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CBO's Model for Forecasting Business Investment

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Abstract

The Congressional Budget Office models most business investment by using a modified neoclassical specification. That specification is similar to the neoclassical model in that the desired capital stock depends positively on output and negatively on the cost of capital, which includes the price of capital, taxes, and rates of return. The specification differs from the neoclassical model in that the capital stock adjusts more rapidly to changes in output than to changes in the cost of capital. In contrast, CBO models investment in capital used by agriculture and extractive industries as depending primarily on output prices. The model contains two important innovations with respect to past modified neoclassical models. First, capital is modeled in a way that allows the productivity of new capital to be observed by using the productivity of existing capital, making it possible to estimate a time-varying coefficient of capital in the production function. Second, capital income is separated into that generated by "measured capital"—equipment, structures, intellectual property products, inventories, and land—and that generated by other sources—market power and unmeasured capital. An increase in the relative importance of unmeasured sources of capital income helps explain why business investment has not kept pace in recent years with high levels of capital income.

Keywords: investment, capital income, forecasting

JEL Classification: E22, E27

Contents

Introduction

Investment by businesses, consisting of private purchases of equipment, structures, and intellectual property products (IPPs), as well as the change in business inventories, is a crucial part of the Congressional Budget Office's economic forecast. Business investment accounts for a disproportionate share of fluctuations in gross domestic product (GDP); despite averaging less than 13 percent of GDP, business investment has accounted for nearly half of the variation of year-over-year growth of real GDP since 1950. In addition, business investment determines growth of the capital stock, a key element of CBO's forecast of potential GDP and thus CBO's forecast of actual GDP.

CBO models most types of business investment by using a modified neoclassical specification for capital. Bernanke, Bohn, and Reiss (1988) and Oliner, Rudebusch, and Sichel (1995) found that such a model performs better than other alternatives, including the neoclassical model. In a modified neoclassical model, as in the neoclassical model, the desired capital stock depends positively on output and negatively on the cost of capital, which includes the price of capital, taxes, and rates of return. However, unlike the neoclassical model, the modified neoclassical model incorporates the assumption that the capital stock responds more rapidly to changes in output than to changes in the cost of capital, leading to superior performance. As a result, short-run movements in investment depend primarily on changes in the growth of output. By contrast to the modified neoclassical approach, CBO models short-run movements in investment in capital specific to mining and farming as depending primarily on changes in expectations of prices.

CBO's modeling strategy differs from that of past modified neoclassical models in two main ways. The first is how capital is specified. In past modified neoclassical models, the productivity of new capital is independent of the productivity of existing capital. That approach makes gleaning capital's coefficient in the production function (the importance of capital in comparison with that of labor) from macroeconomic data impossible. This paper uses a different approach that enables the productivity of new capital to be observed from the productivity of existing capital. That method allows the coefficient of capital in production to be estimated.

The second difference with past modified neoclassical models is the separation of capital income into that generated by "measured capital" and that from other sources. Measured capital consists of factors used by the Bureau of Labor Statistics to construct its measure of the capital input to production—equipment, structures, IPPs, inventories, and land. (Business investment consists entirely of measured capital.) Other sources of capital income include market power and unmeasured capital. Recent work by Autor and colleagues (2017), Barkai (2016), and De Loecker and Eeckhout (2017) highlights the growing importance of those other sources of capital income. A decline in the share of capital income earned by measured capital helps resolve the puzzle of why investment in measured capital has not kept pace in recent years with high levels of overall capital income.

Investment in capital specific to mining and farming is modeled primarily as a function of the real prices of crude oil, natural gas, and farm output because planned production in commodities industries (such as mining and farming) depends strongly on prices. The increase in the importance of hydraulic fracturing (fracking) has more than doubled the elasticity of investment in new oil wells with respect to oil prices over the past 15 years.

The model of investment described here forms part of the structural macroeconometric model that CBO uses to forecast the U.S. economy (Arnold 2018). Many factors that drive investment, such as output, the real prices of various types of capital, and productivity, are determined in that model. Investment then feeds back into that model as an input to variables such as GDP and potential GDP.

The paper is organized as follows. Section 1 presents the conceptual framework for the model, including a basic modified neoclassical model of business investment, a discussion of capital income from unmeasured sources, and a procedure to estimate a basic equation for business investment. Section 2 incorporates many types of capital and gradual adjustment into the basic model of investment. Section 3 compares the model with other models of investment. Section 4 provides the background of the estimation, and section 5 contains the empirical results. Section 6 shows how investment in farming and mining capital is modeled and estimated. Section 7 shows how the model is used to forecast investment.

1. Framework

This section presents a basic modified neoclassical model of investment and shows how it is estimated. A firm's investment in new capital depends, in both the modified neoclassical and neoclassical models, on demand for the firm's output and the real after-tax cost of new capital. The response of investment to changes in demand is similar in the two models, but the response to changes in the cost of capital differs. Subsection 1.1 provides some intuition of why that is the case based on how the two models work. Subsection 1.2 then describes in mathematical terms how capital is modeled in the modified neoclassical model.

The magnitudes of the responses of investment in measured capital to changes in demand and the cost of capital depend on the importance of measured capital in the production of goods and services. That importance is typically estimated using capital's share of total income. However, if capital income includes income earned by sources other than measured capital, that share is too large. Evidence that the importance of such income has increased significantly since 2000 makes adjusting capital's share of income for such income even more essential. Subsection 1.3 discusses capital income from unmeasured sources, which is incorporated into the analysis in the rest of section 1.

In subsection 1.4, profit maximization is used to derive an equation for investment in measured capital as a function of the factors determining it. To simplify that equation, there is only one type of capital and there is no time lag between changes in the factors determining investment and the response of investment. One of those factors is the expected after-tax rate of return, which is an important component of the cost of capital. Because of difficulties posed by using bond yields and dividend yields to estimate that rate, it is instead estimated using the market value of capital, as shown in subsection 1.5. Subsection 1.6 provides an overview of the iterative process needed to estimate the equation for investment.

1.1. How the Modified Neoclassical and Neoclassical Models View Capital. In the modified neoclassical model, originally developed by Johansen (1959), Solow (1962), and Phelps (1963), the capital stock consists of distinct units of capital—for example, aircraft, factory equipment, computers, stores, oil wells, software programs, and movies. Those units may be used together—for example, computers and software—but each unit retains its individual characteristics and is used in combination with a constant number of workers throughout its service life. The fixed ratio of capital to labor over the service life of a unit of capital is known as "putty–clay" or "ex post fixed proportions." The capital stock grows over time as depreciating units are replaced by units of higher quality and as the number of units rises to accommodate an increasing number of workers.

In the neoclassical model, such as that of Jorgenson (1963), the capital stock consists of amorphous lumps of each type of capital—for example, a lump of aircraft capital and a lump of computer capital. Each lump is continually spread evenly across the total number of workers, so that the number of workers using a given stock of capital depends on the contemporaneous ratio of the cost of labor to the cost of capital.

The response of investment to a change in demand for a firm's output is roughly the same in both models. The firm wants to employ more workers and more capital to meet the increase in demand for its output. Investment also follows the same time pattern in each model. For example, consider a permanent increase in the level of demand. As a business adjusts its capital to the new level of demand, investment temporarily overshoots its long-run level before settling back down after the adjustment of capital is complete. That response of the level of investment to the growth, rather than to the level, of output is known as the "accelerator."

In contrast, the response of investment to a change in the cost of capital differs in the two models. In the neoclassical model a firm can allocate its lump of capital in any way it chooses. If the cost of capital falls, for example, by an amount that boosts a firm's desired capital stock by 10 percent relative to the number of workers, the firm modifies its stock of aircraft to give each worker 10 percent more, its stock of computers to give each worker 10 percent more, and so on. Consequently, the *level* of investment depends on the *change* in the cost of capital.

In the modified neoclassical model, however, the quality of existing capital is fixed. If the cost of capital falls, a firm will replace depreciating capital with better capital than they otherwise would have but cannot modify existing capital. Consequently, the level of investment depends on the level of the cost of capital rather than on the change in the cost of capital. In practical terms, that property causes investment to respond more gradually to changes in the cost of capital than to changes in demand in CBO's model.

No theoretical basis exists on which to choose between the two models. Although the fixed quality over service life of the modified neoclassical model may seem more intuitive for equipment, businesses do modify existing structures, as in the neoclassical model. Consequently, the choice must be made on an empirical basis. Model comparisons such as those of Bernanke, Bohn, and Reiss (1988) and Oliner, Rudebusch, and Sichel (1995) argue in favor of the modified neoclassical model.

1.2. Measured Capital and the Production Function. Capital is composed of discrete amounts, designated "units" of capital. Units of capital are normalized so that each worker uses one unit of capital. For example, the unit of capital for an office worker might consist of a desk, chair, computer, some software packages, and 1 percent of a 100-person office building. Thus, N is both the number of workers and the number of units of capital. The real dollars of capital embodied in a new unit of capital is its "quality," denoted k. The quality of a unit of capital can be freely chosen (or putty) before the unit is purchased but is fixed (or clay) afterward.

Individual units of capital are combined into a total capital stock, K, by multiplying the number of units of capital by a geometric average of the quality of each unit of capital ($\bar{k}$). Indexing the units from 1 to N, with unit j having quality kj, total quality per unit is given by

$$\bar{k} = k1^{1/N} k2^{1/N} ... kN^{1/N}.$$

The capital input to production, K, is $\bar{k}N$. Productivity of new capital depends positively on the quality of existing units of capital even though the quality of a new unit of capital experiences diminishing returns. For example, having better software increases the gain from upgrading to a better personal computer, but the return on an upgrade decreases for each additional terabyte of memory.

That formulation links the productivity of new capital to the productivity of existing capital, which can be measured from observable data. The ability to express demand for new capital as a function of the observable productivity of existing capital preserves a key mathematical advantage of the neoclassical model. As shown below, that enables us to estimate capital's coefficient in production and the market value of new capital, which is then used to estimate the cost of capital. In the modified neoclassical (putty–clay) models of Johansen (1959), Phelps (1963), Calvo (1976), and Gilchrist and Williams (2005), the productivity of new capital is independent of the productivity of existing capital.[1] In that case, the productivity of new capital cannot be estimated from observable data.

As capital ages, it depreciates and must be replaced. In the neoclassical model, a percentage of the existing lump of capital depreciates every year. In CBO's modified neoclassical model, depreciation is assumed to take the form of retiring existing units of capital at rate δ. That holds the quality of each unit of capital constant over its service life. Realistically, the productivity of old units of capital can fall so far below that of new units that firms

[1] Aruga (2009) presents a model in which the aggregate stocks of two types of capital are complementary. In that model, however, individual units of one type of capital are complementary only with the other type of capital.

could profitably discard and replace those older units even while they are still functional. However, with endogenous replacement, the level of investment depends on the change in the cost of capital—for example, a lower cost of capital leading firms to discard less efficient older units—a property of the neoclassical model at variance with the empirical evidence.

Let F_t denote the flow of new units of capital at time t, equal to growth of the number of workers plus replacement demand:

$$(1) \qquad F_t = \frac{dN_t}{dt} + \delta N_t.$$

Real investment at time t, I_t, is the product of k_t and F_t; that is, $I_t = k_t F_t$. In the rest of this section, investment is modeled using continuous time in order to simplify the math. The above equation for $\bar{k}$ is a discrete approximation used for exposition. The actual expression for $\bar{k}$ is given by equation (2) below.

Because $N_t = \int_{i=t-\infty}^{t} F_i \exp\left(-\delta\left(t-i\right)\right) di$, total quality per unit of capital is

$$(2) \qquad \log\left(\bar{k}_t\right) = \frac{\int_{i=t-\infty}^{t} F_i \exp\left(-\delta\left(t-i\right)\right) \log\left(k_i\right) di}{N_t}.$$

Differentiating equation (2) with respect to t yields

$$(3) \qquad \frac{d\bar{k}_t}{dt} = \bar{k}_t \frac{F_t}{N_t} \log\left(\frac{k_t}{\bar{k}_t}\right).$$

Adding in the growth rate of N yields

$$(4) \qquad \frac{dK_t}{dt} = K_t \left[\frac{F_t}{N_t} \log\left(\frac{k_t}{\bar{k}_t}\right) + \frac{dN_t/N_t}{dt}\right].$$

Businesses produce output (Y) by using measured capital (K), workers (N), and technology, or total factor productivity (TFP; denoted A). Production is expressed mathematically using a Cobb–Douglas production function with constant returns to scale:

$$d \log Y_t = d \log A_t + \alpha_t \; d \log K_t + (1 - \alpha_t) \; d \log N_t. \tag{5}$$

The production function is expressed in differences because capital's coefficient in production (α) can vary over time.

1.3. Unmeasured Sources of Capital Income. Capital income is any income from the production of goods and services that does not go to labor, including primarily profits, interest, depreciation, rental income, and the portion of proprietors' income not attributable to proprietors' labor. Most capital income is earned by measured capital, as estimated below in section 5. However, a significant proportion of capital income is probably derived from other sources, notably in high-tech industries. For example, Apple earned \$46 billion of net noninterest income in fiscal year 2017 on just \$56 billion of measured capital, including its stock of research and development.[2] Unless Apple's measured capital earns an implausibly high rate of return, most of Apple's earnings come from something other than its measured capital.

Much past work on unmeasured sources of capital income (USCIs) has focused on estimating investment in unmeasured intangible capital. Hall (2001) uses increases in the market value of firms not explained by measured investment to estimate investment in intangible

[2]Net noninterest income is net income less net interest and dividend income. Measured capital is the average of end-of-year values for 2016 and 2017 of net property, plant, and equipment plus a perpetual-inventory estimate of research and development capital (using a depreciation rate of 35 percent) plus acquired intangible assets. Values are from Apple's 10-K filing for fiscal year 2017.

capital. Corrado, Hulten, and Sichel (2009) estimate investment and income for several forms of intangible capital, including software, research and development, and "economic competencies," including brand equity and firm-specific resources. Many of those forms are now part of measured capital. McGrattan and Prescott (2010) use fluctuations in hours worked unattributable to movements in compensation per hour to infer fluctuations of investment in unmeasured capital. De Loecker and Eeckhout (2017), approaching the issue from another angle, interpret changes in the relationship of output to variable costs as measuring changes in market power. Instead, this paper estimates income from unmeasured sources as the portion of capital income not generated by measured capital.

