1 Calculations

In this chapter you will learn how to …

- use non-calculator methods to calculate with positive and negative numbers.
- perform operations in the correct order based on mathematical conventions.
- recognise inverse operations and use them to simplify and check calculations.

For more resources relating to this chapter, visit GCSE Mathematics Online.

Using mathematics: real-life applications

Everyone uses numbers on a daily basis often without really thinking about them. Shopping, cooking, working out bills, paying for transport and measuring all rely on a good understanding of numbers and calculation skills.

Tip

You probably already know most of the concepts in this chapter. They have been included so that you can revise concepts if you need to and check that you know them well.

"Number puzzles and games are very popular and there are mobile apps and games available for all age groups. I use an app with my GCSE classes where they have to work in the correct order to solve different number puzzles." *(Secondary School Teacher)*

Before you start …

KS3	You should be able to add, subtract, multiply and divide positive and negative numbers.	**1** Copy and complete each statement to make it true. Use only $<$, $=$ or $>$. **a** $2 + 3 \square 4 - 7$ **b** $-3 + 6 \square 4 - 7$ **c** $-1 - 4 \square 20 \div -4$ **d** $-6 \times 2 \square -7 - (-5)$	
KS3	You should know the rules for working when more than one operation is involved in a calculation (BODMAS).	**2** Spot the mistake in each calculation and correct the answers. **a** $3 + 8 + 3 \times 4 = 56$ **b** $3 + 8 \times 3 + 4 = 37$ **c** $3 \times (8 + 3) \times 4 = 130$	

KS3	You should understand that addition and subtraction, and multiplication and division are inverse operations.	**3** Identify the inverse operation by choosing the correct option.
		a $14 \times 4 = 56$
		    A $56 \times 4 = 14$ **B** $14 \div 4 = 56$ **C** $56 \div 4 = 14$
		b $200 \div 10 = 20$
		    A $200 \div 20 = 10$ **B** $200 = 10 \times 20$ **C** $10 \times 200 = 2000$
		c $27 + 53 = 80$
		    A $80 = 4 \times 20$ **B** $80 - 27 = 53$ **C** $80 + 27 = 107$	

Find answers at: cambridge.org/ukschools/gcsemaths-studentbookanswers

Assess your starting point using the Launchpad

STEP 1

1 Calculate, without using a calculator, and show your working.

a 647 + 786

b 1406 − 289

c 45 × 19

d 414 ÷ 23

GO TO
Section 1:
Basic calculations

STEP 2

2 Choose the correct answer.

a 9 ÷ (2 + 1) − 2

A 9   **B** $3\frac{1}{2}$ **C** 1   **D** 0

b (3 × 8) ÷ 4 + 8

A 2   **B** 30   **C** 16   **D** 14

c 12 − 6 × 2 + 11

A 78   **B** 23   **C** 1   **D** 11

d [5 × (9 + 1)] − 3

A 53   **B** 47   **C** 40   **D** 43

e (6 + 5) × 2 + (15 − 2 × 3) − 6

A 40   **B** 20   **C** 32   **D** 25

GO TO
Section 2:
Order of operations

STEP 3

3 The perimeter of a square is equal to four times the length of a side. If the perimeter is 128 cm, what is the length of a side?

4 What should you add to 342 to get 550?

5 If a number divided by 45 is 30, what is the number?

GO TO
Section 3:
Inverse operations

GO TO
Chapter review

Section 1: Basic calculations

You will not always have a calculator so it is useful to know how to do calculations using mental and written strategies.

It is best to use a method that you are confident with and always **show your working**.

Remember that when a question asks you to find the:

- **sum** you need to add
- **difference** you need to subtract the smaller number from the larger number
- **product** you need to multiply
- **quotient** you need to divide.

> **Tip**
>
> Some examination papers will not allow you to use your calculator.

WORK IT OUT 1.1

Look at these calculations carefully.

Discuss with a partner what methods these students have used to find the answer.

Which method would you use to do each of these calculations? Why?

