CBSE NCERT Solutions for Class 11 Mathematics Chapter 16

Back of Chapter Questions

Exercise 16.1

1. Describe the sample space for the indicated experiment: A coin is tossed three times
Hint: When a coin is toss three times the total number of possible outcomes is $2^3 = 8$

Solution:

Solution Step 1: A coin has two faces: head or tail

When a coin is toss three times,

The total number of possible outcomes is $2^3 = 8$ Thus,

When a coin is toss three times, the sample terms is:

$S = \{ \text{HHH}, \text{HHT}, \text{HTH}, \text{HTT}, \text{THH}, \text{THT}, \text{TTH}, \text{TTT}\}$

2. Describe the sample space for the indicated experiment: A die is thrown two times.
Hint: when a die is thrown 2 times the total number of outcome will be 36.

Solution:

Solution step 1:When a die is thrown, the possible outcomes are $1, 2, 3, 4, 5,6$.

When a die is thrown two times, the sample space is given by

$$S = \{(x, y): x, y = 1, 2, 3, 4, 5, 6\}$$

The number of elements in this sample space is $6 \times 6 = 36$, the sample space is given by:

$$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),$$
$$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),$$
$$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

3. Describe the sample space for the indicated experiment: A coin is tossed four times
Hint: When a coin is tossed four times, the total number of possible outcomes is $2^4 = 16$

Solution:

Solutions step 1: When a coin is tossed once, there are two possible outcomes: head H and tail T

When a coin is tossed four times, the total number of possible outcomes is $2^4 = 16$

Thus, when a coin is tossed four times, the sample space is given by:

$S = \{HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH,$

$THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT\}$

4. Describe the sample space for the indicated experiment: A coin is tossed, and a die is throw
Hint:When a coin and a die is thrown together the total number of possible outcomes is 12.

Solution:

Solutions step 1:

A coin has two faces: head (H) and tail (T).

A die has six faces that are numbered from 1 to 6, with one number on each face.

Thus, when a coin is tossed and a die is thrown, the sample space is given by:

$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

5. Describe the sample space for the indicated experiment: A coin is tossed and then a die is rolled only in case a head is shown on the coin.

Hint:When a coin and a die is thrown together the total number of possible outcomes is 12.

Solution:

Solutions step 1:A coin has two faces: head (H) and tail (T).

A die has six faces that are numbered from 1 to 6, with one number on each face.

Thus, when a coin is tossed and then a die is rolled only in case a head is shown on the coin, the sample space is given by: $S = \{H1, H2, H3, H4, H5, H6, T\}$

6. 2 boys and 2 girls are in Room X, and 1 boy and 3 girls in Room Y. Specify the sample space for the experiment in which a room is selected and then a person.

Solution:

Solutions step 1**:** Let us denote 2 boys and 2 girls in room X as B_1, B_2 and G_1, G_2 respectively. Let us denote 1 boy and 3 girls in room Y as B_3, and G_3, G_4, G_5 respectively.

Accordingly, the required sample space is given by

$$S = \{XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5\}$$

7. One die of red colour, one of white colour and one of blue colour are placed in a bag. One die is selected at random and rolled, its colour and the number on its uppermost face is noted.Describe the sample space.
Hint: Total number of outcome is 18.

Solution:

Solutions step 1: A die has six faces that are numbered from 1 to 6, with one number on each face. Let us denote the red, white, and blue dices as R, W, and B respectively.

Accordingly, when a die is selected and then rolled, the sample space is given by

$$S = \{R1, R2, R3, R4, R5, R6, W1, W2, W3, W4, W5, W6, B1, B2, B3, B4, B5, B6\}$$

8. (i)An experiment consists of recording boy-girl composition of families with 2 children.

(i) What is the sample space if we are interested in knowing whether it is a boy or girl in the order of their births?

Solution:

Solutions step 1:When the order of the birth of a girl or a boy is considered, the sample space is given by $S = \{GG, GB, BG, BB\}$

(ii) What is the sample space if we are interested in the number of girls in the family?

Hint: Total number of outcome is 3.

Solution:

Solutions step 1:Since the maximum number of children in each family is 2, a family can either have 2 girls or

1 girl or no girl. Hence, the required sample space is $S = \{0, 1, 2\}$

9. A box contains 1 red and 3 identical white balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment.
Hint: Total number of outcome is 3.

Solution:

Solutions step 1: It is given that the box contains 1 red ball and 3 identical white balls. Let us denote the red ball withRand a white ball with W.

When two balls are drawn at random in succession without replacement, the sample space is given by

$S = \{RW, WR, WW\}$

10. An experiment consists of tossing a coin and then throwing it second time if a head occurs. If a tail occurs on the first toss, then a die is rolled once. Find the sample space.

Solution:

Solutions step 1: A coin has two faces: head (H) and tail (T).

A die has six faces that are numbered from 1 to 6, with one number on each face.

Thus, in the given experiment, the sample space is given by

$S = \{HH, HT, T1, T2, T3, T4, T5, T6\}$

11. Suppose 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non-defective (N). Write the sample space of this experiment?
Hint: Total number of outcome is 4.

Solution:

Solution step 1:3 bulbs are to be selected at random from the lot. Each bulb in the lot is tested and classified as defective (D) or non-defective (N).

The sample space of this experiment is given by

$S = \{DDD, DDN, DND, DNN, NDD, NDN, NND, NNN\}$

12. A coin is tossed. If the outcome is a head, a die is thrown. If the die shows up an even number, the die is thrown again. What is the sample space for the experiment?
Hint: Total number of outcome is 22.

Solution:

Solutions step 1: When a coin is tossed, the possible outcomes are head (H) and tail (T).

When a die is thrown, the possible outcomes are $1, 2, 3, 4, 5$, or 6.

Thus, the sample space of this experiment is given by:

$$S = \{T, H1, H3, H5, H21, H22, H23, H24, H25, H26, H41, H42, H43, H44,$$
$$H45, H46, H61, H62, H63, H64, H65, H66\}$$

13. The numbers $1, 2, 3$ and 4 are written separately on four slips of paper. The slips are put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the sample space for the experiment.
Hint: Total number of outcome is 10.

Solution:

Solutions step 1: If 1 appears on the first drawn slip, then the possibilities that the number appears on the second drawn slip are $2, 3$, or 4. Similarly, if 2 appears on the first drawn slip, then the possibilities that the number appears on the second drawn slip are $1, 3$, or 4. The same holds true for the remaining numbers too.

Thus, the sample space of this experiment is given by

$S = \{(1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1),$

$(4,2), (4,3)\}$

14. An experiment consists of rolling a die and then tossing a coin once if the number on the die is even. If the number on the die is odd, the coin is tossed twice. Write the sample space for this experiment.
Hint: Total number of outcome is 18.

