GCSE

Rewarding Learning

CCEA GCSE Specimen Assessment Materials for

Further Mathematics

Version 2: 01 March 2019

For first teaching from September 2017
For first assessment in Summer 2018
For first award in Summer 2019
Subject Code: 2330

Foreword

CCEA has developed new specifications which comply with criteria for GCSE qualifications. The specimen assessment materials accompanying new specifications are provided to give centres guidance on the structure and character of the planned assessments in advance of the first assessment. It is intended that the specimen assessment materials contained in this booklet will help teachers and students to understand, as fully as possible, the markers' expectations of candidates' responses to the types of tasks and questions set at GCSE level. These specimen assessment materials should be used in conjunction with CCEA's GCSE Further Mathematics specification.

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GCSE Further Mathematics
Specimen Assessment Materials

Contents

Subject Code	2330
QAN	603/1054/6

GCSE_FMATHS_SAMs_V2_01.03.19

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Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2018

Further Maths

Unit 1

Pure Mathematics

[CODE]

SPECIMEN PAPER

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in blue or black ink only. **Do not write with a gel pen.**
All working should be clearly shown in the spaces provided since marks may be awarded for partially correct solutions.
Answer **all fifteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on page 4.

For Examiner's use only	
Question Number	**Marks**
1	
2	
3	
4	
5	
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8	
9	
10	
11	
12	
13	
14	
15	
Total Marks	

Formula Sheet

PURE MATHEMATICS

Quadratic equations: If $ax^2 + bx + c = 0$ $(a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Differentiation: If $y = ax^n$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = nax^{n-1}$

Integration: $\displaystyle\int ax^n \,\mathrm{d}x = \frac{ax^{n+1}}{n+1} + c$ $(n \neq -1)$

Logarithms: If $a^x = n$ then $x = \log_a n$

$\log (ab) = \log a + \log b$

$\log \left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det \mathbf{A} = ad - bc$

and $\mathbf{A}^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $(ad - bc \neq 0)$

1 The matrices **A**, **B**, **C** and **D** are defined by

$$\mathbf{A} = \begin{bmatrix} 2 & -3 \\ 4 & 7 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -2 & 11 \\ 1 & -3 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 8 \\ -5 \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} 7 & -2 \end{bmatrix}$$

Find

(i) $\mathbf{A}^2$

Answer _____ [2]

(ii) $\mathbf{B} - 2\mathbf{A}$

Answer _____ [2]

(iii) CD

Answer _____ [2]

2 A function is defined by $f(x) = x^2 - 12x + 40$

(i) Use the method of completing the square to rewrite $f(x)$ in the form $(x + a)^2 + b$.

Answer _____ [2]

(ii) Hence find

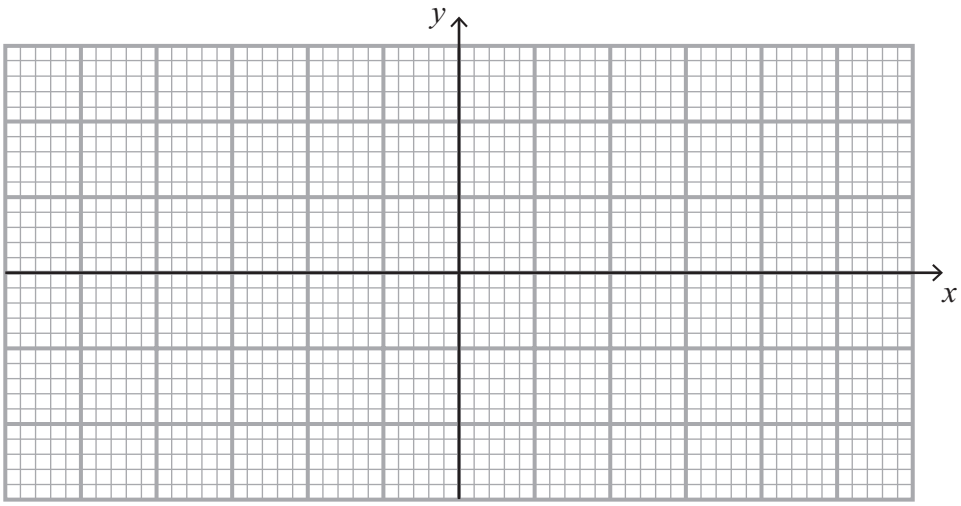
(a) the minimum value of $f(x)$.

Answer _____ [1]

(b) the value of x for which this minimum occurs.

Answer _____ [1]

3 **(a)** Sketch the graph of $y = \sin x$ for $-180° \leqslant x \leqslant 180°$ on the grid below.



[2]

(b) Solve the equation

$\sin(3x + 24°) = -0.5$ for $-60° \leqslant x \leqslant 60°$

Answer _____ ° [6]

4 If $y = \frac{5}{6}x^3 - x + \frac{4}{3x^2}$

find $\frac{d^2y}{dx^2}$ giving your answer in its simplest form.

Answer _____ [5]

5 Solve the inequality $2x^2 + x - 15 < 0$

Answer _____ [4]

6 The matrix **P** is defined by

$$\mathbf{P} = \begin{bmatrix} 12 & -5 \\ 6 & 3 \end{bmatrix}$$

(i) Find the matrix $\mathbf{P}^{-1}$ the inverse of **P.**

Answer _____ [2]

(ii) Hence, **using a matrix method**, solve the simultaneous equation

$12x - 5y = 13$

$6x + 3y = -4.5$

Answer x _____, y = _____ [4]

7 Solve the equation $7^{3x-4} = 4$

Answer _____ [4]

8 The gradient function of a curve is given by $\frac{dy}{dx} = 6x^2 + \frac{1}{x^4} - 3$

The curve passes through the point (–1, 4).

Find the equation of the curve.

Answer _____ [6]

9 The share price P, in pounds, of a company since 2012 can be modelled by

$$P = 3 + 11t - 2t^2$$

where t is the time in years since 2012.

Using calculus, work out the **maximum** share price achieved by the company since 2012, justifying your answer.

Answer £ _____ [6]

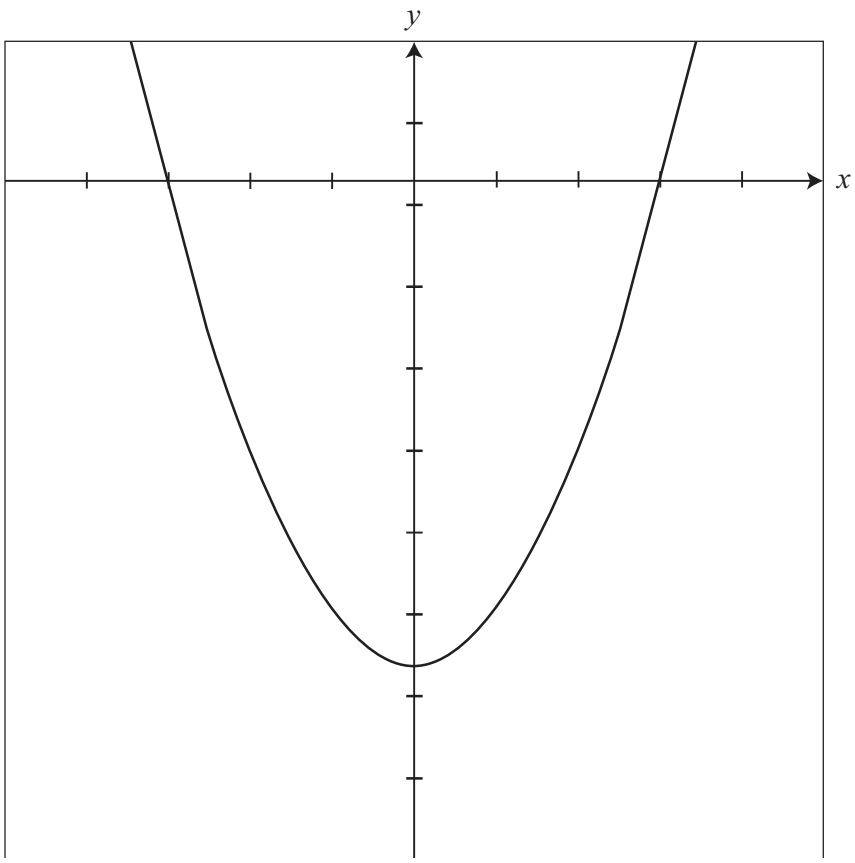
10 Show that there are two points on the curve

$$y = 2x^3 - 9x^2 - 14x + 2$$

at which the tangents to the curve are parallel to the straight line $y = 10x - 3$

[6]

11 The curve $y = 2x^2 - 18$ is shown below.



Work out the area bounded by the curve, the y-axis, the x-axis and the line $x = 2$

Answer _____ [5]

12 Alison had twelve £2 coins, eight £1 coins and twenty 20p coins. The total mass of the coins was 320 g.

Let x, y and z represent the masses, in grams, of a £2 coin, a £1 coin and a 20p coin respectively.

(i) Show that x, y and z satisfy the equation

$$3x + 2y + 5z = 80$$

[1]

Brian had twenty-five £2 coins, thirty £1 coins and fifteen 20p coins. The total mass of these coins was 660 g.

(ii) Show that x, y and z also satisfy the equation

$$5x + 6y + 3z = 132$$

[1]

Christine had eighteen 50p coins, twelve £1 coins and twenty-seven 20p coins.
The total mass of these coins was 393 g.

The mass of a 50p coin is $\frac{2}{3}$ that of a £2 coin.

(iii) Show that x, y and z also satisfy the equation

$$4x + 4y + 9z = 131$$

[2]

(iv) Solve these equations to find the masses of all four coins, i.e. a £2 coin, a £1 coin, a 50p coin and a 20p coin. Show clearly each stage of your solution.

Answer mass of £2 coin = _____ g mass of £1 coin = _____ g

mass of 50p coin = _____ g mass of 20p coin = _____ g [7]

David had twenty £2 coins, some of which were counterfeit. Each counterfeit coin has a mass of 10 g. The total mass of David's coins was 228 g.

(v) Calculate how many counterfeit coins David had.