The two USCIs are market power and unmeasured capital. Ultimately, market power comes from barriers to entry, including legal monopolies and licensing requirements. Slower formation of new businesses since 2000 may have increased the market power of incumbent firms. Tightening of lending standards for start-ups after the 2007–2009 recession could have acted as a barrier to entry.

The other USCI is unmeasured capital. Examples are reputation, brand equity, and firm-specific TFP—a firm's systems and technology that boost its productivity above that of competing firms. Firm-specific TFP can give rise to "superstar" firms, as discussed in Autor and colleagues (2017).

In the absence of comprehensive estimates of market power and unmeasured capital, USCIs are modeled as factors causing the elasticity of demand for a firm's output (η) to be less than infinite. That treatment leads capital income to exceed income from measured capital. As in Abel and Eberly (2011), the inverse demand function for the representative firm's output is

$$\frac{p_t}{\tilde{p}_t} = \left(\frac{Y_t}{\omega \tilde{Y}_t}\right)^{-\frac{1}{\eta_t}},$$

where ω is the firm's share of the aggregate demand curve, p is the firm's price, and $\tilde{p}$ and $\tilde{Y}$ denote total levels. The variable η is a time-varying price elasticity of demand common to all firms. Because all firms face the same profit-maximization decision, subject to scaling factor ω, and because labor and measured capital experience constant returns to scale, all firms charge the same price.

1.4. Profit Maximization and Investment. Investment in measured capital is modeled by examining the profit-maximization decisions of a representative firm. That firm takes $\tilde{p}$, $\tilde{Y}$, the after-tax price of new capital (v), compensation per worker (w), and the after-tax rate of return as given. The firm chooses the rate of growth of workers (dN/dt) and the quality of new capital (k). Those choices determine output and the price the firm receives for its output. To allow the ratio of capital goods prices to the overall price level to change over time, firms are assumed to produce a single type of intermediate output, which is then sold to a firm that transforms it into various types of final goods and services, including capital.

The firm seeks to maximize the present discounted value (PDV) of net after-tax receipts:

$$VR_t = \int_{j=t}^{\infty} \left[(1-u_j)\left(p_j Y_j - w_j N_j\right) - v_j k_j F_j\right] \exp\left(-\int_{i=t}^{j} r_i di\right) dj, \tag{6}$$

where u is the tax rate on business income and r is the expected after-tax rate of return, a weighted average of the after-tax returns to debt and equity.[3] The appendix (section A4) shows that the firm's maximization problem would be the same if, instead, it maximized after-tax profits (by subtracting after-tax interest) and discounted using only the cost of equity. The variable v is the after-tax price of new capital purchased at time t,

$$v_t = p_{kt}\left[1 - itc_t - u_t z_t\right],$$

where p_k is the price of new capital, itc is the rate of investment tax credit or research tax credit, and z is the PDV of depreciation allowances per dollar of new capital. The relevant tax rate is the one at the time depreciation is taken, so u_t is actually an expected value at time t.

Readers familiar with recent specifications of the neoclassical model will note the absence of adjustment costs and q from equation (6). In the neoclassical model, adjustment costs are necessary to prevent explosive swings in investment in response to changes in r or v as businesses recalibrate the capital-to-labor ratio of their existing capital. In the modified neoclassical model, however, recalibration of existing capital is impossible, so adjustment costs are unnecessary. A need still exists to model lags of investment behind changes in Y, which is shown below in section 3.3.

Maximizing VR_t with respect to k_t yields the first-order condition

$$\int\limits_{j=t}^{\infty} \alpha_j \left(1 - 1/\eta_j\right) p_j \left(1 - u_j\right) \frac{Y_j}{N_j} e^{-\delta(j-t)} \exp\left(-\int_{i=t}^{j} r_i di\right) dj = v_t k_t.$$

[3]Revenues minus labor costs are assumed to be the source of business transfer payments. To simplify notation, u in the $1-u$ terms throughout the paper includes both business income taxes and business transfer payments.

To simplify that, assume that the firm expects α and η to remain at their time-t values; that future r_i can be replaced by a single rate, r_t; and that the firm expects p and Y/N to grow at rates $\dot{p}_t$ and $\dot{y}_t$, respectively. The first-order condition then becomes

$$k_t = \alpha_t \left(1 - 1/\eta_t\right) \frac{p_t}{v_t} \frac{Y_t}{N_t} \frac{1 - u_t}{\delta + r_t^*}, \tag{7}$$

with $r_t^* \equiv r_t - \dot{p}_t - \dot{y}_t$. The variable r_t^* is the real after-tax rate of return less expected productivity growth. The real user cost of capital is the inverse of $\frac{p_t}{v_t}\frac{1-u_t}{\delta+r_t^*}$.

The quality of a new unit of capital is proportional to labor productivity (Y/N) and is greater the greater is capital's coefficient in production (α) and the greater is η. Quality is inversely proportional to the real after-tax price of new capital (v/p) and is lower the greater the rate at which capital depreciates (δ). A higher real after-tax rate of return ($r - \dot{p}$) reduces investment, whereas higher expected growth of productivity ($\dot{y}$) boosts investment.

Equation (7) can be used to compare the canonical version of the neoclassical model (Jorgenson 1963) with the modified neoclassical model. If the quality of a unit of capital can be adjusted costlessly after it has been produced, as in the neoclassical model, then multiplying both sides of equation (7) by N_t yields the neoclassical capital stock ($k_t N_t$) on the left-hand side of the resulting equation. Most of the terms on the right-hand side are familiar from Jorgenson's model. The exceptions are the allowance for USCI through η and two differences in r^*: In the neoclassical model, expected growth of productivity does not affect investment and $\dot{p}$ is replaced by $\dot{p}_k$.

To determine total F_t, first note that because all firms charge the same price, the growth rate of each firm's output is the same as the growth rate of total output, $\dot{Y}$. Because all firms have the same TFP and choose the same k, they also have the same growth rate of output per worker, $\dot{y}$, where $y \equiv Y/N$. Thus, the growth rate of employment ($\dot{N}$) is equivalent to $\dot{Y}_t - \dot{y}_t$.[4] Using that expression, (1) becomes

$$F_t = \left(\dot{Y}_t - \dot{y}_t + \delta\right) N_t. \tag{8}$$

Multiplying (7) by (8) yields an equation for investment:

$$I_t = k_t F_t = \alpha_t \left(1 - 1/\eta_t\right) \frac{p_t}{v_t} Y_t \frac{1 - u_t}{\delta + r_t^*} \left(\dot{Y}_t - \dot{y}_t + \delta\right). \tag{9}$$

1.5. Method of Determining r^*. The standard method of determining the real after-tax rate of return $r - \dot{p}$ (part of r^*) is by subtracting an estimate of expected inflation from a combination of yields on stocks and bonds. However, that method poses several difficulties. First, Rognlie (2015) notes that expected inflation is difficult to observe. Second, the dividend-discount model used to estimate the equity portion of r incorporates the assumption that a stable relationship exists between current dividends and investors' expectation of future profits. However, Chetty and Saez (2005) find that firms boosted dividend payments after the difference between the tax rates on dividends and capital gains was reduced. Also, dividends may be endogenous to the investment decision if firms reduce dividends to fund additional investment and may be a poor indicator of future profits when firms slash dividends to preserve liquidity.

Instead, r^* is determined using the market value of debt and equity. The method is to

[4] If investment occurs simultaneously with hiring, as in this basic model, the impact of investment on $\dot{y}$ must be taken into account. In the full model, however, investment lags behind hiring, so $\dot{y}$ can be considered exogenous with respect to the choice of k.

determine what that market value would be if r^* was equal to its sample average ($\hat{r}^*$) and then to use the difference between that value and the actual market value to determine the difference between k and the value of k at $\hat{r}^*$ ($\hat{k}$). Using the actual market value of capital means that the estimates incorporate actual risk premia.

Let V be the market value of debt and equity and $\hat{V}$ be what that value would be if r^* was equal to $\hat{r}^*$, all else equal. The relationship between V and r^* is approximately

$$V_t \approx \hat{V}_t + (r_t^* - \hat{r}^*) \frac{d\hat{V}_t}{d\hat{r}^*}, \text{ or}$$

$$r_t^* - \hat{r}^* \approx \frac{V_t - \hat{V}_t}{d\hat{V}_t / d\hat{r}^*}.$$

Substituting that expression into a linearized version of the relationship between k and r^* implies

$$k_t \approx \hat{k}_t + (r_t^* - \hat{r}^*) \frac{d\hat{k}_t}{d\hat{r}^*} \approx \hat{k}_t + \frac{V_t - \hat{V}_t}{d\hat{V}_t / d\hat{r}^*} \frac{d\hat{k}_t}{d\hat{r}^*}. \tag{10}$$

The value of $\hat{k}_t$ can be found by substituting $\hat{r}^*$ for r_t^* in (7). All that remains to express r_t^* in terms of observable variables is to determine $\hat{V}$.

To determine the portion of $\hat{V}$ from measured capital, one can express the firm's maximization problem as in Hayashi (1982) by using the Hamiltonian function

$$H_t = \left[(1 - u_t)(p_t Y_t - w_t N_t) - v_t k_t F_t + \lambda_t \frac{dK_t}{dt} \right] \exp\left(-\int_{i=0}^{t} r_i di \right), \tag{11}$$

in which k_t is the control variable affecting the state variable K_t and λ_t is the shadow price of K_t in generating future after-tax revenues. After substitution for dK_t/dt from (4), and some rearranging, the first-order condition $\frac{\partial H_t}{\partial k_t} = 0$ yields

$$k_t = \frac{\lambda_t K_t}{N_t} \frac{1}{v_t}. \tag{12}$$

The appendix (section A1) shows that $\lambda_t K_t$, hereafter denoted V_t^M, equals the market value of existing measured capital adjusted for the difference between measured capital's coefficient in production and measured capital's share of the income of labor and measured capital (see below). Combining (7) and (12), we have

$$\hat{V}_t^M = \hat{k}_t v_t N_t = \alpha_t \left(1 - 1/\eta_t\right) p_t Y_t \frac{1 - u_t}{\delta + \hat{r}^*}. \tag{13}$$

The link between the market value of capital and investment allows us to compare the modified neoclassical model with a well-known variant of the neoclassical model—the Tobin's q model. To express investment as a function of Tobin's q, multiply (8) by (12) and divide through by K_t^R, the current cost of replacing the existing capital stock, to obtain

$$\frac{I_t}{K_t^R} = q_t \left(\dot{Y}_t - \dot{y}_t + \delta\right), \tag{14}$$

with $q_t = \frac{V_t^M}{v_t K_t^R}$. In the standard Tobin's q model, investment is determined solely by q. In CBO's model, however, investment depends on growth of aggregate demand as well as on q. As discussed in the next section, q is interpreted differently in those two models.

Estimating the portion of $\hat{V}$ from USCIs, V^U, requires an estimate of η. The appendix (section A1) shows that, using the same assumptions used to derive (7),

$$\frac{1}{\eta_t} = 1 - \frac{w_t N_t}{(1-\alpha_t x_t)\, p_t Y_t}, \tag{15}$$

where $x_t \equiv 1 + \log\left(\frac{k_t}{\bar{k}_t}\right) \frac{\dot{n}_t - r_t^*}{\delta + r_t^*}$. Because USCIs boost the price by $\frac{1}{\eta}p$, capital income from unmeasured sources is $\frac{1}{\eta}pY$.

To estimate $\hat{V}^U$, assume that existing USCIs depreciate at rate δ_U. The number of representative firms is assumed constant, and investors expect the revenue from undepreciated USCIs to grow with the output of the firm. Thus, investors expect real income of undepreciated USCIs to grow at rate $\dot{y}_t + \dot{n}_t^U$, where $\dot{n}_t^U$ is the growth rate of employees expected over the service life of USCIs beginning at time t. With those assumptions, the market value of existing USCIs evaluated at $\hat{r}^*$ is

$$\hat{V}_t^U = \frac{1}{\eta_t}(1-u_t)\frac{p_t Y_t}{\delta_U + \hat{r}^* - \dot{n}_t^U}. \tag{16}$$

To determine total $\hat{V}$, note that (15) can be rearranged to parse pretax capital income as

$$p_t Y_t - w_t N_t = \frac{1}{\eta_t} p_t Y_t + \alpha_t\left(1 - \frac{1}{\eta_t}\right) p_t Y_t + \alpha_t\left(1 - \frac{1}{\eta_t}\right)(x_t - 1)\, p_t Y_t.$$

The after-tax PDVs of the first two right-hand-side terms are V_t^U and V_t^M. The final right-hand-side term of that equation is required because capital's share of the income generated by a unit of capital and the corresponding worker falls over time. Economywide, labor income per unit of capital and capital income per unit of capital rise at roughly the same rate. For an individual unit of capital, however, capital income per unit of capital rises more slowly than that economywide rate because k for that unit is fixed while $\bar{k}$ is rising. Consequently, the capital income from a new unit of capital is front-loaded with respect to total income from that unit. The ratio of the PDV of capital income to the PDV of total income from a unit—equal to α from profit maximization—is thus greater than the average of capital's share of income from that unit. That difference grows with r. However, the more rapid is $\dot{n}$, the younger is the stock of capital, and thus the greater is the share of units with a relatively high capital share of income.

