin{cases} \lambda = \dfrac{2gDA^2 h_f}{LQ^2} \\ k = 3.71\left(\dfrac{1}{\sqrt{10^{1/\sqrt{\lambda}}}} - \dfrac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right) \end{cases}$$

2 Estimation of roughness height and uncertainty

2.2 Uncertainty of λ

The dispersions of the value λ can be estimated from the above equation, on the basis of the theory of uncertainty propagation for independent variables (EA-4/02, 1999), as

$$\Delta\lambda = \frac{\partial\lambda}{\partial h_f}\Delta h_f + \frac{\partial\lambda}{\partial Q}\Delta Q + \frac{\partial\lambda}{\partial D}\Delta D + \frac{\partial\lambda}{\partial L}\Delta L$$

$$\begin{cases} \dfrac{\partial\lambda}{\partial h_f} = \dfrac{2gDA^2}{LQ^2} = \dfrac{\lambda}{h_f} \\ \dfrac{\partial\lambda}{\partial L} = -\dfrac{2gDA^2 h_f}{L^2Q^2} = -\dfrac{\lambda}{L} \ldots\ldots \end{cases}$$

It may be written as

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta h_f}{h_f} - 2\frac{\Delta Q}{Q} + 5\frac{\Delta D}{D} - \frac{\Delta L}{L}$$

In uncertainty analysis, the values $\frac{\Delta\lambda}{\lambda}$ $\frac{\Delta h_f}{h_f}$ $\frac{\Delta Q}{Q}$ $\frac{\Delta D}{D}$ $\frac{\Delta L}{L}$ are referred to as dimensionless standard uncertainties and have the notation $u^*(\lambda)$ $u^*(h_f)$ $u^*(Q)$ $u^*(D)$ $u^*(L)$

Since the uncertainties are independent of each other, its summation can be obtained as

$$\boxed{u^*(\lambda) = \sqrt{u^*(h_f)^2 + 4u^*(Q)^2 + 25u^*(D)^2 + u^*(L)^2}}$$

2.3 Uncertainty of *k*

Similarly, the dispersions of the values ***k*** can also be expressed as

$$\Delta k = \frac{\partial k}{\partial \lambda}\Delta\lambda + \frac{\partial k}{\partial Q}\Delta Q + \frac{\partial k}{\partial D}\Delta D$$

$$\begin{cases} \dfrac{\partial k}{\partial \lambda} = \dfrac{3.71D}{2}\left(\dfrac{0.5\ln 10}{\lambda^{1.5}\sqrt{10^{1/\sqrt{\lambda}}}} + \dfrac{2.51}{\mathrm{Re}\,\lambda^{1.5}}\right) \\ \dfrac{\partial k}{\partial Q} = 3.71D\dfrac{2.51}{Q\,\mathrm{Re}\sqrt{\lambda}} \\ \dfrac{\partial k}{\partial D} = 3.71\left(\dfrac{1}{\sqrt{10^{1/\sqrt{\lambda}}}} - 2\dfrac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right) \end{cases}$$

It may be written as

$$\frac{\Delta k}{k} = a_1\frac{\Delta h_f}{h_f} - a_2\frac{\Delta Q}{Q} + a_3\frac{\Delta D}{D} - a_4\frac{\Delta L}{L}$$

Thus, the dimensionless standard uncertainty of *k* should be

$$\boxed{u^*(k) = \sqrt{a_1^2u^*(h_f)^2 + a_2^2u^*(Q)^2 + a_3^2u^*(D)^2 + a_4^2u^*(L)^2}}$$

2 Estimation of roughness height and uncertainty

2.4 Uncertainty of Q for suppressed sharp-crested weirs

The Rehbock equation (Rehbock, 1929; BS ISO 1438, 2008) is recommended for the flow rate for the weir form. It may be written as follows

$$Q = \left(1.782 + 0.24\frac{H}{P}\right)B\left(H + 0.0011\right)^{1.5}$$

The dispersion of the value $\boldsymbol{Q}$ can be written as

$$\frac{\Delta Q}{Q} = b_1\frac{\Delta H}{H} + \frac{\Delta B}{B} - b_2\frac{\Delta P}{P}$$

$$\begin{cases} b_1 = \left(\dfrac{1.782 + 0.24\frac{1}{P}}{1.782 + 0.24\frac{H}{P}} + \dfrac{1.5}{H + 0.0011}\right)H \\ b_2 = 0.24\dfrac{H}{\left(1.782 + 0.24\frac{H}{P}\right)P} \end{cases}$$

the dimensionless standard uncertainty of $\boldsymbol{Q}$ should be

$$\boxed{u^*(Q) = \sqrt{b_1^2 u^*(H)^2 + u^*(B)^2 + b_2^2 u^*(P)^2}}$$

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2 Estimation of roughness height and uncertainty