Answer _____ [2]

13 Various members of the cat family were examined and for each animal the mass of its body, m, was compared with the mass of its brain, b. The results are given in the table below:

Animal	Body mass m (g)	Brain mass b (g)		
Domestic cat	2500	23.8		
Wildcat	4600	34.6		
Bobcat	15 800	73.5		
Puma	50 100	148.8		
Lion	126 000	261.4		

It is believed that a relationship of the form

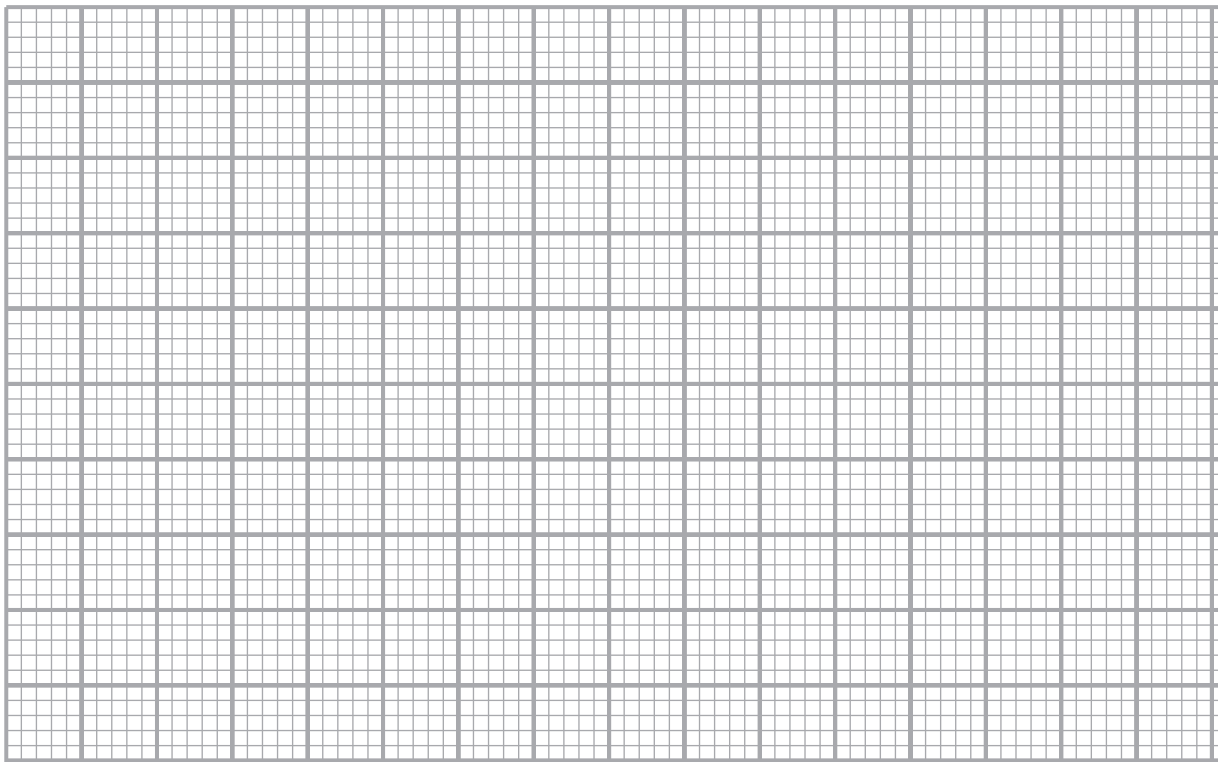
$$b = km^n$$

exists, where k and n are constants.

(i) Verify that a relationship of the form $b = km^n$ exists by drawing a suitable straight line graph on the grid below.

Show clearly the values used, correct to 3 decimal places, in the table on page 21.

Hence find the values of k and n, correct to 2 decimal places.



Answer k = _____, n = _____ [11]

Leopards and Siberian tigers are also members of the cat family.

Use the formula $b = km^n$ with your values for k and n to

(ii) calculate the mass of a leopard's brain, given that its body mass is 39 800 g.

Give your answer to 1 decimal place.

Answer _____ g [1]

(iii) calculate the body mass of a Siberian tiger, given that its brain mass is 335.2 g.

Give your answer to 3 significant figures.

State any assumption that you make.

Answer _____ g [2]

Assumption _____

_____ [1]

14 A curve is defined by the equation $y = ax^2 + bx + c$ where a, b and c are constants.

The curve crosses the y-axis at the point $(0, -20)$.

At this point the gradient of the curve is 3.

The curve crosses the x-axis at the point $(-4, 0)$.

Find the values of a, b and c.

Answer $a =$ _____, $b =$ _____, $c =$ _____ [5]

15 (i) Expand and simplify the expression

$$(x + 2)(x - 5)(3x + 10)$$

Answer _____ [3]

(ii) Hence, simplify fully the algebraic expression

$$\frac{(x + 2)(x - 5)(3x + 10) - 3x(x^2 - 20)}{x - 10}$$

Answer _____ [4]

THIS IS THE END OF THE QUESTION PAPER

Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2019

Further Mathematics

Unit 2

Mechanics

[CODE]

SPECIMEN PAPER

TIME

1 hour

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Answer **all seven** questions.

INFORMATION FOR CANDIDATES

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Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on page **28.**

For Examiner's use only	
Question Number	Marks
1	
2	
3	
4	
5	
6	
7	
Total Marks	

FORMULA SHEET

MECHANICS

Quadratic equations: If $ax^2 + bx + c = 0$ $(a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors: Magnitude of $x\mathbf{i} + y\mathbf{j}$ is given by $\sqrt{x^2 + y^2}$

Angle between $x\mathbf{i} + y\mathbf{j}$ and $\mathbf{i}$ is given by $\tan^{-1}\left(\dfrac{y}{x}\right)$

Uniform Acceleration: $v = u + at$ $\quad s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$ $\quad s = ut + \frac{1}{2}at^2$

where u is initial velocity, v is final velocity, a is acceleration, t is time, s is change in displacement

Newton's Second Law: $F = ma$

where F is resultant force, a is acceleration, m is mass

Take g = 10 m/s^2 throughout

1 A particle has an initial velocity of 3 m/s and travels with an acceleration of 0.5 m/s^2 for 10 seconds.

Calculate:

(i) the velocity of the particle after the 10 seconds.

Answer _____ m/s [2]

(ii) the distance travelled by the particle in the 10 seconds.

Answer _____ m [2]

2 The vector $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j}$. Calculate:

(i) $|\mathbf{a}|$

Answer _____ [2]

(ii) the acute angle the vector **a** makes with the **i**-direction.

Answer _____° [2]

3 A car travels at a constant velocity of 20 m/s for 10 seconds.
It then accelerates at 1.2 m/s^2 for 5 seconds to reach a maximum velocity.
It continues at this maximum velocity for 20 seconds before decelerating to rest in a **further** T seconds.

(i) Calculate the maximum velocity of the car.

Answer _____ m/s [2]

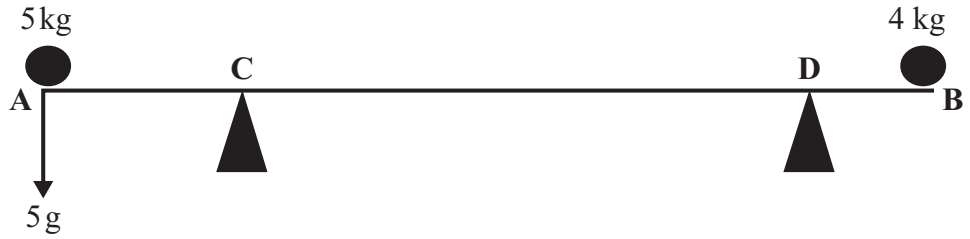
(ii) Draw a velocity–time graph of the journey.

[3]

(iii) Given that the total distance travelled by the car is 1160 metres, calculate the value of T.

Answer _____ [3]

4 A uniform plank AB, of length 7 m and mass 3 kg, is supported by 2 pivots at C and D where AC = 2 m and DB = 1.5 m.
A boulder of mass 5 kg is placed at A and a boulder of mass 4 kg is placed at B as shown in the diagram below.



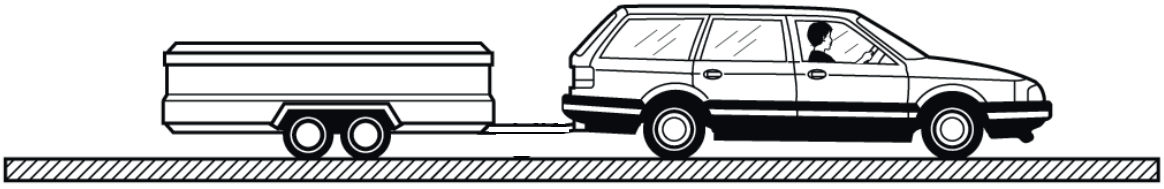
The force at A is shown on the diagram.

(i) Mark on the diagram above all the other forces acting on the plank. [2]

(ii) Calculate the reactions at C and D.

Reaction at C = _____ N, Reaction at D = _____ N [5]

5 A car of mass 1200 kg is towing a trailer of mass 500 kg by means of a light horizontal tow bar along a straight horizontal road. The car and the trailer accelerate uniformly from rest. The tractive force of the car's engine is 1925 N.
The resistance to motion of the car is 0.8 N/kg and the resistance to motion of the trailer is 0.4 N/kg.



(i) Calculate the acceleration of the car and the trailer.

Answer _____ m/s^2 [3]

(ii) Calculate the tension in the tow bar.

Answer _____ N [2]

After travelling for 20 seconds, the tow bar breaks.

(iii) Calculate the velocity of the trailer when the tow bar breaks.

Answer _____ m/s [2]

(iv) Calculate the deceleration of the trailer, given that the resistance to motion of the trailer is unchanged.

Answer _____ m/s^2 [3]

(v) Calculate the additional distance the trailer travels before coming to rest.

Answer _____ m [2]

6 Three forces act at a point **A**. Force **P** is a horizontal force, force **Q** is a vertical force and a third force of 20 N acts at **A** at an angle of 50° to the horizontal as shown in **Fig. 1** below.

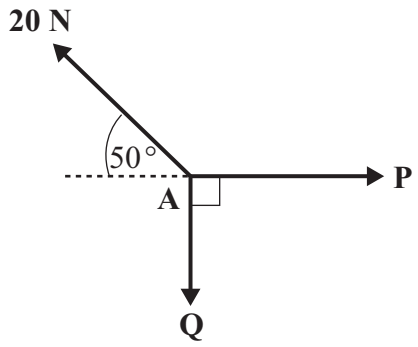


Fig. 1

The **resultant** is a single force of 10 N acting at an angle of 70° to the horizontal as shown in **Fig. 2** below.

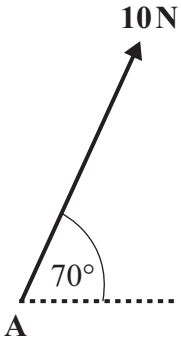
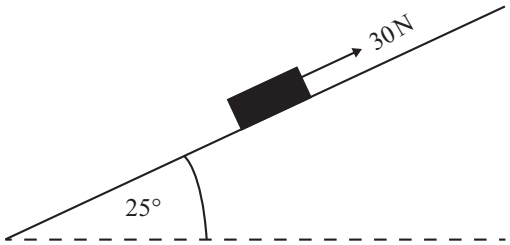


Fig. 2

Calculate the values of the forces **P** and **Q**.

Answer **P** = _____ N, **Q** = _____ N [5]

7 A block of mass 4 kg is pulled up a slope inclined to the horizontal at an angle of 25° by a force of 30 N as shown in the diagram below. The resistance of motion due to friction is 10 N.



(i) Mark on the diagram above all the other forces acting on the block. [2]

(ii) Calculate the acceleration of the block up the slope. Give your answer correct to 2 decimal places.

