Calvert's *Math in Focus*
Bar Model Guide

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Table of Contents

Fractions

Introduction

Bar models are designed to assist students in solving word problems and other math problems that require higher level thinking. They are tools for students to determine which operation to use and how to understand the different parts of a multi-step math problem. They also help students visualize fractions and algebraic concepts.

Bar models are part of the Singapore Math method that helps students gain a deeper understanding of the problems and the steps of the solutions. For basic operations, bar models are used until students gain mastery. Then they no longer need to use the bar models to help them find the solution. If your student has gained a full understanding of a concept, he does not have to create a bar model for every problem assigned in the textbook. However, it is recommended that he still use a bar model at least twice during relevant lessons to reinforce the skill of using bar models so he is prepared when new kinds of bar models are presented for different concepts.

 This companion can be used in several ways:

- Use as a reference guide to learn new types of bar models.
- Review the foundational use of bar models in prior grades.
- Preview the development of bar models over several grades.

In the Table of Contents, the color-coded numbers to the left of the titles represent the grade in which each kind of bar model is used. Since bar modeling skills build on one another, students will also use the bar modeling skills from previous grade(s). If your student has not been exposed to bar modeling or is struggling with a particular type of bar model, look at the models in previous grade(s) and work through those steps.

Students who use bar models to solve problems are able to visualize solutions much more clearly and understand what they need to do in order to arrive at those solutions. Learning and applying bar-modeling techniques can help your student become a more confident and efficient problem-solver.

If you require additional assistance with bar modeling, please contact the Calvert Education Counselors at 1-888-487-4652, option 4.

Addition

Example

Micah has 11 crayons.
He finds 7 more.
How many crayons does Micah have now?

Write an answer sentence.	**Micah has ____ crayons now.**
Draw a bar model.	***Think*: Unequal sets are being put together.**
Draw a bar to show the first set.	
Draw a bar to show joining the second set.	
Label the diagram with what you know.	He finds 7 more. 7 / Micah has 11 crayons. 11
Show what you need to find.	7 / 11 / ? / How many crayons does Micah have now?
Use the completed bar model to help you solve the problem.	***Think*: I know how many are in each set. I need to find the whole.** **I can add the sets.** **11 + 7 = 18**
Complete the answer sentence.	**Micah has 18 crayons now.**
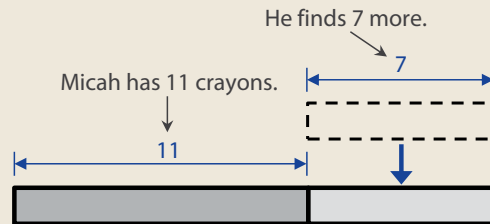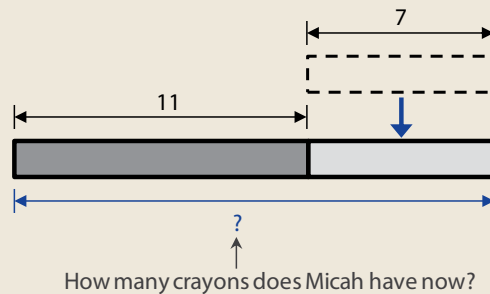
Part-Part-Whole

Example

Amy has 6 dolls.
She has 9 stuffed bears.
How many toys does Amy have in her collection?

Write an answer sentence.	**Amy has ____ toys in her collection.**
Draw a bar model.	**Think**: There is a whole group. It has two unequal parts.
Draw a bar to show Amy's toy collection.	
Mark the bar in two unequal parts.	
Label the diagram with what you know.	Amy has 6 dolls. She has 9 stuffed bears. 6 9
Show what you need to find.	6 9 ? How many toys does Amy have in her collection?
Use the completed bar model to help you solve the problem.	**Think**: I know how many are in each part. I need to find the whole. I can add the parts. 6 + 9 = 15
Complete the answer sentence.	**Amy has 15 toys in her collection.**
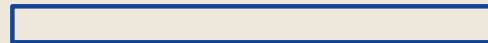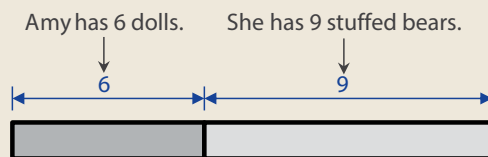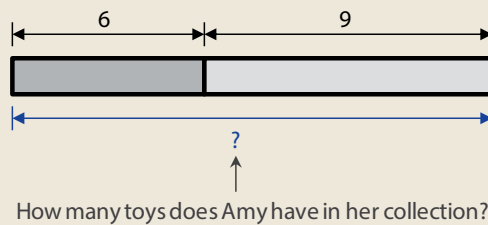
Comparisons with Addition

Example

Kiri has 138 beads in her collection.
Tiffany has 58 more beads in her collection.
How many beads does Tiffany have?

Write an answer sentence.	**Tiffany has ___ beads.**
Draw a bar model.	***Think*: There are two different sets of beads. They are being compared.**
Draw a bar to show Kiri's beads.	Kiri
Draw a bar to show Tiffany's beads. It will be longer because she has more beads.	Kiri Tiffany
Label the diagram with what you know.	Kiri has 138 beads. 138 Tiffany has 58 more beads. 58 Kiri Tiffany
Show what you need to find.	138 Kiri 58 Tiffany ? How many beads does Tiffany have?
Use the completed bar model to help you solve the problem.	***Think*: I know how many beads Kiri has. I know how many more Tiffany has.** **I can add to find out how many Tiffany has.** **138 + 58 = 196**
Complete the answer sentence.	**Tiffany has _196_ beads.**
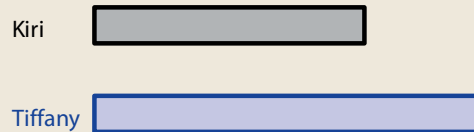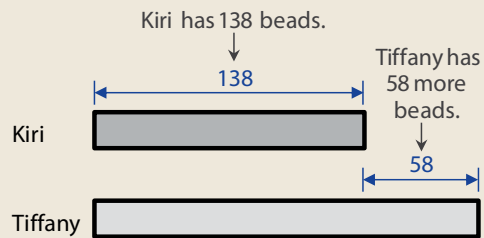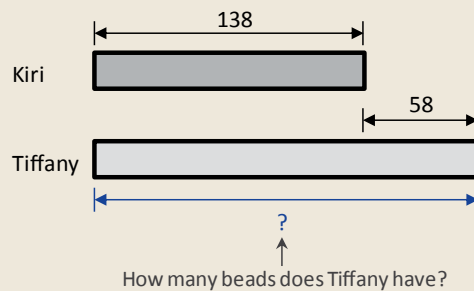
Subtraction

Taking Away Sets

Example

Bella finds 12 seashells on the beach.
She gives 7 seashells to her friends.
How many seashells does Bella have left?

Write an answer sentence.	**Bella has ___ seashells left.**
Draw a bar model.	*Think*: There is a whole group. A set is taken away.
Draw a bar to show the shells Bella had at the beginning.	
Draw a bar to show removing the shells she gave away.	
Label the diagram with what you know.	She gives 7 seashells to her friends. 7 — 12 — Bella finds 12 seashells on the beach.
Show what you need to find.	How many seashells does Bella have left? ? — 7 — 12
Use the completed bar model to help you solve the problem.	*Think*: I know the whole and the set taken away. I need to find what is left. I can subtract. 12 − 7 = 5
Complete the answer sentence.	Bella has 5 seashells left.
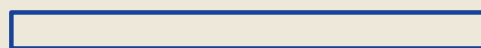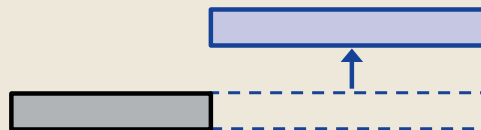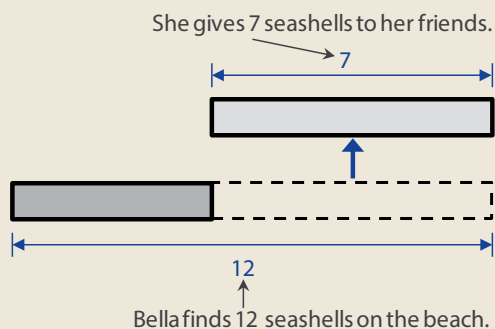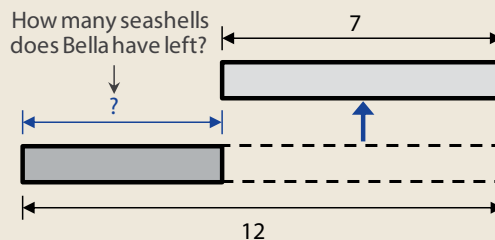
Part-Part-Whole

2 3

Example

Two pages of a stamp collection have 19 stamps.
There are 12 stamps on the first page.
How many stamps are on the other page?

Write an answer sentence.	**There are ___ stamps on the other page.**
Draw a bar model.	**Think: There is a whole group.** **It has two unequal parts.**
Draw a bar to show the whole collection of stamps.	
Mark the bar in two unequal parts.	
Label the diagram with what you know.	There are 12 stamps on the first page. 12 19 Two pages have 19 stamps.
Show what you need to find.	How many stamps are on the other page? 12 ? 19
Use the completed bar model to help you solve the problem.	**Think: I know the whole and one part.** **I need to find the other part.** **I can subtract the part from the whole.** **19 – 12 = 7**
Complete the answer sentence.	**There are _7_ stamps on the other page.**
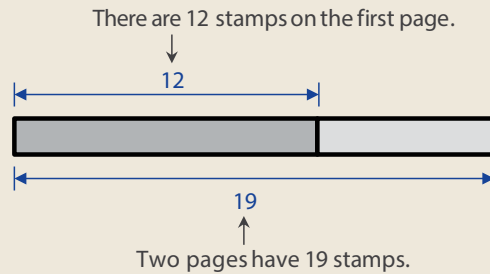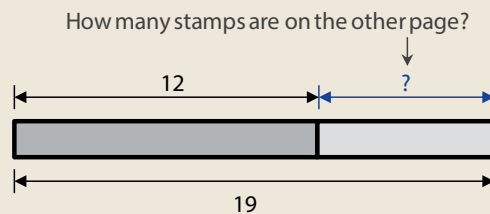
Comparisons with Subtraction

2 **3**

Example

David has 56 crayons.
Ryan has 14 fewer crayons.
How many crayons does Ryan have?

