Scheme of work
Cambridge IGCSE®
Mathematics
0580
For examination from 2015

Cambridge IGCSE

Cambridge Secondary 2

CAMBRIDGE
International Examinations

Contents

Scheme of work – Cambridge IGCSE® Mathematics (0580) from 2015

Overview

This scheme of work provides ideas about how to construct and deliver a course. The syllabus has been broken down into teaching units with suggested teaching activities and learning resources to use in the classroom. This scheme of work, like any other, is meant to be a guideline, offering advice, tips and ideas. It can never be complete but hopefully provides teachers with a basis to plan their lessons. It covers the minimum required for the Cambridge IGCSE course but also adds enhancement and development ideas on topics. It does not take into account that different schools take different amounts of time to cover the Cambridge IGCSE course.

Recommended prior knowledge
It is recommended that candidates have followed the Secondary 1 Mathematics Curriculum Framework which can be found at: http://www.cie.org.uk/qualifications/academic/lowersec/cambridgesecondary1/resources or followed courses which cover the material contained in the UK National Curriculum for Mathematics at Key Stage 3 http://www.education.gov.uk/schools/teachingandlearning/curriculum/secondary/b00199003/mathematics/ks3.

Outline
Whole class **(W)**, group work **(G)**, pair **(P)** and individual activities **(I)** are indicated, where appropriate, within this scheme of work. Suggestions for homework **(H)** and formative assessment **(F)** are also included. The activities in the scheme of work are only suggestions and there are many other useful activities to be found in the materials referred to in the learning resource list.

Opportunities for differentiation are indicated as **basic** and **challenging**. There is the potential for differentiation by resource, length, grouping, expected level of outcome, and degree of support by the teacher, throughout the scheme of work. Timings for activities and feedback are left to the judgment of the teacher, according to the level of the learners and size of the class. Length of time allocated to a task is another possible area for differentiation.

The units within the scheme of work are:

Unit 1: Number
Unit 2: Algebra and graphs
Unit 3: Geometry
Unit 4: Mensuration
Unit 5: Co-ordinate geometry
Unit 6: Trigonometry
Unit 7: Matrices and transformations
Unit 8: Probability
Unit 9: Statistics

Syllabus content
The syllabus references use C for Core and E for Extended curriculum. In this scheme of work they are listed together using **black text** to identify where both the **Core and Extended curriculum cover the same content** and **blue** text for where the content is **only covered by the Extended curriculum**.

Teacher support
Teacher Support (http://teachers.cie.org.uk) is a secure online resource bank and community forum for Cambridge teachers, where you can download specimen and past question papers, mark schemes and other resources. We also offer online and face-to-face training; details of forthcoming training opportunities are posted online.

This scheme of work is available as PDF and an editable version in Microsoft Word format; both are available on Teacher Support at http://teachers.cie.org.uk. If you are unable to use Microsoft Word you can download Open Office free of charge from www.openoffice.org.

Resource list
The resource list for this syllabus, including textbooks endorsed by Cambridge, can be found at www.cie.org.uk and Teacher Support http://teachers.cie.org.uk.

Endorsed textbooks have been written to be closely aligned to the syllabus they support, and have been through a detailed quality assurance process. As such, all textbooks endorsed by Cambridge for this syllabus are the ideal resource to be used alongside this scheme of work as they cover each learning objective.

Textbooks
The most commonly used textbooks referenced in this scheme of work include:

Barton, D	*Essential Mathematics for IGCSE Extended Teacher Resource Kit* (Oxford University Press, 2012)
Haighton, J et al	*Core Mathematics for Cambridge IGCSE (*Nelson Thornes, 2012)
Haighton, J et al	*Extended Mathematics for Cambridge IGCSE (*Nelson Thornes, 2012)
Morrison, K and Hamshaw, N	*Cambridge IGCSE Mathematics Core and Extended Coursebook (with CD-ROM) (*Cambridge University Press, 2012)
Nye, C	*IGCSE Core Mathematics (*Heinemann, 2009)
Nye, C	*IGCSE Extended Mathematics* (Heinemann, 2009)
Pearce, C	*Cambridge IGCSE Maths Student Book* (Collins Educational, 2011)
Pemberton, S	*Essential Mathematics for Cambridge IGCSE Extended (with CD-ROM)* (Oxford University Press, 2012)
Pimental, R and Wall, T	*Cambridge IGCSE Mathematics Second Edition updated with CD* (Hodder Education, 2011)
Rayner, D	*Core Mathematics for Cambridge IGCSE (with CD-ROM)* (Oxford University Press, 2011)
Rayner, D	*Extended Mathematics for Cambridge IGCSE (with CD-ROM*) (Oxford University Press, 2011)
Simpson, A	*Core Mathematics for Cambridge IGCSE* (Cambridge University Press, 2010)
Simpson, A	*Extended Mathematics for Cambridge IGCSE* (Cambridge University Press, 2011)

Websites
This scheme of work includes website links providing direct access to internet resources. Cambridge International Examinations is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services).

The particular website pages in the learning resource column of this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only the particular resources are recommended.

Websites in this scheme of work, and some other useful websites, include:
http://nrich.maths.org/frontpage
http://www.tes.co.uk/teaching-resources/
http://www.bbc.co.uk/schools/gcsebitesize/maths/
http://www.waldomaths.com/
https://www.khanacademy.org/
http://www.geogebra.org/cms/en/
http://quizlet.com
http://www.cimt.plymouth.ac.uk
http://www.math.com/
http://www.mmlsoft.com
http://www.mrbartonmaths.com/jigsaw.htm
http://www.mathsisfun.com/
http://www.mathsrevision.net/
http://www.eclipsecrossword.com/
http://www.basic-mathematics.com/
http://math.about.com/
http://www.youtube.com/
http://resources.woodlands-junior.kent.sch.uk/maths/
http://mrbartonmaths.com/ebook.htm
http://illuminations.nctm.org
http://www.weatherbase.com/
http://www.colinbillett.wordpress.com/
http://www.bgrademaths.blogspot.co.uk/
http://www.timeanddate.com/worldclock/
http://www.springfrog.com/
http://www.timdevereux.co.uk/maths/maths_intro.html
http://www.learner.org/
https://maps.google.com/
http://www.regentsprep.org/regents/math/algtrig/math-ALGTRIG.htm
http://www.tessellations.org/index.shtml
http://www.nationalstemcentre.org.uk/
http://www.onlinenewspapers.com/

Unit 1: Number

Recommended prior knowledge
Learners should be able to add, subtract, multiply and divide confidently with integers and identify the correct operation from a word problem. They should be familiar with directed numbers and have an understanding of a number line involving positive and negative values. They should understand how to round to the nearest whole number, 10, 100 and 1000 and have some familiarity with decimal places. Learners should be familiar with multiplying and dividing whole numbers and decimals by 10 to the power of any positive or negative integer and recognise the equivalence of simple decimals, fractions and percentages, e.g. 0.25, ¼ and 25%. Learners need to understand 12 and 24 hour clock and be able to convert between these. They should also be confident in working with simple fractions and decimals, for example writing a fraction in its simplest form by cancelling common factors; adding and subtracting fractions with the same denominator; adding and subtracting decimals with the same number of decimal places. They should be aware of the order of operations, including brackets and recognise the effects of multiplying and dividing by numbers bigger than 1 or smaller than 1.

Context
This first unit revises and develops mathematical concepts in number that underpin the course. The work is fundamental to the study of all the other units and parts of it will need to be revisited when teaching subsequent units. This unit is appropriate for all learners, with the exception of all of sections 1.2 and 1.17 and the indicated parts of sections 1.5, 1.7, 1.10, 1.11, 1.12, and 1.16 which are only for extended learners. It is anticipated that learners studying the extended syllabus will work through at a faster pace.

Outline
It is intended for the topics in this unit to be studied sequentially, although this is not essential as certain topics, for example Time (1.14), can be studied earlier as it is a simpler and probably quite familiar concept. Also you may want to study unit 1.16 after 1.12, as they are related, or you may choose to leave a gap between them so that percentages can be revised when topic 1.16 is studied. This unit covers all aspects of number from the syllabus, namely fractions, decimals, percentages, ratios, indices, directed numbers, bounds, time, money and finance. Some teachers prefer to not teach number all in one block, it is possible to leave some sections until later in the course, for example upper and lower bounds (1.10) and exponential growth and decay (1.17) could be taught later on in the course, similarly with other topics.

Teaching time
It is recommended that this unit should take approximately 30–35% (Core learners) 15–20% (Extended learners) of the overall IGCSE course.

Ref	Syllabus content	Suggested teaching activities	Learning resources
1.1	Identify and use natural numbers, integers (positive, negative and zero), prime numbers, square numbers, common factors and common multiples, rational and irrational numbers (e.g. π, $\sqrt{2}$), real numbers. Includes expressing numbers as a product of prime factors. Finding the Lowest Common Multiple (LCM) and Highest Common Factor (HCF) of two numbers.	A useful starting point would be to revise positive and negative numbers using a number line and explain the difference between natural numbers and integers. **(W) (Basic)** Learners would find it useful to have a definition of the terms (e.g. factor, multiple, square number) which can be found on the maths revision website. **(W) (I) (Basic)** A fun activity would be to allocate a number to each learner in the class and ask them to stand up if they are, for example, "a multiple of 4", "a factor of 18" etc. Use this to show interesting facts such as prime numbers will have 2 people standing up (emphasises 1 is not prime); square numbers will have an odd number of people standing up. See which are common factors/common multiples for pairs of numbers. This could be extended to HCF and LCM. **(W) (Basic)** A follow-on activity would be for learners to identify a number from a description of its properties. For example, say to the class "which number less than 50 has 3 and 5 as factors and is a multiple of 9?" Learners could then make up their own descriptions and test one another. **(G) (Basic)** Another interesting task is to look a Fermat's discovery that some prime numbers are the sum of two squares, e.g. $29 = 25 + 4 = 5^2 + 2^2$. Learners could see what primes they can form in this way, and any they can't form in this way. Learners can look for a rule which tests whether or not a prime can be made like this. **(I) (Challenging)** Move on to looking at how to write any integer as a product of primes. One method that can be used is the factor tree approach which can be found online or in Pemberton's Essential maths CD. After demonstrating, or showing the presentation, ask learners to practise using the method to write other numbers as products of primes. Then ask learners to look at finding the product of primes of other numbers, for example 60, 450, 42, 315, but this time they can be encouraged to look for alternative methods, for example by researching on the internet. Another useful method is the repeated division method. **(I) (H)** Learners would find it useful to have a definition of the terms *rational*, *irrational* and *real numbers* which can be found on the Maths is Fun website. On the website there are questions on rational and irrational numbers for learners to try. These start simple and soon become more challenging. **(I) (F)**	**Online** http://www.mathsrevision.net/content/numbers http://www.mathsisfun.com/irrational-numbers.html http://vimeo.com/101831240 the factor tree approach **CD-ROM** Pemberton. Unit 1 slides 9 and 10

Ref	Syllabus content	Suggested teaching activities	Learning resources
	Extended curriculum only Use language, notation and Venn diagrams to describe sets and represent relationships between sets. Definition of sets e.g. $A = \{x: x \text{ is a natural number}\}$ $B = \{(x,y): y = mx + c\}$ $C = \{x: a$ Y x Y $b\}$ $D = \{a, b, c, \dots\}$ Notation Number of elements in set A n(A) "…is an element of…" ∈ "…is not an element of…" ∉ Complement of set A A' The empty set ∅ Universal set A is a subset of B $A \subseteq B$ A is a proper subset of B $A \subset B$ A is not a subset of B $A \nsubseteq B$ A is not a proper subset of B $A \not\subset B$ Union of A and B $A \cup B$ Intersection of A and B $A \cap B$	It is useful to start with revising simple Venn diagrams, for example with people who wear glasses in one circle and people with brown hair in another circle asking learners to identify the type of people in the overlapping region. **(W) (Basic)** This can be extended to looking at general Venn diagrams concentrating more on the shading of the regions representing the sets A ∪ B, A ∩ B, A' ∪ B, A ∪ B', A' ∩ B, A ∩ B', A' ∪ B' and A' ∩ B' helping learners to understand the notation. An excellent active-learning resource is the Venn diagrams card sort in Barton's Teacher Resource Kit pages 61-64. Ask learners to work in groups to complete this activity. **(G) (Challenging)** Learners would find it useful to know that (A ∪ B)' is the same as A' ∩ B' and that (A ∩ B)' is the same as A' ∪ B' and to understand the language associated with sets and Venn diagrams. Morrison and Hamshaw's book pages 172-179, for example, uses Venn diagrams to solve problems involving sets. Learners need to be able to distinguish between a subset and a proper subset. The work on Venn diagrams can be extended to look at unions and intersections when there are three sets. **(W) (Challenging)** Ask learners to try the past paper question. **(H) (F)**	**Textbooks** Barton p.61-64 Morrison p.172-179 **Past paper** Paper 41, June 2012, Q8
1.3	Calculate squares, square roots, cubes and cube roots of numbers.	Using simple examples illustrate squares, square roots, cubes and cube roots of integers. **(W) (Basic)** Extend the task by asking more able learners to square and cube fractions and decimals without a calculator, it may be worth doing topic 1.8 first to help with this. **(W) (Challenging)** An interesting activity is to look at finding the square root of an integer by repeated subtraction of consecutive odd numbers until you reach zero. For example, for 25 subtract in turn 1, 3, 5, 7, and then 9 to get to 0. Five odd numbers have been subtracted so the square root of 25 is 5. Ask learners to investigate this method for other, larger, square numbers. **(I) (H)**	**Past paper** Paper 32, June 2012, Q3