Answer _____ m/s^2 [3]

The 30 N force is removed and the block continues to move up the slope before coming to instantaneous rest at a point A.
It then starts to move down the slope.
The resistance to motion caused by friction remains constant at 10 N.

(iii) By redrawing the diagram, or otherwise, calculate how far the block will have moved 2 seconds after leaving the point A.
Give your answer to 2 decimal places.

Answer _____ m [5]

THIS IS THE END OF THE QUESTION PAPER

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Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2019

Further Mathematics

Unit 3

Statistics

[CODE]

SPECIMEN PAPER

TIME

1 hour.

INSTRUCTIONS TO CANDIDATES

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You must answer the questions in the spaces provided.
Complete in blue or black ink only. Do not write with a gel pen.
All working should be clearly shown since marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all seven** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 50.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
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The Formula Sheet is on pages 44 and 45.

For Examiner's use only	
Question Number	Marks
1	
2	
3	
4	
5	
6	
7	
Total Marks	

FORMULA SHEET

STATISTICS

Statistical measures: Mean $= \frac{\Sigma fx}{\Sigma f}$

Standard deviation $= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$

where $\bar{x}$ is the mean

Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

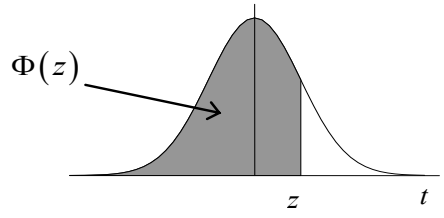
$r = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$

NORMAL PROBABILITY TABLE

Table of $\Phi(z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	(ADD) 1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	8	11	15	19	23	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	21	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	6	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	6	8	11	14	17	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	18	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	19
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	16
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	3	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990	0	0	0	0	0	0	0	0	0

The function tabulated is $\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} \, \mathrm{d}t$. $\Phi(z)$ is the probability that a random variable having a Normal frequency density, with mean zero and variance unity, will be less than z.



1 A dentist records the times, to the nearest minute, spent with patients.
The table below shows a summary of the times.

Time (Minutes)	Frequency			
5 – 9	5			
10 – 14	17			
15 –19	35			
20 – 24	28			
25 – 29	11			
30 – 34	4			

(i) Calculate an estimate of the mean time.

Answer _____ minutes [2]

(ii) Calculate an estimate of the standard deviation of the times.

Answer _____ minutes [3]

2 A total of 120 customers visited a Chippy on Friday.

76 customers bought chips
48 customers bought fish
28 customers did not buy fish or chips

Using a Venn diagram, or otherwise, answer the following questions.

(i) What is the probability that a customer, selected at random, bought fish **and** chips?

Answer _____ [2]

(ii) Given that a customer, selected at random, bought fish, what is the probability that this customer also bought chips?

Answer _____ [2]

3 A shop sold a large number of Christmas trees.
The heights of the trees were normally distributed with mean 1.9 m and standard deviation 0.15 m.
Trees were graded as small, medium or large depending on their height.
Trees measuring over 2.08 m were graded as large.
Find the probability that a tree, chosen at random, was graded as large.

Answer _____ [4]

4 Nine students did a test.
The mean of their nine results was 72% and the standard deviation was 8%.
One student did the test at a later date and scored 76%.

(i) Calculate the mean of all 10 results.

Answer _____ % [2]

(ii) Calculate the standard deviation of all 10 results.

Answer _____ % [4]

5 Out of the first 20 people who voted at a polling station,

4 voted for X, 6 voted for Y and 10 voted for Z.
Two of these people were chosen at random and asked how they voted.

Using a tree diagram, or otherwise, calculate the probability that

(i) they voted for the same person.

Answer _____ [2]

(ii) they voted differently.

Answer _____ [2]

(iii) at least one voted for Z.

Answer _____ [3]

(iv) Given that the first person had voted for X, what is the probability that the second person also voted for X?

Answer _____ [2]

6 The table shows the marks awarded by two judges to the first nine competitors in a dancing competition.

Judge X	8.6	6.6	7.4	6.8	5.4	8.4	9.0	7.8	6.2
Judge Y	7.8	6.0	6.6	6.9	5.2	9.2	8.6	7.6	6.0

(i) Write down the rank orders for Judge X and Judge Y.

[2]

(ii) Calculate Spearman's coefficient of rank correlation.

Answer _____ [4]

(iii) Interpret your answer to part **(ii)**.

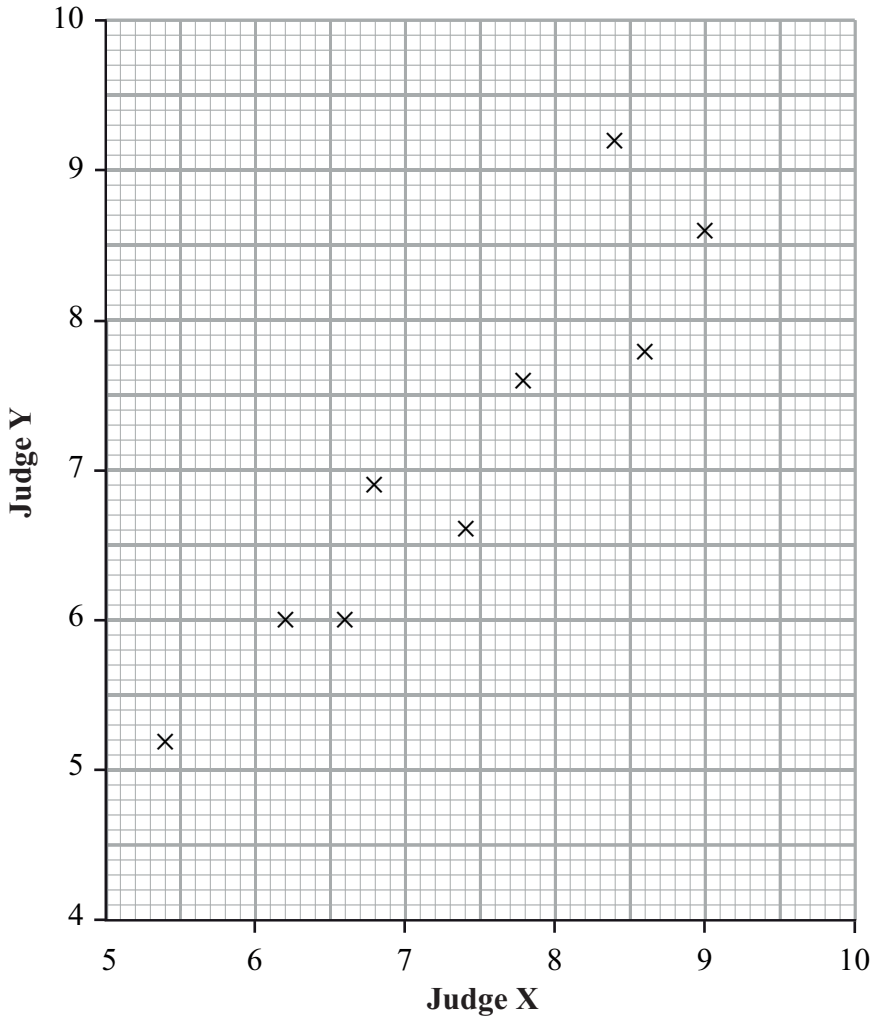
Answer _____ [1]

(iv) Calculate the mean mark for each judge.

Mean for Judge X _____

Mean for Judge Y _____ [1]

The data from the table are plotted on the graph below.



(v) Draw your line of best fit on the graph above. [2]

(vi) Determine the equation of the line of best fit which you have drawn.

Answer _____ [3]

7 **(i)** Using Pascal's triangle, write out the expansion of $(p + q)^5$.

Answer _____ [2]

(ii) A large box contains apples. The probability that an apple, picked at random, is bad equals 0.05. Mary picks 5 apples, chosen at random, from the box.

Find the probability that

(a) none of the apples are bad. Give your answer correct to 2 decimal places.

Answer _____ [3]

(b) at least 2 of the apples are bad. Give your answer correct to 2 decimal places.

Answer _____ [4]

THIS IS THE END OF THE QUESTION PAPER

Rewarding Learning

General Certificate of Secondary Education
2019

Centre Number

Candidate Number

Further Mathematics

Unit 4

Discrete and Decision Mathematics

[CODE]

SPECIMEN PAPER

TIME

1 hour

INSTRUCTIONS TO CANDIDATES

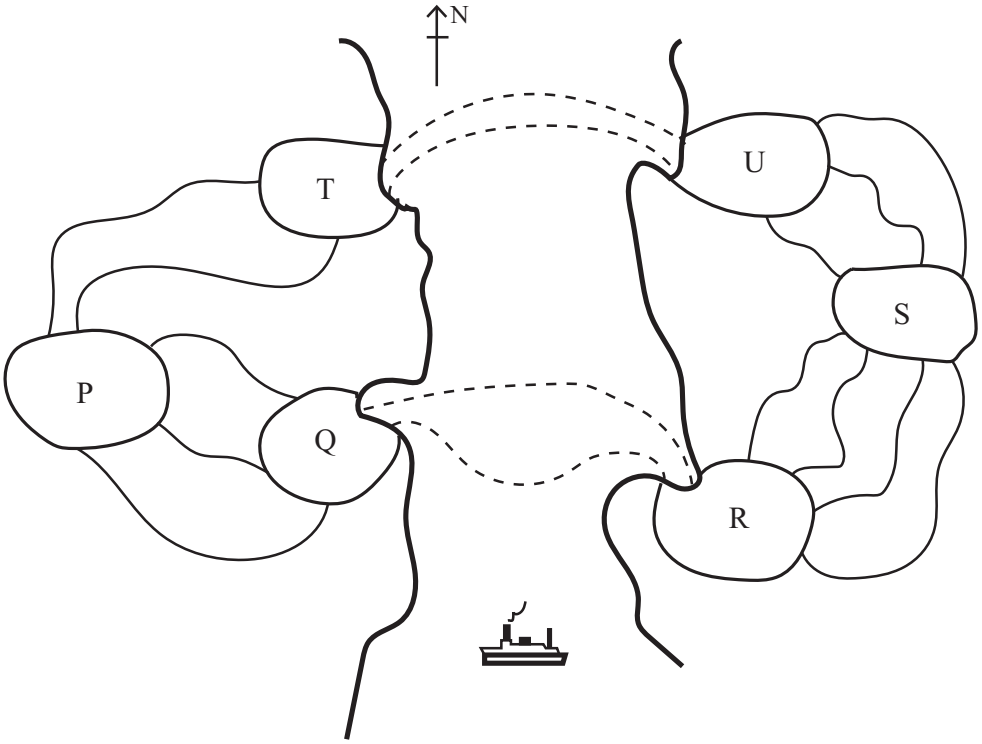
Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in blue or black ink only. **Do not write with a gel pen.**
All working should be clearly shown in the spaces provided since marks may be awarded for partially correct solutions.
Answer **all seven** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 50.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.