Write an answer sentence.	Ryan has ___ crayons.
Draw a bar model.	***Think*: There are two different sets of crayons. They are being compared.**
Draw a bar to show David's crayons.	David
Draw a bar to show Ryan's crayons. It will be shorter because he has fewer crayons.	David Ryan
Label the diagram with what you know.	David has 56 crayons. 56 David Ryan 14 Ryan has 14 fewer crayons.
Show what you need to find.	56 David Ryan ? 14 How many crayons does Ryan have?
Use the completed bar model to help you solve the problem.	***Think*: I know how many crayons David has. I know how many fewer crayons Ryan has. I can subtract to find out how many Ryan has.** **56 + 14 = 42**
Complete the answer sentence.	**Ryan has _42_ crayons.**
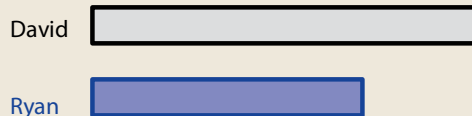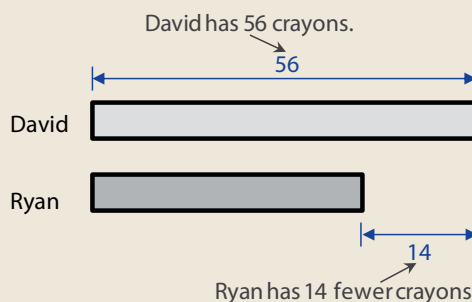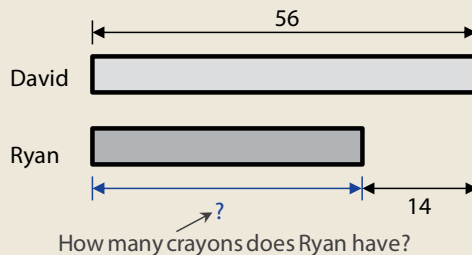
Multiplication

Equal Groups

Example

Mr. Ing needs new tires for 4 bikes.
Each bike has 2 tires.
How many tires does Mr. Ing need?

The unitary method is used to find the value of 1 unit in order to find the value of multiple units in a problem.

Write an answer sentence.	**Mr. Ing needs ___ tires.**
Draw a bar model.	***Think*: Equal groups are being put together.**
Draw a bar to show the first group (one bike).	
Add more bars to show all the groups (four bikes).	There are 4 bikes.
Label the diagram with what you know. Since each group is the same, you only need to label the first group.	Each bike has 2 tires. 2
Show what you need to find.	2 ? How many tires does Mr. Ing need?
Use the completed bar model and the unitary method to help you solve the problem.	***Think*: I know how many are in each equal group. I need to find the whole.** I can multiply. 1 unit → 2 tires 4 units → 4 × 2 = 8 tires
Complete the answer sentence.	**Mr. Ing needs 8 tires.**
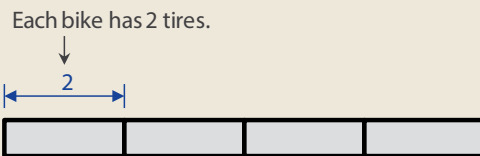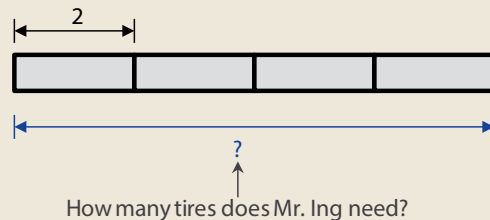
Comparisons with Multiplication

2 **3**

Example

Donna makes 6 greeting cards.
Her sister makes 3 times as many greeting cards.
How many greeting cards does her sister make?

Write an answer sentence.	Her sister makes ____ greeting cards.
Draw a bar model.	**Think**: There are two different sets of greeting cards. They are being compared.
Draw a bar to show Donna's cards.	Donna
	Donna
Draw a bar to show her sister's cards. It should be 3 times as long.	Donna's sister Her sister makes 3 times as many greeting cards.
Label the diagram with what you know.	Donna makes 6 greeting cards. 6 Donna Donna's sister
Show what you need to find.	6 Donna Donna's sister ? How many greeting cards does her sister make?
Use the completed bar model and the unitary method to help you solve the problem.	**Think**: I know how many cards Donna makes. I know that her sister makes 3 times as many. **I can multiply.** **1 unit → 6 cards** **3 units → 3 × 6 = 18 cards**
Complete the answer sentence.	Her sister makes <u>18</u> greeting cards.
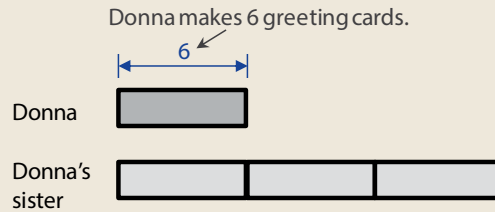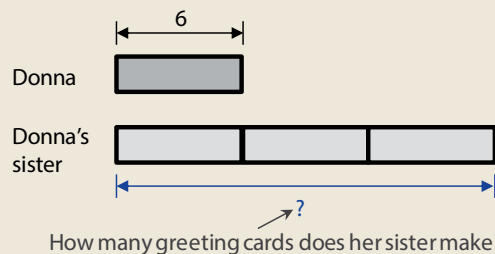
Division

How Many in Each Group?

Example

There are 20 books.
Brennan makes 5 equal piles of books.
How many books are in each pile?

Use this bar model when you know the number of equal groups and want to find the number of objects in each group.

Write an answer sentence.	**There are ___ books in each pile.**
Draw a bar model.	*Think*: **A whole is being divided into equal groups.**
Draw a bar to show the whole.	
Divide the bar into equal parts to show the groups.	There are 5 piles of books.
Label the diagram with what you know.	There are 20 books. 20
Show what you need to find.	20 ? How many books are in each pile?
Use the completed bar model and the unitary method to help you solve the problem.	*Think*: **I know how many are in the whole. I know the number of equal groups.** **I can divide.** **5 units → 20 books** **1 unit → 20 ÷ 5 = 4 books**
Complete the answer sentence.	**There are _4_ books in each pile.**
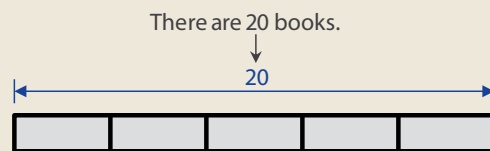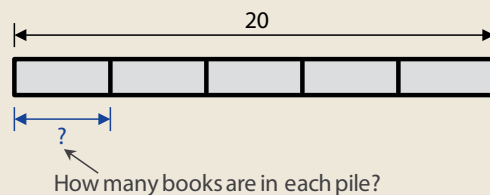
How Many Groups?

Example

An animal shelter has 12 puppies.
Each bag of food feeds 3 puppies.
How many bags of food does the shelter need?

Use this bar model when you know the number of objects in a group and want to find the number of equal groups.

Write an answer sentence.	The shelter needs ____ bags of food.
Draw a bar model.	***Think*: A whole is being divided into equal groups, but I don't know the number of groups.**
Draw a bar to show the first and last groups.	
Draw dotted lines to represent the groups in between.	
Label the diagram with what you know.	An animal shelter has 12 puppies. 12 puppies 3 ... 3 Each bag of food feeds 3 puppies.
Show what you need to find.	12 puppies 3 ... 3 ? bags How many bags of food does the shelter need?
Use the completed bar model to help you solve the problem.	***Think*: I know how many are in the whole. I know how many are in each equal group.** **I can divide.** $12 \div 3 = 4$
Complete the answer sentence.	The shelter needs _4_ bags of food.
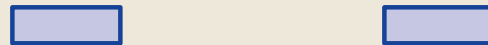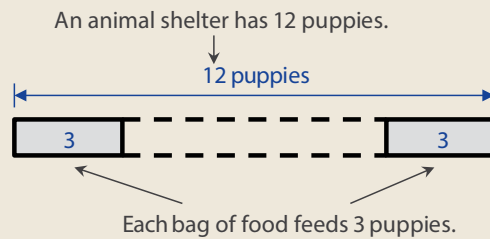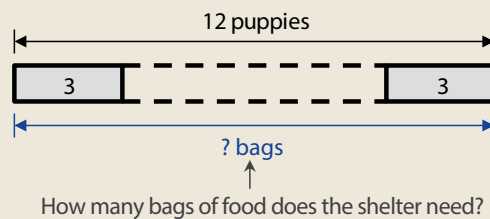
Comparisons with Division   2 3

Example

Jake ate 100 peas.
His little brother ate half as many peas.
How many peas did his little brother eat?

Write an answer sentence.	**His little brother ate ___ peas.**
Draw a bar model.	**Think: There are two different sets of peas. They are being compared.**
Draw a bar to show how many peas Jake ate.	Jake
Draw a bar to show how many peas his brother ate. It should be half as long.	Jake Jake's brother His little brother ate half as many peas.
Label the diagram with what you know.	Jake ate 100 peas. 100 Jake Jake's brother
Show what you need to find.	100 Jake Jake's brother ? How many peas did his little brother eat?
Use the completed bar model and the unitary method to help you solve the problem.	**Think: I know how many peas Jake ate. I know that his brother ate half as many.** **I can divide.** **2 units → 100 peas** **1 unit → 100 ÷ 2 = 50 peas**
Complete the answer sentence.	**His little brother ate 50 peas.**
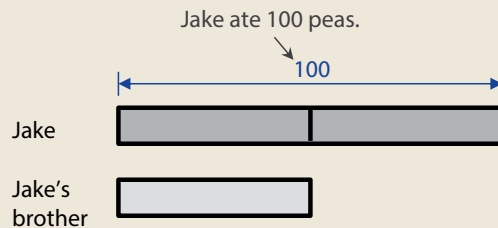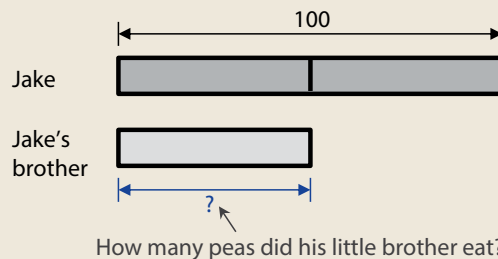
Multi-Step Problems

Example

Mr. Amani sorts 156 letters.
Ms. Jackson sorts 24 fewer letters.
How many letters do they sort in all?

Bar models may contain more than one question mark, even though you are looking for a single answer.

