STATE COMMON CORE STANDARDS

Common Core State Standards

Mathematics I

Integrated Pathway

Student Resource Book

Unit 1

WALCH EDUCATION®

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WALCH EDUCATION

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Introduction to the Program

Welcome to the *CCSS Integrated Pathway: Mathematics I Student Resource Book.* This book will help you learn how to use algebra, geometry, and data analysis to solve problems. Each lesson builds on what you have already learned. As you participate in classroom activities and use this book, you will master important concepts that will help to prepare you for the EOCT and for other mathematics assessments and courses.

This book is your resource as you work your way through the Math I course. It includes explanations of the concepts you will learn in class; math vocabulary and definitions; formulas and rules; and exercises so you can practice the math you are learning. Most of your assignments will come from your teacher, but this book will allow you to review what was covered in class, including terms, formulas, and procedures.

- In **Unit 1: Relationships Between Quantities**, you will learn first about structures of expressions, and then build on this knowledge by creating equations and inequalities in one variable and then in two variables. The unit focuses on real-world applications of linear equations, linear inequalities, and exponential equations. In addition to creating equations, you will also graph them. The unit concludes with manipulating formulas.
- In **Unit 2: Linear and Exponential Relationships**, you will be introduced to function notation, domain and range, rates of change, and sequences. The introduction of these concepts will allow you to deepen your study of linear and exponential functions in terms of comparing, building, and interpreting them. You will also perform operations and transformations on functions.
- In **Unit 3: Reasoning with Equations**, you will begin by solving linear equations and inequalities, as well as exponential equations. With this foundation for solving linear equations, you will move on to solving systems of equations using the substitution and elimination methods. Then, you will explore how to solve systems of linear equations by graphing.
- In **Unit 4: Descriptive Statistics**, you will start with single-measure variables and become fluent with summarizing, displaying, and interpreting data. Then you will build on these practices to include two-variable data. You will also be introduced to two-way frequency tables and fitting linear models to data. After learning how to fit functions to data, you will learn how to interpret the models, including evaluating the fit and learning the difference between correlation and causation.

- In **Unit 5: Congruence, Proof, and Constructions**, you will define transformations in terms of rigid motions. Geometry and algebra merge as you apply rotations, reflections, and translations to points and figures while using function notation. You will also explore triangle congruency, and construct lines, segments, angles, and polygons.
- In **Unit 6: Connecting Algebra and Geometry Through Coordinates**, your study of the links between the two math disciplines deepens as you use algebraic equations to prove geometric theorems involving distance and slope. You will create equations for parallel and perpendicular lines, and use the distance formula to find the perimeter and area of figures.

Each lesson is made up of short sections that explain important concepts, including some completed examples. Each of these sections is followed by a few problems to help you practice what you have learned. The "Words to Know" section at the beginning of each lesson includes important terms introduced in that lesson.

As you move through your Math I course, you will become a more confident and skilled mathematician. We hope this book will serve as a useful resource as you learn.

Lesson 1: Interpreting Structure in Expressions

Common Core State Standards

A–SSE.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.*

Essential Questions

1. How are algebraic expressions different from algebraic equations?
2. How is the order of operations applied to expressions and simple formulas at specific values?
3. How are verbal phrases translated into algebraic expressions?

WORDS TO KNOW

algebraic expression	a mathematical statement that includes numbers, operations, and variables to represent a number or quantity
base	the factor being multiplied together in an exponential expression; in the expression a^b, a is the base
coefficient	the number multiplied by a variable in an algebraic expression
constant	a quantity that does not change
exponent	the number of times a factor is being multiplied together in an exponential expression; in the expression a^b, b is the exponent
factor	one of two or more numbers or expressions that when multiplied produce a given product
like terms	terms that contain the same variables raised to the same power

order of operations	the order in which expressions are evaluated from left to right (grouping symbols, evaluating exponents, completing multiplication and division, completing addition and subtraction)
term	a number, a variable, or the product of a number and variable(s)
variable	a letter used to represent a value or unknown quantity that can change or vary

Recommended Resources

- Math-Play.com. "Algebraic Expressions Millionaire Game."

  http://walch.com/rr/CAU1L1Expressions

  "Algebraic Expressions Millionaire Game" can be played alone or in two teams. For each question, players have to identify the correct mathematical expression that models a given expression.