① $489 + 274$

$400 + 200 \rightarrow 600$
$80 + 70 \rightarrow 150$
$9 + 4 \rightarrow \underline{13}$
763

② $284 - 176$

$\overset{7\ \ 1}{28\!\!\!/4}$
$-\ \underline{176}$
108

③ 29×17

$\rightarrow 30 \times 17 - 17$
$\rightarrow 3 \times 170 - 17$
$\rightarrow 510 - 17$
$\rightarrow 493$

④ 15×62

$= 30 \times 31 \quad 310$
$= 930 \quad\quad 310$
$ \underline{310}$
$ 930$

⑤ 207×47

×	200	0	7
40	8000	0	280
7	1400	0	49

$9400 + 0 + 329$
$= 9729$

⑥ $2394 \div 42$

2394
$-\ \underline{1680}$ (40) $\quad 42 \times 10 = 420$
$\ 714$ $\quad\quad\quad\quad 42 \times 20 = 840$
$-\ \underline{420}$ (10) $\quad 42 \times 40 = 1680$
$\ 294$ $\quad\quad\quad\quad 42 \times 5 = 210$
$-\ \underline{210}$ (5) $\quad\quad 42 \times 2 = 84$
$\ \ 84$
$-\ \underline{84}$ (2)
$= 57$

Problem-solving strategies

There are some useful strategies and techniques that you can use to break down complex problems to help you solve them more easily.

If you follow these steps each time you are faced with a problem, you will become more confident at problem solving and more able to check that your answers are sensible.

These are important skills both for your GCSE courses and for everyday life.

Problem-solving framework

Sally buys, repairs and sells used furniture at a market.

Last week she bought a table for £32 and a bench for £18.

She spent £12 on wood, nails, varnish and glue to fix them up.

She then sold the two items on her stall for £69.

How much profit did she make on the two items?

Steps for solving problems	What you would do for this example
Step 1: Work out what you have to do. Start by reading the question carefully.	Find the profit on the two items.
Step 2: What information do you need? Have you got it all?	Cost of items = £32 + £18 Cost of repairs = £12 Selling price = £69
Step 3: Is there any information that you don't need?	In this problem you don't need to know what she spent the money on, you just need to know how much she spent. Many problems contain extra information that you don't need so as to test your understanding.
Step 4: Decide what maths you can do.	Profit = selling price − cost You can add the costs and subtract them from the selling price.
Step 5: Set out your solution clearly. Check your working and make sure your answer is reasonable.	Cost = £32 + £18 + £12 = £62 Profit = £69 − £62 = £7 Sally made £7 profit.
Step 6: Check that you have answered the question.	Yes. You needed to find the profit and you have found it.

4

EXERCISE 1A

Solve these problems using written methods.

Set out your solutions clearly to show the methods you chose.

1 Nola checked the prices of pens at three different supermarkets. She found that the cheapest pack of pens was £3.90 for three. She bought fifteen pens.

How much did she pay in total and how much did she pay for each pen?

  a What two things are you asked to find here?

  b How many packs of pens did she buy? Why do you need to know this?

  c What operation would you do to find the total cost? Why?

  d How would you work out the cost of each pen?

  e Does a price of £1.30 for a pen seem reasonable to you?

2 Sandra bought a pair of jeans for £34, a scarf for £9.50 and a top for £20.

If she had saved £100 to buy these items, how much money would she have left?

3 How many 16-page brochures can you make from 1030 pages?

4 Jason can type 48 words per minute.

  a How many words can he type in an hour and a half?

  b Approximately how long would it take him to type an article of 2000 words?

5 At the start of a year the population of Greenside Village was 56 309.

During the year 617 people died, 1835 babies were born, 4087 people left the village and 3099 people moved into the village.

What was the population at the end of the year?

6 The Amazon River is 6448 km long, the Nile River is 6670 km and the Severn is 354 km long.

  a How much longer is the Nile than the Amazon?

  b How much shorter is the Severn than the Amazon?

7 What is the combined sum of 132 and 99 plus the product of 36 and 127?

8 What is the result when the difference between 8765 and 3087 is added to the result of 1206 divided by 18?

> **Tip**
>
> You don't always need to write something for the first few steps in the problem-solving framework, but you should still consider these steps mentally when approaching a problem in order to help you decide what to do. You should **always** show your working fully.

> **Did you know?**
>
> The Severn is the longest river in the UK.

Find answers at: cambridge.org/ukschools/gcsemaths-studentbookanswers

Key vocabulary

integers: whole numbers belonging to the set {… −3, −2, −1, 0, 1, 2, 3, …}; they are sometimes called directed numbers because they have a negative or positive sign.

Tip

You will be expected to work with negative and positive values in algebra, so it is important to make sure you can do this early on in your GCSE course.