Write an answer sentence.	**They sort _____ letters in all.**
Draw a bar model.	**Think**: There are two groups of letters. They are being compared.
Draw bars to show Mr. Amani's letters and Ms. Jackson's letters. Ms. Jackson sorts fewer letters, so her bar should be shorter.	Mr. Amani [bar] Ms. Jackson [bar]
Label the diagram with what you know, and show what you need to find. Use a vertical brace to show the total.	156 Mr. Amani [bar] Ms. Jackson [bar] ? ? 24
Use the bar model to solve the first step of the problem.	**Think**: Before I can find how many letters they sort in all, I need to find how many letters Ms. Jackson sorts. **I can subtract.** **156 – 24 = 132** **Ms. Jackson sorts 132 letters.**
Use the completed bar model to help you solve the problem.	**Think**: I know how many letters are sorted by Mr. Amani and by Ms. Jackson. I need to find the total number of letters sorted. **I can add.** **156 + 132 = 288**
Complete the answer sentence.	**They sort 288 letters in all.**
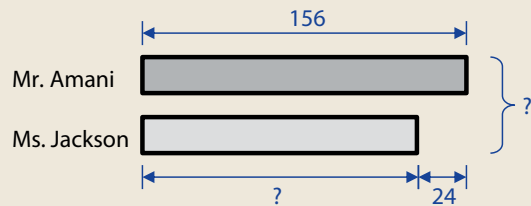
Addition and Subtraction, With Whole Known   3   4

Example

There are 1,347 soccer fans at the game.
682 of the fans are adults.
How many fewer children than adults are at the game?

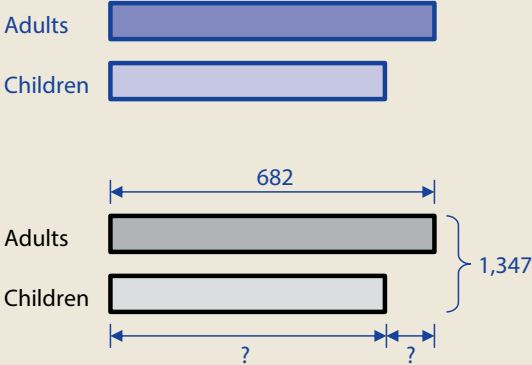|	
Write an answer sentence.	There are ____ fewer children than adults at the game.
Draw a bar model.	**Think:** There are two groups of soccer fans. They are being compared.
Draw bars to show the adults and children at the game. There are fewer children at the game, so that bar should be shorter.	Adults Children
Label the diagram with what you know, and show what you need to find. Use a vertical brace to show the total.	682 Adults Children 1,347 ? ?
Use the bar model to solve the first step of the problem.	**Think:** Before I can find how many fewer children than adults are at the game, I need to find how many children are at the game. I can subtract. **1,347 − 682 = 665** There are 665 children at the game.
Use the completed bar model to help you solve the problem.	**Think:** I know how many adults and how many children are at the game. I need to find how many fewer children than adults. I can subtract. **682 − 665 = 17**
Complete the answer sentence.	There are <u>17</u> fewer children than adults at the game.

Building on a Bar with Addition and Subtraction   3 4

Example

A pizza chef has 571 pounds of mozzarella cheese and 458 pounds of provolone cheese.
He bought 259 more pounds of mozzarella.
How much more mozzarella does he have than provolone?

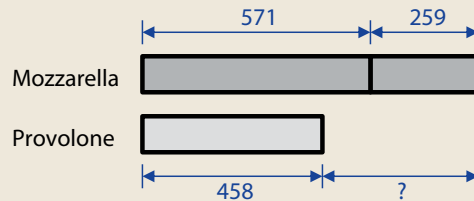|	
Write an answer sentence.	He has ___ more pounds of mozzarella than provolone.
Draw a bar model.	***Think***: There are two types of cheese. They are being compared.
Draw bars to show the different types of cheese. Join two sets to show the amounts of mozzarella.	Mozzarella Provolone
Label the diagram with what you know, and show what you need to find.	571 259 Mozzarella Provolone 458 ?
Use the bar model to solve the first step of the problem.	***Think***: Before I can find how much more mozzarella the chef has, I need to find the total amount of mozzarella. I can add. 571 + 259 = 830 The chef has 830 pounds of mozzarella.
Use the completed bar model to help you solve the problem.	***Think***: I know how much mozzarella and how much provolone the chef has. I need to find how much more mozzarella than provolone. I can subtract. 830 – 458 = 372
Complete the answer sentence.	He has 372 more pounds of mozzarella than provolone.

Multiplication

3 4 5

Example

Miko sold 8 cups of lemonade.
Maya sold 3 times as many.
How many cups of lemonade did they sell in all?

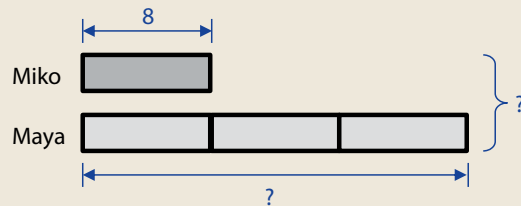|	
Write an answer sentence.	**They sold ____ cups of lemonade in all.**
Draw a bar model.	***Think*: There are two groups of cups of lemonade. They are being compared.**
Draw bars to show Miko's cups and Maya's cups. Maya sold 3 times as many cups, so her bar should be 3 times as long as Miko's.	
Label the diagram with what you know, and show what you need to find. Use a vertical brace to show the total.	
Use the bar model to solve the first step of the problem.	***Think*: Before I can find how many cups of lemonade they sold in all, I need to find how many cups Maya sold.** **I can multiply using the unitary method.** 1 unit → 8 cups 3 units → 3 × 8 = 24 cups **Maya sold 24 cups of lemonade.**
Use the completed bar model to help you solve the problem.	***Think*: I know how many cups of lemonade were sold by Miko and by Maya. I need to find the total number of cups sold.** **I can add.** 8 + 24 = 32
Complete the answer sentence.	**They sold _32_ cups of lemonade in all.**

Division

3 4 5

Example

April and Dante made a total of 68 bracelets.
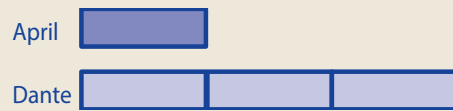Dante made 3 times as many as April.
How many bracelets did Dante make?

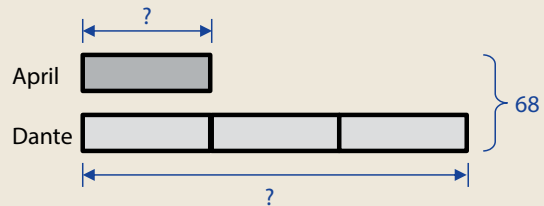|	
Write an answer sentence.	**Dante made ___ bracelets.**
Draw a bar model.	***Think*: There are two groups of bracelets. They are being compared.**
Draw bars to show April's bracelets and Dante's bracelets. Dante made 3 times as many bracelets, so his bar should be 3 times as long as April's.	April [bar] Dante [bar]
Label the diagram with what you know, and show what you need to find. Use a vertical brace to show the total.	April [bar with ?] Dante [bar with ?] } 68
Use the bar model to solve the first step of the problem.	***Think*: Before I can find how many bracelets Dante made, I need to find how many bracelets April made.** **There are 4 equal groups in the bar model, so I can divide the total by 4 using the unitary method.** **4 units → 68 bracelets** **1 unit → 68 ÷ 4 = 17 bracelets** **April made 17 bracelets.**
Use the completed bar model and the unitary method to help you solve the problem.	***Think*: I know how many bracelets April made. I know that Dante made 3 times as many.** **I can multiply.** **1 unit → 17 bracelets** **3 units → 3 × 17 = 51 bracelets**
Complete the answer sentence.	**Dante made _51_ bracelets.**

Three-Step Comparisons

3   4   5

Example

Caleb has 3 times as much money as Dana.
Amber has $18 less than Caleb.
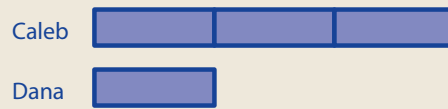Dana has $63.
How much money does Amber have?

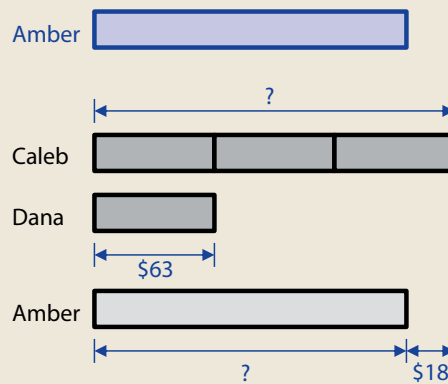|	
Write an answer sentence.	**Amber has $____.**
Draw a bar model.	***Think*: There are three groups of money. They are being compared.**
Draw bars to show the money amounts for Caleb, Dana, and Amber. Caleb has 3 times as much money as Dana, so his bar should be 3 times as long as Dana's. Amber has less money than Caleb, so her bar should be shorter than Caleb's.	Caleb Dana Amber
Label the diagram with what you know, and show what you need to find.	? Caleb Dana $63 Amber ? $18
Use the bar model to solve the first step of the problem.	***Think*: Before I can find much money Amber has, I need to find how much money Caleb has.** **There are 3 equal groups of $63, so I can use the unitary method and multiply.** **1 unit → $63** **3 units → 3 × $63 = $189** **Caleb has $189.**
Use the completed bar model to help you solve the problem.	***Think*: I know how much money Caleb has. I know that Amber has $18 less than Caleb.** **I can subtract.** **189 – 18 = 171**
Complete the answer sentence.	**Amber has $ 171 .**

Using Multiple Bar Models

3 4 5

Example 1

Kailynn has $97.
She wants to buy 5 CDs that each cost $16.
How much money will Kailynn have left?

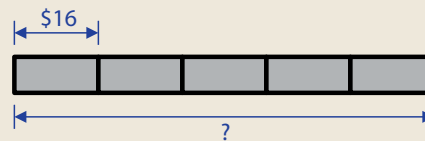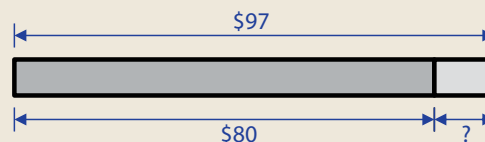|	
Write an answer sentence.	**Kailynn will have $___ left.**
Draw and use a bar model to find the total cost of the CDs.	**Think:** First, I need to find the total cost of the CDs. Then I can find out how much money Kailynn will have left.
Draw a bar to show the cost of the first CD. Add more bars to represent all of the CDs.	
Label the diagram with what you know and what you need to find.	
Use the first completed bar model to solve the first step of the problem.	**Think:** I know the cost of each CD. I need to find the cost of all of the CDs. I can use the unitary method and multiply. 1 unit → $16 5 units → 5 × $16 = $80 The CDs cost a total of $80.
Draw and use a bar model to find how much money Kailynn will have left.	
Draw a bar to show how much money Kailynn has. Mark the bar in two unequal parts. One part shows the cost of the CDs, and the other part shows the amount she has left over.	
Label the diagram with what you know and what you need to find.	
Use the completed bar model to help you solve the problem.	**Think:** I know how much money Kailynn has and the cost of the CDs. I need to find how much money she has left over. I can subtract. 97 − 80 = 17
Complete the answer sentence.	**Kailynn will have $17 left.**

Using Multiple Bar Models   **3 4 5**

Example 2

Eva is making lunch for 6 people.
Each person will get 4 carrots and 3 potatoes.
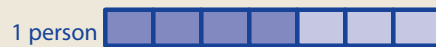How many vegetables does Eva need in all?

