

CASIO Education Workbook Series

CALCULUS

with the

CASIO *fx-9750GII*

WHAT'S INSIDE:

- Limits
- Continuity
- Derivatives
- Derivatives and Slope
- Derivatives and Continuity
- Derivatives and Graphs
- Slope Fields
- Anti-Derivatives
- Integration
- Riemann Sums
- Tangent Lines
- Trigonometric Applications



Topic: Continuity

NCTM Standard

- Organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teacher, and others.

Objectives

The student will be able to develop a visual understanding of how limits and continuity relate and be able to understand and communicate what it means for a function to be continuous at a point.

Getting Started

This activity will have students explore the concept of continuity at a point. It will also allow them to discover that simply having a limit at a point will not guarantee that the function is also continuous. It also explores the idea that having a limit is a necessary, but not a sufficient condition to determine the continuity of a function at a point, and through all points.

Prior to using this activity:

- Students should be able to produce and manipulate graphs of functions manually and a graphing calculator.
- Students should be able to produce split-defined (or piecewise) functions.
- Students should have a basic understanding of the language of limits.

Ways students can provide evidence of learning:

- Students should be able to produce graphs of functions and communicate symbolically, graphically and verbally the relationship between having a limit and being continuous.

Common mistakes to be on the lookout for:

- Students may produce a graph on the calculator and not be able to communicate the concept of a split-defined function because the window chosen may produce the appearance of a single unbroken formula.
- Students may confuse the pixel values with the actual function values.

Definitions:

- Asymptote
- Continuity
- Discontinuity
- Limit
- Parabola
- Vertex

A Graphical Look At Continuity

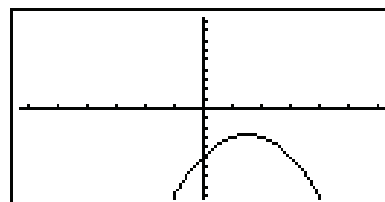
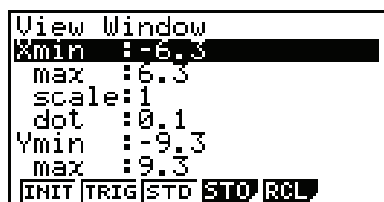
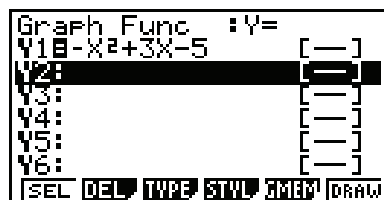
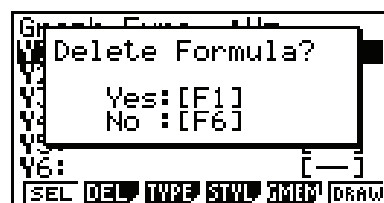
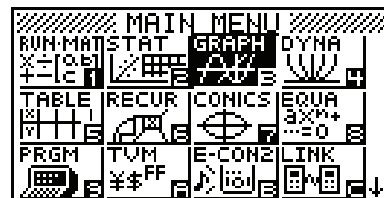
“How-To”

The following will demonstrate how to graph a function, graph a split-defined function and examine its behavior on the CASIO *fx-9750GII*.

Explore the behavior of the function $f(x) = -x^2 + 3x - 5$.

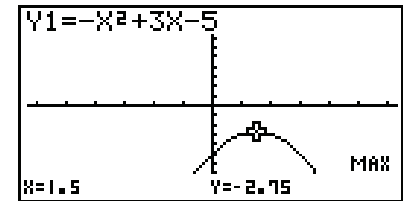
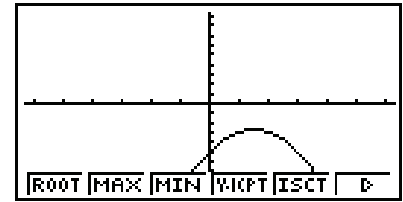
To display a graph of the function:

1. From the Main Menu, highlight the GRAPH icon and press **EXE** or press **3**.
2. To delete any previous equations, highlight the equation and press **F2** (DEL) **F1** (Yes.)
3. Enter the equation in Y1 by pressing **(-)** **X,θ,T** **x²** **+** **3** **X,θ,T** **-** **5** **EXE**.
4. Set the view window to the initial screen by pressing **SHIFT** **F3** (V-Window) **F1** (INIT). Then, change the Ymin to -9.3 and the Ymax to 9.3. This view window will be used in this activity to give the best display of the quadratic functions.
5. Press EXIT to return to the main GRAPH screen.
6. Press **F6** (DRAW) to view the graph of the function.