The variable x averages less than 1 (about 0.94) because k is generally greater than $\bar{k}$ as a result of productivity growth and r^* has averaged greater than $\dot{n}$ historically. Thus, on average, capital is in the stage of its service life at which it earns less than share α of the income of labor and measured capital. Under assumptions similar to those used to derive $\hat{V}_t^M$ and $\hat{V}_t^U$, the PDV of the after-tax adjustment to the market value of measured capital evaluated at $\hat{r}^*$ can be expressed as

$$\hat{V}_t^W = \left(1 - \frac{1}{\eta_t}\right) \alpha_t \left(x_t - 1\right) \left(1 - u_t\right) \frac{p_t Y_t}{\delta + \hat{r}^*}.$$

The variable $\hat{V}_t$ is the sum of $\hat{V}_t^M$, $\hat{V}_t^U$, and $\hat{V}_t^W$.

1.6. Estimation Procedure. To estimate the basic model, one finds the equation for investment by substituting (7), with $\hat{r}^*$ replacing r_t^*, into (10) and multiplying by (8) to yield

$$I_t = \frac{\alpha_t \left(1 - 1/\eta_t\right) \left(1 - u_t\right) p_t Y_t}{v_t \left(\delta + \hat{r}^*\right)} \left(1 - \frac{1}{\delta + \hat{r}^*} \frac{V_t - \hat{V}_t}{d\hat{V}_t / d\hat{r}^*}\right) \left(\dot{Y}_t - \dot{y}_t + \delta\right). \tag{17}$$

After a value for $\hat{r}^*$ is chosen, that equation contains only two variables to be estimated—α_t and η_t. The estimation procedure is iterative. In each iteration: (17) is estimated using a Kalman filter for $\alpha_t\,(1 - 1/\eta_t)$; α_t is calculated from that estimate by rearranging (15) to form

$$\alpha_t = \frac{SV_t}{x_t SV_t + w_t N_t / p_t Y_t}, \tag{18}$$

where SV is the filtered estimate of $\alpha_t\,(1 - 1/\eta_t)$; η_t is calculated using (15); and $\hat{V}_t$ is updated using the new α_t and η_t.

2. Comparison With Other Models of Investment

Economists have developed a few types of models of economywide investment with which many readers are familiar: Jorgensonian neoclassical models, Tobin's q models, and putty–clay models (the best-known type of modified neoclassical model).[5] To better understand CBO's model, this section compares it with those well-known models.

2.1. Comparison With Jorgensonian Neoclassical Models. CBO's model of investment differs from neoclassical models based on Jorgenson (1963) primarily in that the capital stock adjusts more rapidly to changes in output than to changes in the cost of capital. In contrast, a neoclassical model constrains the speeds of adjustment of the capital stock to changes in output and changes in the cost of capital to be the same.

Another way to express that difference is that in the short run a Jorgensonian model

[5]Both Jorgensonian models and Tobin's q models are based on neoclassical theory. In fact, neoclassical models of the economy often contain elements of both. As a practical matter, however, past empirical work on investment tended to use one or the other. Consequently, this section treats them separately.

targets a desired capital stock, whereas the modified neoclassical model targets a desired level of capacity utilization. In both models, firms move swiftly in response to a change in demand—Jorgensonian firms to boost the capital stock to its newly higher desired level and modified neoclassical firms to reduce utilization back to its desired level. Investment in both models is boosted by a higher percentage amount than demand in the short run—the accelerator. For example, in 2016 private nonresidential investment was roughly 10 percent as large as the stock of private nonresidential capital. A 1 percent increase in the desired capital stock or in desired capacity due to a 1 percent higher demand would require a 10 percent rise in investment—from 10 percent of the capital stock to 11 percent of the stock—to achieve the adjustment in a year. (In the real world, the one-year response is much smaller because of time lags and uncertainty about the persistence of demand.)

However, the models respond differently to changes in the cost of capital or productivity. In Jorgenson's model, a rise in the desired stock of capital resulting from a fall in the cost of capital or an increase in productivity calls forth an accelerator-type response of investment. Each 1 percent fall in the cost of capital would require a 10 percent rise in investment to achieve the adjustment of the capital stock in a year. In the modified neoclassical model, however, neither shock changes capacity utilization, so no accelerator effect occurs—only an increase in the quality of new units of capital. A 1 percent rise in productivity or a 1 percent fall in the cost of capital boosts investment in the short run by 1 percent. Because that response is smaller than the short-run response of investment to changes in demand (changes in Y/y) discussed above, a 1 percent increase in y combined with unchanged Y (implying a 1 percent lower Y/y) would cause investment to fall initially, consistent with the finding of Basu, Fernald, and Kimball (2006) that improved technology causes nonresidential investment to fall in the short run.

The procedure for estimating the after-tax rate of return in CBO's model also differs from that in empirical studies using the Jorgenson model. CBO uses market values of existing capital to determine that rate of return. Other empirical studies typically use either the dividend-discount model or a nominal interest rate combined with an assumption about inflation expectations.

2.2. Comparison With Tobin's *q* Models. Another well-known version of the neoclassical model, the q model, relates investment directly to asset prices. In the textbook q model of investment, the ratio of investment to the capital stock depends solely on tax-adjusted Tobin's q, the ratio of the market value of capital to the replacement value of capital adjusted for business taxes. Although a pure q model is theoretically appealing, it fits the data poorly, according to most previous empirical studies. As a result, most empirical work—for example, Fazzari, Hubbard, and Petersen (1988) and Kaplan and Zingales (1997)—adds cash flow (after-tax profits plus depreciation), on the logic that firms are constrained in some way from borrowing in order to invest as much as they would like. Given that nonfinancial corporate businesses have had enough cash flow to be net purchasers of their own shares in most years since the mid-1980s, buying back more than $340 billion in every year from 2011 to 2017, the importance of liquidity constraints in determining total business investment seems limited.

In equation (14), which expresses CBO's model as a function of Tobin's q, investment depends not only on Tobin's q but also on the growth of demand. In light of that model, the empirical importance of cash flow in Tobin's q models reflects an omitted variable—demand—rather than liquidity constraints.

Although investment can be expressed as a function of Tobin's q in both CBO's model and Tobin's q model, the interpretation of q in those models is different. In most q models, deviations of Tobin's q from its equilibrium value arise because the capital stock is out of line with its desired level. Those deviations are eliminated as quickly as firms can adjust the stock to its desired level. By contrast, in a model with ex post fixed proportions, Tobin's q differs from 1 in part because businesses cannot adjust the capital/labor ratio in existing capital. Deviations of Tobin's q resulting from a change in desired $\bar{k}$ are eliminated only as the existing capital stock is replaced and thus can be long lasting.

2.3. Comparison With Putty–Clay Models. Empirically, CBO's model of investment is similar in many ways to standard putty–clay models, such as Bischoff (1971) and Macroeconomic Advisers (2008). Both putty–clay models and the CBO model incorporate the assumption of ex post fixed proportions. The primary difference is that CBO's model allows estimation of the coefficient of capital in production, whereas others do not. That difference arises from CBO's assumptions that new capital is complementary with existing capital and that the productivity of existing capital increases with economywide TFP. Those assumptions allow the productivity of new capital to be observed from the productivity of existing capital, in turn allowing the combined estimation of capital's coefficient in production and firms' elasticity of demand.

3. Modifications of the Basic Model

This section augments the basic model to include many types of capital, gradual adjustment, and modifications to the quality of new capital. Incorporating many types of capital permits special treatment for capital specific to mining and farming, inventories, and land. Assuming that investment reacts gradually to the factors determining it and modifying the quality of new capital greatly improve the fit of the investment equations.

3.1. Treatment of Farming and Mining. Investment specific to farming and mining does not fit the above-developed model well because in that model, investment depends primarily on growth of demand, but investment in those sectors depends primarily on the expected price of their output. (In this paper, "mining" includes extracting oil and natural gas as well as minerals.) For farming, farm output is excluded from Y, and farm-specific investment is discussed in section 6.3.

The production of oil, natural gas, and minerals is far more capital intensive than other production. In fact, if capital in that industry is split into mining-specific capital (such as mines, wells, and drilling rigs) and all other capital, the ratio of all other capital to output in that industry is roughly the same as the capital-output ratio for all other industries using private nonresidential capital. Consequently, mining output is included in Y, non-mining-specific capital used in mining is treated like non-mining capital, and mining-specific capital is treated as increasing the capital-labor ratio of the overall economy.

Mining-specific capital is modeled as an increase in the equilibrium ratio of units of measured capital (M) to workers (N) above 1. Define S as

$$S \equiv \frac{\text{units of nonfarm capital}}{(\text{units of nonfarm capital}) \;-\; (\text{units of mining–specific capital})}.$$

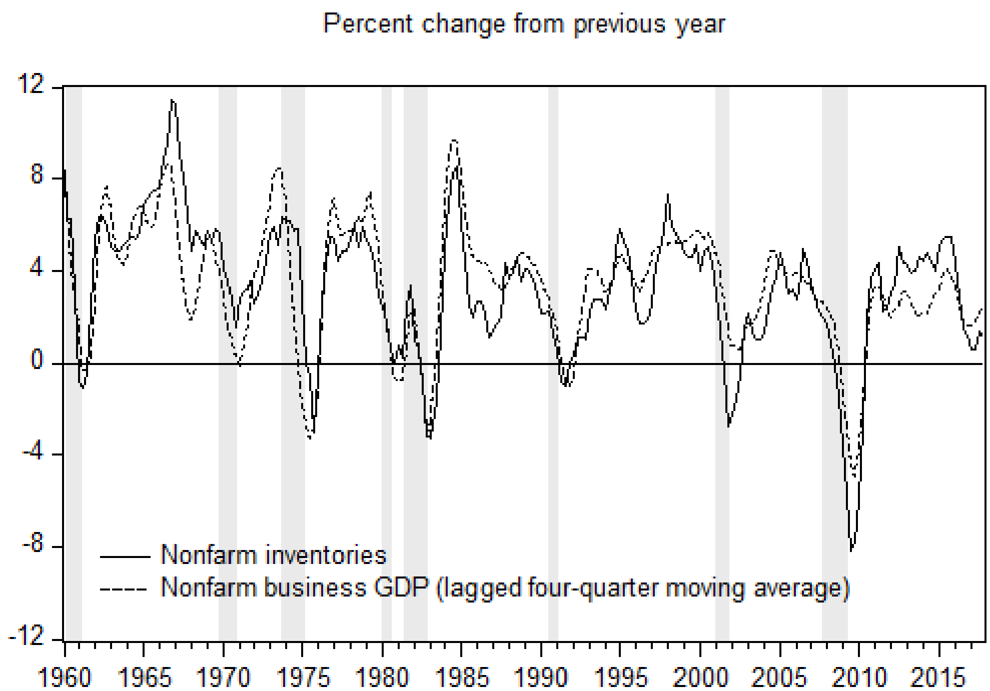
SN replaces N in equations (1) through (4) and α is augmented by S in equation (5). Then α_t/S_t, denoted α_t^*, is the coefficient of non-mining-specific capital in production and M_t/S_t, denoted M_t^*, is the number of non-mining-specific units of capital, which the firm targets to equal N.

3.2. Treatment of Inventories. Inventories are unusual in that they share the same modified neoclassical behavior as fixed capital but without the long service lives that motivate the modified neoclassical model. Like investment in fixed capital, inventory investment is dominated by growth of output over the previous year. Year-over-year growth of a lagged four-quarter moving average of real nonfarm business output explains 68 percent of the variance of year-over-year growth of real nonfarm inventories from 1960 to 2017 (see Figure 1).[6] Also like fixed capital, inventories respond to long-run movements in the interest rate but not to short-run movements, according to Maccini, Moore, and Schaller (2004). However, inventories themselves are not held long enough for ex post fixed proportions during their service life to imply gradual adjustment in response to a change in the cost of capital, the typical holding period being 20 weeks (CBO 2017).

To explain why inventories display the behavior of assets with ex post fixed proportions, it is assumed that the way businesses use inventories—"inventory technologies"—are constant over lengthy periods. For example, the amount of inventory held depends on the type of store a retailer has, a wholesaler's distribution system, or a manufacturer's use of work-in-process inventories. In each case, a business will tend to hold more inventories the higher its sales but will not be willing to incur the cost of changing its way of using inventories in response to short-term movements in real rates of return. For example, a manufacturer would not abandon a just-in-time inventory system just because its rate of return fell.

[6]Because output includes inventory investment, some of that correlation may be spurious. However, even after inventory investment is subtracted from output, the correlation is 60 percent. That subtraction is probably too large a correction because greater production of inventories by one industry will cause industries that supply it to increase their own inventories.

Figure 1. Growth of Real Nonfarm Inventories and Output



Note: Shaded vertical bars indicate recessions.
Source: Bureau of Economic Analysis.

The inventory analogue to a unit of capital is thus a unit of inventory technology. For inventories, k is the amount of real inventory in a new unit of inventory technology, F is new units of inventory technology, and δ is the rate at which units of inventory technology are retired. The upward trend in Y/N that causes k for fixed capital to rise over time also causes k for inventory technologies to rise over time. The downward trend in inventories per dollar of GDP results from a decrease in inventories' share of total units of capital. Because inventories and land do not depreciate, their costs of capital depend partly on their resale value, as discussed in the appendix (section A2).

3.3. Gradual Adjustment and the Number of New Units of Capital. Increased demand for a firm's products is not matched instantly with enough new capital to meet that demand. Firms take time to respond to increased demand with greater orders and to receive delivery of those orders. For structures and the development of intellectual property, a long time can elapse between the beginning of a project and its completion.