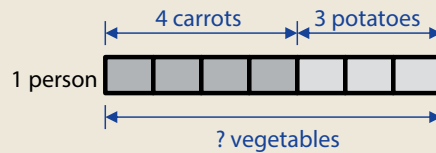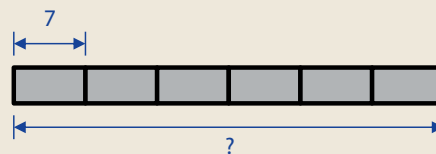|	
Write an answer sentence.	**Eva needs ___ vegetables in all.**
Draw and use a bar model to find how many vegetables each person will get.	***Think*: First, I need to find the number of vegetables each person will get. Then I can find out how many vegetables Eva needs for 6 people.**
Draw a bar to represent each person's carrots. Join it with a bar that shows each person's potatoes. **Label the diagram with what you know and what you need to find.**	
Use the first completed bar model to solve the first step of the problem.	***Think*: I know how many carrots and how many potatoes each person gets. I need to find the total number of vegetables. I can add.** **$4 + 3 = 7$** **Each person gets 7 vegetables.**
Draw and use a bar model to find how many vegetables Eva needs for 6 people.	
Draw a bar to represent each person's vegetables. Add more bars to show all 6 people. **Label the diagram with what you know and what you need to find.**	
Use the completed bar model and the unitary method to help you solve the problem.	***Think*: I know how many vegetables each person gets. I need to find the total number of vegetables. I can use the unitary method and multiply.** **1 unit $\rightarrow$ 7 vegetables** **6 units $\rightarrow 6 \times 7 = 42$ vegetables**
Complete the answer sentence.	**Eva needs <u>42</u> vegetables in all.**

Using Multiple Bar Models

Example 3

Pam bought 4 boxes of candy.
Each box contains 24 pieces of candy.
She separates the candy into bags of 6 pieces each. How many bags does she use?

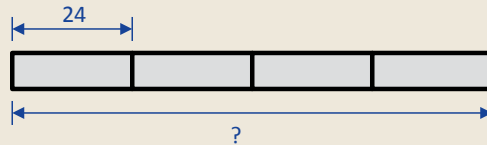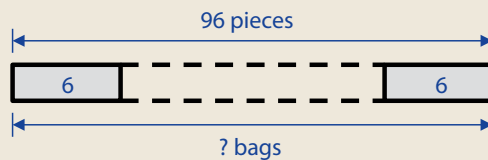|	
Write an answer sentence.	Pam uses ___ bags.
Draw and use a bar model to find how many total pieces of candy.	**Think**: First, I need to find the total number of candy pieces. Then I can find how many bags she uses.
Draw a bar to show the equal groups.	
Label the diagram with what you know and what you need to find.	
Use the first completed bar model to solve the first step of the problem.	**Think**: I know how many boxes and how many pieces of candy in each box. I need to find the total number of pieces. I can use the unitary method and multiply. 1 unit → 24 pieces 4 units → 4 × 24 = 96 pieces **There are 96 total pieces of candy.**
Draw and use a bar model to find how many bags Pam uses.	
Draw a bar to show the first and last groups. Use dotted lines to represent the unknown number of groups in between.	
Label the diagram with what you know and what you need to find.	
Use the completed bar model to help you solve the problem.	**Think**: I know how many total pieces of candy. I know how many pieces are in each bag. I can divide. 96 ÷ 6 = 16
Complete the answer sentence.	Pam uses **16** bags.

Working Backward

Example

Together, Josh and Aiden have 150 baseball cards.
Aiden gave 12 of his cards to Josh, and now Josh has twice as many cards as Aiden.
How many baseball cards did they each have at the beginning?

| **Write an answer sentence.** | At the beginning, Josh had ___ baseball cards and Aiden had ___ baseball cards. |

Think: There are two sets of cards being compared. I know how many the boys have altogether, and I know that Josh now has twice as many cards as Aiden. First, I need to find how many cards Aiden and Josh have now. Then I can find out how many cards they had at the beginning.

Draw and use a bar model to find how many cards Josh and Aiden have now.

Think: I know that Josh and Aiden together have 150 baseball cards. I also know that Josh now has twice as many cards as Aiden.
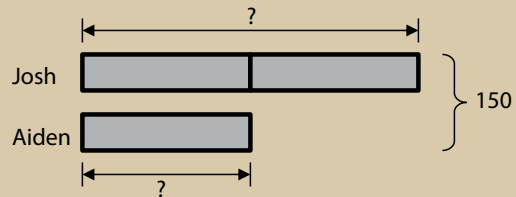
There are 3 equal groups, so I can divide the total by 3.

$150 \div 3 = 50$

Aiden has 50 cards now.

Josh has twice as many cards, or $2 \times 50 = 100$ cards now.

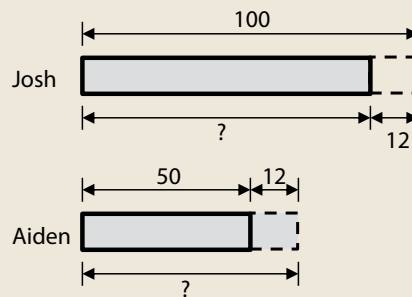**Draw and use a bar model to find how many cards Josh and Aiden had at the beginning.**

Think: Before Aiden gave cards to Josh, Josh had fewer cards and Aiden had more cards.

I can subtract to find how many cards Josh had, and add to find how many cards Aiden had.

Josh: $100 - 12 = 88$

Aiden: $50 + 12 = 62$

| **Complete the answer sentence.** | At the beginning, Josh had _88_ baseball cards and Aiden had _62_ baseball cards. |

Fractions

Adding Like Fractions

Example

Billy ate $\frac{2}{4}$ of a giant cookie.

Maurice ate $\frac{1}{4}$ of it.

What fraction of the giant cookie did they eat in all?

Write an answer sentence.	**They ate ____ of the giant cookie.**
Draw a bar model.	***Think*: The denominators are the same, so these are like fractions.**
Draw a bar to show the entire cookie.	
Divide the bar into 4 equal parts. Each part represents a fourth.	
Label the diagram with what you know.	$\frac{2}{4}$ $\frac{1}{4}$
Show what you need to find.	$\frac{2}{4}$ $\frac{1}{4}$?
Use the completed bar model to help you solve the problem.	***Think*: I know the fractional amount each person ate. I need to find out how much they ate altogether.** **I can add the fractions.** $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$
Complete the answer sentence.	**They ate $\frac{3}{4}$ of the giant cookie.**

Subtracting Like Fractions

Example

Jewel ran $\frac{1}{5}$ of the track before she stops to stretch.
Then she ran more of the track.
She ran $\frac{4}{5}$ of the track in all.
What fraction of the track did Jewel run after her stretch?

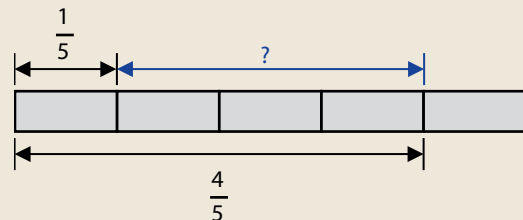|	
Write an answer sentence.	Jewel ran _____ of the track after her stretch.
Draw a bar model.	***Think*: The denominators are the same, so these are like fractions.**
Draw a bar to show the entire track. Divide the bar into 5 equal parts. Each part represents a fifth.	
Label the diagram with what you know.	
Show what you need to find.	
Use the completed bar model to help you solve the problem.	***Think*: I know how much she ran before the stretch and how much she ran in all. I need to find out how much she ran after her stretch.** **I can subtract the two fractions.** $\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$
Complete the answer sentence.	Jewel ran $\frac{3}{5}$ of the track after her stretch.
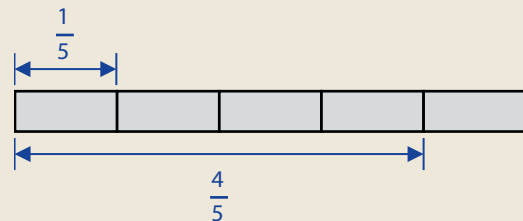
Finding Equivalent Fractions

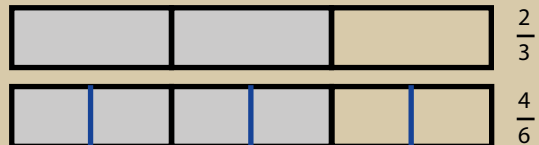**Example**

Find an equivalent fraction for $\frac{2}{3}$.

Draw a bar model.	***Think***: When comparing fractions, they must come from equal sized wholes. So, all of my bar models must be the same size.
Draw two bars that show $\frac{2}{3}$.	
Equally divide each section of the second bar by the same amount.	$\frac{2}{3}$ $\frac{4}{6}$
You can repeat the step to find additional equivalent fractions.	$\frac{8}{12}$
Use the completed bar model to help you solve the problem.	***Think***: Because the bar models and the shaded portions are all the same size, I know these fractions are equivalent. $\frac{2}{3} = \frac{4}{6} = \frac{8}{12}$
Answer the problem.	$\frac{4}{6}$ is equivalent to $\frac{2}{3}$ or $\frac{8}{12}$ is equivalent to $\frac{2}{3}$.

Comparing Like Fractions   3   4

Example

Chuck and Jill have to read the same book for class.
Chuck reads $\frac{4}{5}$ of the book.
Jill reads $\frac{2}{5}$ of the book.
Who reads more of the book?