- Quia. "Algebraic Symbolism Matching Game."

  http://walch.com/rr/CAU1L1AlgSymbolism

  In this matching game, players pair each statement with its algebraic interpretation. There are 40 matches to the provided game.

Lesson 1.1.1: Identifying Terms, Factors, and Coefficients

Introduction

Thoughts or feelings in language are often conveyed through expressions; however, mathematical ideas are conveyed through **algebraic expressions**. Algebraic expressions are mathematical statements that include numbers, operations, and variables to represent a number or quantity. **Variables** are letters used to represent values or unknown quantities that can change or vary. One example of an algebraic expression is $3x - 4$. Notice the variable, x.

Key Concepts

- Expressions are made up of **terms**. A term is a number, a variable, or the product of a number and variable(s). An addition or subtraction sign separates each term of an expression.
- In the expression $4x^2 + 3x + 7$, there are 3 terms: $4x^2$, $3x$, and 7.
- The **factors** of each term are the numbers or expressions that when multiplied produce a given product. In the example above, the factors of $4x^2$ are 4 and x^2. The factors of $3x$ are 3 and x.
- 4 is also known as the **coefficient** of the term $4x^2$. A coefficient is the number multiplied by a variable in an algebraic expression. The coefficient of $3x$ is 3.
- The term $4x^2$ also has an **exponent**. Exponents indicate the number of times a factor is being multiplied by itself. In this term, 2 is the exponent and indicates that x is multiplied by itself 2 times.
- Terms that do not contain a variable are called **constants** because the quantity does not change. In this example, 7 is a constant.

Expression	$4x^2 + 3x + 7$		
Terms	$4x^2$	$3x$	7
Factors	4 and x^2	3 and x	–
Coefficients	4	3	–
Constants	–	–	7

- Terms with the same variable raised to the same exponent are called **like terms**. In the example $5x + 3x - 9$, $5x$ and $3x$ are like terms. Like terms can be combined following the **order of operations** by evaluating grouping symbols, evaluating exponents, completing multiplication and division, and completing addition and subtraction from left to right. In this example, the sum of $5x$ and $3x$ is $8x$.

Guided Practice 1.1.1

Example 1

Identify each term, coefficient, constant, and factor of $2(3 + x) + x(1 - 4x) + 5$.

1. Simplify the expression.

   The expression can be simplified by following the order of operations and combining like terms.

$2(3 + x) + x(1 - 4x) + 5$	Distribute 2 over $3 + x$.
$6 + 2x + x(1 - 4x) + 5$	Distribute x over $1 - 4x$.
$6 + 2x + x - 4x^2 + 5$	Combine like terms: $2x$ and x; 6 and 5.
$11 + 3x - 4x^2$	

   It is common to rearrange the expression so the powers are in descending order, or go from largest to smallest power.

   $-4x^2 + 3x + 11$

2. Identify all terms.

   There are three terms in the expression: $-4x^2$, $3x$, and 11.

3. Identify any factors.

   The numbers or expressions that, when multiplied, produce the product $-4x^2$ are -4 and x^2. The numbers or expressions that, when multiplied, produce the product $3x$ are 3 and x.

4. Identify all coefficients.

   The number multiplied by a variable in the term $-4x^2$ is -4; the number multiplied by a variable in the term $3x$ is 3; therefore, -4 and 3 are coefficients.

5. Identify any constants.

   The number that does not change in the expression is 11; therefore, 11 is a constant.

Example 2

A smartphone is on sale for 25% off its list price. The sale price of the smartphone is \$149.25. What expression can be used to represent the list price of the smartphone? Identify each term, coefficient, constant, and factor of the expression described.

1. Translate the verbal expression into an algebraic expression.

   Let x represent the unknown list price. Describe the situation. The list price is found by adding the discounted amount to the sale price:

   sale price + discount amount

   The discount amount is found by multiplying the discount percent by the unknown list price. The expression that represents the list price of the smartphone is $149.25 + 0.25x$.

