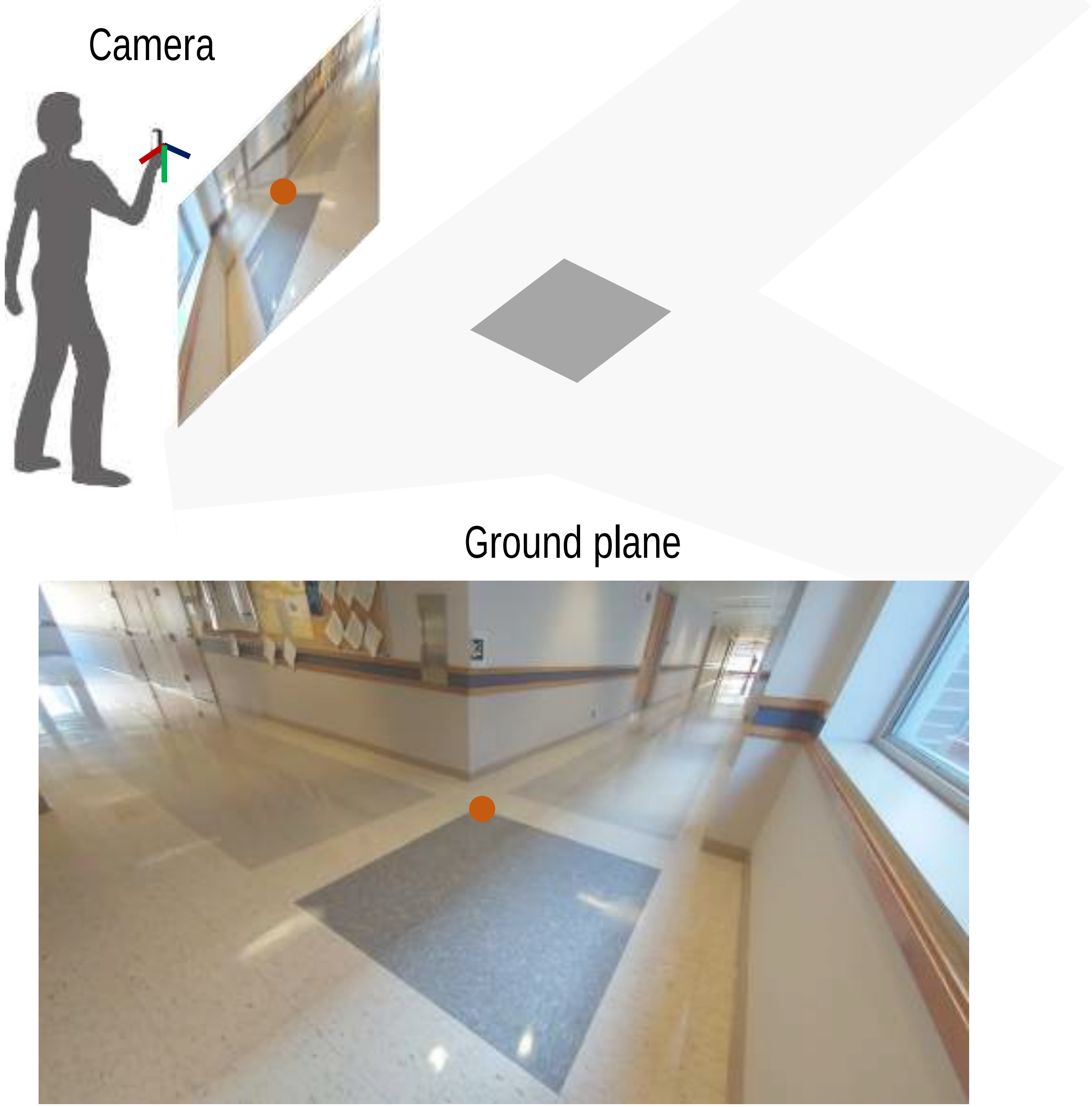
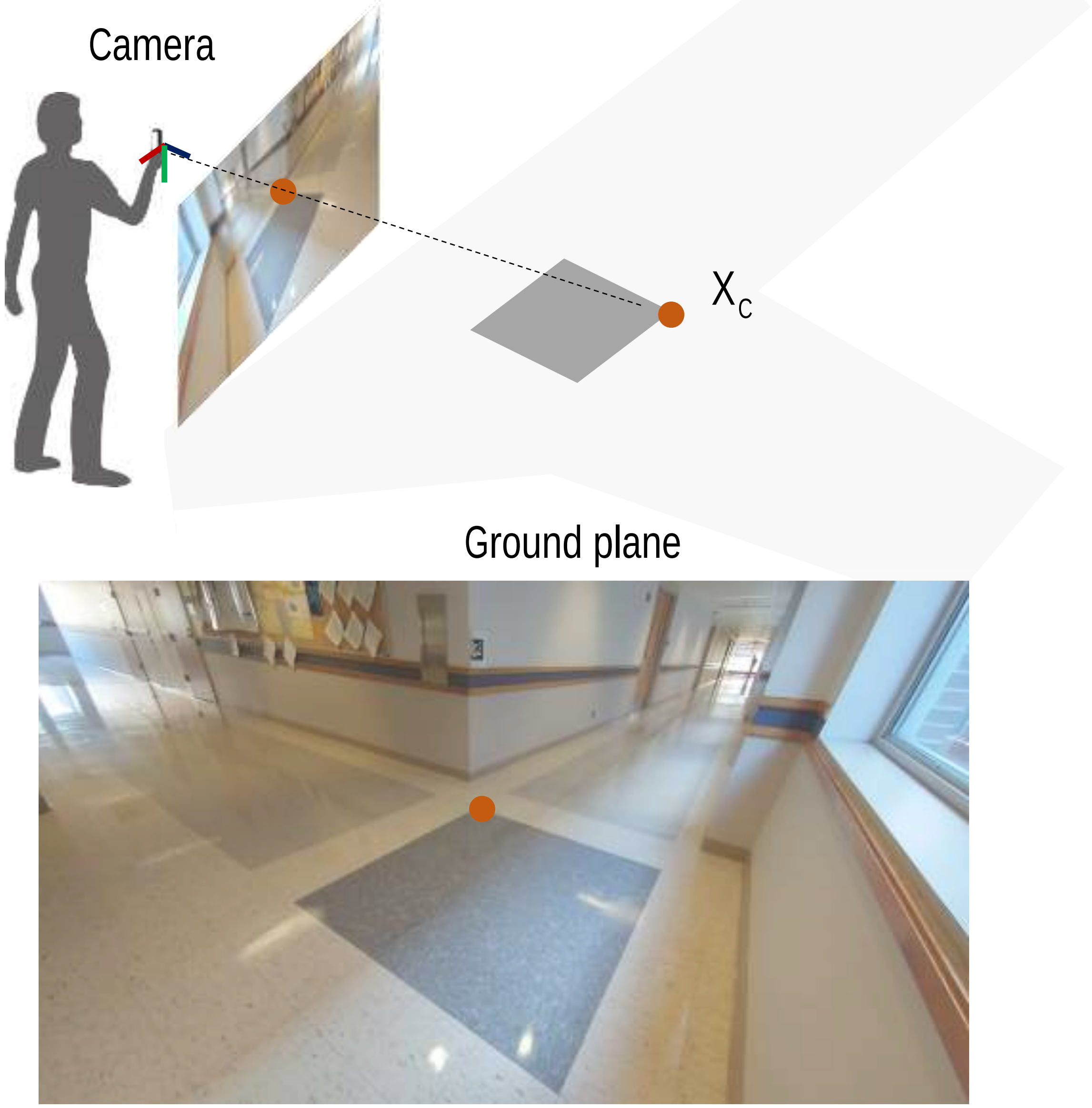
Camera Projection Matrix

Camera Model

Camera Model (1st Person Coordinate)

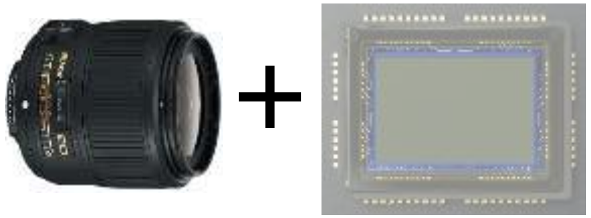


Camera Model (1st Person Coordinate)



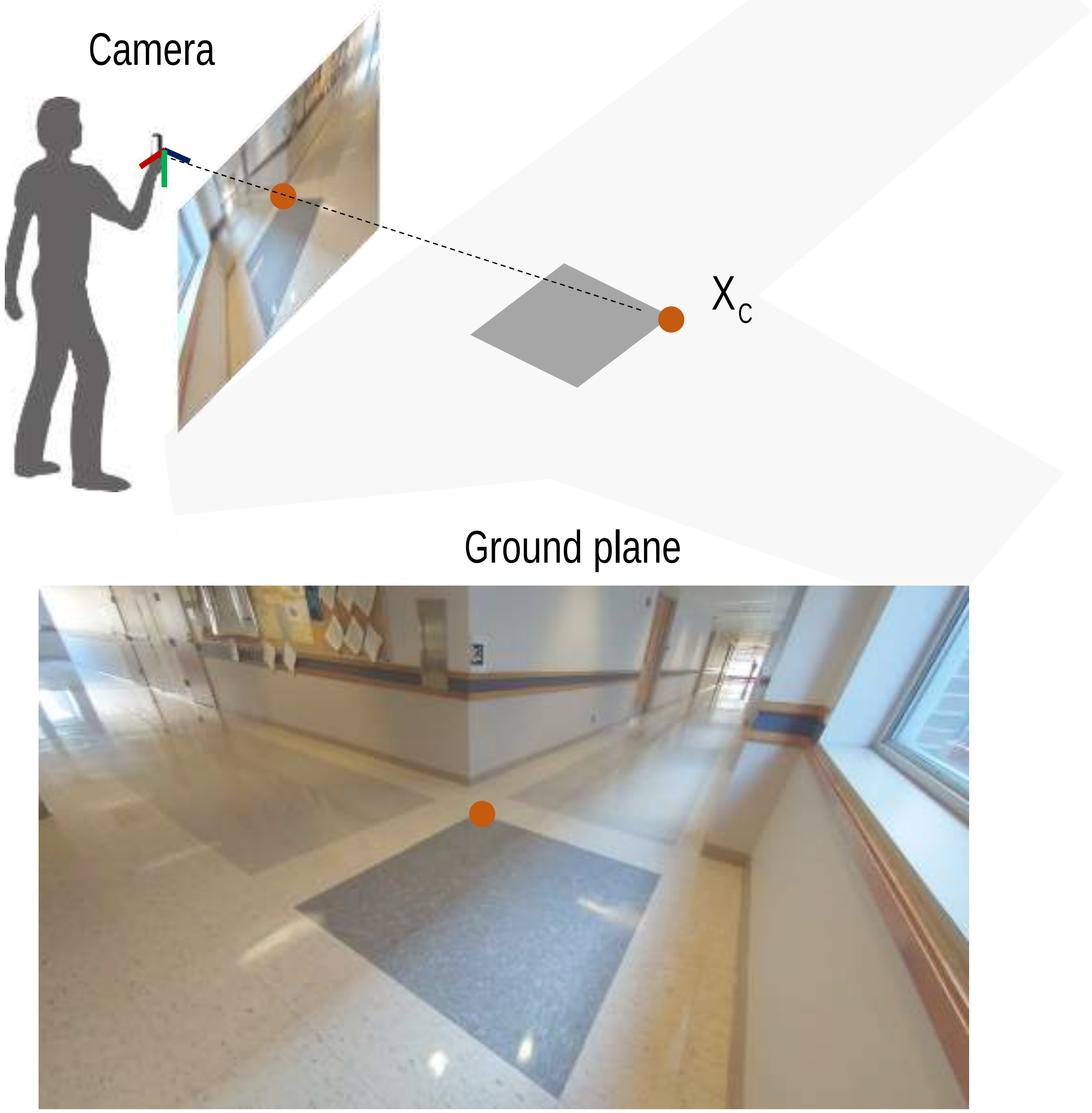
Recall camera projection matrix:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



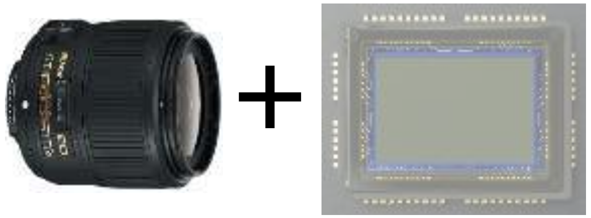
Camera intrinsic parameter
: metric space to pixel space

Camera Model (1st Person Coordinate)



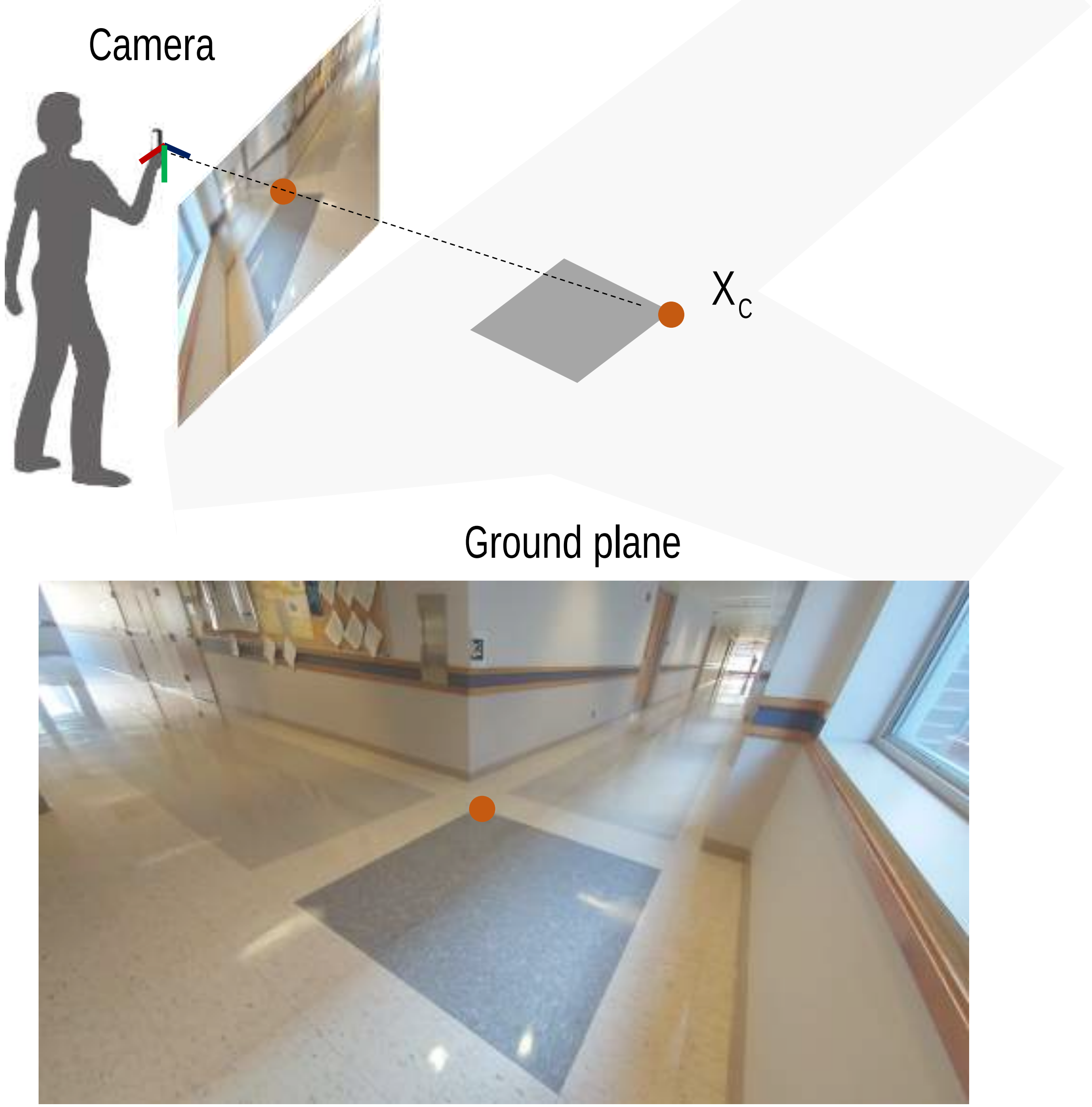
Recall camera projection matrix:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



Camera intrinsic parameter
: metric space to pixel space

Camera Model (1st Person Coordinate)

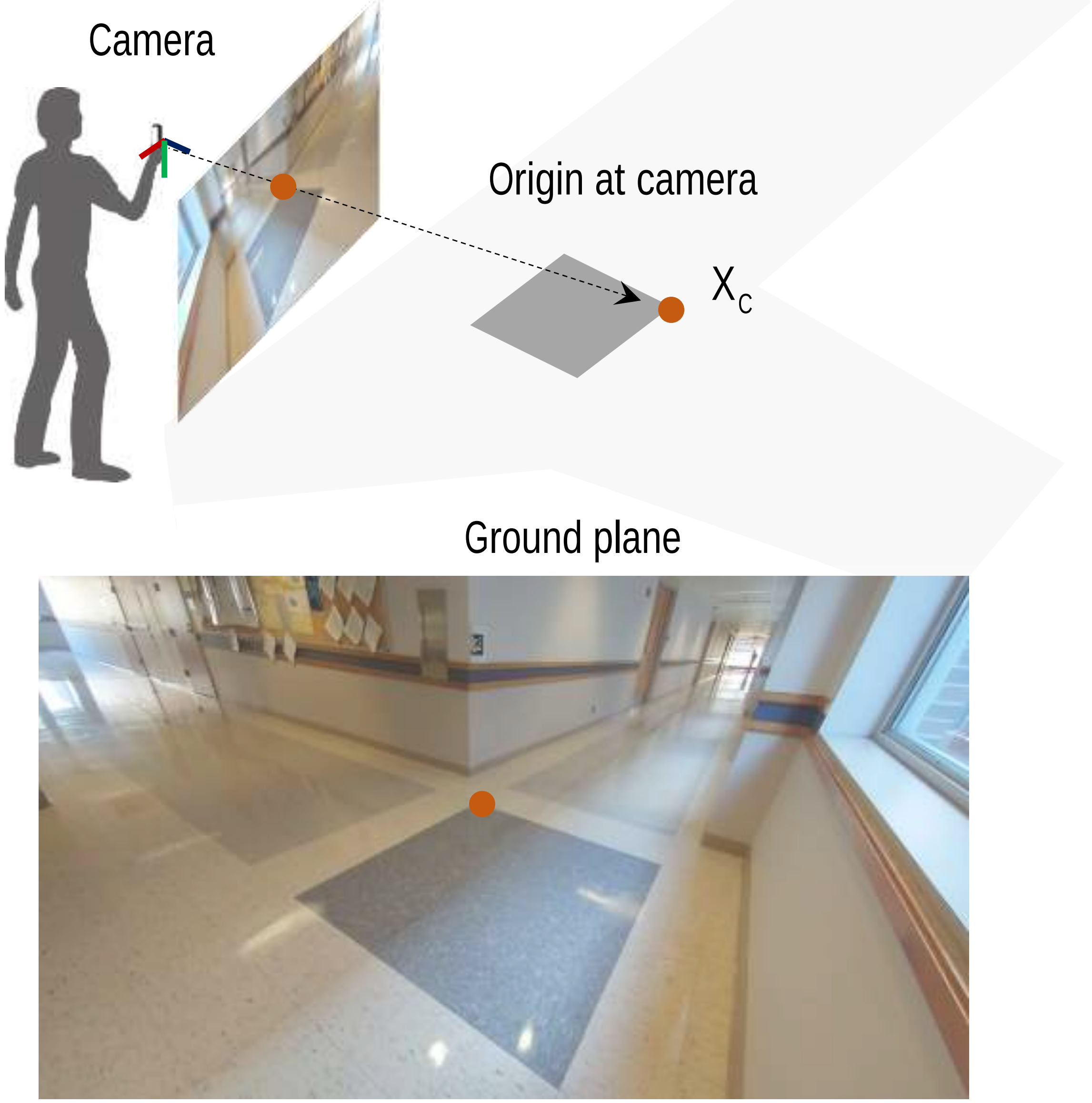


Recall camera projection matrix:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

2D image (pix) 3D world (metric)

Camera Model (1st Person Coordinate)



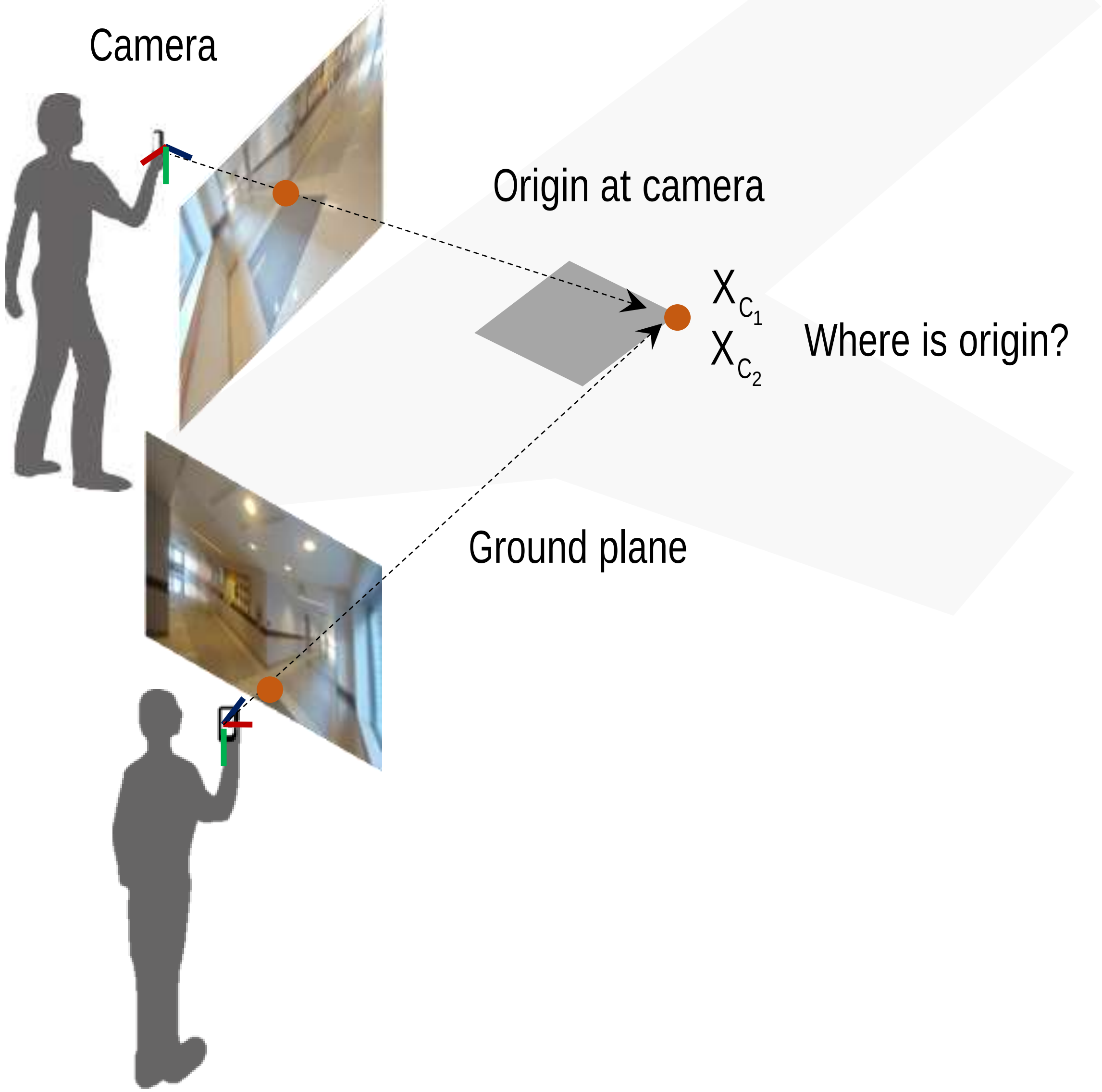
Recall camera projection matrix:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{\mathrm{K}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

2D image (pix) 3D world (metric)

$$\longrightarrow \lambda \mathrm{K}^{-1} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathrm{X}_C$$

Camera Model (1st Person Coordinate)



Recall camera projection matrix:

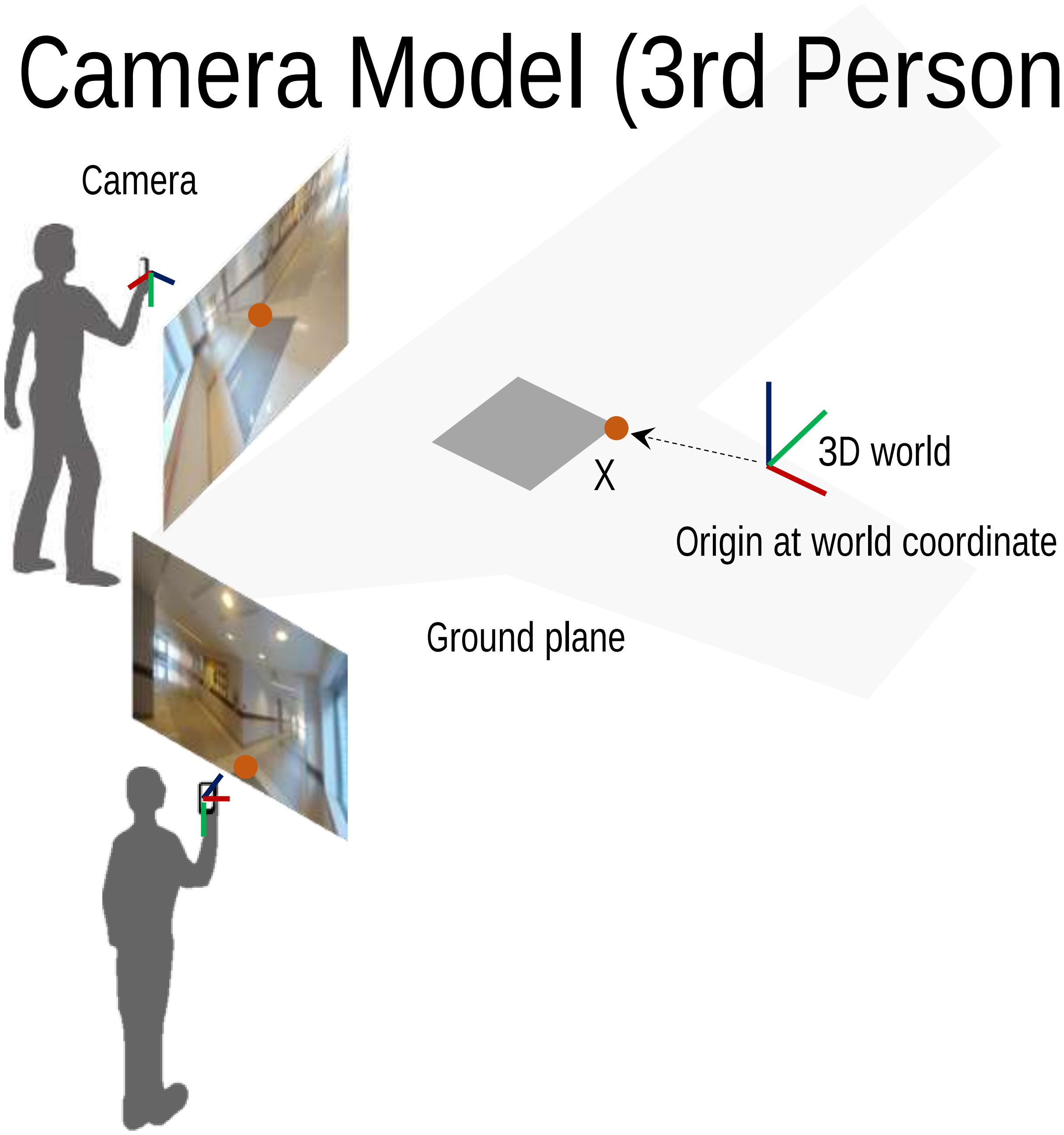
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{\mathrm{K}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

2D image (pix) 3D world (metric)

$$\rightarrow \lambda \mathrm{K}^{-1} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \mathrm{X}_{C_1}$$

$$\lambda \mathrm{K}^{-1} \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \mathrm{X}_{C_2}$$

Camera Model (3rd Person Coord. = World Coord.)