To find the vertex of the graph:

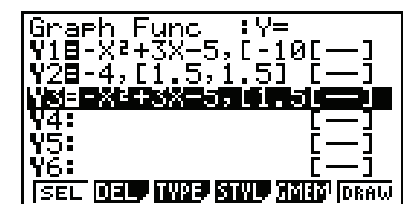
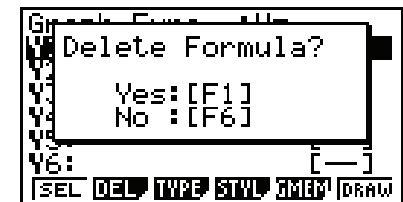
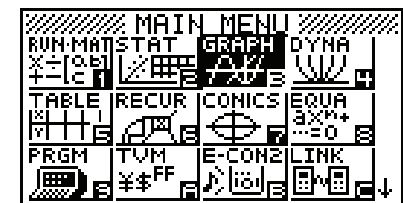
- Press **SHIFT** **F5** (**G-Solv**) **F2** (**MAX**). The coordinates of the vertex will be displayed at the bottom of the screen. [Note: **F2** (**MAX**) was pressed since the vertex of this parabola is the highest, or maximum, point. If the graph of the parabola opened up, the vertex would be the lowest, or minimum, point and you would have chosen **F3** (**MIN**).]



Explore the behavior of the split-defined function $g(x) = \begin{cases} -x^2 + 3x - 5, & x < 1.5 \\ -4, & x = 1.5 \\ -x^2 + 3x - 5, & x > 1.5 \end{cases}$.

To graph a split-defined function:

- From the Main Menu, highlight the GRAPH icon and press **EXE** or press **3**.
- To delete any previous equations, highlight the equation and press **F2** (**DEL**) **F1** (**Yes**).
- Enter each branch of the split-defined (piece-wise) function in its own Y= slot, then create the restrictions by putting the lower and upper bounds in brackets.

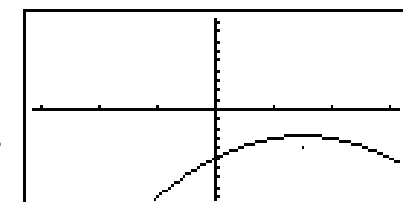


For Y1, press **(←)** **X,θ,T** **x²** **+** **3** **X,θ,T** **-** **5** **,** **SHIFT** **+** **(←)** **1** **0** **,** **1** **0** **5** **SHIFT** **-** **EXE**.

For Y2, press **(←)** **4** **,** **SHIFT** **+** **1** **0** **5** **,** **1** **0** **5** **SHIFT** **-** **EXE**.

For Y3, press **(←)** **X,θ,T** **x²** **+** **3** **X,θ,T** **-** **5** **,** **SHIFT** **+** **1** **0** **5** **,** **1** **0** **5** **SHIFT** **-** **EXE**.

- Press **F6** (**DRAW**) to view the graph of the functions.



A Graphical Look At Continuity

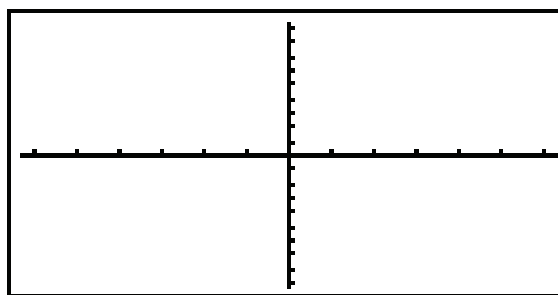
Activity

This activity will have you explore the concept of continuity at a point. It will also allow you to discover that simply having a limit at a point will not guarantee that the function is also continuous. Using the CASIO *fx-9750GII*, you will be working in pairs or small groups.

Questions

Explore the behavior of the function $f(x) = x^2 - x - 6$ around the vertex.

1. Graph the function using the initial view window, changing Ymin to -9.3 and Ymax to 9.3 , and copy on the axes below.



2. Find and record the vertex of the function.

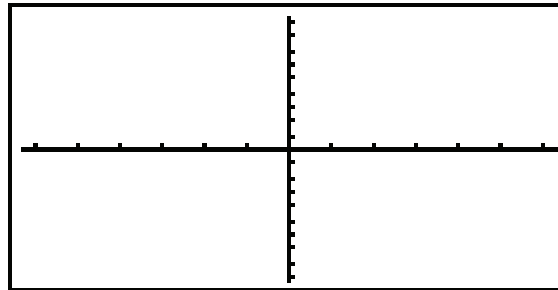
3. Trace to the vertex and zoom in, record what you see.

