CCGPS Frameworks Student Edition

Mathematics

CCGPS Analytic Geometry
Unit 1: Similarity, Congruence, and Proofs

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"Making Education Work for All Georgians"

Unit 1
Similarity, Congruence, and Proofs

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OVERVIEW

In this unit students will:

- verify experimentally with dilations in the coordinate plane.
- use the idea of dilation transformations to develop the definition of similarity.
- determine whether two figures are similar.
- use the properties of similarity transformations to develop the criteria for proving similar triangles.
- use AA, SAS, SSS similarity theorems to prove triangles are similar.
- use triangle similarity to prove other theorems about triangles.
- using similarity theorems to prove that two triangles are congruent.
- prove geometric figures, other than triangles, are similar and/or congruent.
- use descriptions of rigid motion and transformed geometric figures to predict the effects rigid motion has on figures in the coordinate plane.
- know that rigid transformations preserve size and shape or distance and angle; use this fact to connect the idea of congruency and develop the definition of congruent.
- use the definition of congruence, based on rigid motion, to show two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent.
- use the definition of congruence, based on rigid motion, to develop and explain the triangle congruence criteria; ASA, SSS, and SAS.
- prove theorems pertaining to lines and angles.
- prove theorems pertaining to triangles.
- prove theorems pertaining to parallelograms.
- make formal geometric constructions with a variety of tools and methods.
- construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. The first unit of Analytic Geometry involves similarity, congruence, and proofs. Students will understand similarity in terms of similarity transformations, prove theorems involving similarity, understand congruence in terms of rigid motions, prove geometric theorems, and make geometric constructions. During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS

<u>Understand similarity in terms of similarity transformations</u>

MCC9-12.G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:

- **a.** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

MCC9-12.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

MCC9-12.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

<u>Prove theorems involving similarity</u>

MCC9-12.G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

MCC9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Understand congruence in terms of rigid motions

MCC9-12.G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

MCC9-12.G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

MCC9-12.G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems

MCC9-12.G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

MCC9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

MCC9-12.G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Make geometric constructions

MCC9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

MCC9-12.G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

RELATED STANDARDS

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. **Make sense of problems and persevere in solving them.** High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.** High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.** High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations

by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.** High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.** High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.** High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. **Look for and make use of structure.** By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8. **Look for and express regularity in repeated reasoning.** High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who do not have an understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a missing mathematical knowledge effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

ENDURING UNDERSTANDINGS

- Given a center and a scale factor, verify experimentally, that when dilating a figure in a coordinate plane, a segment of the pre-image that does not pass through the center of the dilation, is parallel to its image when the dilation is preformed. However, a segment that passes through the center remains unchanged.
- Given a center and a scale factor, verify experimentally, that when performing dilations of a line segment, the pre-image, the segment which becomes the image is longer or shorter based on the ratio given by the scale factor.
- Use the idea of dilation transformations to develop the definition of similarity.
- Given two figures determine whether they are similar and explain their similarity based on the equality of corresponding angles and the proportionality of corresponding sides.
- Use the properties of similarity transformations to develop the criteria for proving similar triangles: AA.
- Use AA, SAS, SSS similarity theorems to prove triangles are similar.
- Prove a line parallel to one side of a triangle divides the other two proportionally, and its converse.
- Prove the Pythagorean Theorem using triangle similarity.
- Use similarity theorems to prove that two triangles are congruent.
- Use descriptions of rigid motion and transformed geometric figures to predict the effects rigid motion has on figures in the coordinate plane.
- Knowing that rigid transformations preserve size and shape or distance and angle, use this fact to connect the idea of congruency and develop the definition of congruent.
- Use the definition of congruence, based on rigid motion, to show two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent.
- Use the definition of congruence, based on rigid motion, to develop and explain the triangle congruence criteria: ASA, SSS, and SAS.
- Prove vertical angles are congruent.
- Prove when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent.
- Prove points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.
- Prove the measures of interior angles of a triangle have a sum of 180º.
- Prove base angles of isosceles triangles are congruent.
- Prove the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length.
- Prove the medians of a triangle meet at a point.

- Prove properties of parallelograms including: opposite sides are congruent, opposite angles are congruent, diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
- Copy a segment and an angle.
- Bisect a segment and an angle.
- Construct perpendicular lines, including the perpendicular bisector of a line segment.
- Construct a line parallel to a given line through a point not on the line.
- Construct an equilateral triangle so that each vertex of the equilateral triangle is on the circle.
- Construct a square so that each vertex of the square is on the circle.
- Construct a regular hexagon so that each vertex of the regular hexagon is on the circle.

CONCEPTS/SKILLS TO MAINTAIN

Some students often do not recognize that congruence is a special case of similarity. Similarity with a scale factor equal to 1 becomes a congruency. Students may not realize that similarities preserve shape, but not size. Angle measures stay the same, but side lengths change by a constant scale factor. Some students often do not list the vertices of similar triangles in order. However, the order in which vertices are listed is preferred and especially important for similar triangles so that proportional sides can be correctly identified. Dilations and similarity, including the AA criterion, are investigated in Grade 8, and these experiences should be built upon in high school with greater attention to precise definitions, careful statements and proofs of theorems and formal reasoning.

The Pythagorean Theorem and its converse are proved and applied in Grade 8. In high school, another proof, based on similar triangles, is presented. The alternate interior angle theorem and its converse, as well as properties of parallelograms, are established informally in Grade 8 and proved formally in high school.

Properties of lines and angles, triangles and parallelograms are investigated in Grades 7 and 8. In high school, these properties are revisited in a more formal setting, giving greater attention to precise statements of theorems and establishing these theorems by means of formal reasoning.

The theorem about the midline of a triangle can easily be connected to a unit on similarity. The proof of it is usually based on the similarity property that corresponding sides of similar triangles are proportional.

Students should be expected to have prior knowledge/experience related to the concepts and skills identified below. Pre-assessment may be necessary to determine whether instructional time should be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Understand and use reflections, translations, and rotations.
- Define the following terms: circle, bisector, perpendicular and parallel.
- Solve multi-step equations.

- Understand angle sum and exterior angle of triangles.
- Know angles created when parallel lines are cut by a transversal.
- Know facts about supplementary, complementary, vertical, and adjacent angles.
- Solve problems involving scale drawings of geometric figures.
- Draw geometric shapes with given conditions.
- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.
- Draw polygons in the coordinate plane given coordinates for the vertices.

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school children. **Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

- **Adjacent Angles:** Angles in the same plane that have a common vertex and a common side, but no common interior points.

- **Alternate Exterior Angles**: Alternate exterior angles are pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on opposite sides of the transversal and are outside the other two lines. When the two other lines are parallel, the alternate exterior angles are equal.

- **Alternate Interior Angles**: Alternate interior angles are pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on opposite sides of the transversal and are in between the other two lines. When the two other lines are parallel, the alternate interior angles are equal.

- **Angle:** Angles are created by two distinct rays that share a common endpoint (also known as a vertex). ∠ABC or ∠B denote angles with vertex B.
- **Bisector:** A bisector divides a segment or angle into two equal parts.
- **Centroid:** The point of concurrency of the medians of a triangle.
- **Circumcenter:** The point of concurrency of the perpendicular bisectors of the sides of a triangle.
- **Coincidental:** Two equivalent linear equations overlap when graphed.
- **Complementary Angles:** Two angles whose sum is 90 degrees.
- **Congruent:** Having the same size, shape and measure. Two figures are congruent if all of their corresponding measures are equal.
- **Congruent Figures**: Figures that have the same size and shape.
- **Corresponding Angles:** Angles that have the same relative positions in geometric figures.
- **Corresponding Sides:** Sides that have the same relative positions in geometric figures
- **Dilation**: Transformation that changes the size of a figure, but not the shape.
- **Endpoints:** The points at an end of a line segment
- **Equiangular:** The property of a polygon whose angles are all congruent.
- **Equilateral:** The property of a polygon whose sides are all congruent.
- **Exterior Angle of a Polygon:** an angle that forms a linear pair with one of the angles of the polygon.
- **Incenter:** The point of concurrency of the bisectors of the angles of a triangle.
- **Intersecting Lines:** Two lines in a plane that cross each other. Unless two lines are coincidental, parallel, or skew, they will intersect at one point.
- **Intersection:** The point at which two or more lines intersect or cross.

- **Line:** One of the basic undefined terms of geometry. Traditionally thought of as a set of points that has no thickness but its length goes on forever in two opposite directions. $\overleftrightarrow{AB}$ denotes a line that passes through point A and B.

- **Line Segment or Segment:** The part of a line between two points on the line. $\overline{AB}$ denotes a line segment between the points A and B.

- **Linear Pair:** Adjacent, supplementary angles. Excluding their common side, a linear pair forms a straight line.

- **Measure of each Interior Angle of a Regular n-gon:** $\frac{180^\circ(n-2)}{n}$

- **Orthocenter:** The point of concurrency of the altitudes of a triangle.

- **Parallel Lines:** Two lines are parallel if they lie in the same plane and they do not intersect.

- **Perpendicular Lines:** Two lines are perpendicular if they intersect at a right angle.

- **Plane:** One of the basic undefined terms of geometry. Traditionally thought of as going on forever in all directions (in two-dimensions) and is flat (i.e., it has no thickness).

- **Point:** One of the basic undefined terms of geometry. Traditionally thought of as having no length, width, or thickness, and often a dot is used to represent it.

- **Proportion**: An equation which states that two ratios are equal.

- **Ratio**: Comparison of two quantities by division and may be written as r/s, r:s, or r to s.

- **Ray:** A ray begins at a point and goes on forever in one direction.

- **Reflection:** A transformation that "flips" a figure over a line of reflection

- **Reflection Line:** A line that is the perpendicular bisector of the segment with endpoints at a pre-image point and the image of that point after a reflection.

- **Regular Polygon:** A polygon that is both equilateral and equiangular.

- **Remote Interior Angles of a Triangle:** the two angles non-adjacent to the exterior angle.

- **Rotation:** A transformation that turns a figure about a fixed point through a given angle and a given direction.
- **Same-Side Interior Angles**: Pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on the same side of the transversal and are between the other two lines. When the two other lines are parallel, same-side interior angles are supplementary.
- **Same-Side Exterior Angles**: Pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on the same side of the transversal and are outside the other two lines. When the two other lines are parallel, same-side exterior angles are supplementary.
- **Scale Factor**: The ratio of any two corresponding lengths of the sides of two similar figures.
- **Similar Figures**: Figures that have the same shape but not necessarily the same size.
- **Skew Lines:** Two lines that do not lie in the same plane (therefore, they cannot be parallel or intersect).
- **Sum of the Measures of the Interior Angles of a Convex Polygon:** $180^\circ(n - 2)$.
- **Supplementary Angles:** Two angles whose sum is 180 degrees.
- **Transformation**: The mapping, or movement, of all the points of a figure in a plane according to a common operation.
- **Translation:** A transformation that "slides" each point of a figure the same distance in the same direction
- **Transversal:** A line that crosses two or more lines.
- **Vertical Angles:** Two nonadjacent angles formed by intersecting lines or segments. Also called opposite angles.

Similarity in the Coordinate Plane

Adapted from Stretching and Shrinking: Similarity, *Connected Mathematics*, Dale Seymour Publications

Mathematical goals

- Given a center and a scale factor, verify experimentally, that when dilating a figure in a coordinate plane, a segment of the pre-image that does not pass through the center of the dilation, is parallel to its image when the dilation is preformed. However, a segment that passes through the center remains unchanged.
- Given a center and a scale factor, verify experimentally, that when performing dilations of a line segment, the pre-image, the segment which becomes the image is longer or shorter based on the ratio given by the scale factor.
- Use the idea of dilation transformations to develop the definition of similarity.

Common Core State Standards

MCC9-12.G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

MCC9-12.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Plot the ordered pairs given in the table to make six different figures. Draw each figure on a separate sheet of graph paper. Connect the points with line segments as follows:

- For Set 1, connect the points in order. Connect the last point in the set to the first point in the set.
- For Set 2, connect the points in order. Connect the last point in the set to the first point in the set.
- For Set 3, connect the points in order. Do not connect the last point in the set to the first point in the set.
- For Set 4, make a dot at each point (do not connect the dots).