2. Identify all terms.

   There are two terms described in the expression: the sale price of \$149.25, and the discount of 25% off the list price, or 149.25 and $0.25x$.

3. Identify the factors.

   $0.25x$ is the product of the factors 0.25 and x.

4. Identify all coefficients.

   0.25 is multiplied by the variable, x; therefore, 0.25 is a coefficient.

5. Identify any constants.

   The number that does not change in the expression is 149.25; therefore, 149.25 is a constant.

Example 3

Helen purchased 3 books from an online bookstore and received a 20% discount. The shipping cost was $10 and was not discounted. Write an expression that can be used to represent the total amount Helen paid for 3 books plus the shipping cost. Identify each term, coefficient, constant, and factor of the expression described.

1. Translate the verbal expression into an algebraic expression.

   Let x represent the unknown price. The expression used to represent the total amount Helen paid for the 3 books plus shipping is $3x - 0.20(3x) + 10$.

2. Simplify the expression.

   The expression can be simplified by following the order of operations and combining like terms.

$3x - 0.20(3x) + 10$	Multiply 0.20 and $3x$.
$3x - 0.60x + 10$	Combine like terms: $3x$ and $-0.60x$.
$2.4x + 10$	

3. Identify all terms.

   There are two terms in the described expression: the product of 2.4 and x, and the shipping charge of $10: $2.4x$ and 10.

4. Identify the factors.

   $2.4x$ is the product of the factors 2.4 and x.

5. Identify all coefficients.

   2.4 is multiplied by the variable, x; therefore, 2.4 is a coefficient.

6. Identify any constants.

   The number that does not change in the expression is 10; therefore, 10 is a constant.

Practice 1.1.1: Identifying Terms, Factors, and Coefficients

For problems 1–3, identify the terms, coefficients, constants, and factors of the given expressions.

1. $8x^2 - 3x + 6x^2 + 5x - 9$

2. $5(2x + 4) + 3x$

3. $\frac{4x^3}{5} + 9x$

For problems 4 and 5, translate each verbal expression to an algebraic expression then identify the terms, coefficients, and constants of the given expressions.

4. 4 more than the quotient of x squared and 3

5. the sum of x to the sixth power and 3 times x

6. Write an expression with 5 terms, containing the coefficients 12, 15, 18, and 21.

continued

For problems 7–10, write an algebraic expression to describe each situation, and then identify the terms, coefficients, constants, and factors.

7. Colin bought 2 theater tickets and paid a service charge of 5% for buying them from a ticket broker. Write an algebraic expression to represent the total cost of the tickets. Let x represent the cost of each ticket.

8. Eddie purchased 4 packages of light bulbs and received a 15% discount. He also paid \$4.85 in taxes on his purchase. Write an algebraic expression to represent the total amount Eddie paid. Let x represent the cost of each package purchased.

9. The perimeter of a rectangle is found by finding the sum of all the sides. Write an expression to represent the perimeter of a rectangle with length x meters and width 4 meters shorter.

10. Write an algebraic expression that represents $\frac{5}{9}$ of the difference of a given Fahrenheit temperature and 32.

Lesson 1.1.2: Interpreting Complicated Expressions

Introduction

Algebraic expressions, used to describe various situations, contain variables. It is important to understand how each term of an expression works and how changing the value of variables impacts the resulting quantity.

Key Concepts

- If a situation is described verbally, it is often necessary to first translate each expression into an algebraic expression. This will allow you to see mathematically how each term interacts with the other terms.
- As variables change, it is important to understand that constants will always remain the same. The change in the variable will not change the value of a given constant.
- Similarly, changing the value of a constant will not change terms containing variables.
- It is also important to follow the order of operations, as this will help guide your awareness and understanding of each term.

Guided Practice 1.1.2

Example 1

A new car loses an average value of $1,800 per year for each of the first six years of ownership. When Nia bought her new car, she paid $25,000. The expression 25,000 – 1800y represents the current value of the car, where y represents the number of years since she bought it. What effect, if any, does the change in the number of years since Nia bought the car have on the original price of the car?