Uncertainties	Expression
roughness coefficient	$u^*(\lambda)=\sqrt{u^*(h_f)^2+4u^*(Q)^2+25u^*(D)^2+u^*(L)^2}$
roughness height	$u^*(k)=\sqrt{a_1^2u^*(h_f)^2+a_2^2u^*(Q)^2+a_3^2u^*(D)^2+a_4^2u^*(L)^2}$
discharge	$u^*(Q)=\sqrt{b_1^2u^*(H)^2+u^*(B)^2+b_2^2u^*(P)^2}$

In summary, the uncertainties of λ and *k* come from the uncertainties of the measurement for *D, L, H, B* and *P* as well as H_1 and H_2. If electrometric measurements, such as flow meter and pressure transducers, are used for *Q*, H_1 and H_2, $u^*(Q)$, $u^*(H_1)$ and $u^*(H_2)$ equal their accuracy reading. The piezometer tubes are usually used for the measurement of H_1 and H_2, the point gauges for *H*, and measuring scales for *D, L, B* and *P*. The uncertainties of them are basically caused by eyeballing random errors.

3 Application

The following will demonstrate how to determine the value of *k* reasonably by the systematical analysis of the uncertainties for the measured parameters, by taking the ductile cast iron pipes lined with epoxy resin as an example.

- *The calibrated pipeline consisted of 5 pipes. The length of each pipe is 6m and the internal diameter is 0.302 m. The energy loss was measured by two piezometer tubes and the corresponding pipe length is 26.610 m. The eyeballing random errors are* $\Delta H_1 = \pm 0.0005\text{m}$ $\Delta H_2 = \pm 0.0005\text{m}$ $\Delta D = \pm 0.0001\text{m}$ $\Delta L = \pm 0.001\text{m}$
- *The flow rates were measured by a suppressed sharp-crested weir with P=0.526m and B=1.005m. The eyeballing random errors are* $\Delta P = \Delta B = \pm 0.001\text{m}$ $\Delta H = \pm 0.0001\text{m}$

The upstream gauged head above the crest level and the pipe headloss are measured in each case. Thus, the uncertainties of *u*(Q), u*(h*$_f$*) , u*(λ)* and *u*(k)* could be calculated and the results are shown in Figures 1 to 8.