Figure 1	**Figure 2**	**Figure 3**	**Figure 4**	**Figure 5**	**Figure 6**
Set 1	**Set 1**	**Set 1**	**Set 1**	**Set 1**	**Set 1**
(6, 4)	(12, 8)	(18, 4)	(18, 12)	(6, 12)	(8, 6)
(6, -4)	(12, -8)	(18, -4)	(18, -12)	(6, -12)	(8, -2)
(-6, -4)	(-12, -8)	(-18, -4)	(-18, -12)	(-6, -12)	(-4, -2)
(-6, 4)	(-12, 8)	(-18, 4)	(-18, 12)	(-6, 12)	(-4, 6)
Set 2	**Set 2**	**Set 2**	**Set 2**	**Set 2**	**Set 2**
(1, 1)	(2, 2)	(3, 1)	(3, 3)	(1, 3)	(3, 3)
(1, -1)	(2, -2)	(3, -1)	(3, -3)	(1, -3)	(3, 1)
(-1, -1)	(-2, -2)	(-3, -1)	(-3, -3)	(-1, -3)	(1, 1)
(-1, 1)	(-2, 2)	(-3, 1)	(-3, 3)	(-1, 3)	(1, 3)
Set 3	**Set 3**	**Set 3**	**Set 3**	**Set 3**	**Set 3**
(4, -2)	(8, -4)	(12, -2)	(12, -6)	(4, -6)	(6, 0)
(3, -3)	(6, -6)	(9, -3)	(9, -9)	(3, -9)	(5, -1)
(-3, -3)	(-6, -6)	(-9, -3)	(-9, -9)	(-3, -9)	(-1, -1)
(-4, -2)	(-8, -4)	(-12, -2)	(-12, -6)	(-4, -6)	(-2, 0)
Set 4	**Set 4**	**Set 4**	**Set 4**	**Set 4**	**Set 4**
(4, 2)	(8, 4)	(12, 2)	(12, 6)	(4, 6)	(6, 4)
(-4, 2)	(-8, 4)	(-12, 2)	(-12, 6)	(-4, 6)	(-2, 4)

After drawing the six figures, compare Figure 1 to each of the other figures and answer the following questions.

1. Which figures are similar? Use the definition of similar figures to justify your response.

2. Describe any similarities and/or differences between Figure 1 and each of the similar figures.
   - Describe how corresponding sides compare.
   - Describe how corresponding angles compare.

3. How do the coordinates of each similar figure compare to the coordinates of Figure 1? Write general rules for making the similar figures.

4. Is having the same angle measurement enough to make two figures similar? Why or why not?

5. What would be the effect of multiplying each of the coordinates in Figure 1 by ½?

6. Create a similar Figure 7 to Figure 1 where the center of dilation is not the origin but (-6, -4) instead. Also Figure 7 is twice as big as Figure 1. What are the sets of points used to create Figure 7?

Similar Triangles

Mathematical goals

- Discover the relationships that exist between similar figures using the scale factors, length ratios, and area ratios

Common Core State Standards

MCC9-12.G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:

- **c.** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- **d.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

MCC9-12.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

MCC9-12.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
8. Look for and express regularity in repeated reasoning.

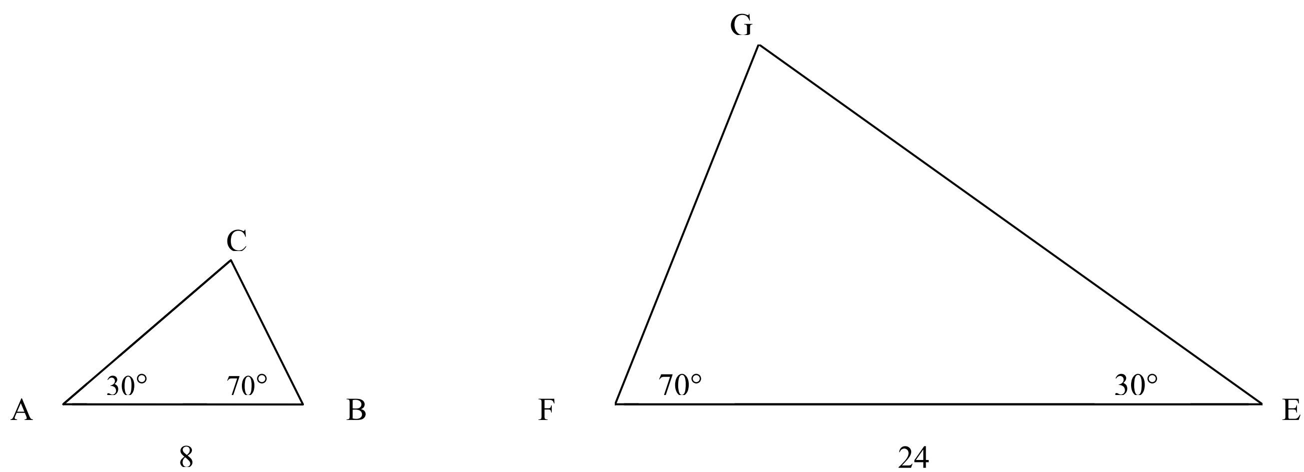
The sketch below shows two triangles, ΔABC and ΔEFG. ΔABC has an area of 12 square units, and its base (AB) is equal to 8 units. The base of ΔEFG is equal to 24 units.

a. How do you know that the triangles are similar?

b. Name the pairs of corresponding sides and the pairs of corresponding angles. How are the corresponding sides related and how are the corresponding angles related? Why is this true?

c. What is the area of Δ EFG? Explain your reasoning.

d. What is the relationship between the area of ΔABC and the area of Δ EFG ? What is the relationship between the scale factor and the ratio of the areas of the two triangles? Use an area formula to justify your answer algebraically.



Shadow Math

Mathematical goals

- Determine missing side lengths and areas of similar figures.

Common Core State Standards

MCC9-12.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

MCC9-12.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

MCC9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

Jeannie is practicing on the basketball goal outside her house. She thinks that the goal seems lower than the 10 ft. goal she plays on in the gym. She wonders how far the goal is from the ground. Jeannie can not reach the goal to measure the distance to the ground, but she remembers something from math class that may help. First, she needs to estimate the distance from the bottom of the goal post to the top of the backboard. To do this, Jeannie measures the length of the shadow cast by the goal post and backboard. She then stands a yardstick on the ground so that it is perpendicular to the ground, and measures the length of the shadow cast by the yardstick. Here are Jeannie's measurements:

Length of shadow cast by goal post and backboard: 5 ft. 9 in.

Length of yardstick's shadow: 1 ft. 6 in.

Draw and label a picture to illustrate Jeannie's experiment. Using her measurements, determine the height from the bottom of the goal post to the top of the backboard.

If the goal is approximately 24 inches from the top of the backboard, how does the height of the basketball goal outside Jeannie's house compare to the one in the gym? Justify your answer.

Proving Similar Triangles

Mathematical goals
- Identify Similar Triangles.
- Use similarity theorems to prove that two triangles are similar.

Common Core State Standards
MCC9-12.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Standards for Mathematical Practice
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

Introduction
This task identifies the three ways to prove that two triangles are similar by showing that they satisfy the two requirements for similar polygons. Examples and practice problems are provided.

You can always prove that two triangles are similar by showing that they satisfy the two requirements for similar polygons.

1.) Show that corresponding angles are congruent AND
2.) Show that corresponding sides are proportional.

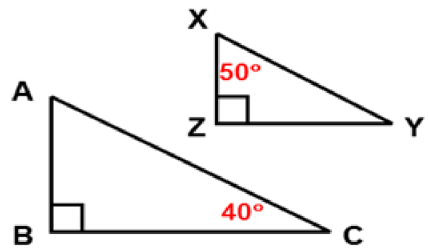
However, there are 3 simpler methods.

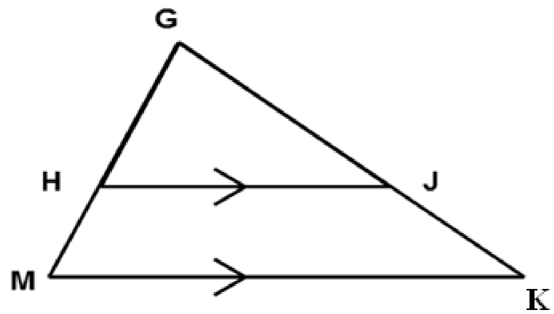
Angle-Angle Similarity Postulate (AA~) If two angles of one triangle are congruent to two angles of another triangle then the triangles are similar.

Examples of AA~

The $\Delta ABC \sim \Delta XZY$ are similar by AA~ because
1) They are both right triangles; therefore they both have a 90 degree angle.
2) All triangles add up to 180 degrees, since angle C is 40 degrees in ΔABC angle A will be 50 degrees. Therefore, $\angle$ A and $\angle$ X are congruent.

The $\Delta GHJ \sim \Delta GMK$ are similar by AA~ because
1) $\angle$ H and $\angle$ M are congruent by Corresponding Angles Postulate.
2) $\angle$ G and $\angle$ G are congruent since they are the same angle.





Side-Side-Side Similarity (SSS~): If the three sides of one triangle are proportional to the three sides of another triangle, then the two triangles are similar.

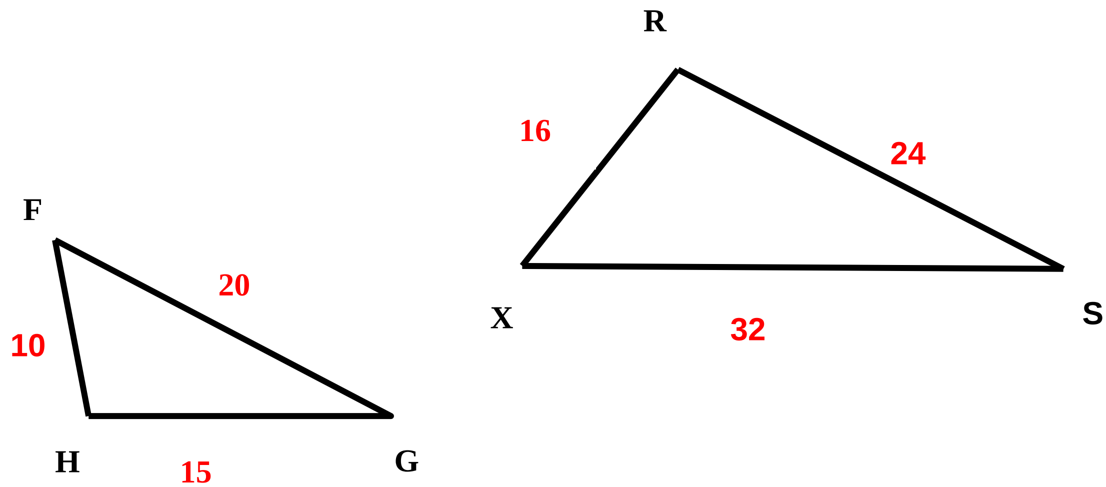
Example of SSS~

ΔFHG ~ ΔXRS because three sides of one triangle are proportional to three sides of another triangle.

$$\frac{FH}{XR} = \frac{10}{16} \equiv \frac{5}{8}$$

$$\frac{HG}{RS} = \frac{15}{24} = \frac{5}{8}$$

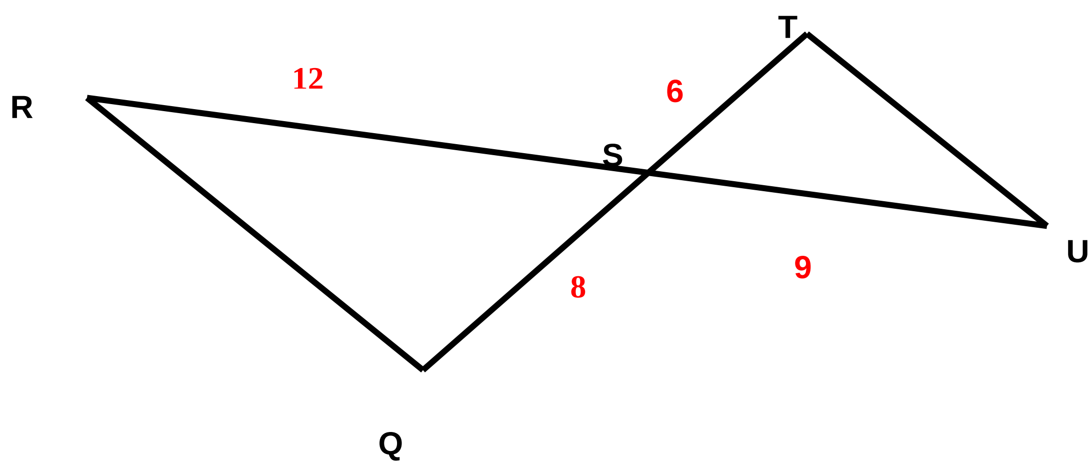
$$\frac{FG}{XS} = \frac{20}{32} = \frac{5}{8}$$



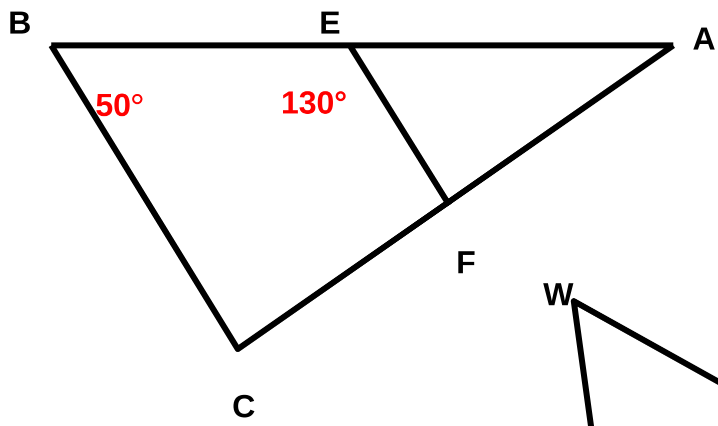
Side-Angle-Side Similarity (SAS~): If two sides of one triangle are proportional to two sides of another triangle and the included angles of these sides are congruent, then the two triangles are similar.