Ref	Syllabus content	Suggested teaching activities	Learning resources
		Another interesting challenge is to look at the palindromic square number 121. (Palindromic means when the digits are reversed it is the same number). Ask learners to find all the palindromic square numbers less than 1000. **(I) (H)** Ask learners to try the past paper question. **(H) (F)**	
1.4	Use directed numbers in practical situations.	An effective start for this topic is to draw a number line from -20 to +20, then point to various numbers (both positive and negative) asking learners, for example, “what is 5 more than this number?”, “What is 6 less than this number?” **(W)** You can keep it simple by using only integers **(Basic)** or extend the task by using decimals or fractions. **(Challenging)** An interesting extension to this is to then look at directed numbers in the context of practical situations. For example, temperature changes, flood levels, bank credits and debits. Learners can see weather statistics for over 29000 cities online at weatherbase.com, which can be used for them to investigate a variety of temperature changes involving positive and negative temperatures**. (G) (Basic)**	**Online** http://www.weatherbase.com/
1.5	Use the language and notation of simple vulgar and decimal fractions and percentages in appropriate contexts. Recognise equivalence and convert between these forms.	Learners would find it useful to have a definition of the terms (e.g. *numerator, denominator, equivalent fractions, simplify, vulgar fraction, improper fraction, mixed number, decimal fraction*, and *percentage*). A fun activity would be to ask learners to produce a crossword with the terms defined. Ask them to add any other terms that they can think of to do with fractions, decimals and percentages. Crosswords can be easily created using the excellent online software at eclipsecrossword.com. **(I) (H)** A useful activity for learners would be using clear examples and questions to try covering converting between fractions, decimals and percentages, such as Metcalf p.84-90. Learners should understand how to use place value (units, tenths, hundredths, etc.) to change a simple decimal into a fraction. For example 0.3 has 3 in the tenths column so it is $\frac{3}{10}$. **(W) (Basic)**	**Online** http://www.eclipsecrossword.com/ **Textbook** Metcalf p.84-90
	Extended curriculum only Includes the conversion of recurring decimals to fractions.	A useful activity for learners is to look at the online lesson at basic-mathematics.com to learn how to convert recurring decimals to fractions. It uses the method: $x = 0.15151515….$ $100x = 15.15151515…$ subtract these to get $99x = 15$ so $x = \frac{15}{99} = \frac{5}{33}$ **(W) (Challenging)**	**Online** http://www.basic-mathematics.com/converting-repeating-decimals-to-fractions.html

Ref	Syllabus content	Suggested teaching activities	Learning resources
1.6	Order quantities by magnitude and demonstrate familiarity with the symbols =, ≠, >, <, ≥ , ≤	A good active learning approach to this topic is to give learners a set of cards with the symbols =, ≠, >, <, ≥, ≤. Ask them to choose which card should go in between pairs of quantities that you give them. For example, 400m and 4000 cm; 20% and 0.2; -8 and -10, etc. **(W) (Basic)** Extend this by asking learners to consider when, or if, more than one card can be used (e.g. ≠ can be used in place of > or <). **(W)** Move on to giving learners a list of fractions, decimals and percentages. Ask them to order these by magnitude using the inequality signs. **(G) (Basic)** To check their understanding, learners can then try the past paper question. **(H) (F)**	**Past paper** Paper 12, June 2013, Q17
1.7	Understand the meaning and rules of indices. Evaluate 2^5, 5^{-2}, 100^0 Work out $2^{-3} \times 2^4$ Use the standard form $A \times 10^n$ where n is a positive or negative integer, and $1 \leq A < 10$. Convert numbers into and out of standard form. Calculate with values in standard form. ***Extended curriculum only*** $5^{½} = \sqrt{5}$, Evaluate $100^{½}$, $8^{-2/3}$	A good starting point is to begin working only with positive indices by revising the meaning of these and the basic rules of indices such as $2^3 \times 2^5 = 2^8$, $5^4 \div 5^3 = 5^1 = 5$ etc. Give simple examples to revise writing an integer as a product of primes including writing answers using index notation. **(W) (Basic)** An interesting challenge for learners is the puzzle "Power Crazy" on the nrich.maths.org website. Ask learners to work in groups to complete the challenge. **(G)** Extend this by working with negative and zero indices, and for extended learners, fractional indices. Useful examples are $2^{-1} = 2^{(2-3)} = \frac{2^2}{2^3} = \frac{1}{2}$ and $2^0 = 2^{(3-3)} = \frac{2^3}{2^3} = 1$. You can move on to introducing fractional indices by relating them to roots (of positive integers), for example $4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^1 = 4$ so $4^{\frac{1}{2}} = \sqrt{4} = 2$. The rules of indices can be used to show how values such as $16^{\frac{3}{4}}$ can be simplified. Learners should try lots of examples and questions, such as in Pearce, Student book p.262-273. **(I) (Challenging)** The next step is to give learners a range of examples showing how to write numbers in standard form and vice-versa. Emphasise to learners that different calculators display standard form in different ways and check that they know how to input numbers in standard form into their particular calculator. **(W) (Basic)** Extend this by using the four rules of calculation with numbers in standard form, both	**Online** http://nrich.maths.org/847 http://www.tes.co.uk/teaching-resource/Standard-Form-Worksheet-6193290/ **Textbook** Pearce, Student book p.262-273

Ref	Syllabus content	Suggested teaching activities	Learning resources
		with and without a calculator. It would be useful for learners if you can emphasise common errors. For example, if learners are asked to work out the answer to $2.4 \times 10^4 - 2 \times 10^4$ in standard form it is common to see an answer of 0.4×10^4. Point out that although $2.4 \times 10^4 - 2 \times 10^4 = 0.4 \times 10^4$ the answer is not in standard form, since 0.4 is less than 1. **(W) (Basic)** A useful activity would be to ask learners to try the standard form worksheet from the tes.co.uk website. **(H)**.	
1.8	Use the four rules for calculations with whole numbers, decimals and vulgar (and mixed) fractions, including correct ordering of operations and use of brackets.	A good starting activity is to ask learners to work in groups to use four 4s and the four rules for calculations to obtain all the whole numbers from 1 to 20, e.g. $4 + 4 \times 4 - 4 = 16$. **(G) (Basic)** The next step is to look at long multiplication and short and long division. You can see the traditional and chunking (repeated subtraction) examples on the bbc.co.uk website. This should be revision for most but is worth spending a bit of time on to ensure learners are confident in the methods. **(W) (Basic)** Extend this by clarifying the order of operations, including the use of brackets pointing out common errors, for example when learners do calculations working from left to right instead of using the order of operations rule, BIDMAS. (Brackets Indices Division Multiplication Addition and Subtraction). Give learners some examples illustrating the rules for multiplying and dividing with negative numbers. **(W) (Basic)** Extend this to using the four rules with fractions (including mixed numbers) and decimals. It is important that learners can do these calculations both with and without the use of a calculator as they may be expected to show working. A useful book with good examples and exercises is Pemberton's Essential Maths p. 2-4 and 10-13. Ask learners to try the questions from the exercises. **(I) (Challenging)**	**Online** http://www.bbc.co.uk/schools/gcsebitesize/maths/number/multiplicationdivisionrev2.shtml **Textbook** Pemberton, Essential maths, p.2-4, 10-13
1.9	Make estimates of numbers, quantities and lengths, give approximations to specified numbers of significant figures and decimal places and round off answers to reasonable accuracy in the context of a given problem.	A simple starting point is to revise rounding numbers to the nearest 10, 100, 1000, etc., or to a set number of decimal places. You can show learners how to round a number to a given number of significant figures explaining the difference and similarities between significant figures and decimal places. **(W) (Basic)** It is useful for learners to explain common misconceptions such as 43.98 to 1 dp is 44.0 not 44. Emphasise that on this syllabus non-exact answers are required to three significant figures unless the question says otherwise. Revision of estimating and rounding can be found at the website math.com. **(W) (Basic)**	**Online** http://www.math.com/school/subject1/lessons/S1U1L3GL.html

Ref	Syllabus content	Suggested teaching activities	Learning resources
1.10	Give appropriate upper and lower bounds for data given to a specified accuracy, e.g. measured lengths.	You will probably want to start this topic with examples to determine upper and lower bounds for data. Start with simple examples and then progressively harder ones, for example, a length, *l*, measured as 3 cm to the nearest millimetre has lower bound 2.95 cm and upper bound 3.05 cm. Emphasise that the bounds, in this case, are not 2.5 and 3.5 which would be a common misconception. Show learners how this information can be written using inequality signs e.g. 2.95 cm ≤ l < 3.05 cm. **(W) (Basic)** An interesting alternative to an exercise is the upper and lower bounds revision wheel in Barton's Teacher Resource Kit p.95-97. **(G)**	**Textbook** Barton p.95-97
	Extended curriculum only Obtain appropriate upper and lower bounds to solutions of simple problems given data to a specified accuracy, e.g. the calculation of the perimeter or the area of a rectangle.	For extended learners move on to looking at upper and lower bounds for quantities calculated from given formulae. A useful exercise can be found in Pearce's Student book p.107-109. **(I) (Challenging).** To check their understanding, learners can then try the past paper question. **(H) (F)**	**Textbook** Pearce, Student book p.107-109 **Past papers** Paper 21, June 2013, Q9 Paper 22, June 2013, Q8
1.11	Demonstrate an understanding of ratio and proportion. Divide a quantity in a given ratio. Direct and inverse proportion. Use common measures of rate. Calculate average speed. Use scales in practical situations.	Learners will find it useful to have a definition of ratio with a practical demonstration, for example the ratio of different coloured beads on a necklace. **(W) (Basic)** The next step is to look at examples illustrating how a quantity can be divided into a number of unequal parts, for example share $360 in the ratio 2 : 3 : 7. You will then want to move on to writing ratios in an equivalent form, for example 6 : 8 can be written as 3 : 4, leading on to the form 1 : *n*. **(W) (Basic)** An interesting alternative to an exercise is the active-learning ratio jigsaw at www.tes.co.uk which learners can work in groups to complete. **(G) (Basic).** A fun homework task would be to ask learners to produce their own jigsaw on ratio similar to this one. **(I) (H) (Basic)** Learners could produce their own jigsaw using blank equilateral triangles and paper. However, if they prefer to do this task electronically then Tarsia software (jigsaw making software) is available for download at mmlsoft.com. **(P) (I) (H) (Basic)** The next step is to look at ratio problems where you are not given the total, for example two lengths are in the ratio 4 : 7 if the shorter length is 48 cm how long is the longer length? **(W) (Basic)**	**Online** http://www.tes.co.uk/teaching-resource/Tarsia-Ratio-general-6107044 http://www.mmlsoft.com/index.php?option=com_content&task=view&id=9&Itemid=10 http://www.colinbillett.wordpress.com/ **Textbook** Pemberton p.218-221

Ref	Syllabus content	Suggested teaching activities	Learning resources
		Extend this to examples where you are given the difference. For example, the mass of two objects are in the ratio 2 : 5, one object is 36 g heavier than the other what is the mass of each object? **(W) (Challenging)** You will then want to look at drawing graphs to determine whether two quantities are in direct proportion. Ask learners to solve a variety of problems involving direct proportion by either the ratio method or the unitary method. Look at quantities in inverse proportion, for example the number of days to perform a job and the number of people working on the job. You will be able to link proportion to measures of rate and scales, for example exchange rates, average speed, density, map scales and other practical examples. For some ideas read the online blog, *it started with a map*, November 2012 at colinbillett.wordpress.com. You can also find some good questions and examples in Pemberton's Essential Maths book pages 218-221. **(G) (P) (Challenging)**	
	Extended curriculum only Increase and decrease a quantity by a given ratio.	For extended learners provide some good examples and questions on increasing and decreasing a quantity by a given ratio, e.g. in Pemberton's Essential Maths book p.222-223. **(P) (I)**	**Textbook** Pemberton p.222-223
1.12	Calculate a given percentage of a quantity. Express one quantity as a percentage of another. Calculate percentage increase or decrease.	The best starting point here is to revise converting between percentages and decimals. You can use examples to find percentages of quantities, for example to find 15% of $24 do 0.15 × 24 = 3.6 so $3.60, (remind learners that in money calculations it is conventional to use 2 dp for dollar answers). You should encourage learners to practice mental arithmetic methods too, for example, divide by 10 to find 10%, halve this to find 5% and add these results to find 15%. **(W) (Basic)** The next step is to use examples to show how to express one quantity as a percentage of another including where there is a mixture of units. **(W) (Basic)** Extend the work on finding percentages of quantities to looking at how to calculate percentage increases and decreases. For example to increase something by 15% multiply by 1.15, to decrease something by 15% multiply by 0.85. Provide practice examples, either write them yourself or from a textbook, e.g. Pimentel p.69-73. **(I) (Basic)** It would be useful to learners if you eliminate the misconception that increasing a quantity by 50% then decreasing the resulting quantity by 50% leads back to the original value. A good way to do this is by using the active-learning card sorting activity	**Textbooks** Pimentel p.69-73 Barton p.40-42

Ref	Syllabus content	Suggested teaching activities	Learning resources
		in D Barton's, Teacher Resource Kit pages 40-42. **(G) (Challenging)**	
	Extended curriculum only Carry out calculations involving reverse percentages, e.g. finding the cost price given the selling price and the percentage profit.	For extended learners you will need to move on to calculations involving reverse percentages. There are two good videos online at youtube.com and at bgrademaths.blocgspot.co.uk explaining two different approaches for reverse percentages questions. Ask learners to compare these methods and to decide which method they think is easier. **(G) (H)**	**Online** http://www.youtube.com/watch?v=OQ9T1-0Up6I http://www.bgrademaths.blogspot.co.uk/2009/06/reverse-percentages.html
1.13	Use a calculator efficiently. Apply appropriate checks of accuracy.	You might want to start this topic using examples to show how to estimate the answer to a calculation by rounding each figure in the calculation to 1 sf. Learners can then check their estimates by doing the original calculation using a calculator. Some good examples and practice on this can be found in Pemberton p.16. **(P) (I) (Basic)** An interesting extension activity, linking this work to percentages, would be to investigate the percentage error produced by rounding in calculations using addition/subtraction and multiplication/division. (You would need to explain percentage error beforehand). **(G) (Challenging)**	**Textbook** Pemberton p.16
1.14	Calculate times in terms of the 24-hour and 12-hour clock. Read clocks, dials and timetables.	A basic starting point would be to revise the units used for measuring time, with examples showing how to convert between hours, minutes and seconds. It is useful to use television schedules and bus/train timetables to help with calculations of time intervals and conversions between 12-hour and 24-hour clock formats. **(W) (Basic)** Ask learners to work in pairs or small groups to create a timetable for a buses or trains running between two local towns. To extend a topic that is relatively easy for more able learners, there is an interesting case study online called scheduling an aircraft, which can be found at cimt.plymouth.ac.uk. **(G) (P) (Basic) (Challenging)** It is useful for learners to look at world time differences and the different time zones. You could ask them to research and annotate a world map with times in various cities assuming it is noon where you live. Times can be found online at timeanddate.com. **(I) (H)** An important point for learners to consider is common misconceptions associated with time calculations. When learners do time calculations on a calculator and have a decimal answer for example, 5.3 emphasise that this means 5 hours 18 minutes not 5 hours 3 minutes or 5 hours 30 minutes. You can illustrate this well using the online	**Online** http://www.cimt.plymouth.ac.uk/resources/res1/schedair.pdf http://www.timeanddate.com/worldclock/ http://www.springfrog.com/converter/decimal-time.htm **Past paper** Paper 33, June 2012, Q1(b)(c)