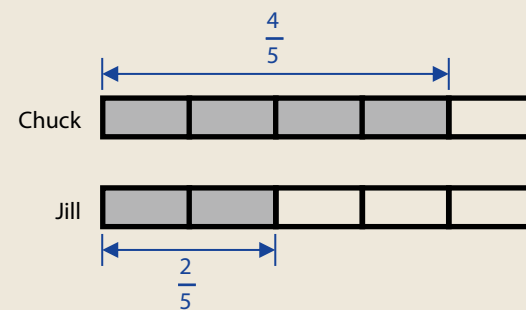|	
Write an answer sentence.	_____ **reads more of the book.**
Draw a bar model.	**Think**: Like fractions have the same denominator, so I only have to compare the numerators.
Draw a bar to show the amount of the book Chuck reads.	Chuck
Draw a second bar to show the amount of the book Jill reads.	Chuck Jill
Label the diagram with what you know.	$\frac{4}{5}$ Chuck Jill $\frac{2}{5}$
Use the completed bar model to help you solve the problem.	**Think**: Because the bars and sections are the same size, I can see that Chuck reads more of the book than Jill. $\frac{4}{5}$ is greater than $\frac{2}{5}$.
Complete the answer sentence.	**Chuck reads more of the book.**

Ordering Like Fractions

Example

Put $\frac{2}{5}$, $\frac{4}{5}$, and $\frac{1}{5}$ in order from least to greatest.

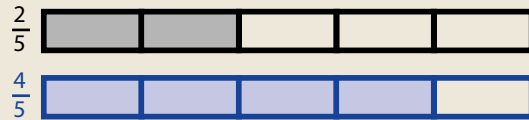|	
Write an answer sentence.	Fractions ___, ___, and ___ are ordered from least to greatest.
Draw a bar model.	***Think***: When ordering like fractions, I only need to compare the numerators.
Draw a bar to represent the first fraction, $\frac{2}{5}$.	$\frac{2}{5}$
Draw a bar to represent the second fraction, $\frac{4}{5}$.	$\frac{2}{5}$ $\frac{4}{5}$
Draw a bar to represent the third fraction, $\frac{1}{5}$.	$\frac{2}{5}$ $\frac{4}{5}$ $\frac{1}{5}$
Use the completed bar model to help you solve the problem.	***Think***: The more shaded parts of a bar, the greater the value of the fraction. $\frac{4}{5}$ has the most shaded parts and $\frac{1}{5}$ has the least.
Complete the answer sentence.	Fractions $\frac{1}{5}$, $\frac{2}{5}$, and $\frac{4}{5}$ are ordered from least to greatest.

Comparing Unlike Fractions

Example

Which is greater, $\frac{3}{4}$ or $\frac{2}{6}$?

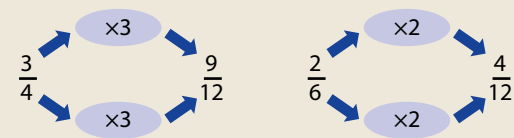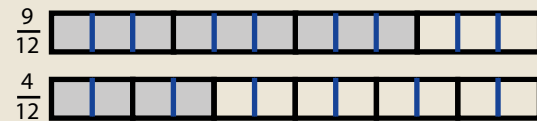|	
Write an answer sentence.	___ is greater than ___.
Draw a bar model.	**Think:** These two fractions have different denominators, so to compare them I must find a common denominator.
Draw bars for each fraction.	$\frac{3}{4}$ $\frac{2}{6}$
Find a common denominator for both fractions.	
Draw additional lines on your bars to show the new fractions.	$\frac{9}{12}$ $\frac{4}{12}$
Use the completed bar model to help you solve the problem.	**Think:** Now that I have made like fractions, I can look for the greatest numerator to determine the greatest fraction. $\frac{9}{12} > \frac{4}{12}$
Complete the answer sentence.	$\frac{3}{4}$ is greater than $\frac{2}{6}$.
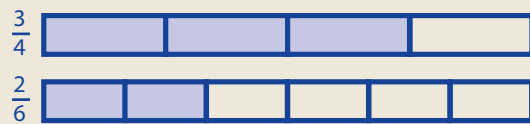
Finding the Fractional Part of a Set   3 4

Example

There are 16 dolls in the collection.
$\frac{3}{4}$ of them are still in the boxes.
How many dolls are still in boxes?

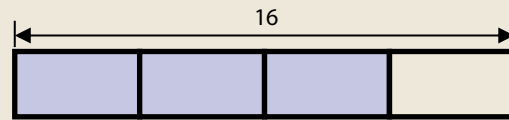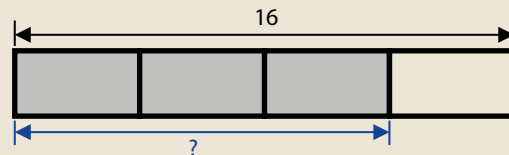|	
Write an answer sentence.	**There are ___ dolls still in boxes.**
Draw a bar model.	***Think*: I will need to divide my bar by 4 because the denominator is 4.**
Draw and label a bar to show the entire collection.	16
Shade $\frac{3}{4}$ of the bar model.	16
Label the diagram to show what you are looking for.	16 ... ?
Use the completed bar model and the unitary method to help you solve the problem.	***Think*: I need to find out how much each segment of the bar is worth. Then I need to find out how much 3 segments are worth.** 4 units → 16 dolls 1 unit → 4 dolls 3 units → 12 dolls So, $\frac{3}{4}$ of 16 is 12.
Complete the answer sentence.	**There are 12 dolls still in boxes.**
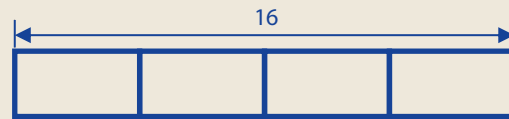
Adding Unlike Fractions

Example

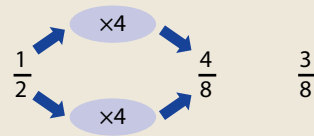What is the sum of $\frac{1}{2}$ and $\frac{3}{8}$?

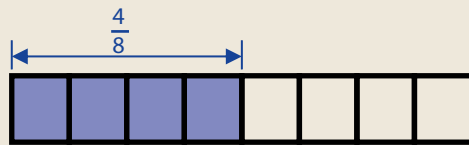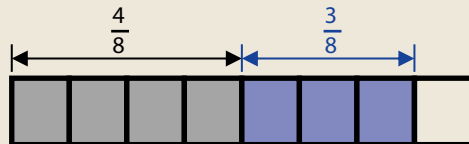|	
Write an answer sentence.	The sum of $\frac{1}{2}$ and $\frac{3}{8}$ is ___.
Draw a bar model.	***Think***: For me to add these two fractions, they must have the same denominator.
First find a common denominator.	$\frac{1}{2}$ ×4 $\frac{4}{8}$ ×4 $\frac{3}{8}$
Draw a bar divided into eighths.	
Label and shade $\frac{4}{8}$ on the bar.	$\frac{4}{8}$
Label and shade $\frac{3}{8}$ on the bar.	$\frac{4}{8}$ $\frac{3}{8}$
Use the completed bar model to help you solve the problem.	***Think***: Now that the fractions have the same denominator, I can add the numerators. $$\frac{4}{8} + \frac{3}{8} = \frac{7}{8}$$
Complete the answer sentence.	The sum of $\frac{1}{2}$ and $\frac{3}{8}$ is $\frac{7}{8}$.

Adding Mixed Numbers

5

Example

Daylynn practices soccer for $2\frac{1}{2}$ hours a week.
She practices tennis for $1\frac{2}{3}$ hours a week.
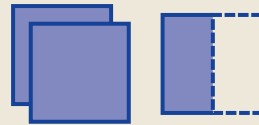How many hours a week does she practice sports?

Write an answer sentence.
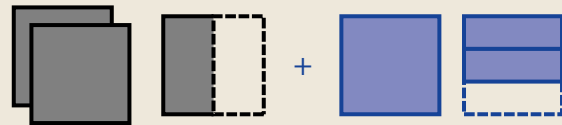
Daylynn practices sports for _____ hours a week.

Draw a bar model.

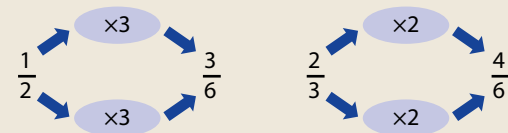*Think*: For me to add these mixed numbers, the fractions must have the same denominator.
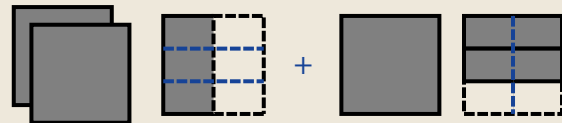
Draw a model of the first mixed number, $2\frac{1}{2}$.

Draw a model of the second mixed number, $1\frac{2}{3}$.

Find a common denominator for the fractions.

$\frac{1}{2}$ ×3 ×3 $\frac{3}{6}$ $\frac{2}{3}$ ×2 ×2 $\frac{4}{6}$

Add additional lines to the fraction portions of your bars to display the new fractions.

Think: Now that the fractions have the same denominator, I can add the mixed numbers.

Use the completed bar model to help you solve the problem.

$2\frac{1}{2} + 1\frac{2}{3} = 2\frac{3}{6} + 1\frac{4}{6}$

$= 3\frac{7}{6}$

$= 4\frac{1}{6}$

Complete the answer sentence.

Daylynn practices sports for $4\frac{1}{6}$ hours a week.

Subtracting Mixed Numbers

5

Example

Mikelle bought $3\frac{2}{5}$ pounds of grapes.
She used $2\frac{1}{3}$ pounds in a fruit salad.
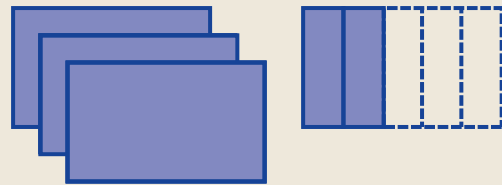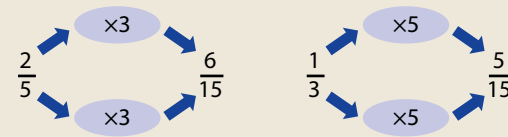How many pounds of grapes does she have left?

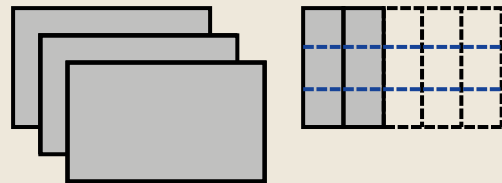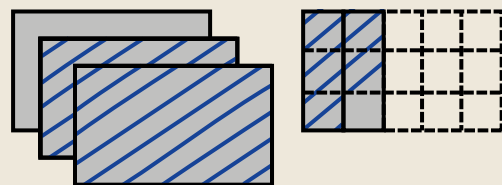|	
Write an answer sentence.	**Mikelle has _____ pounds of grapes left.**
Draw a bar model.	**Think: For me to subtract these mixed numbers, the fractions must have the same denominator.**
Draw a model of the first mixed number, $3\frac{2}{5}$.	
Find a common denominator for the fractions.	$\frac{2}{5} \xrightarrow{\times 3} \frac{6}{15}$ $\frac{1}{3} \xrightarrow{\times 5} \frac{5}{15}$
Add additional lines to the fraction portions of your bars to display the new fractions.	
Cross out the value of the second fraction using the common denominator, $2\frac{5}{15}$.	
Use the completed bar model to help you solve the problem.	**Think: I can count the remaining portions of the bars to find the difference.** $3\frac{2}{5} - 2\frac{1}{3}$ $3\frac{6}{15} - 2\frac{5}{15} = 1\frac{1}{15}$
Complete the answer sentence.	**Mikelle has $1\frac{1}{15}$ pounds of grapes left.**

Multiplying Proper Fractions

Example 1

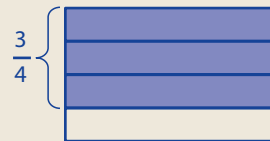What is the product of $\frac{3}{4}$ and $\frac{1}{3}$?