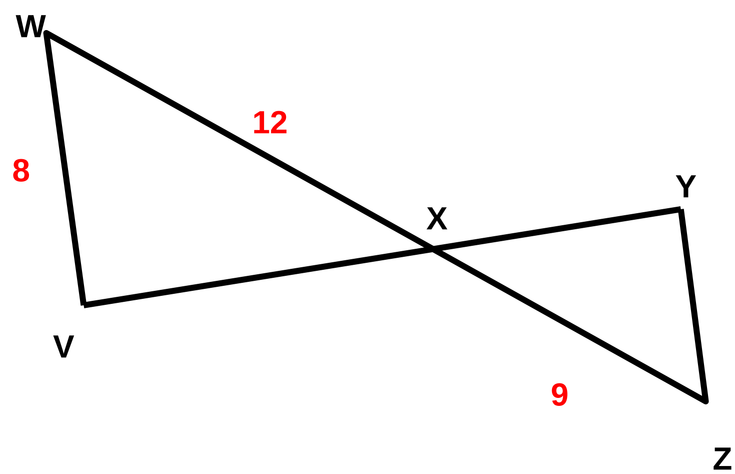
Example of SAS~
$\Delta RSQ \sim \Delta UST$ because

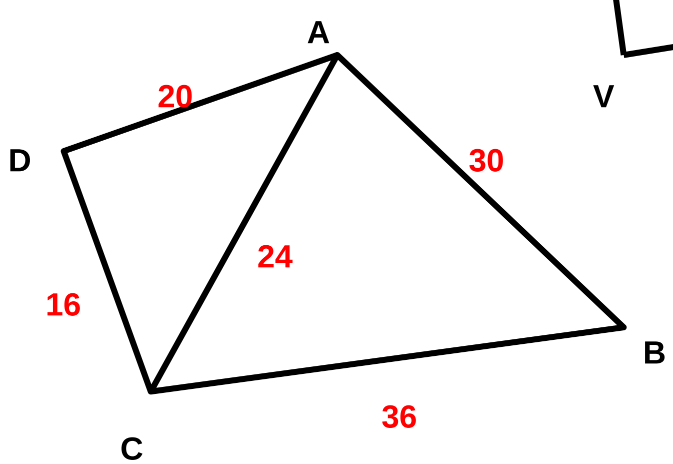
1) $\angle S \cong \angle S$ since Vertical Angles are Congruent
2) $\frac{RS}{US} = \frac{12}{9} = \frac{4}{3}$, Two sides of one triangle are proportional to two sides of another triangle.
$\frac{SQ}{ST} = \frac{8}{6} = \frac{4}{3}$



Can the two triangles shown be proved similar? If so, state the similarity and tell which method you used.







Pythagorean Theorem using Triangle Similarity

Mathematical goals

- Prove the Pythagorean Theorem using triangle similarity.

Common Core State Standards

MCC9-12.G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Introduction

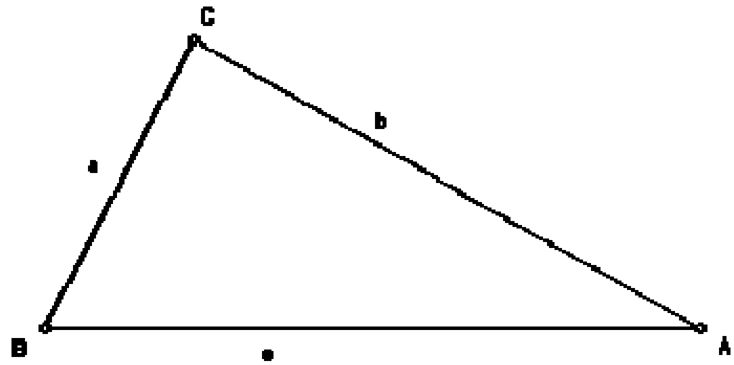
This task has students use cardboard cutouts to illustrate that the altitude to the hypotenuse divides a right triangle into two triangles that are similar to the original triangle. Then use AA to prove this theorem. Then, use this result to establish the Pythagorean relationship among the sides of a right triangle ($a^2 + b^2 = c^2$) and thus obtain an algebraic proof of the Pythagorean Theorem.

Materials

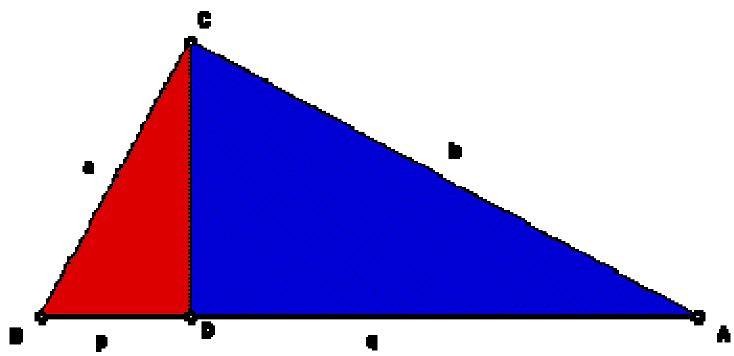
- Cardboard
- Straightedge

Use cardboard cutouts to illustrate the following investigation:

In the next figure, draw triangle ABC as a right triangle. Its right angle is angle C.



Next, draw CD perpendicular to AB as shown in the next figure.

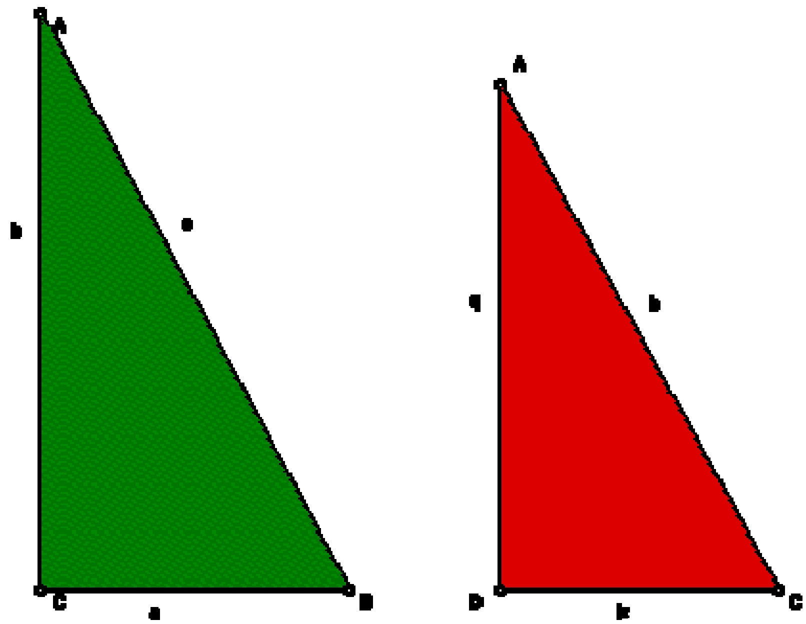


How many triangles do you see in the figure?

Why are the three triangles similar to each other?

<u>Compare triangles 1 and 3</u>:

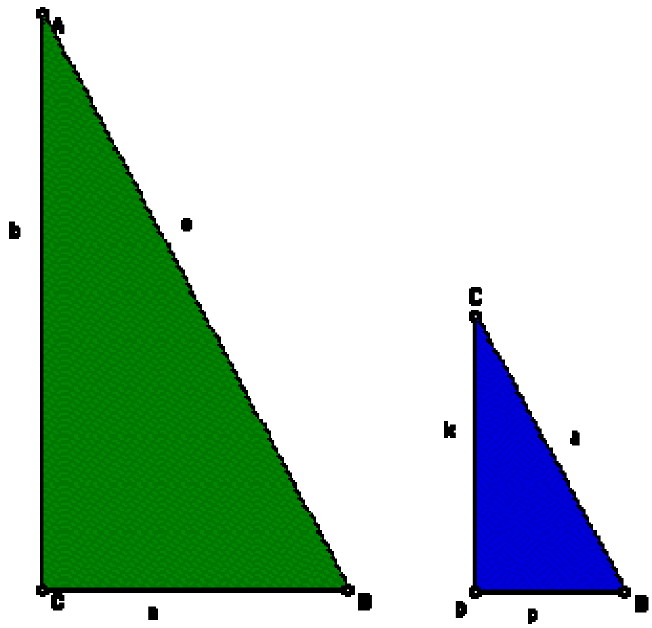
Triangle 1 (green) is the right triangle that we began with prior to constructing CD. Triangle 3 (red) is one of the two triangles formed by the construction of CD.



By comparing these two triangles, we can see that $\frac{c}{b} = \frac{b}{q}$ and $b^2 = cq$

<u>Compare triangles 1 and 2</u>:

Triangle 1 (green) is the same as above. Triangle 2 (blue) is the other triangle formed by constructing CD. Its right angle is angle D.



By comparing these two triangles, we see that $\frac{c}{a} = \frac{a}{p}$ and $a^2 = cp$

By adding the two equations:

$$a^2 + b^2 = cp + cq$$
$$a^2 + b^2 = c(p + q)$$

CD, we have that (p + q) = c. By substitution, we get

$$a^2 + b^2 = c^2$$

Lunch Lines

Mathematical goals

- Prove vertical angles are congruent.
- Understand when a transversal is drawn through parallel lines, special angles relationships occur.
- Prove when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angels are congruent.

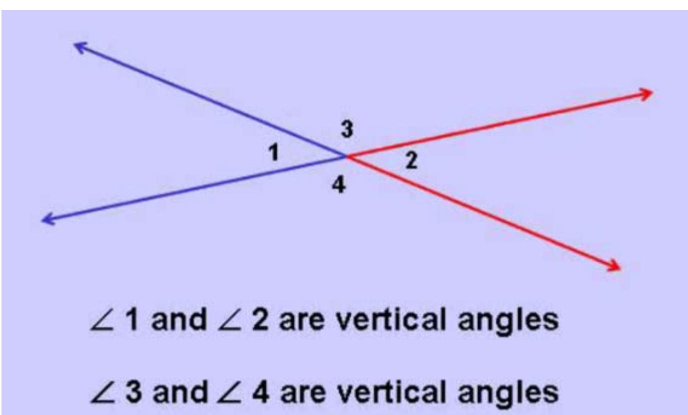
Common Core State Standards

MCC9-12.G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

Two angles are vertical angles if their sides form two pairs of opposite rays.



How do you know that vertical angles are congruent?

$m\angle 1 + m\angle 3 = 180°$ because the Linear Pair postulate
$m\angle 2 + m\angle 3 = 180°$ because the Linear Pair postulate

Set the two equations equal to each other since they both equal 180 degrees.

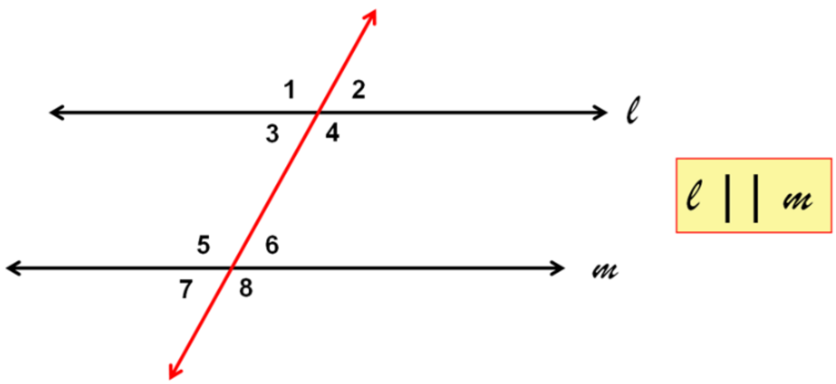
$m\angle 2 + m\angle 3 = m\angle 1 + m\angle 3$
$\quad -m\angle 3 \qquad\qquad -m\angle 3$
$m\angle 2 = m\angle 1$

Therefore: $\angle 2 \cong \angle 1$

Prove that $\angle 3 \cong \angle 4$ using a similar method.