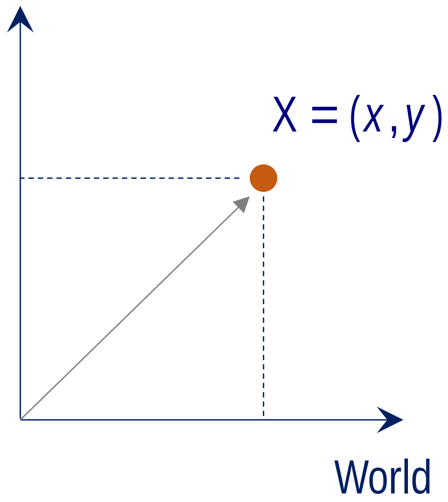
Recall camera projection matrix:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = K \mathbf{X}$$

2D image (pix) 3D world (metric)

Point Rotation

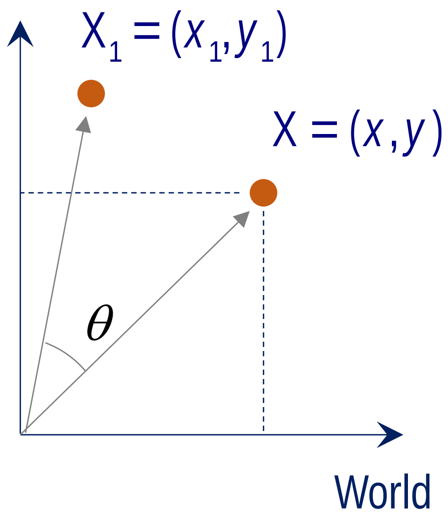
2D rotation



$$\begin{bmatrix} x \\ y \end{bmatrix}$$

Point Rotation

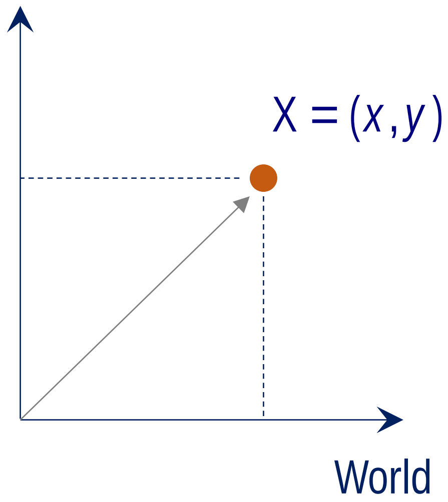
2D rotation



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Coordinate Transform (Rotation)

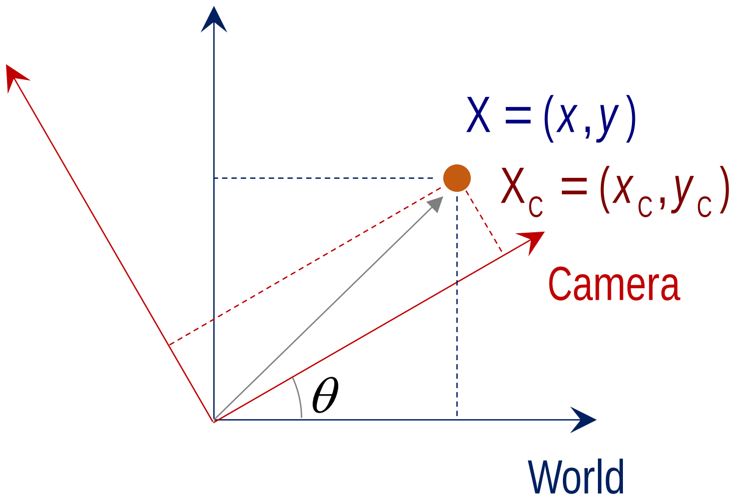
2D coordinate transform:



$$\begin{bmatrix} x \\ y \end{bmatrix}$$

Coordinate Transform (Rotation)

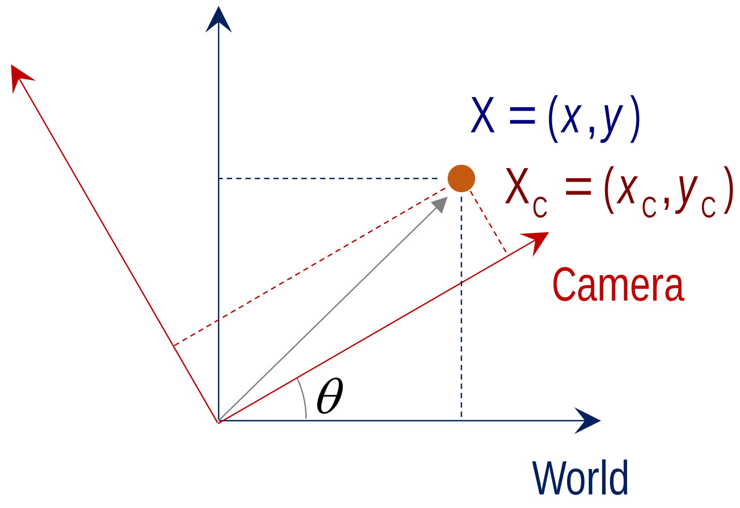
2D coordinate transform:



$$\begin{bmatrix} x_c \\ y_c \end{bmatrix} = \quad ? \quad \begin{bmatrix} x \\ y \end{bmatrix}$$

Coordinate Transform (Rotation)

2D coordinate transform:

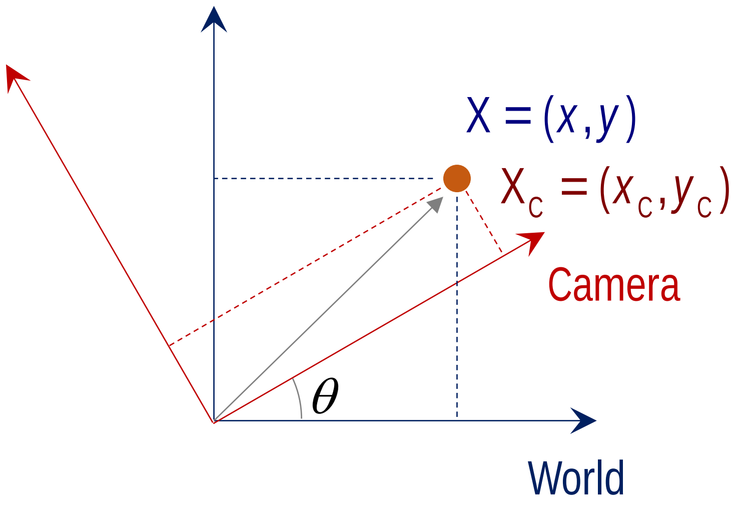


$$\begin{bmatrix} x_c \\ y_c \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Coordinate transformation: Inverse of point rotation

Coordinate Transform (Rotation)

2D coordinate transform:

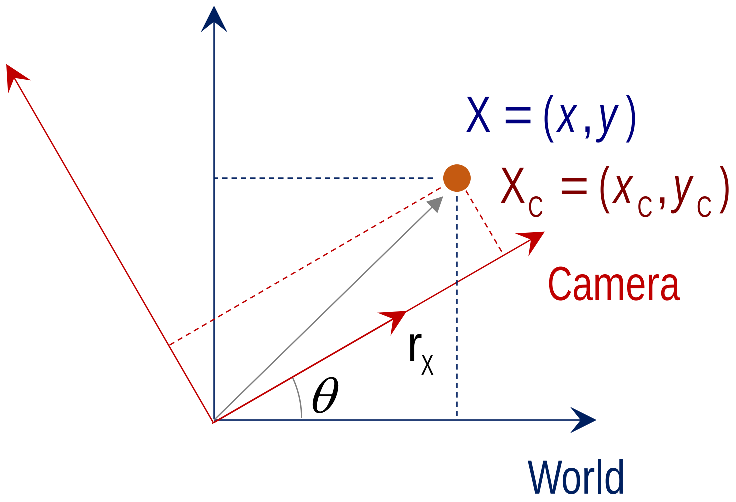


$$\begin{bmatrix} x_c \\ y_c \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\det\left(\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\right) = \cos^2\theta + \sin^2\theta = 1$$

Coordinate Transform (Rotation)

2D coordinate transform:



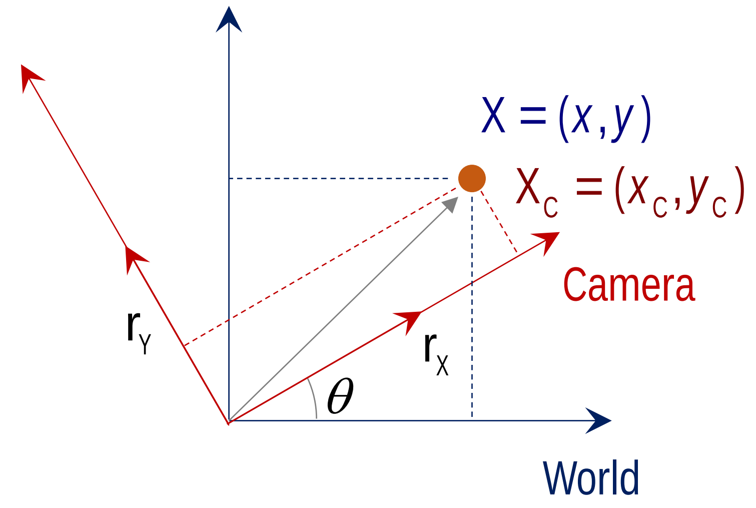
$$\begin{bmatrix} x_c \\ y_c \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

r_x

r_x : x axis of camera seen from the world

Coordinate Transform (Rotation)

2D coordinate transform:



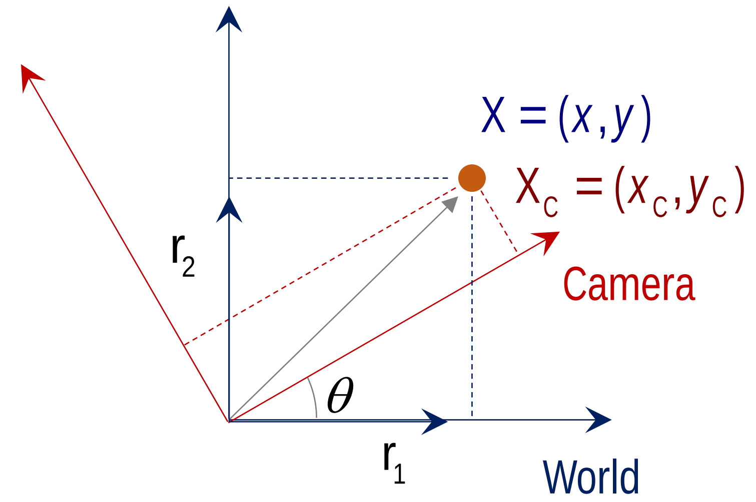
$$\begin{bmatrix} x_c \\ y_c \end{bmatrix} = \begin{array}{c} r_X \\ r_Y \end{array} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

r_X : x axis of camera seen from the world

r_Y : y axis of camera seen from the world

Coordinate Transform (Rotation)

2D coordinate transform:



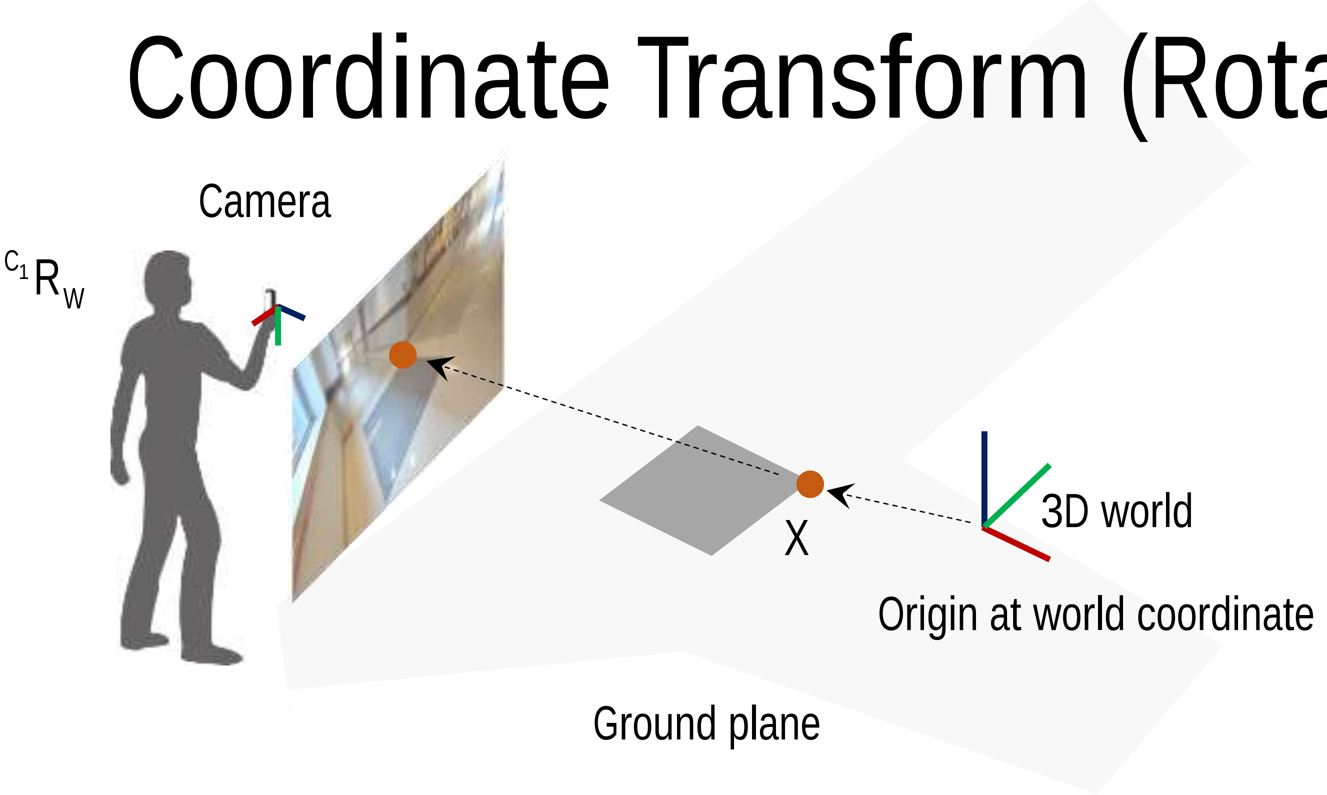
$$\begin{bmatrix} x_c \\ y_c \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

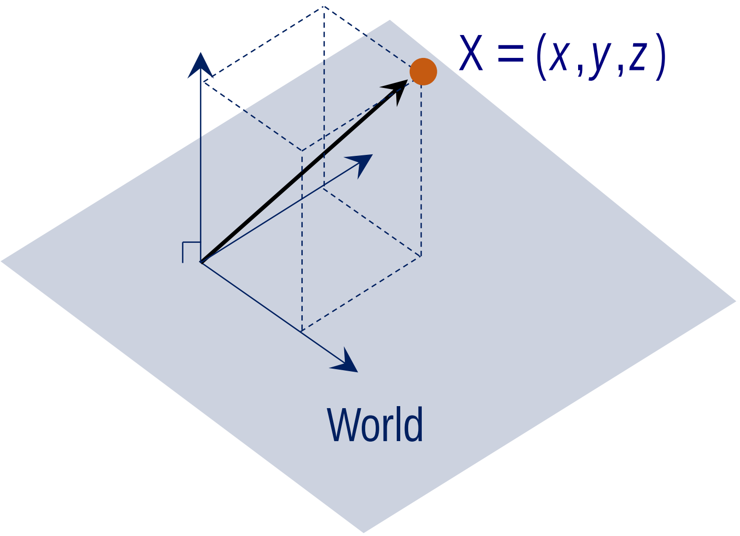
$$\begin{array}{cc} r_1 & r_2 \end{array}$$

r_1 : x axis of world seen from the camera

r_2 : y axis of world seen from the camera

Coordinate Transform (Rotation)

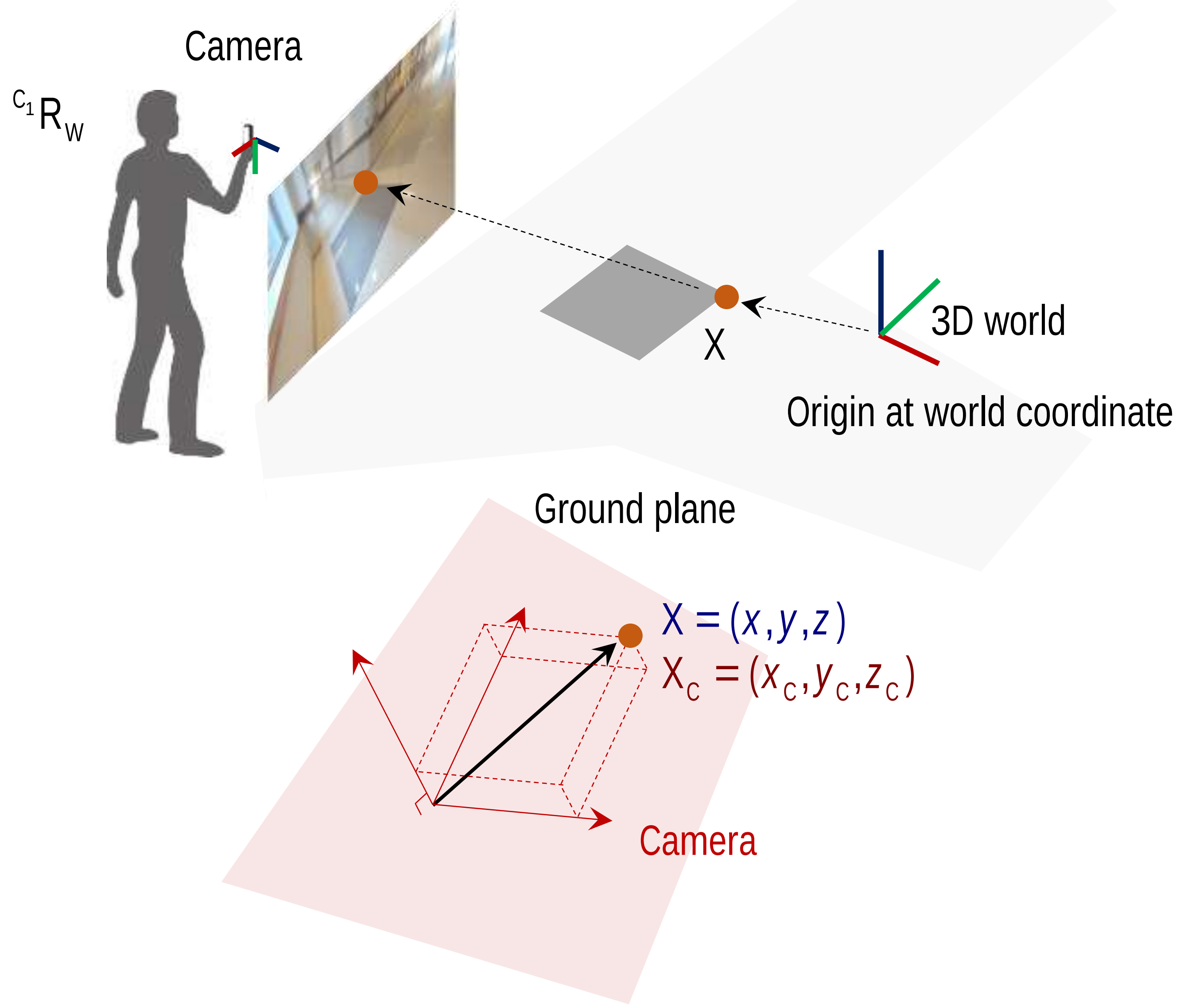




Coordinate transformation from world to camera:

$$X_C = \quad ? \quad X$$

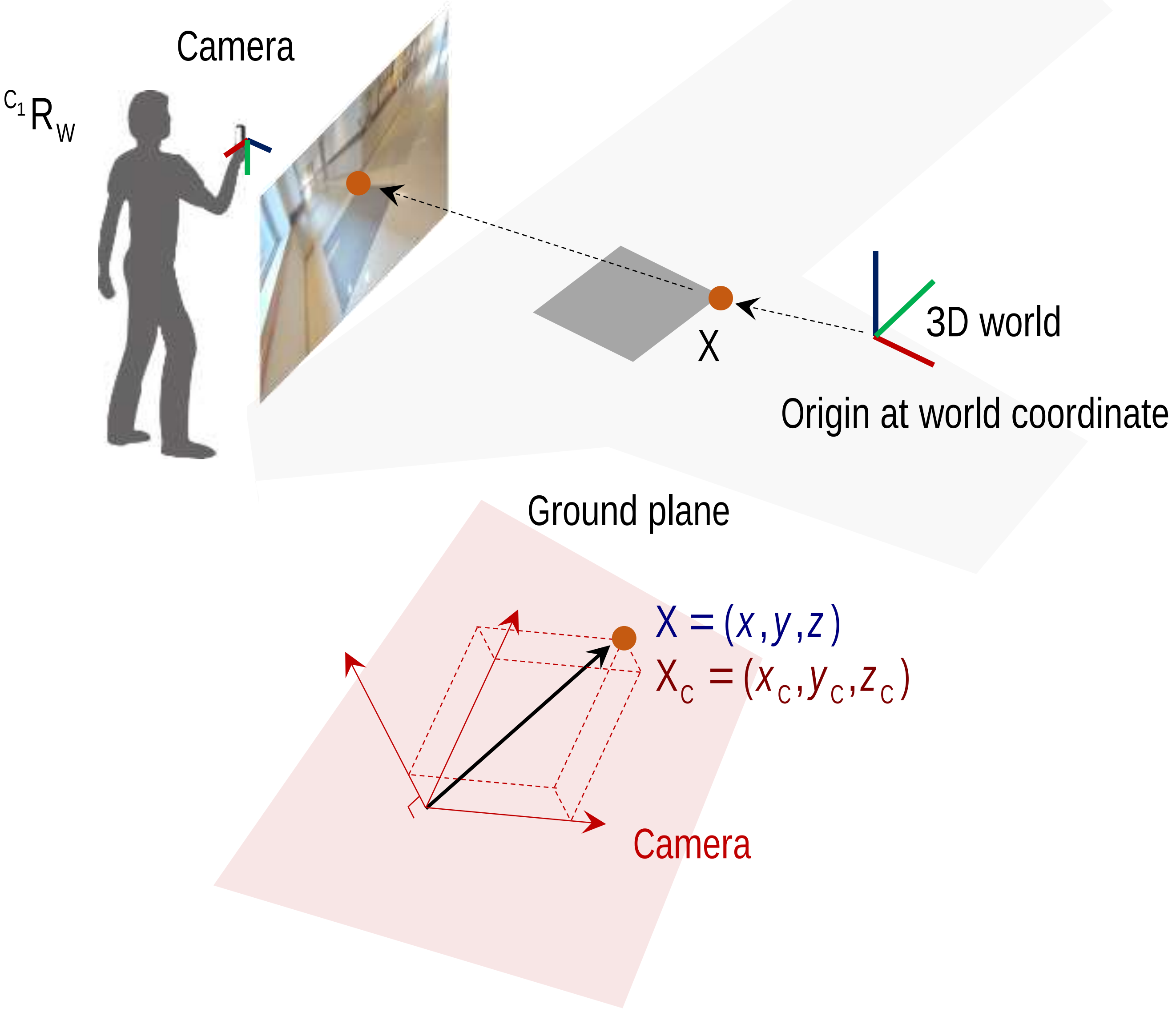
Coordinate Transform (Rotation)



Coordinate transformation from world to camera:

$X_C = \quad ? \quad X$

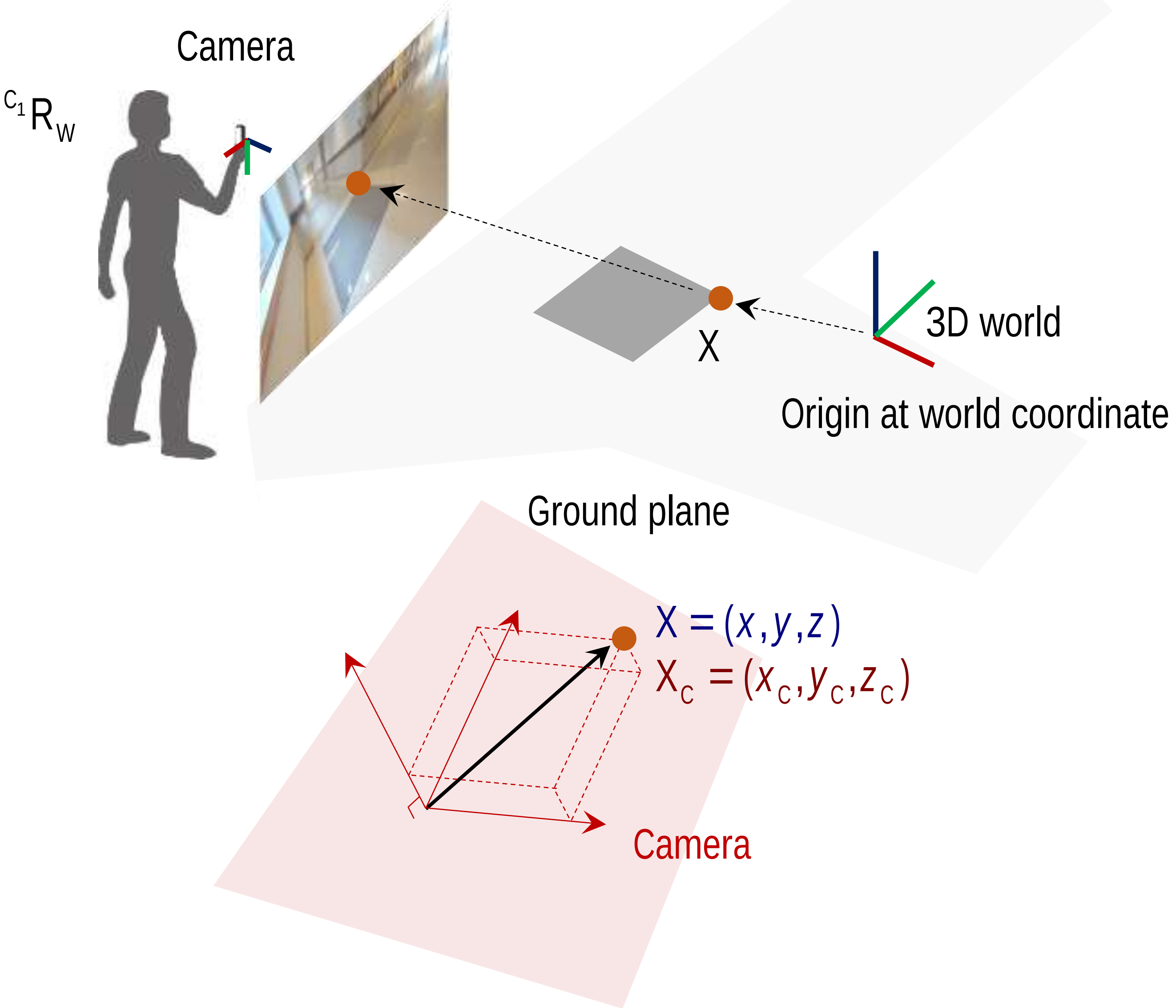
Coordinate Transform (Rotation)



Coordinate transformation from world to camera:

$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = {}^{C}R_W X$$

Coordinate Transform (Rotation)

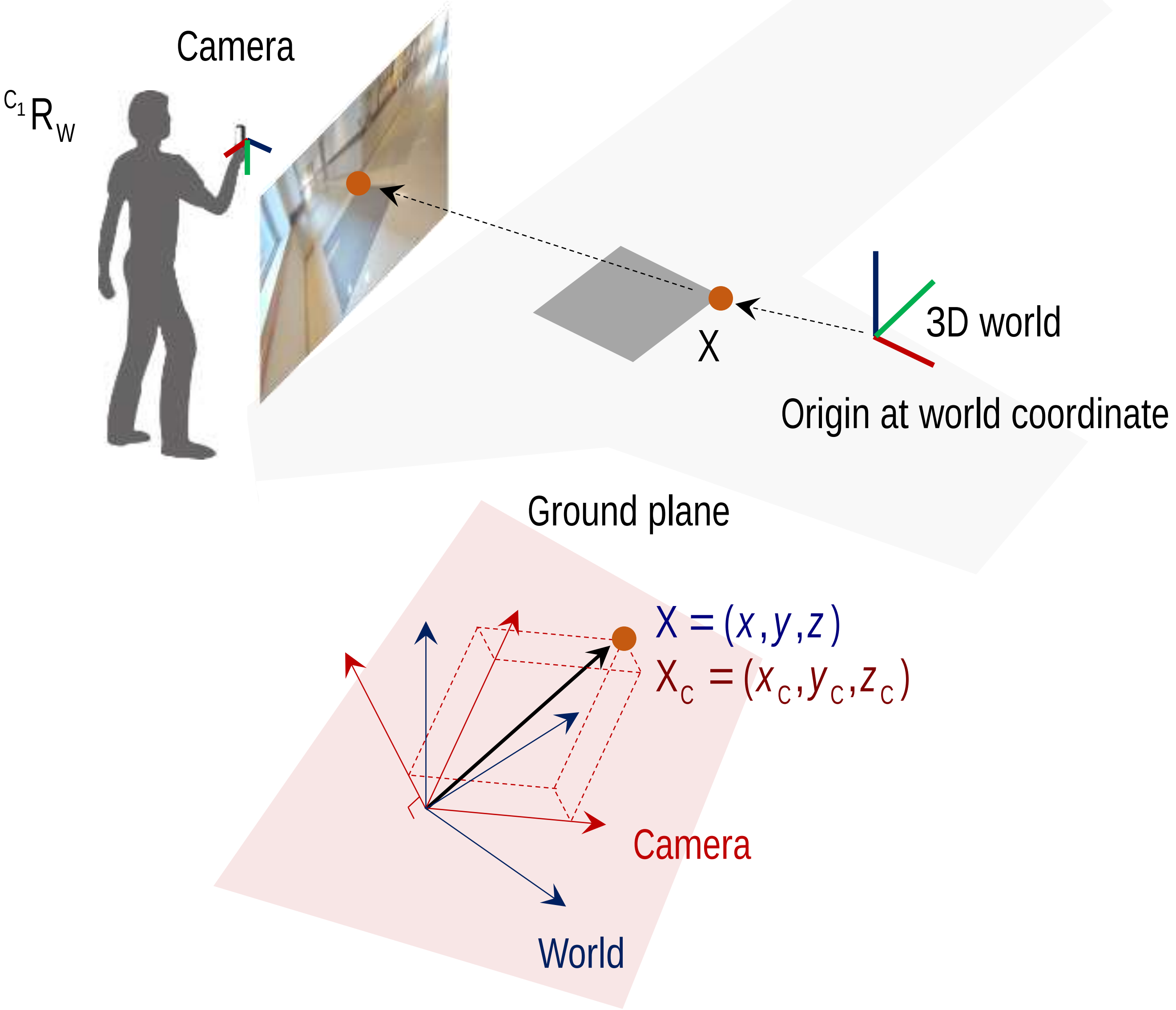


Coordinate transformation from world to camera:

$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = {}^{C}R_{W} X$$

Degree of freedom?