Solution:

Solution step 1:A die has six faces that are numbered from 1 to 6, with one number on each face. Among these numbers, $2, 4$, and 6 are even numbers, while $1, 3$, and 5 are odd numbers.

A coin has two faces: head (H) and tail (T).

Hence, the sample space of this experiment is given by:

$$S = \{2H, 2T, 4H, 4T, 6H, 6T, 1HH, 1HT, 1TH, 1TT, 3HH, 3HT, 3TH, 3TT, 5HH, 5HT, 5TH, 5TT\}$$

15. A coin is tossed. If it shows a tail, we draw a ball from a box which contains 2 red and 3 black balls. If it shows head, we throw a die. Find the sample space for this experiment.
Hint: Total number of outcome is 11.

Solution:

Solution step 1:The box contains 2 red balls and 3 black balls. Let us denote the 2 red balls as R_1, R_2 and the 3 black balls as B_1, B_2, and B_3.

The sample space of this experiment is given by

$S = \{TR_1, TR_2, TB_1, TB_2, TB_3, H1, H2, H3, H4, H5, H6\}$

16. A die is thrown repeatedly until a six comes up. What is the sample space for this experiment?
Hint: Total number of outcome is 216.

Solution:

Solutions step 1: In this experiment, six may come up on the first throw, the second throw, the third throw and so on till six is obtained.

Hence, the sample space of this experiment is given by

$S = \{6, (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (1, 1, 6), (1, 2, 6),$

$\dots, (1, 5, 6), (2, 1, 6), (2, 2, 6), \dots, (2, 5, 6), \dots, (5, 1, 6), (5, 2, 6), \dots\}$

Exercise 16.2

1. A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number".Are E andF mutually exclusive?

Hint:$E \cap F \neq \Phi$ Then, E and F are not mutually exclusive events.

Solution:

Solutions step 1: When a die is rolled, the sample space is given by

$S = \{1, 2, 3, 4, 5, 6\}$

Accordingly, $E = \{4\}$ and $F = \{2, 4, 6\}$

It is observed that $E \cap F = \{4\} \neq \Phi$ Therefore, E and F are not mutually exclusive events.

2. A die is thrown. Describe the following events:
(i)A: a number less than 7

(ii)B: a number greater than 7

(iii)C: a multiple of 3

(iv)D: a number less than 4

(v)E: an even number greater than 4

(vi)F: a number not less than 3

Also find $A \cup B, A \cap B, B \cup C, E \cap F, D \cap E, A - C, D - E, E \cap F', F'$

Hint: Total number of outcome is 6.

Solution:

Solution step 1: When a die is thrown, the sample space is given by $S = \{1, 2, 3, 4, 5, 6\}$. Accordingly$A = \{1, 2, 3, 4, 5, 6\}$

(ii) B: a number greater than 7

Hint: Total number of outcome is 0.

Solution:

Solutions step 1:$B = \Phi$

(iii) a multiple of 3

Hint: Total number of outcome is 2.

Solution:

Solutions step 1:$C = \{3, 6\}$

(iv) a number less than 4

Hint: Total number of outcome is 3.

Solution:

Solutions step 1: $D = \{1, 2, 3\}$

(v) an even number greater than 4

Hint: Total number of outcome is 1.

Solution:

Solutions step 1: $E = \{6\}$

(vi) a number not less than 3

Hint: Total number of outcome is 4.

Solution:

Solutions step 1: $F = \{3, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}, A \cap B = \Phi$

$A \cup C = \{3, 6\}, E \cap F = \{6\}$

$D \cap E = \Phi, A - C = \{1, 2, 4, 5\}$

$D - E = \{1, 2, 3\},$

$F' = \{1, 2\}$

$E \cap F' = \Phi$

3. An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

A: the sum is greater than 8,

B: 2 occurs on either die

C: The sum is at least 7 and a multiple of 3.

Which pairs of these events are mutually exclusive?

Hint: Total number of outcome is 36.

Solution:

Solution step 1: When a pair of dice is rolled, the sample space is given by

$S = \{(x, y): x, y = 1, 2, 3, 4, 5, 6\}$

$$= \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

Accordingly,

$A = \{(3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)\}$

$B = \{(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (1, 2), (3, 2), (4, 2), (5, 2), (6, 2)\}$

$C = \{(3,6),(4,5),(5,4),(6,3),(6,6)\}$

It is observed that

$A \cap B\ = \Phi$

$B \cap C\ = \Phi$

$C \cap A = \{(3,6),(4,5),(5,4),(6,3),(6,6)\} \neq \phi$

Hence, events A and B and events B and C are mutually exclusive.

4. Three coins are tossed once. Let A denote the event 'three heads show", B denote the event "two heads and one tail show". C denote the event "three tails show" and D denote the event 'a head shows on the first coin". Which events are
(i)mutually exclusive?

(ii)simple?

(iii)compound?

Solution:

Solution step 1: When three coins are tossed, the sample space is given by

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Accordingly,

$A = \{HHH\}$

$B = \{HHT, HTH, THH\}$

$C = \{TTT\}$

$D = \{HHH, HHT, HTH, HTT\}$

We now observe that

$A \cap B\ = \Phi, A \cap C\ = \Phi, A \cap D = \{HHH\} \neq \Phi$

$B \cap C\ = \Phi, B \cap D = \{HHT, \{HTH\} \neq \Phi$

$C \cap D = \Phi$

(i) mutually exclusive?
Hint: $E \cap F = \Phi$ Then, E and F are mutually exclusive events.

Solution:

Solutions step 1:Event A and B; event A and C; event B and C; and event C and D are all mutually exclusive.

(ii) simple?
Hint:If an event has only one sample point of a sample space, it is called a simple event.

Solution:

Solutions step 3:If an event has only one sample point of a sample space, it is called a simple event. Thus, A and C are simple events.

(iii) compound?

Hint:If an event has more than one sample point of a sample space, it is called a compound event.

Solution:

Solutions step 4:If an event has more than one sample point of a sample space, it is called a compound event.

Thus, B and D are compound events.

5. Three coins are tossed. Describe
(i)Two events which are mutually exclusive.

(ii)Three events which are mutually exclusive and exhaustive.

(iii)Two events, which are not mutually exclusive.

(iv)Two events which are mutually exclusive but not exhaustive.

(v)Three events which are mutually exclusive but not exhaustive.

Hint: Total number of outcome is 8.

Solution:

Solution step 1:When three coins are tossed, the sample space is given by

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Two events that are mutually exclusive can be

A: getting no heads and B: getting no tails

This is because sets

$A = \{TTT\}$ and $B = \{HHH\}$ are disjoint.

(ii) Three events which are mutually exclusive and exhaustive

Hint: Total number of outcome is 8.