1. Refer to the expression given: 25,000 – 1800y.

   The term 1800y represents the amount of value the car loses each year, y. As y increases, the product of 1800 and y also increases.

2. 25,000 represents the price of the new car.

   As y increases and the product of 1800 and y increases, the original cost is not affected. 25,000 is a constant and remains unchanged.

Example 2

To calculate the perimeter of an isosceles triangle, the expression $2s + b$ is used, where s represents the length of the two congruent sides and b represents the length of the base. What effect, if any, does increasing the length of the congruent sides have on the expression?

1. Refer to the expression given: $2s + b$.

   Changing only the length of the congruent sides, s, will not impact the length of base b since b is a separate term.

2. If the value of the congruent sides, s, is increased, the product of $2s$ will also increase. Likewise, if the value of s is decreased, the value of $2s$ will also decrease.

3. If the value of s is changed, the result of the change in the terms is a doubling of the change in s while the value of b remains the same.

Example 3

Money deposited in a bank account earns interest on the initial amount deposited as well as any interest earned as time passes. This compound interest can be described by the expression $P(1 + r)^n$, where P represents the initial amount deposited, r represents the interest rate, n represents the number of months that pass. How does a change in each variable affect the value of the expression?

1. Refer to the given expression: $P(1 + r)^n$.

   Notice the expression is made up of one term containing the factors P and $(1 + r)^n$.

2. Changing the value of P does not change the value of the factor $(1 + r)^n$, but it will change the value of the expression by a factor of P. In other words, the change in P will multiply by the result of $(1 + r)^n$.

3. Similarly, changing r changes the **base** of the exponent (the number that will be multiplied by itself), but does not change the value of P. This change will affect the value of the overall expression.

4. Changing n changes the number of times $(1 + r)$ will be multiplied by itself, but does not change the value of P. This change will affect the value of the overall expression. ✓

Practice 1.1.2: Interpreting Complicated Expressions

Use your understanding of terms, coefficients, factors, exponents, and the order of operations to answer each of the following questions.

1. Explain why the expression $7 \bullet 3^x$ is not equal to the expression 21^x.

2. Explain why the expression $(5 \bullet 2)^x$ is equal to the expression 10^x.

3. Julio and his sister bought 8 books and m number of magazines and split the cost. The amount of money that Julio spent is represented by the expression $\frac{1}{2}(8+m)$. Does the number of books purchased affect the value of m?

4. Satellite Cell Phone company bills on a monthly basis. Each bill includes a \$19.95 service fee for 500 minutes plus a \$3.95 communication tax and \$0.15 for each minute over 500 minutes. The following expression describes the cost of the cellphone service per month: $23.90 + 0.15m$. If Satellite Cell Phone lowers its service fee, how will the expression change?

5. The expression $\frac{9}{x}$ is given. Describe the value of this expression if the value of x is less than 1, but greater than 0.

continued

6. For what values of x will the result of 0.5^x be greater than 1?

7. A bank account balance for an account with an initial deposit of P dollars earns interest at an annual rate of r. The amount of money in the account after n years is described using the following expression: $P(1 + r)^n$. What effect, if any, does increasing the value of r have on the amount of money after n years?

8. The effectiveness of an initial dose, d, of a particular medicine decreases over a period of time, t, at a rate, r. This situation can be described using the expression: $d(1 - r)^t$. What effect, if any, does decreasing the value of r have on the value of d?

9. The population of a town changes at a rate of r each year. To determine the number of people after n years, the following expression is used: $P(1 + r)^n$, where P represents the initial population, r represents the rate, and n represents the number of years. If the population were declining, what values would you expect for the factor $(1 + r)$?

10. The fine print on the back of a gift card states that a 1% inactivity fee will be deducted each month from the remaining balance if the card has never been used. The expression $x(0.99)^y$ describes this situation. Does the number of months that the gift card remains inactive affect the rate at which the amount is deducted?

Lesson 2: Creating Equations and Inequalities in One Variable

Common Core State Standards

N–Q.2 Define appropriate quantities for the purpose of descriptive modeling.★

N–Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

A–CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Essential Questions

1. How are quantities modeled with equations and inequalities?
2. How are equations and inequalities alike and different?
3. What makes creating an exponential equation different from creating a linear equation?