3 Application

3.1 Uncertainty of *Q*

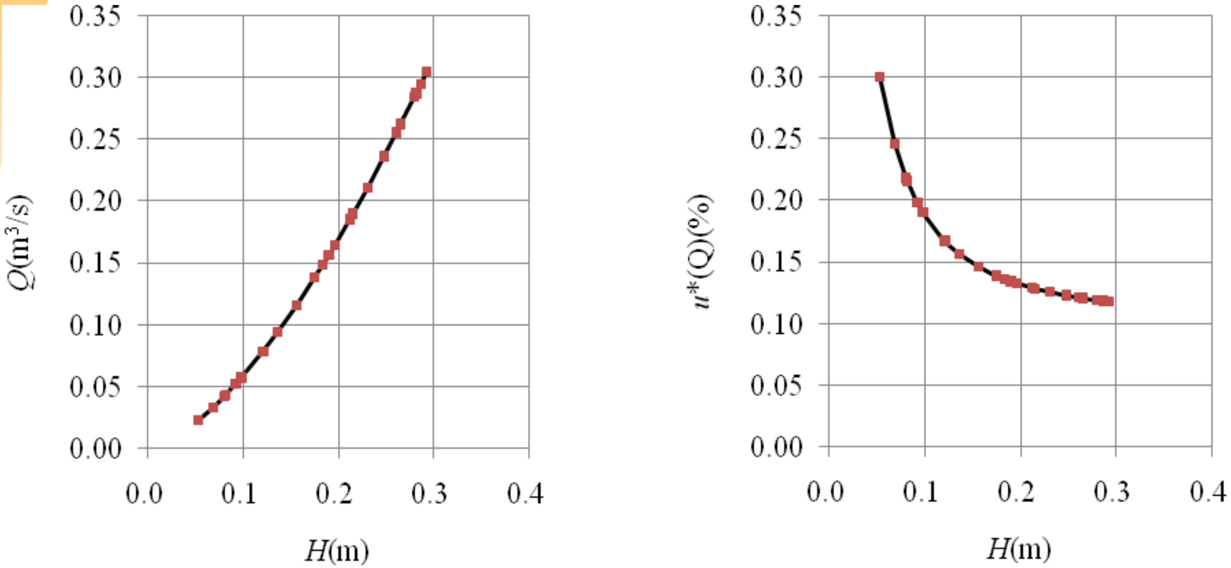


Figure 2. Curves of Q and $u^*(Q)$ versus H

As the weir head H increases from 0.0542 m to 0.2921 m, the flow rate Q varies from 0.0236m^3/s to 0.3056 m^3/s and the dimensionless standard uncertainty $u^*(Q)$ decreases from 0.3% to 0.12% monotonically. It demonstrates that the precision of the flow measurement is quite high.

3 Application

3.2 Uncertainty of h_f

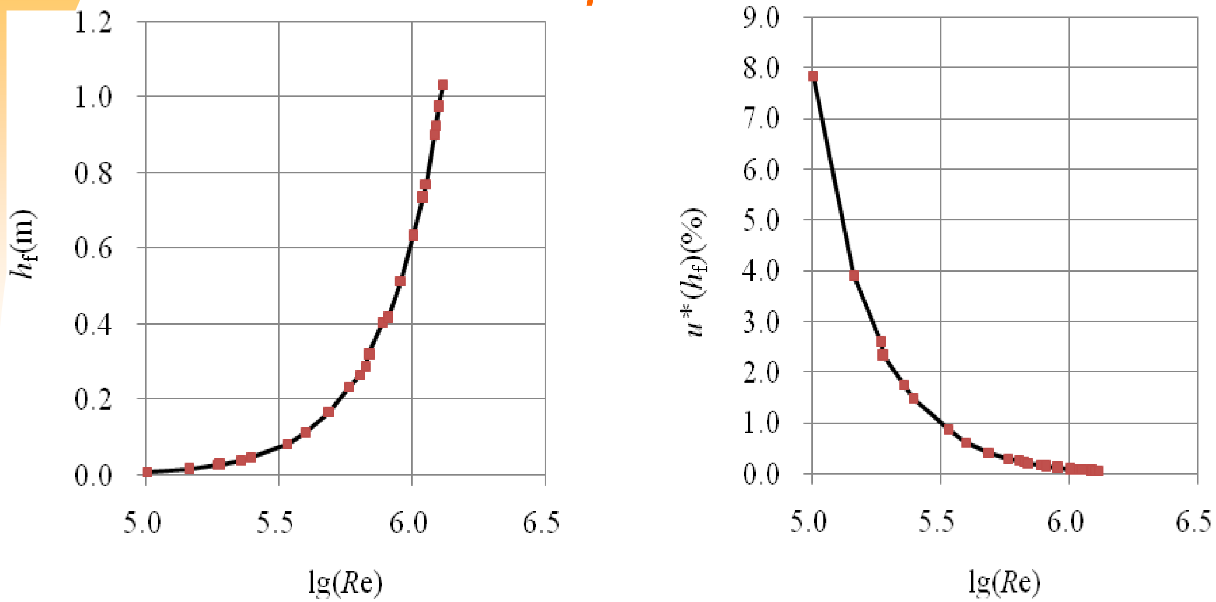


Figure 3. Curves of h_f and $u^*(h_f)$ versus Re

These two figures show the results of calculated head losses and their uncertainties. If the flow Q increases from 0.0236m^3/s to 0.3056m^3/s, the corresponding Reynolds number is between 10^5 and $10^{6.1}$. As Re rises, h_f increases from 0.009m to 1.035m and $u^*(h_f)$ reduces from 7.86% to 0.07% monotonically. It illustrates that the value of $u^*(h_f)$ is much greater when Q is less.