Coordinate Transform (Rotation)



Coordinate transformation from world to camera:

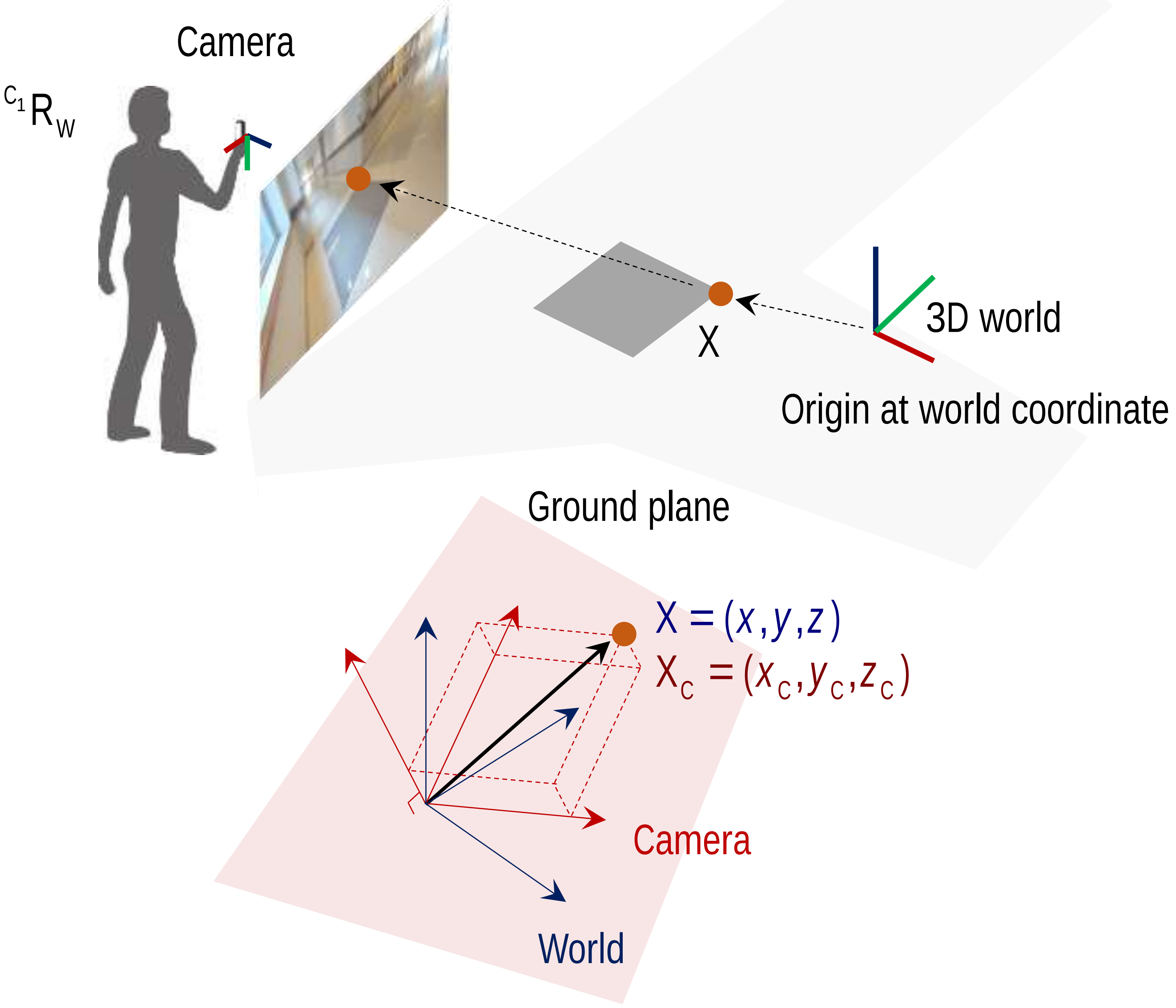
$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = {}^C R_W X$$

Degree of freedom?

$${}^C R_W \in SO(3)$$

- Orthogonal matrix $\longrightarrow$ $\left({}^C R_W\right)^T \left({}^C R_W\right) = I_3, \quad \det\left({}^C R_W\right) = 1$

Coordinate Transform (Rotation)



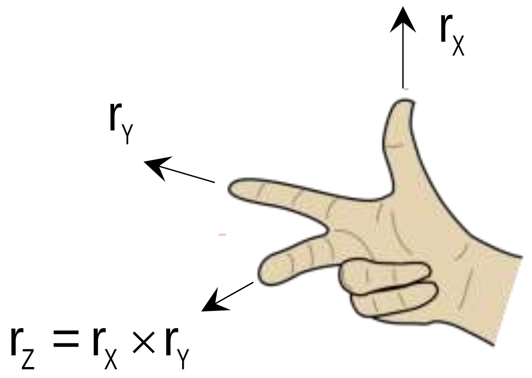
Coordinate transformation from world to camera:

$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = {}^{C}R_W X$$

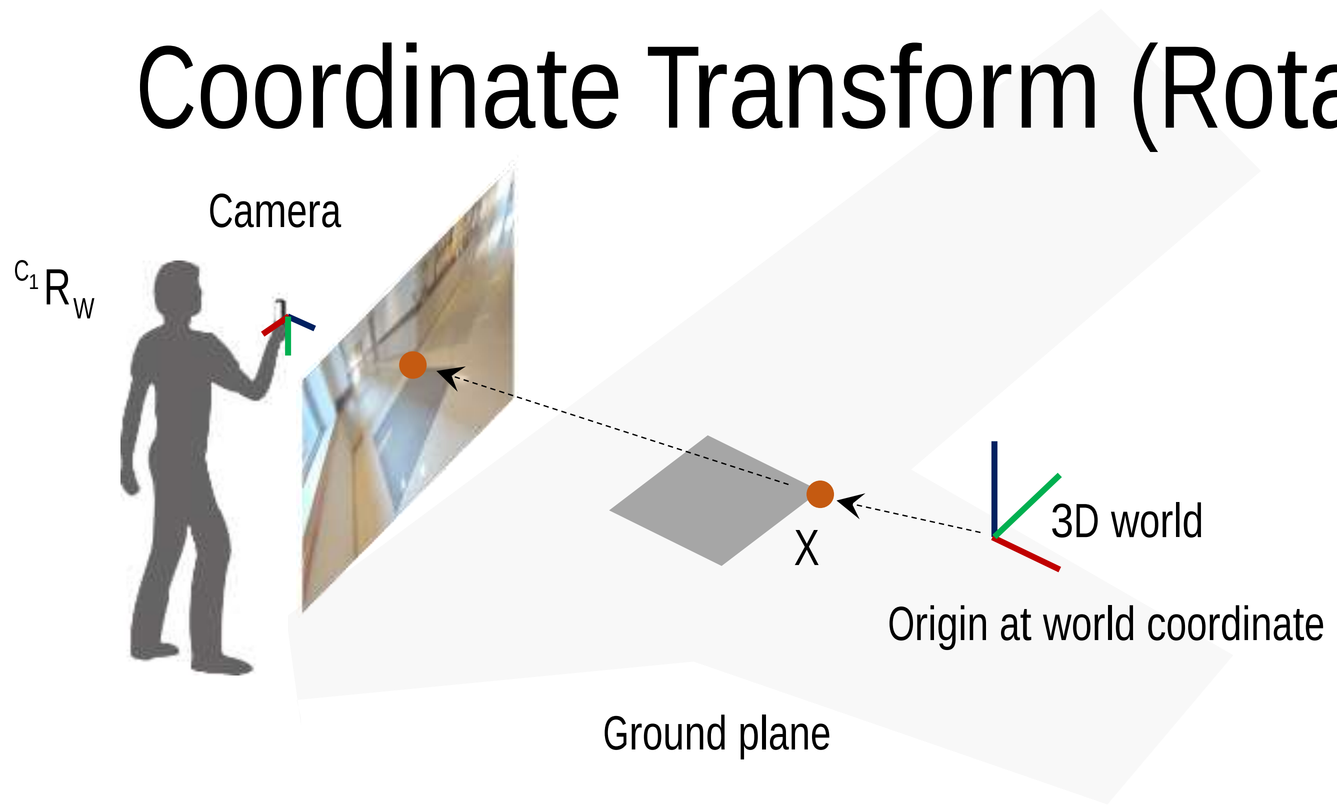
$$\begin{bmatrix} r_X \\ r_Y \\ r_Z \end{bmatrix}$$

${}^{C}R_W \in SO(3)$

- Orthogonal matrix $\longrightarrow \left({}^{C}R_W\right)^T \left({}^{C}R_W\right) = I_3, \quad \det\left({}^{C}R_W\right) = 1$
- Right hand rule



Coordinate Transform (Rotation)

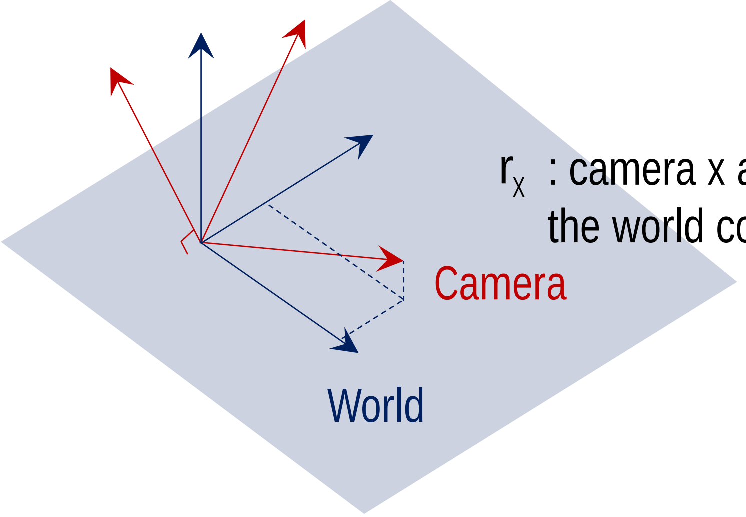


Coordinate transformation from world to camera:

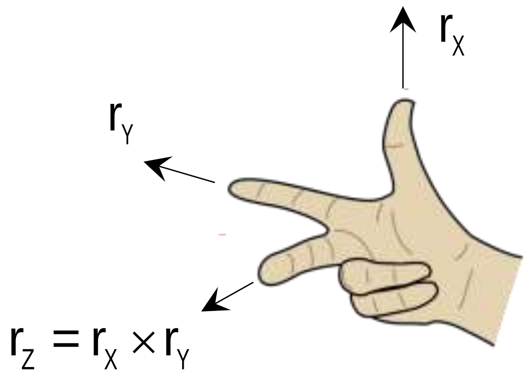
$$X_C = \begin{array}{c} r_X \\ r_Y \\ r_Z \end{array} \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = {}^{C}R_W X$$

$${}^{C}R_W \in SO(3)$$

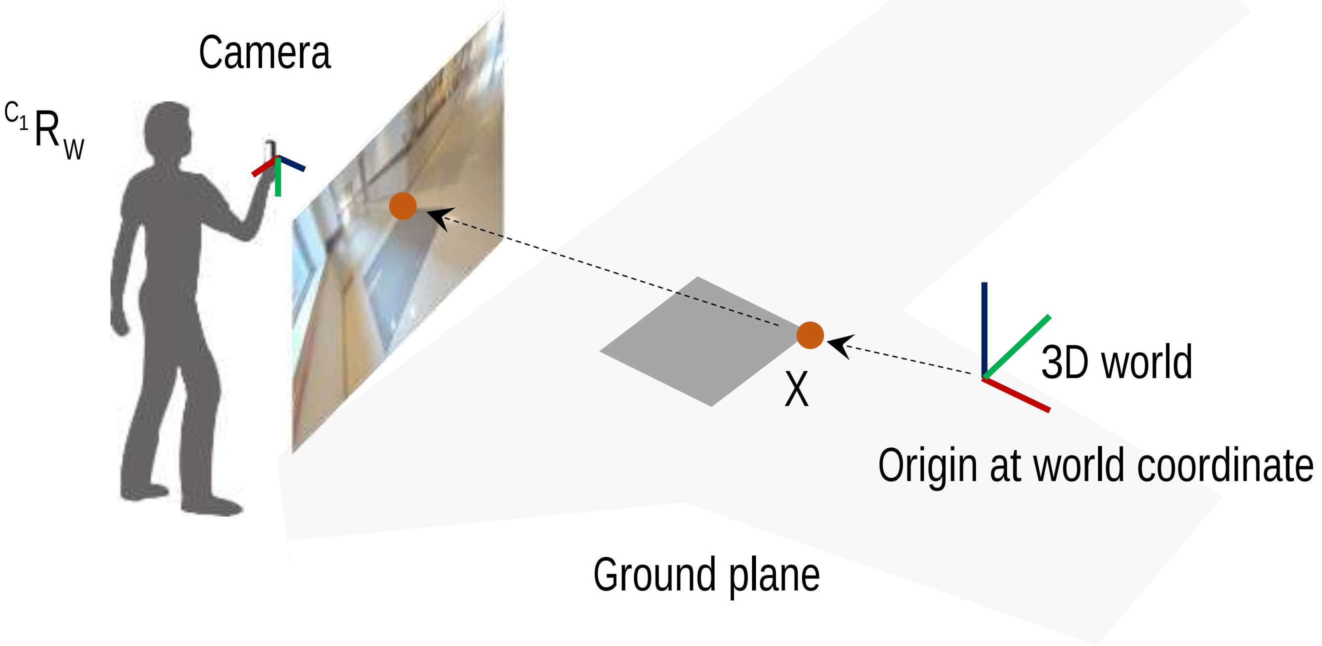
- Orthogonal matrix $\longrightarrow \left({}^{C}R_W\right)^T \left({}^{C}R_W\right) = I_3, \quad \det\left({}^{C}R_W\right) = 1$
- Right hand rule



r_X : camera x axis seen from the world coord.



Coordinate Transform (Rotation)



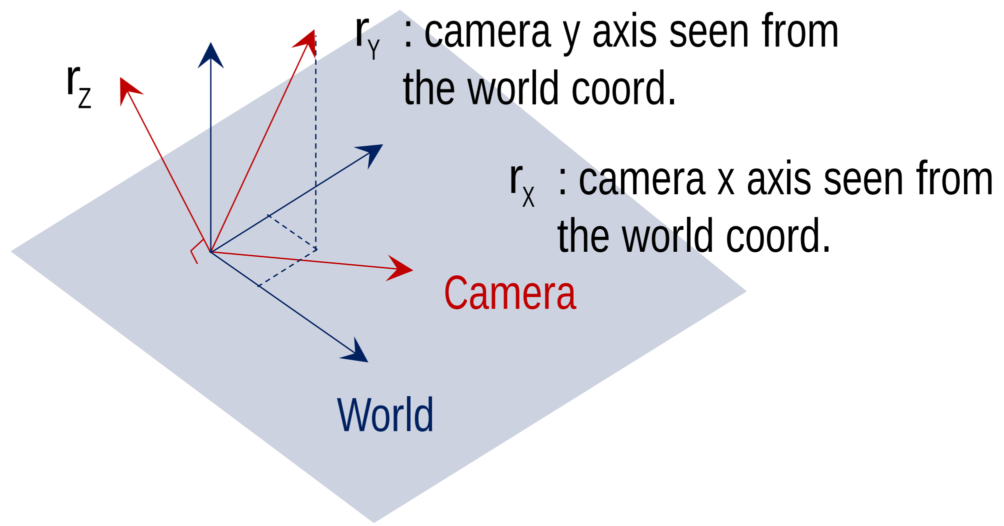
Coordinate transformation from world to camera:

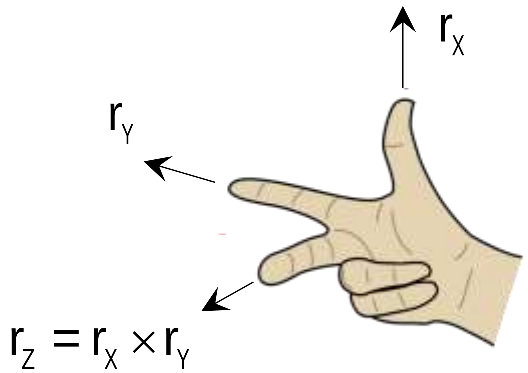
$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = {}^{C}R_W X$$

with the rows labelled r_X, r_Y, r_Z.

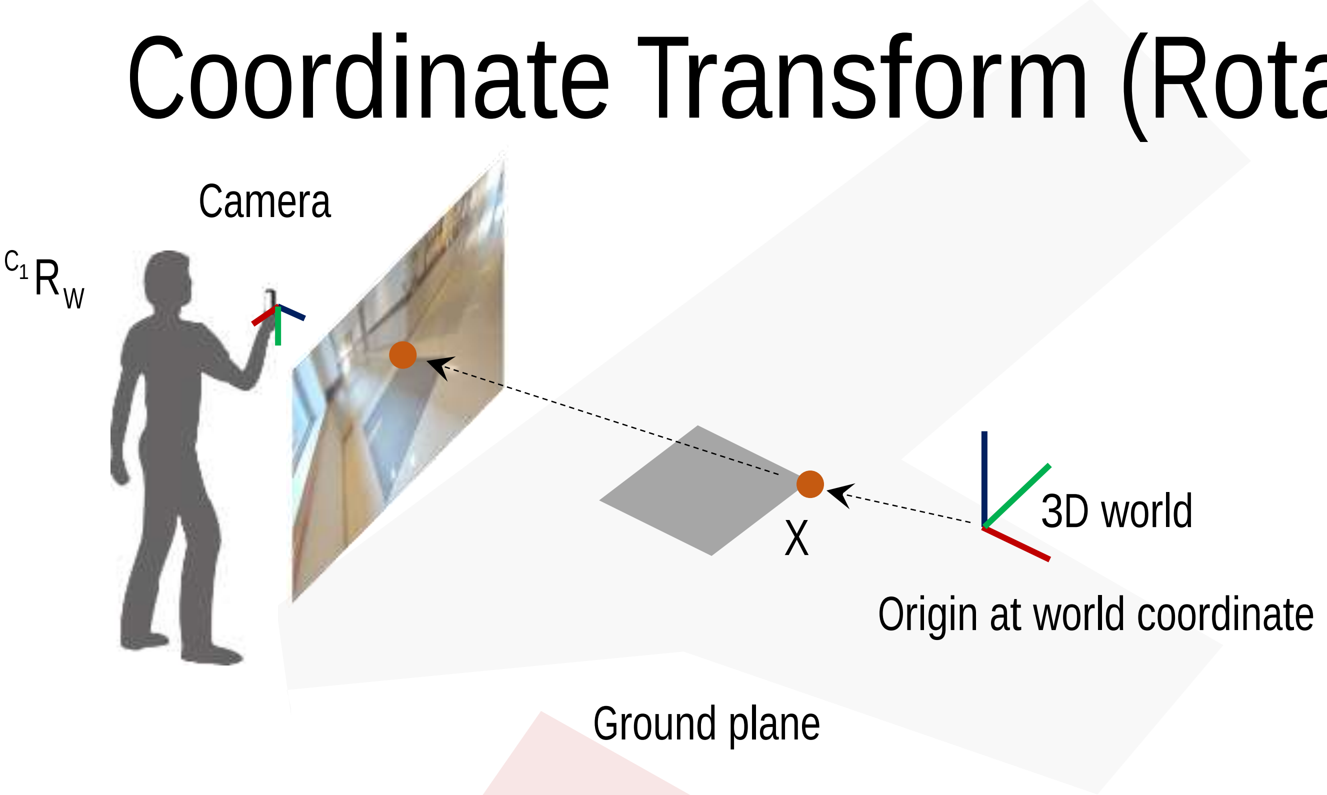
$$^{C}R_W \in SO(3)$$

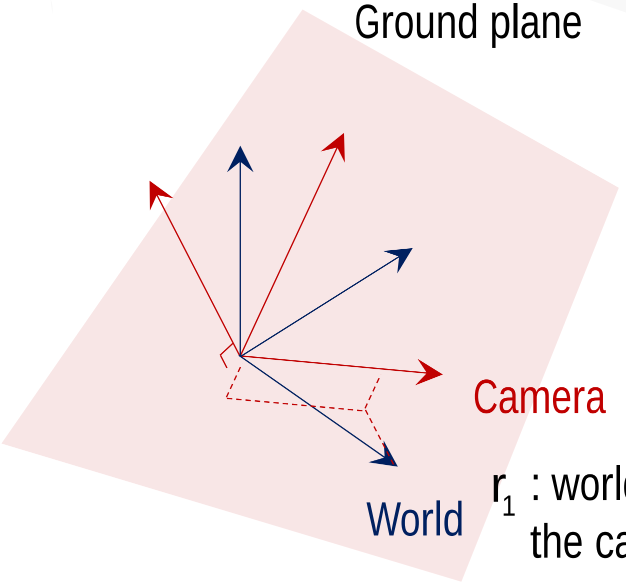
- Orthogonal matrix $\longrightarrow$ $\left({}^{C}R_W\right)^T \left({}^{C}R_W\right) = I_3, \quad \det\left({}^{C}R_W\right) = 1$
- Right hand rule





Coordinate Transform (Rotation)





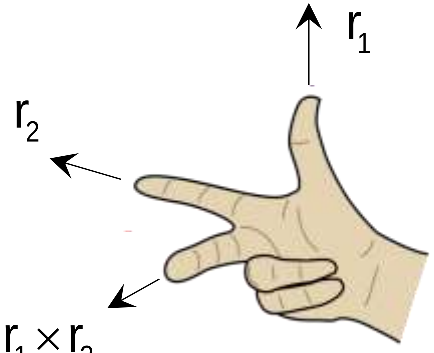
r_1 : world x axis seen from the camera coord.