Ref	Syllabus content	Suggested teaching activities	Learning resources
		decimal time converter at springfrog.com. **(Challenging)** To check their understanding, learners can try the past paper question. **(H) (F)**	
1.15	Calculate using money and convert from one currency to another.	You can use examples showing how to solve straightforward problems involving exchange rates. A good resource for this is Morrison and Hamshaw's Coursebook p.264-265. It is useful for learners if you link this work to syllabus section 2.9 (using conversion graphs). Up-to-date exchange rates can be found from a daily newspaper or online at cnnfn.cnn.com. **(W) (Basic)**	**Online** http://cnnfn.cnn.com/markets/currencies/ **Textbook** Morrison p.264-265
1.16	Use given data to solve problems on personal and household finance involving earnings, simple interest and compound interest. Includes discount, profit and loss. Extract data from tables and charts.	A good place to start is to look at simple problems on personal and household finance, using practical examples where possible. For example, taking information from published tables or advertisements. It would be useful for learners if you introduce a range of simple words and concepts here to describe different aspects of finance, for example tax, percentage profit, deposit, loan, etc. **(W) (Basic)** The next step is to introduce the formula I = PRT (I = interest earned, P is the investment, R is the percentage rate and T is the time) to solve a variety of problems involving simple interest, including those requiring learners to use rearranged versions of the formula. Core learners should also have an understanding of how to work out compound interest, ideally in a single calculation, for example the compound interest earned on an investment of \$500 over 4 years at a rate of 3% interest is 500×1.03^4. **(W) (Basic)** An interesting homework task is to ask learners to research the cost of borrowing money from different banks (or money lenders). **(I) (H)** To check their understanding, learners can try the past paper question. **(H) (F)**	**Past paper** Paper 12, Nov 2012, Q16
	Extended curriculum only Knowledge of compound interest formula is required. Value of investment = $P(1+r/100)^n$ where *P* is the amount invested, *r* is the percentage rate of interest and *n* is the number of years of compound interest.	Emphasise to extended learners that they should know and be able to use the formula $P(1+ r/100)^n$ for compound interest. Some examples and questions can be found in Metcalf's Extended Course book p.290-293 **(I)**	**Textbook** Metcalf p.290-293

Ref	Syllabus content	Suggested teaching activities	Learning resources
1.17	***Extended curriculum only*** Use exponential growth and decay in relation to population and finance, e.g. depreciation, bacteria growth.	To introduce the topic of exponential growth and decay you will find some good examples from the website khanacademy.org which uses the approach $n = ak^t$ where n = number at time t, a is the initial value and k is the rate. Ask learners to compare the similarities between this exponential growth formula and the compound interest formula. **(W) (Challenging)**	**Online** http://www.khanacademy.org/math/trigonometry/exponential_and_logarithmic_func/exp_growth_decay/v/word-problem-solving--exponential-growth-and-decay

Unit 2: Algebra and graphs

Recommended prior knowledge

Learners should be able to understand the concept of using letters to represent to represent unknown numbers or variables; know the meanings of the words *term, expression*, *equation*, *formula* and *function*. They should be confident with the work on directed numbers, rules of indices and order of operations from Unit 1 and know that algebraic operations follow the same order as arithmetic operations. Learners should be able to generate terms of a simple integer sequence and find a term given its position in the sequence; find simple term-to-term rules. They should be able to simplify or transform linear expressions with integer coefficients collect like terms; multiply a single term over a bracket. They should also know how to express simple functions algebraically and represent them in mappings, substitute positive and negative integers into formulae, linear expressions and expressions involving small powers, e.g. $3x^2 + 4$ or $2x^3$. Learners should understand the work from Unit 5 on Cartesian co-ordinates in two dimensions and finding the gradient of a line and be able to draw horizontal, vertical and diagonal lines from equations e.g. $y = 3$, $x = -2$, $y = 5x – 4$ and $x + 3y = 12$. Learners should know how to work out areas of triangles, rectangles and trapeziums.

Context

This unit revises and develops mathematical concepts in algebra that are important in other parts of the course. Approximately half of the unit is for core and extended learners and the other half is for extended learners only which is reflected in the different teaching time recommendations. Sections which are for extended learners only are 2.3, 2.6, 2.8, 2.11 and 2.12, all of the other sections have parts for extended learners only and these are indicated as such throughout. It is anticipated that learners studying the extended syllabus will work through at a faster pace and should have more prior knowledge of aspects of the core syllabus.

Outline

It is intended for the topics in this unit to be studied sequentially as many later topics require knowledge from earlier ones, for example finding the inverse of a function is much easier having studied how to transform equations first. However, some earlier extended topics require the knowledge of later core topics, e.g. to transform a formula where the subject appears more than once (extended section 2.1) requires the skill of factorising (core section 2.2), consequently, for extended learners, it might be useful to cover all the core work thoroughly and sequentially before starting the extended work. The unit covers all aspects of algebra from the syllabus, namely constructing and rearranging equations; expanding and factorising; manipulating a variety of algebraic fractions; working with indices; solving equations - including linear, quadratic and simultaneous equations; inequalities; direct and inverse variation; rates of change - including travel graphs; solving equations graphically; estimating gradients of curves and working with functions. Some teachers prefer to not teach algebra all in one block, it is possible to leave some sections until later in the course for example linear programming (2.6) and direct and inverse variation (2.8) could be taught later on in the course, similarly with many other topics.

Teaching time

It is recommended that this unit should take approximately 20–25% (Core learners) 35–40% (Extended learners) of the overall IGCSE course.

Ref	Syllabus content	Suggested teaching activities	Learning resources
2.1	Use letters to express generalised numbers and express basic arithmetic processes algebraically. Substitute numbers for words and letters in formulae. Transform simple formulae. Construct simple expressions and set up simple equations.	An effective start to this topic is revising basic algebraic notation, for example, $a + a = 2a$, $b \times c = bc$ (emphasising that cb is the same as bc but that the convention is to write letters in alphabetical order). Also look at simple examples with indices $d \times d = d^2$ and $e \times e \times e \times e = e^4$. Explain to learners how to substitute numbers into a formula, including formulae that contain brackets. **(W) (Basic)** An interesting investigation is to ask learners to work in groups to look at the difference between simple algebraic expressions which are often confused. For example, find the difference between $2x$, $2 + x$ and x^2 for different values of x. Ask learners "is there a number that makes them all equal?" **(G) (Basic)** Once the basics are secure move on to transforming simple formulae, for example rearranging $y = ax + b$ to make x the subject. Learners need to understand how to construct simple expressions and equations from word problems. **(W) (Basic)** An excellent extension activity is the puzzle, perimeter expressions, at nrich.maths.org **(I) (H) (Challenging)**	**Online** http://nrich.maths.org/7283
	Extended curriculum only Substitute numbers for words and letters in complicated formulae. Construct and transform complicated formulae and equations, e.g. transform formulae where the subject appears twice.	For extended learners you will need to build on all of the work from the core part of topic 2.1. Moving on to more complicated formulae when substituting, for example those with many orders of operations to consider. You can link the work on transforming formulae to the work on solving equations, asking learners to think about the balance method used in both. **(W) (Challenging)** A useful assessment tool is the past paper, June 2013 Q15. **(I) (H) (F)** Examples of constructing more complicated equations and expressions can be found in the past paper, June 2013 Q5abc. **(F)** The final step is to explain to learners how to transform complex formulae, for example, $x^2 + y^2 = r^2$, $s = ut + ½at^2$, expressions involving square roots, etc. You can use a series of examples to illustrate how to transform formulae containing algebraic fractions, (with possible links to the work in topic 2.3) for example $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ **(W) (Challenging)** Ask learners to try the past paper, June 2013 Q20. **(F)** The most challenging formula to transform, which deserves time spending on it, is	**Past papers** Paper 21, June 2013, Q15 Paper 42, June 2013, Q5(a)(b)(c) Paper 22, June 2013, Q20 Paper 21, Nov 2012, Q16

Ref	Syllabus content	Suggested teaching activities	Learning resources
		where the subject appears twice. Using examples showing the factorising and dividing through methods, you can discuss the benefits of each. You can link this work to topic 2.2. **(W) (Challenging)** A good past paper question on this topic is Nov 2012 Q16. **(F)**	
2.2	Manipulate directed numbers. Use brackets and extract common factors. e.g. expand $3x(2x - 4y)$, $(x + 4)(x - 7)$ e.g. factorise $9x^2 + 15xy$	An important starting point is to revise all aspects of directed numbers with all four operations and link this to positive and negative algebraic terms with the four operations. The inability to deal with negative numbers can otherwise cause an unnecessary stumbling block in algebraic work. **(W) (Basic)** You will need to use examples, with both positive and negative numbers, to illustrate expanding brackets. Start simply with a single term being multiplied over a bracket containing two or more terms. Extend this technique to multiplying two simple linear brackets together for example $(x - 3)(x + 7)$. Learners may find it useful to see a 2×2 algebraic multiplication grid to help with their understanding. **(W) (Basic)** The next step is to use examples, with both positive and negative numbers, to illustrate factorising simple expressions with one bracket. Explain that factorising is the reverse of expanding. A good source of examples and questions can be found in Pimentel p.96-98 and 101-102. **(I) (Basic)** Ask learners to try the past paper June 2013 Q6. **(F)**	**Textbook** Pimentel p.96-98, 101-102 **Past paper** Paper 21, June 2013, Q6
	Extended curriculum only Expand products of algebraic expressions. Factorise where possible expressions of the form: $ax + bx + kay + kby$ $a^2x^2 - b^2y^2$ $a^2 + 2ab + b^2$ $ax^2 + bx + c$	For extended learners move on to examples where they will need to find the products of algebraic expressions, for example $(x^2 + 3x - 4)(x - 5)$. **(W) (Challenging)** Building on the earlier factorising work use examples to show learners how to factorise three term quadratic equations initially where the coefficient of x^2 is 1. Include simple difference of two square examples such as $x^2 - 16$ emphasising that these can be solved using the same method as three term quadratics bearing in mind that the coefficient of the x term is 0. You can give learners some questions practicing factorising simple quadratics from the power point presentation, slide 5, online at tes.co.uk. **(I) (H)** The next step is looking at factorising by grouping. An example of the kind of question learners might see can be found in the past paper June 2013 Q10. **(F)** A really challenging topic is that of factorising quadratics where the coefficient of x^2 is	**Online** http://www.tes.co.uk/teaching-resource/Factorising-Quadratic-Expressions-6320242/ **E-Book** http://mrbartonmaths.com/ebook.htm **Past paper** Paper 21, June 2013, Q10

Ref	Syllabus content	Suggested teaching activities	Learning resources
		not 1. It is worth spending a considerable amount of time on this topic including revisiting it throughout the course to ensure methods are not forgotten. A higher order thinking skill is to ask learners to compare different methods for tackling a question. This is particularly useful for more able learners. Ask them to compare the two different methods for factorising quadratics of the form $ax^2 + bx + c$, where $a \neq 1$. The first method can be found in the power point presentation, slide 16, online at tes.co.uk (which uses splitting the x term into two terms and then factorising by grouping). The second method can be found in Mr Barton's E-book, the section *more factorising quadratics* pages 7 and 8 (which uses a trial and error approach). **(W) (Challenging)** Finally give learners examples how to factorise harder difference of two squares problems, for example. $16x^2 – 25y^2$. It is also worth mentioning two-term quadratic factorising examples such as $18x^2 – 24x$. Emphasise that these are often poorly answered. Point out that because they are quadratics learners often try to use two sets of brackets instead of just the one set of brackets required. **(W) (Challenging)**	
2.3	***Extended curriculum only*** Manipulate algebraic fractions. Factorise and simplify rational expressions.	Building on the work on factorising in topic 2.2 show learners how to factorise and simplify rational expressions such as for example $\frac{x^2-2x}{x^2-5x+6}$ **(W) (Challenging)** Provide learners with plenty of examples and questions (e.g. Nye p.402-403). It is worth linking this work on simplifying rational expressions to the work on using the four rules with algebraic fractions so that learners always give the most simplified answer. **(P) (I) (Challenging)** To assess learners' understanding of this topic ask them to complete the past paper June 2013 Q18. **(F)** Then you will need to spend time revising adding and subtracting simple fractions with learners, for example $\frac{2}{5}+\frac{3}{8}$. Explaining the process of finding a common denominator by, in this case, multiplying the two denominators. Ask learners to discuss when the lowest common denominator doesn't need to be the product of the two denominators, if you need to you can give the example $\frac{3}{10}+\frac{5}{8}$. **(W) (Basic)** The next step is to move on to algebraic fractions starting with numerical denominators, for example $\frac{x}{3}+\frac{x-4}{2}$, $\frac{2x}{3}-\frac{3(x-5)}{2}$ then extending this to algebraic denominators, for example $\frac{1}{x-2}+\frac{2}{x-3}$. You will need to emphasise common errors that occur when	**Textbook** Nye p.402-406 **Past paper** Paper 21, June 2013, Q18 Paper 21, June 2013, Q22

Ref	Syllabus content	Suggested teaching activities	Learning resources
		subtracting algebraic fractions. For example in the question $\frac{3}{x-5} - \frac{4}{x+2}$ explain that it is common to see sign errors on the numerator when $x - 5$ is multiplied by -4. **(W) (Challenging)** After this move on to examples demonstrating multiplying and dividing with numerical fractions, reminding learners that instead of dividing by a fraction you multiply by its reciprocal. **(W) (Basic)** Extend this by looking at algebraic fractions for example $\frac{3a}{4} \times \frac{9a}{10}$, $\frac{3a}{4} \div \frac{9a}{10}$. **(W) (Challenging)** Provide example questions for learners to practice (from textbooks, e.g. Nye p.404-406) and the past paper June 2013 Q22 is also worth doing. **(I) (F)**	
2.4	Use and interpret positive, negative and zero indices. Use the rules of indices.	A good starting point is to give learners examples revising the rules of indices work from Unit 1 topic 1.7. Extend this to using and interpreting positive, negative and zero indices and using the rules of indices with algebraic terms, for example, simplify: $3x^4 \times 5x$, $10x^3 \div 2x^2$, $(x^6)^2$. **(W) (Basic)**	
	Extended curriculum only Use and interpret fractional indices. Use the rules of indices.	For extended learners, move on to looking at fractional indices. For example, simplify: $3x^{-4} \times \frac{2}{3}x^{\frac{1}{2}}$, $\frac{2}{5}x^{\frac{1}{2}} \div 2x^{-2}$, $\left(\frac{2x^5}{3}\right)^3$ and solving exponential simple equations, e.g. solve $32^x = 2$. **(W) (Challenging)** For an excellent active learning resource, that you can use with learners working in groups, see the indices card sorting activity in Barton's Teacher Resource Kit pages 65-66. **(G)** To assess learners' understanding of the topic ask them to complete the past paper June 2013 Q11. **(F)**	**Textbook** Barton p.65-66 **Past paper** Paper 21, June 2013, Q11
2.5	Solve simple linear equations in one unknown. Solve simultaneous linear equations in two unknowns.	Begin this work with revising how to solve simple linear equations, including those with negatives, for example $3x + 2 = -1$. You will also want to include examples showing how to solve linear equations with brackets, for example $5(x + 4) = 3(x + 10)$. **(W) (Basic)** For a fun active learning resource ask learners to work in groups to complete the simple equations 1 jigsaw activity from tes.co.uk. Many more jigsaws are available at mrbartonmaths.com which also contains the link for downloading the Tarsia software to	**Online** http://www.tes.co.uk/ResourceDetail.aspx?storyCode=6108758 http://www.mrbartonmaths.com/jigsaw.htm