3.3 Uncertainty of *pipe roughness coefficient λ*

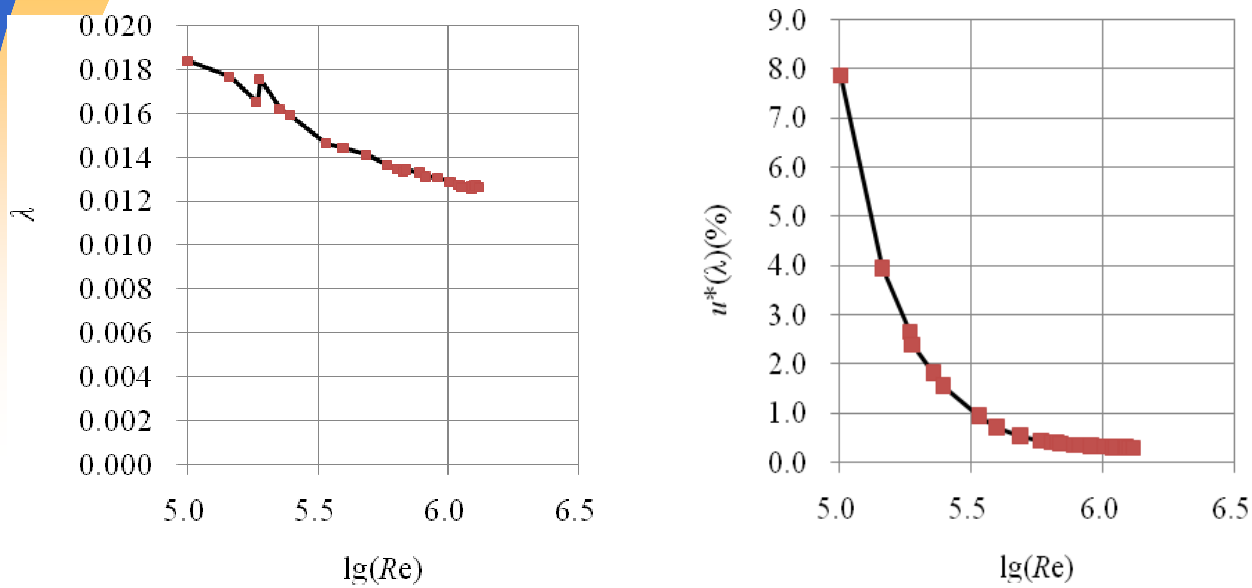
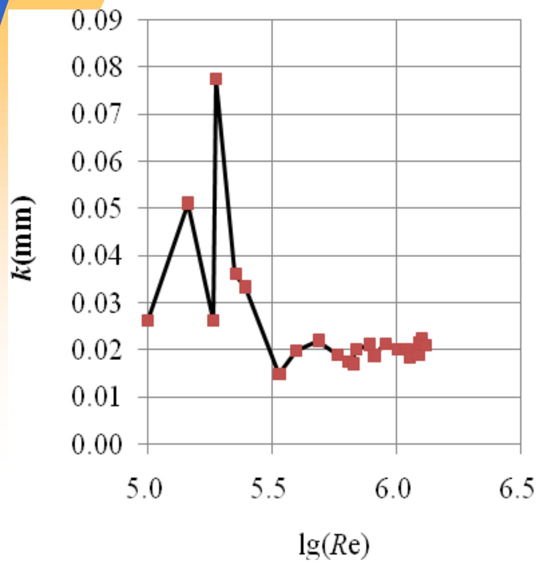


Figure 4. Curves of *λ* and *u*(λ)* versus *Re*

Figure 4 shows the results of calculated Darcy friction factors and their uncertainties. It can be seen that with the increase of Re the pipe roughness coefficient *λ* decreases from 0.0184 to 0.0127, and the uncertainty *u*(λ)* reduces from 7.88% to 0.3% monotonically. It demonstrates that the calibration of *λ* has a greater uncertainty if Re or *Q* is less.