Coordinate transformation from world to camera:

$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix} X = {}^{C}R_W X$$

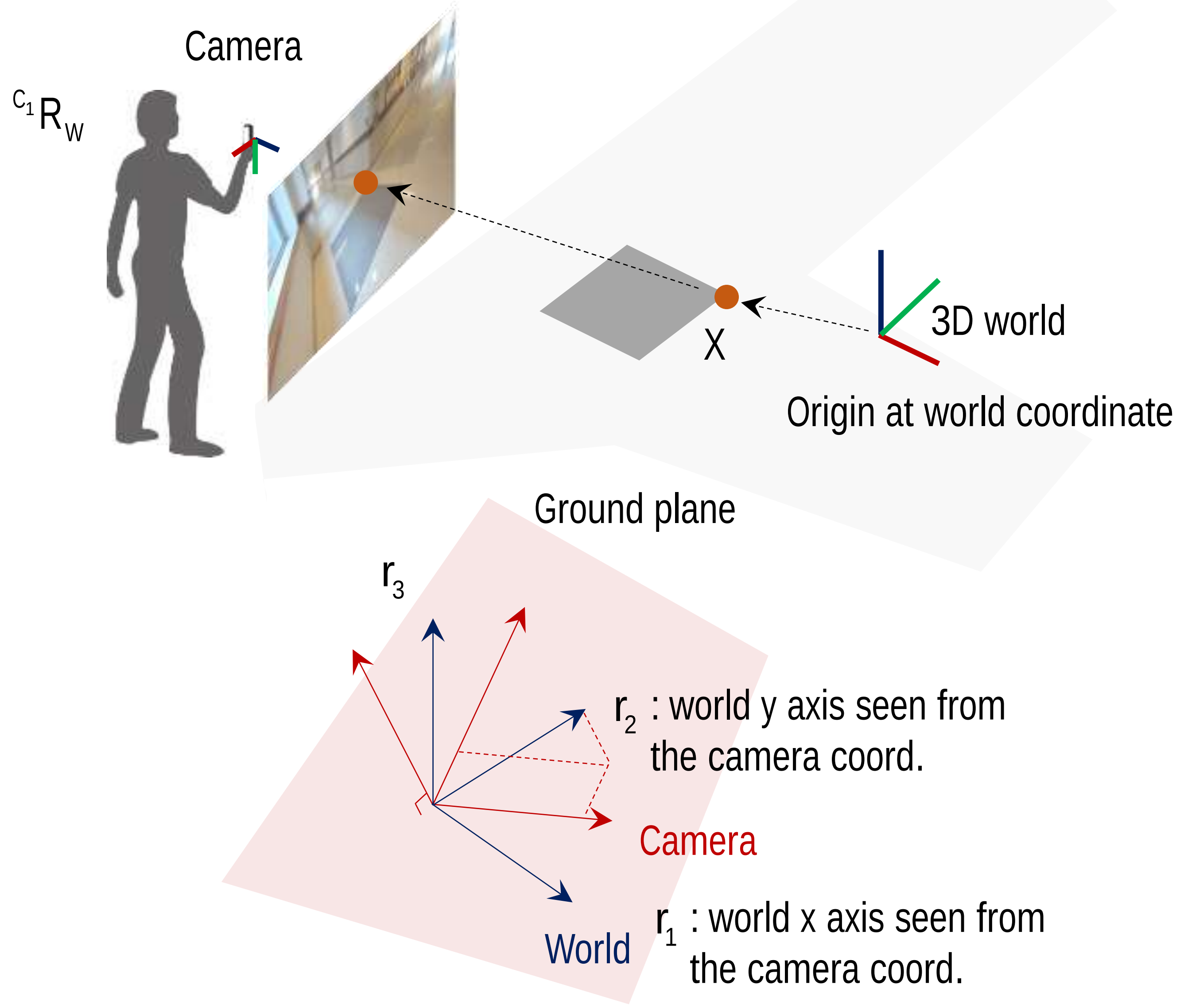
$${}^{C}R_W \in SO(3)$$

- Orthogonal matrix $\longrightarrow \left({}^{C}R_W\right)^T\left({}^{C}R_W\right) = I_3, \quad \det\left({}^{C}R_W\right) = 1$
- Right hand rule



$r_3 = r_1 \times r_2$

Coordinate Transform (Rotation)

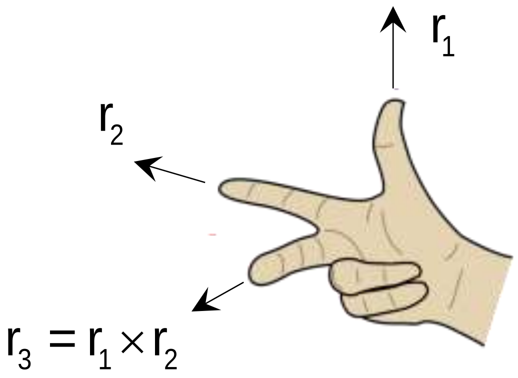


Coordinate transformation from world to camera:

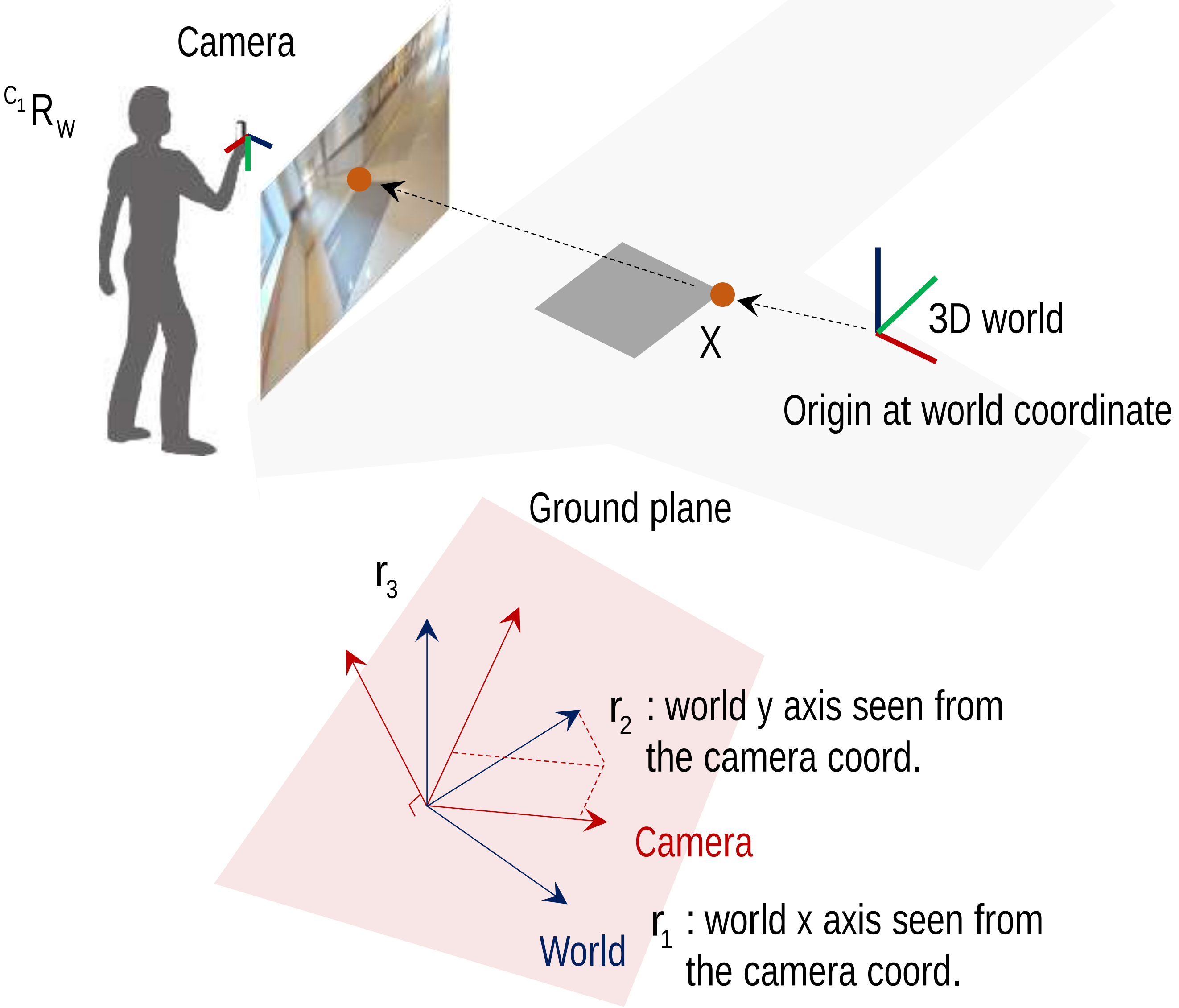
$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix} X = {}^C R_W X$$

$${}^C R_W \in SO(3)$$

- Orthogonal matrix $\longrightarrow \left({}^C R_W\right)^T \left({}^C R_W\right) = I_3, \quad \det\left({}^C R_W\right) = 1$
- Right hand rule



Camera Projection (Pure Rotation)



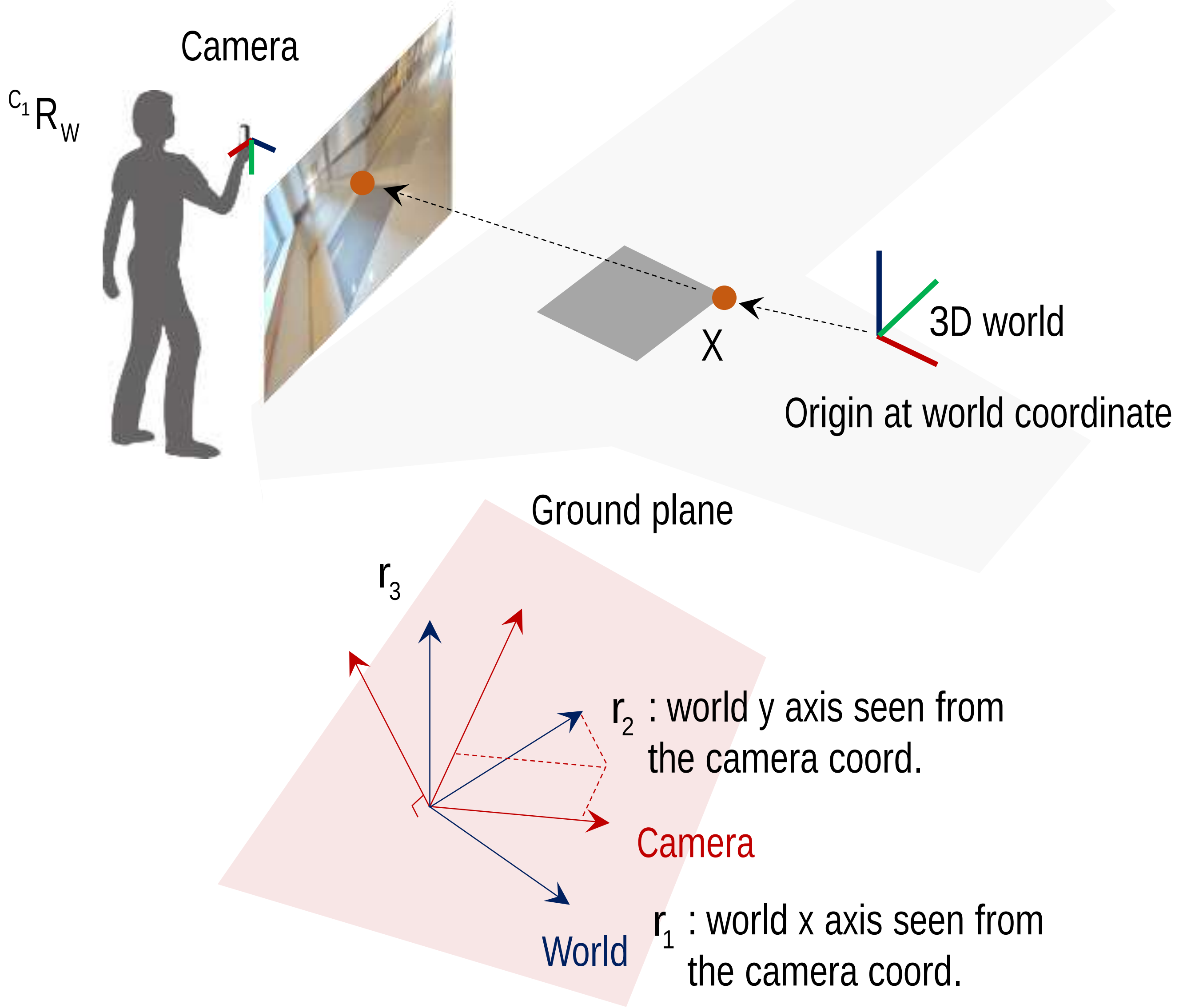
Coordinate transformation from world to camera:

$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix} X = {}^{C}R_W X$$

Camera projection of world point:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix}$$

Camera Projection (Pure Rotation)



Coordinate transformation from world to camera:

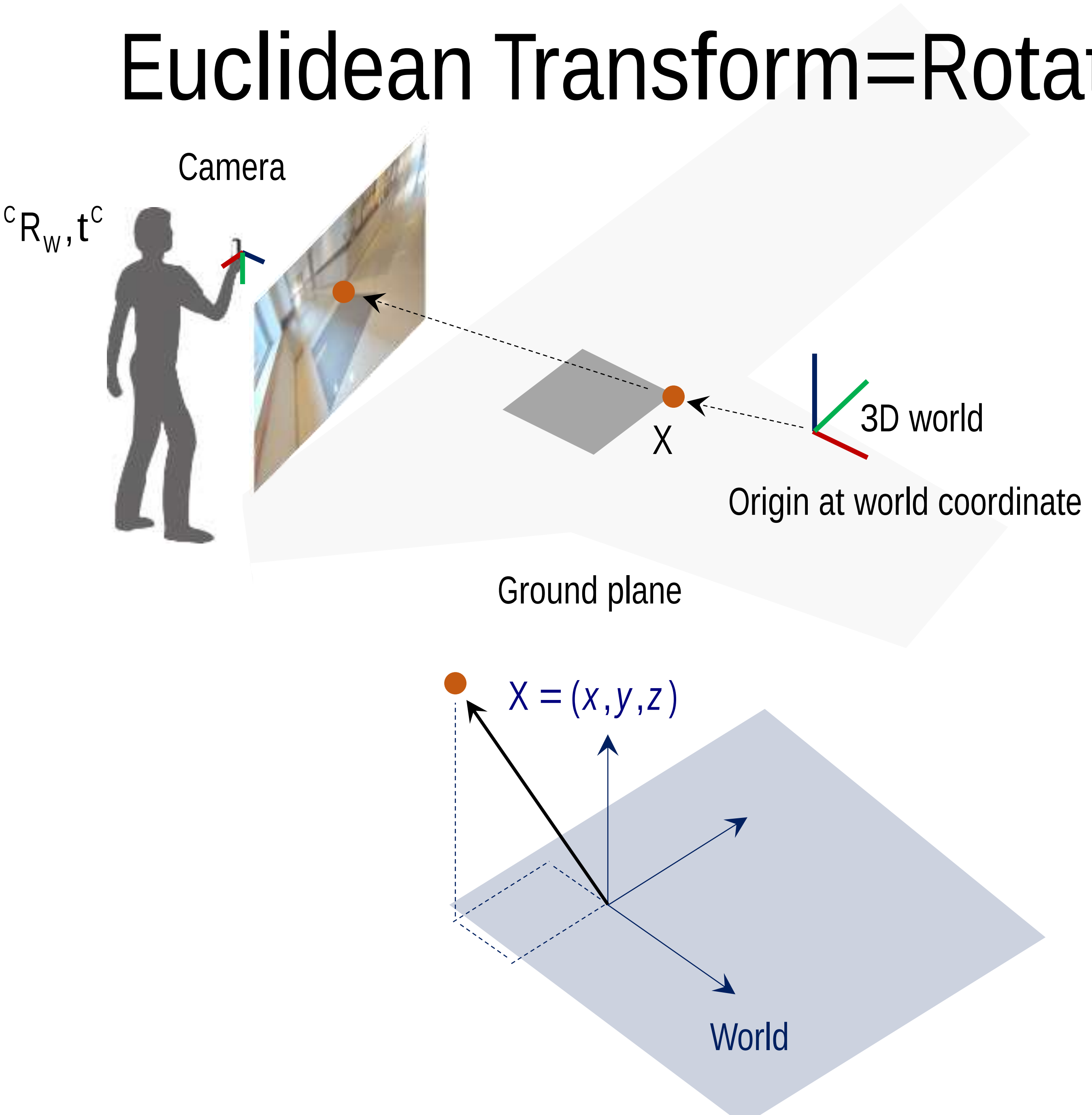
$$X_C = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} X = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix} X = {}^{C}R_W X$$

Camera projection of world point:

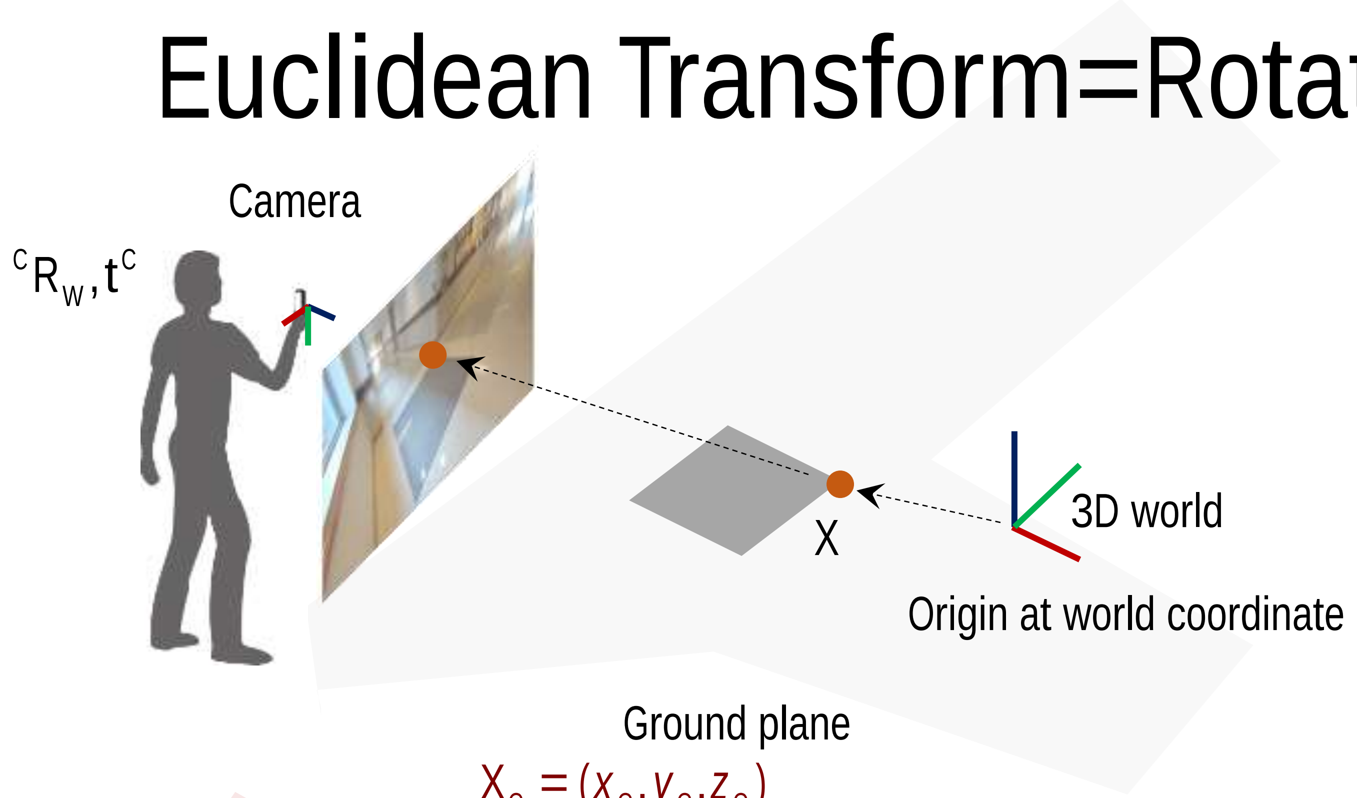
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix} = K \begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix}$$

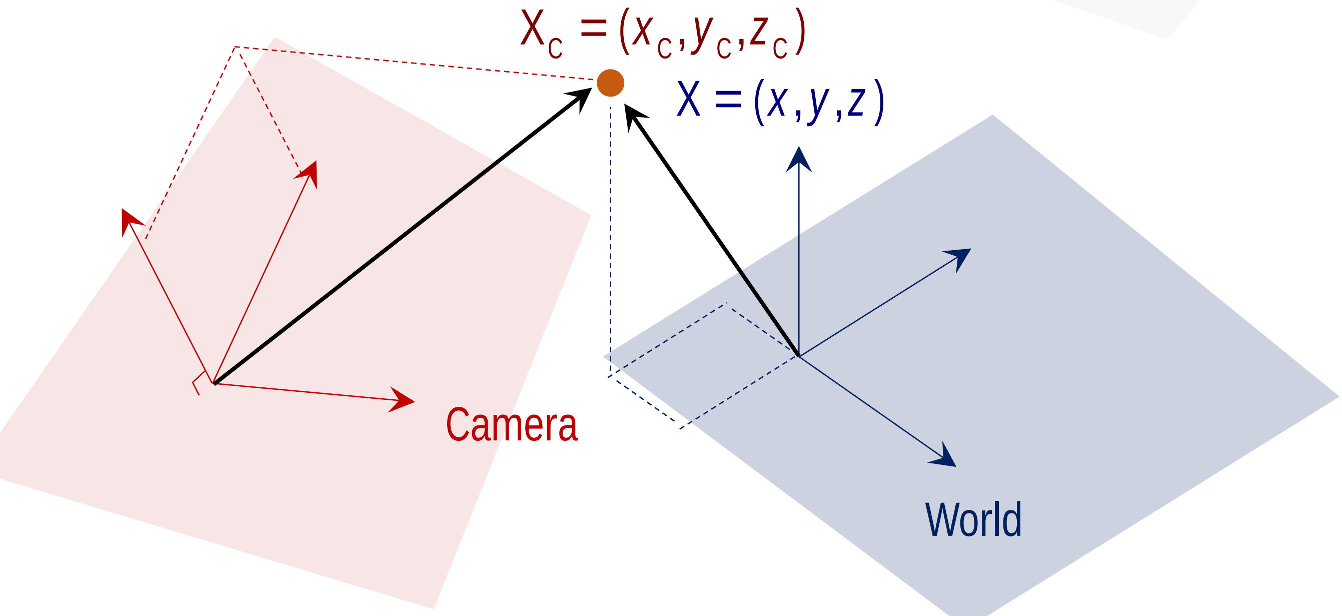
$$= \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = K \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Euclidean Transform=Rotation+Translation



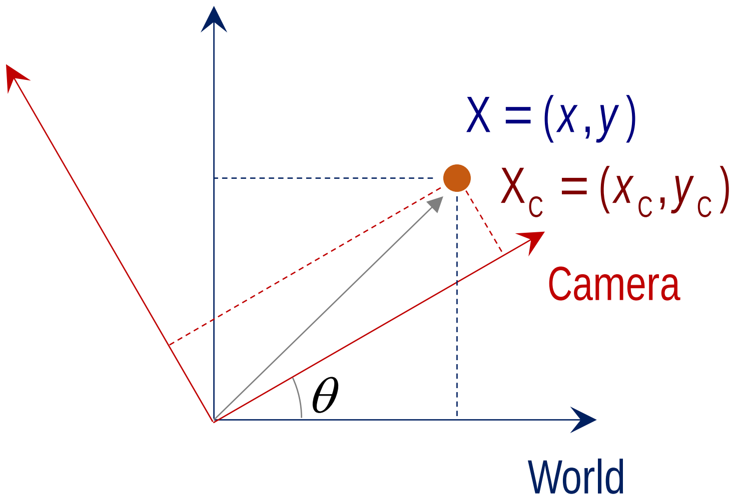
Euclidean Transform=Rotation+Translation





Euclidean Transform=Rotation+Translation

2D coordinate transform:

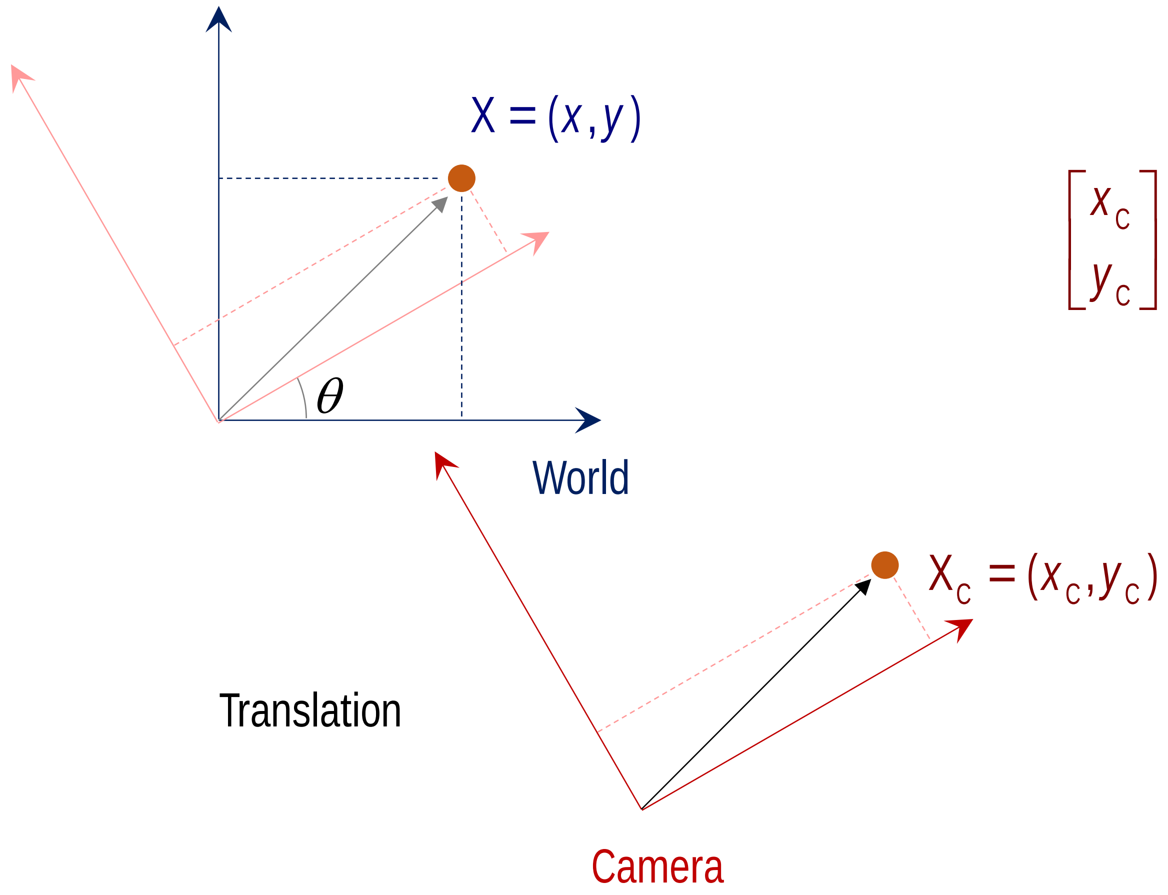


$$\begin{bmatrix} x_c \\ y_c \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Coordinate transformation: Inverse of point rotation

Euclidean Transform=Rotation+Translation

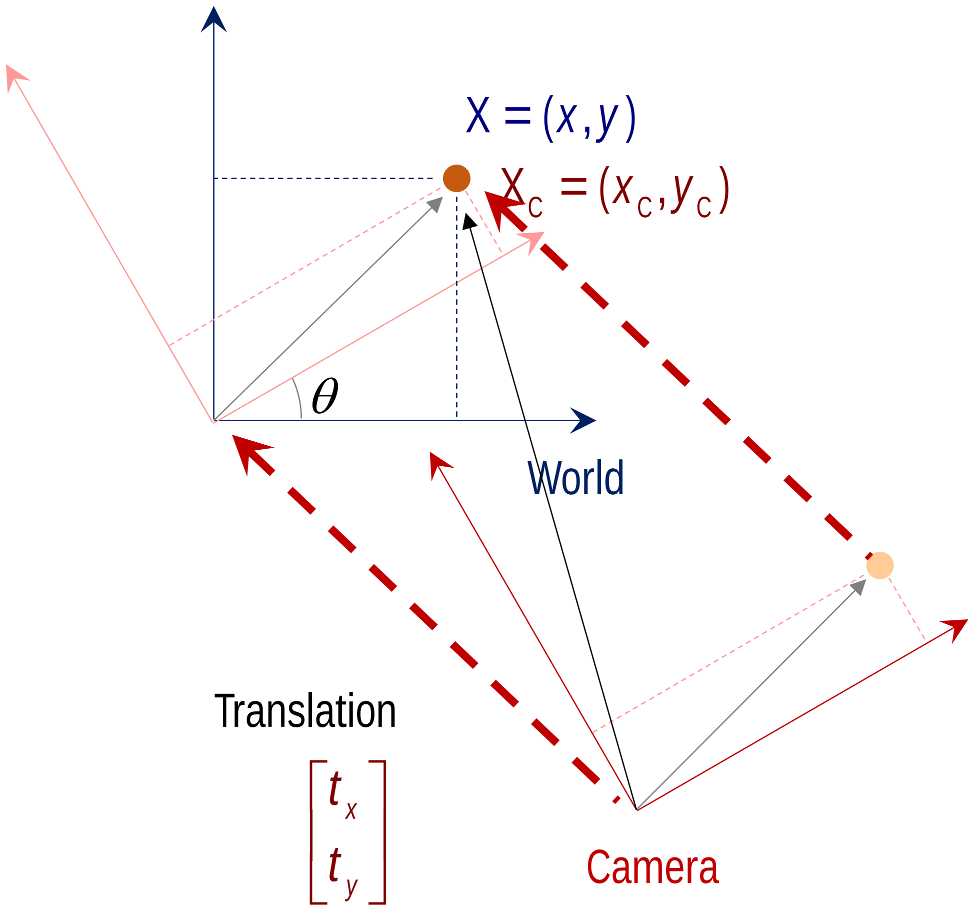
2D coordinate transform:



$$\begin{bmatrix} x_c \\ y_c \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Euclidean Transform=Rotation+Translation