4. What does the value of $\lim_{x \rightarrow 0.5} f(x)$ appear to be? Explain how you arrived at that answer.

5. Explore the behavior of the following split-defined function:

$$g(x) = \begin{cases} x^2 - x - 6, & x < 0.5 \\ -6, & x = 0.5 \\ x^2 - x - 6, & x > 0.5 \end{cases}$$

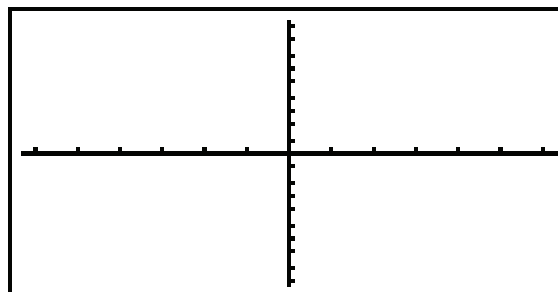
Use the same viewing window as before. Record what you see below.



6. What does the value of $\lim_{x \rightarrow 0.5} g(x)$ appear to be?

7. How does it compare to the $\lim_{x \rightarrow 0.5} f(x)$?

8. Now trace to a value where $x = 0.5$ and zoom in. Describe and record what you see.



9. Find the following limits:

a. $\lim_{x \rightarrow 0.5^+} g(x)$ _____

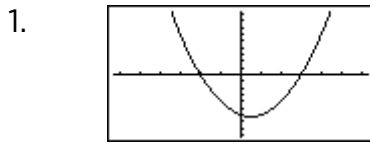
b. $\lim_{x \rightarrow 0.5^-} g(x)$ _____

c. $\lim_{x \rightarrow 0.5} g(x)$ _____

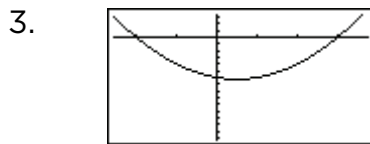
10. Find $g(0.5)$. How does this compare to your answer for $\lim_{x \rightarrow 0.5} g(x)$?

11. Draw a conclusion about the relationship between limits and continuity.

Solutions

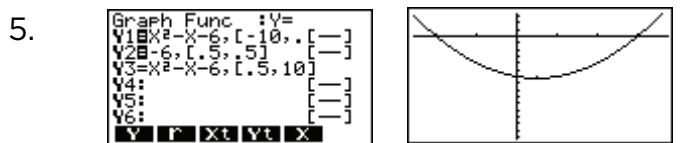


2. (0.5, -6.25)



Nothing unusual should be seen. The function is continuous.

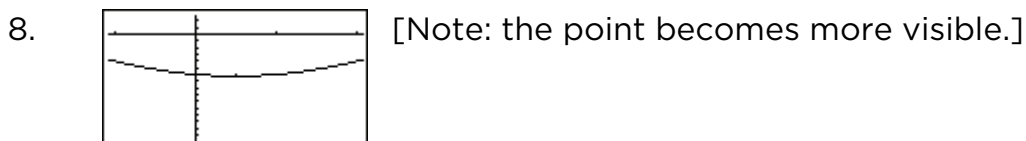
4. The limit is -6.25, the vertical value of the vertex. Answer will vary; care should be taken to point out that simply tracing to a value is not confirmation enough and can be tricky. Direct substitution is a valid explanation. A good answer might also include a mention of “passing through” or even a mention of continuity.



[Note: The discontinuity will not be immediately apparent.]

6. The limit is -6.25.

7. Answers may vary as students begin to get the idea that the change in the definition of the function may be creating some problems, although not with the limit. This is a good checkpoint for the understanding of what it means to be a “limit.”



9. All three limits are -6.25, although some students may try to refine the answers to longer decimals. This provides another good opportunity to stress the idea of “limit” as the value the function approaches.

10. $g(0.5) = -6$, a value different from the limit.

11. Answers will vary; a good answer will include the fact that the function has a gap or a hole or a jump at the point of discontinuity. The idea is to have the students begin to think about the fact that simply having a limit does not guarantee the continuity of a function.