3 Application

3.4 Uncertainty of *pipe roughness height* *k*



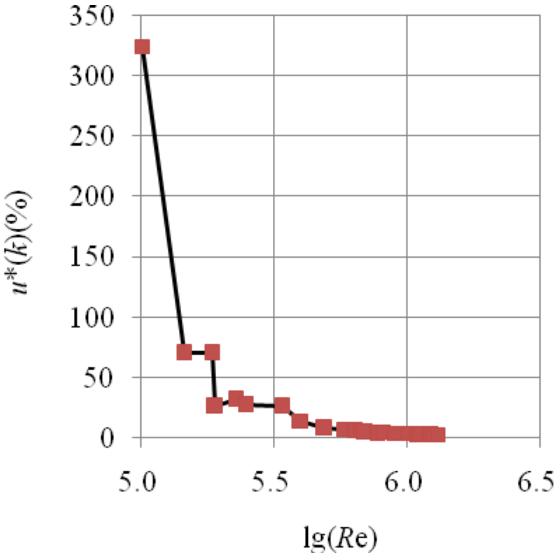


Figure 5. Curves of *k* and *u*(k)* versus *Re*

Figure 5 shows that the value of *k* found by the Colebrook equation varied significantly with the change in Re. When Re increases from 10^5 to $10^{6.1}$, *k* changes between 0.015 mm and 0.078 mm, and the corresponding dimensionless standard uncertainty *u*(k)* decreases from 324% to 3%. When Re is less, *k* is greater and ruleless. When Re is greater, i.e. Re> $10^{5.7}$, *k* approximates a fixed number and *u*(k)* is less than 5%. The fixed number is the calibrated value of *k*.