For Examiner's use only	
Question Number	Marks
1	
2	
3	
4	
5	
6	
7	
Total Marks	

1 A company must deliver a package from P to S.
Three roads connect P to Q.
Two shipping lanes connect Q to R.
Four roads connect R to S.
Two roads connect P to T, two shipping lanes connect T to U, and three roads connect U to S.
These routes are illustrated in the figure below.



Key
_____ road
_ _ _ _ _ shipping lane

How many different routes may the package follow?

Answer _____ [2]

2 By completing the truth tables below, prove that the Boolean expressions
(p **and** q) **or** r and (p **or** r) **and** (q **or** r) are equivalent.

p	q	r	p **and** q	(p **and** q) **or** r
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

p	q	r			
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

[4]

3 Emergency supplies are to be dropped by parachute to refugees.
The supplies consist of boxes of medical supplies and boxes of food packed into containers which are to be dropped.

Each box of medical supplies has a mass of 40 kg and costs £100 to fill.
Each box of food has a mass of 50 kg and costs £50 to fill.
x boxes of medical supplies and y boxes of food are packed into each container.

(i) The selection of the boxes of medical supplies and boxes of food for each container is subject to five restrictions:

1 Each container has a maximum load of 800 kg.

(a) Show that $4x + 5y \leqslant 80$

[1]

2 The budget for each container is £1000.

(b) Show that $2x + y \leqslant 20$

[1]

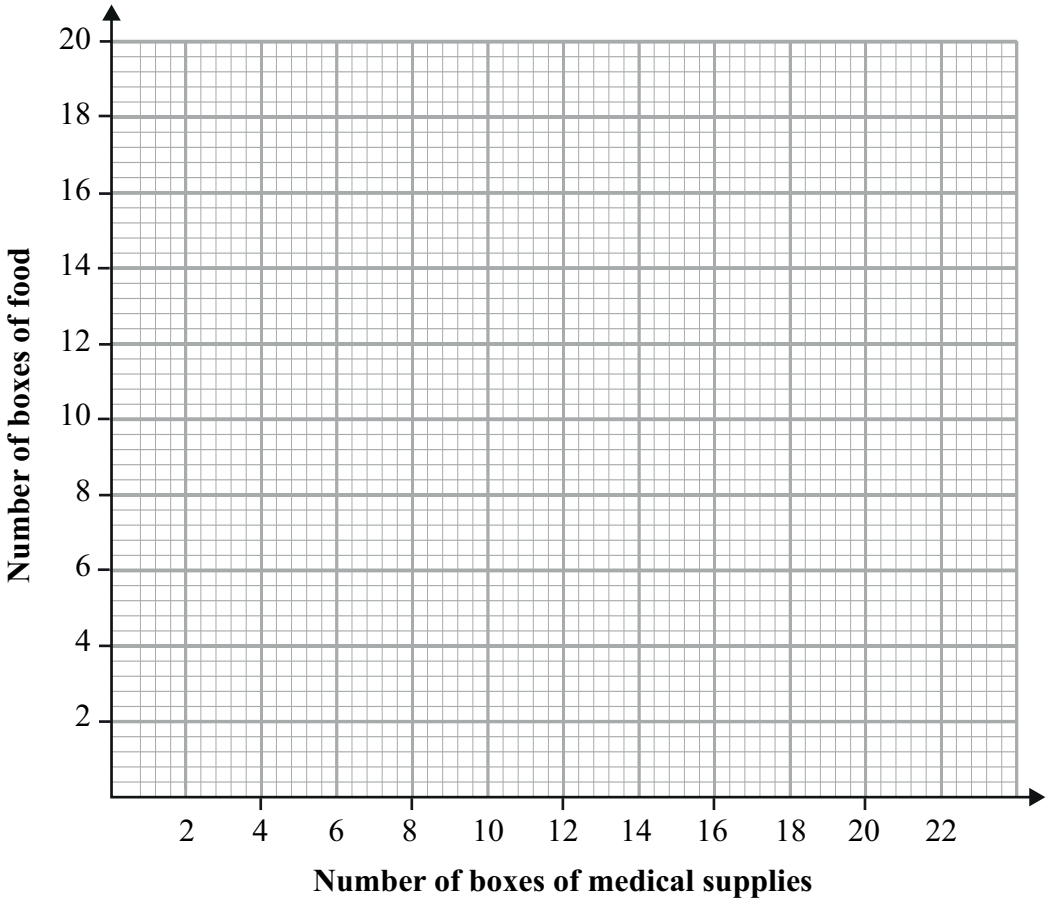
3 There must be at least 8 boxes in total of medical supplies and food; i.e. $x + y \geqslant 8$

4 There must be at least 2 boxes of medical supplies in each container; i.e. $x \geqslant 2$

5 There must be at least 4 boxes of food; i.e. $y \geqslant 4$

(ii) Illustrate the five inequalities by a suitable diagram on the graph below.

Identify the region with the letter R containing the set of points which satisfy all five inequalities. [3]



(iii) (a) Use your solution set to identify the maximum number of boxes that will go into a container.

Answer _____ [1]

(b) Given that the maximum number of boxes found in part **(iii)** (**a**) will be used to fill the container, calculate the minimum cost of filling these boxes.

Answer £_____ [2]

(iv) It takes one person to fill a box of medical supplies and two people to fill a box of food.

Use your solution set to identify the minimum number of people required to fill a container.

Answer _____ people [3]

4 **(i)** By drawing a truth table below prove that the statements
not p **or not** q and **not** (p **and** q) are equivalent.

[4]

The statements "Joe likes cheese" and "Eleanore wears a ring" are true.

(ii) Using the result of **(i)**, write down a simpler statement that is equivalent to the following:

"It is not true that either Joe doesn't like cheese or Eleanore doesn't wear a ring."

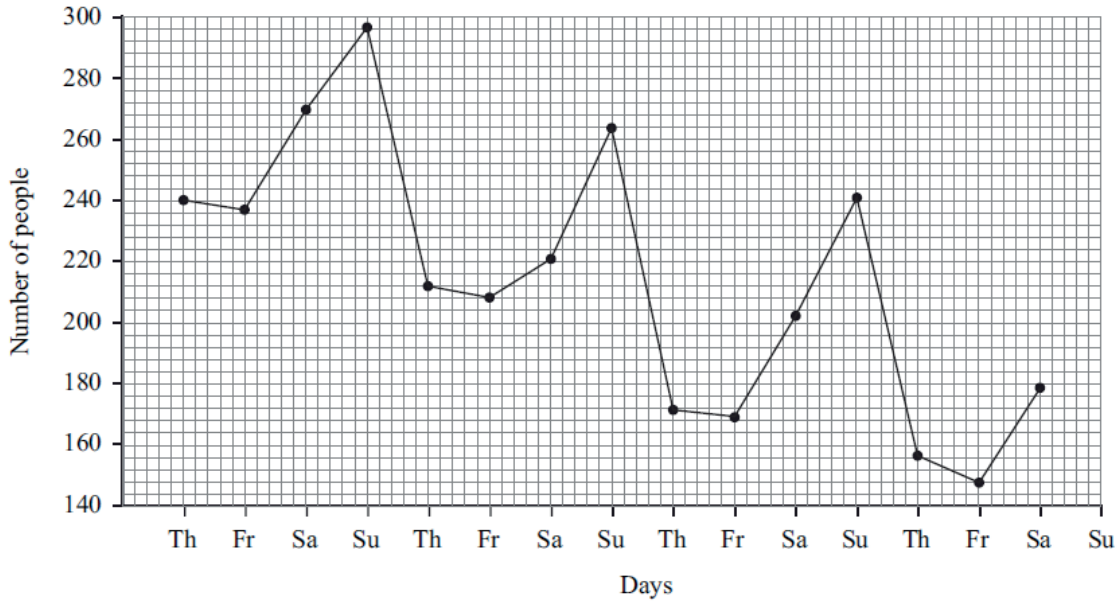
You should show all logical steps taken to make your statement.

Answer _____ [3]

5 Owen organises cycle tours at an outward pursuit centre on four days of each week. The numbers of people taking part in these tours in July 2009 are summarised in the table below.

	Thursday	**Friday**	**Saturday**	**Sunday**
Week 1	240	237	270	297
Week 2	212	208	221	263
Week 3	172	168	202	241
Week 4	157	148	179	

These data have been plotted in the graph below.



(i) Calculate appropriate moving averages using the information below to smooth the data.[2]

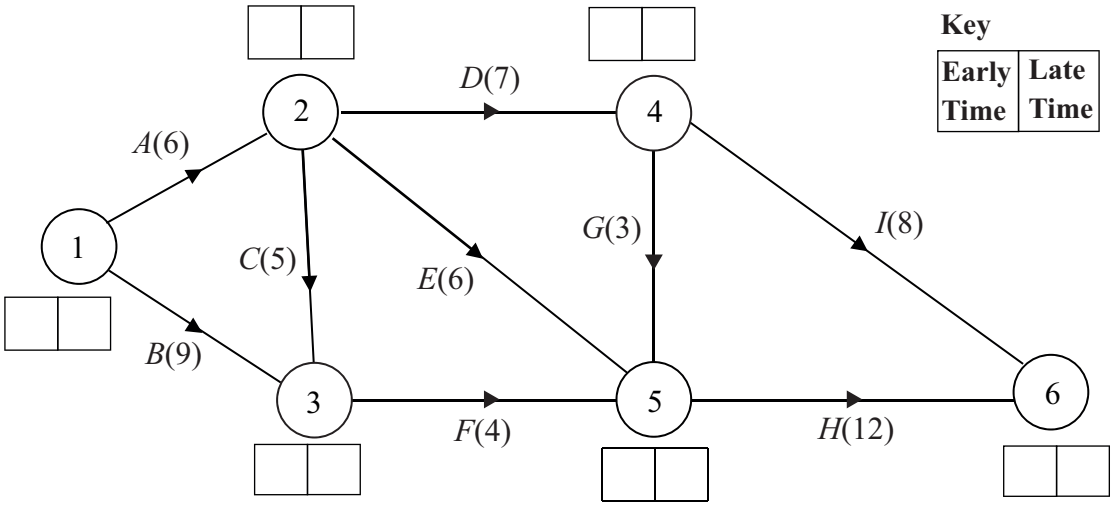
240
237
270
297
212
208
221
263
172
168
202
241
157
148
179

(ii) Plot these averages on the graph on page 66 opposite and draw the trend line. [3]

(iii) Showing clearly where any reading is taken, use the trend line to calculate an estimate of the number of people taking a cycle tour on Sunday in week 4.

Answer _____ [3]

6 The diagram below shows the activity network used to model a small refurbishment project. The activities are represented by the edges and the number in brackets on each edge represents the time, in hours, taken to complete that activity.