When a transversal crosses parallel lines, there are several pairs of special angles. Let's look at a few together.

<u>Corresponding Angle Postulate</u>: If two parallel lines are cut by a transversal, then corresponding angles are congruent.



Using this postulate, name a pair of congruent angles.

How do we know that $\angle 3 \cong \angle 6$?

Alternate Interior Angle Theorem: If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

Prove this theorem using the figure above.

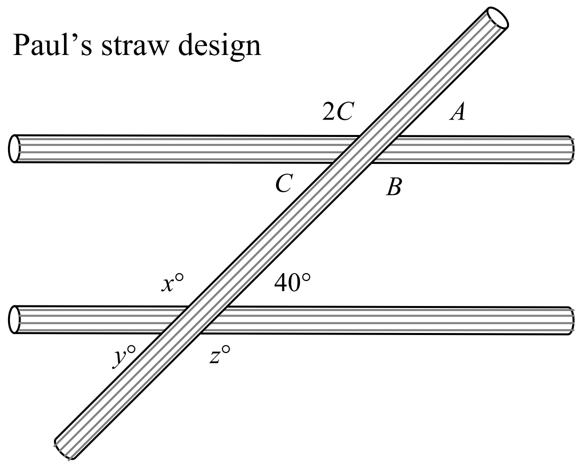
How do we know that ∠3 ≅∠5 are supplementary?

Same-Side Interior Angle Theorem: If two parallel lines are cut by a transversal, then same-side interior angles are supplementary.

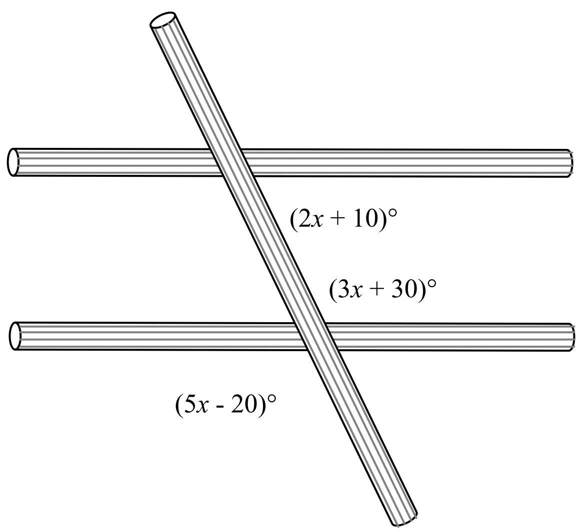
Prove this theorem using the figure above.

Paul, Jane, Justin, and Opal were finished with lunch and began playing with drink straws. Each one was making a line design using either 3 or 4 straws.
They had just come from math class where they had been studying special angles.
Paul pulled his pencil out of his book bag and labeled some of the angles and lines. He then challenged himself and the others to find all the labeled angle measurements in Paul and Justin's straw designs and to determine whether the lines that appear to be parallel really are parallel.

Paul's straw design



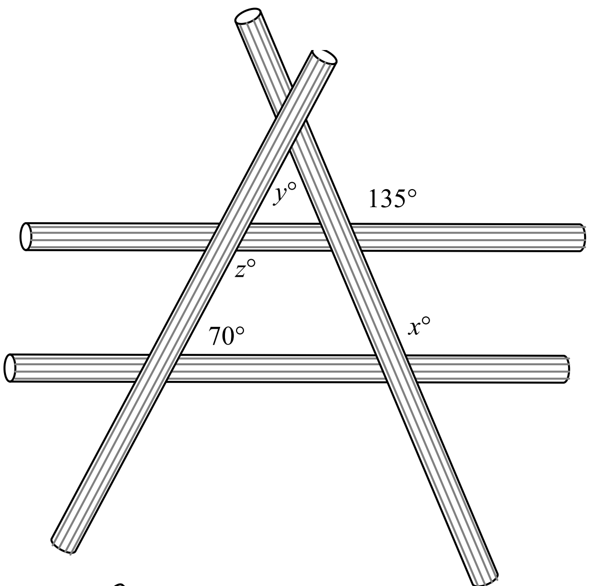
- Find all of the labeled angle measurements.
- Determine whether the lines that appear to be parallel really are parallel.
- Explain the reasoning for your results.



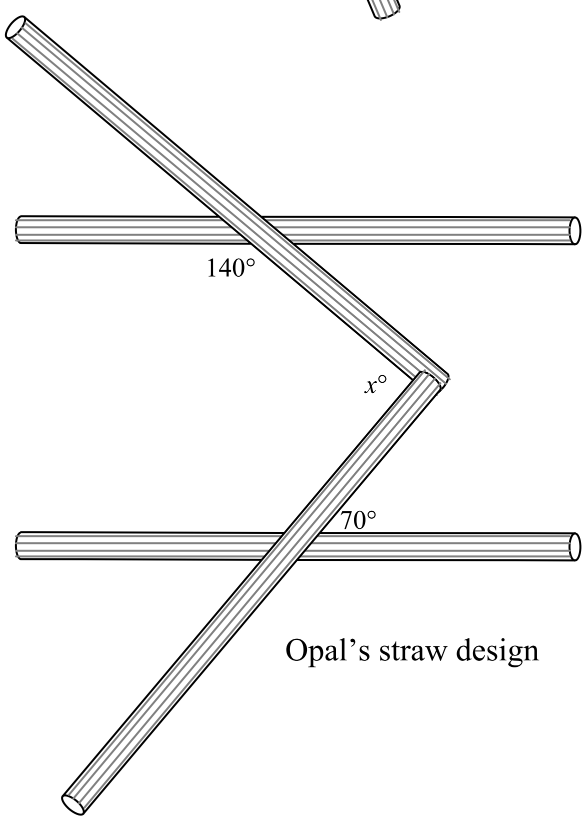
Justin's straw design

Paul then challenged himself and the others to find all the labeled angle measurements in Jane and Opal's straw designs knowing that the lines created by the straws in their designs were parallel.

Jane's straw design



- Find all of the labeled angle measurements (knowing that the lines created by the straws are parallel).
- Explain the reasoning for your results.



Opal's straw design

Triangle Proportionality Theorem

Mathematical goals

- Prove a line parallel to one side of a triangle divides the other two proportionally, and it's converse.

Common Core State Standards

MCC9-12.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

MCC9-12.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

MCC9-12.G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

MCC9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Allow students to use a Dynamic Geometry Software (DGS) package to draw two parallel lines, A and B.

The result should look similar to Figure 1.

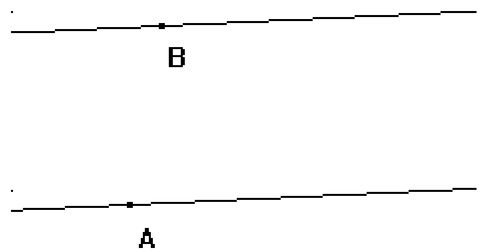


Figure 1

Then have them draw line $\overline{AB}$. Now have the students create a new point C on $\overline{AB}$ and draw another transversal. Label the intersections of this line with the parallel lines as points D and E as shown in Figure 2.

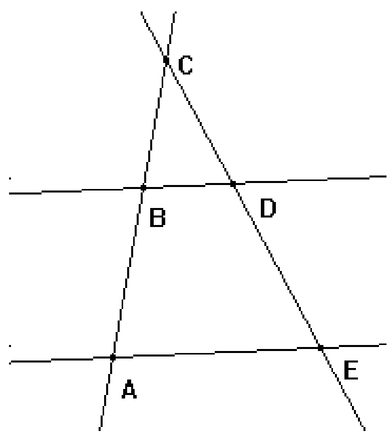


Figure 2

Ask the students to describe what they see.
Be sure that they see two triangles:
ΔCBD and ΔCAE.

Note: Students may benefit from using colored pencils to see both triangle ΔCBD and ΔCAE.

Now have students:

- use the DGS to measure the sides of the two triangles and their perimeters.

- record these values in the first two rows of the table below.
- use the DGS to compute the value of the ratio of the Side 1 measurements, Side 2 measurements, etc. Store each ratio computation on the DGS construction for use in a later question.

	Side 1	Side 2	Side 3	Perimeter
ΔCBD	$CB =$	$CD =$	$BD =$	$Perim_{\Delta CBD} =$
ΔCAE	$CA =$	$CE =$	$AE =$	$Perim_{\Delta CAE} =$
Ratio	$\frac{CB}{CA} =$	$\frac{CD}{CE} =$	$\frac{BD}{AE} =$	$\frac{Perim_{\Delta CBD}}{Perim_{\Delta CAE}} =$

The following questions could be addressed during the course of the activity:

1. Explain why $\overline{AB}$ (illustrated in figure 2) is a transversal.

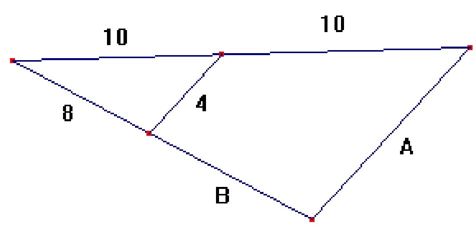
2. Explain why segments $\overline{CB}$ and $\overline{CA}$ are called corresponding segments.

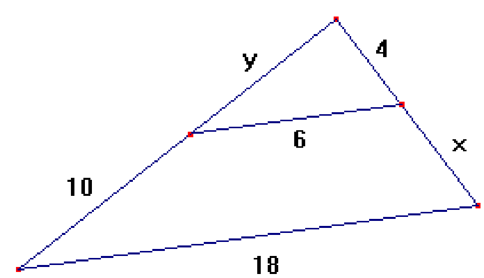
3. In view of the last row of results in the table, what appears to be true about the ratio of lengths defined by two transversals intersecting parallel lines?

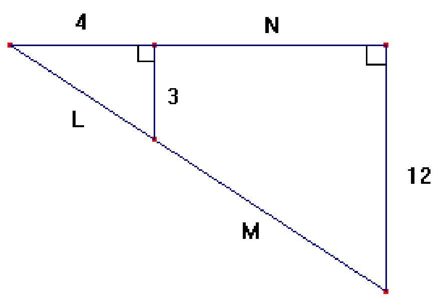
4. Grab different points and lines in the construction and move them around, if possible. While all of the measurements will change, one relationship will continue to hold no matter how the construction is changed. What is that relationship?

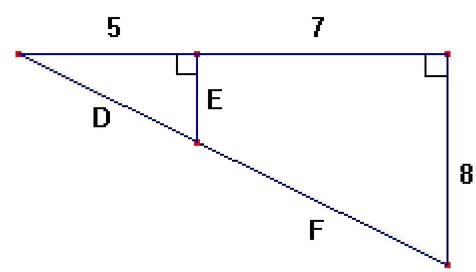
5. Allow students to compare their findings from question 4 to those of a classmate. Did everyone discover the same relationship?

6. Ask students to use the relationship that they have observed to solve for the unknown quantities in each of the following figures. They may assume that lines which look parallel in each figure are parallel.

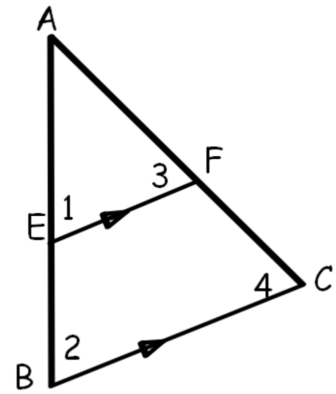








Triangle Proportionality Theorem
If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.



Proof #1:

Given: $\overleftrightarrow{EF} \parallel \overline{BC}$

Prove: $\frac{AE}{EB} = \frac{AF}{FC}$

Complete the proof:

Show that ΔAEF ~ ΔABC
Since $\overleftrightarrow{EF} \parallel \overline{BC}$, you can conclude that ∠1 ≅ ∠2 and ∠3 ≅∠4 by __________

So ΔAEF ~ ΔABC by ____________

Use the fact that corresponding sides of similar triangles are proportional to complete the proof

$\frac{AB}{AE} =$____________ Corresponding sides are proportional

$\frac{AE+EB}{AE} =$____________ Segment Addition Postulate

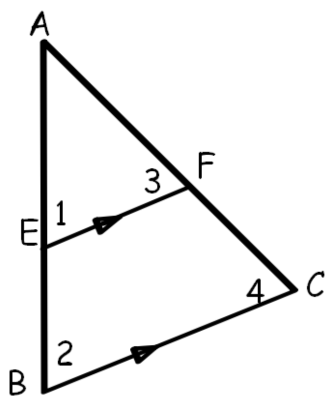
$1 + \frac{EB}{AE} =$_______________ Use the property that $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$\frac{EB}{AE} =$_______________ Subtract 1 from both sides.