WORDS TO KNOW

equation a mathematical sentence that uses an equal sign (=) to show that two quantities are equal

exponential decay an exponential equation with a base, b, that is between 0 and 1 ($0 < b < 1$); can be represented by the formula $y = a(1 - r)^t$, where a is the initial value, $(1 - r)$ is the decay rate, t is time, and y is the final value

exponential equation an equation that has a variable in the exponent; the general form is $y = a \bullet b^x$, where a is the initial value, b is the base, x is the time, and y is the final output value.

Another form is $y = ab^{\frac{x}{t}}$, where t is the time it takes for the base to repeat.

exponential growth an exponential equation with a base, b, greater than 1 ($b > 1$); can be represented by the formula $y = a(1 + r)^t$, where a is the initial value, $(1 + r)$ is the growth rate, t is time, and y is the final value

inequality	a mathematical sentence that shows the relationship between quantities that are not equivalent
linear equation	an equation that can be written in the form $ax + b = c$, where a, b, and c are rational numbers
quantity	something that can be compared by assigning a numerical value
rate	a ratio that compares different kinds of units
solution	a value that makes the equation true
solution set	the value or values that make a sentence or statement true
unit rate	a rate per one given unit
variable	a letter used to represent a value or unknown quantity that can change or vary

Recommended Resources

- APlusMath. "Algebra Planet Blaster."

  http://walch.com/rr/CAU1L2LinEquations

  Players solve each multi-step linear equation to find the correct planet to "blast." Incorrect answers cause players to destroy their own ship.

- Figure This! Math Challenges for Families. "Challenge 24: Gasoline Tanks."

  http://walch.com/rr/CAU1L2Rates

  This website features a description of the math involved, related occupations, a hint to get started, complete solutions, and a "Try This" section, as well as additional related problems with answers, questions to think about, and resources for further exploration.

- Purplemath.com. "Exponential Functions: Introduction."

  http://walch.com/rr/CAU1L2ExpEquations

  This website gives an introduction of exponential equations and provides a few examples of tables of input and output values, with integers as inputs. The introduction goes into more depth about the shapes of the graphs of exponential functions and continues on to develop the concept of compound interest. It also introduces the number *e*.

Lesson 1.2.1: Creating Linear Equations in One Variable

Introduction

Creating equations from context is important since most real-world scenarios do not involve the equations being given. An **equation** is a mathematical sentence that uses an equal sign (=) to show that two quantities are equal. A **quantity** is something that can be compared by assigning a numerical value. In this lesson, contexts will be given and equations must be created from them and then used to solve the problems. Since these problems are all in context, units are essential because without them, the numbers have no meaning.

Key Concepts

- A **linear equation** is an equation that can be written in the form $ax + b = c$, where a, b, and c are rational numbers. Often, the most difficult task in turning a context into an equation is determining what the variable is and how to represent that variable.
- The variables are letters used to represent a value or unknown quantity that can change or vary. Once the equation is determined, solving for the variable is straightforward.
- The **solution** will be the value that makes the equation true.
- In some cases the solution will need to be converted into different units. Multiplying by a unit rate or a ratio can do this.
- A **unit rate** is a rate per one given unit, and a **rate** is a ratio that compares different kinds of units.
- Use units that make sense, such as when reporting time; for example, if the time is less than 1 hour, report the time in minutes.
- Think about rounding and precision. The more numbers you list to the right of the decimal place, the more precise the number is.
- When using measurement in calculations, only report to the nearest decimal place of the least accurate measurement. See Guided Practice Example 5.

Creating Equations from Context
1. Read the problem statement first. 2. Reread the scenario and make a list or a table of the known quantities. 3. Read the statement again, identifying the unknown quantity or variable. 4. Create expressions and inequalities from the known quantities and variable(s). 5. Solve the problem. 6. Interpret the solution of the equation in terms of the context of the problem and convert units when appropriate, multiplying by a unit rate.