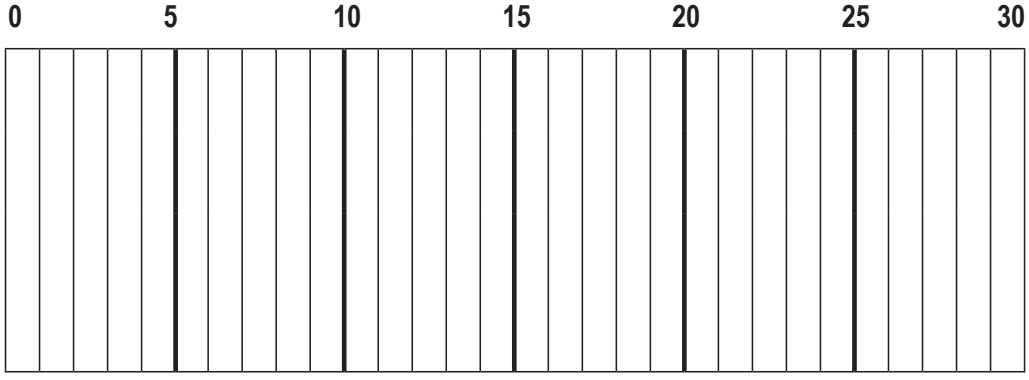
(i) Calculate the early time and the late time for each event. Write your answers in the spaces provided in the diagram above. [4]

(ii) Hence determine the critical activities and the length of the critical path.

Critical activities _____ [1]

Length of critical path _____ [1]

(iii) Schedule the activities for the minimum number of workers using the time line below. Ensure that you make clear the order in which each worker undertakes his activities.



[5]

7 **(a)** A gardener wishes to plant a mixed border with 3 shrubs and 4 roses. She has 5 shrubs and 6 roses available.

How many different combinations of 3 shrubs and 4 roses could be used for planting this border?

Answer _____ [4]

(b) A team of 4 comprising a captain, vice-captain and two other runners has to be chosen from the eight member squad of Aoife, Bart, Clare, Dougan, Evie, Forest, Grainne and Hugh.

In how many different ways can this be done?

Answer _____ [3]

THIS IS THE END OF THE QUESTION PAPER

MARK SCHEMES
DIVIDER FRONT

MARK SCHEMES
DIVIDER BACK

Rewarding Learning

General Certificate of Secondary Education

Further Mathematics

GENERAL MARKING INSTRUCTIONS

General Marking Instructions

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for accurate working, whether in calculation, reading from tables, graphs or answers.

MW indicates marks for combined method and accurate working.

Assessment Objectives

Below are the assessment objectives for GCSE Further Mathematics.

Use and apply standard techniques (AO1)

Candidates should be able to:

- accurately recall facts, terminology and definitions;
- use and interpret notation correctly; and
- accurately carry out routine procedures or set tasks requiring multi-step solutions.

Reason, interpret and communicate mathematically (AO2)

Candidates should be able to:

- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- present arguments and proofs; and
- assess the validity of an argument and critically evaluate a given way of presenting information.

Solve problems within mathematics and in other contexts

Candidates should be able to:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;
- evaluate methods used and results obtained; and
- evaluate solutions to identify how they may have been affected by assumptions made.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be **followed through** from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

It should be noted that where an error trivialises a question, or changes the nature of the skills being tested, then as a general rule, it would be the case that not more than half the marks for that question or part of that question would be awarded; in some cases the error may be such that no marks would be awarded.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier, only a proportion of the marks will be available (based on the professional judgement of the examiner).

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Rewarding Learning

General Certificate of Secondary Education
2019

Further Mathematics

Unit 1

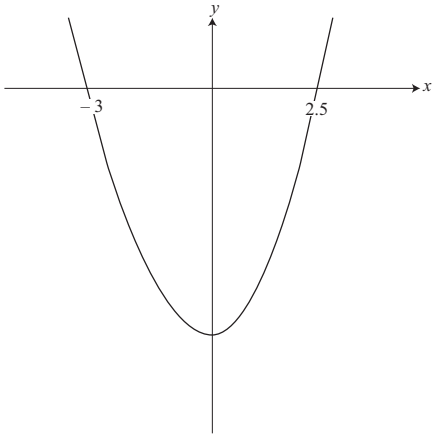
Pure Mathematics

[CODE]

SPECIMEN

MARK SCHEME

			AVAILABLE MARKS
1	**(i)** $\mathbf{A}^2 = \begin{bmatrix}2 & -3\\4 & 7\end{bmatrix}\begin{bmatrix}2 & -3\\4 & 7\end{bmatrix} = \begin{bmatrix}-8 & -27\\36 & 37\end{bmatrix}$	M1, MW1	
	(ii) $\mathbf{B - 2A} = \begin{bmatrix}-2 & 11\\1 & -3\end{bmatrix} - 2\begin{bmatrix}2 & -3\\4 & 7\end{bmatrix} = \begin{bmatrix}-6 & 17\\-7 & -17\end{bmatrix}$	M1, MW1	
	(iii) $\mathbf{CD} = \begin{bmatrix}8\\-5\end{bmatrix}[7 \quad -2] = \begin{bmatrix}56 & -16\\-35 & 10\end{bmatrix}$	M1, MW1	6
2	**(i)** $x^2 - 12x + 40$		
	$= (x-6)^2 - 36 + 40$ (for $(x-6)^2$)	M1	
	$= (x-6)^2 + 4$	W1	
	(ii) **(a)** min value = 4	MW1	
	(b) value of $x = 6$	MW1	4
3	**(a)** correct shape	M1	
	(–180, 0), (–90, –1), (0, 0), (90, 1) and (180, 0) plotted correctly	W1	
	(b) let $\theta = 3x + 24°$ giving $\sin\theta = -0.5$	M1	
	$\theta = -30°$ or $-150°$	MW1, MW1	
	so $3x + 24° = -30°$	M1	
	$x = -18°$	W1	
	or $3x + 24° = -150°$		
	$x = -58°$	W1	8
4	$y = \frac{5}{6}x^3 - x + \frac{4}{3x^2}$		
	$y = \frac{5}{6}x^3 - x + \frac{4}{3}x^{-2}$		
	$\frac{dy}{dx} = \frac{5}{2}x^2 - 1 - \frac{8}{3}x^{-3}$	MW1, MW1, MW1	
	$\frac{d^2y}{dx^2} = 5x + 8x^{-4}$ or $5x + \frac{8}{x^4}$	MW1, MW1	5

			AVAILABLE MARKS
5	Find roots		
	$2x^2 + x - 15 = 0$	M1	
	$(2x - 5)(x + 3) = 0$	M1	
	$x = 2.5$ or $x = -3$	W1	
			
	Using positive curve shape (may be implied)		
	$-3 < x < 2.5$	MW1	4
6	**(i)** $\mathbf{P}^{-1} = \frac{1}{66}\begin{bmatrix} 3 & 5 \\ -6 & 12 \end{bmatrix}$	MW1, MW1	
	(ii) $12x - 5y = 13$		
	$6x + 3y = -4.5$ equals $\begin{bmatrix} 12 & -5 \\ 6 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ -4.5 \end{bmatrix}$	M1, W1	
	$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{66}\begin{bmatrix} 3 & 5 \\ -6 & 12 \end{bmatrix}\begin{bmatrix} 13 \\ -4.5 \end{bmatrix}$	M1	
	$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{66}\begin{bmatrix} 16.5 \\ -132 \end{bmatrix} = \begin{bmatrix} 0.25 \\ -2 \end{bmatrix}$ or $x = 0.25$ and $y = -2$	W1	6
7	$(3x - 4)\log 7 = \log 4$	M1, M1	
	$3x \log 7 - 4 \log 7 = \log 4$		
	$3x \log 7 = \log 4 + 4 \log 7$		
	$x = \dfrac{\log 4 + 4 \log 7}{3 \log 7}$	MW1	
	$x = 1.57$	W1	4

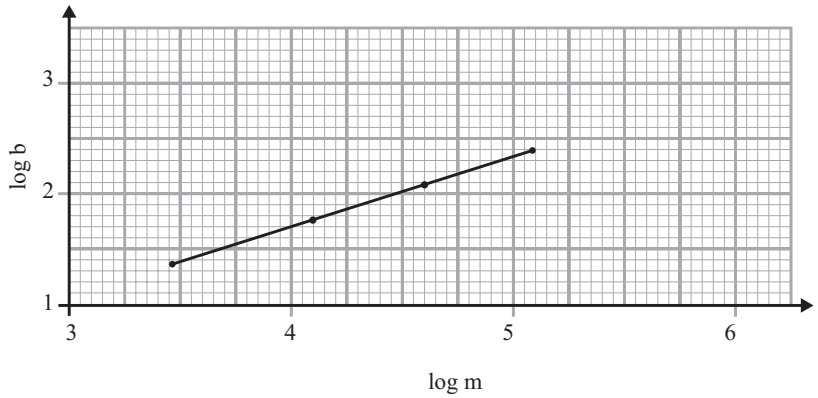
			AVAILABLE MARKS
8	$y = \int(6x^2 + \frac{1}{x^4} - 3)\mathrm{d}x$	M1	
	$= \int 6x^2 + x^{-4} - 3)\mathrm{d}x$		
	$y = 2x^3 - \frac{1}{3}x^{-3} - 3x + c$	MW1, MW1, MW1	
	$4 = -2 + \frac{1}{3} + 3 + c$	M1	
	$2\frac{2}{3} = c$		
	$y = 2x^3 - \frac{1}{3}x^{-3} - 3x + 2\frac{2}{3}$	W1	6
9	$\frac{\mathrm{d}P}{\mathrm{d}t} = 11 - 4t$	M1, W1	
	$11 - 4t = 0$	M1	
	$t = 2.75$	W1	
	$P = 3 + 11(2.75) - 2(2.75)^2$		
	$P = 18.125 = £18.13$	MW1	
	$\frac{\mathrm{d}^2P}{\mathrm{d}t^2} = -4$ therefore the value is a maximum	M1	6
10	$y = 2x^3 - 9x^2 - 14x + 2$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 - 18x - 14$	M1, W1	
	gradient of line = 10	W1	
	$6x^2 - 18x - 14 = 10$	M1	
	$6x^2 - 18x - 24 = 0$		
	$x^2 - 3x - 4 = 0$	MW1	
	$(x - 4)(x + 1) = 0$		
	$x = 4$ or -1 (so 2 points)	MW1	6
11	$\int_0^2 (2x^2 - 18)\mathrm{d}x$	M1	
	$= \left[\frac{2}{3}x^3 - 18x\right]_0^2$	MW1	
	$= \left(\frac{16}{3} - 36\right) - (0)$	M1	
	$= -30\frac{2}{3}$	W1	
	Area = $30\frac{2}{3}$	W1	5