Ref	Syllabus content	Suggested teaching activities	Learning resources
		view the jigsaws. **(G) (Basic)** A good introduction to simultaneous equations is a non-algebraic approach, for example 3 coffees and 2 teas cost \$6.50 and 5 coffees and 2 teas cost \$9.50. Showing learners how the simultaneous equation from these statements can be formed and emphasising that the cost of tea and coffee does not change. **(W) (Basic)** Extend this by looking at examples to illustrating how to solve simultaneous linear equations with two unknowns by elimination, substitution and finding approximate solutions using graphical methods (linking to topic 2.10). **(W)** A sample question can be found in the past paper June 2013 Q10. **(I) (F)**	**Past paper** Paper 22, June 2013, Q10
	Extended curriculum only Solve quadratic equations by factorisation, completing the square or by use of the formula. Solve simple linear inequalities.	Extended learners will then need to explore all the different methods for solving quadratic equations, namely by factorisation, using the quadratic formula and completing the square (for real solutions only). The best starting point is using examples of the form $ax^2 + bx + c = 0$ then extend this by looking at equations requiring rearranging into this form first, e.g. Rayner p.79-86. **(W) (G) (P) (I)** A more challenging activity involves learners needing to construct their own equations from information given and then solve them to find the unknown quantity or quantities. This could involve the solution of linear equations, simultaneous equations or quadratic equations. **(W) (Challenging)** To introduce the topic of solving linear inequalities it is a good idea starting with just numbers, for example 7 > 5, showing that that multiplying or dividing an inequality by a negative number reverses the inequality sign, i.e. -7 < -5. You can use examples to illustrate how to solve simple linear inequalities including representing the inequality on a number line, e.g. from textbooks such as Pemberton CD Unit 2. **(W) (Challenging)** The most challenging inequalities for learners to solve are those where the inequality needs to be split into two parts and each part solved separately (see the final slide of the power point presentation). The June 2013 past paper Q18 is a good examples. **(W) (F) (Challenging)**	**Textbook** Rayner p.79-86 **CD-Rom** Pemberton Unit 2 – algebra, solving linear inequalities **Past paper** Paper 22, June 2013, Q18
2.6	***Extended curriculum only*** Represent inequalities graphically and use this representation in the	A good starting point is to begin by asking learners to draw a number of straight lines on a set of axes, possibly on mini white boards, for example $y = 2$, $x = -5$, $y = 3x$ and $x + 2y = 10$. Ask learners to consider a point on one side of each of these lines, the origin if possible, and use substitution to see if the inequalities $y < 2$, $x > -5$, $y < 3x$ and	**Textbook** Pemberton p.428-431 **Past paper**

Ref	Syllabus content	Suggested teaching activities	Learning resources
	solution of simple linear programming problems. Note: The conventions of using broken lines for strict inequalities and shading unwanted regions will be expected.	$x + 2y > 10$ are true for that particular point. Ask learners to work in groups to do their own examples similar to those already suggested. **(G) (Basic)** Extend this work by asking learners to look at examples illustrating how to solve linear programming problems by graphical means, highlighting the convention of using broken lines for strict inequalities < and > and solid lines for the inequalities ≤ and ≥. **(W) (Challenging)** Finally, learners will need to understand how to construct inequalities from constraints given, showing that a number of possible solutions to a problem exist, indicated by the unshaded region on a graph. Create your own examples and questions or use those in textbooks, e.g. Pemberton p.428-431. **(W) (Challenging)**	Paper 43, June 2013, Q3
2.7	Continue a given number sequence. Recognise patterns in sequences and relationships between different sequences. Find the *n*th term of linear sequences, simple quadratic and cubic sequences.	Learners will find it useful to have the definition of a sequence of numbers. Begin by asking learners to work in groups to investigate some simple sequences, for example finding the next two numbers in a sequence of even, odd, square, triangle or Fibonacci numbers, etc. **(G) (Basic)** Extend this to looking at finding the term-to-term rule for a sequence, for example the sequence 3, 9, 15, 21, 27, ..., has a term-to-term rule of +6; the sequence 40, 20, 10, 5, 2.5, …, has a term-to-term rule of ÷2. Learners will need to have some appreciation of the limitations of a term-to-term rule, i.e. that they are not very useful for finding terms that are a long way down the sequence. This leads on nicely to finding the position-to-term rule for a sequence by examining the common difference, for example the *n*th term in the sequence 3, 9, 15, 21, 27, …, is $6n - 3$. **(W) (Basic)** Ask learners to try the past paper June 2013 Q3. **(F)** An interesting investigation is to look at square tables placed in a row so that 4 people can sit around one table, 6 people can sit around 2 tables joined, 8 people can sit around 3 tables joined, and so on. Ask learners to work out how many people can sit around *n* tables. To add extra challenge ask learners to investigate tables of different shapes and sizes and to try to relate the *n*th term formula to the practical situation explaining how the numbers in the formula relate to the arrangements of the tables. **(P) (I) (H)** With more able learners you could look at deriving the formula for a linear sequence *n*th term = $a + (n - 1)d$ where a is the first term and d is the common difference, this formula	**Past paper** Paper 22, June 2013, Q3

Ref	Syllabus content	Suggested teaching activities	Learning resources
		is not essential knowledge. **(P) (I) (Challenging)**. Another approach is looking at patterns and relationships between different sequences. For example, the sequence 2, 5, 10, 17, 26, …, is the square numbers + 1. You can give learners several examples of these asking them to find the *n*th term, using just simple quadratic and cubic sequences i.e. of the form $an^2 \pm c$ or $an^3 \pm c$. **(P) (I) (H)**	
	Extended curriculum only Find the nth term quadratic and cubic sequences, exponential sequences and simple combinations of these.	For extended learners extend the core work by looking at examples of finding the *n*th term of harder quadratic sequences. A useful resource is the video and in particular the applet which can be found online at waldomaths.com. Learners can work in groups using the applet to investigate finding the *n*th term of harder quadratic sequences. **(G) (Challenging)** For even more challenge you can extend this further still by investigating cubic sequences using the video and applets again. **(G) (Challenging)** Other methods for finding *n*th terms are possible. Ask learners to search online for alternative methods for finding *n*th terms. **(P) (I) (H) (Challenging)** To assess learners understanding of this topic you can use the past paper June 2013 Q10. **(F)** Finally, learners will need to look at exponential sequences with a common multiplier (or ratio) instead of a common difference. With more able learners derive the formula for the *n*th term = $ar^{(n-1)}$ where a is the first term and r is the common ratio, this formula is not essential knowledge. **(Challenging)**	**Online** Video quadratic sequences: http://www.waldomaths.com/video/QuadSeq01/QuadSeq01.jsp Applet quadratic sequences: http://www.waldomaths.com/QuadSeq2L.jsp Video cubic sequences: http://www.waldomaths.com/video/CubSeq01/CubSeq01.jsp Applet cubic sequences: http://www.waldomaths.com/CubSeq1L.jsp **Past paper** Paper 41, June 2013, Q10
2.8	***Extended curriculum only*** Express direct and inverse variation in algebraic terms and use this form of expression to find unknown quantities.	Learners will need to be able to solve a variety of problems involving direct or inverse variation. Efficient notation to be encouraged is moving from the question, for example, *y* varies directly with *x* (or *y* is directly proportional to *x*) to each of these steps in turn $y \propto x \Rightarrow y = kx$. Another useful example is *t* varies inversely as the square root of $v \Rightarrow t \propto \frac{1}{v^2} \Rightarrow t = \frac{k}{v^2}$, where k is a constant. Emphasise the common error of reversing direct and inverse variation. Once the formula has been established ask learners to use given values to work out the value of the constant, k, and then use the formulae with the evaluated k. Examples and questions for learners to practise this can be found in textbooks, e.g. Morrison p.463-467 and Barton p.89-91. **(W) (Basic)** To assess understanding, ask learners to try the past paper June 2013 Q19. **(I) (H) (F)**	**Textbooks** Morrison p.463-467 Barton p.89-91 **Past paper** Paper 21, June 2013, Q19

Ref	Syllabus content	Suggested teaching activities	Learning resources
2.9	Interpret and use graphs in practical situations including travel graphs and conversion graphs. Draw graphs from given data.	A good starting point is to draw and use straight line graphs to convert between different units, for example between metric and imperial units or between different currencies. Exchange rates can be found at cnnfn.cnn.com which can be useful for setting questions. Learners need to be confident in solving problems using compound measures. It will be useful to learners to link this work to the work from topic 1.11 and 1.15 of the syllabus. **(W) (Basic)** It is important for learners to be able to draw a variety of graphs from given data, for example to determine whether two quantities (*y* and *x* or for more able learners *y* and x^2, etc.) are in proportion. You will be able to link this to the work in topic 2.8 on direct and inverse variation (for extended learners). Provide learners with examples and questions, either prepared yourself or from textbooks, e.g. Metcalf p.273-274. **(W) (Basic)**	**Online** http://cnnfn.cnn.com/markets/currencies/ **Textbooks** Metcalf p.273-279 Barton p.73-75
	Extended curriculum only Apply the idea of rate of change to easy kinematics involving distance-time and speed-time graphs, acceleration and deceleration. Calculate distance travelled as area under a linear speed-time graph.	For extended learners you will want to provide examples of how to draw and use distance-time graphs to calculate average speed (linking this to the calculating gradients work in topic 5.2). Learners should be able to interpret the information shown in travel graphs and to be able to draw travel graphs from given data. Ask learners to draw a travel graph for an imaginary journey and to write a set of questions about this journey. For example "what was the average speed?" **(I) (H)** When learners have drawn their graphs and written their question they can then give these to other members of a group to answer. **(G) (P)** You will need to ensure that learners have studied topic 4.2 and that they can confidently calculate areas of rectangles, triangles, trapeziums and compound shapes derived from these. **(W) (Basic)** Extend this work by looking at examples of speed-time graphs being used to find acceleration and deceleration and to calculate distance travelled as area under a linear speed-time graph. Useful examples and questions can be found in Metcalf's Extended Course book p.275-279. **(W)** Challenge can be provided by looking at examples where learners are required to convert between different units. For example, where different units are being used in the question and in the graph. **(W) (Challenging)** An interesting group activity is to ask learners to look at the distance-time and speed-	**Past paper** Paper 21, June 2013, Q25

Ref	Syllabus content	Suggested teaching activities	Learning resources
		time card sorting activity from Barton's Teacher Resource Kit p.73-75. **(G)** To assess their understanding of this work ask learners to try the past paper question. **(I) (H) (F)**	
2.10	Construct tables of values for functions of the form $ax + b$, $\pm x^2 + ax + b$, a/x $(x \neq 0)$, where *a* and *b* are integral constants. Draw and interpret such graphs. Solve linear and quadratic equations approximately by graphical methods.	Begin this topic by drawing a series of lines with x = constant and y = constant. Ask learners to identify the equations of the lines that you have drawn. Emphasise the importance of using a ruler and a sharp pencil in mathematical diagrams throughout this topic. **(W) (Basic)** Move on to examples of drawing diagonal straight line graphs from a table of values where the gradient and intercept are integers. You can link this to the work on gradient in topic 5.2. **(W) (Basic)** As an extension activity you could ask learners to find out how to use the gradient and intercept to draw a line, an example can be found in the online lesson at math.com. Learners can work as a group to explore and compare the methods for drawing lines from equations. **(G) (Challenging)** Extend this to looking at drawing quadratic functions of the form $\pm x^2 + ax + b$, and simple reciprocal functions such as a/x $(x \neq 0)$. Learners should be able to draw a variety of these graphs confidently and accurately from a table of values, introduce the terms parabola and hyperbola (although these are not required). You can then discuss with learners the symmetry properties of a quadratic graph and how this is useful. **(W)** The next step is to show how the solutions to a quadratic equation may be approximated using a graph. Extending this work to show how the solution(s) to pairs of equations (for example $y = x^2 - 2x - 3$ and $y = x$) can be estimated using a graph. This work can be linked to the work on simultaneous equations from topic 2.5. **(W) (Challenging)**	**Online** http://www.math.com/school/subject2/lessons/S2U4L3GL.html

Ref	Syllabus content	Suggested teaching activities	Learning resources
	Extended curriculum only Construct tables of values and draw graphs for functions of the form ax^n, where a is a rational constant, and n = –2, –1, 0, 1, 2, 3, and simple sums of not more than three of these and for functions of the form a^x, where a is a positive integer. Solve associated equations approximately by graphical methods. Draw and interpret graphs representing exponential growth and decay problems.	Software drawing packages such as Geogebra are useful here for learners to use to investigate different features of graphs. Geogebra is free to download, online from geogebra.org. You will probably want to start by asking learners to draw functions of the form a/x^2; a/x; ax^3; a^x; where a is a constant, using a graph drawing package like Geogebra. Ask learners to work in groups to use the software to gain an awareness of what each of the different types of graphs look like. Learners should be in a position to recognise common types of functions from their graphs, for example from the parabola, hyperbola, quadratic, cubic and exponential graphs. **(G) (Basic)** Then move on to asking learners to draw the graphs from tables of values. A useful video example lesson can be found online at khanacademy.org. Extend the work to include simple sums of not more than three functions in the form ax^n, where a is a rational constant, and n = –2, –1, 0, 1, 2, 3. Ask learners to solve associated equations approximately using these graphs. **(W) (Challenging)** For assessment purposes the past paper question is very useful but it does require topic 2.11 to have also been studied. **(F)** The final step is to look at examples of how to draw and interpret graphs representing exponential growth and decay problems. It will be useful to learners to link this to the work from topic 1.17. **(W) (Challenging)**	**Online** http://www.geogebra.org/cms/en/ http://www.khanacademy.org/math/trigonometry/exponential_and_logarithmic_func/exp_growth_decay/v/graphing-exponential-functions **Past paper** Paper 41, June 2013, Q2
2.11	***Extended curriculum only*** Estimate gradients of curves by drawing tangents.	Ensure learners have studied topic 5.2 (finding the gradient of a straight line) before beginning this topic. They should already be able to confidently find the gradient of a straight line. Learners will find it useful to have a definition of the term tangent. **(W) (Basic)** Move on to looking at examples showing how to find the gradient at a point on a curve by drawing a tangent at that point. **(W) (Challenging)** Learners can try the past paper question, bearing in mind this question also requires topic 2.10 to have been studied. **(F)**	**Past paper** Paper 41, June 2013, Q2
2.12	***Extended curriculum only*** Use function notation, e.g. f(x) = 3x – 5, f: $x \rightarrow 3x - 5$, to describe simple functions.	A useful staring point is to give learners a definition of a function, f(x) i.e. that it is a rule applied to values of x. Look at evaluating simple functions, for example linear functions, for specific values, describing the functions using f(x) notation and mapping notation. **(W) (Basic)** The next step is to introduce the inverse function as an operation which 'undoes' the	**Textbook** Pearce p.294-303 **Online** http://www.khanacademy.org/math/algebra/algebra-