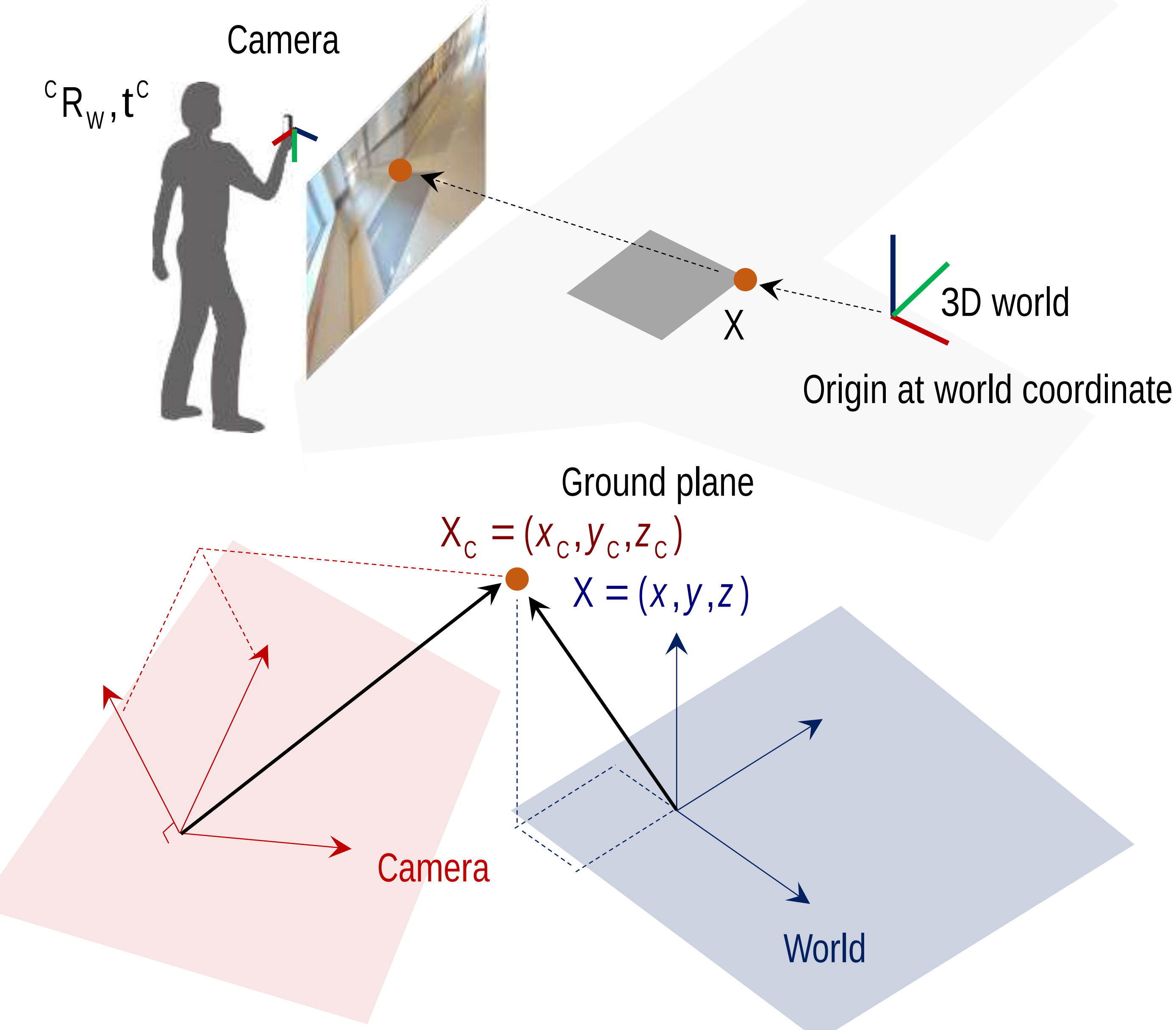
2D coordinate transform:



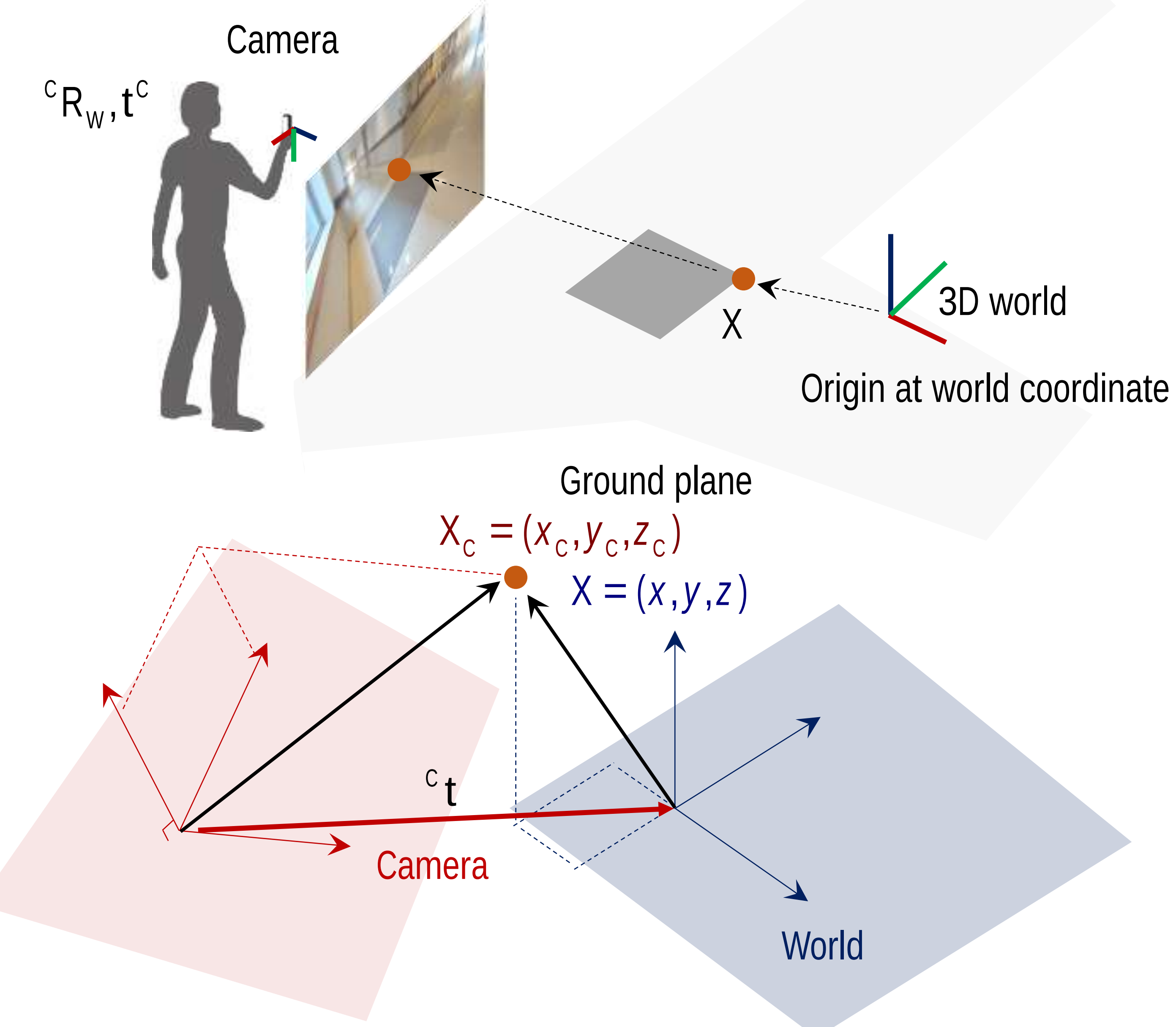
$$\begin{bmatrix} x_c \\ y_c \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$\begin{bmatrix} t_x \\ t_y \end{bmatrix}$: the location of world coordinate seen from camera coord.

Euclidean Transform=Rotation+Translation



Euclidean Transform=Rotation+Translation

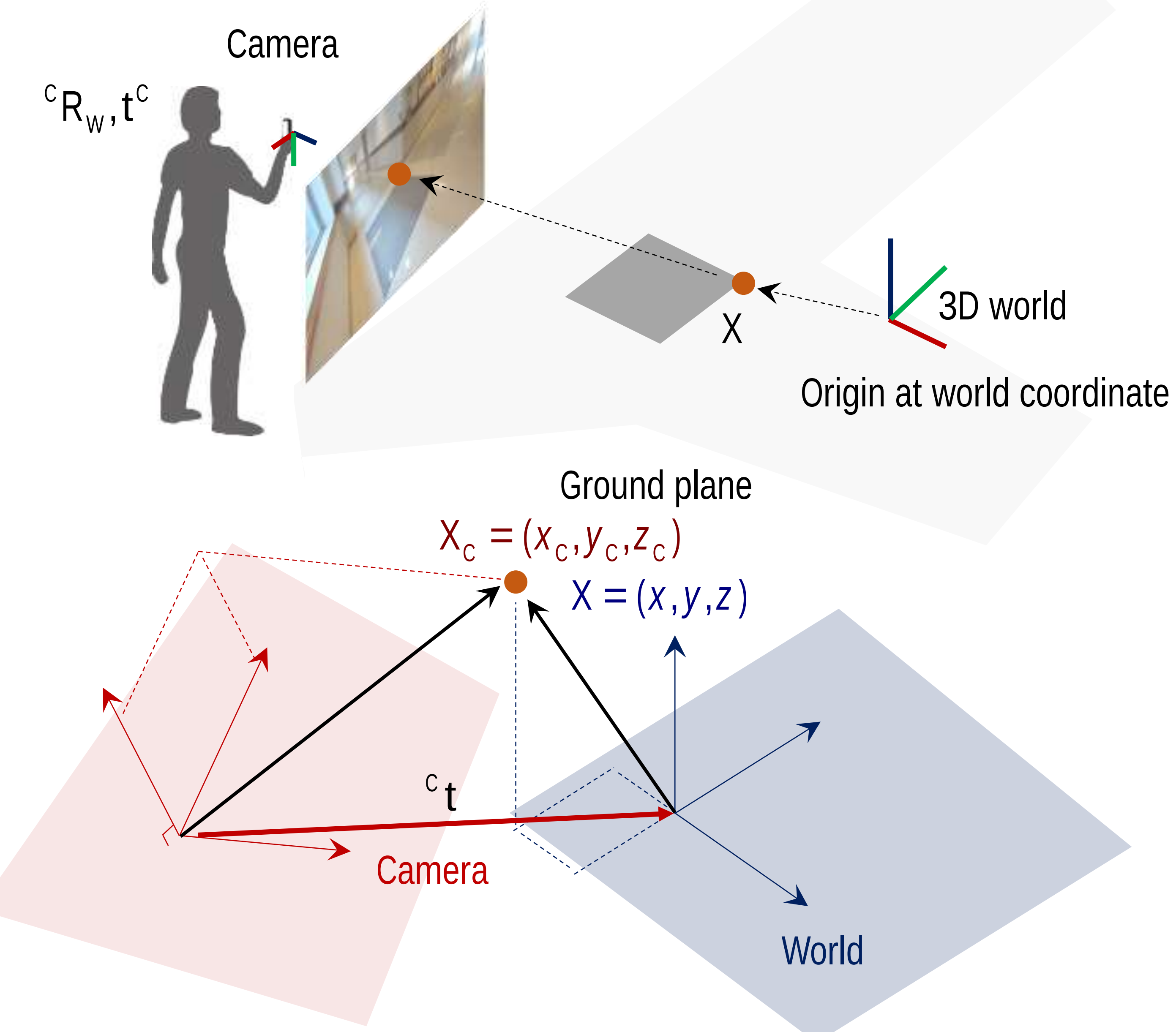


Coordinate transformation from world to camera:

$$X_C = {}^{C}R_W X + {}^{C}t$$

where ${}^{C}t$ is the world orgin seen from camera.

Euclidean Transform=Rotation+Translation

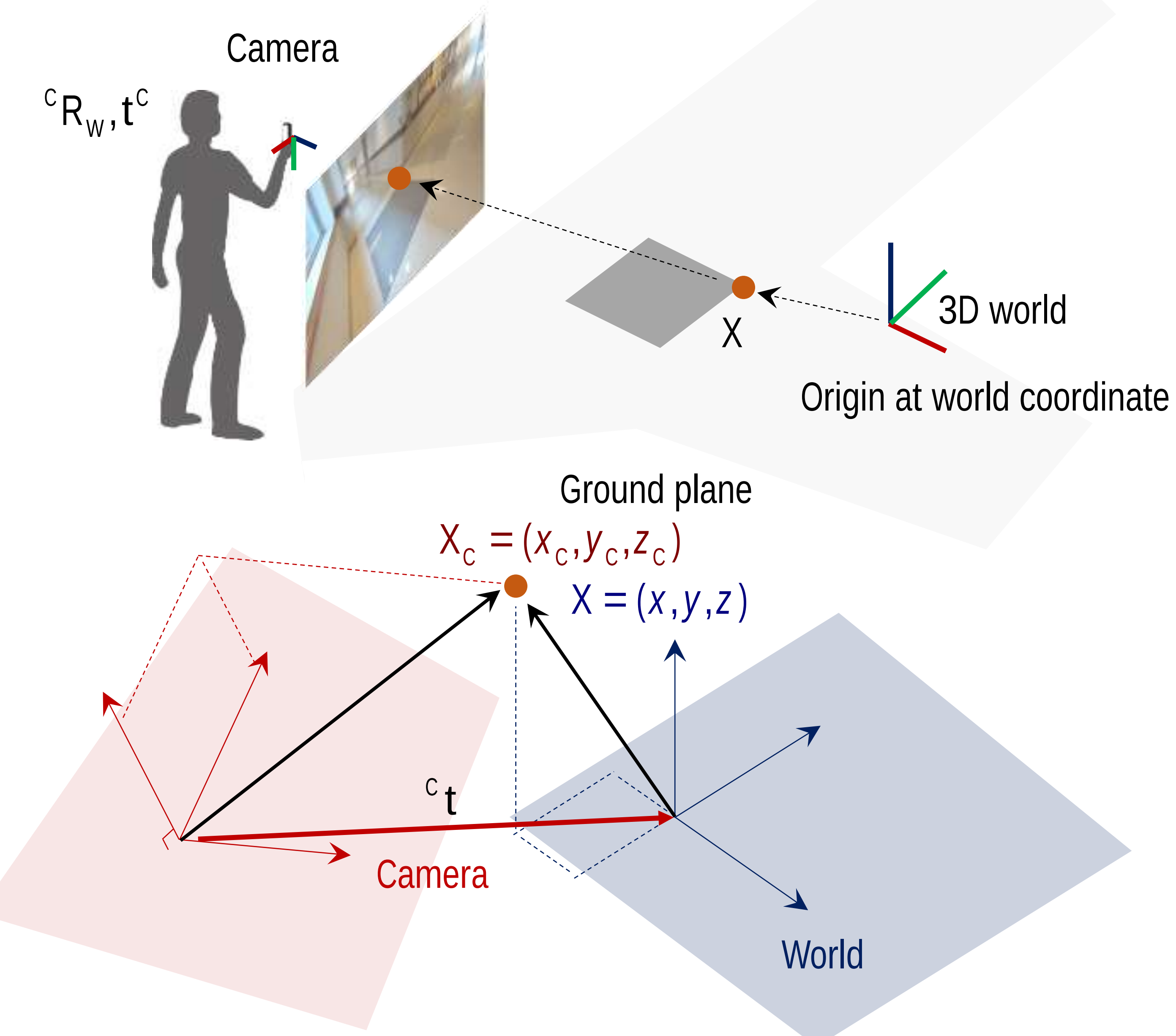


Coordinate transformation from world to camera:

$$X_c = {}^cR_w X + {}^c t = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \\ r_{z1} & r_{z2} & r_{z3} & t_z \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix}$$

where ${}^c t$ is the world orgin seen from camera.

Geometric Interpretation



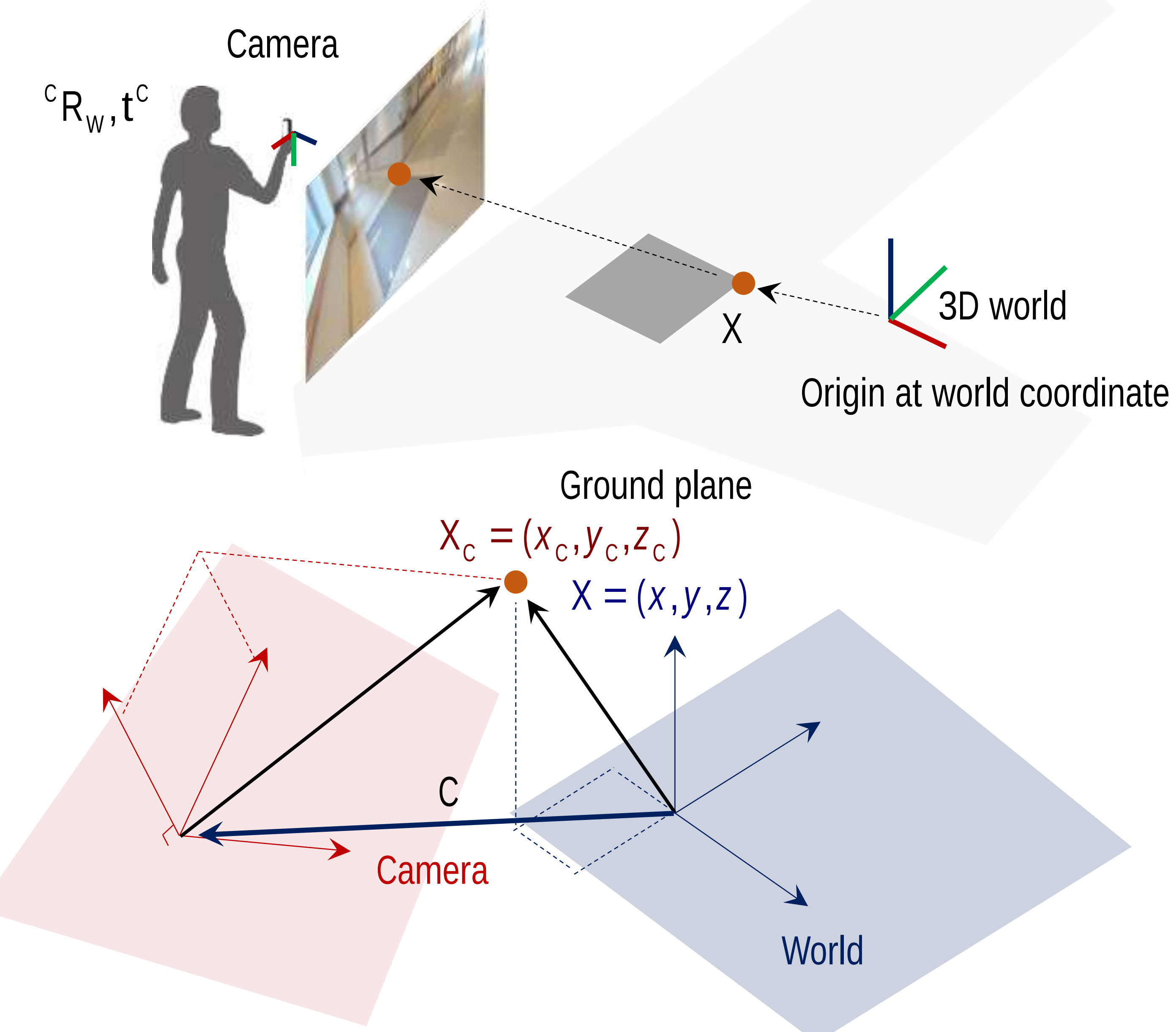
Coordinate transformation from world to camera:

$$X_C = {}^{C}R_W X + {}^{C}t = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \\ r_{z1} & r_{z2} & r_{z3} & t_z \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix}$$

where ${}^{C}t$ is the world orgin seen from camera.

Rotate and then, translate.

Geometric Interpretation



Coordinate transformation from world to camera:

$$X_C = {}^{C}R_{W}X + {}^{C}t = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \\ r_{z1} & r_{z2} & r_{z3} & t_z \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix}$$

where ${}^{C}t$ is the world orgin seen from camera.

Rotate and then, translate.

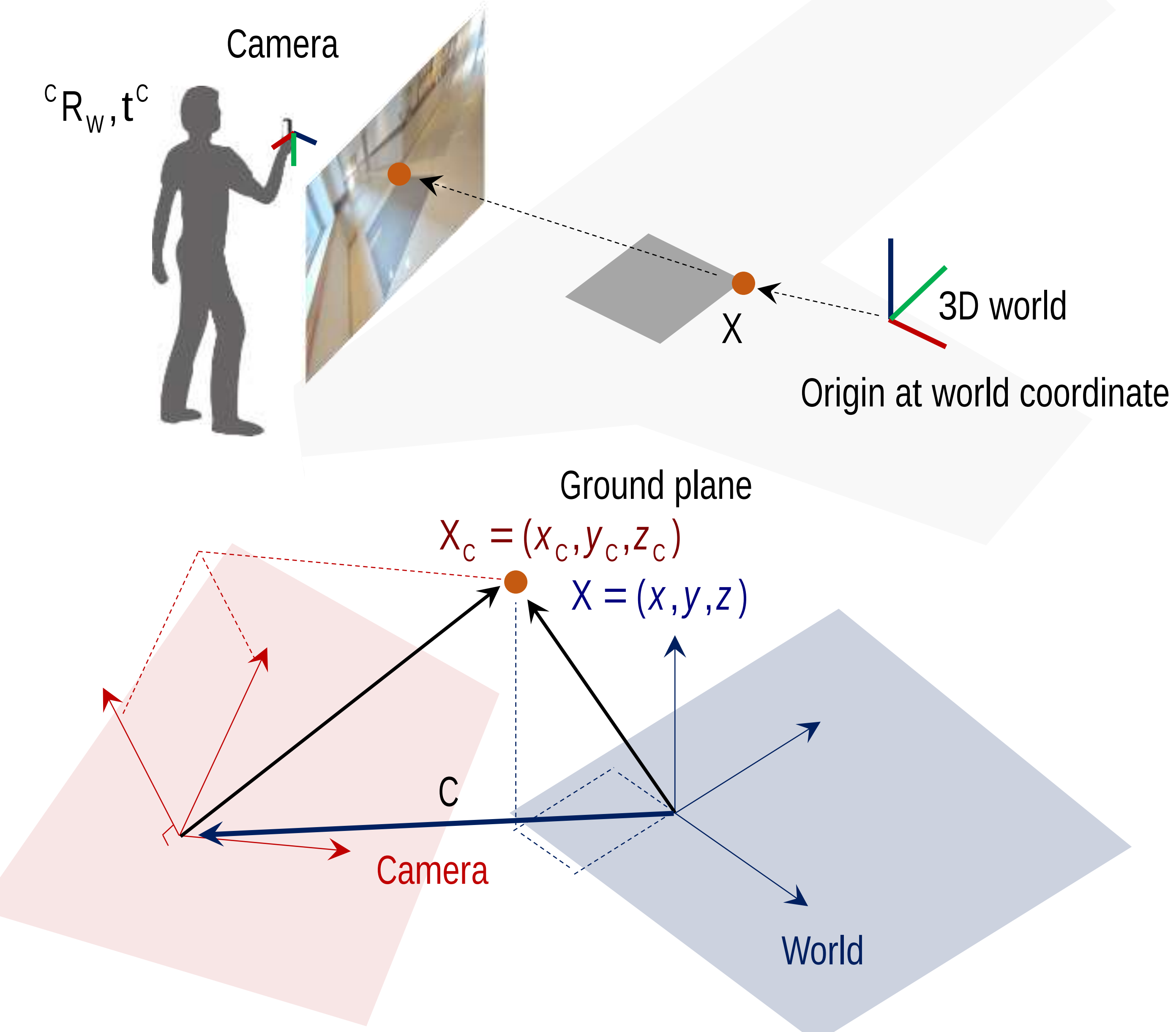
---

cf) Translate and then, rotate.

$$X_C = {}^{C}R_{W}(X - C) = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} \begin{bmatrix} 1 & & & -C_x \\ & 1 & & -C_y \\ & & 1 & -C_z \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix}$$

where C is the camera location seen from world.