Because individual firms' investments are "lumpy"—that is, clustered together rather than spread smoothly over time—assume that firms order capital once every T periods.[7] (Then, the representative firm is representative only of firms in a particular investment cycle.) Orders placed at time t are delivered uniformly over τ periods from $t+1$ to $t+\tau$. As an approximation, assume that firms target the number of units of nonmining capital to equal the expected ratio of output to productivity midway through the order cycle, at $t+T/2$. Denoting time t expectations of variable X at time $t+T/2$ as ${}_tX_{t+T/2}$ yields

$$M^*_{t+T/2} = {}_tY_{t+T/2}/{}_ty_{t+T/2}.$$

Because orders must cover replacement demand plus desired growth of M^* since orders were last placed, new orders placed by the fraction $1/T$ of firms ordering at time t equal

$$\frac{1}{T}\left(\frac{{}_tY_{t+T/2}}{{}_ty_{t+T/2}} - \frac{{}_{t-T}Y_{t-T/2}}{{}_{t-T}y_{t-T/2}}\right) + replacements. \tag{19}$$

Define output at full employment, $\bar{Y}$, as the product of employment at full employment, $\bar{N}$, and cyclically adjusted productivity per worker, $\bar{y}$. (Estimation of $\bar{N}$ and $\bar{y}$ is discussed

[7] For evidence that investment is lumpy, see Doms and Dunne (1998) and Cooper, Haltiwanger, and Power (1999).

in the appendix, section A3.) Firms expect that y will equal $\bar{y}$ by the time that orders are delivered, so ${}_ty_{t+T/2}$ equals ${}_t\bar{y}_{t+T/2}$. Firms also expect total output Y to gradually converge toward potential $\bar{Y}$:

$$\frac{{}_tY_{t+T/2}}{{}_t\bar{Y}_{t+T/2}} = \psi_T \frac{Y_t}{\bar{Y}_t} + (1 - \psi_T)\,,$$

with a higher ψ reflecting a slower rate of expected convergence. Firms are assumed to expect $\bar{N}$ to grow at rate $\bar{n}$, so that ${}_t\bar{N}_{t+T/2} = \bar{N}_t\,(1 + \bar{n}_t)^{T/2}$. After those substitutions are made in expression (19), new orders at time t equal

$$\frac{1}{T}\;\psi_T\;\Delta_T\!\left[\frac{Y_t}{\bar{y}_t}\,(1+\bar{n}_t)^{T/2}\right] + \frac{1}{T}\,(1-\psi_T)\,\Delta_T\!\left[\bar{N}_t\,(1+\bar{n}_t)^{T/2}\right] + replacements,$$

where, to simplify notation, $\Delta_T[X_t]$ denotes $X_t - X_{t-T}$.

To account for many types of businesses and capital, assume that fraction ϕ_{mj} of firms have an order cycle for type-m capital T_{mj} periods long with a corresponding ψ_{mj} but that all orders for type-m capital are delivered over τ_m periods. Define σ_{mt} as type-m capital's share of total units of capital delivered at time t adjusted for differences in cyclical effects on orders across m. Then the number of new units of type-m capital delivered in period t is

$$\begin{aligned}
(20) \qquad F_{mt} \;=\;& \sigma_{mt}\sum_{i=1}^{\tau_m}\sum_j \beta_{mj}\frac{\Delta_{T_{mj}}\!\left[(Y_{t-i}/\bar{y}_{t-i})\,(1+\bar{n}_{t-i})^{T_{mj}/2}\right]}{\tau_m \times T_{mj}} \\
&+\sigma_{mt}\sum_{i=1}^{\tau_m}\sum_j \gamma_{mj}\frac{\Delta_{T_{mj}}\!\left[\bar{N}_{t-i}\,(1+\bar{n}_{t-i})^{T_{mj}/2}\right]}{\tau_m \times T_{mj}} + \sigma_{mt}\sum \delta_m M_{m,t-1},
\end{aligned}$$

where $\beta_{mj} = \phi_{mj}\psi_{mj}$, $\gamma_{mj} = \phi_{mj}\left(1-\psi_{mj}\right)$, and $\sum_j \phi_{mj} = 1$. Note that investment depends on changes in both demand and labor supply.

3.4. Modifications to the Quality of New Units of Capital. To reconcile the theory to the real world, several modifications must be made to the quality of new units of capital k. Those adjustments include modifying the depreciation rate when estimating the market value of existing capital, modifying the responses of quality to taxes and the market value of capital, and adjusting for different effects of higher oil prices in the 1970s and 1980s on the market values of new and existing capital.

With many types of capital and $y_t = \bar{y}_t$ as a result of delivery lags, (7) becomes

$$k_{mt} = \frac{\alpha_t^* \left(1 - 1/\eta_t\right) \left(1 - u_t\right) p_t \bar{y}_t}{v_{mt} \left(\delta_m + r_t^*\right)}. \tag{21}$$

With geometric depreciation, the depreciation rate used to determine the market value of existing capital is the same as that for new capital, δ. No matter how old it is, every unit of capital is expected to last another $1/\delta$ years, on average. However, if units of capital have a fixed service life when new, then on average the expected remaining service life of existing capital is just $1/\left(2\delta\right)$ years, meaning the depreciation rate needed to determine market value is approximately 2δ. Under the assumption that the truth lies midway between those alternatives, a depreciation rate of 1.5δ is used to estimate the market value of existing capital. Markets valuing capital by using higher depreciation rates than those commonly used to calculate book value and replacement cost could explain why Rognlie (2015) finds that Tobin's q averages less than 1 over history.

Tax variables are constructed using the contemporaneous corporate tax code (see the appendix, section A4, for details). However, firms may not expect tax rates to remain at current levels.[8] In addition, tax provisions for noncorporate businesses can have important effects on investment. For example, partnerships' share of investment in structures increased significantly in the 1980s, at least in part because of more favorable treatment of passive losses for individuals.

To capture those effects, contemporaneous tax provisions receive a weight of $c1$ and their sample averages a weight of $1 - c1$.[9] The coefficient $c2$ measures the impact of favorable tax treatment for structures in the 1980s not captured in the corporate tax code. With those assumptions, v_{mt} in (21) becomes

$$v_{mt} = \frac{p_{mt}}{1 + c2\ D_{80s}} \left(1 - itc_{mt} - u_t z_{mt}\right)^{c1} \left(1 - \widehat{itc}_m - \hat{u}\hat{z}_m\right)^{1-c1}, \tag{22}$$

where p_m is the price index of type-m capital; $\widehat{itc}_m$, $\hat{u}$, and $\hat{z}_m$ denote sample averages; and D_{80s} is a dummy variable for 1981–1986 with $c2$ nonzero for structures only. In (21) $1 - u_t$ becomes $(1 - u_t)^{c1} (1 - \hat{u})^{1-c1}$.

The response of the quality of new units of capital k_{mt} to the market value of existing type-m capital V_{mt} might be less than theory predicts for several reasons, including mismeasurement of market value, incorrect assumptions about the factors determining market value, differences between average q and marginal q, and different expectations for r^* between firms and investors. To account for a response less than theory predicts, the market-value term of (10) is multiplied by the parameter $c3$ when estimating k_{mt}. After that change and the

[8] Firms may expect tax rates to change either because rates are scheduled to change under current law (for example, the scheduled increase in tax rates for pass-through businesses at the end of 2025 under the Tax Cuts and Jobs Act of 2017) or because firms anticipate changes to current law.

[9] Sample averages are used so that the estimate of $c1$ does not affect the average level of the v_{mt} and thus the average level of α.

modification for partial response to current taxes, equation (21) becomes

$$k_{mt} = \frac{\alpha_t^* \left(1 - 1/\eta_t\right) \left(1 - u_t\right)^{c1} \left(1 - \hat{u}\right)^{1-c1} p_t \bar{y}_t}{v_{mt} \left(\delta_m + \hat{r}^*\right)} \left[1 - c3 \frac{V_t - \hat{V}_t}{d\hat{V}_t / d\hat{r}^*} \frac{1}{\delta_m + \hat{r}^*}\right], \tag{23}$$

with v_{mt} given by (22). To account for nongeometric depreciation of existing capital, the components of $\hat{V}_t$ and $d\hat{V}_t/d\hat{r}^*$ are calculated using $1.5\delta_m$ instead of δ_m.

To account for lags between order and delivery, a moving average of length τ_m is applied to $V_t - \hat{V}_t$ in (23) and to tax terms in v_{mt}, and $\bar{y}_t$ is replaced by $\bar{y}_{t-1}$ times its average quarterly growth.[10] However, firms are assumed to know p_t and p_{mt}. In the full model, V_t includes V_t^M, V_t^U, V_t^W, and the resale values of inventories and land, as well as the PDV of the tax value of unused depreciation allowances from past investment.

One factor that could have caused a wedge between average q and marginal q in the past was the elevated level of crude oil prices in the 1970s and 1980s. As those prices rose, existing energy-intensive capital became more expensive to operate than new energy-efficient capital. That reduced the market value of existing capital in comparison with that of new capital, opening a wedge between V and a hypothetical V relevant for new capital.

To adjust for that, the ratio of the price deflator of imports of petroleum and products to p is detrended, creating po. The term $0.4\left(po_t - \overline{po}\right)\bar{Y}$, where $\overline{po}$ is po in the third quarter of 1973, is then added to V from the fourth quarter of 1973 through the second quarter of 1986, after which $po_t - \overline{po}$ is negative. The coefficient 0.4 optimizes the fit of the equations. In practical terms, that modification boosts estimated $c3$ by helping explain why investment remained strong in the late 1970s and early 1980s despite low asset prices.

[10]No moving averages are applied to the v_{mt} for equipment. Time to build is short enough (two quarters) that firms generally know what tax rates will be when orders are delivered.

4. Background of the Estimation

4.1. Data. Output is the gross value added by sectors using private nonfarm nonresidential capital: nonfarm business less tenant-occupied housing plus nonprofit institutions. (Tenant-occupied housing is part of nonfarm business but uses residential capital.) Taxes on production and imports, which are not part of the net revenue of the firm, are subtracted from value added as a reduction in p. Employment is the sum of employment in the nonfarm business and nonprofit institution sectors.

The market value of the nonfarm nonfinancial corporate sector is calculated using data from the Federal Reserve Board's *Financial Accounts of the United States*. According to Hall (2001), the value of capital equals the value of debt and equity less the value of financial assets. Debt and assets are valued at par in the *Financial Accounts*, so they are converted to market values as shown in the appendix (section A5). Because the data for market value are only for nonfinancial corporations, the $\hat{V}_{mt}$ used to calculate $\hat{V}_t$ and $d\hat{V}_t/d\hat{r}^*$ in (23) are multiplied by the nonfinancial share of the stock of type-m capital. In (16) for $\hat{V}_t^U$, $\bar{Y}_t$ times a Hodrick–Prescott (H-P) filter of the ratio of Y_t for nonfinancial corporations to Y_t is substituted for Y_t.

Service lives ($1/\delta$) for most types of capital are from the Bureau of Economic Analysis (2003) and Li (2012). Determined partly from goodness of fit, the service life for inventory technology is assumed to be 12 years and the service life of sources of unmeasured capital income ($1/\delta_U$) is assumed to be 10 years. That service life is longer than lives for most types of research and development but is probably shorter than the duration of firm-specific TFP, reputation, or barriers to entry. A long service life is supported by Furman and Orszag's

(2015) finding that 85 percent of firms with a return on invested capital above 25 percent in 2003 still had a return above 25 percent in 2013. As discussed below, however, the choice of δ_U has little effect on the results.

4.2. Estimation Procedure. In the first step of the iterative estimation procedure, equations are estimated for four broad categories of investment in private nonfarm nonmining nonresidential capital: equipment, structures, IPPs, and inventories. The left-hand side of each of the first three equations is total nominal investment (IN) in the broad category, whereas the right-hand side is the sum of the products of p_{mt}, k_{mt}, and F_{mt} for each type of capital within the broad category.[11] For inventories, IN is inventory investment plus inventories in depreciating units of inventory technology minus the increase in inventories in existing units of inventory technology due to increased y over the previous five quarters.[12] To correct for heteroskedasticity, both sides are divided by the CBO (2018a) estimate of nominal potential GDP ($\overline{gdp_t}$).[13]

Consequently, the equation for investment in broad investment category E is

$$\text{(24)} \qquad \frac{IN_{Et}}{\overline{gdp_t}} = \frac{\sum_{m \in E} p_{mt} k_{mt} F_{mt}}{\overline{gdp_t}} + \epsilon_{Et},$$

[11] Types of equipment include computers, communications equipment, and other nonfarm nonmining equipment. Types of IPPs include software, research and development, and entertainment, literary, and artistic originals. Nonfarm nonmining structures and inventories are simple aggregates. Depreciation rates and tax terms are weighted averages using H-P–filtered shares of investment as weights. Investment in new units of land is assumed to be a simple function of investment in new units of structures, as discussed in the appendix (section A2).

[12] The assumption that the amount of inventory associated with a unit of inventory technology grows with y is necessary so that, as for fixed capital, an increase in Y and y that leaves Y/y unchanged does not change the number of existing units of capital.