Guided Practice 1.2.1

Example 1

James earns $15 per hour as a teller at a bank. In one week he pays 17% of his earnings in state and federal taxes. His take-home pay for the week is $460.65. How many hours did James work?

1. Read the statement carefully.

2. Reread the scenario and make a list of the known quantities.

   James earns $15 per hour.

   James pays 17% of his earning in taxes.

   His pay for the week is $460.65.

3. Read the statement again and look for the unknown or the variable.

   The scenario asks for James's hours for the week. The variable to solve for is hours.

4. Create expressions and inequalities from the known quantities and variable(s).

   James's pay for the week was $460.65.

   $\underline{\qquad\qquad} = 460.65$

   He earned $15 an hour. Let h represent hours.

   $15h$

   He paid 17% in taxes.

   $-0.17(15h)$

   Put this information all together.

   $15h - 0.17(15h) = 460.65$

5. Solve the equation.

$15h - 0.17(15h) = 460.65$	Multiply -0.17 and $15h$.
$15h - 2.55h = 460.65$	Combine like terms $15h$ and $-2.55h$.
$12.45h = 460.65$	Divide both sides by 12.45.

$$\frac{12.45h}{12.45} = \frac{460.65}{12.45}$$

$h = 37$ hours

James worked 37 hours.

6. Convert to the appropriate units if necessary.

The scenario asked for hours and the quantity given was in terms of hours. No unit conversions are necessary.

Example 2

Brianna has saved $600 to buy a new TV. If the TV she wants costs $1,800 and she saves $20 a week, how many years will it take her to buy the TV?

1. Read the statement carefully.

2. Reread the scenario and make a list of the known quantities.

The TV costs $1,800.

Brianna saved $600.

Brianna saves $20 per week.

3. Read the statement again and look for the unknown or the variable.

The scenario asks for the number of years. This is tricky because the quantity is given in terms of weeks. The variable to solve for first, then, is weeks.

4. Create expressions and inequalities from the known quantities and variable(s).

Brianna needs to reach $1,800.

$\_\_\_\_\_\_ = 1800$

Brianna has saved $600 so far and has to save more to reach her goal.

$600 + \_\_\_\_\_\_\_ = 1800$

Brianna is saving $20 a week for some unknown number of weeks to reach her goal. Let x represent the number of weeks.

$600 + 20x = 1800$

5. Solve the problem for the number of weeks it will take Brianna to reach her goal.

$$\begin{aligned} 600 + 20x &= 1800 \\ -600 \qquad &\quad -600 \\ \hline 20x &= 1200 \\ \frac{20x}{20} &= \frac{1200}{20} \\ x &= 60 \text{ weeks} \end{aligned}$$

Brianna will need 60 weeks to save for her TV.

6. Convert to the appropriate units.

The problem statement asks for the number of years it will take Brianna to save for the TV. There are 52 weeks in a year.

$$\frac{1 \text{ year}}{52 \text{ weeks}}$$

$$60 \text{ weeks} \bullet \frac{1 \text{ year}}{52 \text{ weeks}}$$

$$60 \cancel{\text{ weeks}} \bullet \frac{1 \text{ year}}{52 \cancel{\text{ weeks}}} \approx 1.15 \text{ years}$$

Brianna will need approximately 1.15 years, or a little over a year, to save for her TV.

Example 3

Suppose two brothers who live 55 miles apart decide to have lunch together. To prevent either brother from driving the entire distance, they agree to leave their homes at the same time, drive toward each other, and meet somewhere along the route. The older brother drives cautiously at an average speed of 60 miles per hour. The younger brother drives faster, at an average speed of 70 mph. How long will it take the brothers to meet each other?

1. Read the statement carefully.

2. Reread the scenario and make a table of the known quantities.

   Problems involving "how fast," "how far," or "how long" require the distance equation, $d = rt$, where d is distance, r is rate of speed, and t is time.

   Complete a table of the known quantities.

	Rate (r)	**Distance (d)**
Older brother	60 mph	55 miles
Younger brother	70 mph	55 miles

3. Read the statement again and look for the unknown or the variable.

   The scenario asks for how long, so the variable is time, t.