Camera Projection Matrix



Coordinate transformation from world to camera:

$$X_C = {}^{C}R_{W}X + {}^{C}t = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \\ r_{z1} & r_{z2} & r_{z3} & t_z \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix}$$

Camera projection of world point:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \left[\underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{{}^{C}R_{W}} \; \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{{}^{C}t} \right] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Image Projection

$$C = \begin{bmatrix} -r \\ 0 \\ 0 \end{bmatrix} \qquad R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

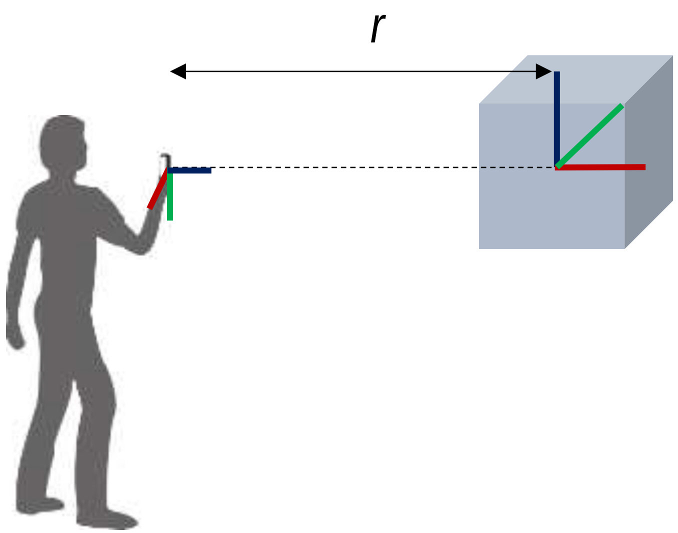
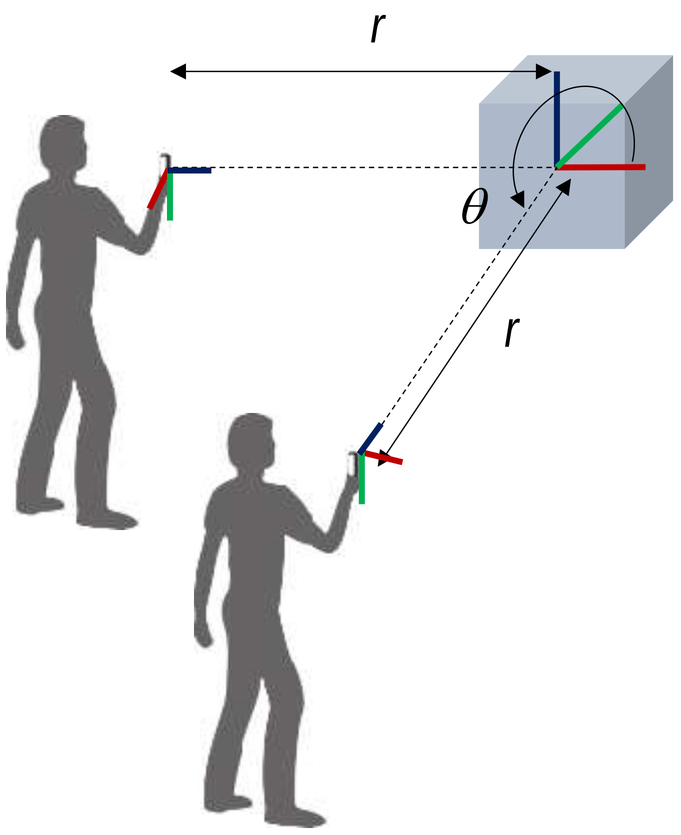
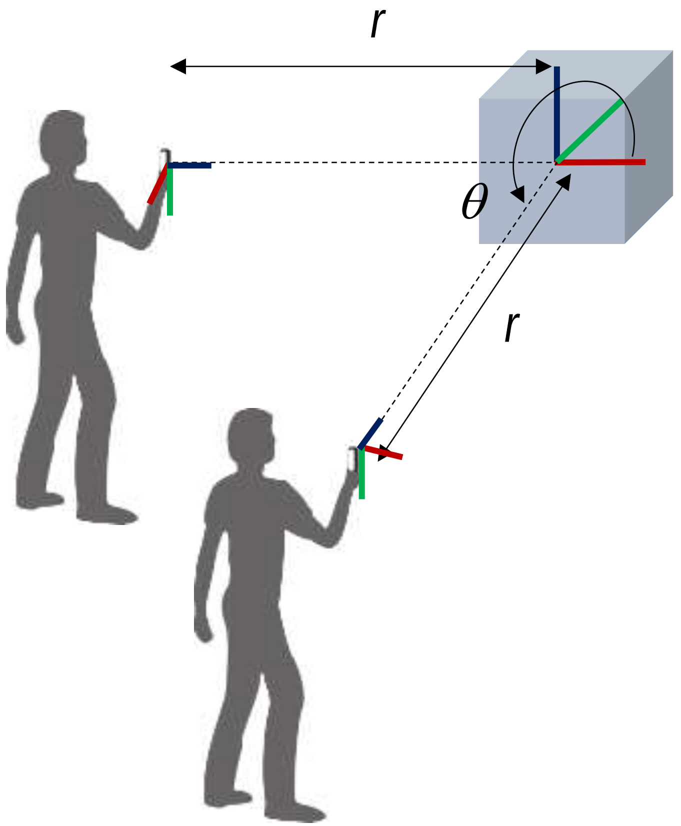


Image Projection



$$C = \begin{bmatrix} -r \\ 0 \\ 0 \end{bmatrix} \qquad R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

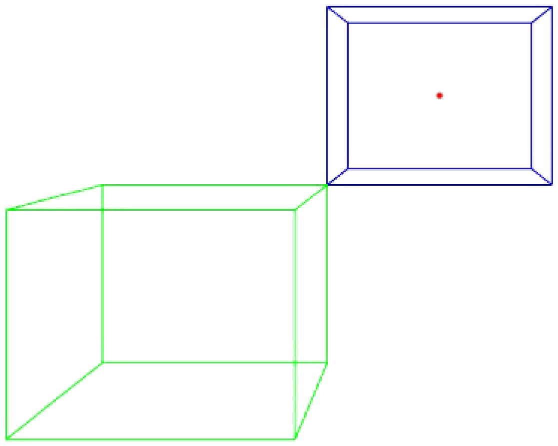
Image Projection



$$C = \begin{bmatrix} -r \\ 0 \\ 0 \end{bmatrix} \qquad R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} r\cos\theta \\ r\sin\theta \\ 0 \end{bmatrix} \qquad R = \begin{bmatrix} -\sin\theta & \cos\theta & 0 \\ 0 & 0 & -1 \\ -\cos\theta & -\sin\theta & 0 \end{bmatrix}$$

Image Projection



RotateCamera.m

```
K = [200 0 100;
    0 200 100;
    0 0 1];

radius = 5;

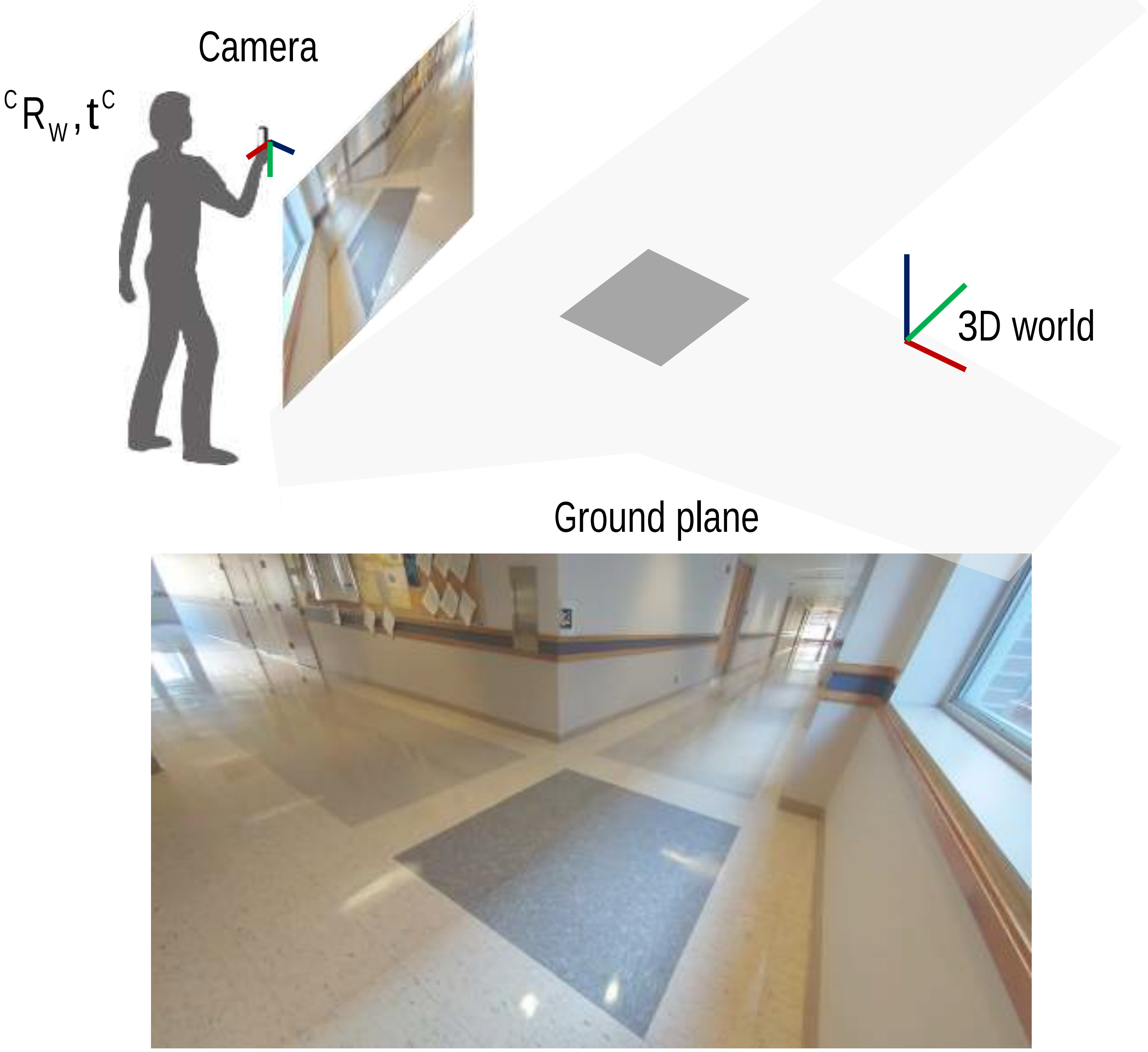
theta = 0:0.02:2*pi;

for i = 1 : length(theta)
   camera_offset = [radius*cos(theta(i)); radius*sin(theta(i)); 0];
   camera_center = camera_offset + center_of_mass';

   rz = [-cos(theta(i)); -sin(theta(i)); 0];
   ry = [0 0 -1]';
   rx = [-sin(theta(i)); cos(theta(i)); 0];
   R = [rx'; ry'; rz'];
   C = camera_center;
   P = K * R * [ eye(3) -C];

   proj = [];
   for j = 1 : size(sqaure_point,1)
      u = P * [sqaure_point(j,:)';1];
      proj(j,:) = u'/u(3);
   end
end
```

Geometric Interpretation



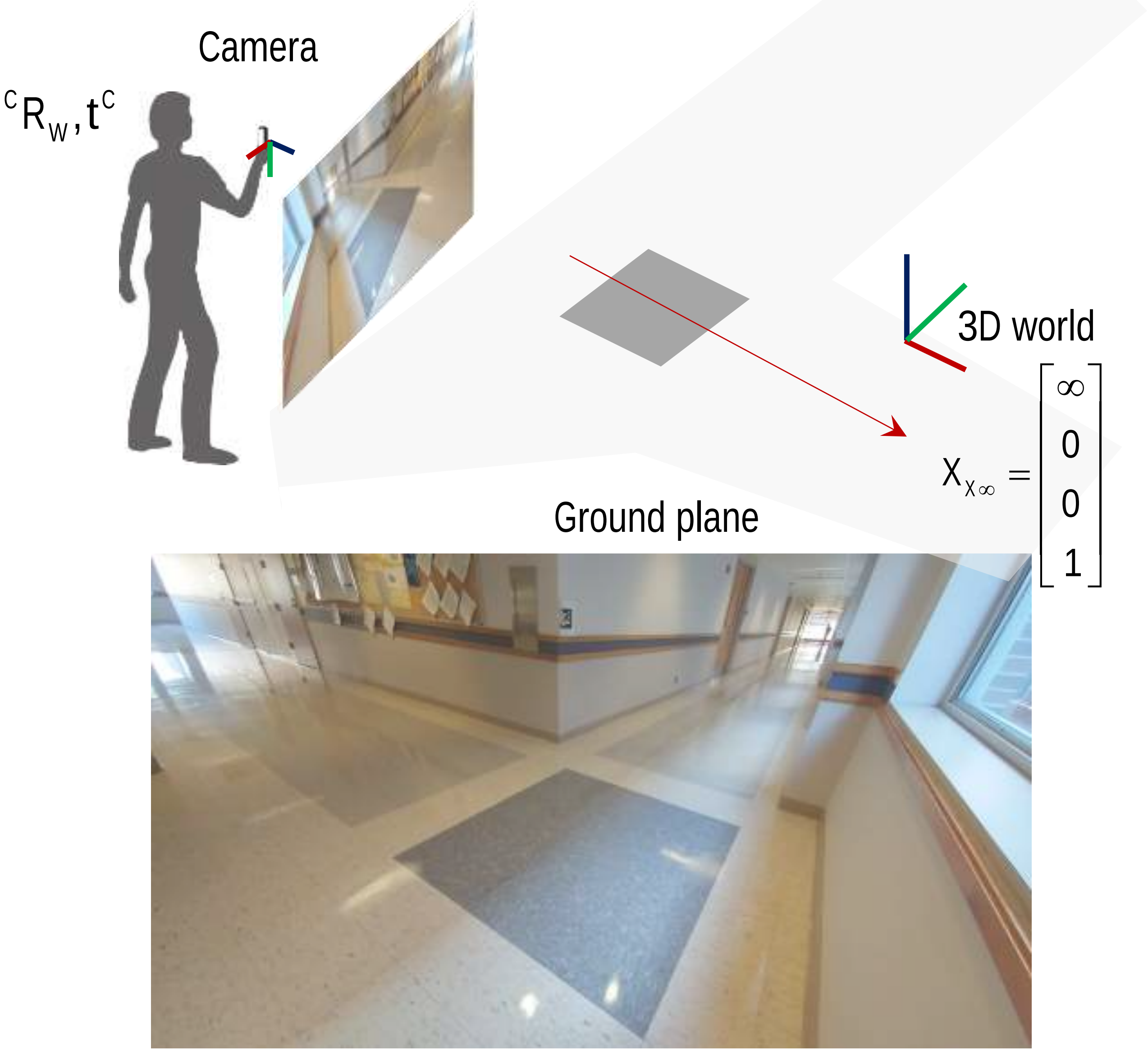
Camera projection of world point:

$$\lambda\begin{bmatrix}u\\v\\1\end{bmatrix}=\underbrace{\begin{bmatrix}f & & p_x\\ & f & p_y\\ & & 1\end{bmatrix}}_{K}\left[\underbrace{\begin{matrix}r_{x1} & r_{x2} & r_{x3}\\ r_{y1} & r_{y2} & r_{y3}\\ r_{z1} & r_{z2} & r_{z3}\end{matrix}}_{^{C}R_{W}}\ \underbrace{\begin{matrix}t_x\\t_y\\t_z\end{matrix}}_{^{C}t}\right]\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}=P\begin{bmatrix}\mathrm{X}\\1\end{bmatrix}$$

$$\longrightarrow\ \lambda\begin{bmatrix}u\\v\\1\end{bmatrix}=\begin{bmatrix}p_{11} & p_{12} & p_{13} & p_{14}\\ p_{21} & p_{22} & p_{23} & p_{24}\\ p_{31} & p_{32} & p_{33} & p_{34}\end{bmatrix}\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$$

What does each number mean?

Geometric Interpretation



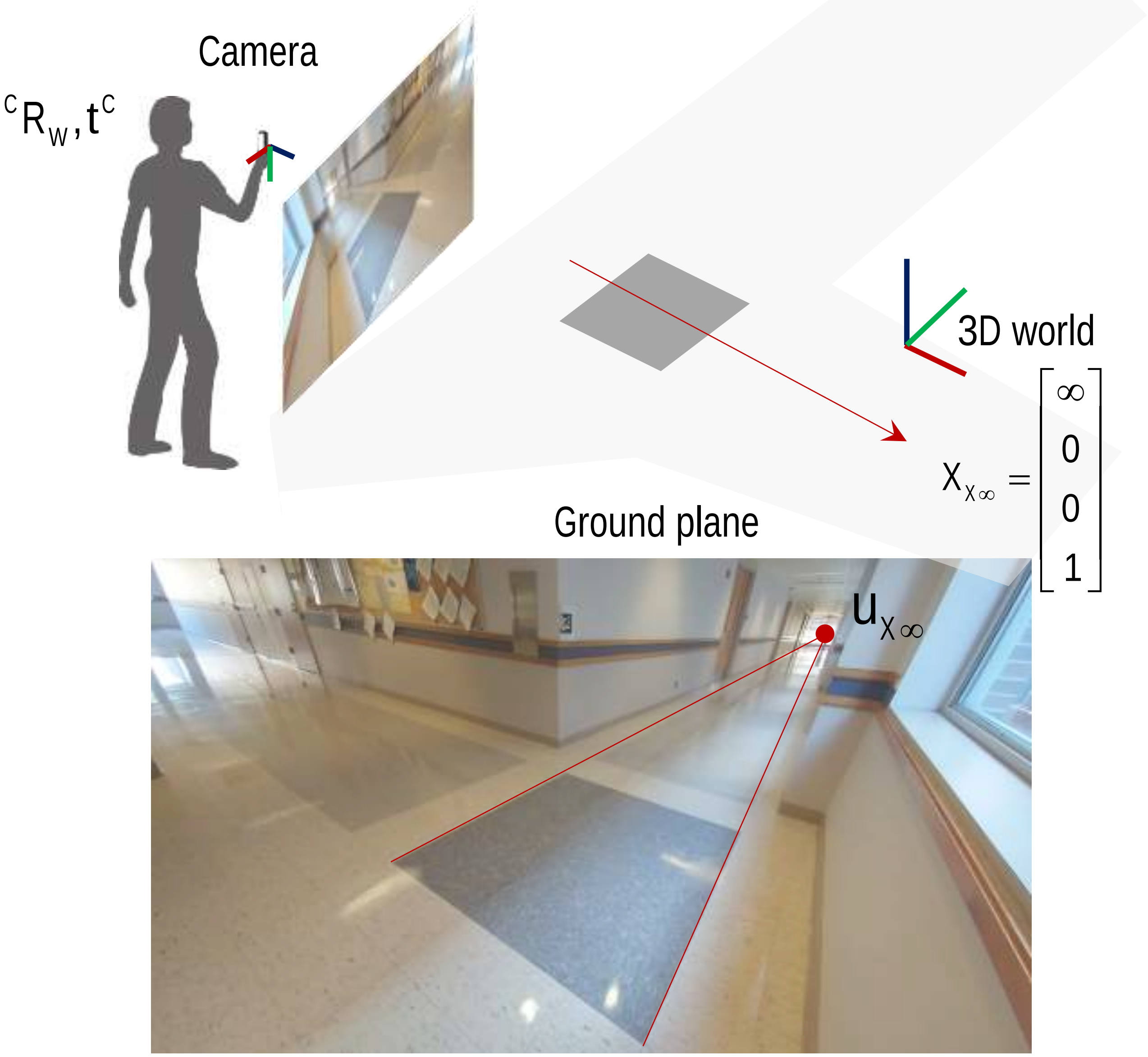
Camera projection of world point:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \left[\underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{{}^{c}R_{w}} \;\; \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{{}^{c}t} \right] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = P \begin{bmatrix} X \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} \infty \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

What is point at infinity in world x direction?

Geometric Interpretation



Camera projection of world point:

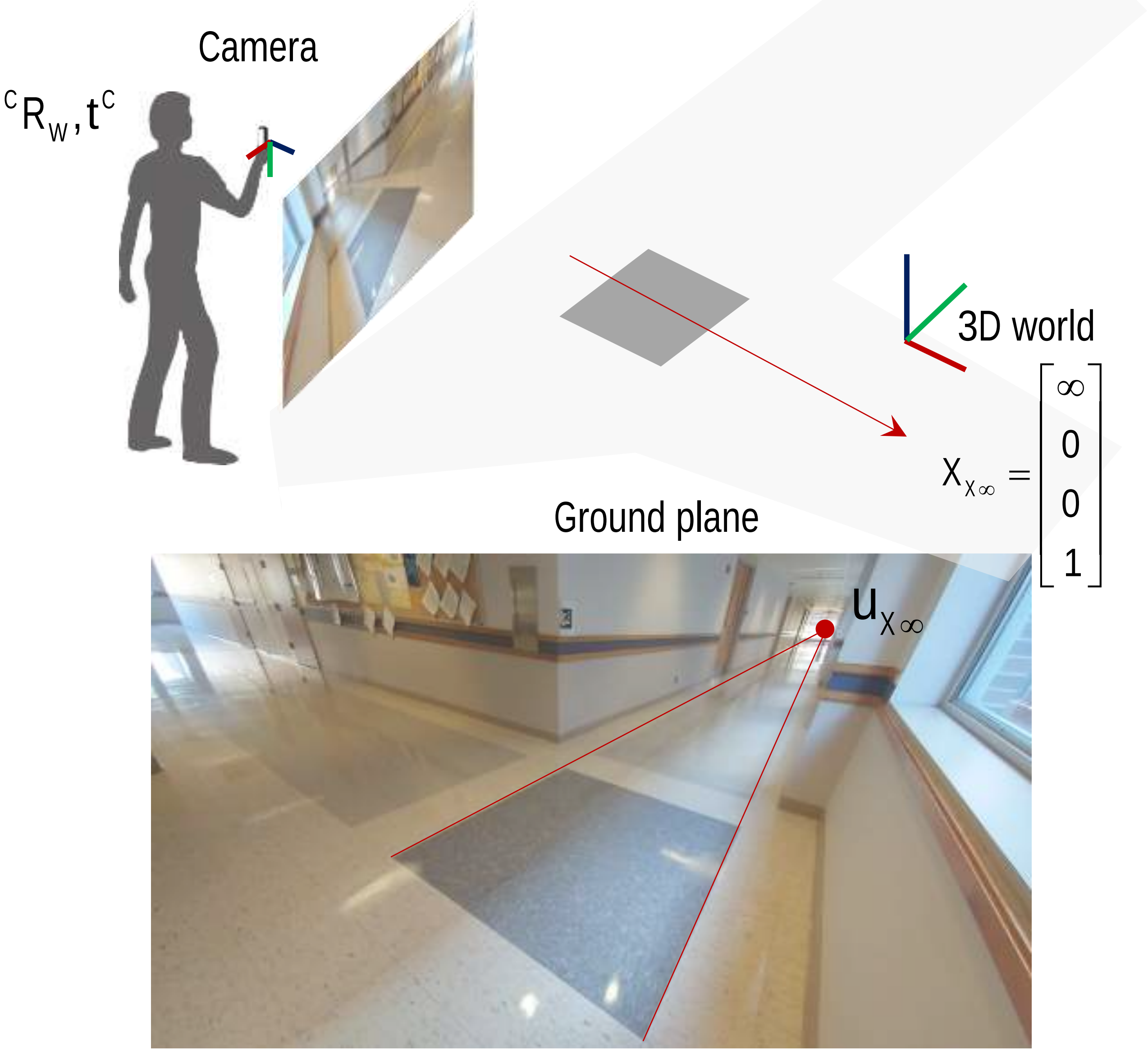
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \left[\underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{^{c}R_{W}} \;\; \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{^{c}t} \right] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = P \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} \infty \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

What is point at infinity in world x direction?

This point is at infinite but finite in image.

Geometric Interpretation



Camera projection of world point:

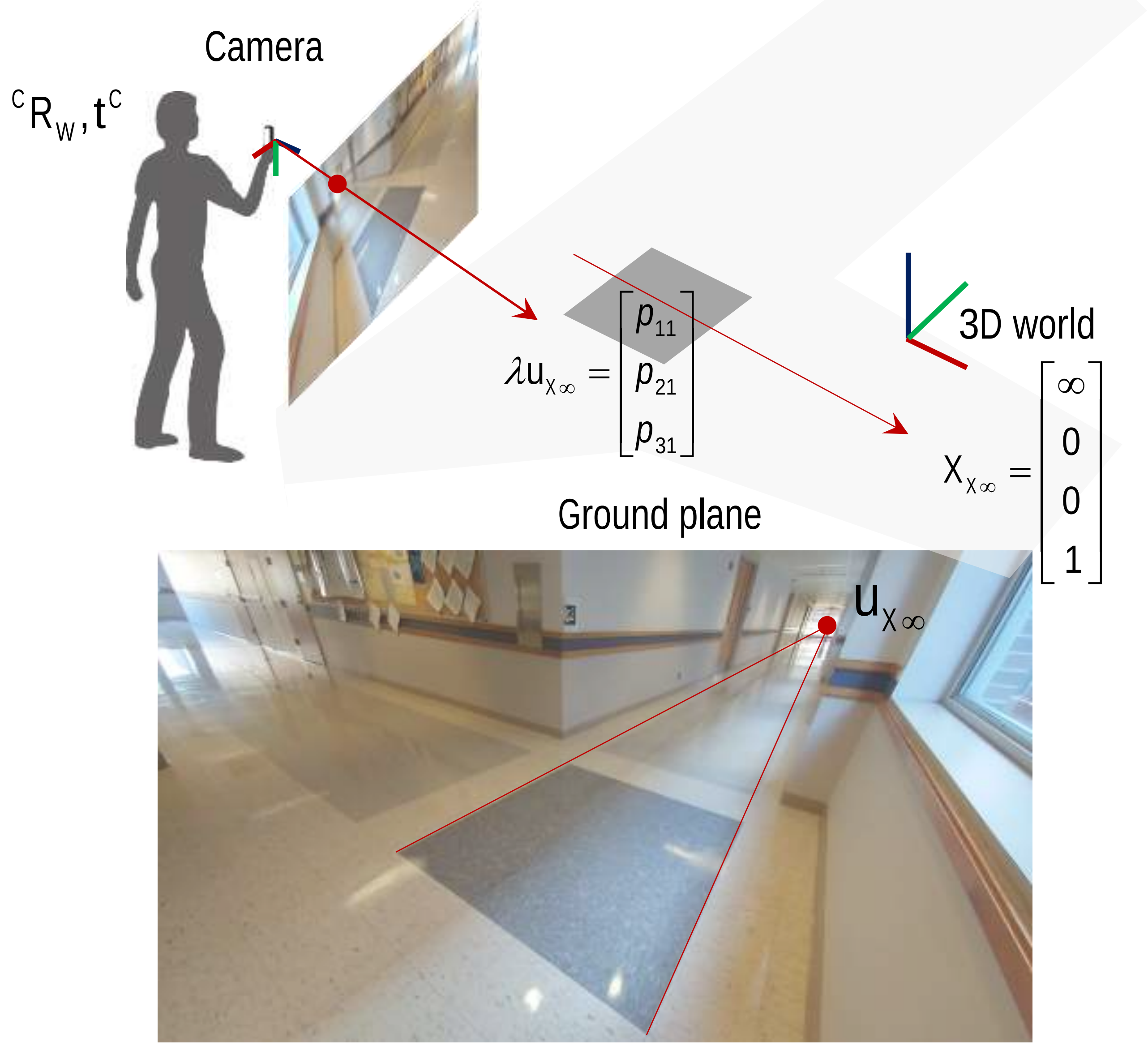
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \left[\underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{^{C}R_{W}} \;\; \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{^{C}t} \right] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = P \begin{bmatrix} X \\ 1 \end{bmatrix}$$

$$\rightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} \infty \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\rightarrow \quad u = \lim_{X \to \infty} \frac{p_{11}X + p_{14}}{p_{31}X + p_{34}} = \frac{p_{11}}{p_{31}}$$

$$v = \lim_{X \to \infty} \frac{p_{21}X + p_{24}}{p_{31}X + p_{34}} = \frac{p_{21}}{p_{31}}$$

Geometric Interpretation



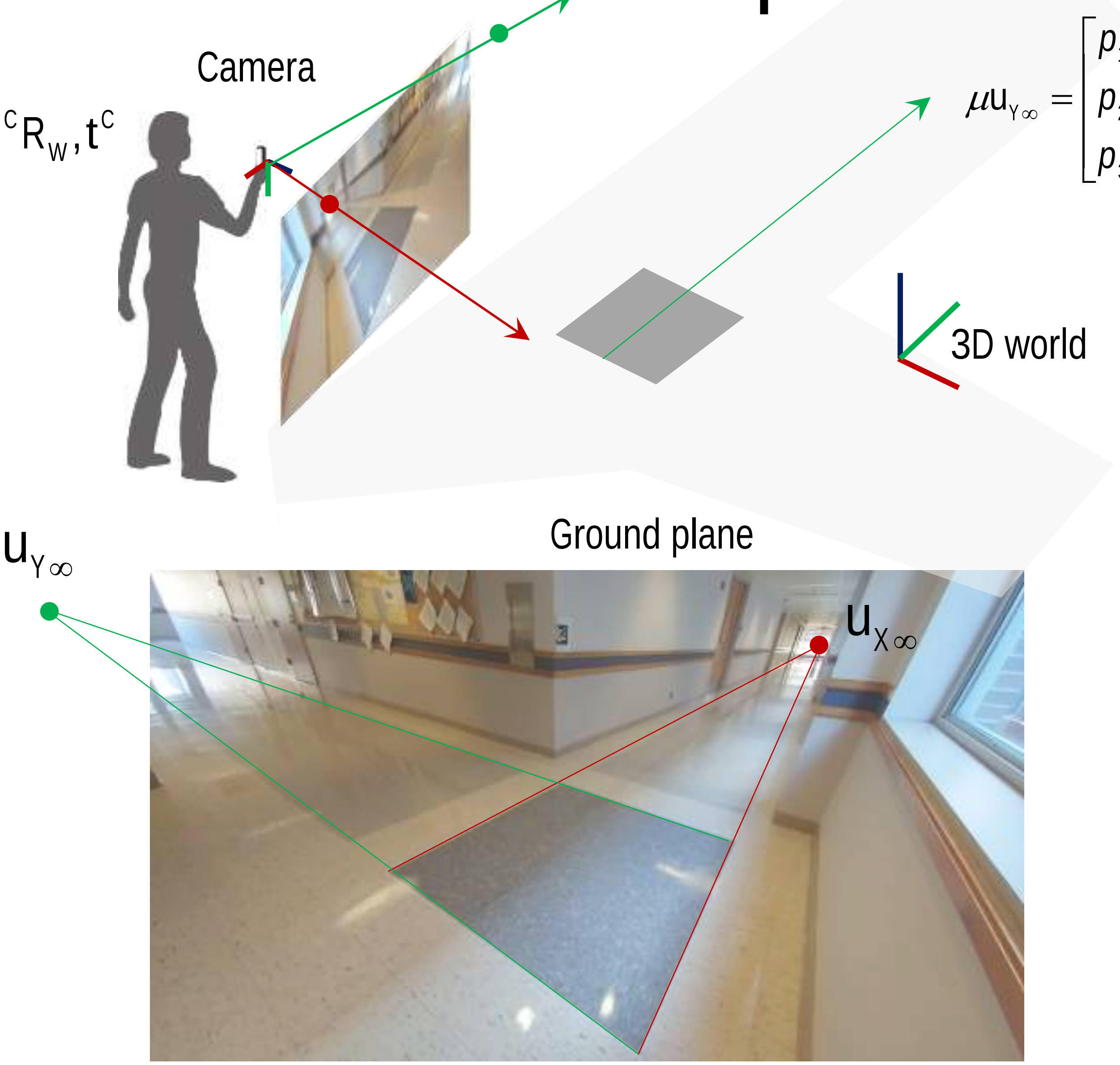
Camera projection of world point:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{K} \Bigg[\underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{^{C}R_{W}} \ \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{^{C}t} \Bigg] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = P \begin{bmatrix} X \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} \infty \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\longrightarrow \begin{aligned} u &= \lim_{X \to \infty} \frac{p_{11}X + p_{14}}{p_{31}X + p_{34}} = \frac{p_{11}}{p_{31}} \\ v &= \lim_{X \to \infty} \frac{p_{21}X + p_{24}}{p_{31}X + p_{34}} = \frac{p_{21}}{p_{31}} \end{aligned} \longrightarrow \lambda u_{X\infty} = \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} \\ p_{21} \\ p_{31} \end{bmatrix}$$