[13] That estimate of potential GDP was based on the data available before revisions released in late July 2018, whereas the investment equations use revised data. To reconcile the two, the estimate of potential GDP is multiplied by an H-P filter of the ratio of post-revision GDP to pre-revision GDP.

where k_{mt} is expressed using (23) modified for time to build, or

$$(25) \qquad k_{mt} = \frac{\alpha_t^* \left(1 - 1/\eta_t\right) \left(1 - \frac{1}{\tau_m} \sum_{i=0}^{\tau_m - 1} u_{t-i}\right)^{c1} \left(1 - \hat{u}\right)^{1-c1} p_t \bar{y}_t}{\left[1 - \frac{1}{\tau_m} \sum_{i=0}^{\tau_m - 1} \left(itc_{m,t-i} + u_{t-i} z_{m,t-i}\right)\right]^{c1} \left(1 - \widehat{itc}_m - \hat{u}\hat{z}_m\right)^{1-c1}} \times \frac{1}{\delta_m + \hat{r}^*} \frac{1 + c2\ D_{80s}}{p_{mt}} \left[1 - c3 \sum_{i=0}^{\tau_m - 1} \frac{V_{t-i} - \hat{V}_{t-i}}{d\hat{V}_{t-i}/d\hat{r}^*} \frac{1}{\delta_m + \hat{r}^*}\right];$$

F_{mt} is expressed using (20), or

$$F_{mt} = \sigma_{mt} \sum_{i=1}^{\tau_m} \sum_j \beta_{mj} \frac{\Delta_{T_{mj}} \left[\left(Y_{t-i}/\bar{y}_{t-i}\right) \left(1 + \bar{n}_{t-i}\right)^{T_{mj}/2}\right]}{\tau_m \times T_{mj}} + \sigma_{mt} \sum_{i=1}^{\tau_m} \sum_j \gamma_{mj} \frac{\Delta_{T_{mj}} \left[\bar{N}_{t-i} \left(1 + \bar{n}_{t-i}\right)^{T_{mj}/2}\right]}{\tau_m \times T_{mj}} + \sigma_{mt} \sum \delta_m M_{m,t-1};$$

and ϵ_{Et} is an error term. Note that the p_{mt} in (24) and in the expression for k_{mt} cancels out. The investment equations are estimated as a system using maximum likelihood, with $c1$, $c2$, $c3$, the β_{mj}, and the γ_{mj} being estimated parameters and $\alpha_t^* \left(1 - 1/\eta_t\right)$ estimated using a Kalman filter. Data are quarterly, and the equations are estimated from the first quarter of 1960 through the fourth quarter of 2017.

Because firms jointly determine their investment and their output, the equation for F_{mt} could suffer from simultaneity. That potential problem is mitigated by two factors: only lagged values of output enter the equation for investment; and the level of investment depends on the growth, rather than the level, of output.

The second step in the iterative estimation procedure is to recalculate α_t^*, $1/\eta_t$, the components of $\hat{V}_{t-i}$, and the σ_{mt} by using information from the investment equations. Through the use of an H-P filter for labor's share of income, α_t^* is calculated using (18) and $1/\eta_t$

is calculated using (15).[14] Labor income is labor compensation plus a share of proprietors' income for sectors using private nonfarm nonresidential capital equal to labor's share of the gross value added of nonfinancial corporate business excluding taxes on production and imports less subsidies. The variable x_t is calculated using sample averages for all variables except M_{mt}/M_t. Raw values for σ_{mt} are calculated by setting $F_{mt} = I_{mt}/k_{mt}$ and solving (20) for σ_{mt}. Final σ_{mt} are H-P–filtered values of $\bar{F}_{mt}/\sum_m \bar{F}_{mt}$, where the $\bar{F}_{mt}$ (cyclically adjusted F_{mt}) are calculated using $\bar{N}_{t-i}$ instead of $Y_{t-i}/\bar{y}_{t-i}$ in (20). The estimates of M_{mt} are calculated using a perpetual inventory method, and S_t is M_t/M_t^*.

4.3. Parameters. The standard error for the quarterly innovation of α_t^* is set to 0.0005, or about 0.05 percent of output. The estimate of $\hat{r}^*$ is 4.2 percent, the average after-tax return over the sample period less average $\dot{y}$. So that V_t averages roughly to $\hat{V}_t$, $\frac{c_V p_t \bar{Y}_t}{1.5/\delta_U + \hat{r}^*}$ is added to $\hat{V}_t$, where c_V is set at 0.001 so that $\frac{V_t - \hat{V}_t}{p_t \bar{Y}_t}$ averages to zero. That expression can be interpreted as the market value of the income from unmeasured capital that is not part of GDP and thus not included in V^U.

The lengths of order cycles are chosen starting from 4, 8, 16, and 24 quarters for T_{mj} in the output portion of (20) and 8 and 24 quarters for T_{mj} in the labor supply portion. Order cycles with negative coefficients are removed. The variables β_{mj} are positive for order cycles of 4, 8, and 16 quarters for equipment and IPPs and for order cycles of 8, 16, and 24 quarters for structures. The variables γ_{mj} are positive for order cycles of 8 and 24 quarters for equipment, for an order cycle of 24 quarters for IPPs, and for no order cycles for structures. The sum of the β_{mj} and γ_{mj} in each equation is constrained to 1. The time to delivery, τ_m, that

[14]Because capital's share of income differs between nonfinancial corporations and all businesses using nonresidential fixed capital, at least one of α^* or η also must differ. Very little correlation exists between the ratios of K/Y and capital's share of income between the two sectors, so it is η that is assumed to differ.

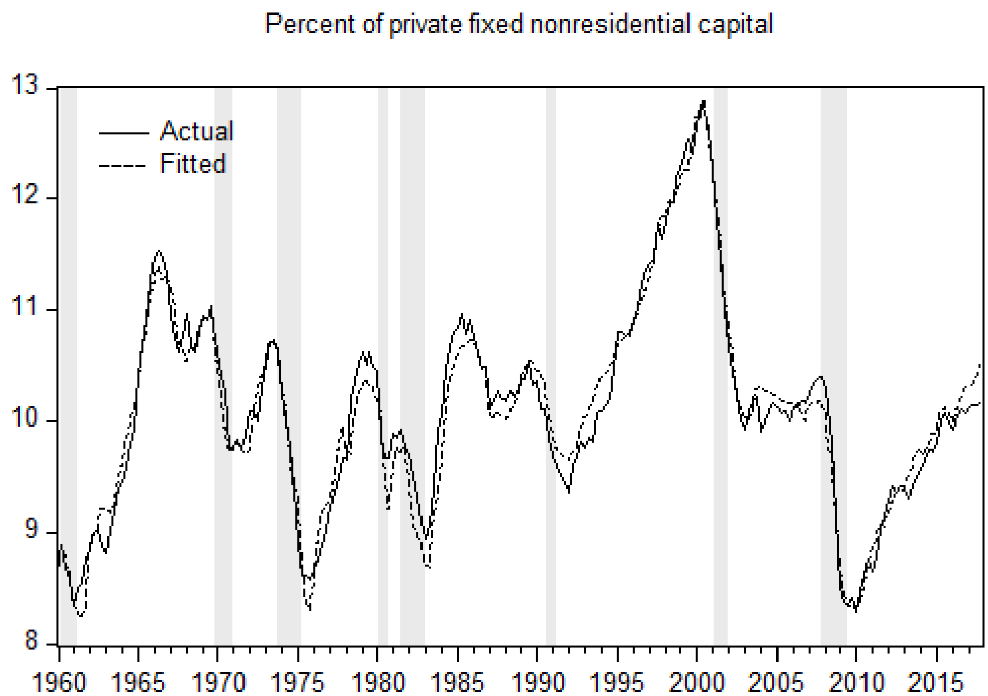
maximizes the log likelihood of the system is 2 quarters for equipment and 5 quarters for structures and IPPs. Consistent with Figure 1, β_{m4} for inventory technology is set to 1 and the other β_{mj} and γ_{mj} variables are zero.

5. Empirical Results

5.1. Estimated Coefficients. The model fits the investment data well, which is not surprising because $\alpha^* (1 - 1/\eta_t)$ is estimated using a Kalman filter (see Figure 2). The estimated coefficient on contemporaneous tax law ($c1$), 0.60, is significantly positive but also significantly less than 1 (see Table 1). That coefficient is below the midpoint of the range of 0.5 to 1.0 found in the survey by Hassett and Hubbard (2002), possibly because measurement error from using only corporate tax law biases down the estimate. The estimated coefficient on the market value of capital, $c3$, is much smaller than that on tax law despite having a much larger z-statistic. Optimal lags on the factors determining the number of new units of capital are much longer than those on the factors determining the quality of new units of capital.

The model fits the data less well when $\alpha^* (1 - 1/\eta)$ is constrained to be constant over the sample (see Figure 3). Even so, the model fits well for most of the period after 1990, despite the absence of a lagged dependent variable, and does a good job identifying turning points. Estimates of the β and γ, shown in Table 1, differ little from those obtained when $\alpha^* (1 - 1/\eta)$ is estimated using a Kalman filter. However, the estimate of $c3$ is smaller than when estimated using a Kalman filter, probably because less accurate estimates of α^* and η lead to less accurate estimates of the $\hat{V}$ used to estimate $c3$. The coefficient $c2$ is constrained to its value in the baseline case because it is otherwise unrealistically large with respect to

Figure 2. actual and fitted investment



Note: Investment is private, nonresidential fixed investment excluding agricultural machinery; mining and oilfield machinery; mining exploration, shafts, and wells; and farm structures. Fitted values are from the system defined by (24). Shaded vertical bars indicate recessions.
Source: Data for actual investment and the capital stock are from the Bureau of Economic Analysis.

the rise in the noncorporate share of investment in structures in the 1980s.

5.2. Capital's Coefficient in Production. The filtered estimate of $\alpha^* (1 - 1/\eta)$ is the dashed line (labeled "Input to investment equations") in Figure 4. The estimate of α^* derived from that estimate (labeled "Nonmining capital") rose in the first half of the 1980s, declined in the first half of the 1990s, and rose temporarily between around 2006 and the mid-2000s. Without the adjustment to V for high oil prices in the 1970s and 1980s, the estimated coefficient on asset prices, $c3$, is 24 percent smaller, so that a portion of the boom in investment in the late 1990s actually due to high asset prices is instead attributed to a sizeable temporary increase in α^*. (A small temporary increase in α^* is still evident during

Table 1—Estimated Coefficients from a System of Investment Equations

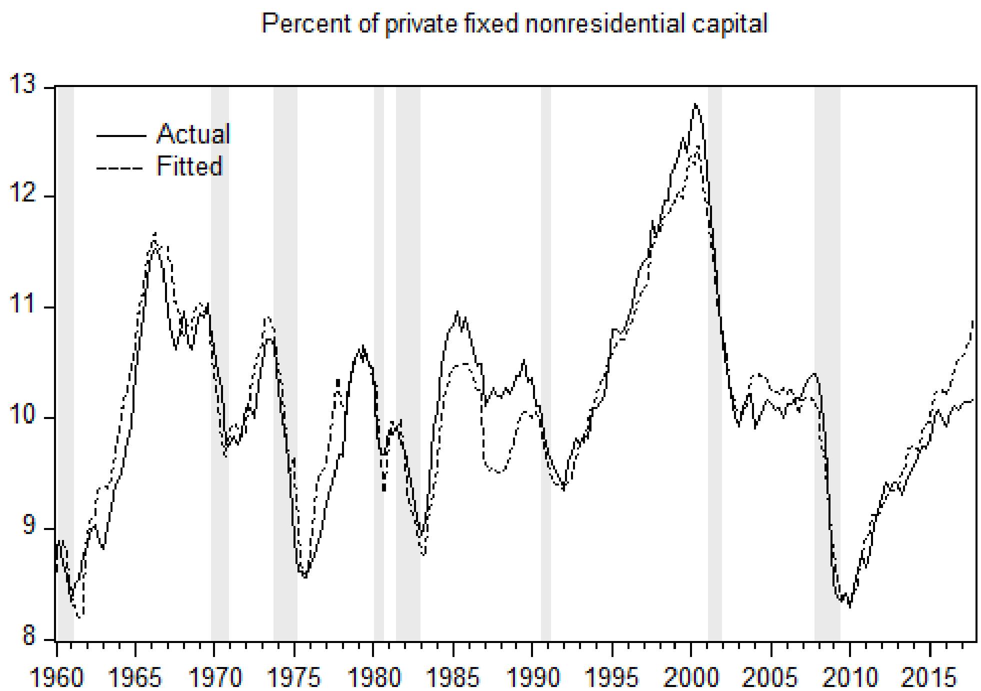
		Using Kalman filter		Constant $\alpha^*(1-1/\eta)$	
		Estimate	z-statistic	Estimate	z-statistic
	c1	0.60*	14.8	0.92*	20.1
	c2	0.18*	19.7	0.18	
	c3	0.18*	23.1	0.16*	18.3
Equipment	β_{m4}	0.10*	8.9	0.08*	4.3
	β_{m8}	0.19*	8.3	0.21*	6.5
	β_{m16}	0.08*	2.7	0.12*	3.2
	γ_{m8}	0.21	2.2	0.14	1.2
	γ_{m24}	0.43		0.45	
	R^2	0.95		0.90	
Structures	β_{m8}	0.18*	7.5	0.19*	4.1
	β_{m16}	0.42*	9.5	0.44*	4.9
	β_{m24}	0.39		0.37	
	R^2	0.95		0.94	
IPP	β_{m4}	0.05*	3.8	0.04	0.7
	β_{m8}	0.03	1.3	0.03	0.3
	β_{m16}	0.06*	3.0	0.05	0.6
	γ_{m24}	0.86		0.88	
	R^2	0.997		0.993	
Inventories	β_{m4}	1.00		1.00	
	R^2	0.56		0.55	

Note: The table shows the results from estimating the system of equations defined by (24) from the first quarter of 1960 through the fourth quarter of 2017. The dependent variables exclude investment in agricultural machinery; mining and oilfield machinery; mining exploration, shafts, and wells; and farm structures. IPP is intellectual property products. * denotes significance at the 99 percent level.

that period, suggesting that true $c3$ may be larger than estimated.) Strong growth of output in the oil and gas extraction sector boosted α (labeled "All measured capital") in relation to α^* after the early years of the 2000s.

The variable $\alpha^*(1-1/\eta)$ is a key driver of investment in nonfarm nonmining capital. In contrast to α^*, it has declined since 2000 as a result of falling η, restraining investment despite little net change in capital's coefficient in production during that period. The estimated fall in the elasticity of demand η echoes the finding of De Loecker and Eeckhout (2017) that capital's share of income has declined as a result of greater market power.