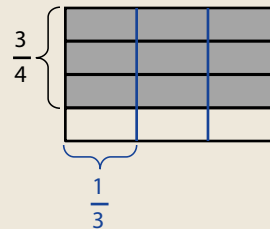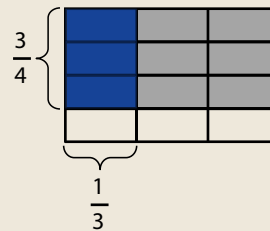| Write an answer sentence.	The product of $\frac{3}{4}$ and $\frac{1}{3}$ is ___.
Draw a bar model.	***Think*: I need to multiply the fractions.**
Label and shade a bar to show $\frac{3}{4}$.	
Label $\frac{1}{3}$ on the same bar.	
Shade the portion of $\frac{1}{3}$ that overlaps the shaded parts of $\frac{3}{4}$.	
Use the completed bar model to help you solve the problem.	***Think*: To find the product I can count the overlapping parts. This amount will be the numerator, and the amount of parts total is the denominator. Then I will need to simplify the fraction.** $\frac{3}{12} = \frac{1}{4}$
Complete the answer sentence.	The product of $\frac{3}{4}$ and $\frac{1}{3}$ is $\frac{1}{4}$.

Multiplying Proper Fractions (using the Unitary Method)   5   6

Example 2

Javier had $\frac{3}{4}$ quart of rice.
He used $\frac{1}{3}$ of it to make dinner.
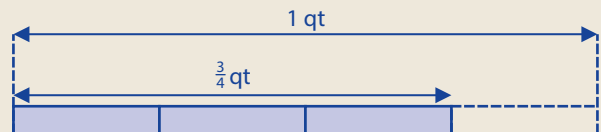How much rice did he use to make dinner? How much is left?

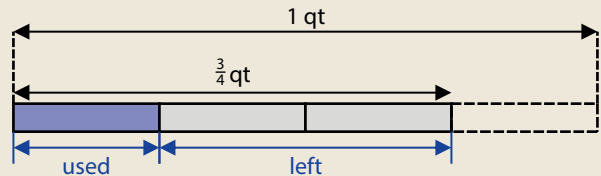|	
Write an answer sentence.	Javier used _____ quart of rice to make dinner. He has _____ quart left.
Draw a bar model.	**Think:** To determine the amount of rice that was used, I need to multiply the fractions.
Draw and label a bar to show the amount of rice Javier began with, $\frac{3}{4}$ quart.	
Draw and label the amounts of the rice that Javier used and has left.	
Use the unitary method to determine the amounts used and left.	4 units → 1 qt 1 unit → $\frac{1}{4}$ qt 2 units → $\frac{1}{2}$ qt
Use the completed bar model to help you solve the problem.	**Think:** I know that Javier used one unit of rice, which equals $\frac{1}{4}$ quart. I also know he has 2 units left, which equals $\frac{2}{4}$, or $\frac{1}{2}$.
Complete the answer sentence.	Javier used $\frac{1}{4}$ quart of rice to make dinner. He has $\frac{1}{2}$ quart left.

Multiplying Improper Fractions by Proper Fractions

Example
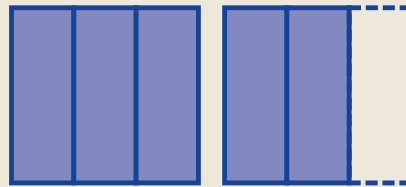
$$\frac{5}{3} \times \frac{3}{4} = \underline{\hspace{2em}}$$

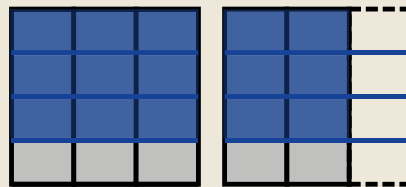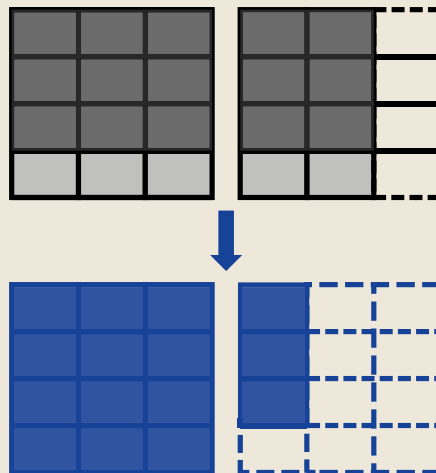|	
Write an answer sentence.	$\frac{5}{3} \times \frac{3}{4} = \underline{\hspace{2em}}$
Draw a bar model.	*Think*: Because the larger fraction is an improper fraction the model will have more than one bar.
Draw a bar to show the improper fraction.	
Draw and shade the overlap of $\frac{3}{4}$ on the same bars.	
Create a new bar model that displays the final fraction.	
Use the completed bar model to help you solve the problem.	*Think*: To find the product, I can count the overlapping parts. This amount will be the numerator, and the amount of parts total is the denominator. Then I will need to simplify the fraction. $$\frac{15}{12} = 1\frac{3}{12}$$ $$= 1\frac{1}{4}$$
Complete the answer sentence.	$\frac{5}{3} \times \frac{3}{4} = 1\frac{1}{4}$

Multiplying Mixed Numbers by Whole Numbers

Example

A team of 5 children wins a tug-o-war competition.
The grand prize is a huge jar of jellybeans. Each teammate wins $1\frac{1}{2}$ pounds of the jellybeans.
How many pounds of jellybeans are in the jar?

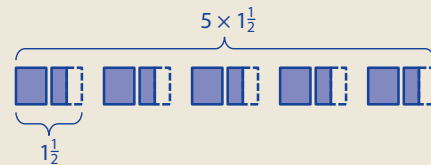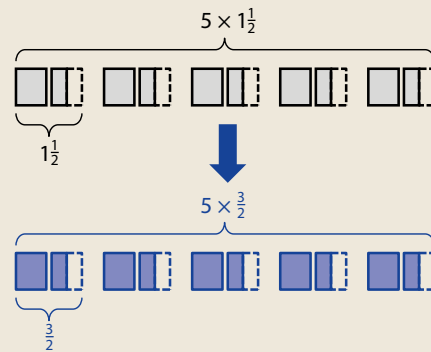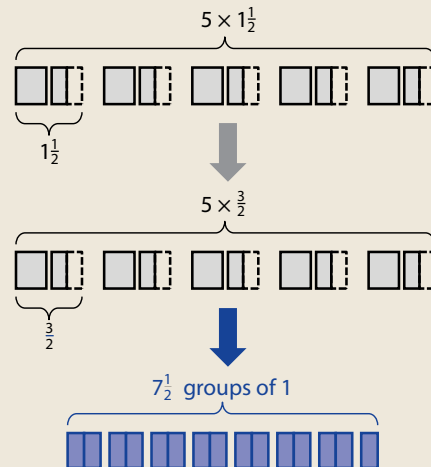|	
Write an answer sentence.	**There are ___ pounds of jellybeans in the jar.**
Draw a bar model.	***Think*: I need to multiply the winnings by the number of teammates to find out the original amount of jellybeans.**
Write the multiplication problem that needs to be solved.	$5 \times 1\frac{1}{2} = $ _____
Draw a bar model to represent the multiplication problem.	$5 \times 1\frac{1}{2}$ $1\frac{1}{2}$
Replicate the bar model, but change the mixed number to an improper fraction.	$5 \times 1\frac{1}{2}$ $1\frac{1}{2}$ $5 \times \frac{3}{2}$ $\frac{3}{2}$
Create new groups of 1s from the $\frac{1}{2}$s.	$5 \times 1\frac{1}{2}$ $1\frac{1}{2}$ $5 \times \frac{3}{2}$ $\frac{3}{2}$ $7\frac{1}{2}$ groups of 1
Use the completed bar model to help you solve the problem.	***Think*: I can count the 1s that are created to find the product.** $5 \times 1\frac{1}{2} = 7\frac{1}{2}$
Complete the answer sentence.	**There are $7\frac{1}{2}$ pounds of jellybeans in the jar.**

Dividing Whole Numbers by Proper Fractions

5   6

Example

A gardener uses 15 gallons of water on his roses each day.
He sprays $\frac{3}{10}$ of a gallon on each rose bush in the nursery.
How many rose bushes does he water each day?

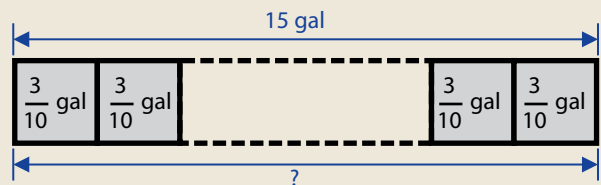|	
Write an answer sentence.	The gardener waters _____ rose bushes each day.
Draw a bar model.	***Think:*** When dividing fractions, I can multiply by the reciprocal.
Draw a bar to represent the amount of water used on each rose bush.	
Label the bar to show the total amount of water used. Then show what you need to find.	
Use the completed bar model to help you solve the problem.	***Think:*** $15 \div \frac{3}{10} = 15 \times \frac{10}{3}$ $= 50$
Complete the answer sentence.	The gardener waters _50_ rose bushes each day.

Dividing Fractions by Fractions

Example

A piece of fabric is $\frac{3}{4}$ yard long.
A seamstress cuts it into pieces that are $\frac{1}{8}$ yard long.
Into how many pieces does she cut the fabric?

