The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

cemc.uwaterloo.ca

Canadian Intermediate Mathematics Contest

Wednesday, November 20, 2019
(in North America and South America)

Thursday, November 21, 2019
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 2 hours

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Do not open this booklet until instructed to do so.
There are two parts to this paper. The questions in each part are arranged roughly in order of increasing difficulty. The early problems in Part B are likely easier than the later problems in Part A.

PART A

1. This part consists of six questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.**
For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

PART B

1. This part consists of three questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Be sure to write your name, school name, and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

At the completion of the contest, insert your student information form inside your answer booklet.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on the Web site, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some students may be shared with other mathematical organizations for other recognition opportunities.

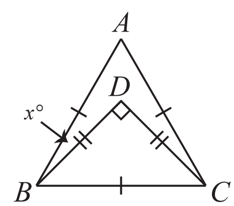
Canadian Intermediate Mathematics Contest

NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write solutions in the answer booklet provided.
3. Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.
4. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
5. Diagrams are not drawn to scale. They are intended as aids only.
6. No student may write both the Canadian Senior Mathematics Contest and the Canadian Intermediate Mathematics Contest in the same year.

PART A

For each question in Part A, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded only if relevant work is shown in the space provided in the answer booklet.

1. In the diagram, $\triangle ABC$ is equilateral. Point D is inside $\triangle ABC$ so that $\triangle BDC$ is right-angled at D and has $DB = DC$. If $\angle ABD = x^\circ$, what is the value of x?



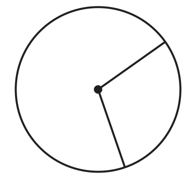
2. Binh has 20 quarters, worth 25 cents each. Abdul has 20 dimes, worth 10 cents each, as well as some quarters. If Binh and Abdul have the same amount of money, how many quarters does Abdul have?

3. Suppose that a, b and c are positive integers with $2^a 3^b 5^c = 36\,000$. What is the value of $3a + 4b + 6c$?

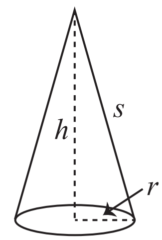
4. Ali plays a trivia game with 5 categories, each with 3 questions. She earns 1 point for each correct answer. If she answers all 3 questions in a category correctly, she earns 1 bonus point. Ali answers exactly 12 questions correctly and the remaining 3 questions incorrectly. What are her possible total scores?

5. The absolute value of a number x is equal to the distance from 0 to x along a number line and is written as $|x|$. For example, $|8| = 8$, $|-3| = 3$, and $|0| = 0$. For how many pairs (a, b) of integers is $|a| + |b| \leq 10$?

6. A sector is cut out of a paper circle, as shown. The two resulting pieces of paper are "curled" to make two cones. The sum of the circumferences of the bases of these cones equals the circumference of the original paper circle. If the ratio of the lateral surface areas of the cones is $2:1$, what is the ratio of the volumes of the cones?



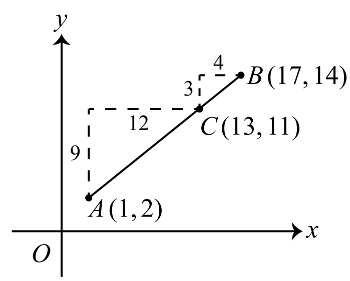
(The *lateral surface area* of a cone is the area of the external surface not including the circular base. A cone with radius r, height h and slant height s has lateral surface area equal to $\pi r s$ and volume equal to $\frac{1}{3}\pi r^2 h$.)



PART B

For each question in Part B, your solution must be well-organized and contain words of explanation or justification. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

1. The point $C(13,11)$ is on the line segment joining $A(1,2)$ and $B(17,14)$. Point C has the property that the ratio of lengths $AC : CB$ equals $3:1$ because $\dfrac{13-1}{17-13} = \dfrac{12}{4} = \dfrac{3}{1}$ and $\dfrac{11-2}{14-11} = \dfrac{9}{3} = \dfrac{3}{1}$.