3 Application

3.4 Uncertainty of *pipe roughness height k*

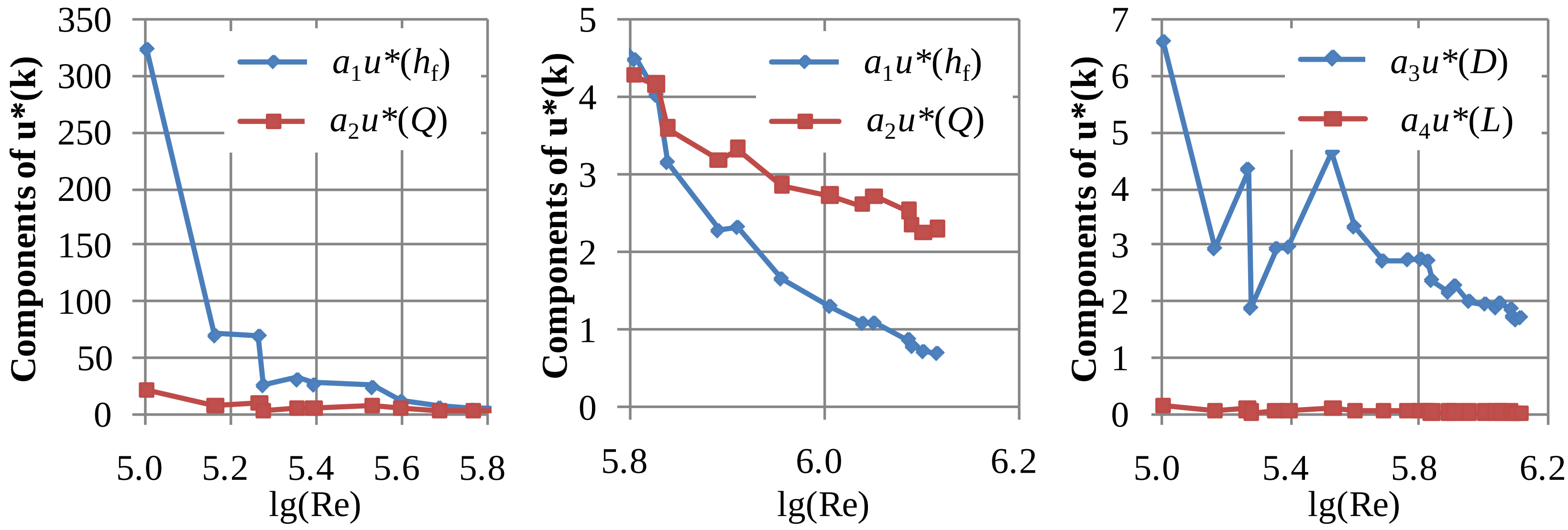


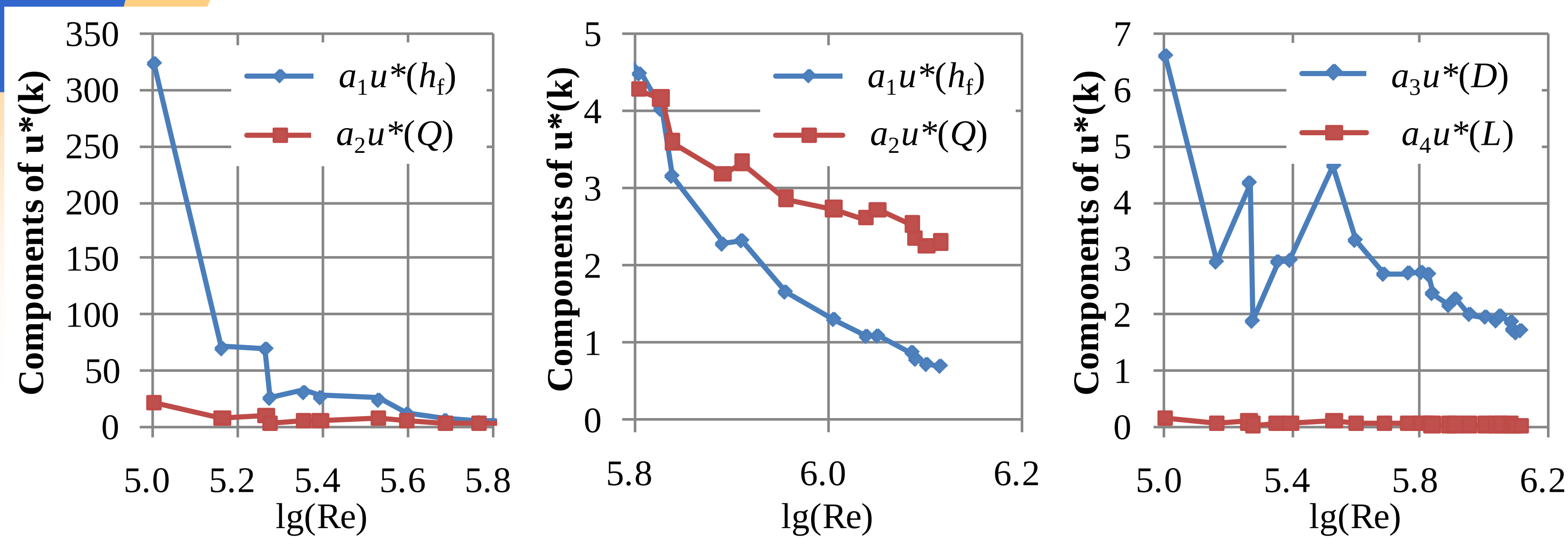
Figure 7 Effect of $u^*(h_f)$, $u^*(Q)$, $u^*(D)$ and $u^*(L)$ on $u^*(k)$

Obviously, the effect of $u^*(L)$ on $u^*(k)$ is negligible. If Re is less, $u^*(k)$ mainly depends on $u^*(h_f)$. If Re is greater, $u^*(k)$ is dominated by $u^*(h_f)$, $u^*(Q)$ and $u^*(D)$.

The characteristics of the Colebrook Equation is that when Re is greater, or the flow is in fully rough regime, the value of λ mainly depends on k rather than Re ; but if the flow is in smooth rough regime, Re is often a dominating effect.