Ref	Syllabus content	Suggested teaching activities	Learning resources
	Find inverse functions $f^{-1}(x)$. Form composite functions as defined by $gf(x) = g(f(x))$.	effect of a function. Demonstrate how learners can evaluate simple inverse functions for specific values, describing the functions using the $f^{-1}(x)$ notation and mapping notation. It will be useful to learners to link this to the work on transforming formulae from topic 2.1. Explain to learners that to find the inverse of the function $f(x) = 2x – 5$, a useful method is to rewrite this as $y = 2x – 5$, then to interchange the x and y to get $x = 2y – 5$, then to make y the subject $y = (x + 5)/2$ and finally to re-write using the inverse function notation as $f^{-1}(x) = (x + 5)/2$. **(W) (Challenging)** Using linear and/or quadratic functions, $f(x)$ and $g(x)$, show learners how to form composite functions such as $gf(x)$, and how to evaluate them for specific values of x. It will be useful for learners to investigate, for a variety of different functions $gf(x)$ and $fg(x)$ in order to see that these are not very often the same. Emphasise that it is important that learners know the correct order to apply the functions. **(W) (Challenging)** Provide learners with examples and questions, either prepared yourself or from textbooks, e.g. Pearce, or the video at khanacademy.org which also talks about what the graph of an inverse function looks like. Knowing that the graph of an inverse function is a reflection in the line $y = x$ is not required knowledge but is a useful extension for the more able learners. **(W)** Ask learners to try the past paper question to assess their understanding on this topic. **(F)**	functions/function_inverses/v/function-inverse-example-1 **Past paper** Paper 22, June 2013, Q2

Unit 3: Geometry

Recommended prior knowledge

Learners should be able to confidently use a ruler, protractor and pair of compasses, the first two as measuring tools as well as for drawing. Learners should also know the names of common 2D and 3D shapes such as quadrilateral, rectangle, triangle (equilateral, isosceles, right angled and scalene), parallelogram, trapezium, kite, circle, sphere, cylinder, prism, pyramid, cube, cuboid and cone. They should also know the names of the parts of a circle: radius, diameter, circumference and chord, understand the terms perpendicular and parallel and be familiar with the use of these in relation to special quadrilaterals' sides and diagonals.

Context

This unit revises and develops mathematical concepts in geometry. It is appropriate for all learners, with the exception of the indicated parts of sections 3.4, 3.5 and 3.6 which are only for extended learners. It is anticipated that learners studying the extended syllabus will work through at a faster pace.

Outline

It is intended for the topics in this unit to be studied sequentially; however it is possible to study sections as stand-alone topics. The unit covers all aspects of geometry from the syllabus, namely working with angles, constructions, similar figures, symmetry and loci. Some teachers prefer to not teach geometry all in one block, it is possible to leave some sections until later in the course, for example loci could be taught later on in the course, similarly with other topics.

Teaching time

It is recommended that this unit should take approximately 7% of the overall IGCSE course.

Ref	Syllabus content	Suggested teaching activities	Learning resources
3.1	Use and interpret the geometrical terms: point, line, parallel, bearing, right angle, acute, obtuse and reflex angles, perpendicular, similarity and congruence. Use and interpret vocabulary of triangles, quadrilaterals, circles, polygons and simple solid figures including nets.	Use flashcards at quizlet.com to look at the geometrical terminology or the Geometry crossword in Barton's Teacher Resource Kit p.70-72. **(W) (Basic)** Introduce the terminology for bearings, similarity and congruence briefly, (similar shapes and three figure bearings will be studied in more detail in topics 3.4 and 6.1). Illustrate common solids, e.g. cube, cuboid, tetrahedron, cylinder, cone, sphere, prism, pyramid, etc. Define the terms vertex, edge and face. Explore some geometric solids and their properties at illuminations.nctm.org. **(I) (H)**	**Online** http://quizlet.com/26701221/flashcards http://illuminations.nctm.org/ActivityDetail.aspx?ID=70 http://illuminations.nctm.org/Lessons/imath/GeoSolids/GeoSolids-AS1.pdf **Textbook** Barton p.70-72
3.2	Measure lines and angles. Construct a triangle given the three sides using ruler and pair of compasses only. Construct other simple geometrical figures from given data using ruler and protractor as necessary. Construct angle bisectors and perpendicular bisectors using straight edge and pair of compasses only.	Reinforce accurate measurement of lines and angles through various exercises. For example, each learner draws two lines that intersect. Measure the length of each line to the nearest millimetre and one of the angles to the nearest degree. Each learner should then measure another learner's drawing and compare answers. Also, draw any triangle, measure the three angles and check that they add up to 180°. **(G) (Basic)** Show how to: • construct a triangle using a ruler and compasses only, given the lengths of all three sides; • bisect an angle using a straight edge and compasses only; • construct a perpendicular bisector using a straight edge and compasses only. • Construct a range of simple geometrical figures from given data, e.g. construct a circle passing through three given points. See various constructions online at mathsisfun.com. **(W)** Provide examples and practice questions on all of the above, either prepared by you or from a textbook. **(I)** Give learners a written description of a compound shape (with no side lengths) and ask them to recreate the shape. For example, draw a rectangle, construct an equilateral triangle using the top edge of the rectangle as one of the sides, etc. **(I) (H)** Use the past paper question. **(I) (H) (F)**	**Online** http://www.mathsisfun.com/geometry/construct-ruler-compass-1.html http://www.mathsisfun.com/geometry/constructions.html **Textbook** Metcalf p.272-281 **Past paper** Paper 11, June 2012, Q12

Ref	Syllabus content	Suggested teaching activities	Learning resources
3.3	Read and make scale drawings.	Use an example to revise the topic of scale drawing. Show how to calculate the scale of a drawing given a length on the drawing and the corresponding real length. Point out that measurements do not need to be included on a scale drawing and that many scale drawings usually have a scale written in the form 1 : *n*. **(W) (Basic)** Use the online resources at tes.co.uk for some examples and questions. **(G)** Draw various situations to scale and interpret results. For example, ask learners to draw a plan of a room in their house to scale and use it to determine the area of carpet needed to cover the floor, plan an orienteering course, etc. **(P) (I) (H)**	**Online** http://www.tes.co.uk/ResourceDetail.aspx?storyCode=6280918
3.4	Calculate lengths of similar figures. ***Extended curriculum only:*** Use the relationships between areas of similar triangles, with corresponding results for similar figures and extension to volumes and surface areas of similar solids.	Discuss the conditions for congruent triangles. Point out that in naming triangles which are congruent it is usual to state letters in corresponding order, i.e. ΔABC is congruent to ΔEFG implies that the angle at *A* is the same as the angle at *E*. **(W) (Basic)** Extend the work on congruent shapes to introduce similar triangles/shapes. Use the fact that corresponding sides are in the same ratio to calculate the length of an unknown side. Link this work to work on transformations since rotation, reflection and translation leave shapes congruent and enlargements form similar shapes. **(W) (Basic)** For extended learners, expand on the work on calculating lengths of similar figures to using the relationships between areas, surface areas and volumes of similar shapes and solids. **(W) (Challenging)** Use the past paper questions. **(F)**	**Textbook** Morrison p.216-224 **Past papers** Paper 41, June 2013, Q8(d) Paper 42, June 2013, Q11
3.5	Recognise rotational and line symmetry (including order of rotational symmetry) in two dimensions. Note: Includes properties of triangles, quadrilaterals and circles directly related to their symmetries. ***Extended curriculum only:*** Recognise symmetry properties of the	Define the terms line of symmetry and order of rotational symmetry for two dimensional shapes. Revise the symmetries of triangles (equilateral, isosceles) and quadrilaterals (square, rectangle, rhombus, parallelogram, trapezium, kite) including considering diagonal properties. Discuss the infinite symmetry properties of a circle. **(W) (Basic)** For extended learners, define the terms plane of symmetry and order of rotational symmetry for three dimensional shapes. Use diagrams to illustrate the symmetries of cuboids (including a cube), prisms (including a cylinder), pyramids (including a cone). Look at diagrams for the symmetry properties of a circles paying attention to chords and tangents. **(W) (Challenging)**	**CD-ROM** Pemberton, Essential maths for Cambridge IGCSE CD. Unit 1, shape and space; symmetry **Textbook** Nye p.325-328

Ref	Syllabus content	Suggested teaching activities	Learning resources
	prism (including cylinder) and the pyramid (including cone). Use the following symmetry properties of circles: • equal chords are equidistant from the centre • the perpendicular bisector of a chord passes through the centre • tangents from an external point are equal in length.		
3.6	Calculate unknown angles using the following geometrical properties: • angles at a point • angles at a point on a straight line and intersecting straight lines • angles formed within parallel lines • angle properties of triangles and quadrilaterals • angle properties of regular polygons • angle in a semi-circle • angle between tangent and radius of a circle. ***Extended curriculum only:*** • angle properties of irregular polygons • angle at the centre of a circle is twice the angle at the circumference • angles in the same segment are equal • angles in opposite segments	Revise basic angle properties by drawing simple diagrams which illustrate angles at a point; angles on a straight line and intersecting lines; angles formed within parallel lines and angle properties of triangles and quadrilaterals. **(W) (Basic)** Define the terms irregular polygon, regular polygon, concave and convex. Use examples that include: triangles, quadrilaterals, pentagons, hexagons and octagons. Show that each exterior angle of a regular polygon is 360°/n, where n is the number of sides, and that the interior angle is 180° minus the exterior angle. Solve a variety of problems that use these formulae. Draw a table of information for regular polygons. Use as headings: number of sides, name, exterior angle, sum of interior angles, interior angle. **(I) (H)**. Use diagrams to show the angle in a semi-circle and the angle between tangent and radius of a circle are 90°. Use the dynamic pages on timdevereux.co.uk to see the circle theorems come to life. **(W)** Provide the solution to an examination style question on the topic of angles that contains a mistake in the working. Ask learners to identify the mistake. **(G)** For extended learners move on to look at angle properties of irregular polygons. By dividing an n-sided polygon into a number of triangles show that the sum of the interior angles is 180(n - 2) degrees and that the interior and exterior angles sum to 180°. **(W)** Explain the theory that angles in opposite segments are supplementary. Investigate cyclic quadrilaterals. For example, explain why all rectangles are cyclic quadrilaterals. What other quadrilateral is always cyclic? Is it possible to draw a parallelogram that is	**Book** Barton p.67-69 **Online** http://www.timdevereux.co.uk/maths/geompages/8theorem.php http://www.tes.co.uk/teaching-resource/Circle-Theorems-GCSE-Higher-KS4-with-answers-6181909/ **Past papers** Paper 12, Nov 2012, Q15, 20 Paper 21, Nov 2012, Q6

Ref	Syllabus content	Suggested teaching activities	Learning resources
	are supplementary; cyclic quadrilaterals. Note: Candidates will be expected to use the correct geometrical terminology when giving reasons for answers.	cyclic? etc. Use examples to show that the angle at the centre of a circle is twice the angle at the circumference and that angles in the same segment are equal. Again use the dynamic pages on timdevereux.co.uk to see the circle theorems come to life. **(W) (Challenging)** Solve a variety of problems using all the circle theorems making sure that learners know the correct language for describing the reasoning for their answers. Try the worksheet online at tes.co.uk with a large number of questions in examination style on circle theorems. Also use the past paper questions. **(I) (H) (F)**	
3.7	Use the following loci and the method of intersecting loci for sets of points in two dimensions which are: • at a given distance from a given point • at a given distance from a given straight line • equidistant from two given points • equidistant from two given intersecting straight lines.	To introduce the concept of loci ask learners in the class to stand up if they fulfil certain criteria, e.g. if they are exactly 2 m from the door, less than 2 m from the board, etc. A 2 m long piece of string can help if learners are not confident with estimating lengths. Move on to drawing simple diagrams to illustrate the four different loci. Use the convention of a broken line to represent a boundary that is not included in the locus of points. **(W) (Basic)** Look at the online example of a circle rolling around a square at nrich.maths.org. Use a similar idea of an example of a rectangular card being 'rolled' along a flat surface. Work out the locus of one of the vertices of the rectangle as it moves. **(I) (H)** Extend the work to look at overlapping regions of intersecting loci, use the past paper questions. **(F)**	**Textbook** Pimentel p.191-197 **Online** http://nrich.maths.org/2159 **Past paper** Paper 43, June 2013, Q2(a)

Unit 4: Mensuration

Recommended prior knowledge
Learners should be able to choose suitable units of measurement to estimate, measure, and solve problems in a range of simple contexts, including units of mass, length, area, volume or capacity. They should know abbreviations for, and some relationships between, metric units: kilometres (km), metres (m), centimetres (cm), millimetres (mm); tonnes (t), kilograms (kg) and grams (g); litres (l) and millilitres (ml). They should be able to read the scales on a range of analogue and digital measuring instruments and be familiar with the terms perimeter, area and volume. They should also know the names of common 2D and 3D shapes such as rectangle, triangle (equilateral, isosceles, right angled and scalene), parallelogram, trapezium, kite, circle, sphere, cylinder, prism, pyramid, cube, cuboid and cone. Learners should know the names of the parts of a circle: radius, diameter and circumference and understand the terms perpendicular and parallel and be familiar with the use of these in relation to special quadrilaterals' sides and diagonals. It is suggested that Unit 3 is studied before Unit 4, in particular syllabus reference 3.1.