Solution:

Solution step 1:Three events that are mutually exclusive and exhaustive can be

A: getting no heads

B: getting exactly one head C: getting at least two heads

i.e.,

$A = \{TTT\}$

$B = \{HTT, THT, TTH\}$

$C = \{HHH, HHT, HTH, THH\}$

This is because $A \cap B = B \cap C = C \cap A = \Phi$ and $A \cup B \cup C = S$

(iii) Two events, which are not mutually exclusive

Hint: Total number of outcome is 5.

Solution:

Solution step 1:Two events that are not mutually exclusive can be

A: getting three heads B: getting at least 2 heads i.e.,

$A = \{HHH\}$

$B = \{HHH, HHT, HTH, THH\}$

This is because $A \cap B = \{HHH\} \neq \Phi$

(iv) Two events which are mutually exclusive but not exhaustive.

Hint: Total number of outcome is 6.

Solution:

Solution step 1: Two events which are mutually exclusive but not exhaustive can be

A: getting exactly one head

B: getting exactly one tail That is

$A = \{HTT, THT, TTH\}$

$B = \{HHT, HTH, THH\}$

It is because, $A \cap B = \Phi$, but $A \cup B \neq S$

(v) Three events which are mutually exclusive but not exhaustive.

Hint: Total number of outcome is 7.

Solution:

Solution step 1:Three events that are mutually exclusive but not exhaustive can be

A: getting exactly three heads

B: getting one head and two tails

C: getting one tail and two heads

i.e.,

$A = \{HHH\}$

$B = \{HTT, THT, TTH\}$

$C = \{HHT, HTH, THH\}$

This is because $A \cap B = B \cap C = C \cap A = \Phi$, but $A \cup B \cup C \neq S$

6. Two dice are thrown. The events A, B and C are as follows:
A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5

Describe the events

(i)A'

(ii)not B

(iii)A or B

(iv)A and B

(v)A but not C

(vi)B or C

(vii)B and C

(viii)$A \cap B' \cap C'$

Hint: Total number of outcome is 36.

Solution:

Solutions step 1:When two dice are thrown, the sample space is given by

$S = \{(\square, \square): \square, \square = 1, 2, 3, 4, 5, 6\}$

$$= \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

Accordingly,

$$A = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3) \\ (4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \end{Bmatrix}$$

$$B = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3) \\ (3,4),(3,5),(3,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \end{Bmatrix}$$

$$C = \{(,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)\}$$

$$A' = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3) \\ (3,4),(3,5),(3,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \end{Bmatrix} = B$$

(ii) not B

Hint: Total number of outcome is 18.

Solution:

Solutions step 2:Not $B = B' = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3) \\ (4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \end{Bmatrix} = A$

(iii) A or B

Hint: Total number of outcome is 36.

Solution:

Solutions step 1:A or $B = A \cup B = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix} = S$

(iv)A and B

Hint: Total number of outcome is 0.

Solution:

Solution step 1:A and B $= A \cap B = \phi$

(v)A but not C

Hint: Total number of outcome is 8.

Solution:

Solutions step 1:A but not C $= A - C$

$$= \begin{Bmatrix} (2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5), \\ (4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \end{Bmatrix}$$

(vi)B or C

Hint: Total number of outcome is 22.

Solution:

Solution step 6: (vi)B or C $= B \cup C$

$$= \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2), \\ (2,3),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \end{Bmatrix}$$

(vii)B and C

Hint: Total number of outcome is 6.

Solution:

Solutions step 1:B and C $= B \cap C\{(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)\}$

(viii)$A \cap B' \cap C'$

Hint: Total number of outcome is 26.

Solution:

Solutions step 1: $C' = \begin{Bmatrix} (1,5),(1,6),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6),(4,2) \\ (4,3),(4,4)(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \end{Bmatrix}$

$\therefore\ A \cap B' \cap C' = A \cap A \cap C' = A \cap C'$

$= \begin{Bmatrix} (2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5), \\ (4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \end{Bmatrix}$

7. (i)Two dice are thrown. The events A, B and C are as follows:
A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice $\leq\ 5$

State true or false: (give reason for your answer)

(i)A and B are mutually exclusive

(ii)A and B are mutually exclusive and exhaustive

(iii)$A = B'$

(iv)A and C are mutually exclusive are mutually exclusive

(v)A and B' are mutually exclusive.

(vi)A', B' and Care mutually exclusive and exhaustive

Hint: Total number of outcome is 0.

Solution:

Solutions step 1A $= \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3) \\ (4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \end{Bmatrix}$

$B = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3) \\ (3,4),(3,5),(3,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \end{Bmatrix}$

$C = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)\}$

(i)It is observed that $A \cap B = \Phi$

$\therefore$ A and B are mutually exclusive.

Thus, the given statement is true.

(ii)A and B are mutually exclusive and exhaustive

Hint: Total number of outcome is 0.

Solution:

Solutions step 1:It is observed that $A \cap B = \Phi$ and $A \cup B = S \therefore$ A and B are mutually exclusive and exhaustive.Thus, the given statement is true

(iii)$A = B'$

Hint: Total number of outcome is 18.

Solution:

Solutions step 1:It is observed that

$$B' = \begin{Bmatrix} (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3) \\ (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6), \end{Bmatrix} = A$$

Thus, the given statement is true.

(iv)A and C are mutually exclusive are mutually exclusive

Hint: Total number of outcome is 4.

Solution:

Solutions step 1: It is observed that $A \cap C = \{(2,1), (2,2), (2,3), (4,1)\} \neq \Phi$

$\therefore$ A and C are not mutually exclusive.

Thus, the given statement is false.

(v)A and B' are mutually exclusive.

Hint: Total number of outcome is 0.

Solution:

Solution step 1: $A \cap B' = A \cap A = A$

$\because A \cap B' \neq \Phi$

$\therefore$ A and B' are not mutually exclusive.

Thus, the given statement is false.

(vi) A', B' and Care mutually exclusive and exhaustive

Hint: Total number of outcome is 4.

Solution:

Solutions step 6:It can be observed that: $A' \cup B' \cup C = S$

However, $B' \cap C = \{(2,1),(2,2),(2,3),(4,1)\} \neq \Phi$

Therefore, events A', B' and Care not mutually exclusive and exhaustive.

Thus, the given statement is false.

Exercise: 16.3

1. Which of the following cannot be valid assignment of probabilities for outcomes of sample space $S = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7,\}$

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(a)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(b)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
(c)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
(d)	-0.1	0.2	0.3	0.4	-0.2	0.1	0.3
(e)	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

Solution:

Solutions step 1:(a)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
0.1	0.01	0.05	0.03	0.01	0.2	0.6

Here, each of the numbers $p(\omega_1)$ is positive and less than 1.

Sum of probabilities

$= p(\omega_1) + p(\omega_1) + p(\omega_3) + p(\omega_4) + p(\omega_5) + p(\omega_6) + p(\omega_7)$

$= 0.1 + 0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6$

$= 1$

Thus, the assignment is valid.