		AVAILABLE MARKS
12 **(i)**	$12x + 8y + 20z = 320$	
	Dividing by 4 gives $3x + 2y + 5z = 80$ (1) MW1	
(ii)	$25x + 30y + 15z = 660$	
	Dividing by 5 gives $5x + 6y + 3z = 132$ (2) MW1	
(iii)	$18(\frac{2}{3}x) + 12y + 27z = 393$ MW1	
	$12x + 12y + 27z = 393$	
	Dividing by 3 gives $4x + 4y + 9z = 131$ (3) MW1	
(iv)	$4x + 12z = 108$ $3(1) - (2)$	
	$x + 3z = 27$ M1, W1	
	$2x + z = 29$ $2(1) - (3)$ M1, W1	
	$5z = 25$ W1	
	$z = 5$ $x = 27 - 3 = 12$ (back substitution) M1	
	masses of coins are: £2 = 12 g £1 = 9.5 g 50p = 8 g 20p = 5 g W1	
(v)	Let n = number of counterfeit coins.	
	Then $10n + (20 - n)12 = 228$ M1	
	$-2n = -12$	
	$n = 6$ i.e. 6 counterfeit coins W1	13

13 (i) $\log b = \log k + n \log m$ M1 W1

Body mass m	Brain mass b	$\log m$	$\log b$
2500	23.8	3.398	1.377
4600	34.6	3.663	1.539
15 800	73.5	4.199	1.866
50 100	148.8	4.700	2.173
126 000	261.4	5.100	2.417

M1 for taking logs
W1 for all answers correct to 3 decimal places



W1 for labels
W1 for all points plotted accurately
W1 for straight line drawn through these points

$n = \dfrac{(2.417 - 1.377)}{(5.100 - 3.398)} = 0.611 \rightarrow 0.61$ (2 d.p.) M1, W1

Then using $b = km^n$ $k = 73.5/15\,800^{0.611} = 0.20$ M1, W1

$b = 0.2m^{0.61}$

(ii) Leopard's brain mass $= 0.2 \times 39\,800^{0.61} = 127.9$ g (1 d.p.) MW1

(iii) Siberian tiger's mass $= (335.2 \div 0.2)^{1/0.61} = 193\,000$ g (3 s.f.) M1, W1

Assumption: The relationship holds outside the range M1 15

			AVAILABLE MARKS
14	$y = ax^2 + bx + c$		
	Substitute (0, –20) therefore $c = -20$	MW1	
	$\frac{dy}{dx} = 2ax + b$	M1	
	$x = 0$, gradient = 3 giving $b = 3$	MW1	
	$y = ax^2 + 3x - 20$		
	Substituting (–4,0) we obtain $0 = 16a - 12 - 20$	M1	
	$16a = 32$		
	$a = 2$	W1	5
15	**(i)** $(x - 5)(3x + 10) = 3x^2 - 5x - 50$	MW1	
	$(x + 2)(x - 5)(3x + 10) = 3x^3 - 5x^2 - 50x + 6x^2 - 10x - 100$	M1	
	$(x + 2)(x - 5)(3x + 10) = 3x^3 + x^2 - 60x - 100$	W1	
	(ii) $(x + 2)(x - 5)(3x + 10) - 3x(x - 20)$	MW1	
	$= (x^2 - 100) = (x - 10)(x + 10)$	MW2	
	So full expression		
	$= \frac{(x + 2)(x - 5)(3x + 10) - 3x(x^2 - 20)}{x - 10} = (x + 10)$	W1	7
		Total	**100**

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Rewarding Learning

General Certificate of Secondary Education
2019

Further Mathematics

Unit 2

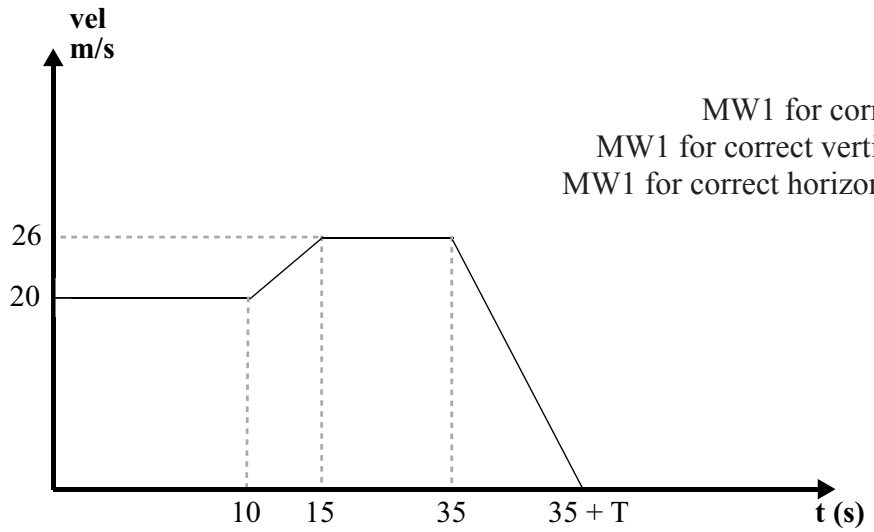
Mechanics

[CODE]

SPECIMEN

MARK SCHEME

			AVAILABLE MARKS
1	**(i)** $u = 3$, $a = 0.5$, $t = 10$		
	$v = u + at$	MW1	
	$v = 3 + 0.5 \times 10$		
	$v = 8$ m/s	W1	
	(ii) $s = ut + \frac{1}{2} at^2$		
	$s = 3 \times 10 + \frac{1}{2} \times 0.5 \times 10^2$	MW1	
	$s = 55$ m	W1	4
2	**(i)** $\lvert \mathbf{a} \rvert = \sqrt{3^2 + (-2)^2}$	MW1	
	$\lvert \mathbf{a} \rvert = \sqrt{13} = 3.61$	W1	
	(ii) angle between **a** and **i**		
	is $\tan^{-2}(\frac{2}{3})$	MW1	
	$= 33.69°$	W1	4
3	**(i)** $u = 20$, $a = 1.2$, $t = 5$, $v = V$		
	$v = u + at$		
	$V = 20 + 1.2 \times 5$	MW1	
	$V = 26$ m/s	W1	
	(ii)		

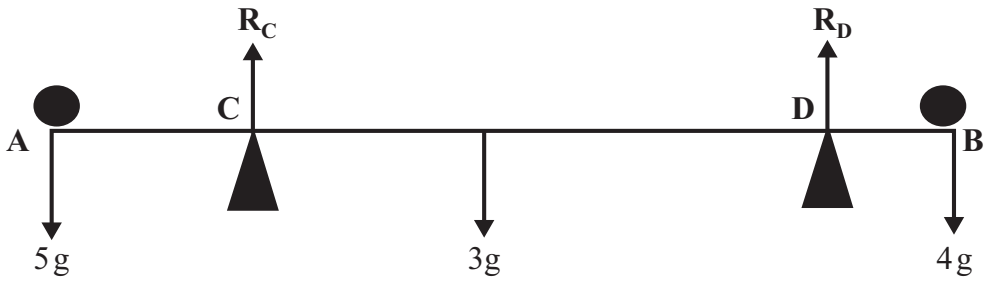


MW1 for correct shape
MW1 for correct vertical labels
MW1 for correct horizontal labels

	(iii) $1160 = 20 \times 10 + \frac{1}{2}(20 + 26)5 + 26 \times 20 + \frac{1}{2} \times 26 \times T$	M1, W1	
	$1160 = 200 + 115 + 520 + 13T$		
	$T = 25$ s	W1	8

AVAILABLE MARKS

4 **(i)**



MW2 for all correct forces
(Allow MW1 for any 3 correct forces)

(ii) Moments about C:

$3\,g \times 1.5 + 4\,g \times 5 = R_D \times 3.5 + 5\,g \times 2$ MW2
$45 + 200 = 3.5R_D + 100$
$145 = 3.5R_D$
$41.43 = R_D$ W1

$R_C + R_D = 5\,g + 3\,g + 4\,g$ MW1
$R_C + 41.43 = 120$
$R_C = 78.57$ N W1

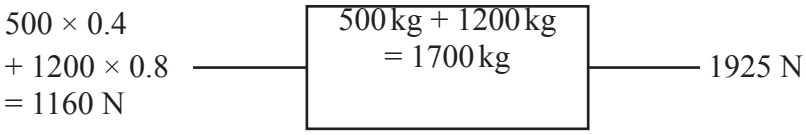
or

Moments about D:

$3\,g \times 2 + 5\,g \times 5.5 = R_C \times 3.5 + 4\,g \times 1.5$ MW2
$60 + 275 = 3.5R_C + 60$
$275 = 3.5R_C$
$78.57 = R_C$ W1

$R_C + R_D = 5g + 3g + 4g$ MW1
$78.57 + R_D = 120$
$R_D = 41.43$ N W1 7

		AVAILABLE MARKS
5	For combined car and trailer:	



(i) Using $F = ma$ for the combined car and trailer:

$1925 - 1160 = 1700\,a$ MW1, MW1

$a = 0.45$ m/s^2 W1

(ii) For trailer:

$T - 200 = 500 \times 0.45$ MW1

$T = 425$ N W1

(iii) $u = 0$, $a = 0.45$, $t = 20$

$v = u + at$

$v = 0 + 0.45 \times 20$ MW1

$v = 9$ m/s W1

(iv) Using $F = ma$ for the trailer:

$0 - 200 = 500\,a$ MW1, MW1

$a = -0.4$ or dec $= 0.4$ m/s^2 W1

(v) $u = 9$, $v = 0$, $a = -0.4$

$v^2 = u^2 + 2as$

$0 = 9^2 + 2 \times s \times -0.4$ MW1

$0.8\,s = 81$

$s = 101.25$ m W1 — 12

6 Resolving horizontally for both diagrams: M1

$P - 20 \cos 50° = 10 \cos 70°$ MW1

$P = 10 \cos 70° + 20 \cos 50°$

$P = 16.28$ N W1

Resolving vertically for both diagrams:

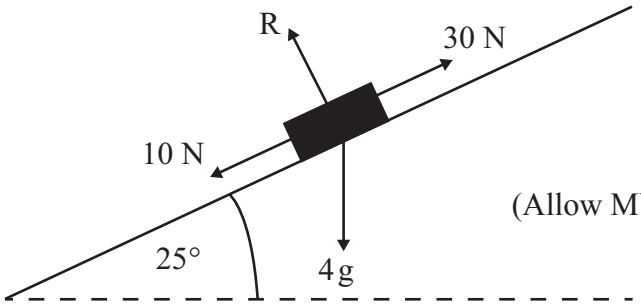
$20 \sin 50° - Q = 10 \sin 70°$ MW1

$Q = 20 \sin 50° - 10 \sin 70°$

$Q = 5.92$ N W1 — 5

	AVAILABLE MARKS

7 (i)



MW2 for all correct forces
(Allow MW1 for any 2 correct forces)

(ii) Applying $F = ma$ to the block going up the slope:

$30 - 10 - 4\,g \sin 25° = 4a$ MW1, MW1

$a = 0.77$ m/s^2 W1

(iii) Calculating the acceleration of the block down the slope:

$4\,g \sin 25° - 10 = 4a$ MW1, MW1

$a = 1.726$ m/s^2 W1

$u = 0$, $a = 1.726$, $t = 2$

$s = ut + \frac{1}{2}at^2$

$s = 0 \times 2 + \frac{1}{2} \times 1.726 \times 2^2$ MW1

$s = 3.452$

$s = 3.45$ m W1 — 10

Total — **50**

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Rewarding Learning

General Certificate of Secondary Education
2019

Further Mathematics

Unit 3

Statistics

[CODE]

SPECIMEN

MARK SCHEME

		AVAILABLE MARKS

1

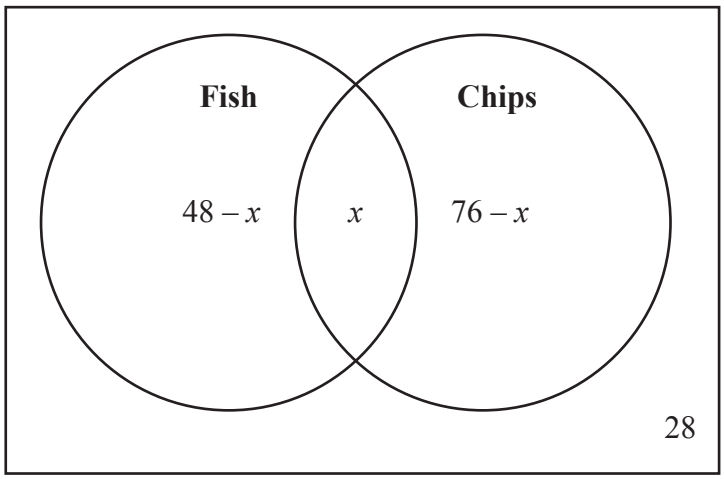
Time (minutes)	Frequency (f)	Mid-point x	fx	fx^2
5–9	5	7	35	245
10–14	17	12	204	2448
15–19	35	17	595	10115
20–24	28	22	616	13552
25–29	11	27	297	8019
30–34	4	32	128	4096
	100		1875	38475

(i) $\dfrac{\Sigma fx}{\Sigma f} = \dfrac{1875}{100} = 18.75$ M1, W1

(ii) $\sqrt{\left(\dfrac{38475}{100} - (18.75)^2\right)}$ M1, W1

5.76 W1 5

2 **(i)**



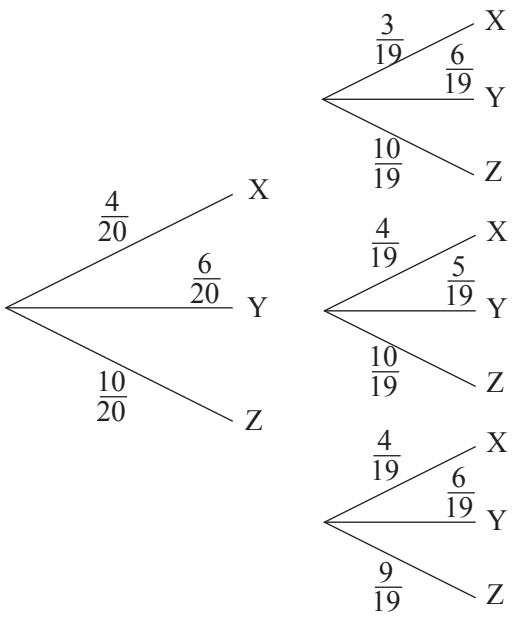
$(76 + 48 + 28) - 120 = 32$ MW1

$\dfrac{32}{120}$ or $\dfrac{4}{15}$ MW1

(ii) $\text{P(chips} \mid \text{fish)} = \dfrac{\text{P(chips and fish)}}{\text{P(fish)}}$ M1

$\dfrac{32}{48}$ or $\dfrac{2}{3}$ W1 4

			AVAILABLE MARKS
3	$Z = \frac{2.08 - 1.9}{0.15} = 1.2$	M1, W1	
	$P(Z > 1.2) = 1 - P(Z < 1.2)$	M1	
	0.12	W1	4
4	**(i)** $72 \times 9 + 76 = 724$	MW1	
	$\frac{724}{10} = 72.4$	MW1	
	(ii) $8^2 = \frac{\sum x^2}{9} - 72^2$	MW1	
	$\sum x^2 = 47\,232$	MW1	
	New $\sum x^2 = 53\,008$		
	New standard deviation $= \sqrt{\left(\frac{53\,008}{10} - (72.4)^2\right)}$	MW1	
	7.68	W1	6

			AVAILABLE MARKS
5	**(i)**		
	or		
		$\frac{4}{20} \times \frac{3}{19} + \frac{6}{20} \times \frac{5}{19} + \frac{10}{20} \times \frac{9}{19}$ M1	
		$\frac{12}{380} + \frac{30}{380} + \frac{90}{380} = \frac{132}{380}$ or $\frac{33}{95}$ W1	
	(ii)	$1 - \frac{132}{380}$ M1	
		$\frac{248}{380}$ or $\frac{62}{95}$ W1	
		or	
		$\frac{4}{20} \times \frac{16}{19} + \frac{6}{20} \times \frac{14}{19} + \frac{10}{20} \times \frac{10}{19}$ M1	
		$\frac{64}{380} + \frac{84}{380} + \frac{100}{380} = \frac{248}{380}$ or $\frac{62}{95}$ W1	
	(iii)	$1 - \text{P(neither voted for Z)} = 1 - \frac{10}{20} \times \frac{9}{19}$ M1, W1	
		$\frac{290}{380}$ or $\frac{145}{190}$ or $\frac{29}{38}$ W1	
		or	
		$\frac{10}{20} \times \frac{10}{19} + \frac{10}{20} \times \frac{10}{19} + \frac{10}{20} \times \frac{9}{19}$ M1, W1	
		$\frac{100}{380} + \frac{100}{380} + \frac{90}{380} = \frac{290}{380}$ or $\frac{145}{190}$ or $\frac{29}{38}$ W1	
	(iv)	$\frac{3}{19}$ M1, W1	9

6 (i)

Ranks X	8	3	5	4	1	7	9	6	2
Ranks Y	7	2.5	4	5	1	9	8	6	2.5

or

Ranks X	2	7	5	6	9	3	1	4	8
Ranks Y	3	7.5	6	5	9	1	2	4	7.5

MW1, MW1

(ii)

d^2	1	0.25	1	1	0	4	1	0	0.25

$r = 1 - \dfrac{6\sum d^2}{n(n^2-1)}$ M1, W1

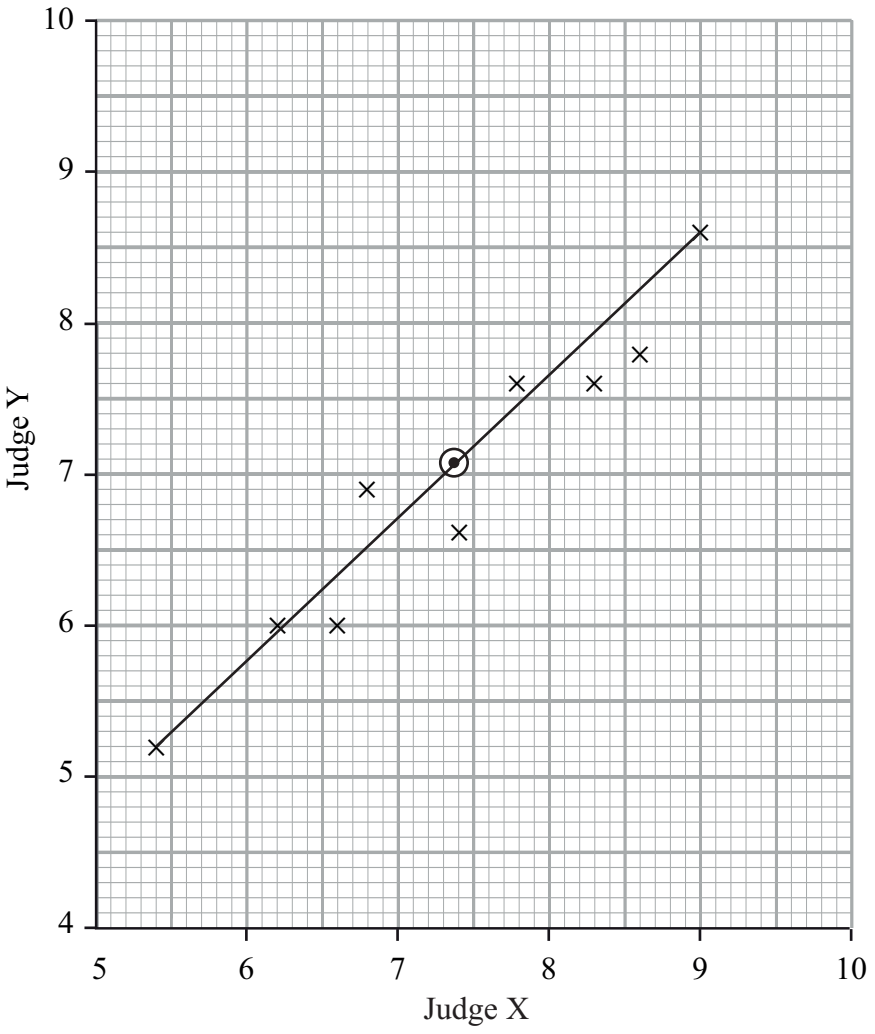
$r = 1 - \dfrac{6(8.5)}{9(80)}$ M1

$r = 0.93$ W1

(iii) (Strong) positive correlation M1

(iv) Mean for Judge X = 7.36
Mean for Judge Y = 7.1 MW1

(v)



MW1 (through means)
MW1 (slope)

AVAILABLE MARKS

			AVAILABLE MARKS
(vi)	Gradient = $\frac{(8.75 - 7.1)}{(9 - 7.36)} = 1.01$	MW1	
	Using means $7.1 = 1.01 \times 7.36 + c$		
	$-0.33 = c$	M1	
	Equation is $y = 1.01x - 0.33$	MW1	13
7 **(i)**	1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1	MW1	
	$p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$	MW1	
(ii)	**(a)** $1 - 0.05 = 0.95$	MW1	
	$(0.95)^5$	M1	
	0.77	W1	
	(b) $(0.95)^5 + 5(0.95)^4(0.05)$	M1, W1	
	$1 - \{(0.95)^5 + 5(0.95)^4(0.05)\}$	M1	
	0.02	W1	9
		Total	**50**

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Further Mathematics

Unit 4

Discrete and Decision Mathematics

[CODE]