Context
This unit revises and develops mathematical concepts in mensuration. It is appropriate for all learners, with the exception of the indicated parts of sections 4.3 and 4.4 which are only for extended learners. It is anticipated that learners studying the extended syllabus will work through at a faster pace. Learners should use calculators where appropriate; however, it is recommended that regular non-calculator work is completed to strengthen learners' mental arithmetic.

Outline
It is intended for the topics in this unit to be studied sequentially, as latter topics require the knowledge from earlier ones, for example a learner will not be able to work out the volume of a cylinder without having first learned how to calculate the area of a circle. The unit covers all aspects of mensuration from the syllabus, namely mass, length, perimeter, area, volume and capacity. Some teachers prefer to not teach mensuration all in one block, it is possible to leave some sections until later in the course for example areas and volumes of compound shapes could be taught later on in the course, similarly with other topics.

Teaching time
It is recommended that this unit should take approximately 7% of the overall IGCSE course.

Ref	Syllabus content	Suggested teaching activities	Learning resources
4.1	Use current units of mass, length, area, volume and capacity in practical situations and express quantities in terms of larger or smaller units. Convert between units including units of area and volume.	A good starting point is to use practical examples to illustrate how to convert between: millimetres, centimetres, metres and kilometres; grams, kilograms and tonnes; millilitres, centilitres and litres. For example looking at various measuring scales. **(W) (Basic)** Extend this work to looking at converting between units of area mm^2, cm^2 and m^2 and volume mm^3, cm^3 and m^3. A simple lesson explaining this can be found in the online video lesson on youtube.com. **(W) (Challenging)** More able learners will probably find it interesting to explore the link between the work on converting between area units to the work on ratio and similar shapes and can look at using scales on maps to work with areas. **(G) (Challenging)** To assess learners understanding, ask them to try the past paper question **(I) (H) (F)**	**Online** http://www.youtube.com/watch?v=dnSgdT2yNNI **Past paper** Paper 23, Nov 2012, Q15
4.2	Carry out calculations involving the perimeter and area of a rectangle, triangle, parallelogram and trapezium and compound shapes derived from these.	You might want to begin this topic by reminding learners how to calculate the perimeter and area of a rectangle, square and a triangle. **(W) (Basic)** This can then be extended by looking at how to calculate the area of a parallelogram and a trapezium and a variety of compound shapes. **(W) (Basic)** An interesting investigation is to look at using isometric dot paper to find the area of shapes that have a perimeter of 5, 6, 7, ... , units. **(G)** Ask learners to find out what shape quadrilateral has the largest area when the perimeter is, for example 24 cm. **(H)**	**CD-ROM** Pemberton, Essential maths for Cambridge IGCSE CD. Unit 2, shape and space; perimeter and area
4.3	Carry out calculations involving the circumference and area of a circle.	A useful starting point is revising, using straightforward examples, how to calculate the circumference and area of a circle. Learners are expected to know the formulae. **(W) (Basic)** Extend this by looking at how to find compound areas involving circles, for example, a circle with the radius of 5.3 cm is drawn touching the sides of a square. Ask learners "What area of the square is not covered by the circle?" the question can be extended to consider the area of waste material when cutting several circles of this size out of an A4 sheet of paper. **(I) (H)** 	

Ref	Syllabus content	Suggested teaching activities	Learning resources
	Extended curriculum only Solve problems involving the arc length and sector area as fractions of the circumference and area of a circle.	For extended learners the next step is to use examples to illustrate how to calculate the arc length and the sector area by using fractions of full circles. **(W) (Challenging)** Learners will need to combine their work on sector area with area of a triangle work (syllabus reference 6.3) to find segment areas. For an example of this see the past paper question. **(F) (Challenging)**	**Textbook** Nye, Extended Mathematics p.67-69 **Past paper** Paper 23, June 2013, Q18
4.4	Carry out calculations involving the volume of a cuboid, prism and cylinder and the surface area of a cuboid and a cylinder.	Starting with simple examples draw the nets of a variety of solids asking learners if they are able to identify the solid from the net. It is useful for learners to understand that there are many different right and wrong ways to draw the net of a cube. Less able learners might appreciate the opportunity to work in groups to draw nets on card and to use these to make various geometrical shapes. **(G) (Basic)** The next step is to demonstrate a purpose and use for drawing nets. For example in the packaging industry there are many different interesting nets used to create boxes, particularly those that require little or no glue. An interesting homework activity would be to ask learners to collect lots of different packaging boxes to investigate the nets used to create them. **(H) (Basic)** You can then ask learners to look at how to calculate the surface area of a cuboid and a cylinder, using the nets to help. Extend this to illustrating how to calculate the volume of a cuboid and a variety of prisms, including cylinders. Learners will find it useful to know the formula volume of prism = cross-sectional area x length. A useful resource on this topic is the online resource, surface area and volume, at www.learner.org. **(W) (Basic)** Ask learners to try the past paper question 14n **(F).**	**Online** http://www.learner.org/interactives/geometry/area.html http://math.about.com/od/formulas/ss/surfaceareavol.htm **Textbook** Rayner, Extended Mathematics p.107-123 **Past papers** Paper 11, June 2013, Q14 Paper 21, June 2013, Q15
	Extended curriculum only Carry out calculations involving the surface area and volume of a sphere, pyramid and cone. Note: Formulae will be given for the surface area and volume of the sphere, pyramid and cone.	For extended learners move on to using nets to illustrate how to calculate the surface area of a triangular prism, a pyramid and a cone. It will be useful for learners to understand how to obtain the formula $\pi r(r + s)$ for the surface area of a cone (where s = slant length). You will also want to explain how to calculate the surface area of a sphere using the formula $4\pi r^2$. **(W) (Basic)** The next step is to use examples to illustrate how to calculate the volume of a pyramid (including a cone) using the formula $\frac{1}{3}$ × area of base × perpendicular height. Also look at how to calculate the volume of a sphere using the formula $\frac{4}{3}\pi r^3$. Diagrams and formulae can be found at www.math.about.com. Emphasise to learners that they should	**Online** http://math.about.com/od/formulas/ss/surfaceareavol.htm **Textbook** Rayner, Extended Mathematics p.107-123 **Past paper** Paper 21, June 2013, Q15

Ref	Syllabus content	Suggested teaching activities	Learning resources
		know which formulae to learn and which will be given. Whilst some formulae will be given, you could challenge those with good memories to learn the given formulae too. **(W) (Challenging)** Examples and practice questions on this topic can be found in textbooks. **(P) (I)** Learners should be prepared to use formulae to find lengths too. Show them the past paper question 15 as an example. **(P) (I) (H) (F)** An interesting task is to show learners a sheet of A4 paper. Explain to them that this can be rolled into a cylinder in two ways. Ask learners "Which gives the biggest volume?" To extend this task ask learners to investigate what width and length gives the maximum cylinder volume if the area of paper remains constant but the length and width can vary **(I) (H)**	
4.5	Carry out calculations involving the areas and volumes of compound shapes.	The final section is all about extending all the work from sections 4.1 to 4.4 to find the surface area and volume of a wide variety of composite shapes. There are a number of past paper questions for learners to try demonstrating the kind of questions that they may see. **(P) (I) (H) (F)**	**Past papers** Paper 31, June 2013, Q11 Paper 43, June 2012, Q5 Paper 42, June 2012, Q11

Unit 5: Co-ordinate geometry

Recommended prior knowledge
Learners should be able to understand the concept of using letters to represent unknown numbers or variables; know the meanings of the words *term,* and *equation*. They should be confident with the work on directed numbers from Unit 1 and be able to simplify or transform linear expressions with integer coefficients and substitute positive and negative integers into linear expressions. Learners should also understand the work from Unit 6 on Pythagoras' theorem (extended learners only) and have an awareness of horizontal, vertical and diagonal line equations, e.g. $y = 3$, $x = -2$, $y = 5x – 4$.

Context
This unit revises and develops mathematical concepts in co-ordinate geometry that are important in other parts of the course. It is appropriate for all learners, with the exception of all of sections 5.3 and 5.6 and the indicated part of section 5.2 which are only for extended learners. It is anticipated that learners studying the extended syllabus will work through at a faster pace.

Outline
It is intended for the topics in this unit to be studied sequentially, as many later topics require knowledge from earlier ones, for example interpreting the equation of a straight line graph requires knowledge of the gradient. The unit covers all aspects of co-ordinate geometry from the syllabus, namely Cartesian co-ordinates, gradient of a straight line, lengths of line segments, midpoint of a line, equations of lines including parallel and perpendicular lines. Some teachers prefer to not teach co-ordinate geometry all in one block, it is possible to leave some sections until later in the course for example finding the gradient of parallel and perpendicular lines (5.6) could be taught later on in the course, similarly with other topics.

Teaching time
It is recommended that this unit should take approximately 7% of the overall IGCSE course.

Ref	Syllabus content	Suggested teaching activities	Learning resources
5.1	5.1 Demonstrate familiarity with Cartesian co-ordinates in two dimensions.	Revise co-ordinates in two dimensions see resources.woodlands-junior.kent.sch.uk for online games to do with co-ordinates. **(W) (Basic)** Draw a picture by joining dots on a square grid. Draw *x* and *y* axes on the grid and write down the coordinates of each dot. **(I) (H)** Ask other learners to draw these pictures from a list of coordinates only. **(G)**	**Online** http://resources.woodlands-junior.kent.sch.uk/maths/shapes/coordinates.html
5.2	5.2 Find the gradient of a straight line. *Core*: Note: Problems will involve finding the gradient where the graph is given. ***Extended curriculum only:*** Calculate the gradient of a straight line from the co-ordinates of two points on it.	Define, with diagrams, a line with a positive gradient as one sloping up and a line with a negative gradient as one sloping down. Use simple examples to show how to calculate the gradient (positive, negative or zero) of a straight line from a graph using vertical distance divided by horizontal distance in a right angled triangle. **(W) (Basic)** Extend this to consider the gradient of the line *x* = constant. **(W)** Use examples to show how to calculate the gradient of a straight line from the co-ordinates of two points on it, firstly by drawing the line and then without drawing the line. Use gradient = change in *y* coordinates over change in *x* coordinates. Explain the common error of subtracting the coordinates the opposite way round on the numerator to the denominator causing the sign to be incorrect. **(W) (Challenging)**	**Textbook** Pimentell, Cambridge IGCSE Maths p.139-140 **CD-ROM** Pemberton, Essential maths for Cambridge IGCSE CD, Unit 1, algebra; gradients and straight line graphs, slides 1-14
5.3	5.3 ***Extended curriculum only:*** Calculate the length and the co-ordinates of the midpoint of a straight line from the co-ordinates of its end points.	Revise Pythagoras' theorem from Unit 6. Use examples to show how to calculate the length of a straight line segment from the co-ordinates of its end points. **(W)** Do this firstly by using a sketch **(Basic)** and then to extend the learners use the formula $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Note that knowledge of this formula is not essential) **(Challenging)**. Use the past paper question. **(F)** Use examples to show how to find the co-ordinates of the midpoint of a straight line from the co-ordinates of its end points. Include examples working backwards, e.g. when an end point and a midpoint are known find the other end point. **(W) (I)**	**Textbook** Morrison Coursebook p.196-198 **Past Paper** Paper 22, June 2012, Q17a
5.4	5.4 Interpret and obtain the equation of a straight line graph in the form *y* = m*x* + c.	Revise drawing a graph of *y* = m*x* + c from a table of values. Interpret the meaning of m and c from the equation using the terms gradient and intercept. Starting with a straight line graph show how its equation (*y* = m*x* + c) can be obtained. **(W) (I) (Basic).** To interpret the meaning of an equation, explain how an equation simply gives the relationship between the *x* and *y* co-ordinates on the line, e.g. for the equation *y* = 2*x*	**Textbook** Metcalf, Core Course book p.301-302

Ref	Syllabus content	Suggested teaching activities	Learning resources
	Core: Note: Problems will involve finding the equation where the graph is given.	this means the *y* ordinate is always double the *x* ordinate. Use this to identify if a point lies on the line, e.g. which of these points: (2, 8), (-4, 8), (7, 14), (20, 10), (0, 0) lie on the line $y = 2x$? Ask learners to come up similar questions. **(I) (H)** Then give these questions to others in a group to identify which points do not lie on a given line. **(G) (P)**	
5.5	5.5 Determine the equation of a straight line parallel to a given line. e.g. find the equation of a line parallel to $y = 4x – 1$ that passes through (0, –3).	Use examples to show how to find the equation of a straight line parallel to a given line, e.g. find the equation of a line parallel to $y = 4x – 1$ that passes through (0, –3). **(W)** Use the past paper question (this also covers work on gradient). **(F)**	**Textbook** Pemberton, Essential Maths book p.68-70 **Past paper** Paper 11, November 2012, Q18
5.6	5.6 ***Extended curriculum only*** Find the gradient of parallel and perpendicular lines.	Use examples to show that parallel lines have the same gradient. Include examples where the equation is given implicitly, e.g. which of these lines are parallel? $y = 2x$, $y + 2x = 10$, $y – 2x + 3$, $2y = 2x + 7$, etc. **(W)** Use an odd-one-out activity, e.g. which is the odd one out (because the line is not parallel to the others), giving three or more examples? Ask learners to come up with their own set of odd one out examples. **(G)** Find the gradient of perpendicular lines by using the fact that if two lines are perpendicular the product of their gradients is -1, e.g. find the gradient of a line perpendicular to $y = 3x + 1$. **(W)** Use a variety of examples linking earlier topics from this unit, e.g. find the equation of a line perpendicular to one passing through the co-ordinates (1, 3) and (–2, –9). **(W) (Challenging)** Use the online examples and questions at bbc.co.uk. **(I) (H)**	**Online** http://www.bbc.co.uk/schools/gcsebitesize/maths/algebra/graphshirev1.shtml

Unit 6: Trigonometry

Recommended prior knowledge
Learners should be familiar with the work on angles from Unit 3 and have some understanding of how to label a triangle, with the convention that vertices are labelled with capital letters and opposite sides are labelled with lower case corresponding letters. Learners should also understand how an angle is named using three letters and be familiar with the term *right-angle triangle*, know that the longest side in a right angled triangle is called the hypotenuse and that angles in a triangle add up to 180°. Learners should be comfortable with three dimensional diagrams including being able to visualise parallel and perpendicular sides in them and be able to work with squares and square roots and to be able to solve equations similar to those arising from using Pythagoras' theorem. Extended learners should know how to work out a sector area using the work from section 4.3.

Context
This unit develops mathematical concepts in trigonometry. Section 6.1 and indicated parts of section 6.2 are appropriate for all learners. The rest of the unit is only for extended learners. It is anticipated that learners studying the core syllabus will spend closer to 3% of their time on this unit as they are only completing the first two sections of the syllabus, extended learners may require closer to 5% as there is a higher content for them to study.