In general, suppose that line segment AB is not horizontal or vertical, that C is on AB, and that C is not at A or B. The ratio of lengths $AC : CB$ equals $m : n$ exactly when the x-coordinate of C splits the x-coordinates of A and B in the ratio $m : n$ and the y-coordinate of C splits the y-coordinates of A and B in the ratio $m : n$.

(a) In the example above, determine the length of AC and the length of CB.

(b) The point $J(5,5)$ is on the line segment joining $G(11,2)$ and $H(1,7)$. Determine the ratio of the lengths $GJ : JH$.

(c) Determine the coordinates of the point F on the line segment joining $D(1,6)$ and $E(7,9)$ so that the ratio of lengths $DF : FE$ equals $1:2$.

(d) The point $M(7,5)$ is on the line segment joining $K(1,q)$ and $L(p,9)$ so that the ratio of lengths $KM : ML$ equals $3:4$. Determine the values of p and q.

2. In this problem, every 2×2 grid contains four non-zero positive digits, one in each cell. The sum of the four two-digit positive integers created by the two rows and two columns of a grid $\begin{array}{|c|c|}\hline a & b \\ \hline c & d \\ \hline\end{array}$ is denoted $\left\langle \begin{array}{|c|c|}\hline a & b \\ \hline c & d \\ \hline\end{array} \right\rangle$.

   For example, $\left\langle \begin{array}{|c|c|}\hline 3 & 4 \\ \hline 9 & 8 \\ \hline\end{array} \right\rangle = 34 + 98 + 39 + 48 = 219$.

   (a) Determine the value of $\left\langle \begin{array}{|c|c|}\hline 7 & 3 \\ \hline 2 & 7 \\ \hline\end{array} \right\rangle$.

   (b) Determine all pairs (x, y) of digits for which $\left\langle \begin{array}{|c|c|}\hline 5 & b \\ \hline c & 7 \\ \hline\end{array} \right\rangle$ equals $\left\langle \begin{array}{|c|c|}\hline x & b+1 \\ \hline c-3 & y \\ \hline\end{array} \right\rangle$.

   (c) Determine all possible values of $\left\langle \begin{array}{|c|c|}\hline a & b \\ \hline c & d \\ \hline\end{array} \right\rangle$ minus $\left\langle \begin{array}{|c|c|}\hline a+1 & b-2 \\ \hline c-1 & d+1 \\ \hline\end{array} \right\rangle$.

   (d) Determine all grids for which $\left\langle \begin{array}{|c|c|}\hline a & b \\ \hline c & d \\ \hline\end{array} \right\rangle$ equals 104, and explain why there are no more such grids.

3. There are three tables in a room. The left table has L kg of chocolate on it, the middle table has M kg of chocolate on it, and the right table has R kg of chocolate on it, for some positive integers L, M and R. People enter the room one at a time. Each person determines the table at which they will sit by determining the table at which they would currently receive the greatest mass of chocolate, assuming that the people at each table would share the chocolate at that table equally. If at least two tables would result in the same share of the chocolate, the person will sit at whichever of these tables is farther to the left.

   For example, suppose that $L = 5$, $M = 3$ and $R = 6$. Person 1 sits at the right table, because their share would be 6 kg instead of 5 kg or 3 kg at the other two tables. Person 2 then sits at the left table, because their share would be 5 kg instead of 3 kg at each of the other two tables. The chart to the right shows the possible shares in kg and the final decision for each of the first four people.

Left		Middle		Right	
5		3		6	P1
5	P2	3		3	
$\frac{5}{2}$		3	P3	3	
$\frac{5}{2}$		$\frac{3}{2}$		3	P4

   (a) Suppose that $L = 5$, $M = 3$ and $R = 6$ and that 7 people enter the room in total. Continue the chart above to show where the 7 people sit and explain why each of Person 5, Person 6 and Person 7 make their choice.

   (b) Explain why there cannot be positive integers L, M and R for which the first 6 people who enter the room sit at the tables in the following order: Left, Middle, Right, Left, Left, Left.

   (c) Suppose that $L = 9$, $M = 19$ and $R = 25$. Determine, with justification, where Person 2019 will sit.

2019
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(English)