$\frac{AE}{EB} =$________________ Take the reciprocal of both sides.

Converse of the Triangle Proportionality Theorem
If a line divides two sides of a triangle proportionally, then it is parallel to the third side

Proof #2



Given: $\frac{AE}{EB} = \frac{AF}{FC}$

Prove: $\overleftrightarrow{EF} \parallel \overline{BC}$

Complete the proof. Show that ΔAEF ~ ΔABC
It is given that $\frac{AE}{EB} = \frac{AF}{FC}$ and taking the reciprocal of both sides show that __________________.

Now add 1 to both sides by adding $\frac{AE}{AE}$ to the left side and $\frac{AF}{AF}$ to the right side.

____

Adding and using the Segment Addition Postulate gives _______________________.

Since $\angle A \cong \angle A, \Delta AEF \sim \Delta ABC$ by ____________________.

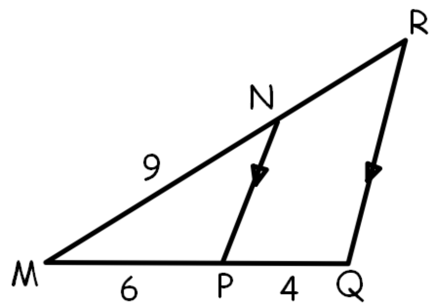
As corresponding angles of similar triangles, ∠AEF ≅ ________________.

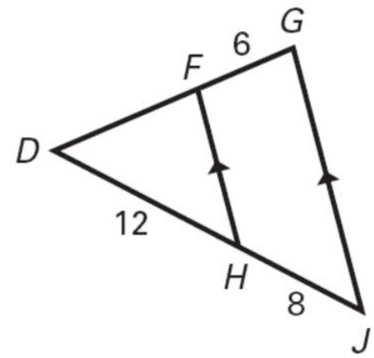
So, $\overleftrightarrow{EF} \parallel \overline{BC}$ by _________________.

Let's practice finding the length of a segment since you know how to prove the Triangle Proportionality Theorem and its converse.

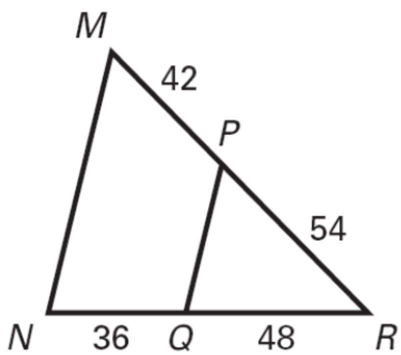
1. What is the length of NR?

2) What is the length of DF?





3. Given the diagram, determine whether $\overline{MN}$ is parallel to $\overline{PQ}$.



Challenges from Ancient Greece

Mathematical goals

- Make formal geometric constructions with a variety of tools and methods.
- Use congruent triangles to justify geometric constructions.

Common Core State Standards

MCC9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Standards for Mathematical Practice

4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Introduction

In this task, students will investigate and perform geometric constructions using Euclidean tools. The focus of this task is to learn how to copy line segments, copy an angle, bisect a segment, and bisect an angle.

It is highly recommended that students have access to tools such as a Mira™ or reflective mirror and Patty Paper™ to assist developing conceptual understandings of the geometry. During construction activities, students should also use technology, such as Geometer's Sketchpad to reinforce straight edge and compass work and to assist with dexterity challenges.

Materials

- compass and straightedge
- Mira™ or reflective mirror
- graph paper
- patty paper or tracing paper (optional)

The study of Geometry was born in Ancient Greece, where mathematics was thought to be embedded in everything from music to art to the governing of the universe. Plato, an ancient philosopher and teacher, had the statement, "Let no man ignorant of geometry enter here," placed at the entrance of his school. This illustrates the importance of the study of shapes and logic during that era. Everyone who learned geometry was challenged to construct geometric objects using two simple tools, known as Euclidean tools:

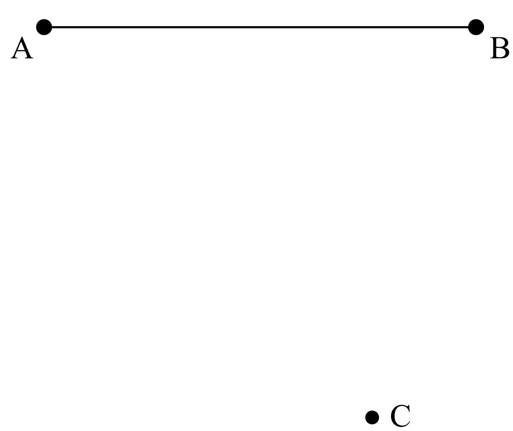
- A straight edge without any markings
- A compass

The straight edge could be used to construct lines, the compass to construct circles. As geometry grew in popularity, math students and mathematicians would challenge each other to create constructions using only these two tools. Some constructions were fairly easy (Can you construct a square?), some more challenging, (Can you construct a regular pentagon?), and some impossible even for the greatest geometers (Can you trisect an angle? In other words, can you divide an angle into three equal angles?). Archimedes (287-212 B.C.E.) came close to solving the trisection problem, but his solution used a marked straight edge. What constructions can you create?

Your First Challenge: Can you copy a line segment?

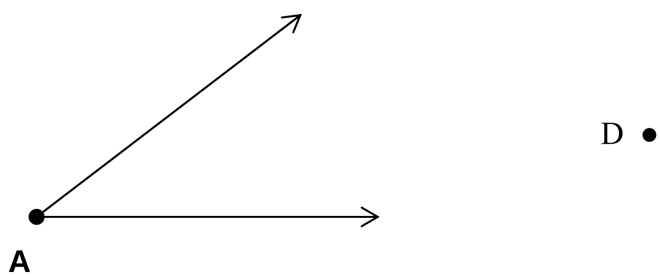
Step 1	Construct a circle with a compass on a sheet of paper.
Step 2	Mark the center of the circle and label it point A.
Step 3	Mark a point on the circle and label it point B.
Step 4	Draw $\overline{AB}$.

Your Second Challenge: Can you copy any line segment?
Below is a line segment $\overline{AB}$. Using only an unmarked straight edge and compass, can you construct another line segment the same length beginning at point C? Write instructions that explain the steps you used to complete the construction. (*Hint: An ancient geometer would require you to "cut off from the greater of two lines" a line segment equal to a given segment.*)

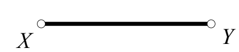
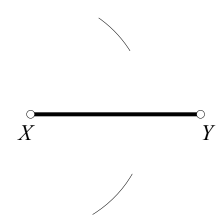
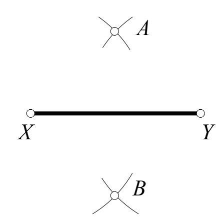


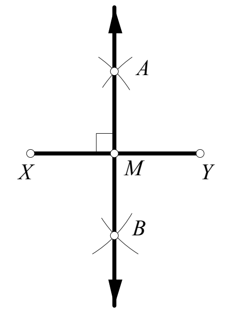
Your Third Challenge: Can you copy an angle?

Now that you know how to copy a segment, copying an angle is easy. How would you construct a copy of an angle at a new point? Discuss this with a partner and come up with a strategy. Think about what congruent triangles are imbedded in your construction and use them to justify why your construction works. Be prepared to share your ideas with the class.



Your Fourth Challenge: Can you bisect a segment?

1. Begin with line segment *XY*.	
2. Place the compass at point *X*. Adjust the compass radius so that it is more than (½)*XY*. Draw two arcs as shown here.	
3. Without changing the compass radius, place the compass on point *Y*. Draw two arcs intersecting the previously drawn arcs. Label the intersection points *A* and *B*.	

4. Using the straightedge, draw line *AB*. Label the intersection point *M*. Point *M* is the midpoint of line segment *XY*, and line *AB* is perpendicular to line segment *XY*.	

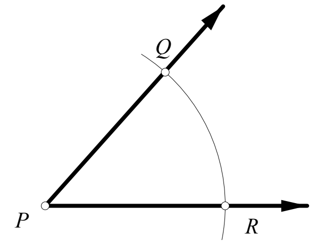
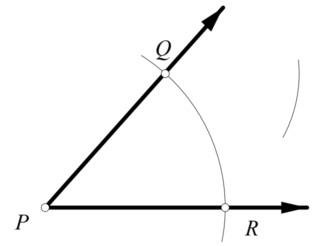
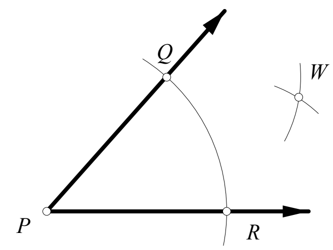
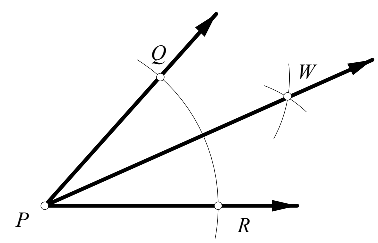
Construct the perpendicular bisector of the segments. Mark congruent segments and right angles. Check your work with a protractor.

1.

2.

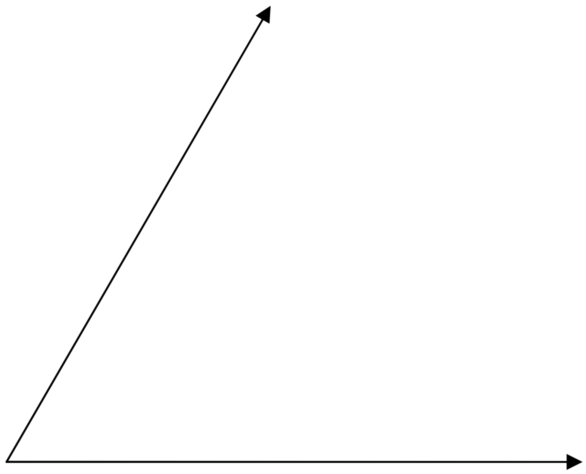
3.

Your Fifth Challenge: Can you bisect an angle?

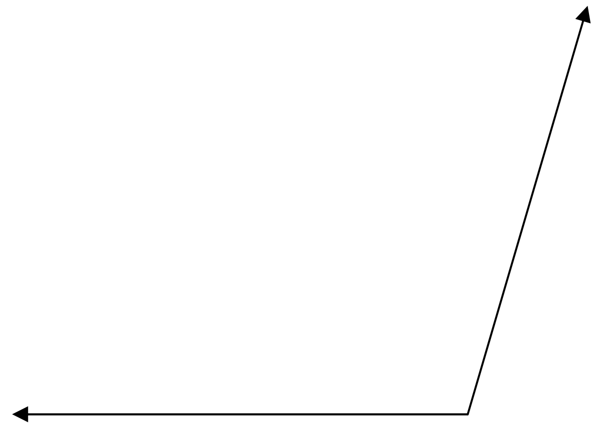
1. Let point *P* be the vertex of the angle. Place the compass on point *P* and draw an arc across both sides of the angle. Label the intersection points *Q* and *R*.	
2. Place the compass on point *Q* and draw an arc across the interior of the angle.	
3. Without changing the radius of the compass, place it on point *R* and draw an arc intersecting the one drawn in the previous step. Label the intersection point *W*.	
4. Using the straightedge, draw ray *PW*. This is the bisector of ∠*QPR*.	

Construct the angle bisector. Mark congruent angles. Check your construction by measuring with a protractor.

1.



2.



Constructing Parallel and Perpendicular Lines

Mathematical goals

- Make formal geometric constructions with a variety of tools and methods.
- Use congruent triangles to justify geometric constructions.