Working with negative and positive integers

When doing calculations involving positive and negative **integers**, you need to remember the following:

- Adding a negative number is the same as subtracting the number:
  $4 + -3 = 1$
- Subtracting a negative number is the same as adding a positive number:
  $5 - -3 = 8$
- Multiplying or dividing the same signs gives a positive answer:
  $-4 \times -2 = 8$ and $\dfrac{-4}{-2} = 2$
- Multiplying or dividing different signs gives a negative answer:
  $4 \times -2 = -8$ and $\dfrac{-4}{2} = -2$

EXERCISE 1B

1 Calculate.

   a $12 - 5 + 8$ **b** $-3 - 4 - 8$ **c** $3 + 5 - 6$ **d** $-2 - 8 + 5$

   e $14 - 3 - 9$ **f** $9 - 3 - 4$ **g** $-34 + 18 - 12$ **h** $25 - 19 - 42$

2 Calculate.

   a $-9 - (-7)$ **b** $-3 - (-10)$ **c** $-4 - (-12)$ **d** $8 - (-9)$

   e $9 - (-8)$ **f** $-14 - (-14)$ **g** $-3 - 8 - (-9)$ **h** $-12 + 4 - (-8)$

3 Calculate.

   a $-2 \times -4 \times -4$ **b** $-4 \times 3 \times -6$ **c** $-3 \times -4 \times 3$ **d** $-4 \times -8 \times 3$

   e $3 \times 6 \times -4$ **f** $12 \times 2 \times -3$ **g** $1 \times -1 \times 10$ **h** $-3 \times -8 \times 9$

4 Calculate.

   a $24 \div 3$ **b** $-24 \div 3$ **c** $-48 \div -6$ **d** $400 \div -40$

   e $-22 \div -22$ **f** $-33 \div 11$ **g** $45 \div -9$ **h** $-64 \div -8$

5 Calculate.

   a $\dfrac{-40}{5}$ **b** $\dfrac{-28}{-4}$ **c** $\dfrac{30}{-5}$ **d** $\dfrac{12}{-2}$

   e $\dfrac{-48}{-6}$ **f** $\dfrac{-63}{7}$ **g** $\dfrac{-60}{-20}$ **h** $\dfrac{60}{-6}$

6 Apply the operations in the first row to the given number to complete each table.

a

	− 10	× −2	+ 4	÷ −2	− 8	+ 1
−5						

b

	× −4	÷ −5	+ 8	− 3	× 2	− 9
10						

c

	− 10	× −2	+ 4	÷ −2	− 8	+ 1
0						

7 Here are some bank transactions.

Calculate the new balance in each case.

a Balance of £230, withdraw £100.

b Balance of £250.50, withdraw £300.

c Balance of −£450, deposit £900 then withdraw £300.

d Balance of −£100, deposit £2000 then withdraw £550.

8 The damaged opening of an oil well, 5000 feet below sea level, caused a massive oil spill in the Gulf of Mexico in 2010.

The oil well itself extended to a depth of 13 000 feet.

Express the answers to these questions as directed numbers.

a How deep was the deepest part of the oil well below the sea bed?

b How far did oil travel from the bottom of the well to reach the surface of the water?

c The oil company involved estimated that they were losing money at the rate of $15 000 000 per day. Use an integer to express the change in the company's balance after:

i one week   **ii** thirteen weeks.

9 Here is a set of integers.

$\{-8, -6, -3, 1, 3, 7\}$

a Find two numbers with a difference of 9.

b Find three numbers with a sum of 1.

c Find two numbers whose product is −3.

d Find two numbers which, when divided, will give an answer of −6.

10 One more than −6 is added to the product of 7 and 6 less than 3.
What is the result?

> **Tip**
>
> The foot (plural feet) is a standard unit of imperial measurement for length; the metric measurement for length is metres. You will learn about **metric** measurements in Chapter 10.

Find answers at: cambridge.org/ukschools/gcsemaths-studentbookanswers

11 The temperature in Inverness is 4 °C at 7 pm at night.

By 1 am the same night, it has dropped by 12 degrees.

a What is the temperature at 1 am?

b What is the average hourly change in the temperature?

c By noon the next day, the temperature is 7 °C. How many degrees warmer is this than it was at 1 am?

Section 2: Order of operations

Jose posted this calculation on his wall on social media.

JOSE:
$$24 + 6 \div 2 - 1 \times 4 = ?$$

💬 COMMENT ✓ LIKE ↪ SHARE

JOANNA: 56

PETER: 11

LUCIA: 23

DIPAK: 104

Within minutes, his friends had posted four different answers.

Which one (if any) do you think is correct? Why?

There is a set of rules that tell you the order in which you need to work when there is more than one operation.

The order of operations is:

1 Do any operations in brackets first.

2 If there are any '**powers of**' or '**fractions of**' in the calculation, do them next.

3 Do division and multiplication next, working from left to right.

4 Do addition and subtraction last, working from left to right.

Brackets and other grouping symbols

Brackets are used to group operations. For example:

$(3 + 7) \times (30 \div 2)$

When there is more than one set of brackets, you work from the **innermost set** to the **outermost set**.