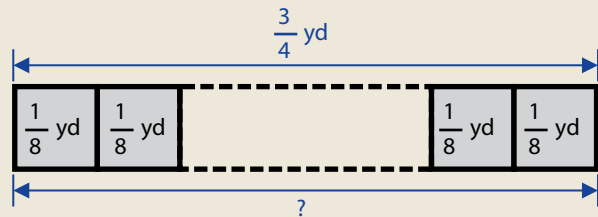|	
Write an answer sentence.	**The seamstress cuts ___ pieces of fabric.**
Draw a bar model.	***Think*: When dividing fractions, I can multiply by the reciprocal.**
Draw a bar to represent the amount of pieces the seamstress cuts.	
Label the bar to show the total amount of fabric. Then show what you need to find.	
Use the completed bar model to help you solve the problem.	***Think*:** $\frac{3}{4} \div \frac{1}{8} = \frac{3}{4} \times 8$ $= 6$
Complete the answer sentence.	**The seamstress cuts 6 pieces of fabric.**

Dividing Fractions by Whole Numbers

Example

Li cuts a loaf of bread into thirds.
Then he divides $\frac{2}{3}$ of the loaf into 4 equal parts.
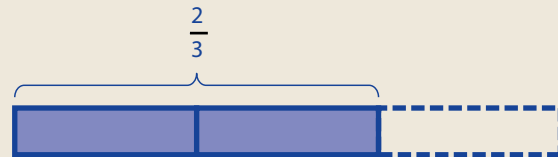What fraction of the loaf is each of the 4 parts?

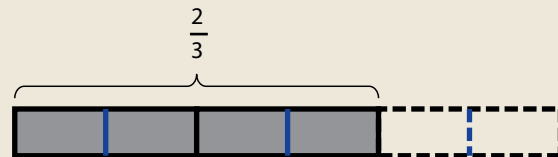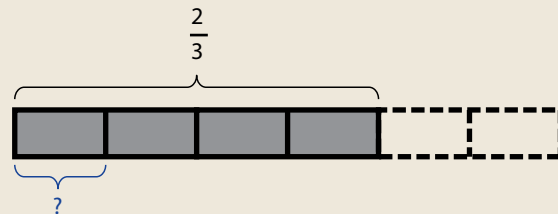|	
Write an answer sentence.	**Each part is ___ of the whole loaf of bread.**
Draw a bar model.	***Think***: Dividing a fraction means dividing each fractional part into smaller units.
Draw a bar to represent the loaf. Shade and label $\frac{2}{3}$ of the bar.	$\frac{2}{3}$
Divide the bar so that the shaded portion is cut into 4 equal parts.	$\frac{2}{3}$
Show what you need to find.	$\frac{2}{3}$?
Use the completed bar model to help you solve the problem.	***Think***: I can count the segments to find the denominator. Since I am looking for the fractional amount for one slice, or part, of the loaf, the numerator will be 1. $\frac{1}{6}$
Complete the answer sentence.	**Each part is $\frac{1}{6}$ of the whole loaf of bread.**

Rates

Example 1

A copier printed 176 pages in 8 minutes.
How many pages can the copier print per minute?

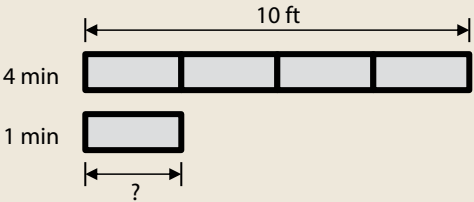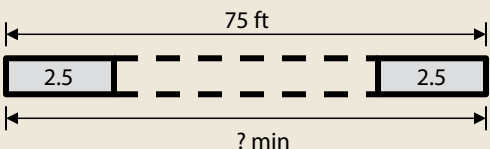|	
Write an answer sentence.	**The copier can print ___ pages per minute.**
Draw a bar model.	***Think:*** I am given a rate and need to find the unit rate.
Draw a bar to represent the given rate.	8 min
Draw another bar to represent the unit rate.	8 min 1 min
Label the diagram with what you know and **what you need to find.**	176 pages 8 min 1 min ?
Use the completed bar model and the unitary method **to help you solve the problem.**	***Think:*** I know the total number of pages printed in 8 minutes. I need to find the number of pages printed in 1 minute. I can use the unitary method and divide. 8 units → 176 pages 1 unit → 176 ÷ 8 = 22 pages
Complete the answer sentence.	**The copier can print 22 pages per minute.**

Rates

Example 2

A snail traveled 10 feet in 4 minutes.
At that rate, how long will it take the snail to travel 75 feet?

Write an answer sentence.	It will take the snail ____ minutes to travel 75 feet.
Draw and use a bar model to find the snail's speed.	***Think***: First, I need to find the speed of the snail as a unit rate in feet per minute. Then I can find out how long it will take the snail to travel 75 feet.
Draw a bar to represent the given rate. Draw another bar to represent the unit rate. **Label the diagram with what you know and what you need to find.**	
Use the bar model to help you find the unit rate.	***Think***: I know the distance traveled in 4 minutes. I need to find the distance traveled in 1 minute. I can use the unitary method and divide. 4 units → 10 ft 1 unit → 10 ÷ 4 = 2.5 ft The snail travels at a speed of 2.5 feet per minute.
Draw and use a bar model to find how many minutes it will take the snail to travel 75 feet.	
Draw a bar to show the first and last groups. Use dotted lines to represent the unknown number of groups in between. **Label the diagram with what you know and what you need to find.**	
Use the completed bar model to help you solve the problem.	***Think***: I know the snail's speed per minute and the distance it will travel. I need to find how long it will take the snail to travel that distance. I can divide to find the number of groups of 2.5 ft in 75 ft. 75 ÷ 2.5 = 30
Complete the answer sentence.	It will take the snail <u>30</u> minutes to travel 75 feet.

Percents

Example 1

The regular price of a bicycle is $495.
During a sale, Marcus bought the bicycle at a discount of 15%.
How much did he save by buying the bicycle during the sale?

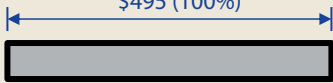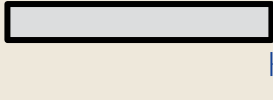|	
Write an answer sentence.	**Marcus saved $___ by buying the bicycle during the sale.**
Draw a bar model.	**Think:** The amount that Marcus saved is the dollar amount of the discount during the sale. There are two dollar amounts (the regular price and the sale price), and they are being compared.
Draw a bar to represent the regular price of the bicycle.	Regular price
Draw a shorter bar to represent the price of the bicycle on sale.	Regular price Sale price
Label the diagram with what you know and what you need to find.	$495 (100%) Regular price Sale price $? (15%)
Use the completed bar model and the unitary method to help you solve the problem.	**Think:** I can use the unitary method to find the amount of the discount. $100\% \rightarrow \$495$ $1\% \rightarrow \$495 \div 100 = \$4.95$ $15\% \rightarrow 15 \times \$4.95 = \$74.25$
Complete the answer sentence.	**Marcus saved $ _74.25_ by buying the bicycle during the sale.**

Percents

Example 2

Mr. Patel bought a new refrigerator that cost $1,175.
The sales tax was 5%.
How much did Mr. Patel pay for the refrigerator, including tax?

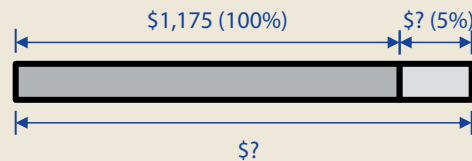|	
Write an answer sentence.	**Mr. Patel paid $___ for the refrigerator, including tax.**
Draw a bar model.	***Think*: First, I need to find the amount of sales tax. Then, I can find the total amount that Mr. Patel paid for the refrigerator, including tax.**
Draw a bar to represent the cost of the refrigerator.	
Join a bar to add on the amount of sales tax.	
	$1,175 (100%) $? (5%)
Label the diagram with what you know and what you need to find.	$?
Use the completed bar model and the unitary method to help you solve the problem.	***Think*: I can use the unitary method to find the amount of sales tax.**
	100% → $1,175 1% → $1,175 ÷ 100 = $11.75 5% → 5 × $11.75 = $58.75
	The amount of sales tax is $58.75.
	To find the amount he paid for the refrigerator including the sales tax, I can add.
	$1,175 + $58.75 = $1,233.75
Complete the answer sentence.	**Mr. Patel paid $1,233.75 for the refrigerator, including tax.**

Algebra

Adding Like Terms

Example

Simplify $4x + 2x$.

Write an answer sentence.	The sum of **4x** and **2x** is _____.
Draw a bar model.	**Think:** 4x and 2x are like terms.

$$\overbrace{\boxed{x}\ \boxed{x}\ \boxed{x}\ \boxed{x}}^{4x}\ \overbrace{\boxed{x}\ \boxed{x}}^{2x}$$
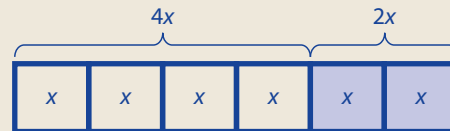
Use the completed bar model to help you solve the problem.	**Think:** $4x + 2x = x + x + x + x + x + x$ $= 6x$
Complete the answer sentence.	The sum of **4x** and **2x** is <u>6x</u>.

Subtracting Like Terms

Example

Simplify $6y - 4y$.

Write an answer sentence.	The difference between **6y** and **4y** is _____.
Draw a bar model.	**Think:** 6y and 4y are like terms.
Draw a bar to represent **6y**.	$\overbrace{\boxed{y}\ \boxed{y}\ \boxed{y}\ \boxed{y}\ \boxed{y}\ \boxed{y}}^{6y}$
Shade in and label **4y**.	$\overbrace{\boxed{y}\ \boxed{y}\ \underbrace{\boxed{y}\ \boxed{y}\ \boxed{y}\ \boxed{y}}_{4y}}^{6y}$
Use the completed bar model to help you solve the problem.	**Think:** $6y - 4y = 2y$
Complete the answer sentence.	The difference between **6y** and **4y** is <u>2y</u>.
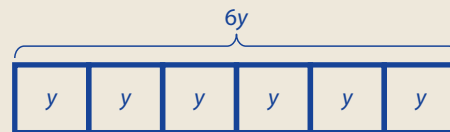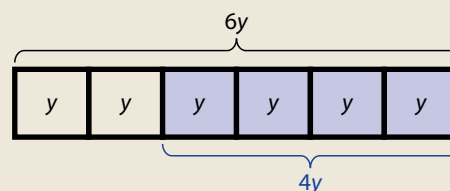
Using Variables to Write Expressions

Example 1

Tammy had 9 racecars. She gives *z* racecars to her brother.
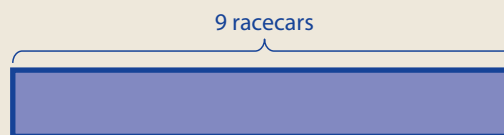How many racecars does Tammy have left?