(b)

(b)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

Hint: Sum of all probabilities is equal to 1.

Solution:

Solutions step 1:(b)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

Here each of the numbers $p(\omega_1)$ is positive and less than 1.

Sum of the probabilities

$= p(\omega_1) + p(\omega_1) + p(\omega_3) + p(\omega_4) + p(\omega_5) + p(\omega_6) + p(\omega_7)$

$= \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = 7 \times \frac{1}{7} = 1$

Thus, the assignment is valid.

(iii)

(c)	0.1	0.2	0.3	0.4	0.5	0.6	0.7

Hint: Sum of all probabilities is equal to 1.

Solution:

Solutions step 1: (c)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
0.1	0.2	0.3	0.4	0.5	0.6	0.7

Here each of the numbers $p(\omega_1)$ is positive and less than 1.

Sum of probabilities

$= p(\omega_1) + p(\omega_1) + p(\omega_3) + p(\omega_4) + p(\omega_5) + p(\omega_6) + p(\omega_7)$

$= 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7$

$= 2.8 \neq 1$

Thus, the assignment is not valid

(iv)

(d)	-0.1	0.2	0.3	0.4	-0.2	0.1	0.3

Hint: Sum of all probabilities is equal to 1.

Solution:

Solutions step 1:(d)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
-0.1	0.2	0.3	0.4	-0.2	0.1	0.3

Here, $p(\omega_1)$ and $p(\omega_2)$ are negative.

Hence, the assignment is not valid.

(v)

(e)	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

Hint: Sum of all probabilities is equal to 1.

Solution:

Solutions step 1:(e)

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

Here, $p(\omega_7) = \frac{15}{14} > 1$

Hence, the assignment is not valid.

2. A coin is tossed twice, what is the probability that at least one tail occurs?

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:When a coin is tossed twice, the sample space is given by

$S = \{HH, HT, TH, TT\}$

Let A be the event of the occurrence of at least one tail.

Accordingly, $A = \{HT, TH, TT\}$

$\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

$= \frac{n(A)}{n(S)}$

$= \frac{3}{4}$

(i) A die is thrown, find the probability of following events:

(i)A prime number will appear,

(ii)A number greater than or equal to 3 will appear,

(iii)A number less than or equal to one will appear,

(iv)A number more than 6 will appear,

(v)A number less than 6 will appear.

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step1:The sample space of the given experiment is given by

$S = \{1, 2, 3, 4, 5, 6\}$

Let A be the event of the occurrence of a prime number.

Accordingly, $A = \{2, 3, 5\}$

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}} = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(ii)A number greater than or equal to 3 will appear,

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step1:Let B be the event of the occurrence of a number greater than or equal to 3.
Accordingly, $B = \{3, 4, 5, 6\}$

$$\therefore P(B) = \frac{\text{Number of outcomes favourable to B}}{\text{Total Number of possible outcomes}} = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

(iii)A number less than or equal to one will appear, Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:Let C be the event of the occurrence of a number less than or equal to one.
Accordingly, $C = \{1\}$

$$\therefore P(C) = \frac{\text{Number of outcomes favourable to C}}{\text{Total Number of possible outcomes}} = \frac{n(C)}{n(S)} = \frac{1}{6}$$

(iv)A number more than 6 will appear,

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

solutions step 1:Let D be the event of the occurrence of a number greater than 6.

Accordingly, $D = \Phi$

$$\therefore P(D) = \frac{\text{Number of outcomes favourable to D}}{\text{Total Number of possible outcomes}} = \frac{n(D)}{n(S)} = \frac{0}{6} = 0$$

(v)A number less than 6 will appear.

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

solutions step 1:Let E be the event of the occurrence of a number less than 6.

Accordingly, $E = \{1, 2, 3, 4, 5\}$

$$\therefore P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total Number of possible outcomes}} = \frac{n(E)}{n(S)} = \frac{5}{6}$$

3. A card is selected from a pack of 52 cards.
(a)How many points are there in the sample space?

(b)Calculate the probability that the card is an ace of spades.

(c)Calculate the probability that the card is (i) an ace (ii) black card.

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:(a)When a card is selected from a pack of 52 cards, the number of possible outcomes is 52 i.e., the sample space contains 52 elements.

Therefore, there are 52 points in the sample space.

(b)Calculate the probability that the card is an ace of spades.

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:(b)Let A be the event in which the card drawn is an ace of spades.

Accordingly, $n(A) = 1$

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}} = \frac{n(A)}{n(S)} = \frac{1}{52}$$

(c)Calculate the probability that the card is (i) an ace (ii) black card.

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1: (c)(i)Let E be the event in which the card drawn is an ace.

Since there are 4 aces in a pack of 52 cards, $n(E) = 4$

$$\therefore P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total Number of possible outcomes}} = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(c) (ii)black card.

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:Let F be the event in which the card drawn is black.

Since there are 26 black cards in a pack of 52 cards, $n(F) = 26$

$$\therefore P(F) = \frac{\text{Number of outcomes favourable to F}}{\text{Total Number of possible outcomes}} = \frac{n(F)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

4. A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is (i) 3 (ii) 12

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:Since the fair coin has 1 marked on one face and 6 on the other, and the die has six faces that are numbered 1, 2, 3, 4, 5, and 6, the sample space is given by

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

Accordingly, $n(S) = 12$

Let A be the event in which the sum of numbers that turn up is 3.

Accordingly, $A = \{(1,2)\}$

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}} = \frac{n(A)}{n(S)} = \frac{1}{12}$$

(ii) 12

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:Let B be the event in which the sum of numbers that turn up is 12.

Accordingly, $B = \{(6, 6)\}$

$$\therefore P(B) = \frac{\text{Number of outcomes favourable to B}}{\text{Total Number of possible outcomes}} = \frac{n(B)}{n(S)} = \frac{1}{12}$$

5. There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:There are four men and six women on the city council.

As one council member is to be selected for a committee at random, the sample space contains 10 $(4 + 6)$ elements.

Let A be the event in which the selected council member is a woman.

Accordingly, $n(A) = 6$

$$\text{l}\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}} = \frac{n(A)}{n(S)} = \frac{6}{10} = \frac{3}{5}$$

6. A fair coin is tossed four times, and a person win Re 1 for each head and lose Rs 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.
Hint:$\bar{p} = (1 - p)$

Solution:

Solutions step 1:Since the coin is tossed four times, there can be a maximum of 4 heads or tails.

When 4 heads turn up,₹$1 +$ ₹$1 +$ ₹$1 +$ ₹$1 = 4$ is the gain.

When 3 heads and 1 tail turn up,

₹ $1 +$ ₹ $1 +$ ₹ $1 -$ ₹ $1.50 =$ ₹ $3 -$ ₹ $1.50 =$ ₹ 1.50 is the gain.