Outline
It is intended for the topics in this unit to be studied sequentially as later topics, for example trigonometrical problems in three dimensions, require knowledge from previous areas. The unit covers all aspects of trigonometry and related work from the syllabus, namely bearings, Pythagoras' theorem, trigonometrical problems in two and three dimensions (including angels of elevation and depression), using the sine and cosine rule and the formula for the area of a triangle. Some teachers prefer to not teach trigonometry all in one block, it is possible to leave some sections until later in the course for example the trigonometrical problems in three dimensions (6.4) could be taught later on in the course, similarly with other topics.

Teaching time
It is recommended that this unit should take approximately 3–5% of the overall IGCSE course.

Ref	Syllabus content	Suggested teaching activities	Learning resources
6.1	Interpret and use three-figure bearings. Note: Measured clockwise from the North i.e. 000°–360°	Introduce three figure bearings and use examples of measuring and drawing involving bearings. Pearce's student book p.344-347 has useful examples. You may want to link this work to that on scale drawings in topic 3.3. **(W) (Basic)** Use examples to show how to calculate bearings, e.g. calculate the bearing of B from A if you know the bearing of A from B. **(W) (Basic)** Use a map to determine distance and direction (bearing) between two places, e.g. learners' home and school, etc. Maps from around the world can be found online at maps.google.com **(I) (H)**	**Textbook** Pearce, Student book p.344-347 **Online** https://maps.google.com/
6.2	Apply Pythagoras' theorem and the sine, cosine and tangent ratios for acute angles to the calculation of a side or of an angle of a right-angled triangle. Note: Angles will be quoted in, and answers required in, degrees and decimals to one decimal place. ***Extended curriculum only:*** Solve trigonometrical problems in two dimensions involving angles of elevation and depression. Extend sine and cosine values to angles between 90° and 180°.	Revise squares and square roots. Use simple examples involving right angled triangles to illustrate Pythagoras' theorem. Start with finding the length of the hypotenuse then move on to finding the length of one of the shorter sides. See examples online at mathsisfun.com. **(W) (Basic)** Extend this work to cover diagrams where the right angled triangle isn't explicitly drawn or the problem is presented without a diagram, e.g. find the diagonal length across a rectangular field or the height of a building. **(W) (Basic)** When introducing trigonometry spend some time on labelling the sides of triangles with a marked angle: adjacent, hypotenuse and opposite. Ask learners to work in groups to draw right angle triangles with a 30° angle of various sizes. Ask them to work out the ratio opposite side ÷ adjacent side for all the different triangles to find they should all be a similar value. **(G)** Then use examples involving the sine, cosine and tangent ratios to calculate the length of an unknown side of a right-angled triangle given an angle and the length of one side. Use a mix of examples, some examples where division is required and some examples where multiplication is required. For learners who struggle with rearranging the trigonometrical ratios it is possible to use the formula triangle approach (see the worksheet online at tes.co.uk). **(W) (G) (P)** For more able learners encourage the rearranging approach. Move on to examples involving inverse ratios to calculate an unknown angle given the length of two sides of a right-angled triangle. **(W) (G) (P)** Solve a wide variety of problems in context using Pythagoras' theorem and	**Online** http://www.mathsisfun.com/pythagoras.html http://www.tes.co.uk/ResourceDetail.aspx?storyCode=6344255 **Textbooks** Barton, Teacher Resource Kit p.45-47 Morrison, Core and Extended Course book p.303-321 **CD-ROM** Pemberton, Essential maths for Cambridge IGCSE CD. Unit 8, shape and space; Sine and cosine ratios for angles up to 180° **Past Paper** Paper 31, June 2013, Q7abc

Ref	Syllabus content	Suggested teaching activities	Learning resources
		trigonometric ratios (include work with any shape that may be partitioned into right-angled triangles). **(I) (Challenging)** Use examples to illustrate how to solve problems involving bearings using trigonometry. Use the past paper question. **(F)** For extended learners define angles of elevation and depression. Use examples to illustrate how to solve problems involving angles of elevation and depression using trigonometry. **(W)** Draw a sine curve and discuss its properties. Use the curve to show, for example, sin 150° = sin 30°. Repeat for the cosine curve. **(W) (Challenging)**	
6.3	***Extended curriculum only*** Solve problems using the sine and cosine rules for any triangle and the formula area of triangle = $\frac{1}{2}ab \sin C$.	Rearrange the formula for the area of a triangle (½bh) to the form ½absinC (see the online resource at regentsprep.org). Illustrate its use with a few simple examples. Explain that the letters in the formula may change from problem to problem, so learners should try to remember the pattern of two sides and the sine of the included angle. **(W)** Extend this to see if learners can use the formula to work out other problems, e.g. calculate the area of a segment of a circle given the radius and the sector angle (using their knowledge of sector area work from section 4.3) or calculate the area of a parallelogram given two adjacent side lengths and any angle. **(I) (Challenging)** Use examples to show how to solve problems using the sine rule explaining that the version $\frac{a}{\sin A} = \frac{b}{\sin B}$ is preferable for finding a side and the version $\frac{\sin A}{a} = \frac{\sin B}{b}$ is preferable for finding an angle. Use examples to show how to solve problems using the cosine rule making sure that learners either learn both rearrangements of the formula namely to find a side $a^2 = b^2 + c^2 - 2bc\cos A$ and to find an angle $\cos A = \frac{b^2+c^2-a^2}{2bc}$ or can confidently rearrange from one to the other. Give learners a set of questions where they can either use the sine rule or the cosine rule. Ask them not to work out the answers but instead to decide which rule to use (see the video online at youtube.com for an example activity). **(G)** Explain how learners can tell whether they need the sine rule or the cosine rule, i.e. use the cosine rule when you know all three sides in a triangle or an enclosed angle and two sides otherwise use the sine rule. **(W) (Challenging)** Use the past paper questions. **(P) (I) (H) (F)**	**Textbook** Rayner, Extended Mathematics p.209-213 **Online** http://www.regentsprep.org/regents/math/algtrig/ATT13/areatriglesson.htm http://www.youtube.com/watch?v=ZeMSUFsh8hs **Past Papers** Paper 41, June 2013, Q6 Paper 23, June 2013, Q18

Ref	Syllabus content	Suggested teaching activities	Learning resources
6.4	***Extended curriculum only*** Solve simple trigonometrical problems in three dimensions including angle between a line and a plane.	Introduce problems in three dimensions by finding the length of the diagonal of a cuboid and determining the angle it makes with the base. Extend by using more complex figures, e.g. a pyramid. **(W) (Challenging)** Use the past paper question. **(P) (I) (H) (F)**	**Textbook** Pimentel, Cambridge IGCSE Maths p.240-247 **Past Paper** Paper 21, June 2013, Q23

Unit 7: Matrices and transformations

Recommended prior knowledge
Learners should be familiar with the work on Cartesian co-ordinates and equations of straight line graphs from Unit 5 and have some understanding of the terms *reflect*, *rotate*, *translate* and *enlarge*. They should recognise line and rotation symmetry in two-dimensional shapes and patterns; draw lines of symmetry and complete patterns with two lines of symmetry; identify the order of rotational symmetry and be familiar with the work on symmetry in Unit 3. They should also be familiar with the work on similarity and congruence from Unit 3, in particular know that shapes remain congruent after rotation, reflection and translation but are similar after an enlargement, and be able to use Pythagoras' theorem from Unit 6.

Context
This unit revises and develops mathematical concepts in transformations and introduces vectors and matrices. Sections 7.1 and 7.2 are appropriate for all learners, with the exception of the negative scale factors parts. The rest of the unit is only for extended learners. It is anticipated that learners studying the core syllabus will spend closer to 5% of their time on this unit as they are only completing the first two sections, extended learners may require closer to 8% as there is a higher content for them to study.

Outline
It is intended for the topics in this unit to be studied sequentially, although this is not essential as certain topics for example matrices (7.4) can be studied earlier. The unit covers all aspects of matrices and transformations from the syllabus, namely drawing and describing rotations, reflections, translations and enlargements (for extended learners describing transformations using matrices); vectors, and matrices. Some teachers prefer to not teach matrices and transformations all in one block, it is possible to leave some sections until later in the course for example vectors (7.3) could be taught later on in the course, similarly with other topics.

Teaching time
It is recommended that this unit should take approximately 5–8% of the overall IGCSE course.

Ref	Syllabus content	Suggested teaching activities	Learning resources
7.1	Describe a translation by using a vector represented by e.g. $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$, or **a**. Add and subtract vectors. Multiply a vector by a scalar.	Use the concept of translation to explain a vector. Use simple diagrams to illustrate column vectors in two dimensions, explaining the significance of positive and negative numbers. Introduce the various forms of vector notation. Show how to add and subtract vectors algebraically illustrate by making use of a vector triangle. Show how to multiply a column vector by a scalar and illustrate this with a diagram. Use, for example, the examples online at mymaths.co.uk (subscription required for this website). **(W) (Basic)**	**Textbook** Barton, Teacher Resource Kit p.105-107 **Online** http://www.mymaths.co.uk/tasks/library/loadLesson.asp?title=vectors/vectorspart1 **Past paper** Paper 11, June 2013, Q5
7.2	Reflect simple plane figures in horizontal or vertical lines. Rotate simple plane figures about the origin, vertices or midpoints of edges of the figures, through multiples of 90°. Construct given translations and enlargements of simple plane figures. Recognise and describe reflections, rotations, translations and enlargements. Note – ***Core***: Positive and fractional scale factors for enlargements only. Note – ***Extended curriculum only:*** Negative scale factors for enlargements.	Draw an arrow shape on a squared grid. Use this to illustrate: reflection in a line (mirror line), rotation about any point (centre of rotation) through multiples of 90° (in both clockwise and anti-clockwise directions) and translation by a vector. Several different examples of each transformation should be shown. Use the word image appropriately. **(W) (Basic)** Investigate how transformations are used to make tessellations and produce an Escher-type drawing. For inspiration and step by step guides see tessellations.org. **(I) (H)** Use a pre-drawn shape on (x, y) coordinate axes to complete a number of transformations using the equations of lines to represent mirror lines and coordinates to represent centres of rotation. Work with (x, y) coordinate axes to show how to find: the equation of a simple mirror line given a shape and its (reflected) image, the centre and angle of rotation given a shape and its (rotated) image, the vector of a translation. Emphasize all the detail that is required to describe each of the transformations. **(W) (G) (Basic)** Draw a triangle on a squared grid. Use this to illustrate enlargement by a positive integer scale factor about any point (centre of enlargement). Use both of the methods: counting squares and drawing rays. Show how to find the centre of enlargement given a shape and its (enlarged) image. **(W)** For extended learners show how to draw enlargements using negative and/or fractional scale factors. **(W) (Challenging)** Use the past paper questions. **(F)**	**CD-ROM** Pemberton, Essential maths for Cambridge IGCSE CD. Unit 3, shape and space **Online** http://www.tessellations.org/index.shtml **Past papers** Paper 42, June 2013, Q2a (i) (ii) Paper 31, November 2012, Q10 **Textbook** Barton, Teacher Resource Kit p.29-34

Ref	Syllabus content	Suggested teaching activities	Learning resources
7.3	***Extended curriculum only*** Calculate the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$. Represent vectors by directed line segments. Use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors. Use position vectors.	Revise the work from section 7.1. Use diagrams to help show how to calculate the magnitude of a vector, link this to the work on Pythagoras' theorem from topic 6.2. Explain the notation required i.e. $\overrightarrow{AB}$ or **a** for vectors and for their magnitudes $\lvert\overrightarrow{AB}\rvert$or |**a**| (with modulus signs). **(W) (G) (P)** Define a position vector and solve various problems in vector geometry. Explain to learners that in their answers to questions, they are expected to indicate **a** in some definite way, e.g. by an arrow or by underlining, thus $\overrightarrow{AB}$ or $\underline{a}$. **(W) (G) (P) (Challenging).** Use the past paper question. **(F)**	**Textbook** Morrison, Core and Extended Course book p.500-511 **Past papers** Paper 21, June 2013, Q20 Paper 23, June 2013, Q23
7.4	***Extended curriculum only*** Display information in the form of a matrix of any order. Calculate the sum and product (where appropriate) of two matrices. Calculate the product of a matrix and a scalar quantity. Use the algebra of 2 × 2 matrices including the zero and identity 2 × 2 matrices. Calculate the determinant |**A**| and inverse $\mathbf{A}^{-1}$ of a non-singular matrix **A**.	Use simple examples to illustrate that information can be stored in a matrix. For example, the number of different types of chocolate bar sold by a shop each day for a week. Define the order/size of a matrix as the number of rows × number of columns. **(W) (Basic)** Explain how to identify matrices that you may add/subtract or multiply together with reference to the order of the matrices. Use examples to illustrate how to add/subtract and multiply matrices together. Define the identity matrix and the zero matrix for 2 × 2 matrices. Use simple examples to illustrate multiplying a matrix by a scalar quantity. Use examples to illustrate how to calculate the determinant and the inverse of a non-singular 2 × 2 matrix, explaining the term non-singular Explain when a matrix will have no inverse. **(W) (G)** As an extension for the more able, investigate how to use matrices to help solve simultaneous equations **(Challenging)**. Use the past paper questions **(F)**.	**Past papers** Paper 23, June 2013, Q17 Paper 42, June 2013, Q7 **Textbook** Barton, Teacher Resource Kit p.43-44

Ref	Syllabus content	Suggested teaching activities	Learning resources
7.5	***Extended curriculum only*** Use the following transformations of the plane: reflection (M), rotation (R), translation (T), enlargement (E), and their combinations. Identify and give precise descriptions of transformations connecting given figures. Describe transformations using co-ordinates and matrices (singular matrices are excluded). Note: If M(*a*) = *b* and R(*b*) = *c*, the notation RM(*a*) = *c* will be used. Invariants under these transformations may be assumed.	Starting with a letter E drawn on (*x*, *y*) coordinate axes, perform combinations of the following transformations: translation, rotation, reflection and enlargement. **(W) (Basic)** Use a unit square and the base vectors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ to identify matrices which represent the various transformations met so far, e.g. $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$ represents a rotation about (0, 0) through 90° anti-clockwise. Work with a simple object drawn on (*x*, *y*) coordinate axes to illustrate how it is transformed by a variety of given matrices. Use one of these transformations to illustrate the effect of an inverse matrix. Use the online resource at nationalstemcentre.org.uk for lots of interactive and dynamic examples to help learners to describe transformations using matrices. **(W) (Challenging)** Ask learners to investigate the connection between the area scale factor of a transformation and the determinant of the transformation matrix, e.g. using the matrix $\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$ and working with a rectangle drawn on (*x*, *y*) coordinate axes. **(G) (P)** Use the past paper question. **(F)**.	**Past paper** Paper 42, June 2013, Q2a (iii) **Online** http://www.nationalstemcentre.org.uk/elibrary/resource/530/aqa-fp1-matrices-transformations

Unit 8: Probability

Recommended prior knowledge
Learners should be familiar with the work from unit 1 on fractions, decimals and percentages including the four rules with them. They should also know the language of probability e.g. certain, impossible, even chance, likely, unlikely, outcomes, experiment, biased, fair and random and know how to extract information from tables and graphs. Learners should also understand that greater than 7 does not include 7 but that greater than or equal to 7 does. Extended learners should have studied the Venn diagrams work from unit 1, including drawing and interpreting them.