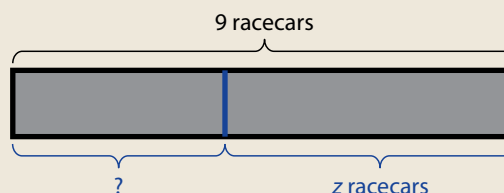|	
Write an answer sentence.	**Tammy has ___ racecars left.**
Draw a bar model.	**Think:** I can subtract to find this answer.
Draw a bar to represent the amount of cars that Tammy had in the beginning.	9 racecars
Label the amount she gives away. Show what you need to find.	9 racecars ? *z* racecars
Use the completed bar model to help you solve the problem.	**Think:** 9 − *z*
Complete the answer sentence.	**Tammy has _9 − z_ racecars left.**

Example 2

There are *p* golf balls in a box.
How many golf balls are in 6 boxes?

|	
Write an answer sentence.	**There are ___ golf balls in 6 boxes.**
Draw a bar model.	**Think:** I can multiply to find this answer.
Draw a bar to represent the 6 boxes.	
Label each box with a *p* to represent the amount of golf balls in each box.	*p* *p* *p* *p* *p* *p*
Use the completed bar model to help you solve the problem.	**Think:** $p \times 6 = 6p$
Complete the answer sentence.	**There are _6p_ golf balls in 6 boxes.**

Simplifying Algebraic Expressions

5 6 7

Example

The figure shows a model train track, with the length of 6 pieces labeled.
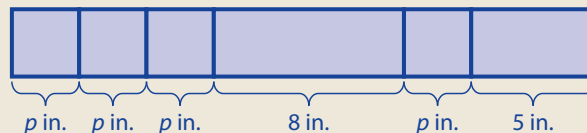Find the total length of the track in terms of p.
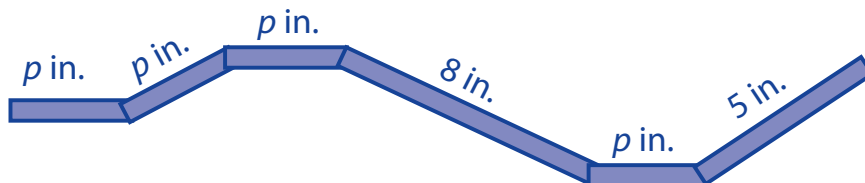
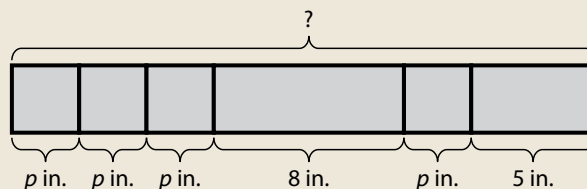|	
Write an answer sentence.	The length of the track is _____ inches.
Draw a bar model.	*Think:* I do not know the value of p, but I can look at the figure and see that it is less than 5 and 8, so the segments on my bar that represent p should be shorter than the segment for 5.
Draw a bar to represent the lengths of all of the track pieces.	
Show what you need to find.	
Use the completed bar model to help you solve the problem.	*Think:* I will add together all of the variables, then add together all the numbers. $p + p + p + p + 8 + 5 = (4 \cdot p) + 8 + 5$ $= 4p + 13$
Complete the answer sentence.	The length of the track is _4p + 13_ inches.

Expanding Algebraic Expressions

Example

Expand $3(t + 3)$.

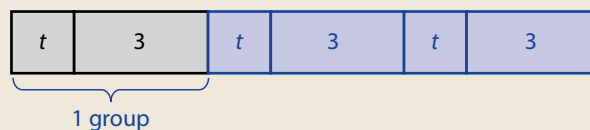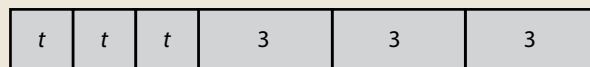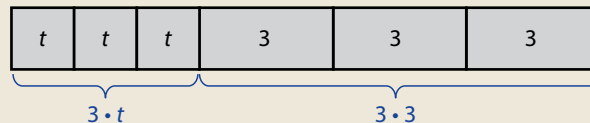|	
Write an answer sentence.	**The expanded form of $3(t + 3)$ is ____.**
Draw a bar model.	***Think:*** **I will need to represent the expression inside the parentheses three times.**
Draw a bar to represent the expression in the parentheses.	t │ 3
Draw the remainder of the bar by repeating the *group* for a total of 3 times.	t │ 3 │ t │ 3 │ t │ 3 1 group
Rearrange to collect like terms.	t t t 3 3 3
Label the groups.	t t t 3 3 3 $3 \cdot t$ $\qquad$ $3 \cdot 3$
Use the completed bar model to help you solve the problem.	***Think:*** $3(t + 3) = 3 \cdot (t + 3)$ $= 3 \cdot t + 3 \cdot 3$ $= 3t + 9$
Complete the answer sentence.	**The expanded form of $3(t + 3)$ is $\underline{3t + 9}$.**

Simplifying with Multiple Terms and Decimals

Example

Simplify $0.3x + 0.4x + 3$.

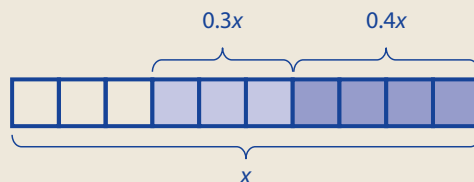*This same process is also used when working with fractional coefficients.*

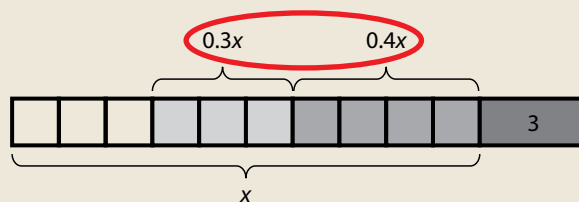|	
Write an answer sentence.	**The simplified form of $0.3x + 0.4x + 3$ is _____.**
Draw a bar model.	***Think:*** **First I will need to identify the like terms and then add them.**

Draw a bar to represent the decimals with the variable x in the expression. Because the decimals are to the tenths place, the bar should be divided into 10 equal parts.	
Draw another part of the bar to represent 3.	
Add the like terms.	
Use the completed bar model to help you solve the problem.	***Think:*** $0.3x + 0.4x + 3 = 0.7x + 3$
Complete the answer sentence.	**The simplified form of $0.3x + 0.4x + 3$ is $0.7x + 3$.**
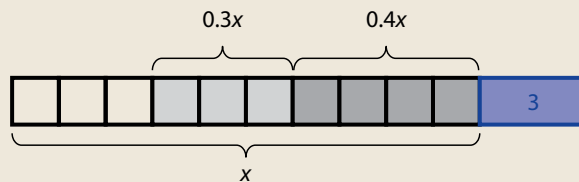
Expanding Algebraic Expressions with Fractional Factors   7

Example

Expand $\frac{1}{3}(6x + 15)$.

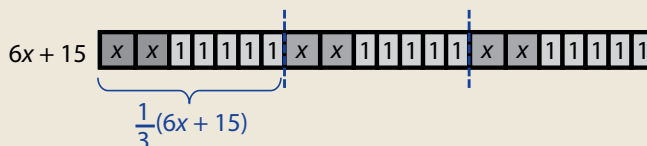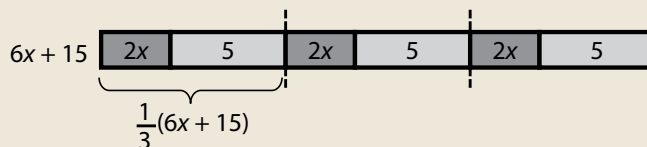|	
Write an answer sentence.	The expanded form of $\frac{1}{3}(6x + 15)$ is _____.
Draw a bar model.	***Think:*** I will need to create 3 equal groups within the bar model.
Draw a bar to represent the expression in the parentheses.	$6x + 15$ $\boxed{x\,|\,x\,|\,x\,|\,x\,|\,x\,|\,x\,|\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1}$
Rearrange the bar model to show 3 equal groups. There are three groups because the denominator of the fraction is 3.	$6x + 15$ $\boxed{x\,x\,1\,1\,1\,1\,1\,|\,x\,x\,1\,1\,1\,1\,1\,|\,x\,x\,1\,1\,1\,1\,1}$ $\underbrace{\qquad}$ $\frac{1}{3}(6x + 15)$
Another way to draw the bar model is with the like terms grouped together in each section.	$6x + 15$ $\boxed{2x\ |\ 5\ |\ 2x\ |\ 5\ |\ 2x\ |\ 5}$ $\underbrace{\qquad}$ $\frac{1}{3}(6x + 15)$
Use the completed bar model to help you solve the problem.	***Think:*** $\frac{1}{3}(6x + 15) = 2x + 5$
Complete the answer sentence.	The expanded form of $\frac{1}{3}(6x + 15)$ is <u>$2x + 5$</u>.

Factoring Algebraic Expressions with Multiple Variables

7

Example

Factor $4x + 2y$.

Write an answer sentence.	The factored form of $4x + 2y$ is ____.
Draw a bar model.	**Think:** I will need to create equal groups of both variables to factor this expression.
Draw a bar to represent the expression.	$4x + 2y$ | x | x | x | x | y | y |
Rearrange the bar model to show 2 identical groups.	$2(2x + y)$ | x | x | y | | x | x | y |
Use the completed bar model to help you solve the problem.	**Think:** $4x + 2y = 2(2x + y)$
Complete the answer sentence.	The factored form of $4x + 2y$ is $\underline{2(2x + y)}$.

Elimination Method

Example

Consider the system of linear equations:
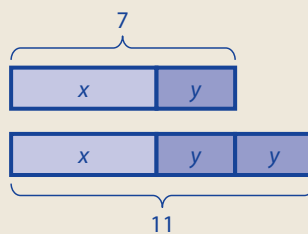$x + y = 7$
$x + 2y = 11$
Find the value of x.

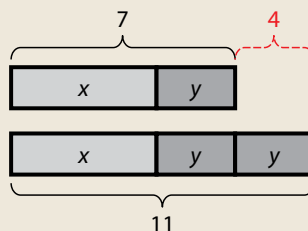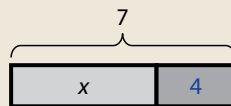|	
Write an answer sentence.	In the set of linear equations, x equals ___.
Draw a bar model.	***Think:*** To find the solution for x, I can first find the solution for y.
Draw a bar model to represent both linear equations.	
Notice the difference in the two bar models is 4. The second model is also one y-bar longer.	
Replace the y with a 4 in the top bar model.	
Use the completed bar model to help you solve the problem.	***Think:*** $x + 4 = 7$ $x = 3$
Complete the answer sentence.	In the set of linear equations, x equals _3_.