FIGURE 3. ACTUAL AND FITTED INVESTMENT ASSUMING CONSTANT SHARES OF INCOME



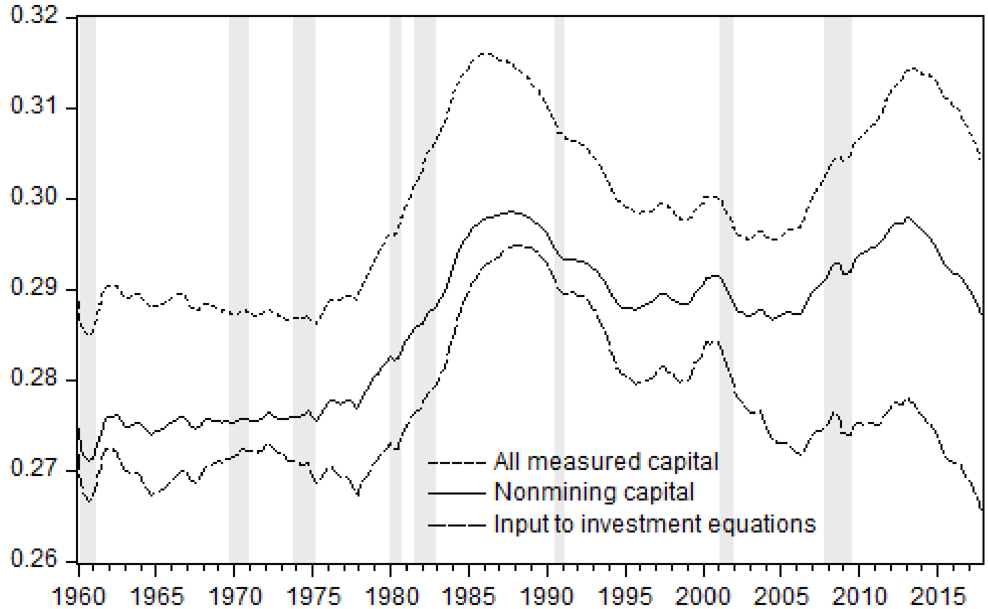
Note: Same as for Figure 2.

5.3. Sources of the Increase in Capital's Share of Income. Figure 5 shows the estimated composition of capital income. An increase in capital's share of income in the 1980s stemmed from a rise in measured capital's share of income, corresponding to a higher value of α. However, the increase in capital's share of income since about 2000 is due entirely to a sharp increase in the income from unmeasured sources, reflecting smaller values of η.

5.4. Sensitivity Analysis. When the assumed service life for USCIs $(1/\delta_U)$ is increased from 10 to 15 years, the estimated coefficients are little changed. Estimated income of USCIs is also little changed, but the longer service life for USCIs boosts their market value. The main effect is a fall in c_V from 0.001 to –0.006.

Increasing the assumed standard error of $\alpha^* (1 - 1/\eta)$ in the Kalman filter from 0.0005

Figure 4. estimates of the coefficient of measured capital in production

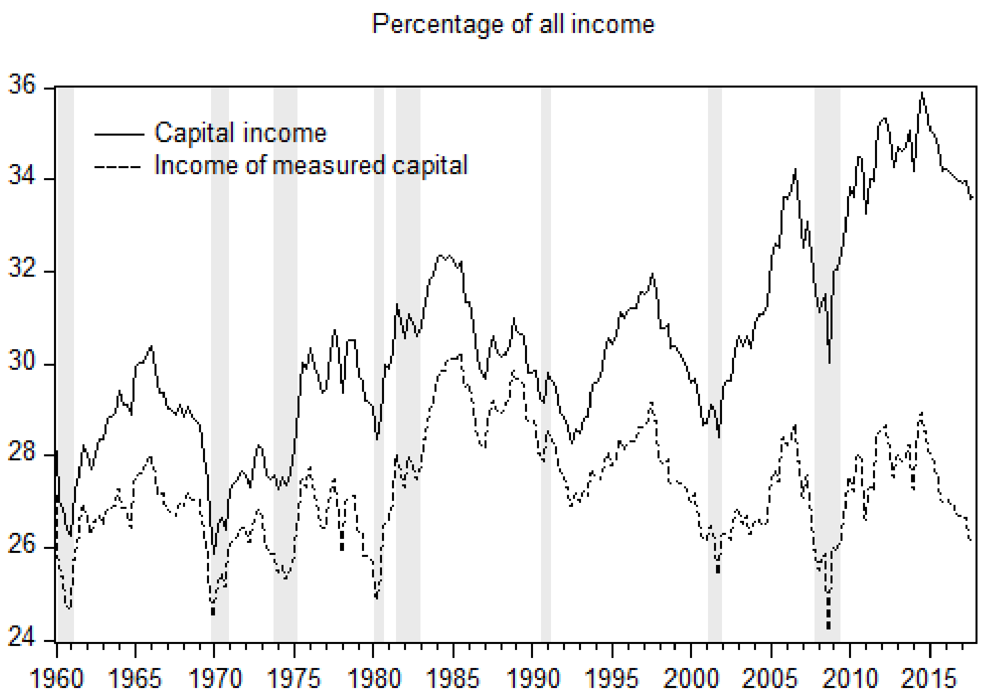


Note: The line labeled "All measured capital" is the estimate of α. The line labeled "Nonmining capital" is the estimate of α^*. The line labeled "Input to investment equations" is $\alpha^*(1 - 1/\eta)$. Shaded vertical bars indicate recessions.

to 0.001 improves the overall fit at the expense of implausible movements in the estimated income of USCIs. The increase in the standard error allows α^* to capture more of the movements in investment, including the rise in the mid-1980s. However, the share of income attributable to USCIs becomes volatile, plunging by 65 percent between mid-1983 and mid-1985.

Reducing the value of $\hat{r}^*$ from 4.2 percent to 3.2 percent also has little impact on the estimated coefficients or on the estimated patterns of α^* and η. However, the estimated levels of α^* and η are different. The reduction in $\hat{r}^*$ reduces α^* by about 0.025. The consequent reduction in income from measured capital boosts $1/\eta$, more than doubling estimated income of unmeasured capital during much of the sample period. The main takeaways from the

Figure 5. Capital's share of income

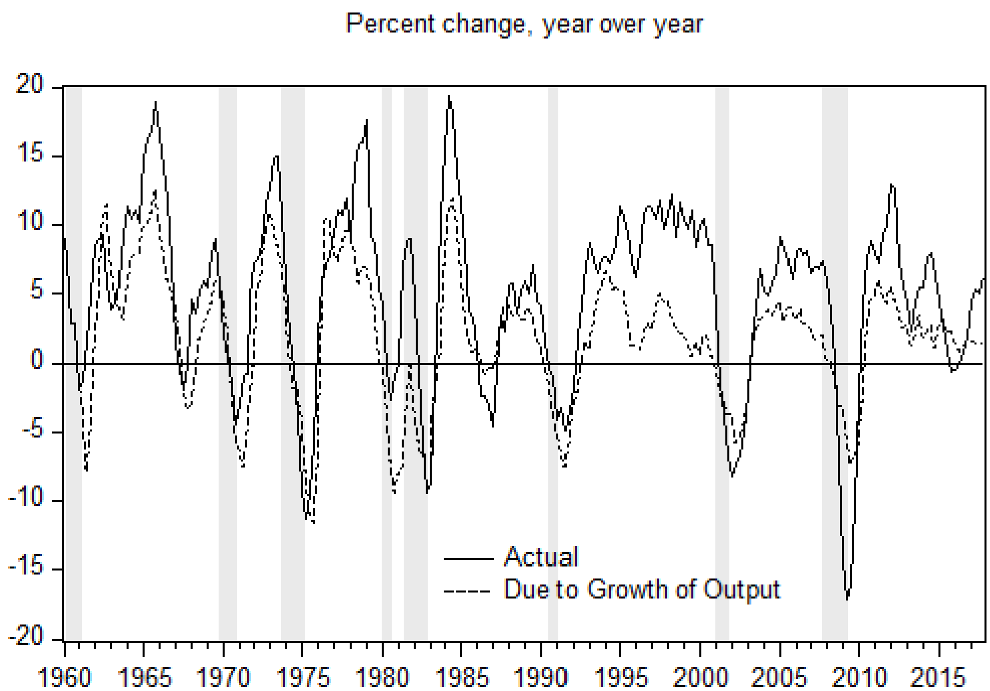


Note: Income is output of nonfarm business less tenant-occupied housing plus nonprofit institutions less taxes on production and imports less the statistical discrepancy between GDP and gross domestic income. Capital income is that income less compensation of labor and a portion of proprietors' income. Income of measured capital is capital income less income of unmeasured capital, which is $1/\eta$ of total income. Shaded vertical bars indicate recessions.
Source: Income data are from the Bureau of Economic Analysis.

sensitivity analysis are that the level of capital income from unmeasured sources is sensitive to the choice of parameters, but the pattern of that income and the estimated coefficients in the equations are not.

5.5. What Drives Business Investment? Changes in the growth of business output and labor supply dominate short-run movements in investment. Taken together, variations in growth of output less worker productivity ($Y/\bar{y}$), labor supply ($\bar{N}$), and productivity ($\bar{y}$) directly accounted for 59 percent of variations in the year-over-year growth of real private nonresidential fixed investment during the sample period (see Figure 6). Although changes in asset prices have had a smaller direct impact on investment, that impact has been important at times. For example, lower asset prices directly caused more than 5 percentage points of the 18 percent drop in real private nonresidential fixed investment between the second quarter of 2008 and the third quarter of 2009. That direct impact excludes how asset prices affected other components of GDP—for example, wealth affecting consumer spending. According to the estimates discussed below, variation in real oil prices has accounted for 6 percent of the variation in year-over-year growth of real private nonresidential fixed investment since 1995.

FIGURE 6. CONTRIBUTION OF THE GROWTH OF OUTPUT TO NONRESIDENTIAL FIXED INVESTMENT



Note: The solid line is year-over-year growth of real nonresidential fixed investment. The dashed line is the contribution to that growth from variations in $Y/\bar{y}$, $\bar{N}$, and $\bar{y}$ as estimated using (24). Shaded vertical bars indicate recessions.

6. Investment in Capital for Mining and Farming

Oil wells, gas wells, mines, and farms generally produce commodities, which are sold at the market price. The incentive to invest depends on the size of the resource base, for example, the availability of shale resources, and the expected price to be received for additional output. For investment in capital specific to mining—broadly defined as the extraction of oil, natural gas, and minerals—the key prices are those of oil and natural gas. For investment in capital specific to farming, the key price is the price index for farm output. Investment in mining exploration, shafts, and wells, abbreviated as "investment in mining structures" in this paper, is the most important category, accounting for 60 percent of investment in capital specific to mining and farming, on average, since 2000.

6.1. Investment in Mining Structures. Investment in mining structures depends on the prices of oil and natural gas. (Prices of minerals probably also have an effect, but they are not part of CBO's model; and structures for mineral production account for less than 10 percent of investment in mining structures.) The relevant price is the price at which production from a new well will be sold, which is assumed to be a weighted average of the current price and a long-run equilibrium price independent of the current price. The sooner production is expected to occur after drilling, the larger the weight on the current price. Wells employing hydraulic fracturing typically produce oil or gas much more rapidly than conventional wells; that is, such wells are depleted more quickly than conventional wells and thus depend more heavily on the current price. Because horizontal drilling is typically used in association with hydraulic fracturing, CBO models the price elasticity of investment as an increasing function of the fraction of active drilling rigs using horizontal drilling (from

Baker Hughes), denoted h.

The advent of hydraulic fracturing has also increased the number of potential wells that are profitable at a given price of oil. Let R be the number of active drilling rigs, hR the number of active rigs employing horizontal drilling, R_0 the number of rigs that would be active if h was zero, and θ the number of conventional rigs crowded out per rig employing horizontal drilling. Then $R = R_0 + (1-\theta)hR$. The ratio of active rigs to rigs that would be active in the absence of horizontal drilling, R/R_0, is $1/[1-(1-\theta)h]$.

Although the Bureau of Economic Analysis does not separately estimate investment in oil structures and investment in natural gas structures, CBO's equation for real investment in mining structures (I_{sm}) is composed of a piece depending on the price of natural gas at Henry Hub (pg) and a piece depending on the price of West Texas Intermediate crude oil (pc). That equation is

(26)

$$I_{sm,t} = \left[c_g \left(\frac{pg_t}{\tilde{p}_t^{e_2} \tilde{p}_{sm,t}^{1-e_2}} \right)^{e_0+e_1 h_{t-1}} + \frac{c_c}{1-(1-\theta)\, h_{t-1}} \left(\frac{pc_t}{\acute{p}_t^{e_2} \acute{p}_{sm,t}^{1-e_2}} \right)^{e_0+e_1 h_{t-1}} \right] (1 - c_r \; rdf_t) \, ,$$

where $\tilde{p}$ is the GDP price index (p) multiplied by the sample average of pg/p, $\tilde{p}_{sm}$ is the price index for investment in mining structures (p_{sm}) multiplied by the sample average of pg/p_{sm}, $\acute{p}$ is p multiplied by the sample average of pc/p, and $\acute{p}_{sm}$ is p_{sm} multiplied by the sample average of pc/p_{sm}. The adjustments to p and p_{sm} are made so that a change in the price elasticity of investment ($e_0 + e_1 h$) does not affect the level of investment for a given level of prices. Lagged values of a three-quarter moving average are used for both price ratios. The price elasticity of investment is assumed to be the same for both natural gas and crude oil. Parameters c_g and c_c scale the effects of natural gas prices and oil prices on investment.

The variable rdf is a lagged two-quarter moving average of the spread between yields on BBB-rated bonds and 10-year Treasury notes. The parameter c_r measures the impact of the risk premium on mining investment, a relatively risky category of investment.

The equation is estimated from the fourth quarter of 1994 through the fourth quarter of 2017. The sample period is limited by the availability of data for pg. Estimated coefficients are shown in Table 2. As horizontal drilling became more important, the increase in real investment caused by a 1 percent rise in energy prices increased from about 0.5 percent during the 1994–2003 period to more than 1.3 percent in 2017. That larger response of investment means that an increase in oil prices would have a more positive (or less negative) effect on U.S. GDP now than it would have 15 years ago. The small estimate for θ indicates that most horizontally drilled oil wells add to investment without crowding out conventional drilling. The equation provides a good fit for the data despite using only prices, the share of rigs using horizontal drilling, and a risk spread (see Figure 7).