Common Core State Standards

MCC9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

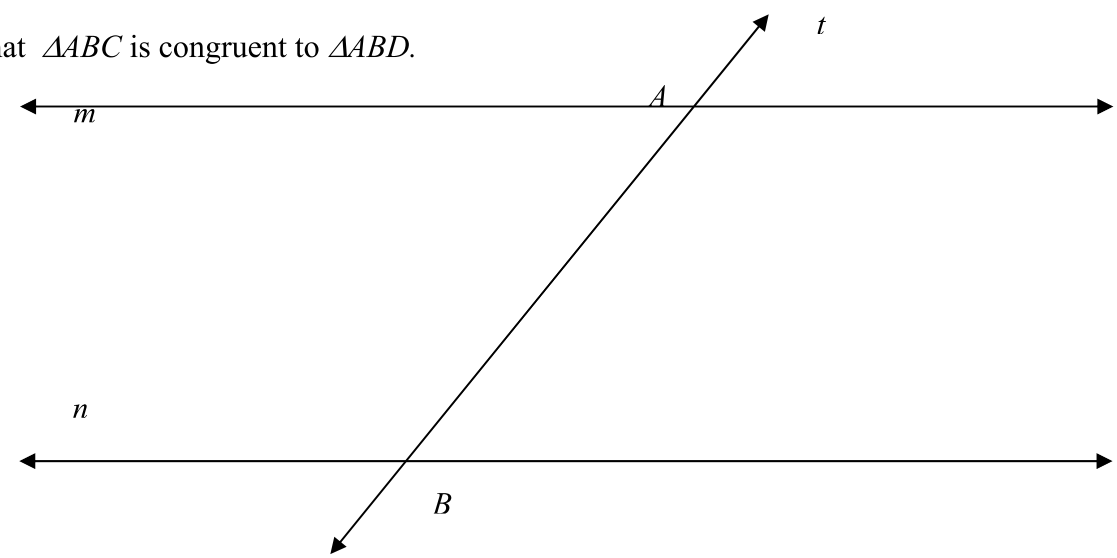
Standards for Mathematical Practice

4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Let's start by exploring features of parallel lines.

In the figure below, lines *m* and *n* are parallel and the line *t* intersects both.

- Label a new point *C* anywhere you choose on the line *m*. Connect *B* and *C* to form ΔABC.
- Construct a point *D* on line *n* so that points D and C are on opposite sides of line *t* and $AC = BD$.
- Verify that ΔABC is congruent to ΔABD.

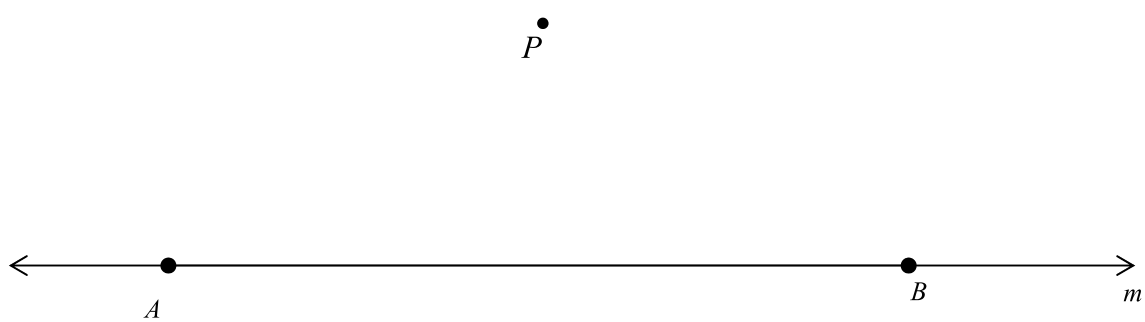


1. Name all corresponding and congruent parts of this construction.

2. What can you conclude about ∠CAB and ∠DBA? Will this always be true, regardless of where you choose *C* to be? Does it matter how line *t* is drawn? (*In other words could line t be perpendicular to both lines? Or slanted the other way*?)

3. What type of quadrilateral is *CADB*? Why do you think this is true?

Drawing a line that intersects two parallel lines creates two sets of four congruent angles. Use this observation to construct a parallel line to $\overline{AB}$ through a given point P.



4. Construct a perpendicular line to $\overline{AB}$ that passes through *P*. Label the intersection with line *m* as Q.

Constructions Inscribed in a Circle

Adapted from http://www.mathopenref.com/ 2009 Copyright Math Open Reference.

Mathematical goals

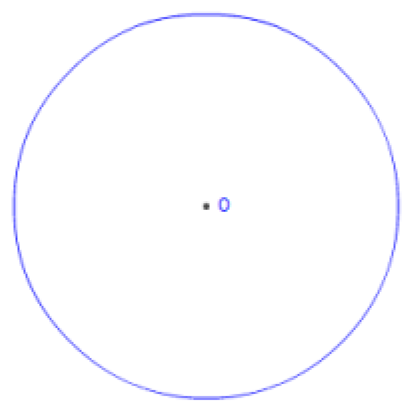
- Make formal geometric constructions with a variety of tools and methods.
- Use congruent triangles to justify geometric constructions.

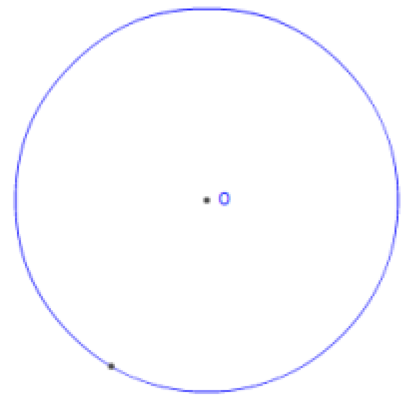
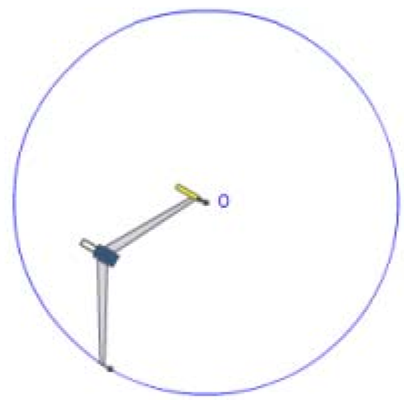
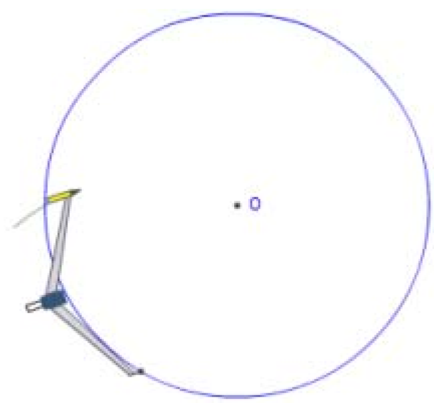
Common Core State Standards

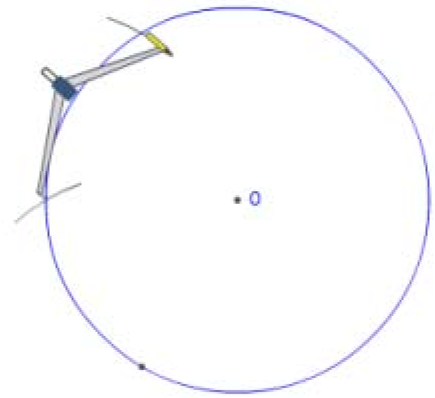
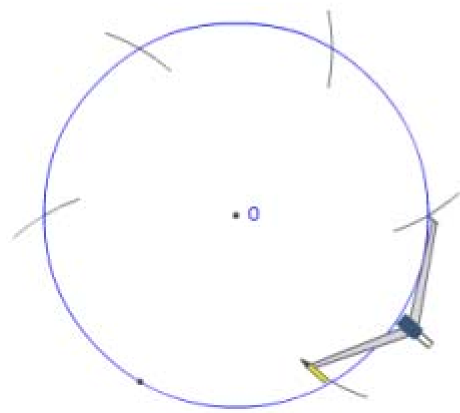
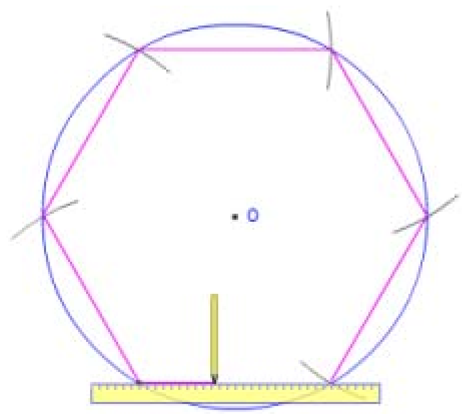
MCC9-12.G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Standards for Mathematical Practice

4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

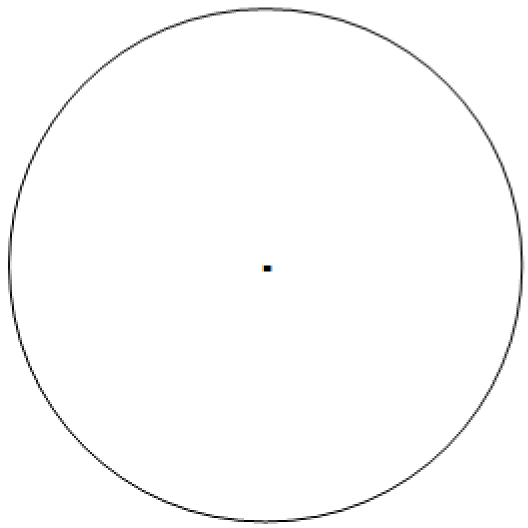
We start with the given circle, center O.	

1. Mark a point anywhere on the circle. Label this point P. This will be the first vertex of the hexagon.	
2. Set the compass on point P and set the width of the compass to the center of the circle O. The compass is now set to the radius of the circle $\overline{OP}$.	
3. Make an arc across the circle. This will be the next vertex of the hexagon. Call this point Q. (It turns out that the side length of a hexagon is equal to its circumradius - the distance from the center to a vertex).	

4. Move the compass on to the next vertex Q and draw another arc. This is the third vertex of the hexagon. Call this point R.	
5. Continue in this way until you have all six vertices. PQRSTU	
6. Draw a line between each successive pairs of vertices, for a total of six lines.	
7. Done. These lines form a regular hexagon inscribed in the given circle. Hexagon PQRSTU	

Try the example below using the steps to construct a hexagon inscribed in a circle using a compass and straightedge. Then brainstorm with a partner on how to construct an equilateral triangle inscribed in a circle.

1. Construct the largest regular hexagon that will fit in the circle below.



2. How would you construct an equilateral triangle inscribed in a given circle?

Proving Two Triangles are Congruent

Mathematical goals

- Knowing that rigid transformations preserve size and shape or distance and angle, use this fact to connect the idea of congruency and develop the definition of congruent.
- Use the definition of congruence, based on rigid motion, to show two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent.
- Use the definition of congruence, based on rigid motion, to develop and explain the triangle congruence criteria; ASA, SSS, and SAS, AAS.

Common Core State Standards
MCC9-12.G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

MCC9-12.G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

MCC9-12.G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Standards for Mathematical Practice
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

<u>Triangle Investigations: Station 1</u>

1. Draw a triangle of any size and shape and label it ABC.
2. Using the ruler and protractor, measure and record the angles and sides of the triangle. Remember the total degrees in any triangle should be _______?
3. Draw ΔDOG where AB=DO, BC=OG, and AC=DG.
4. Measure and record the angles of Δ DOG, how are they related to the angles of ABC?
5. What can you say about Δ ABC and Δ DOG?

Triangle Investigations: Station 2

1. Draw Δ KIT where KI=5 cm.
2. Using a protractor and point K as a vertex draw a 60^o angle with side length 11cm (label point T).
3. Connect point T to point I.
4. Measure the sides and angles of Δ KIT.
5. Now draw a line segment of 11cm. Name it FG.
6. Using point F as your vertex draw a 60^o angle with side length 5 cm. label point H.
7. Connect point G to point H.
8. Measure the sides and angles of Δ FGH.
9. Are triangles KIT and FGH congruent?

Triangle Investigations: Station 3

1. Draw a line segment that is 7cm long. Label it ML.
2. Using point L as a vertex draw a 38^o angle.
3. Draw a line segment beginning at point M that is 5 cm long and hits the side of the angle?
4. Repeat steps 1-2. This time connect the 5 cm segment at a different point on the side of the angle.
5. Are the 2 triangles congruent?

Triangle Investigations: Station 4

1. Draw a line segment LM that is 7 inches long.
2. Using point L as a vertex draw a 35^oangle.
3. Using point M as a vertex draw a 57^o angle.
4. Label the point of intersection of the two angles as N. This is triangle LMN.
5. Draw line segment ST that is 7 inches long.
6. Using point S as a vertex draw a 35^o angle.
7. Using point T as a vertex draw a 57^o angle.
8. Label the point of intersection of the two angles as U. This is triangle STU
9. Hold up the two triangles - are they congruent?

<u>Triangle Investigations: Station 5</u>

Practice with Triangle Congruence

State whether each pair of triangles is congruent by SSS, SAS, ASA, AAS, or HL; if none of these methods work, write N. If congruent, make a congruence statement for the triangles.