Context
This unit revises and develops mathematical concepts in probability. It is appropriate for all learners, with the exception of part 8.5 which is only for extended learners. It is anticipated that learners studying the extended syllabus will work through at a faster pace. Learners should use calculators where appropriate; however, it is recommended that regular non-calculator work is completed to strengthen learners' mental arithmetic.

Outline
It is intended for the topics in this unit to be studied sequentially, as latter topics require the knowledge from earlier ones, for example a learner will not be able to work out the probability of combined events without having first learned how to work with probability of single events. The unit covers all aspects of probability from the syllabus, namely understanding the probability scale, working with single and combined events, understanding how to work out the probability of an even not happening, using relative frequency as an estimate of probability and using tree diagrams. Some teachers prefer to not teach probability all in one block, it is possible to leave some sections until later in the course for example calculating the probability of combined events could be taught later on in the course, similarly with other topics.

Teaching time
It is recommended that this unit should take approximately 7% of the overall IGCSE course.

Ref	Syllabus content	Suggested teaching activities	Learning resources
8.1	Calculate the probability of a single event as either a fraction, decimal or percentage. Note: Problems could be set involving extracting information from tables or graphs.	Use theoretical probability to predict the likelihood of a single event. For example, find the probability of choosing the letter M from the letters of the word *MATHEMATICS*. Use the formula: probability = favourable outcomes/possible outcomes. **(W) (Basic)** Discuss when fractions, decimals or percentages are preferable for representing probabilities, e.g. if the probability is 2/3 then a fraction is preferable because it is exact. **(W) (Basic)** Learners use example questions that you've prepared or from textbooks. **(P) (I)** Use the past paper questions. **(F)** For extended learners use Venn diagrams for probability questions, for example see Pemberton's Essential Maths book p.327. **(G) (P) (I) (H) (Challenging)**	**Textbooks** Pearce, Student book p.570-573 Pemberton, Essential Maths p.327 **Past papers** Paper 13, Nov 2012, Q20b Paper 21, Nov 2012, Q18a
8.2	Understand and use the probability scale from 0 to 1.	Discuss probabilities of 0 and 1, leading to the outcome that a probability lies between these two values. Revise the language of probability associated with the probability scale. Use the probability scale by estimating frequencies of events occurring based on probabilities. **(W) (Basic)** Ask learners to produce their own probability scale with events marked on it, see an example at tes.co.uk. **(I) (H)** Ask learners to find out the meaning of mutually exclusive and exhaustive. **(I) (H)**	**Online** http://www.tes.co.uk/ResourceDetail.aspx?storyCode=6143121 **Textbook** Barton, Teacher Resource Kit p.37-39
8.3	Understand that the probability of an event occurring = 1 – the probability of the event not occurring.	Use examples to show the probability of an event occurring = 1 – the probability of the event not occurring, including those where there are only two outcomes and those when there are more than two outcomes. **(W) (Basic)** Try the past paper question **(F).**	**Past paper** Paper 12, June 2013, Q9
8.4	Understand relative frequency as an estimate of probability.	Compare estimated experimental probabilities, or relative frequency, with theoretical probabilities. Learners need to recognise that when experiments are repeated different outcomes may result and increasing the number of times an experiment is repeated generally leads to better estimates of probability. **(W) (Basic)** Conduct a class experiment into rolling dice 300 times, e.g. 15 pairs of learners rolling a dice 20 times each. Collect and combine results from groups to create a large sample set, show how estimates change as more data is added to the set. **(W) (G) (P) (Basic)**	**Online** http://www.mathsisfun.com/activity/buffons-needle.html **Past paper** Paper 33, June 2013, Q6a

Ref	Syllabus content	Suggested teaching activities	Learning resources
		Repeat the experiment where the theoretical probability is not known, e.g. the chance of a drawing pin landing point down when thrown in the air. Try Buffon's needle experiment at mathsisfun.com. **(W)** Carry out experiments to sample the number of unknown coloured counters in a bag. Ask learners to suggest how many of each type of coloured counter there are in the bag, given the known total. **(G)** Use the past paper question. **(F)**	
8.5	***Extended curriculum only*** Calculate the probability of simple combined events, using possibility diagrams and tree diagrams where appropriate. Note: In possibility diagrams, outcomes will be represented by points on a grid, and in tree diagrams, outcomes will be written at the end of branches and probabilities by the side of the branches.	Roll two different dice or two spinners and list all of the outcomes. Use simple examples to illustrate how possibility diagrams and tree diagrams can help to organise data. **(W) (Challenging)** Use possibility diagrams and tree diagrams to help calculate probabilities of simple combined events, paying particular attention to how diagrams are labelled. **(W) (P)) (Challenging)** Solve problems involving independent and dependent events, e.g. picking counters from a bag with and without replacement. **(W) (P) (Challenging)** Discuss conditional probability. **(W) (Challenging)** Try the "In a box" probability problem online at nrich.maths.org. **(I) (H)**	**CD-ROM** Pemberton, Essential maths for Cambridge IGCSE CD. Unit 6, probability and statistics; probability 2 **Online** http://nrich.maths.org/919

Unit 9: Statistics

Recommended prior knowledge
Learners should be able to decide which data would be relevant to an inquiry and collect and organise the data, as well as design and use a data collection sheet or questionnaire for a simple survey. They should have some familiarity with the three averages of mean, median and mode, and the measure of spread and range, all in simple cases. They should also know how to draw and interpret simple bar line graphs and bar charts, pie charts and pictograms and be familiar with the terms *discrete* and *continuous* data. Learners should be able to use a calculator effectively and be confident working with Cartesian coordinates and choosing appropriate scales on axes.

Context
This unit revises and develops mathematical concepts in statistics. It is appropriate for all learners, with the exception of all of sections 9.4 and 9.5 and the work on unequal intervals in histograms in section 9.2 which are only for extended learners. It is anticipated that learners studying the extended syllabus will work through at a faster pace.

Outline
It is intended for the topics in this unit to be studied sequentially, although this is not essential as certain topics for example correlation and lines of best fit (9.6 and 9.7) can be studied earlier. The unit covers all aspects of statistics from the syllabus, namely tables and statistical diagrams, averages, range, cumulative frequency, scatter diagrams and correlation. Some teachers prefer to not teach statistics all in one block, it is possible to leave some sections until later in the course for example correlation and lines of best fit (9.6 and 9.7) could be taught later on in the course, similarly with other topics.

Teaching time
It is recommended that this unit should take approximately 7% of the overall IGCSE course.

Ref	Syllabus content	Suggested teaching activities	Learning resources
9.1	Collect, classify and tabulate statistical data. Read, interpret and draw simple inferences from tables and statistical diagrams.	Use simple examples to revise collecting data and presenting it in a frequency (tally) chart. For example, record the different makes of car in a car park, record the number of words on the first page of a series of different books etc. **(W) (Basic)** Ask learners to conduct an experiment of this type, tabulating their data. **(G) (P)** Use examples to classify data using statistical terminology, e.g. discrete, continuous, numerical (quantitative) non-numerical (qualitative), etc. Use examples to show how to draw simple inferences from statistical diagrams, and tables including two-way tables. Use examples from textbooks and the past paper question. **(W) (P) (I) (H) (F) (Basic)**	**Textbook** Metcalf Core Course book p.139-145 **Past paper** Paper 12, June 2013, Q6
9.2	Construct and read bar charts, pie charts, pictograms, simple frequency distributions, histograms with equal intervals and scatter diagrams. ***Extended curriculum only:*** Construct and read histograms with unequal intervals. For unequal intervals on histograms, areas are proportional to frequencies and the vertical axis is labelled 'frequency density'.	Use the data collected, in section 9.1, to construct a pictogram, a bar chart and a pie chart. Point out that the bars in a bar chart can be drawn apart. **(W) (Basic)** Use an example to show how discrete data can be grouped into equal classes. **(W) (Basic)** Draw a histogram to illustrate the data (i.e. with a continuous scale along the horizontal axis). Point out that this information could also be displayed in a bar chart (i.e. with bars separated) because data is discrete. Explain how to draw scatter diagrams with simple examples (you may choose to do this at the same time as topic 9.6). **(W) (Basic)** Investigate the length of words used in two different newspapers and present the findings using statistical diagrams (links to newspapers can be found online at onlinenewspapers.com). **(G) (P) (Basic)** Record sets of continuous data, e.g. heights, masses etc., in grouped frequency tables. Use examples that illustrate equal class widths for core learners and unequal class widths for extended learners. Draw the corresponding histograms. Emphasize the fact that for continuous data bars of a histogram must touch. Use the bar charts and histograms section of the e-book, to illustrate why frequency density is a fairer way to represent data than frequency on the vertical axis. Label the vertical axis of a histogram as 'frequency density' and show that the area of each bar is proportional to the frequency. Show how to calculate frequency densities from a frequency table with grouped data and how to calculate frequencies from a given histogram. **(W) (Challenging)** Try the past paper question **(I) (H) (F)**	**Textbook** Morrison, Coursebook p.426 **Online** http://www.onlinenewspapers.com/ http://mrbartonmaths.com/ebook.htm the maths e-book of notes and examples **Past paper** Paper 41, June 2013, Q5(c)

Ref	Syllabus content	Suggested teaching activities	Learning resources
9.3	Calculate the mean, median, mode and range for individual and discrete data and distinguish between the purposes for which they are used.	From data collected, show how to work out the mean, the median and the mode from a list of data or from a frequency table. Explain, that if there are two middle values, how to find the half-way point for the median as there can only be one median but that there can be more than one mode or no mode. Use simple examples to highlight how these averages may be used. For example, in a discussion about average wages the owner of a company with a few highly paid managers and a large work force may wish to quote the mean wage rather than the median. See the online examples of which is the best average to use for different situations at mathsteacher.com. **(W) (Basic)** Include examples where the mean is given and the number of people, total or an individual value need to be found. **(Challenging)** Use the past paper question. **(I) (H) (F)** Explain how the mode can be recognised from a frequency diagram. **(G)** Try the 'Bat Wings' problem online at nrich.maths.org. **(I) (H)**	**Online** http://www.mathsteacher.com.au/year8/ch17_stat/02_mean/mean.htm http://nrich.maths.org/505 **Textbook** Barton, Teacher Resource Kit p.21-22 **Past paper** Paper 31, Nov 2012, Q3(a)
9.4	***Extended curriculum only*** Calculate an estimate of the mean for grouped and continuous data. Identify the modal class from a grouped frequency distribution.	Use examples to show how to calculate an estimate for the mean of data in a grouped frequency table using the mid-interval values. Explain how the modal class can be found in a grouped frequency distribution. **(W)** Look at the examples and questions online at cimt.plymouth.ac.uk. **(I) (H)**. Explain how to find the interval that contains the median but that working out the median is not required at this level, for more able learners you could show them the idea of linear interpolation as extension work. **(Challenging)** Try the past paper question. **(F)**	**Textbook** Pimentell, Cambridge IGCSE Maths p.348-349 **Online** http://www.cimt.plymouth.ac.uk/projects/mepres/book9/bk9i16/bk9_16i2.html **Past paper** Paper 22, June 2013, Q20ab
9.5	***Extended curriculum only*** Construct and use cumulative frequency diagrams. Estimate and interpret the median, percentiles, quartiles and inter-quartile range.	Explain cumulative frequency and use an example to illustrate how a cumulative frequency table is constructed. Draw the corresponding cumulative frequency curve emphasising that points are plotted at upper class limits, the curve must always be increasing and highlight its distinctive shape. Explain that this can be approximated by a cumulative frequency polygon. **(W) (Basic)** Use a cumulative frequency curve to help explain and interpret percentiles. Introduce the names given to the 25th, 50th and 75th percentiles and show how to estimate these from a graph. Show how to estimate the inter-quartile range from a cumulative	**Textbook** Pemberton, Essential maths p.368-375 **Past paper** Paper 41, June 2013, Q3

Ref	Syllabus content	Suggested teaching activities	Learning resources
		frequency diagram. Explain how to use a cumulative frequency curve to complete a frequency table. **(W) (Challenging)** Use the past paper question and exercises from a textbook. **(W) (P) (I) (H) (F) (Challenging)**	
9.6	Understand what is meant by positive, negative and zero correlation with reference to a scatter diagram.	Use simple examples of scatter diagrams to explain the terms and meanings of positive, negative and zero correlation. Revise drawing scatter diagrams and describe the resulting correlation. Explain why and where scatter graphs are useful, e.g. in making predictions. **(W) (Basic)** Ask learners to collect some bivariate data of their choice and to predict the correlation, if any, that they expect to find, e.g. height and arms span for members of the class. Use collected data to draw a scatter diagram and to then look for the expected correlation. Discuss results. **(G) (P)** Explain that if there are too few points on a scatter diagram a correlation may appear apparent when in fact there is no real relationship between the variables. **(W) (G)** Look at the online David and Goliath problem at nrich.maths.org. **(I) (H)** Try the past paper question. **(I) (H) (F)**	**Textbook** Rayner, Core Mathematics p.138-142 **Online** http://nrich.maths.org/7360 **Past paper** Paper 13, June 2013, Q18
9.7	Draw a straight line of best fit by eye.	Explain, with diagrams, that the purpose of a good line of best fit is to have the sum of the vertical distances from each point to the line as small as possible. In simpler terms ask learners to aim for a similar number of points on each side of the line and as many points as possible on the line or as close to it as possible. **(W) (G) (P)** Draw diagrams showing bad lines of best fit explaining what is wrong with them. For example, the common error when a learner draws the line through the origin when that doesn't fit with the trend of the data. Use a textbook for example exercises. **(W) (Basic)** Try the past paper question. **(I) (H) (F)**	**Textbook** Rayner, Core Mathematics p.143-144 **Past paper** Paper 22, June 2013, Q17