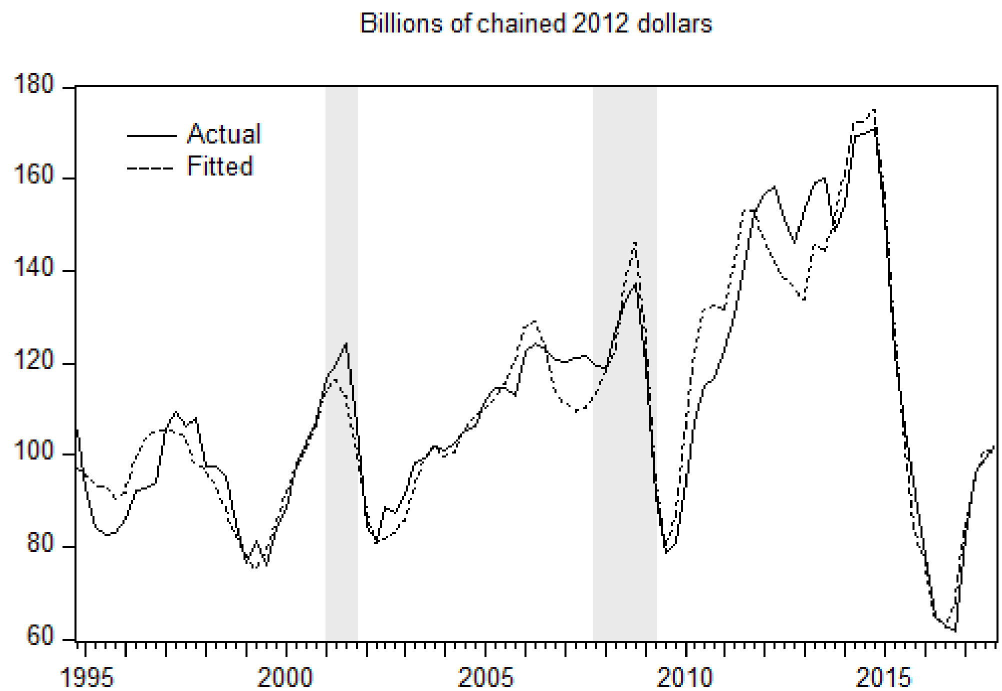
Table 2—Estimated Coefficients From an Equation for Investment in Mining Structures

	Estimate	t-statistic
c_g	52.5	11.9
c_c	51.4	10.1
e_0	0.42	6.9
e_1	1.09	7.2
e_2	0.35	8.0
θ	0.13	2.7
c_r	0.05	5.7
R^2	0.92	

Note: The table shows the results from estimating equation (26) from the fourth quarter of 1994 through the fourth quarter of 2017. The dependent variable is real investment in mining exploration, shafts, and wells. All coefficients are significant at the 99 percent level.

6.2. Investment in Mining Equipment. Investment in mining and oilfield machinery consists of new equipment used to produce new shafts and wells. Consequently, such invest-

FIGURE 7. REAL INVESTMENT IN MINING STRUCTURES



Note: Real investment in mining exploration, shafts, and wells. Fitted values are from (26). Shaded vertical bars indicate recessions.
Source: Data for actual investment are from the Bureau of Economic Analysis.

ment is modeled using a simplified version of a modified neoclassical model of investment, with investment in mining structures as the measure of output. The estimated equation for real investment in mining and oilfield machinery, I_{em}, is

$$\frac{p_{em,t} I_{em,t}}{\overline{gdp}_t} = c_1 \sum_{i=0}^{6} \frac{p_{sm,t-i} \Delta I_{sm,t-i}}{\overline{gdp}_{t-i}} + c_2 \sum_{i=0}^{5} \frac{p_{sm,t-7-i} \Delta I_{sm,t-7-i}}{\overline{gdp}_{t-7-i}} + c_3 \sum_{i=2}^{25} \frac{p_{sm,t-i} I_{sm,t-i}}{\overline{gdp}_{t-i}},$$

where subscript em refers to mining equipment and subscript sm refers to mining structures. The final expression is used as a proxy for depreciation. The lag structure was chosen through experimenting with polynomial distributed lags; moving averages were used to simplify the equation. Estimated over the same period as the equation for mining structures, the fit is good, with an R^2 of 0.95. The coefficient estimates (with t-statistics in parentheses) are 0.157 (18.6) for c_1, 0.119 (10.6) for c_2, and 0.0096 (79.7) for c_3.

6.3. Investment in Agricultural Capital. Investment in agricultural capital is modeled as a function of replacement demand and past changes in the price of farm output (p_a) multiplied by real farm output (Y_a). Just as for mining equipment, replacement demand is estimated using past output, and the lag structure was chosen based on initial specifications using polynomial distributed lags.

The equation for real investment in agricultural machinery, I_{ea}, is

$$\frac{p_{ea,t} I_{ea,t}}{\overline{gdp}_t} = c_{a1}\left(1 + c_{a3}T\right) \sum_{i=1}^{14} \frac{\Delta p_{a,t-i} Y_{a,t-i-1}}{\overline{gdp}_{t-i}} + c_{a2}\left(1 + c_{a3}T\right) \sum_{i=1}^{16} \frac{p_{a,t-i-1} Y_{a,t-i-1}}{\overline{gdp}_{t-i}},$$

where p_{ea} is the price index for agricultural machinery and T is a time trend equal to 1 in the first quarter of 1959 and rising by 1 per quarter. A time trend is added because the ratio of investment in agricultural machinery to farm output has trended upward. Estimated from the first quarter of 1966 through the fourth quarter of 2017, the fit is only moderately good, with an R^2 of 0.84. The equation overpredicts in the mid-1980s and underpredicts in the late 1970s and from 2012 to 2014. The coefficient estimates (and t-statistics) are 0.054 (9.3) for c_{a1}, 0.008 (30.5) for c_{a2}, and 0.003 (7.2) for c_{a3}.

The equation for investment in farm structures (I_{sa}) excludes the time trend:

$$\frac{p_{sa,t} I_{sa,t}}{\overline{gdp}_t} = c_{a4} \sum_{i=1}^{28} \frac{\Delta p_{a,t-i-1} Y_{a,t-i-1}}{\overline{gdp}_{t-i}} + c_{a5} \sum_{i=1}^{21} \frac{p_{a,t-i-1} Y_{a,t-i-1}}{\overline{gdp}_{t-i}}.$$

Estimated from the second quarter of 1966 to the fourth quarter of 2017, the R^2 is 0.91.

Coefficient estimates (and t-statistics) are 0.040 (15.7) for c_{a4} and 0.0028 (65.2) for c_{a5}.

7. How CBO Uses the Model to Forecast Investment

7.1. CBO's Macro Model. Business investment is forecast within CBO's primary forecasting model, a large-scale macro model similar to those used by private entities and other government agencies that forecast the economy at an economywide level (Arnold 2018). In that model, many variables are simultaneously determined, meaning that changes in a particular variable affect other variables in the model that might, in turn, influence the first variable. For example, business investment depends on the growth of GDP in prior quarters. However, because business investment is a component of GDP, the level of business investment then depends in part on its own growth in prior quarters.

CBO forecasts the components of business investment on a disaggregated basis, in real terms. Those components include five categories of producers' durable equipment (computers and peripherals, mining and oilfield machinery, agricultural machinery, communication equipment, and other equipment), three categories of IPPs (software; research and development; and entertainment, literary, and artistic originals), three categories of nonresidential structures (mining, farm, and other structures), and the change in private inventories. The equations for those variables are discussed in the preceding sections of this paper and are driven primarily by output growth, asset prices, tax law, productivity, and prices for energy and farm output.

CBO's forecast for investment affects CBO's forecast for GDP in two ways. First, investment affects GDP directly because it is a component of GDP. Second, investment affects GDP through its effect on the capital stock. CBO's forecast of GDP in the long run is driven primarily by the supply-side factors that determine potential GDP, one of which is the capital stock. CBO's analyses of the effects of tax policy on investment use more detail on the tax treatment of capital than is present in CBO's macro model, as discussed in CBO (2018b).

7.2. Inputs to Model Equations. Several important determinants of investment are exogenous to the rest of CBO's macro model. Tax variables (u, itc_m, and the determinants of z_m) are set according to current law. The cyclically adjusted shares of each type of capital in new units of capital (σ_m) and the fraction of rigs using horizontal drilling are forecast based on recent and longer-term trends. Depreciation rates (δ_m) are weighted averages of depreciation rates at the finest level of detail, each of which is held constant at the last historical value. Futures prices for oil and natural gas are used to guide forecasts for the prices of oil and natural gas.

The forecasts for capital's coefficient in production (α) and firms' elasticity of demand (η) should be jointly consistent with the forecast for capital's share of total income. That can be seen by rearranging (15) as

$$1 - \frac{w_t N_t}{p_t Y_t} = \frac{1}{\eta_t} + \alpha_t x_t \left(1 - \frac{1}{\eta_t}\right).$$

A rise in capital's share of income is consistent with a lower η, a higher α, or both. Over longer periods, and especially since 2000, movements in η have accounted for most of the variation in capital's share of income. Consequently, most fluctuations in capital's share of income from one forecast to the next are assumed to come from changes in η.

Many key drivers of investment are themselves determined by equations within CBO's macro model. Those drivers include all the price indexes, both for investment and for the output of businesses using private capital; real output of businesses using capital (Y); cyclically adjusted productivity per worker ($\bar{y}$); and employment at full employment ($\bar{N}$). Those variables are linked to conditions in the broader economy. To a limited extent, price indexes for investment depend on lags of real investment. CBO's forecasts for the corporate bond yield and the stock price index are used to forecast $(V - \hat{V})/(d\hat{V}/d\hat{r}^*)$.

CBO's forecast of investment also involves an element of judgment capturing developments not reflected in the equations. Those adjustments are made using what are called add factors. An add factor is a term added to an equation to adjust the forecast of that equation without respecifying or reestimating it. In practice, most add factors largely reflect errors in the final quarter of history and monthly information for the first quarter of the forecast and are then phased toward zero over the forecast period. In addition, commercial forecasts of investment help to inform CBO's forecast and can thus influence the choice of add factors. However, because CBO's forecasts assume current law but outside forecasts often do not, CBO's forecasts are not overly reliant on commercial forecasts.

Appendix

The appendix consists of five sections. Section A1 contains a proof that $\lambda_t K_t$, as derived in (12), is the adjusted market value of measured capital in production. Section A2 derives the after-tax price of new inventories and land. Section A3 shows how $\bar{N}$ and $\bar{y}$ are estimated. Section A4 presents the sources of data used to measure the tax treatment of new capital. Section A5 shows how market values of debt and assets are derived from par values.

A1. *Proof That $\lambda_t K_t$ Is the Adjusted Market Value of Measured Capital in Production*

Lemma: Derivation of $1/\eta$ *and* x. The value of net after-tax receipts at time t discounted to time t_0 is

$$R_t = \{(1-u_t)\left(p_t Y_t - w_t N_t\right) - v_t k_t F_t\} \exp\left(-\int\limits_{i=t_0}^{t} r_i di\right).$$

To solve that as an Euler equation, R_t is expressed as a function of state variables $\bar{k}_t$ and N_t by replacing k_t using (3) and F_t using (1). The Euler equation for the choice of N_t is evaluated assuming static expectations for $k_t/\bar{k}_t$. Assuming static expectations for α, η, and r^*, (7) implies that $\frac{d(p_{kt}k_t)}{dt} = p_{kt}k_t\left(\dot{p}+\dot{y}\right)$. With those assumptions, the solution to the Euler equation can be written as

$$(1-u_t)\left[\left(1-\frac{1}{\eta_t}\right)\frac{p_t Y_t}{N_t} - w_t\right] - v_t k_t\left(\delta + r_t^*\right) - \ v_t k_t \log\left(\frac{k_t}{\bar{k}_t}\right)\left(\dot{n}_t - r_t^*\right) = 0$$

with $\dot{n} \equiv \frac{dN/N}{dt}$.

To convert that equation into an expression for $1/\eta$, $v_t k_t$ is replaced using (7). After further rearranging, that yields

$$\frac{1}{\eta_t} = 1 - \frac{w_t N_t}{\left(1-\alpha_t x_t\right) p_t Y_t},$$

where $x_t \equiv 1 + \log\left(\frac{k_t}{\bar{k}_t}\right)\frac{\dot{n}_t - r_t^*}{\delta + r_t^*}$.

Proposition: Suppose the firm is a price maker in the output market. If the production function is homogeneous in employment, N*, and total capital,* K*, then*

$$\text{(A1)} \qquad \lambda_t K_t \equiv V_t^M = V_t - V_t^W - V_t^U.$$

Proof.[15] The production function implies that $(1-\alpha_j)\,Y_j = \frac{\partial Y_j}{\partial N_j}N_j$, so

$$(1-\alpha_j)\left(1-\frac{1}{\eta_j}\right)p_jY_j = \left(1-\frac{1}{\eta_j}\right)p_j\frac{\partial Y_j}{\partial N_j}N_j.$$

The solution to the Euler equation for N_t, (15), can be rearranged as

$$(1-\alpha_j)\left(1-\frac{1}{\eta_j}\right)p_jY_j - w_jN_j = \alpha_j\left(1-\frac{1}{\eta_j}\right)(x_j-1)\,p_jY_j.$$

Combining those equations and integrating both sides over j,

$$\text{(A2)} \qquad \int_{j=t}^{\infty}\left\{\left(1-\frac{1}{\eta_j}\right)p_j\frac{\partial Y_j}{\partial N_j}N_j - w_jN_j\right\}\exp\left(-\textstyle\int_{i=t}^{j} r_i di\right)dj = V_t^W + V_t^{WF},$$

where

$$V_t^W + V_t^{WF} = \int_{j=t}^{\infty}\alpha_j\left(1-\frac{1}{\eta_j}\right)(x_j-1)\,p_jY_j\exp\left(-\textstyle\int_{i=t}^{j} r_i di\right)dj.$$

V_t^{WF} is the difference between the present discounted values (PDVs) of the income of measured capital yet to be created and that capital's contribution to future revenues arising from $x \neq 1$.