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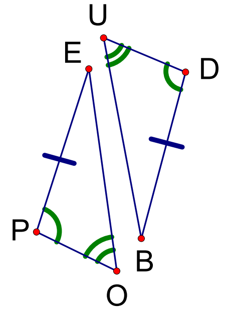

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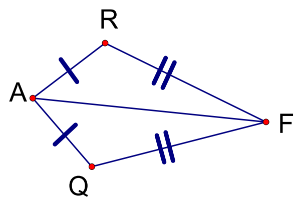

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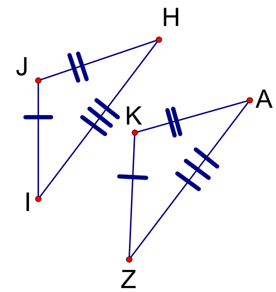

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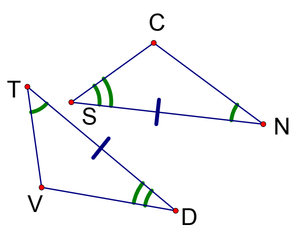

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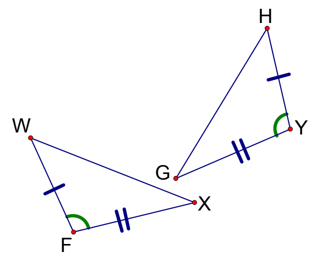

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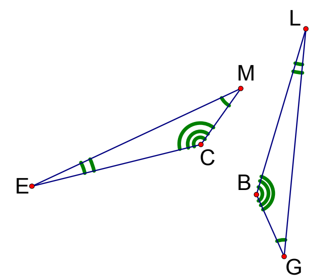

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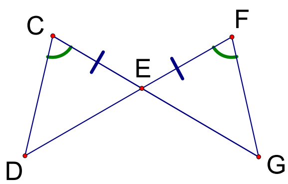

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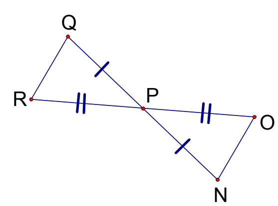

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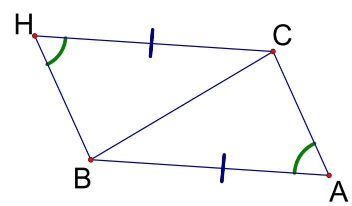

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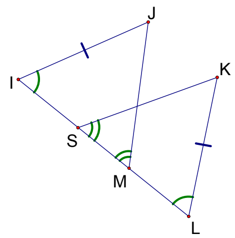

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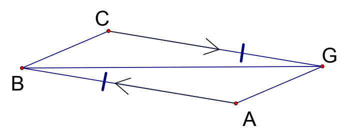

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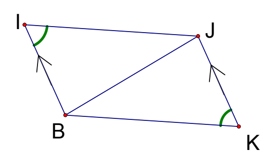


Triangle Proofs

Mathematical goals

- Prove theorems pertaining to triangles.

Common Core State Standards

MCC9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Introduction

This task provides students an opportunity to prove several triangles theorems including the measure of interior angles of a triangle sum to 180 degrees, base angles of isosceles triangles are congruent, and the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length. Students will review CPCTC (Corresponding Parts of Congruent Triangles are Congruent) before beginning the proofs. Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words.

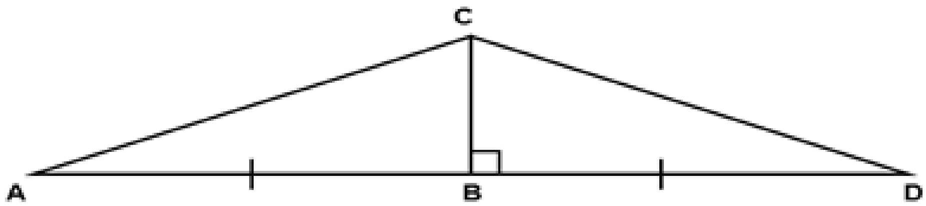
Materials

- Patty paper

Corresponding
Parts of
Congruent
Triangles are
Congruent

Remember the definition of congruent figures?
If two geometric figures are congruent, then their corresponding parts are congruent.

Example: In the figure below, how do we know that $\Delta ABC \cong \Delta DBC$?



Statements	Reason
1. $\overline{AB} \cong \overline{BD}$	1. Given
2. $\angle ABC \cong \angle DBC$	2. By Euclid's Perpendicular Postulate
3. $\overline{CB} \cong \overline{CB}$	3. Reflexive Property of Congruence
4. $\Delta ABC \cong \Delta DBC$	4. SAS Postulate

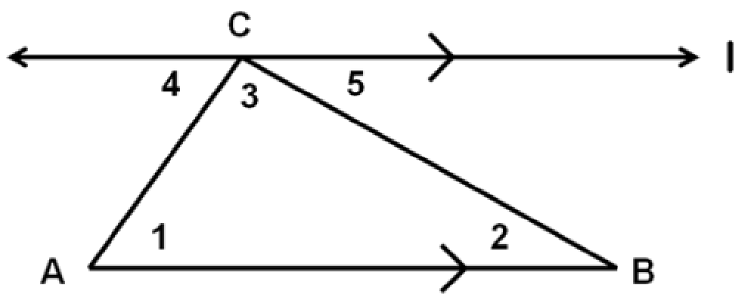
. . . And now that we know that the two triangles are congruent then by CPCTC all the other corresponding parts are congruent as well.

$\angle A \cong \angle D$
$\angle ACB \cong \angle DCB$
$\overline{AC} \cong \overline{DC}$

Let's start proving theorems about triangles using two column proofs. Fill in the missing statements and reasons in the proof below.

Theorem: The sum of the measure of the angles of any triangle is 180°.

Proof:
Given: The top line (that touches the top of the triangle) is running parallel to the base of the triangle.
Prove: $m\angle 1 + m\angle 3 + m\angle 2 = 180°$

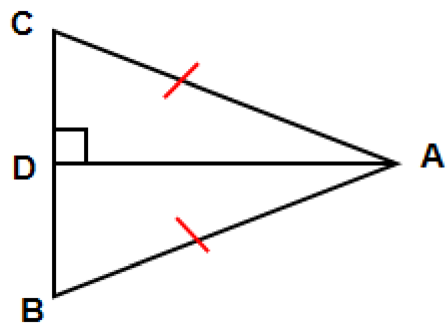


Statements	Reason
1. $m\angle 4 = m\angle 1$	1.
2. $m\angle 5 = m\angle 2$	2.
3.	3. Three angles form one side of the straight line
4. $m\angle 1 + m\angle 3 + m\angle 2 = 180°$	4.

<u>Isosceles Triangles</u>
Theorem: If two sides of a triangle are congruent, than the angles opposite those sides are congruent.

Fill in the following proof using postulates, theorems, and properties that you have learned.

Proof:
Given: $\overline{AC} \cong \overline{AB}$
Prove: $\angle C \cong \angle B$

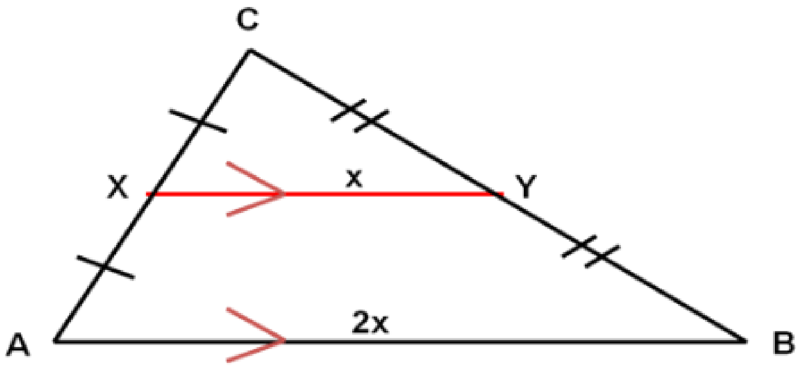


Statements	Reason
1.	1. Given
2. Draw $\overline{AD} \perp \overline{CB}$	2.
3.	3. Reflexive Property of Congruence
4. $\Delta CDA \cong \Delta BDA$	4.
5.	5.

Definition: A line segment whose endpoints are the midpoint of two sides of a triangle is called a midsegment of the triangle.

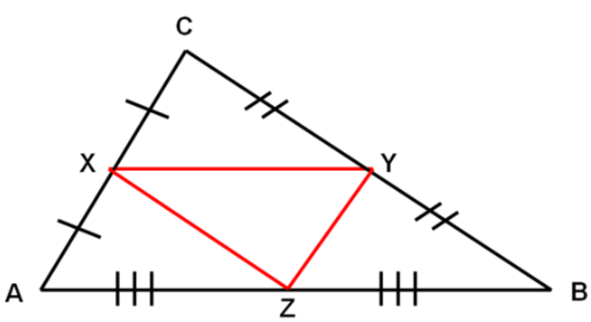
Theorem: The segment connecting the midpoints of two sides of the triangle is parallel to the third side and half the length of the third side.

Using the figure below. $\overline{XY} \parallel \overline{AB}$ and XY = ½ (AB) or AB = 2(XY)



Let's prove this theorem using a sheet of patty paper.

1) Draw Δ ABC on a sheet of patty paper.
2) Fold and pinch to locate the three midpoints of the triangle.
3) Draw and label the three midpoints X, Y, Z.
4) Draw segments $\overline{XY}$, $\overline{YZ}$, and $\overline{XZ}$.



Using your construction, verify:

$\overline{XY} \parallel \overline{AB}$, $\overline{YZ} \parallel \overline{XY}$, $\overline{CA}$, and $\overline{XZ} \parallel \overline{CB}$

Centers of Triangles

Mathematical goals

- Prove the medians of a triangle meet at a point.
- Bisect a segment and an angle.
- Construct perpendicular lines, including the perpendicular bisector of a line segment.

Common Core State Standards

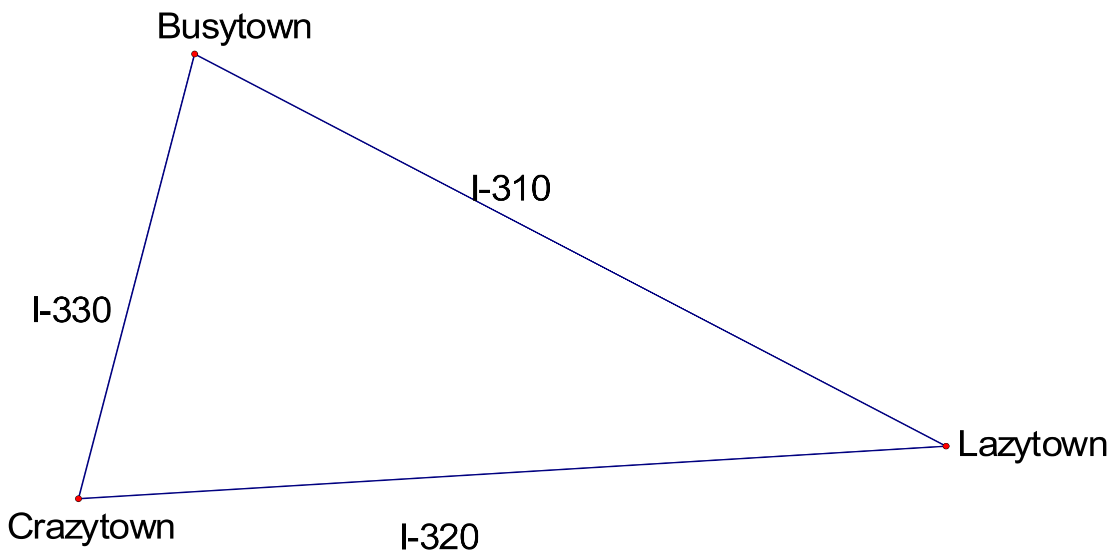
MCC9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Triangle Center:	Point of Concurrency of:	Significance of:
Incenter	Angle bisectors	Center of inscribed circle Equidistant from the sides of the triangle
Circumcenter	Perpendicular bisectors	Center of the circumscribing circle Equidistant from the vertices of the
Orthocenter	Altitudes	
Centroid	Medians	Center of balance or gravity The distance from a vertex to the centroid is twice the distance from the centroid to the opposite side.

A developer plans to build an amusement park but wants to locate it within easy access of the three largest towns in the area as shown on the map below. The developer has to decide on the best location and is working with the ABC Construction Company to minimize costs wherever possible. No matter where the amusement park is located, roads will have to be built for access directly to the towns or to the existing highways.



1. Just by looking at the map, choose the location that you think will be best for building the amusement park. Explain your thinking.