When 2 heads and 2 tails turn up,

₹ $1 +$ ₹$1 -$ ₹ $1.50 -$ ₹ $1.50 = -$₹ 1, i.e.,₹1 is the loss.

When 1 head and 3 tails turn up,

₹ 1 − ₹1.50 − ₹ 1.50 − ₹ 1.50 = −₹ 3.50, i.e., ₹ 3.50 is the loss.

When 4 tails turn up,

₹ 1.50 − ₹ 1.50 − ₹ 1.50 − ₹ 1.50 = − ₹ 6.00, i.e.,₹ 6.00 is the loss.

There are 24 = 16 elements in the sample space S, which is given by:

$S = \{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH,$

$TTHH, HTHT, THTH, THHT, HTTT, THTT, TTHT, TTTH, TTTT\}$

$\therefore n(S) = 16$

The person wins ₹ 4.00 when 4 heads turn up, i.e., when the event {HHHH} occurs.

∴ Probability (of winning ₹ 4.00)= $\frac{1}{16}$

The person wins ₹ 1.50 when 3 heads and one tail turn up, i.e.,

when the event {HHHT, HHTH, HTHH, THHH} occurs.

∴ Probability (of winning ₹ 1.50) = $\frac{4}{16} = \frac{1}{4}$

The person loses ₹ 1.00 when 2 heads and 2 tails turn up, i.e.,

when the event {HHTT, HTTH, TTHH, HTHT, THTH, THHT} occurs.

∴ Probability (of losing ₹ 1.00)= $\frac{6}{16} = \frac{3}{8}$

The person loses ₹3.50 when 1 head and 3 tails turn up, i.e.,

when the event {HTTT, THTT, TTHT, TTTH} occurs.

∴ Probability (of losing ₹ 3.50) = $\frac{4}{16} = \frac{1}{4}$

The person loses₹ 6.00 when 4 tails turn up, i.e.,

when the event {TTTT} occurs.

Probability (of losing ₹ 6.00) = $\frac{1}{16}$

(i)Three coins are tossed once. Find the probability of getting

(i)3 heads

(ii)2 heads

(iii)at least 2 heads

(iv)at most 2 heads

(v)no head

(vi)3 tails

(vii)exactly two tails

(viii)no tail

(ix)at most two tails.

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}}$

Solution:

Solutions step 1:When three coins are tossed once, the sample space is given by

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$\therefore$ Accordingly, $n(S) = 8$

It is known that the probability of an event A is given by

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}} = \frac{n(A)}{n(S)}$$

Let B be the event of the occurrence of 3 heads. Accordingly,

$B = \{HHH$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{1}{8}$$

(ii)2 heads

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}}$

Solution:

Solutions step 1:Let C be the event of the occurrence of 2 heads. Accordingly,

$C = \{HHT, HTH, THH\}$

$$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{1}{8}$$

(iii)at least 2 heads

$$\text{Hint:}P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}}$$

Solution:

Solutions step 1:(iii)Let D be the event of the occurrence of at least 2 heads.

Accordingly,

$$D = \{HHH, HHT, HTH, THH\}$$

$$\therefore P(D) = \frac{n(D)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(iv)at most 2 heads

$$\text{Hint:}P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}}$$

Solution:

Solutions step 1:(iv)Let E be the event of the occurrence of at most 2 heads.

Accordingly,

$$E = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}$$

(v)no head

$$\text{Hint:}P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}}$$

Solution:

Solutions step 1:(v)Let F be the event of the occurrence of no head.

Accordingly,

$$F = \{TTT\}$$

$\therefore P(F) = \frac{n(F)}{n(S)} = \frac{1}{8}$

(vi)3 tails

Hint:$P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}}$

Solution:

Solutions step 1: (vi)Let G be the event of the occurrence of 3 tails.

Accordingly,

$G = \{TTT\}$

$\therefore P(G) = \frac{n(G)}{n(S)} = \frac{1}{8}$

(vii) exactly two tails

Hint:$P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}}$

Solution:

Solutions step 1:(vii)Let H be the event of the occurrence of exactly 2 tails.

Accordingly,

$H = \{HTT, THT, TTH\}$

$\therefore P(H) = \frac{n(H)}{n(S)} = \frac{3}{8}$

(viii)

Hint:$P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}}$

Solution:

Solutions step 1:(viii)Let I be the event of the occurrence of no tail.

Accordingly,

$I = \{HHH\}$

$\therefore P(I) = \frac{n(I)}{n(S)} = \frac{1}{8}$

(ix)

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}}$

Solution:

Solutions step 1:(ix)Let J be the event of the occurrence of at most 2 tails.

Accordingly,

$J = \{HHH, HHT, HTH, THH, HTT, THT, TTH\}$

$$\therefore P(J) = \frac{n(J)}{n(S)} = \frac{7}{8}$$

8. If $\frac{2}{11}$ is the probability of an event, what is the probability of the event 'not A'.

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}}$

Solution:

Solutions step 1:It is given that $P(A) = \frac{2}{11}$.

Accordingly,

$$P(\text{not } A) = 1 - P(A) = 1 - \frac{2}{11} = \frac{9}{11}$$

9. A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is (i) a vowel (ii) an consonant

Hint:$P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total Number of possible outcomes}}$

Solution:

Solutions step 1:There are 13 letters in the word ASSASSINATION.

$\therefore$ Hence, $n(S) = 13$

(i)There are 6 vowels in the given word.

$\therefore$ Probability (vowel) $= \frac{6}{13}$

(ii)There are 7 consonants in the given word.

$\therefore$ Probability (consonant) $= \frac{7}{13}$

10. In a lottery, person choses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game? [Hint: order of the numbers is not important.]

Hint: ${}^{n}C_{r} = \frac{\lfloor n}{\lfloor r \times \lfloor n-r}$

Solution:

Solutions step 1:Total number of ways in which one can choose six different numbers from 1 to 20

$$= {}^{20}C_6 = \frac{\lfloor 20}{\lfloor 6 \lfloor 20-6} = \frac{\lfloor 20}{\lfloor 6 \lfloor 14} = \frac{20 \times 19 \times 18 \times 17 \times 16 \times 15}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = 38760$$

Hence, there are 38760 combinations of 6 numbers.

Out of these combinations, one combination is already fixed by the lottery committee.

$\therefore$ Required probability of winning the prize in the game $= \frac{1}{38760}$

11. Check whether the following probabilities P(A) and P(B) are consistently defined
(i)$P(A) = 0.5, P(B) = 0.7, P(A \cap B) = 0.6$

(ii)$P(A) = 0.5, P(B) = 0.4, P(A \cup B) = 0.8$

Hint:If E and F are two events such that $E \subset F$, then$P(E) \leq P(F)$.