After-tax capital income from unmeasured sources is $\frac{1}{\eta_t}p_tY_t$. Let V^{UF} be the PDV of the after-tax profits due to unmeasured sources of capital income yet to be created. Then

$$V_t^U + V_t^{UF} = \int_{j=t}^{\infty}\frac{1}{\eta_j}p_jY_j\exp\left(-\textstyle\int_{i=t}^{j} r_i di\right)dj.$$

The market value of the firm, V_t, is the PDV of future net receipts less the PDV of measured

[15] Taxes are omitted to simplify the proof.

capital and unmeasured sources of capital income yet to be created, or $VR_t - V_t^{UF} - V_t^{WF}$.

The first-order condition $\frac{\partial\left(\lambda_t \exp\left[-\int_{i=0}^{t} r_i di\right]\right)}{\partial t} = -\frac{\partial H_t}{\partial K_t}$ for (11) yields

$$\text{(A3)} \qquad \frac{\partial \lambda_t}{\partial t} - \lambda_t r_t = -\frac{d\left(p_t Y_t\right)}{dK_t} + \lambda_t K_t \frac{F_t}{N_t}\frac{1}{K_t} - \lambda_t \left[\frac{F_t}{N_t}\log\left(\frac{k_t N_t}{K_t}\right) + \frac{dN_t/N_t}{dt}\right]$$

after dividing through by $\exp\left[-\int_{i=0}^{t} r_i di\right]$. Because demand is inelastic,

$$\text{(A4)} \qquad \frac{d\left(p_t Y_t\right)}{dK_t} = \left(1 - \frac{1}{\eta_t}\right) p_t \frac{\partial Y_t}{\partial K_t}.$$

Multiplying (A3) through by K_t, substituting for $\frac{d(p_t Y_t)}{dK_t}$ using (A4), for $\lambda_t K_t$ using (12), and for the last term of (A3) using (4), and combining terms,

$$\frac{\partial\left(\lambda_t K_t\right)}{\partial t} - r_t\left(\lambda_t K_t\right) = -\left(1 - \frac{1}{\eta_t}\right) p_t \frac{\partial Y_t}{\partial K_t} K_t + p_{kt} k_t F_t.$$

The solution to that differential equation is

$$\text{(A5)} \qquad \lambda_t K_t = \int_{j=t}^{\infty} \left\{\left(1 - \frac{1}{\eta_j}\right) p_j \frac{\partial Y_j}{\partial K_j} K_j - p_{kj} k_j F_j\right\} \exp\left(-\int_{i=t}^{j} r_i di\right) dj.$$

Because the production function is homogeneous of degree one in N_j and K_j,

$$Y_j = \frac{\partial Y_j}{\partial N_j} N_j + \frac{\partial Y_j}{\partial K_j} K_j.$$

Summing equations (A2) and (A5) and making that substitution yields

$$\lambda_t K_t = \int\limits_{j=t}^{\infty} \left\{ \left(1 - \frac{1}{\eta_j}\right) p_j Y_j - w_j N_j - p_{kj} k_j F_j \right\} \exp\left(-\int_{i=t}^{j} r_i di\right) dj - V_t^{W} - V_t^{WF}.$$

The value of the integral is $VR_t - V_t^U - V_t^{UF}$, so the right-hand side of that equation is $VR_t - V_t^U - V_t^{UF} - V_t^W - V_t^{WF}$. Because $V_t = VR_t - V_t^{UF} - V_t^{WF}$, the proposition is proved.

A2. The After-Tax Cost of New Inventories and Land

Inventories combine characteristics of intermediate inputs and capital. Like intermediate inputs, inventories of materials and supplies and work in progress are eventually incorporated into the goods that firms produce. Like capital, inventories must earn a positive return to cover the cost of the debt or equity needed to finance them between the time they are purchased and the time they are sold. To be consistent with its treatment of other types of capital, the Congressional Budget Office's model treats inventories as capital when they are held and as having a net after-tax price governed by the purchase price less the discounted sales price.

Following CBO (2006), the real after-tax cost of holding a dollar of inventory is $r_t^* + \dot{y}_t + u_t \dot{p}_t / 2$, which incorporates the assumption that inventories are taxed by using a 50–50 mix of LIFO (last in, first out) and FIFO (first in, first out). The after-tax cost of a unit of inventory technology, v, is determined by extending and discounting the after-tax cost until the technology is retired. The amount of inventory associated with a unit of inventory technology grows at rate $\dot{y}$, whereas the purchase price of those inventories rises at rate $\dot{p}$. Inventory technologies are retired at rate δ_m. Under those assumptions, the after-tax cost of inventory technology is

$$v_{mt} = p_{mt}\frac{r_t^* + \dot{y}_t + u_t\dot{p}_t/2}{\delta_m + r_t^*}.$$

Like inventories, land can be sold after its current use is complete. Thus, land's net after-tax price is its purchase price minus its resale value. (Land cannot be depreciated, so the after-tax price equals the pretax price.) Because of the scarcity of land, its price is assumed to rise at rate $\dot{p} + \dot{y}$. Under the assumption that land is retired from its current use at the depreciation rate of the associated structure, δ_m, the PDV of the resale value for land placed into service at time t is

$$p_{mt}\int_{j=t}^{\infty}\delta_m e^{-(\delta_m + r_t - \dot{p}_t - \dot{y})(j-t)}dj = p_{mt}\frac{\delta_m}{\delta_m + r_t^*}.$$

The after-tax cost is p_{mt} minus that expression, or $p_{mt}\frac{r_t^*}{\delta_m + r_t^*}$.

No data exist on investment in land consistent with the definition in this paper, so a proxy must be found. Equation (13) implies that, when new, the market value of a unit of land is $\frac{\delta_m + \hat{r}^*}{\hat{r}^*}$ as large as the market value of a unit of the associated structure. (For land, δ_m in (13) vanishes because land does not depreciate.) As a result of nongeometric depreciation, the ratio of market values for units in service is $\frac{1.5\delta_m + \hat{r}^*}{\hat{r}^*}$. On the basis of data from the Bureau of Labor Statistics, the market value of all land is roughly 29 percent as large as the market value of all structures in the nonfarm business sector. Consequently, the ratio of the number of units of land to the number of units of structures is $0.29\frac{\hat{r}^*}{1.5\delta_m + \hat{r}^*}$.

A3. Estimates of $\bar{N}$ and $\bar{y}$

For estimating $\bar{N}$, the ratio $N/\bar{N}$ is assumed to be negatively related to the current and lagged differences between the actual unemployment rate (ru) and the underlying long-term rate of unemployment ($\overline{ru}$):

$$\text{(A6)} \qquad \log\left(\frac{N_t}{\bar{N}_t}\right) = -\zeta_1 \left(ru_t - \overline{ru}_t\right) - \zeta_2 \left(ru_{t-1} - \overline{ru}_{t-1}\right).$$

Data for $\overline{ru}_t$ are from Congressional Budget Office (2018a). Assume $\log\left(\bar{N}_t\right)$ follows a random walk with drift

$$\Delta \log\left(\bar{N}_t\right) = \zeta_0 + \epsilon_t,$$

where Δ denotes a first difference. The ζ are estimated using

$$\Delta\log\left(N_t\right) = \zeta_0 - \zeta_1 \; \Delta(ru_t - \overline{ru}_t) - \zeta_2 \; \Delta(ru_{t-1} - \overline{ru}_{t-1}) + \epsilon_t$$

and then used to calculate $\bar{N}_t$ by inverting (A6).

For estimating $\bar{y}$, the ratio $y/\bar{y}$ is assumed to be positively related to the ratio of hours per worker to its Hodrick–Prescott (H-P)-filtered value ($H/\bar{H}$) and negatively related to current and lagged growth of $ru - \overline{ru}$:

$$\log\left(\frac{y_t}{\bar{y}_t}\right) = \xi_0 + \xi_1 \log\left(\frac{H_t}{\bar{H}_t}\right) - \xi_2 \; \Delta(ru_t - \overline{ru}_t) - \xi_3 \; \Delta(ru_{t-1} - \overline{ru}_{t-1}).$$

$\bar{y}$ is then estimated in an analogous manner to $\bar{N}$.

A4. Sources of Data for the Tax Treatment of New Capital

The tax treatment of new capital incorporates the assumption that the firm maximizes net after-tax receipts discounted by a weighted average of the after-tax returns to debt and equity. That is equivalent to the firm's maximizing after-tax profits discounted by the after-tax return to equity. To see that, consider a simple example in which prices and real quantities are constant. Then, (6) can be expressed as

$$VR_t = \frac{(1-u_t)(p_tY_t - w_tN_t) - v_tk_tF_t}{r_t}. \tag{A7}$$

With the same notation, the PDV of after-tax profits is

$$\begin{aligned} VE_t &= \int_{j=t}^{\infty} [(1-u_j)(p_jY_j - w_jN_j - rd_jD_j) - v_jk_jF_j] \exp\left(-\int_{i=t}^{j} re_i di\right) dj \\ &= \frac{(1-u_t)(p_tY_t - w_tN_t - rd_tD_t) - v_tk_tF_t}{re_t}, \end{aligned}$$

where rd is the pretax cost of debt D and re is the pretax cost of equity. Then

$$re_tVE_t + (1-u_t)\, rd_tD_t = (1-u_t)(p_tY_t - w_tN_t) - v_tk_tF_t.$$

That can be reexpressed as

$$\left[re_t\frac{VE_t}{VE_t + D_t} + (1-u_t)\, rd_t\frac{D_t}{VE_t + D_t}\right](VE_t + D_t) = (1-u_t)(p_tY_t - w_tN_t) - v_tk_tF_t.$$

The portion of the left-hand-side of that equation multiplying $VE_t + D_t$ is a weighted average of the after-tax returns to equity and debt and is thus equal to r_t. Comparing the result with (A7), we can see that $VR_t = VE_t + D_t$. For either fixed D or D that varies in proportion to VE, maximizing VR is identical to maximizing VE.

Data for the methods of depreciation taken, tax lifetimes, declining-balance parameters, rates of investment tax credit, and the basis adjustment for the investment credit are taken from Gravelle (1994), updated for provisions allowing bonus depreciation since 2002. Take-up rates for bonus depreciation and expensing under Section 179 for 2002 to 2014 are taken from Kitchen and Knittel (2016). The use of Section 179 expensing before 2002 and after 2014 is assumed to vary with the limitation on the amount of investment qualifying for Section 179.

Ideally, the tax rate on new capital is a combination of the rates faced by C corporations, pass-through businesses, and nonprofits. A time series for tax rates of pass-through businesses back to 1960 is not available at present, so only the rate for C corporations is used. That rate is multiplied by a factor less than 1 for a few reasons. First, the tax rate for nonprofit institutions is zero. Second, loss-making firms may not be able to take advantage of depreciation allowances or may have to defer them. Third, not all income is subject to tax, even for profitable firms. On the basis of those considerations, the corporate tax rate is multiplied by a factor g, which is 0.80 for all businesses investing in nonresidential capital and 0.85 for nonfinancial corporations.

The tax rate on business income is then

$$u_t = g \times [uf_t + usl_t \times (1 - uf_t)],$$

where uf_t is the federal statutory corporate income tax rate, from Gravelle (1994), and

usl_t is the average state and local corporate tax rate, equal to state and local corporate tax collections divided by corporate profits before tax for domestic industries. The latter is multiplied by $(1 - uf_t)$ because state and local taxes are deductible from federal tax. The business transfer portion of both $1 - u$ and $1 - \hat{u}$ is estimated as the H-P–filtered ratio of business transfer payments to capital income.

Tax rates faced by holders of equity and debt should be captured in asset prices and so do not need to enter the model separately. For example, a reduction in the personal tax rate on dividends should be fully reflected in higher stock prices and thus in V.

The PDV of depreciation allowances requires an estimate of the nominal rate of return r_t, which equals the model estimate of $\hat{r}^* + c3\,(r_t^* - \hat{r}^*)$ plus estimates of $\dot{p}_t$ and $\dot{y}_t$. From the fourth quarter of 1968 on, raw $\dot{p}_t$ $(\dot{p}_t^r)$ equals the change in the GDP price index expected over the next year from the Philadelphia Federal Reserve Bank's Survey of Professional Forecasters plus the average difference in growth rates between p_t and the GDP price index over that period. Because expected inflation has to cover the entire tax life of an asset, not just the next year, the estimate of $\dot{p}_t$ is an average of $\dot{p}_t^r$ and the sample average of $\dot{p}_t^r$. Estimates of $\dot{p}_t$ before the fourth quarter of 1968 are based on a regression of $\dot{p}_t$ on $ru_t - \overline{ru}_t$ and current and lagged growth of p_t during 1968 to mid-2017. Expected $\dot{y}$, set equal to the sample average, is just over 1.5 percent.

A5. Market Values of Debt and Assets

For converting the par value of interest-bearing liabilities into market value, the ratio of interest paid to the par value of those liabilities is regressed on past interest rates (measured as the average of the yields on bonds rated AAA and BAA) to determine the average maturity, denoted T_M, which is assumed constant over time. Rather than make a separate calculation for each maturity, all the liabilities held at time t are assumed to have been issued at time $t - T_M$ at the average interest rate over the period $t - 2T_M$ to t. If one denotes that average rate as ri_0 and the actual interest rate at time t as ri_t, the market value of existing debt is the par value times $\frac{ri_0}{ri_t}\left(1 - \frac{1}{(1+ri_t)^{T_M}}\right) + \frac{1}{(1+ri_t)^{T_M}}$. The market value of assets is calculated using a similar procedure.

*

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