SPECIMEN

MARK SCHEME

1 Method of multiplying routes in series

AVAILABLE MARKS

$3 \times 2 \times 4 = 24$ routes and $2 \times 2 \times 3 = 12$ routes MW1

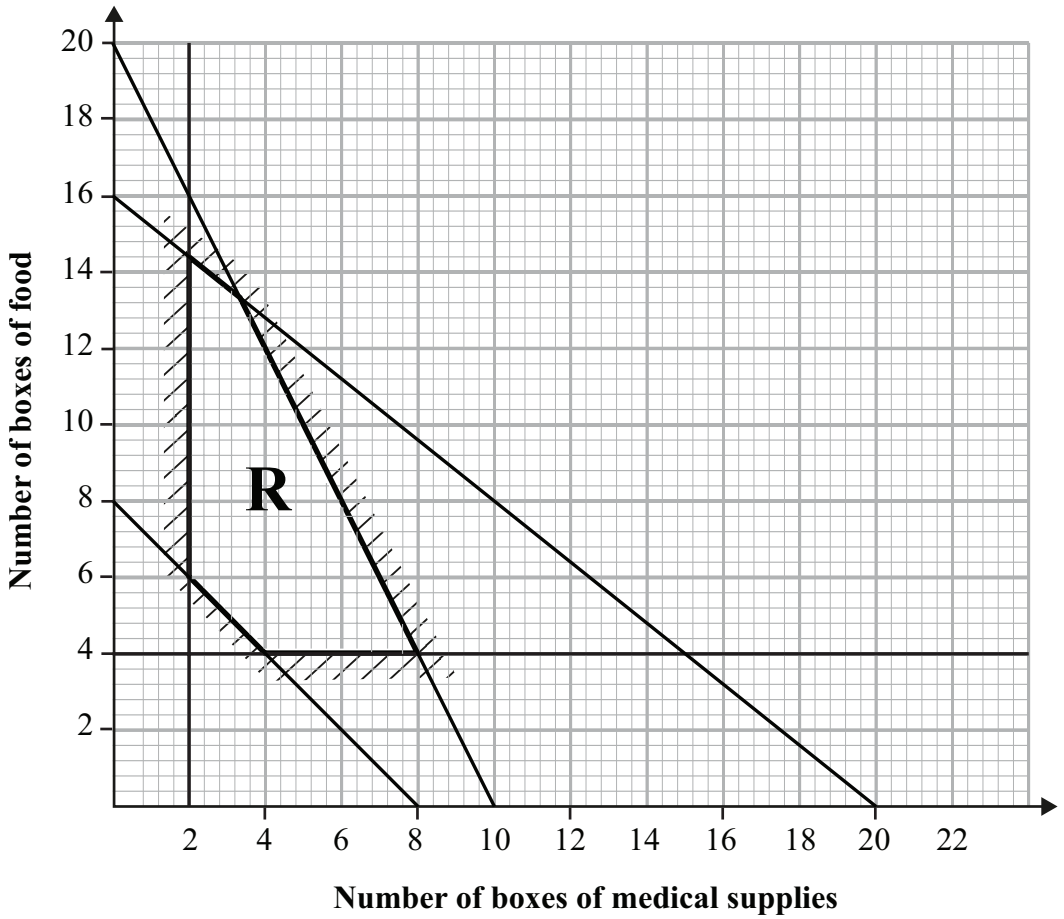
Method of adding routes in parallel $24 + 12 = 36$ routes MW1 2

2 Truth tables:

p	q	r	p **and** q	(p **and** q) **or** r
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F
			MW1	MW1

p	q	r	p **or** r	q **or** r	(p **or** r) **and** (q **or** r)
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	F	F
F	T	T	T	T	T
F	T	F	F	T	F
F	F	T	T	T	T
F	F	F	F	F	F
			MW1 (both)		MW1

Thus they are equivalent. 4

		AVAILABLE MARKS
3 **(i)** **(a)** $40x + 50y \leqslant 800$ $4x + 5y \leqslant 80$ MW1		
(b) $100x + 50y \leqslant 1000$ $2x + y \leqslant 20$ MW1		
(ii) 		
line through (0,16) and (20,0) and correct side shaded MW1 line through (0,20) and (10,0) and correct side shaded MW1 (line through (0,8) and (8,0) and correct side shaded, and line $x = 2$ drawn and correct side shaded, and line $y = 4$ drawn and correct side shaded) MW1		
(iii) **(a)** 16 boxes [(2,14) (3,13) (4,12)] MW1		
(b) Identifying (2,14) as the cheapest MW1		
to give a cost of 2 × £100 + 14 × £50 = £900 W1		
(iv) Identification of the point (4,4) as the minimum M1, W1		
8 people W1		11

4 **(i)**

p	q	**not** p	**not** q	**not** p **or not** q	p **and** q	**not** (p **and** q)
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

AVAILABLE MARKS

Basic structure (four rows and values of p, q columns) M1

not p, **not** q correct and (p **and** q) correct MW1

not p **or** NOT q correct MW1

not (p **and** q) correct MW1

(ii) p = "Joe likes cheese" and q = "Eleanore wears a ring"

"It is not true that either Joe doesn't like cheese or Eleanore doesn't wear a ring" = **not** (**not** p **or not** q) MW1

Using the result from part **(i)**
= **not not** (p **and** q) M1

= p **and** q
Therefore "Joe likes cheese and Eleanore wears a ring" W1 7

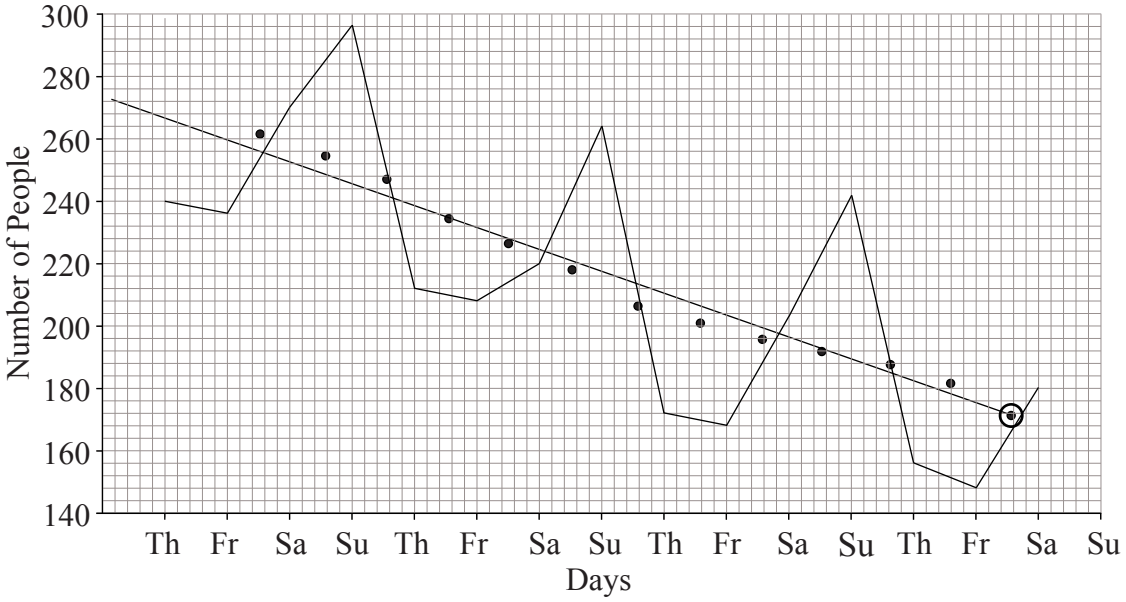
AVAILABLE MARKS

5 **(i)** Attempted four point moving averages using correct points M1

240	
237	
270	261
297	254
212	246.75
208	234.5
221	226
263	216
172	206
168	201.25
202	195.75
241	192
157	187
148	181.25
179	

W1

(ii)



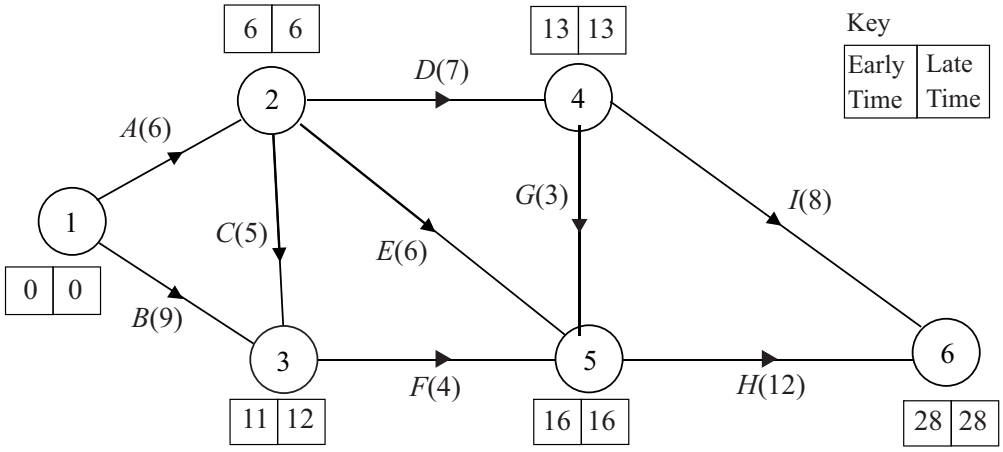
set M2
line W1

(iii) $\frac{x + 179 + 148 + 157}{4} = 172$ M1, W1

$x = 204$ W1 8

6 (i)

AVAILABLE MARKS



E_1, E_2, E_3, E_4 MW1

E_5, E_6 MW1

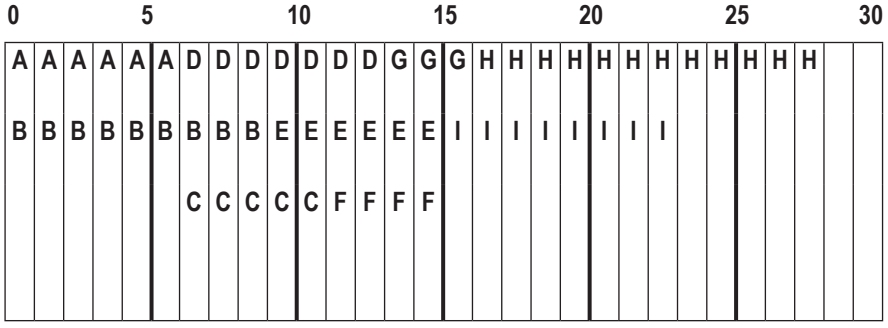
L_6, L_5 MW1

L_4, L_3, L_2, L_1 MW1

(ii) Critical activities are A, D, G, H. MW1

Length of critical path is 6 + 7 + 3 + 12 = 28 hours MW1

(iii)



M1, W1

M1, W1

MW1

11

				AVAILABLE MARKS
7	**(a)**	Choice of 3 out of 5 = ${}_5C_3$ (or use 5th row of Pascal's triangle)	M1	
		= 10	W1	
		Choice of 4 out of 6 = ${}_6C_4 = 15$ (or use 6th row of Pascal's triangle)	MW1	
		Multiply answers $15 \times 10 = 150$ ways	MW1	
	(b)	Choice of 2 out of 6 = ${}_6C_2 = 15$ (or use 6th row of Pascal's triangle)	M1	
		Method of permutations $8 \times 7 \times$	M1	
		$8 \times 7 \times {}_6C_2 = 840$ ways	W1	7
			Total	**50**

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