Solution:

Solutions step 1:(i)$P(A) = 0.5, P(B) = 0.7, P(A \cap B) = 0.6$

It is known that if E and F are two events such that $E \subset F$, then

$P(E) \leq P(F)$.

However, here, $P(A \cap B) > \square(\square)$.

Hence, P(A) and P(B) are not consistently defined.

(ii)$P(A) = 0.5, P(B) = 0.4, P(A \cup B) = 0.8$

Hint:If E and F are two events such that $E \subset F$, then$P(E) \leq P(F)$.

Solution:

Solutions step 1: (ii)$P(A) = 0.5, P(B) = 0.4, P(A \cup B) = 0.8$

It is known that if E and F are two events such that $E \subset F$, then

$P(E) \leq P(F)$.

Here, it is seen that $P(A \cup B) > \square(\square)$ and $P(A \cup B) > \square(\square)$. Hence, P(A) and P(B) are consistently defined.

(i)Fill in the blanks in following table:

	$P(A)$	$P(B)$	$P(A \cap B)$	$P(A \cup B)$
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	...
(ii)	0.35	...	0.25	0.6
(iii)	0.5	0.35	...	0.7

Hint:$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Solution:

Solutions step 1:(i)Here $P(A) = \frac{1}{3}, P(B) = \frac{1}{5}, P(A \cap B) = \frac{1}{15}$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A \cup B) = \frac{1}{3} + \frac{1}{5} + -\frac{1}{15} = \frac{5+3-1}{15} = \frac{7}{15}$$

(ii)

(ii)	0.35	...	0.25	0.6

Hint:$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Solution:

Solutions step 1:Here, $P(A) = 0.35, P(A \cap B) = 0.25, P(A \cup B) = 0.6$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore\ 0.6 = 0.35 + P(B) - 0.25$

$\Rightarrow\ P(B) = 0.6 - 0.35 + 0.25$

$\Rightarrow\ P(B) = 0.5$

(iii)

(iii)	0.5	0.35	...	0.7

Hint:$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Solution:

Solutions step 1:(iii)Here, $P(A) = 0.5, P(B) = 0.35, P(A \cup B) = 0.7$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore\ 0.7 = 0.5 + 0.35 - P(A \cap B)$

$\Rightarrow\ P(A \cap B) = 0.5 + 0.35 - 0.7$

$\Rightarrow\ P(A \cap B) = 0.15$

12. Given $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$. Find P(A or B), if A and B are mutually exclusive events.

Hint:$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Solution:

Solutions step 1:Here, $P(A) = \frac{3}{5}, P(B) = \frac{1}{5}$

For mutually exclusive events A and B,

$P(A \text{ or } B) = P(A) + P(B)$

$\therefore\ P(A \text{ or } B) = \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

13. If E and F are events such that $P(E) = \frac{1}{4}, P(F) = \frac{1}{2}$ and P(E and F)=1/8, find:
(i)P(E or F),

(ii)P(not E and not F).

Hint:$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$

Solution:

Solutions step 1:Here, $P(E) = \frac{1}{4}, P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$

(i)We know that $P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$

$$\therefore\ P(E \text{ or } F) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2 + 4 - 1}{8}$$

(ii)P(not E and not F).
Hint:P(E or F) = P(E) + P(F) − P(E and F)

Solution:

Solutions step 1:(ii)From (i), P(E or F) = P (E ∪ F) = $\frac{5}{8}$

We have $(E \cup F)' = (E' \cap F')$[By De Morgan's law]

$\therefore P(E' \cap F') = P(E \cup F)'$

Now, $P(E \cup F') = 1 - P(E \cup F) = 1 - \frac{5}{8} = \frac{3}{8}$

$P(E' \cap F') = \frac{3}{8}$

Thus P (not E and not F)

14. Events E and F are such that P(not E or not F) = 0.25, State whether E and F are mutually exclusive.

Hint:P(E or F) = P(E) + P(F) − P(E and F)

Solution:

Solutions step 1:It is given that P (not E or not F) = 0.25

i.e.,$P(E' \cup F') = 0.25$

$\Rightarrow P(E \cap F)' = 0.25 \quad [E' \cup F' = (E \cap F)']$

Now, $P(E \cap F) = 1 - P(E \cap F)'$

$\Rightarrow P(E \cap F) = 1 - 0.25$

$\Rightarrow P(E \cap F) = 0.75 \neq 0$

$\Rightarrow E \cap F \neq 0$

Thus, E and F are not mutually exclusive.

Marks for step 1: 2
Difficulty level 1: E

15. A and B are events such that $P(A) = 0.42, P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$. Determine (i) P(not A), (ii) P (not B) and (iii) P(A or B).

Hint:$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Solution:

Solutions step 1:It is given that $P(A) = 0.42, P(B) = 0.48, P(A \text{ and } B) = 0.16$

(i)$P(\text{not } A) = 1 - P(A) = 1 - 0.42 = 0.58$

(ii)$P(\text{not } B) = 1 - P(B) = 1 - 0.48 = 0.52$

(iii)We know that $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$\therefore\ P(A \text{ or } B) = 0.42 + 0.48 - 0.16 = 0.74$$

16. In Class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

Hint:$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Solution:

Solutions step 1:Let A be the event in which the selected student studies Mathematics and B be the event in which the selected student studies Biology.

Now,

$$P(A) = 40\% = \frac{40}{100} = \frac{2}{5}$$

$$P(B) = 30\% = \frac{30}{100} = \frac{3}{10}$$

$$P(A \text{ and } B) = 10\% = \frac{10}{100} = \frac{1}{10}$$

We know that $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$\therefore P(A \text{ or } B) = \frac{2}{5} + \frac{3}{10} - \frac{1}{10} - \frac{6}{10} = 0.6$$

Thus, the probability that the selected student will be studying Mathematics or Biology is 0.6.

In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Hint: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Solution:

Solutions step 1:Let A and B be the events of passing first and second examinations respectively.

Accordingly,

$P(A) = 0.8, P(B) = 0.7$ and $P(A \text{ or } B) = 0.95$

We know that $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\therefore\ 0.95 = 0.8 + 0.7 - P(A \text{ and } B)$

$\Rightarrow\ P(A \text{ and } B) = 0.8 + 0.7 - 0.95 = 0.55$

Thus, the probability of passing both the examinations is 0.55

17. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Hint: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Solution:

Solutions step 1:Let A and B be the events of passing English and Hindi examinations respectively.

Accordingly, $P(A \text{ and } B) = 0.5$, $P(\text{not } A \text{ and not } B) = 0.1$, i.e.,$P(A' \cap B') = 0.1$

$P(A) = 0.75$

Now, $(A \cup B)' = (A' \cap B')$[De Morgan's law]

$\therefore P(A \cup B)' = P(A' \cap B') = 0.1$

$P(A \cup B) = 1 - P(A \cup B)' = 1 - 0.1 = 0.9$

We know that $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\therefore 0.9 = 0.75 + P(B) - 0.5$

$\Rightarrow$ P(B) = 0.9 − 0.75 + 0.5

$\Rightarrow$ P(B) = 0.65

Thus, the probability of passing the Hindi examination is 0.65.

18. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

(i)The student opted for NCC or NSS.

(ii) The student has opted neither NCC nor NSS.

(iii)The student has opted NSS but not NCC.

Hint:P(A or B) = P(A) + P(B) − P(A and B)

Solution:

Solutions step 1:Let A be the event in which the selected student has opted for NCC and B be the event in which the selected student has opted for NSS.

Total number of students = 6

Number of students who have opted for NCC=30

$\therefore\ P(A) = \frac{30}{60} = \frac{1}{2}$

Number of students who have opted for NSS=32

$\therefore P(B) = \frac{32}{60} = \frac{8}{15}$

Number of students who have opted for both NCC and NSS = 24

$\therefore P(A \text{ and } B) = \frac{20}{60} = \frac{2}{5}$

(i)The student opted for NCC or NSS.
Hint:P(A or B) = P(A) + P(B) − P(A and B)

Solution:

Solutions step 1:(i)We know that P(A or B) = P(A) + P(B) − P(A and B)

$\therefore P(A \text{ or } B) = \frac{1}{2} + \frac{8}{15} - \frac{2}{5} = \frac{15 + 16 - 12}{30} = \frac{19}{30}$

Thus, the probability that the selected student has opted for NCC or NSS is $\frac{19}{30}$.

P (not A and not B)

$= P(A' \text{ and } B')$

$= P(A' \cap B')$

$= P(A \cup B)' \left[(A' \cup B') = (A \cup B)' (\text{by De Morgan's law})\right]$

$= 1 - P(A \cup B)$

$= 1 - P(A \text{ or } B)$

$= 1 - \frac{19}{30} = \frac{11}{30}$

(ii)The student has opted neither NCC nor NSS.

Hint:$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Solution:

Solutions step 1:(ii)

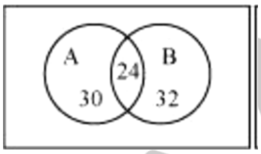
Thus, the probability that the selected students has neither opted for NCC nor NSS is $\frac{11}{30}$.

(iii)The student has opted NSS but not NCC.

Hint:$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Solution:

Solutions step 1:(iii)The given information can be represented by a Venn diagram as it.



It is clear that

Number of students who have opted for NSS but not NCC

$= n(B - A) = n(B) - n(A \cap B) = 32 - 24 = 8$

Thus, the probability that the selected student has opted for NSS but not for NCC $= \frac{8}{60} = \frac{2}{15}$

Miscellaneous Exercise on Chapter 16

1. (i)A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that

(i) all will be blue? (ii) atleast one will be green?

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:Total number of marbles $= 10 + 20 + 30 = 60$

Number of ways of drawing 5 marbles from 60 marbles $=$ $^{60}\square_5$

(i) All the drawn marbles will be blue if we draw 5 marbles out of 20 blue marbles. 5 blue marbles can be drawn from 20 blue marbles in $20_{\square_5}$ ways.

$\therefore$ Probability that all marbles will be blue $= \frac{20\square_5}{60\square_5}$

(ii)atleast one will be green?

Hint: $\mathrm{P(A)} = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:(ii) Number of ways in which the drawn marble is not green $=$ $^{(20+10)}\square_5$

$\therefore$ Probability that no marble is green $= \frac{30\square_5}{60\square_5}$

Probability that at least one marble is green $= 1 - \frac{30\square_5}{60\square_5}$

2. 4 cards are drawn from a well-shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?

Hint: $\mathrm{P(A)} = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:Number of ways of drawing 4 cards from 52 cards $=$ $^{52}C_4$

In a deck of 52 cards, there are 13 diamonds and 13 spades.

$\therefore$ Number of ways of drawing 3 diamonds and one spade $=$ $^{13}C_5 \times {}^{13}C_1$

Thus, the probability of obtaining 3 diamonds and one spade $= \frac{^{13}C_5 \times {}^{13}C_1}{^{52}C_4}$

(i)A die has two faces each with number ‘1’, three faces each with number ‘2’ and one face with number 3’. If die is rolled once, determine

(i) P(2)

(ii) P(1 or 3)

(iii) P(not3)

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:Total number of faces = 6

(i) Number faces with number ‘2’= 3

$$\therefore \square(2) = \frac{3}{6} = \frac{1}{2}$$

(ii)

Questions body:P(1 or 3)
Full marks: 1

Overall difficulty level: E
Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:(ii) $P(1 \text{ or } 3) = P(\text{not } 2) = 1 - P(2) = 1 - \frac{1}{2} = \frac{1}{2}$

(iii)P(not3)
Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:(iii) Number of faces with number 3’ = 1

$\therefore P(3) = \frac{1}{6}$

Thus, $P(\text{not } 3) = 1 - P(3) = 1 - \frac{1}{6} = \frac{5}{6}$

3. (i)In a certain lottery, 10,000 tickets are sold, and ten equal prizes are awarded. What is the probability of not getting a prize if you buy?

(a) one ticket

(b) two tickets

(c) 10 tickets?

Hint: P’=1-P

Solution:

Solutions step 1:Total number of tickets sold $= 10{,}000$

Number prizes awarded $= 10$

(i) If we buy one ticket, then

$P\text{ (getting a prize)} = \frac{10}{10000} = \frac{1}{1000}$

$P\text{ (not getting a prize)} = 1 - \frac{1}{1000} = \frac{999}{1000}$

(ii)two tickets
Hint: P’=1-P

Solution:

Solutions step 1:(ii) If we buy two tickets, then

Number of tickets not awarded $= 10{,}000 - 10 = 9990$

$P\text{ (not getting a prize)} = \frac{{}^{9990}C_2}{{}^{1000}C_2}$

(iii)10 tickets?
Hint: P’=1-P

Solution:

Solutions step 1:(iii) If we buy 10 tickets, then

P (not getting a prize) $= \frac{{}^{9990}C_2}{{}^{1000}C_2}$

4. Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that

(a) you both enter the same sections?

(b) you both enter the different sections?

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:My friend and 1 are among the 100 students.

Total number of ways of selecting 2 students out of 100 students = ${}^{100}C_2$

(a) The two of us will enter the same section if both of us are among 40 students or among 60 students.

∴ Number of ways in which both of us enter the same section = ${}^{40}C_2 + {}^{60}C_2$

∴ Probability that both of us enter the same section

$$= \frac{{}^{40}C_2 + {}^{40}C_2}{{}^{100}C_2} = \frac{\frac{\lfloor 40}{\lfloor 2 \lfloor 38} + \frac{\lfloor 60}{\lfloor 2 \lfloor 58}}{\frac{\lfloor 100}{\lfloor 2 \lfloor 98}} = \frac{(39 \times 40) + (59 \times 60)}{99 \times 100} = \frac{17}{33}$$

(ii)you both enter the different sections?