Tip

Many people remember these rules using the letters **BODMAS**. (or sometimes BIDMAS).
Brackets
Of ('powers **o**f' or 'fractions **o**f'; in BIDMAS, I stands for Indices)
Divide and/or **M**ultiply
Add and/or **S**ubtract.

8

WORKED EXAMPLE 1

Calculate $2((4 + 2) \times 2 - 3(1 - 3) - 10)$

$2 ((4 + 2) \times 2 - 3(1 - 3) - 10)$ Highlight the different pairs of brackets to help if you need to.

$$2((4 + 2) \times 2 - 3(1 - 3) - 10)$$
$$= 2(6 \times 2 - 3(-2) - 10)$$
$$= 2(6 \times 2 - 3 \times -2 - 10)$$

The red brackets are the innermost, so do the calculations inside these ones first. There are two lots of red brackets, so work from left to right. **Note** that you can leave -2 inside brackets if you prefer because $3(-2)$ is the same as 3×-2.

$$2(6 \times 2 - 3 \times -2 - 10)$$
$$= 2(12 - -6 - 10)$$
$$= 2(8)$$
$$= 2 \times 8$$
$$= 16$$

Blue brackets are next. Do the multiplications first from left to right, then the subtractions from left to right.

Often a different style of bracket will be used to make it easier to identify each pair.

For example, the following different types of brackets have been used below: $(\ ), [\ ], \{\ \}$.

$$\{2 - [4(2 - 7) - 4(3 + 8)] - 2\} \times 8$$

Other symbols can also be used to group operations.

For example:

Fraction bars: $\dfrac{5 - 12}{3 - 8}$

Roots: $\sqrt{16 + 9}$

These symbols are treated like brackets when you do a calculation.

Tip

$\dfrac{5 - 12}{3 - 8}$ is the same calculation as $(5 - 12) \div (3 - 8)$.

Find answers at: cambridge.org/ukschools/gcsemaths-studentbookanswers

9

Calculator tip

Most modern calculators are programmed to use the correct order of operations. Check your calculator by entering $2 + 3 \times 4$. You should get 14.

If the calculation has brackets, you need to enter the brackets into the calculator to make sure it does these first.

WORK IT OUT 1.2

Which of the solutions is correct in each case?

Find the mistakes in the incorrect option.

	Option A	Option B
1	$7 \times 3 + 4$ $= 21 + 4$ $= 25$	$7 \times 3 + 4$ $= 7 \times 7$ $= 49$
2	$(10 - 4) \times (4 + 9)^2$ $= 6 \times 16 + 81$ $= 96 + 81$ $= 177$	$(10 - 4) \times (4 + 9)^2$ $= 6 \times (13)^2$ $= 6 \times 169$ $= 1014$
3	$45 - [20 \times (4 - 3)]$ $= 45 - [20 \times 1]$ $= 45 - 21$ $= 24$	$45 - [20 \times (4 - 3)]$ $= 45 - 20 \times 1$ $= 45 - 20$ $= 25$
4	$30 - 4 \div 2 + 2$ $= 26 \div 2 + 2$ $= 13 + 2$ $= 15$	$30 - 4 \div 2 + 2$ $= 30 - 2 + 2$ $= 30$
5	$\dfrac{18 - 4}{4 - 2}$ $= \dfrac{18}{2}$ $= 9$	$\dfrac{18 - 4}{4 - 2}$ $= \dfrac{14}{2}$ $= 7$
6	$\sqrt{36 \div 4} + 40 \div 4 + 1$ $= \sqrt{9} + 10 + 1$ $= 3 + 11$ $= 14$	$\sqrt{36 \div 4} + 40 \div 4 + 1$ $= \sqrt{9} + 40 \div 5$ $= 3 + 8$ $= 11$

EXERCISE 1C

1 Calculate, showing the steps in your working.

a $5 \times 10 + 3$ **b** $5 \times (10 + 3)$ **c** $2 + 10 \times 3$

d $(2 + 10) \times 3$ **e** $23 + 7 \times 2$ **f** $6 \times 2 \div (3 + 3)$

g $10 - 4 \times 5$ **h** $12 + 6 \div 2 - 4$ **i** $3 + 4 \times 5 - 10$

j $18 \div 3 \times 5 - 3 + 2$ **k** $5 - 3 \times 8 - 6 \div 2$ **l** $7 + 8 \div 4 - 1$

m $\dfrac{15 - 5}{2 \times 5}$ **n** $(17 + 1) \div 9 + 2$ **o** $\dfrac{16 - 4}{4 - 1}$