Geometric Interpretation



$$\mu u_{Y\infty} = \begin{bmatrix} p_{12} \\ p_{22} \\ p_{32} \end{bmatrix}$$

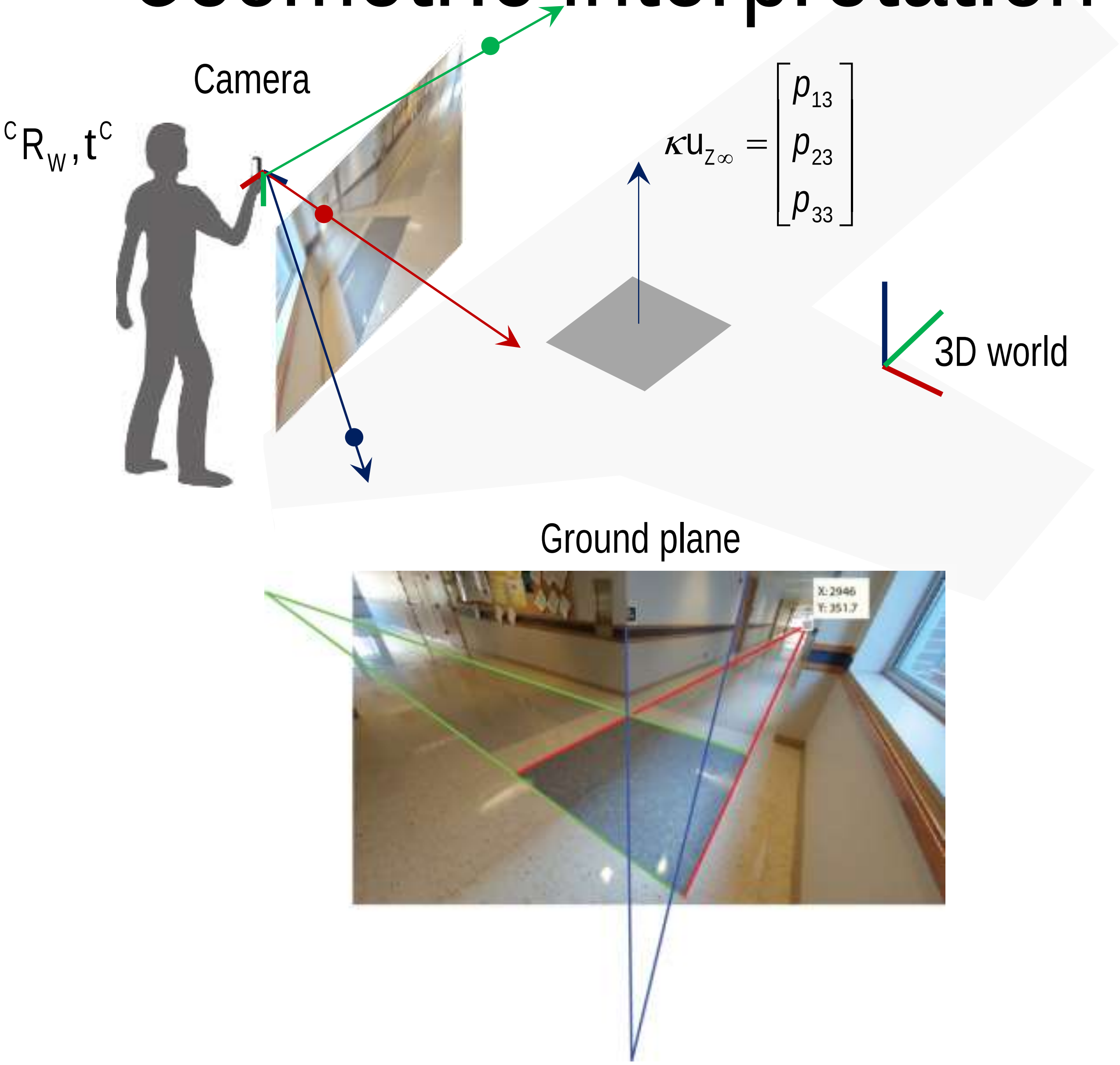
Camera projection of world point:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underset{K}{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}} \left[\underset{{}^{C}R_W}{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}} \;\; \underset{{}^{C}t}{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}} \right] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = P \begin{bmatrix} X \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} 0 \\ \infty \\ 0 \\ 1 \end{bmatrix}$$

$$\longrightarrow \begin{aligned} u &= \lim_{X \to \infty} \frac{p_{12}Y + p_{14}}{p_{32}Y + p_{34}} = \frac{p_{12}}{p_{32}} \\ v &= \lim_{X \to \infty} \frac{p_{22}Y + p_{24}}{p_{32}Y + p_{34}} = \frac{p_{22}}{p_{32}} \end{aligned} \longrightarrow \mu u_{Y\infty} = \mu \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{12} \\ p_{22} \\ p_{32} \end{bmatrix}$$

Geometric Interpretation



Camera projection of world point:

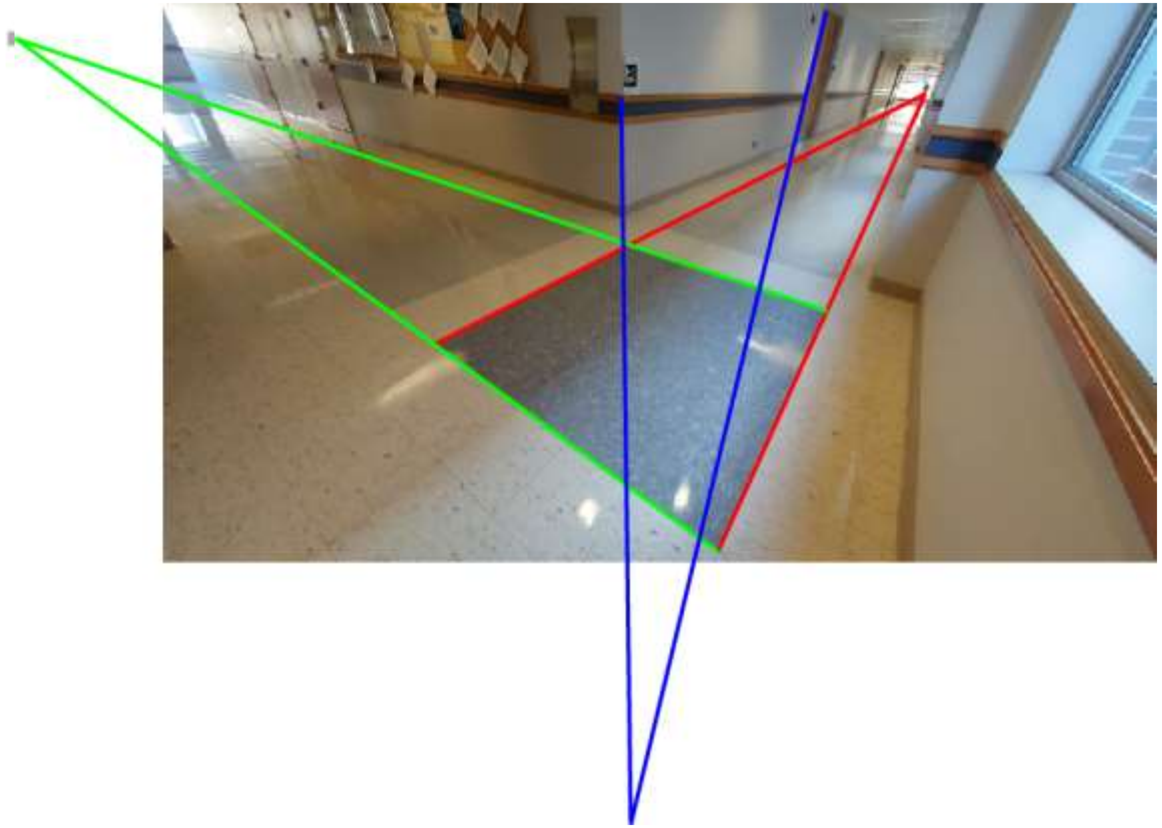
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{\mathrm{K}} \begin{bmatrix} \underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{{}^{C}\mathrm{R}_{W}} & \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{{}^{C}\mathrm{t}} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathrm{P} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \infty \\ 1 \end{bmatrix}$$

$$\longrightarrow \begin{aligned} u &= \lim_{X \to \infty} \frac{p_{13} Z + p_{14}}{p_{33} Z + p_{34}} = \frac{p_{13}}{p_{33}} \\ v &= \lim_{X \to \infty} \frac{p_{23} Z + p_{24}}{p_{33} Z + p_{34}} = \frac{p_{23}}{p_{33}} \end{aligned} \longrightarrow \kappa \mathrm{u}_{Z\infty} = \kappa \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{13} \\ p_{23} \\ p_{33} \end{bmatrix}$$

Practice

PredictVanishingPoint.m



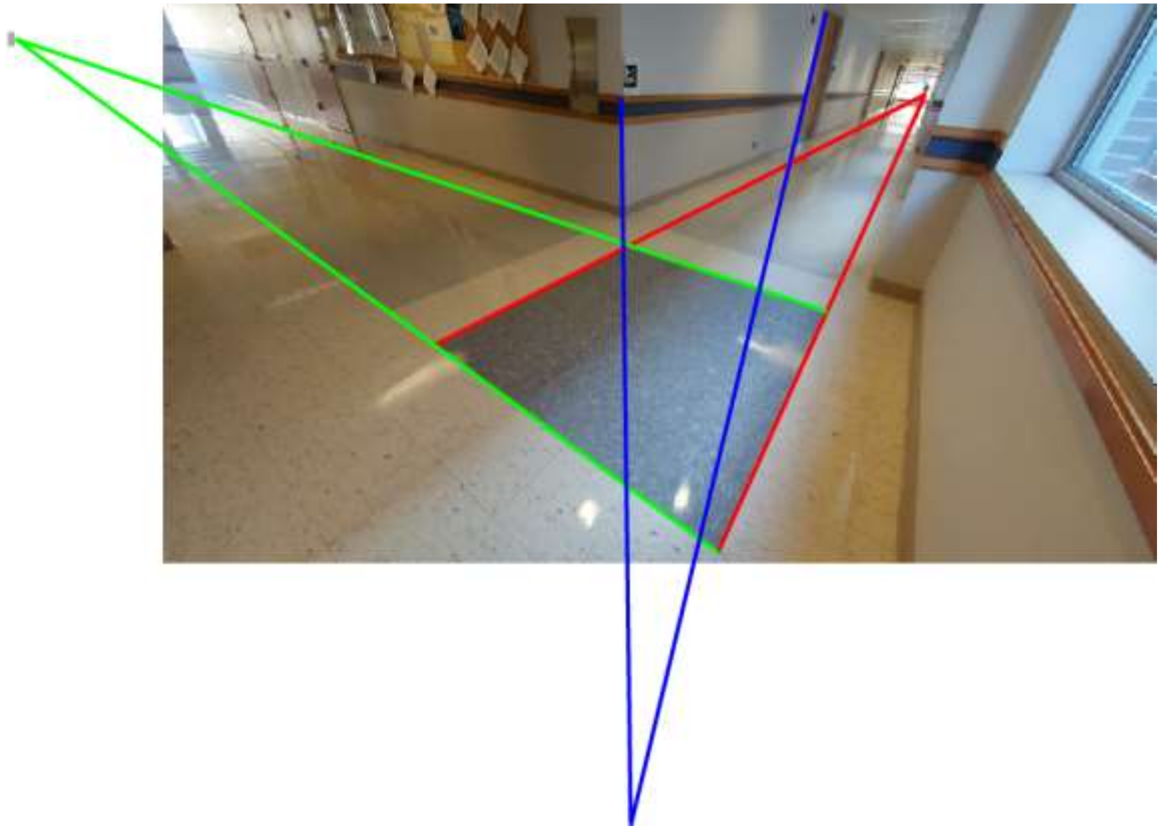
$$f = f_m \frac{W_{img}}{W_{ccd}} = 0.00153 \frac{3840}{0.0048} = 1224\,\text{pix}$$

$$p_x = \frac{W_{img}}{2} = \frac{3840}{2} = 1920\,\text{pix} \qquad p_y = \frac{H_{img}}{2} = \frac{2160}{2} = 1080\,\text{pix}$$

$$\text{C} = \begin{bmatrix} 0.0070 \\ 0.7520 \\ -0.2738 \end{bmatrix} \qquad \text{R} = \begin{bmatrix} -0.8496 & 0.0498 & 0.5731 \\ -0.3216 & -0.8203 & -0.4067 \\ 0.4180 & -0.5299 & 0.6835 \end{bmatrix}$$

Practice

PredictVanishingPoint.m



$$f = f_m \frac{W_{img}}{W_{ccd}} = 0.00153 \frac{3840}{0.0048} = 1224\,\text{pix}$$

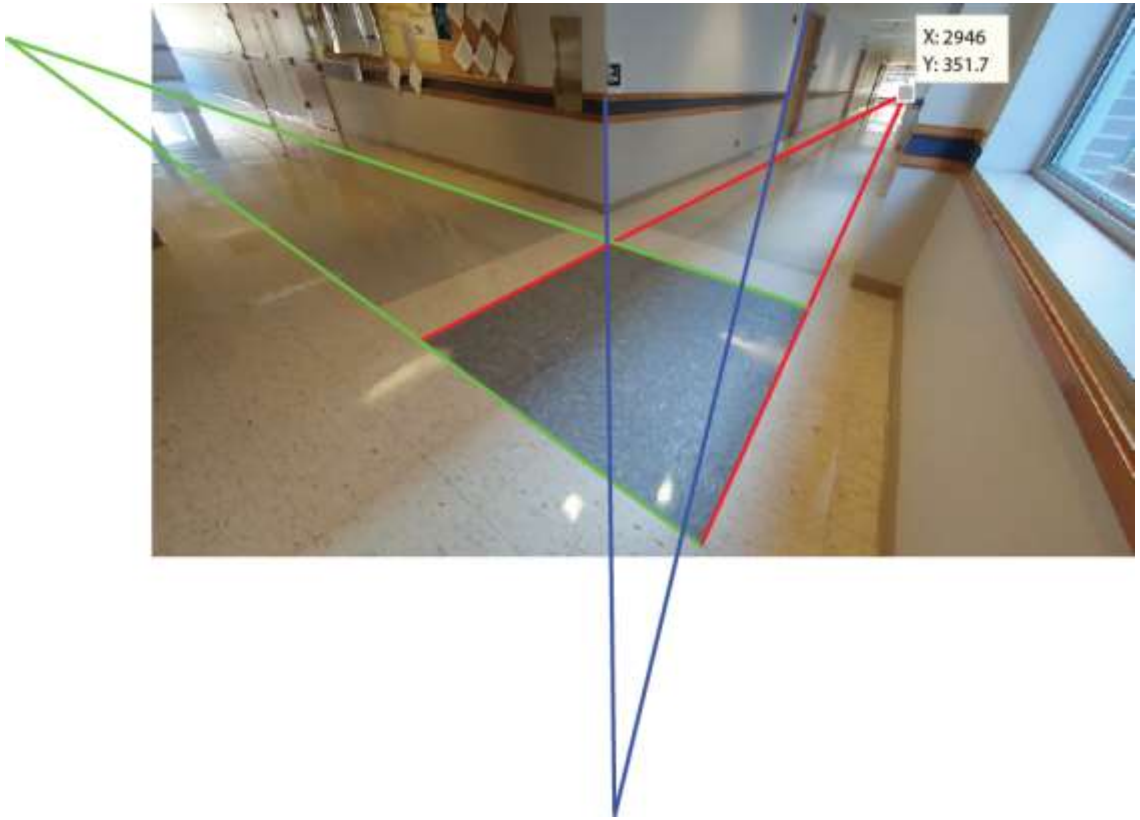
$$p_x = \frac{W_{img}}{2} = \frac{3840}{2} = 1920\,\text{pix} \qquad p_y = \frac{H_{img}}{2} = \frac{2160}{2} = 1080\,\text{pix}$$

$$C = \begin{bmatrix} 0.0070 \\ 0.7520 \\ -0.2738 \end{bmatrix} \qquad R = \begin{bmatrix} -0.8496 & 0.0498 & 0.5731 \\ -0.3216 & -0.8203 & -0.4067 \\ 0.4180 & -0.5299 & 0.6835 \end{bmatrix}$$

$$P = KR\begin{bmatrix} I_3 & -C \end{bmatrix} = \begin{bmatrix} -0.2374 & -0.9565 & 2.0138 & 1.2723 \\ 0.0578 & -1.5763 & 0.2404 & 1.2508 \\ 0.0004 & -0.0005 & 0.0007 & 0.0006 \end{bmatrix}$$

Practice

PredictVanishingPoint.m



$$f = f_m \frac{W_{img}}{W_{ccd}} = 0.00153 \frac{3840}{0.0048} = 1224\,\text{pix}$$

$$p_x = \frac{W_{img}}{2} = \frac{3840}{2} = 1920\,\text{pix} \qquad p_y = \frac{H_{img}}{2} = \frac{2160}{2} = 1080\,\text{pix}$$

$$C = \begin{bmatrix} 0.0070 \\ 0.7520 \\ -0.2738 \end{bmatrix} \qquad R = \begin{bmatrix} -0.8496 & 0.0498 & 0.5731 \\ -0.3216 & -0.8203 & -0.4067 \\ 0.4180 & -0.5299 & 0.6835 \end{bmatrix}$$

$$P = KR\begin{bmatrix} I_3 & -C \end{bmatrix} = \begin{bmatrix} -0.2374 & -0.9565 & 2.0138 & 1.2723 \\ 0.0578 & -1.5763 & 0.2404 & 1.2508 \\ 0.0004 & -0.0005 & 0.0007 & 0.0006 \end{bmatrix}$$

```
R = [-0.8496    0.0498    0.5731
   -0.3216   -0.8203   -0.4067
    0.4180   -0.5299    0.6835];

C = [0.0070
    0.7520
   -0.2738];

P = K * R * [eye(3) -C]
u_x = P(1:2,1)/P(3,1)
u_y = P(1:2,2)/P(3,2)
u_z = P(1:2,3)/P(3,3)
```

```
u_x =
 -567.8239
  138.2813

u_y =
   1.0e+03 *
    1.8050
    2.9748

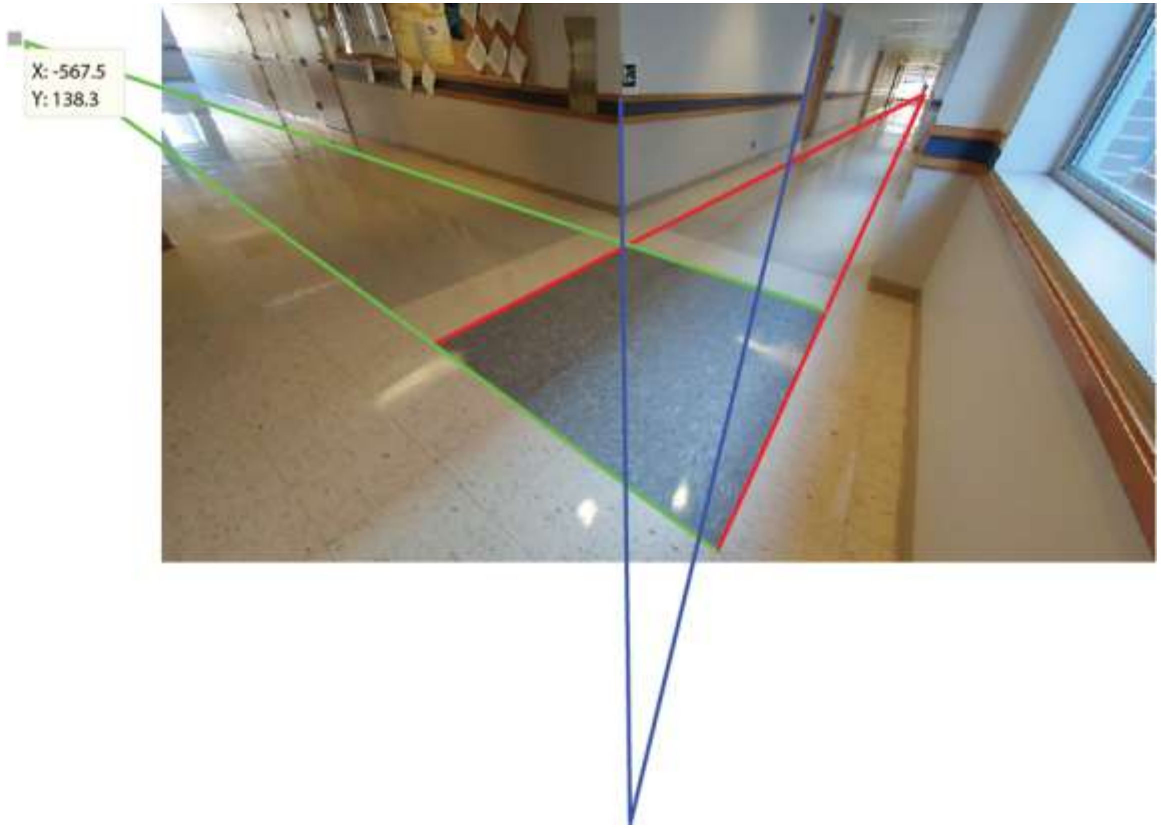
u_z =
   1.0e+03 *
    2.9463
    0.3517
```

Practice

$$f = f_m \frac{W_{img}}{W_{ccd}} = 0.00153 \frac{3840}{0.0048} = 1224\,\text{pix}$$

$$p_x = \frac{W_{img}}{2} = \frac{3840}{2} = 1920\,\text{pix} \qquad p_y = \frac{H_{img}}{2} = \frac{2160}{2} = 1080\,\text{pix}$$

$$C = \begin{bmatrix} 0.0070 \\ 0.7520 \\ -0.2738 \end{bmatrix} \qquad R = \begin{bmatrix} -0.8496 & 0.0498 & 0.5731 \\ -0.3216 & -0.8203 & -0.4067 \\ 0.4180 & -0.5299 & 0.6835 \end{bmatrix}$$



$$P = KR\begin{bmatrix} I_3 & -C \end{bmatrix} = \begin{bmatrix} \boxed{\begin{matrix} -0.2374 \\ 0.0578 \\ 0.0004 \end{matrix}} & \begin{matrix} -0.9565 \\ -1.5763 \\ -0.0005 \end{matrix} & \begin{matrix} 2.0138 \\ 0.2404 \\ 0.0007 \end{matrix} & \begin{matrix} 1.2723 \\ 1.2508 \\ 0.0006 \end{matrix} \end{bmatrix}$$

```
R = [-0.8496    0.0498    0.5731
   -0.3216   -0.8203   -0.4067
    0.4180   -0.5299    0.6835];

C = [0.0070
    0.7520
   -0.2738];

P = K * R * [eye(3) -C]
u_x = P(1:2,1)/P(3,1)
u_y = P(1:2,2)/P(3,2)
u_z = P(1:2,3)/P(3,3)
```

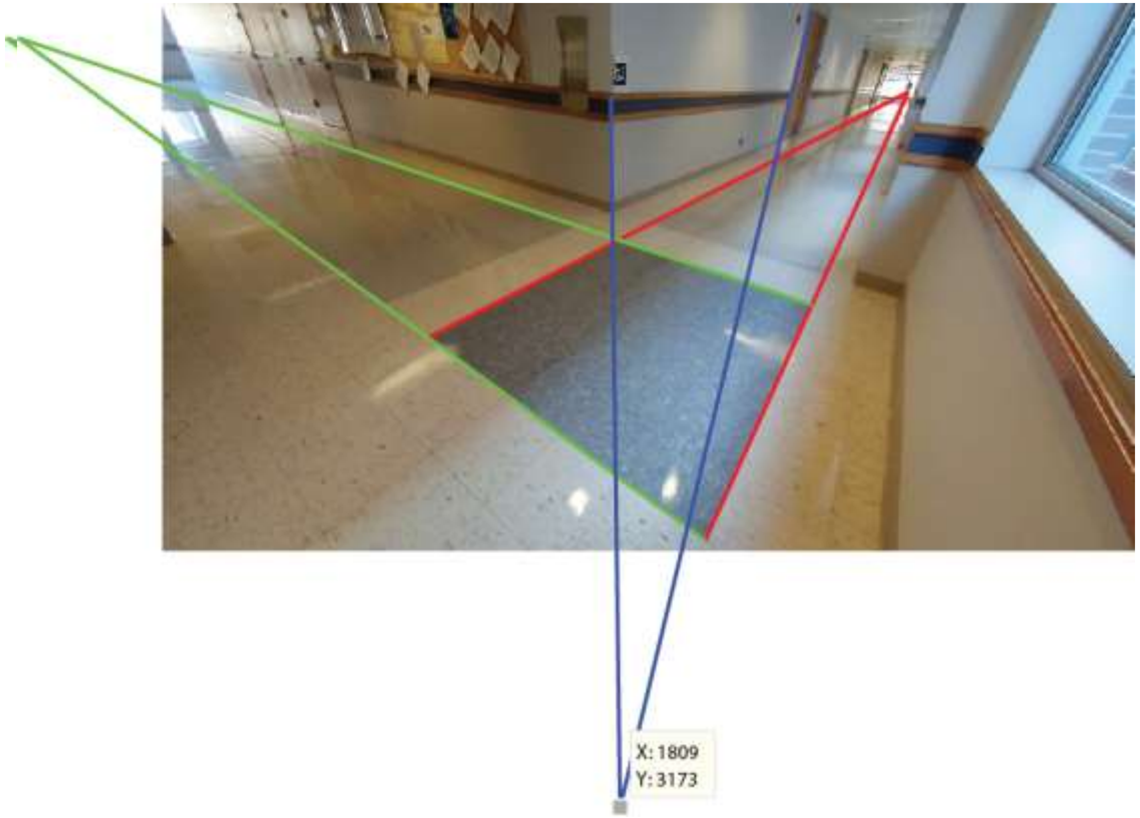
```
u_x =
 -567.8239
  138.2813

u_y =
   1.0e+03 *
    1.8050
    2.9748

u_z =
   1.0e+03 *
    2.9463
    0.3517
```