3 Application

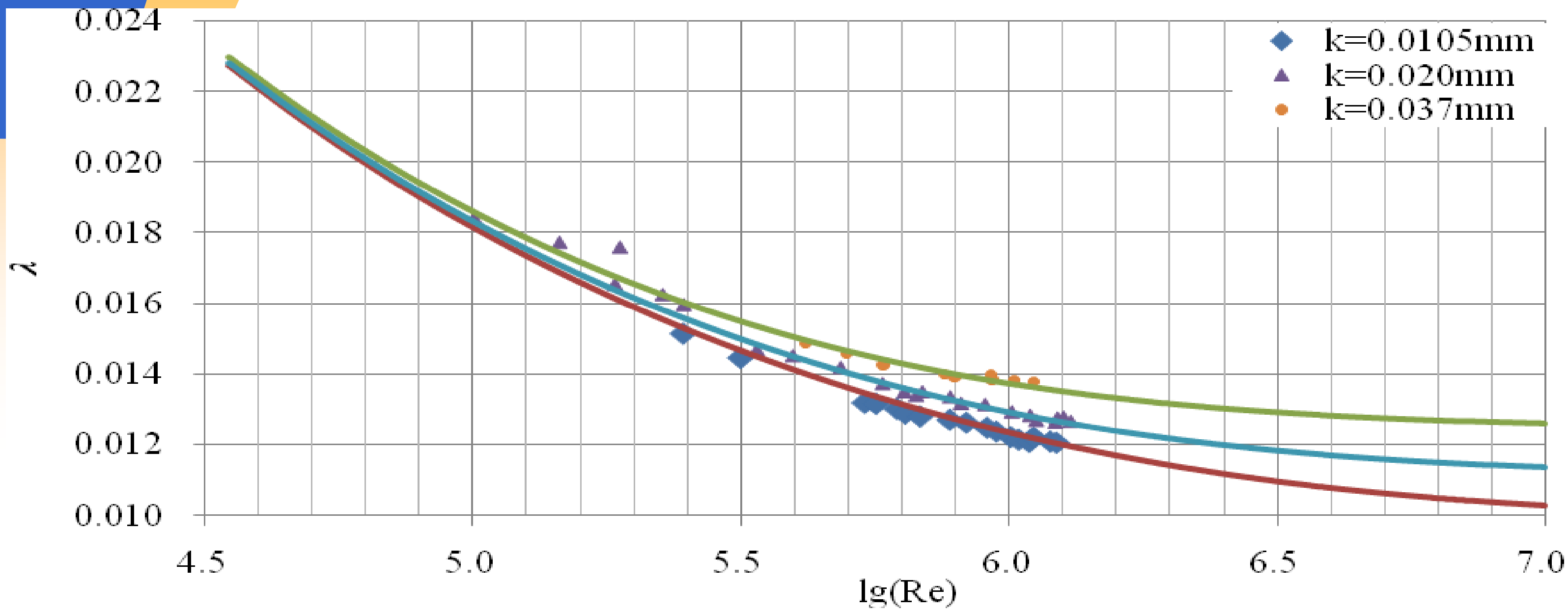
3.4 Uncertainty of *pipe roughness height* k



As mentioned above, one comes to a conclusion that for the ductile cast iron pipes lined with epoxy resin k=0.02mm and $u^*(k)$<5%. Similarly, it is calibrated that if $u^*(k)$<5%, k=0.0105mm for the ductile cast iron pipes lined with polyethylene and k=0.037mm for the ductile cast iron pipes lined with cement mortar.

3 Application

3.4 Uncertainty of *pipe roughness height* *k*



The above Figure shows the curves of λ versus Re , in which the discrete points are drawn by the measured data and the solid lines are computed by the Colebrook Equation at k =0.0105mm, 0.02mm and 0.037mm, respectively. Obviously, except some positions in which there are the greater deviations when Re is less, the deviations between the measured data and computed one is less than 1% in most of the positions.

4 Conclusions

This paper presents the equations for calculation of the uncertainties of the roughness coefficient, average roughness height and the flow rate of suppressed sharp-crested weirs, on the basis of the uncertainties of the measurements of pipe diameter, length and headloss or peizometer head as well as width, height and head above crest level of weir for flow rate. It demonstrates how to determine the value of *k* reasonably by the systematical analysis of the uncertainties, Some important conclusions are obtained as follows:

1) **The dimensionless standard uncertainties of headloss, flow rate and roughness coefficient all decrease monotonically with the increase of Re;**
2) **When Re>4x10^5 , with the increase of Re the roughness height varies slightly and its uncertainty reduces greatly;**
3) **The tiny uncertainties of measured headloss, flow rate, pipe diameter and length can result in a quite great uncertainty of roughness height.**

Thank you