Hint: P’=1-P

Solution:

Solutions step 1:(b) P (we enter different sections)

$= 1 - \text{P}$ (we enter the same section)

$= 1 - \frac{17}{36} = \frac{16}{33}$

5. Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Hint: $\text{P(A)} = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:Let $\text{L}_1, \text{L}_2, \text{L}_3$ be three letters and E_1, E_2, and E_3 be their corresponding envelops respectively.

There are 6 ways of inserting 3 letters in 3 envelops. These are as follows:

□$_1$□$_1$, □$_2$□$_3$, □$_3$□$_2$
□$_2$□$_2$, □$_1$□$_3$, □$_3$□$_1$
□$_3$□$_3$, □$_1$□$_2$, □$_2$□$_1$
□$_1$□$_1$, □$_2$□$_2$, □$_3$□$_3$
□$_1$□$_2$, □$_2$□$_3$, □$_3$□$_1$
□$_1$□$_3$, □$_2$□$_1$, □$_3$□$_2$

There are 4 ways in which at least one letter is inserted in a proper envelope.

Thus, the required probability is $\frac{4}{6} = \frac{2}{3}$

6. A and B are two events such that $P(A) = 0.54, P(B) = 0.69$ and $P(A \cap B) = 0.35$. Find

(i) □(□ ∩ □)

(ii) □(□′ ∩ □′)

(iii) □(□ ∩ □′)

(iv) □(□ ∩ □′)

Hint:P(A or B) = P(A) + P(B) – P(A and B)

Solution:

Solutions step 1:It is given that □(□) = *0.54*, □(□) = *0.69*, □(□ ∩ □) = *0.35*

(i) We know that □(□ ∪ □) = □(□) + □(□) − □(□ ∩ □)

∴ □ (□ ∪ □) = *0.54* + *0.69* − *0.35* = *0.88*

(ii)$P(A' \cap B')$
Hint:P(A or B) = P(A) + P(B) – P(A and B)

Solution:

Solutions step 1:(ii) □′ ∩ □′ = (□ ∪ □)′ [by De Morgan's law]

∴ □(□′ ∩ □′) = □(□ ∪ □)′ = *1* − □(□ ∪ □) = *1* − *0.88* = *0.12*

(iii)$P(A \cap B')$
Hint:P(A or B) = P(A) + P(B) – P(A and B)

Solution:

Solutions step 1:(iii) □(□ ∩ □′) = □(□) − □(□ ∩ □) = *0.54* − *0.35* = *0.19*

(iv)$P(B \cap A')$
Hint:P(A or B) = P(A) + P(B) – P(A and B)

Solution:

Solutions step 1:(iv) We know that: □(□ ∩ □′) = □(□) − □(□ ∩ □)

⇒ □(□ ∩ □′) / □(□) = □(□) / □(□) − □(□ ∩ □) / □(□)

$$\therefore \square(\square \cap \square') = \square(\square) - \square(\square \cap \square)$$

$$\therefore \square(\square \cap \square') = 0.69 - 0.35 = 0.34$$

7. From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S.No.	Name	Sex	Age in years
	Harish	M	30
	Rohan	M	33
	Sheetal	F	46
	Alis	F	28
	Salim	M	41

A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years?

Hint:P(A or B) = P(A) + P(B) – P(A and B)

Solution:

Solutions step 1:event in which the spokesperson will be a male and F be the event in which the spokesperson will be over 35 years of age.

Accordingly, $P(E) = \frac{3}{5}$ and $P(F) = 2/5$

Since there is only one male who is over *35* years of age,

$$P(E \cap F) = \frac{1}{5}$$

We know that: $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$P(E \cup F) = \frac{3}{5} + \frac{2}{5} - \frac{1}{5} = \frac{4}{5}$$

Thus, the probability that the spokesperson will either be a male or over 35 years of age is $\frac{4}{5}$.

8. (i)If 4-digit numbers greater than 5,000 are randomly formed from the digits $0, 1, 3, 5$, and 7, what is the probability of forming a number divisible by 5 when,

(i) the digits are repeated?

(ii) the repetition of digits is not allowed?

Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:(i) When the digits are repeated

Since four-digit numbers greater than 5000 are formed, the leftmost digit is either 7 or 5. The remaining 3 places can be filled by any of the digits $0, 1, 3, 5$, or 7 as repetition of digits is allowed.

Total number of 4-digit numbers greater than $5000 = 2 \times 5 \times 5 \times 5 - 1$

$= 250 - 1 = 249$

[In this case, 5000 cannot be counted, so 1 is subtracted]

A number is divisible by 5 if the digit at its units place is either 0 or 5.

$\therefore$ Total number of 4-digit numbers greater than 5000 that are divisible by 5

$$= 2 \times 5 \times 5 \times 2 - 1 = 100 - 1 = 99$$

Thus, the probability of forming a number divisible by 5 when the digits are repeated is $= \frac{99}{249} = \frac{33}{83}$

(ii)the repetition of digits is not allowed?

Hint: Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:(ii) When repetition of digits is not allowed

The thousands place can be filled with either of the two digits 5 or7.

The remaining 3 places can be filled with any of the remaining 4 digits.

$\therefore$ Total number of 4-digit numbers greater than $5000 = 2 \times 4 \times 3 \times 2 = 48$

When the digit at the thousands place is 5, the units place can be filled only with 0 and the tens and hundreds of places can be filled with any two of the remaining 3 digits.

Here, number of 4-digit numbers starting with 5 and divisible by 5

$= 3 \times 2 = 6$

When the digit at the thousands place is 7, the units place can be filled in two ways (0 or 5) and the tens and hundreds of places can be filled with any two of the remaining 3 digits. Here, number of 4-digit numbers starting with 7 and divisible by 5

$= 1 \times 2 \times 3 \times 2 = 12$

$\therefore$Total number of 4-digit numbers greater than 5000 that are divisible by 5

$= 6 + 12 = 18$

Thus, the probability of forming a number divisible by 5 when the repetition of digits is not allowed is $\frac{18}{48} = \frac{3}{8}$

9. The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

Hint: Hint: $P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$

Solution:

Solutions step 1:The number lock has4 wheels, each labelled with ten digits i.e., from *0* to *9*. Number of ways of selecting 4 different digits out of the *10* digits = ${}^{10}\square_{4}$

Now, each combination of *4* different digits can be arranged in *4*! Ways.

Number of four digits with no repetitions = $4!\ \ {}^{10}\square_{4} = 5040$

There is only one number that can open the suitcase.

Thus, the required probability is $\frac{1}{5040}$