2. Now you will use some mathematical concepts to help you choose a location for the tower. Investigate the problem above by constructing the following:
   a) all 3 medians of the triangle
   b) all 3 altitudes of the triangle
   c) all 3 angle bisectors of the triangle
   d) all 3 perpendicular bisectors of the triangle

   You have four different kinds of tools at your disposal- patty paper, MIRA, compass and straight edge, and Geometer's Sketch Pad. Use a different tool for each of your constructions.

3. Choose a location for the amusement park based on the work you did in part 2. Explain why you chose this point.

4. How close is the point you chose in part 3, based on mathematics, to the point you chose by observation?

You have now discovered that each set of segments resulting from the constructions above always has a point of intersection. These four points of intersection are called the ***points of concurrency*** of a triangle.

The intersection point of the medians is called the ***centroid*** of the triangle.
The intersection point of the angle bisectors is called the ***incenter*** of the triangle.
The intersection point of the perpendicular bisectors is called the ***circumcenter*** of the triangle.
The intersection point of the altitudes is called the ***orthocenter*** of the triangle.

5. Can you give a reasonable guess as to why the specific names were given to each point of concurrency?

6. Which triangle center did you recommend for the location of the amusement park?

7. The president of the company building the park is concerned about the cost of building roads from the towns to the park. What recommendation would you give him? Write a memo to the president explaining your recommendation.

Constructing with Diagonals

Mathematical goals

- Prove theorems pertaining to lines and angles.
- Prove theorems pertaining to triangles.
- Prove theorems pertaining to parallelograms.

Common Core State Standards

MCC9-12.G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

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Introduction

This task provides a guided discovery and investigation of the properties of quadrilaterals. Students will determine which quadrilateral(s) can be constructed based on specific information about the diagonals of the quadrilateral(s).
Sample proofs are given for each problem. The samples provided are not the only correct way these proofs can be written. Students should realize that proofs can be logically organized with differing orders of steps. They should also be given the opportunity to decide which type of proof they prefer writing.

Materials

There are many ways students can approach this task and the supplies will depend upon the method you choose for your students.

- Hands-on manipulatives like spaghetti noodles, straws, pipe cleaners, d-stix, etc. can be used to represent the lengths of the sides. Protractors will be needed to create the indicated angles between the sides and clay or play dough can be used to hold the sides together.
- Students can use compasses, straightedges and protractors to construct the triangles.
- Geometer's Sketchpad, or similar software, is a good tool to use in these investigations.

1. Construct two segments of different length that are perpendicular bisectors of each other. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints? Justify your answer.

2. Repeat #1 with two congruent segments. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints? Justify your answer.

3. Construct two segments that bisect each other but are not perpendicular. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints? Justify your answer.

4. What if the two segments in #3 above are congruent in length? What type of quadrilateral is formed? What names can be used to describe the quadrilaterals formed using these constraints? Justify your answer.

5. Draw a segment and mark the midpoint. Now construct a segment that is perpendicular to the first segment at the midpoint but is not bisected by the original segment. Connect the four end points to form a quadrilateral. What names can be used to describe the quadrilaterals formed using these constraints? Justify your answer.

6. In the above constructions you have been discovering the properties of the diagonals of each member of the quadrilateral family. Stop and look at each construction. Summarize any observations you can make about the special quadrilaterals you constructed. If there are any quadrilaterals that have not been constructed yet, investigate any special properties of their diagonals.

7. Complete the chart below by identifying the quadrilateral(s) for which the given condition is necessary.

Conditions	Quadrilateral(s)	Explain your reasoning
Diagonals are perpendicular.		
Diagonals are perpendicular and only one diagonal is bisected.		
Diagonals are congruent and intersect but are not perpendicular.		
Diagonals bisect each other.		
Diagonals are perpendicular and bisect each other.		

Diagonals are congruent and bisect each other.		
Diagonals are congruent, perpendicular and bisect each other.		

8. As you add more conditions to describe the diagonals, how does it change the types of quadrilaterals possible? Why does this make sense?

9. Name each of the figures below using as many names as possible and state as many properties as you can about each figure.

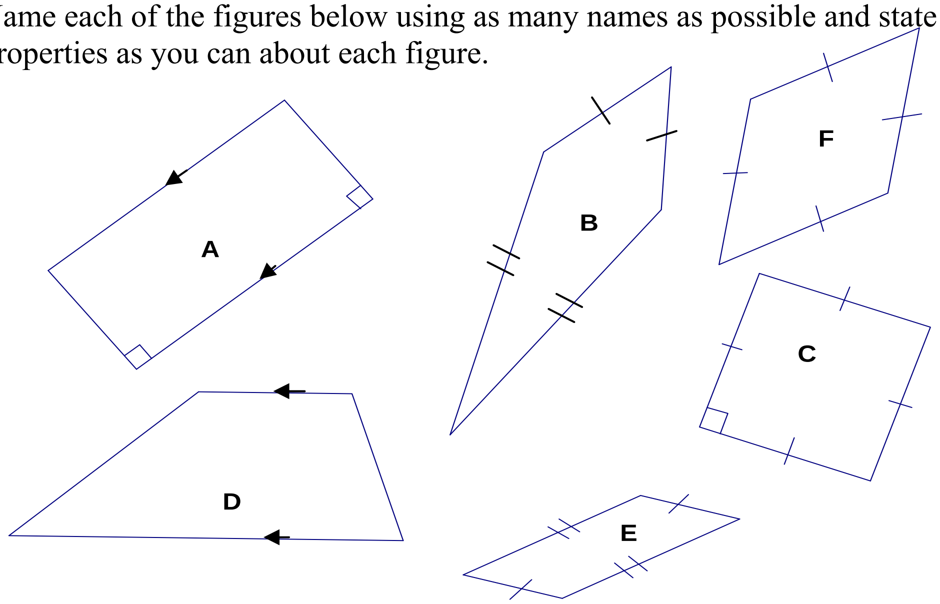

Figure	Names	Properties
A		
B		
C		
D		
E		
F		

10. Identify the properties that are always true for the given quadrilateral by placing an X in the appropriate box.

Property	Parallelogram	Rectangle	Rhombus	Square	Isosceles Trapezoid	Kite
Opposite sides are parallel.						
Only one pair of opposite sides is parallel.						
Opposite sides are congruent.						
Only one pair of opposite sides is congruent.						
Opposite angles are congruent.						
Only one pair of opposite angles is congruent.						
Each diagonal forms 2 $\cong$ triangles.						
Diagonals bisect each other.						
Diagonals are perpendicular.						
Diagonals are congruent.						
Diagonals bisect vertex angles.						
All $\angle$s are right $\angle$s.						
All sides are congruent.						
Two pairs of consecutive sides are congruent.						

11. Using the properties in the table above, list the **minimum** conditions necessary to prove that a quadrilateral is:
    a. a parallelogram
    b. a rectangle
    c. a rhombus
    d. a square
    e. a kite
    f. an isosceles trapezoid

Proving Quadrilaterals in the Coordinate Plane

Mathematical goals

- Prove theorems pertaining to lines and angles.
- Prove theorems pertaining to triangles.
- Prove theorems pertaining to parallelograms.

Common Core State Standards

MCC9-12.G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

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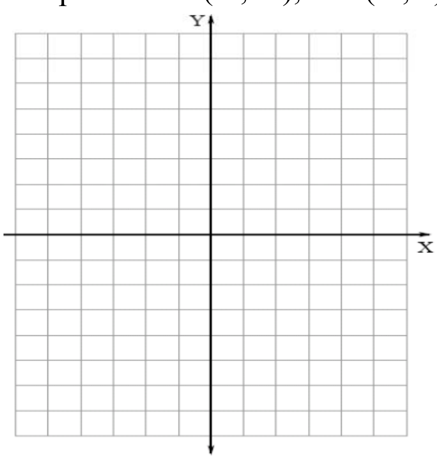
Plot points A = (-3, -1), B = (-1, 2), C = (4, 2), and D = (2, -1).



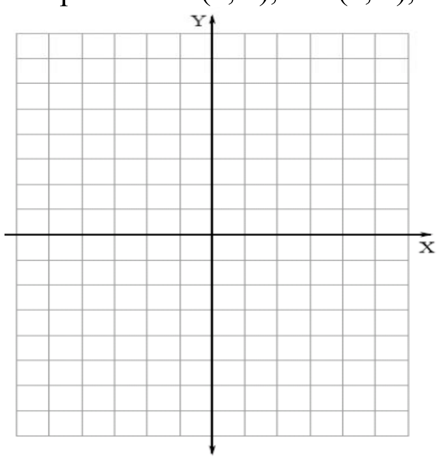
1. What specialized geometric figure is quadrilateral ABCD? Support your answer mathematically.

2. Draw the diagonals of ABCD. Find the coordinates of the midpoint of each diagonal. What do you notice?

3. Find the slopes of the diagonals of ABCD. What do you notice?

4. The diagonals of ABCD create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

Plot points E = (1, 2), F = (2, 5), G = (4, 3) and H = (5, 6).



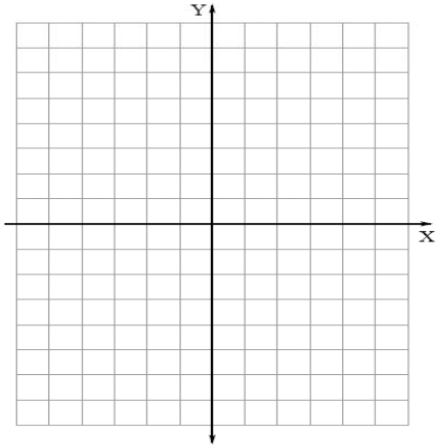
5. What specialized geometric figure is quadrilateral EFHG? Support your answer mathematically using two different methods.

6. Draw the diagonals of EFHG. Find the coordinates of the midpoint of each diagonal. What do you notice?

7. Find the slopes of the diagonals of EFHG. What do you notice?

8. The diagonals of EFHG create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

Plot points P = (4, 1), W = (-2, 3), M = (2,-5), and K = (-6, -4).



9. What specialized geometric figure is quadrilateral PWKM? Support your answer mathematically.

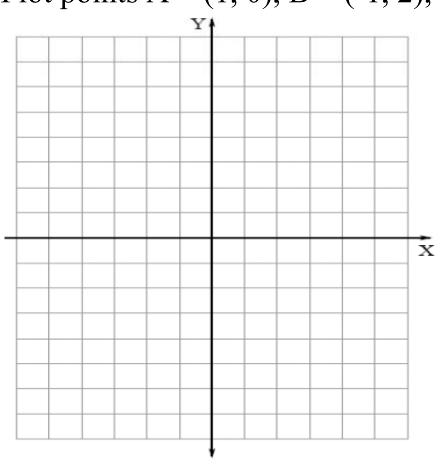
10. Draw the diagonals of PWKM. Find the coordinates of the midpoint of each diagonal. What do you notice?

11. Find the lengths of the diagonals of PWKM. What do you notice?

12. Find the slopes of the diagonals of PWKM. What do you notice?

13. The diagonals of ABCD create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

Plot points A = (1, 0), B = (-1, 2), and C = (2, 5).



14. Find the coordinates of a fourth point D that would make ABCD a rectangle. Justify that ABCD is a rectangle.

15. Find the coordinates of a fourth point D that would make ABCD a parallelogram that is not also a rectangle. Justify that ABCD is a parallelogram but is not a rectangle.

Culminating Task: Company Logo

Adapted from Common Core-Aligned Task with Instruction Supports: Silicon Valley Mathematics Initiative, SCALE, New York City DOE, 2011

Mathematical goals

- Mathematicians recognize congruence of plane figures and are able to prove congruence using geometric theorems.
- Congruence of plane figures can be verified through rigid motions.
- Congruence can be used to solve problems and prove geometric relationships.

Common Core State Standards

MCC9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

MCC9-12.G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

MCC9-12.G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

MCC9-12.G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

MCC9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

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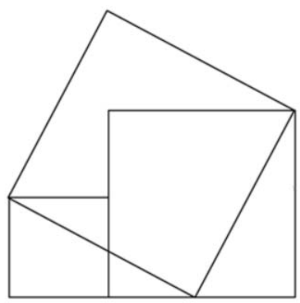
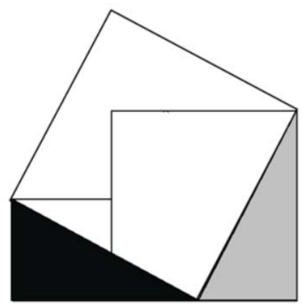
A company has designed a new logo using overlapping squares.





1. How many squares do you see in the logo?
   Describe where you see the squares.

2. The logo designer colored two triangles in the logo.
   How are the two triangles related?
   Justify your answer.

3. What are the relationships between the sizes of the squares in the original logo? Explain your findings.