Practice

PredictVanishingPoint.m



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R = [-0.8496   0.0498   0.5731
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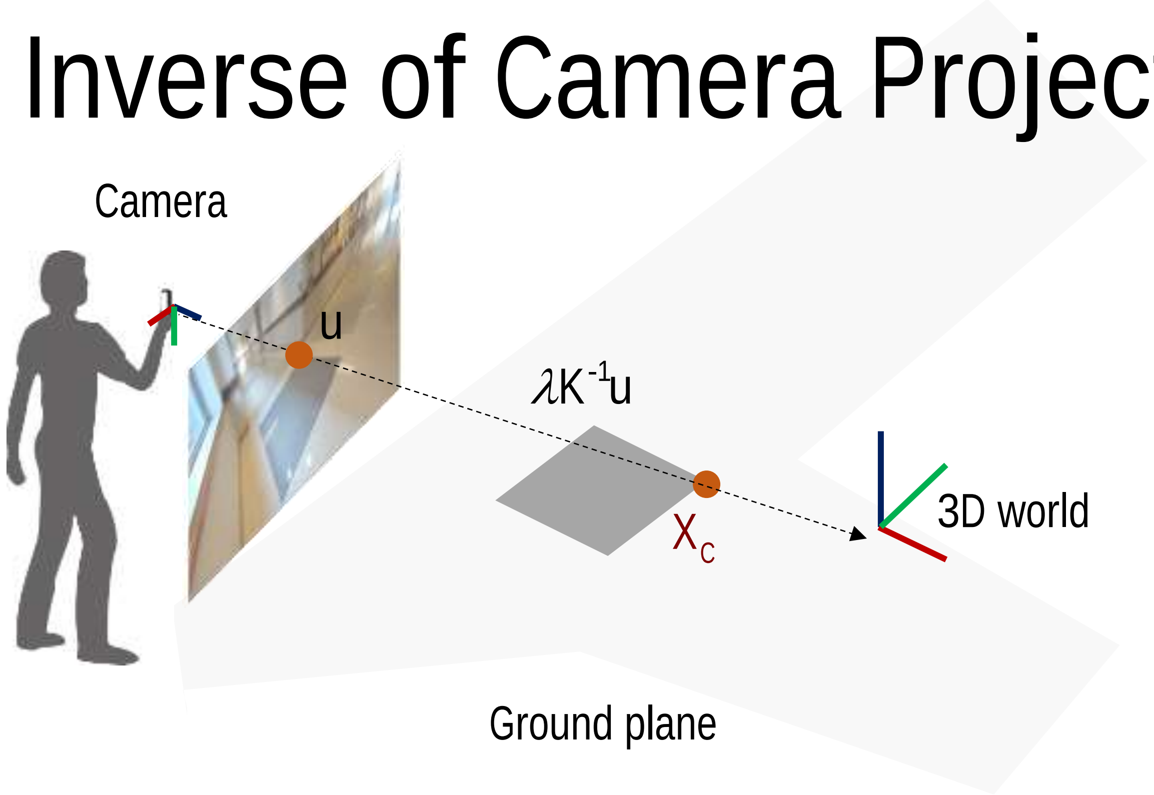
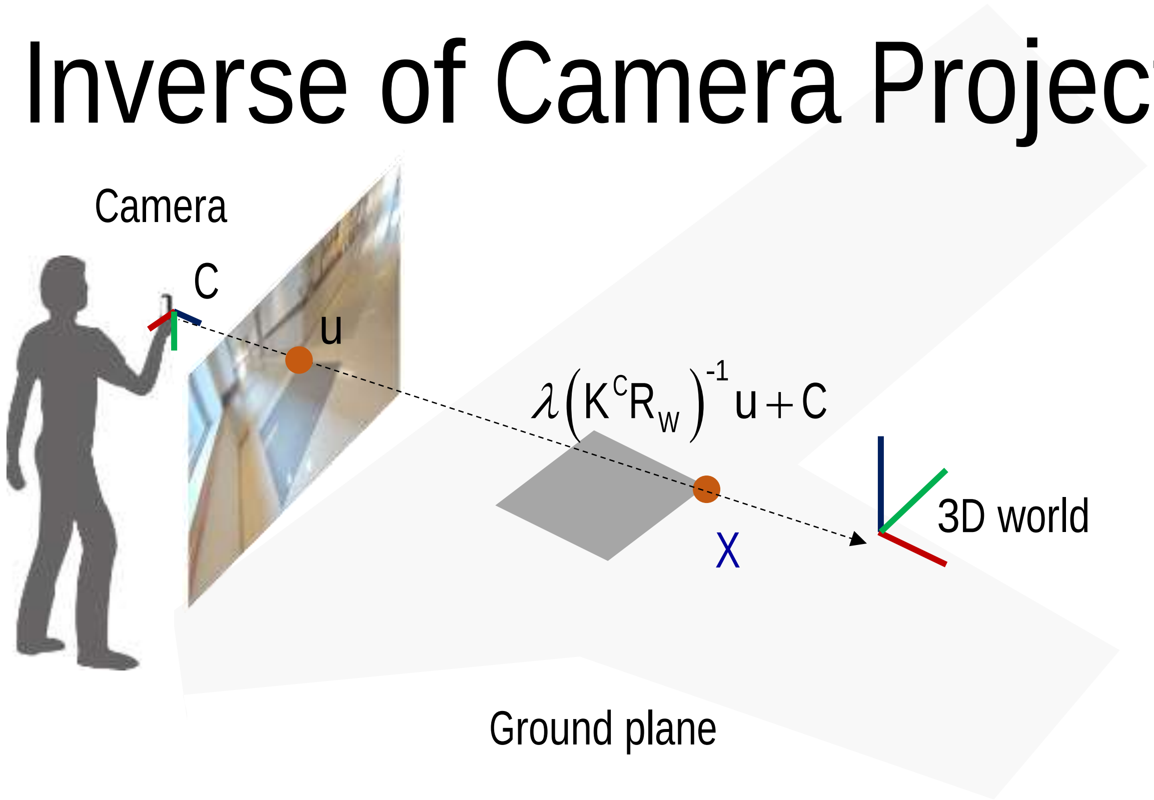
   1.0e+03 *
   2.9463
   0.3517
```

Inverse of Camera Projection Matrix



$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathrm{K}\mathrm{X}_{\mathrm{C}}$$

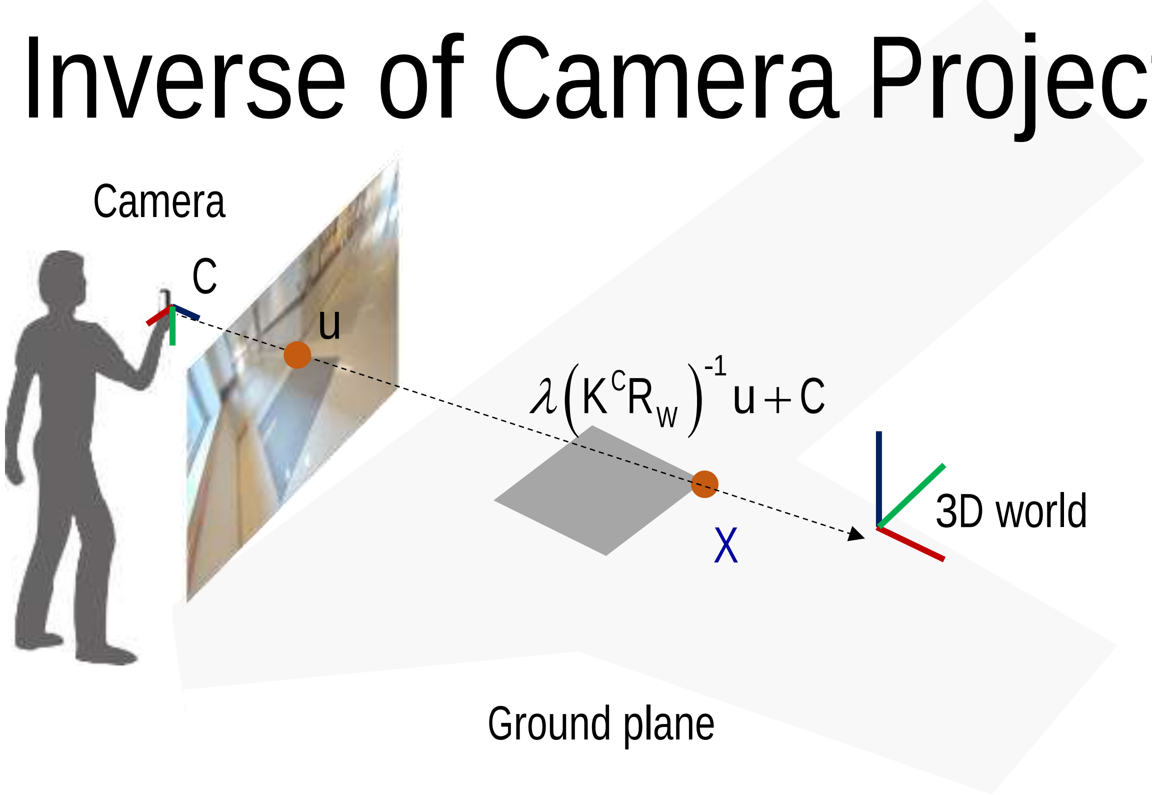
Inverse of Camera Projection Matrix



$$\lambda\begin{bmatrix}u\\v\\1\end{bmatrix} = KX_{C}$$

$$= K^{C}\left(R_{W}X + {}^{C}t\right) = K\,{}^{C}R_{W}(X - C)$$

Inverse of Camera Projection Matrix

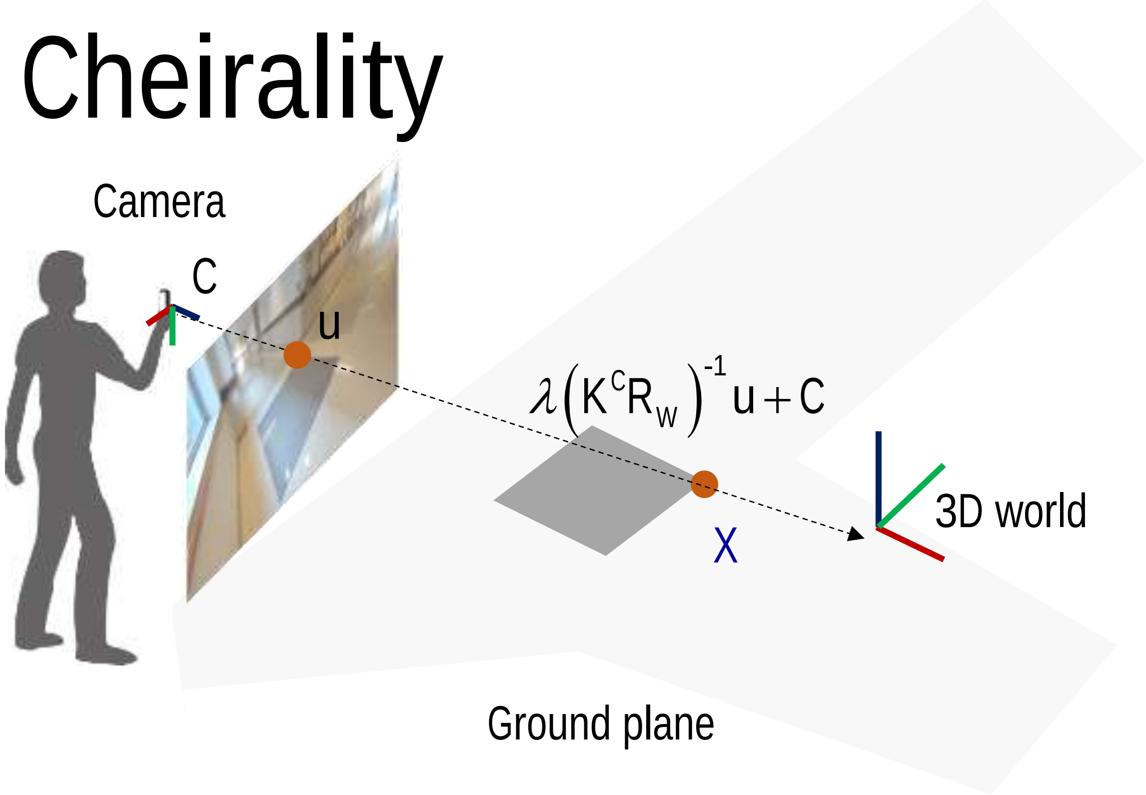


$$\lambda\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K X_C$$

$$= K\,{}^{C}\left(R_W X + {}^{C}t\right) = K\,{}^{C}R_W(X - C)$$

$$\longrightarrow \quad \underbrace{X = \lambda\left(K\,{}^{C}R_W\right)^{-1}\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}}_{\text{3D ray direction}} + \underbrace{C}_{\text{3D ray origin}}$$

Cheirality



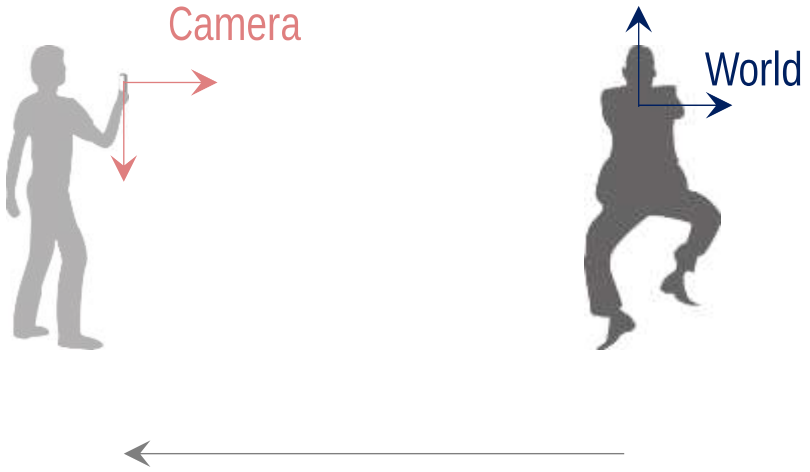
$$\lambda\begin{bmatrix}u\\v\\1\end{bmatrix}=K X_C$$

$$=K\left({}^{C}R_{W}X+{}^{C}t\right)=K\,{}^{C}R_{W}(X-C)$$

$$\longrightarrow\quad X=\underbrace{\lambda\left(K\,{}^{C}R_{W}\right)^{-1}\begin{bmatrix}u\\v\\1\end{bmatrix}}_{\text{3D ray direction}}+\underbrace{C}_{\text{3D ray origin}}$$

where $\lambda>0$

Perspective Camera

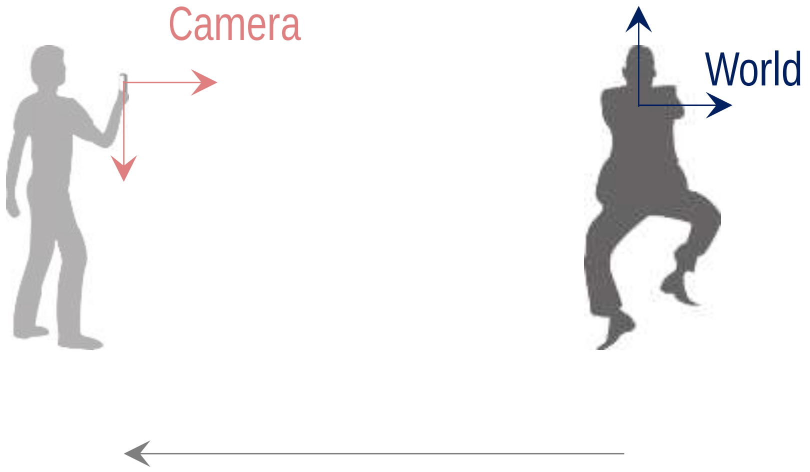



Strong perspectiveness

Perspective camera model:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathrm{P} \begin{bmatrix} \mathrm{X} \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} \mathrm{X} \\ 1 \end{bmatrix}$$

Affine Camera




Strong perspectiveness

Perspective camera model:

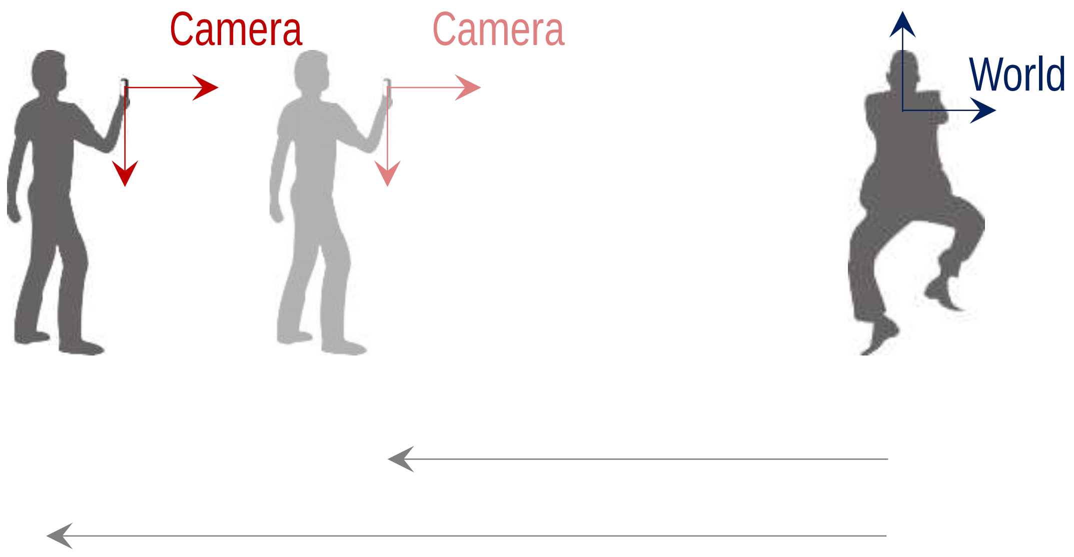
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathrm{P} \begin{bmatrix} \mathrm{X} \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} \mathrm{X} \\ 1 \end{bmatrix}$$

Affine camera model:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathrm{P}_{\mathrm{A}} \begin{bmatrix} \mathrm{X} \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{X} \\ 1 \end{bmatrix}$$

$$\longrightarrow \quad \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{23} \end{bmatrix} \begin{bmatrix} \mathrm{X} \\ 1 \end{bmatrix}$$

Affine Camera



Weak perspectiveness

Strong perspectiveness

Perspective camera model:

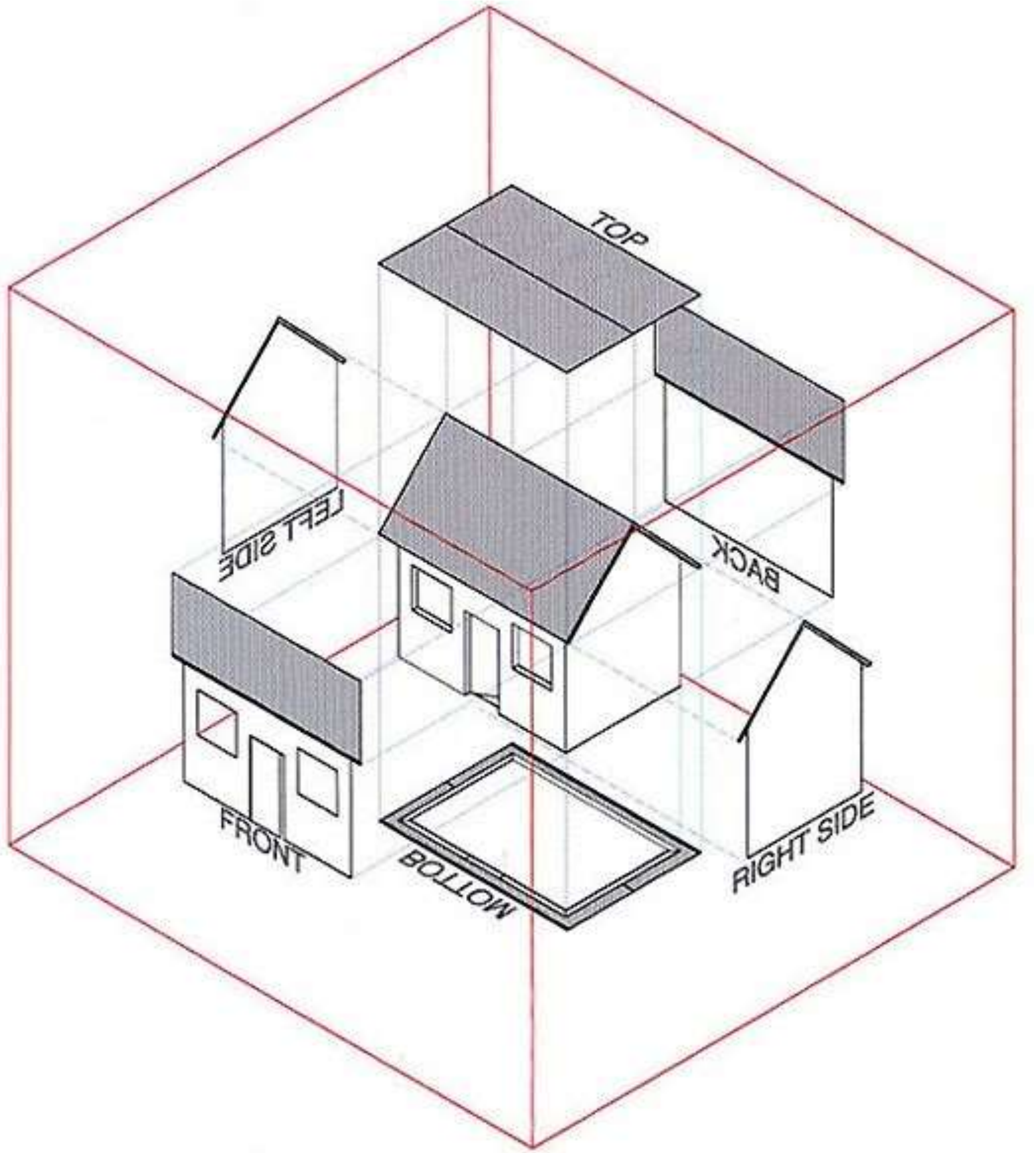
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathrm{P} \begin{bmatrix} \mathrm{X} \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} \mathrm{X} \\ 1 \end{bmatrix}$$

Affine camera model:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathrm{P}_{\mathrm{A}} \begin{bmatrix} \mathrm{X} \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{X} \\ 1 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{23} \end{bmatrix} \begin{bmatrix} \mathrm{X} \\ 1 \end{bmatrix}$$

Orthographic Camera



Affine camera:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{23} \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix}$$

Orthographic camera:

$$f = 1 \quad p_X = p_Y = 0$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix}$$

Camera Anatomy

Lens configuration (internal parameter)

$$\begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix} = L\left(\underbrace{\mathbf{K}}_{} \underbrace{\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}}_{} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} \right)$$

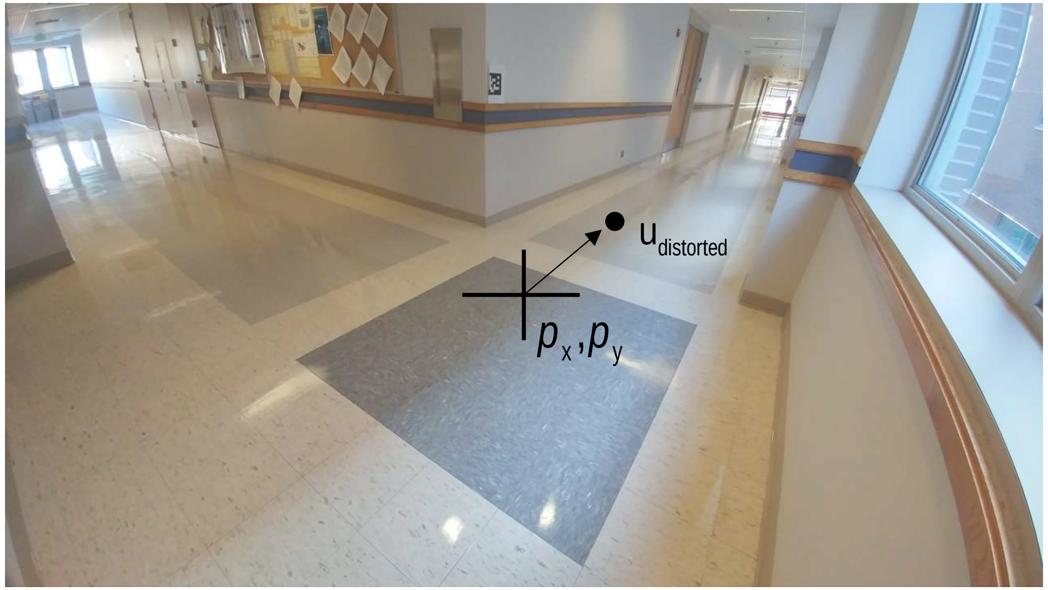
Spatial relationship between sensor and pinhole (internal parameter)

Camera body configuration (extrinsic parameter)

Lens Radial Distortion

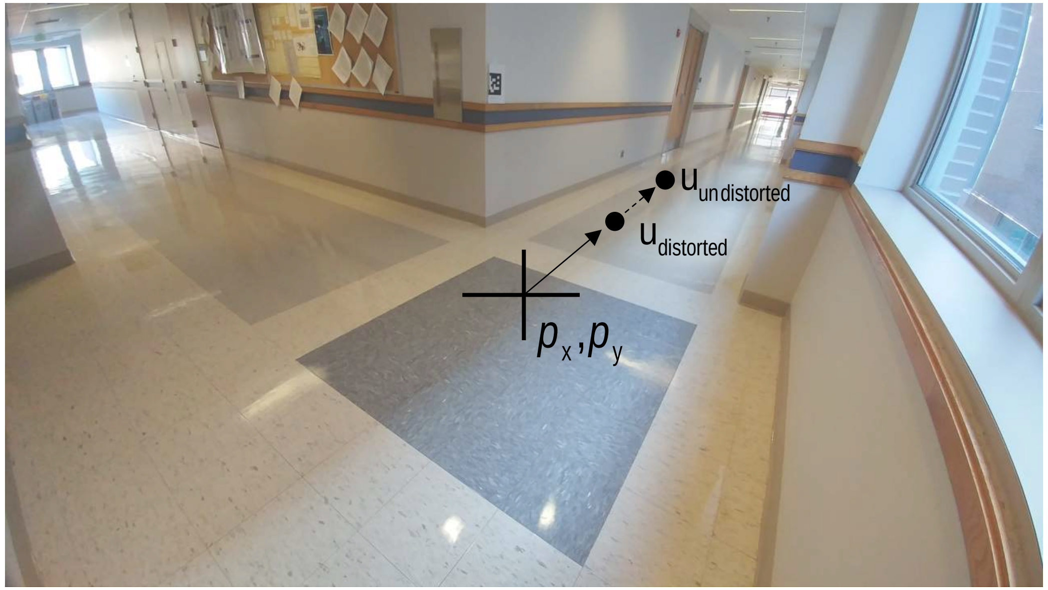
Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.



Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.



$$\overline{\mathrm{u}}_{\mathrm{distorted}} = L(\rho)\overline{\mathrm{u}}_{\mathrm{undistorted}}$$

where $$\rho = \left\| \overline{\mathrm{u}}_{\mathrm{distorted}} \right\|$$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \cdots$$

Radial Distortion Model

$$\bar{u}_{\text{distorted}} = L(\rho)\bar{u}_{\text{undistorted}}$$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \cdots$$

$k_1 < 0$

$k_1 > 0$