4. Create expressions and inequalities from the known quantities and variable(s).

   Step 2 showed that the distance equation is $d = rt$ or $rt = d$. Together the brothers will travel a distance, d, of 55 miles.

   (older brother's rate)(t) + (younger brother's rate)(t) = 55

   The rate r of the older brother = 60 mph and the rate of the younger brother = 70 mph.

   $60t + 70t = 55$

   Expand the table from step 2 to see this another way.

	Rate (r)	**Time (t)**	**Distance (d)**
Older brother	60 mph	t	$d = 60t$
Younger brother	70 mph	t	$d = 70t$

   Together, they traveled 55 miles, so add the distance equations based on each brother's rate.

   $60t + 70t = 55$

5. Solve the problem for the time it will take for the brothers to meet each other.

   $60t + 70t = 55$

   $130t = 55$

   $\frac{130t}{130} = \frac{55}{130}$

   $t \approx 0.42$ hours

   It will take the brothers 0.42 hours to meet each other.

   Note: The answer was rounded to the nearest hundredth of an hour because any rounding beyond the hundredths place would not make sense. Most people wouldn't be able to or need to process that much precision. When talking about meeting someone, it is highly unlikely that anyone would report a time that is broken down into decimals, which is why the next step will convert the units.

6. Convert to the appropriate units if necessary.

Automobile speeds in the United States are typically given in miles per hour (mph). Therefore, this unit of measurement is appropriate.

However, typically portions of an hour are reported in minutes unless the time given is $\frac{1}{2}$ of an hour.

Convert 0.42 hours to minutes using 60 minutes = 1 hour.

$$60 \text{ min} = 1 \text{ hr}$$

$$0.42 \text{ hr} \bullet \frac{60 \text{ min}}{1 \text{ hr}}$$

$$0.42 \cancel{\text{hr}} \bullet \frac{60 \text{ min}}{1 \cancel{\text{hr}}} = 25.2 \text{ minutes}$$

Here again, rarely would a person report that they are meeting someone in 25.2 minutes. In this case, there is a choice of rounding to either 25 or 26 minutes. Either answer makes sense.

The two brothers will meet each other in 25 or 26 minutes. ✓

Example 4

Think about the following scenarios. In what units should they be reported? Explain the reasoning.

a. Water filling up a swimming pool

A swimming pool, depending on the size, has between several gallons and hundreds of thousands of gallons of water.

Think about water flowing out of a faucet and picture filling up a milk jug. How long does it take? Less than a minute? The point is that gallons of water can be filled in minutes.

Report the filling of a swimming pool in terms of gallons per minute.

b. The cost of tiling a kitchen floor

Think about how big rooms are. They can be small or rather large, but typically they are measured in feet. When calculating the area, the measurement units are square feet.

Tiles cost in the dollar range.

Report the cost of tiling a kitchen floor in dollars per square foot.

c. The effect of gravity on a falling object

Think about how fast an object falls when you drop it from shoulder-height. How far is it traveling from your shoulder to the ground? It travels several feet (or meters).

How long does it take before the object hits the ground? It only takes a few seconds.

Report gravity in terms of feet or meters per second.

d. A snail traveling across the sidewalk

The context of the problem will determine the correct units. Think about how slowly a snail moves. Would a snail be able to travel at least one mile in an hour? Perhaps it makes more sense to report the distance in a smaller unit. Report the snail traveling across the sidewalk in feet per minute.

If comparing speeds of other animals to the snail's rate, and the animals' rates are being reported in miles per hour, then it makes sense to report the snail's rate in miles per hour, too.

e. Painting a room

Think about how long it takes to paint a room. It takes longer than several minutes. It would probably take hours.

How is the surface area of a wall typically measured? It's usually measured in square feet.

Report the painting of a room in square feet per hour.

Example 5

Ernesto built a wooden car for a soap box derby. He is painting the top of the car blue and the sides black. He already has enough black paint, but needs to buy blue paint. He needs to know the approximate area of the top of the car to determine the size of the container of blue paint he should buy. He measured the length to be 9 feet $11\frac{1}{4}$ inches, and the width to be $\frac{1}{2}$ inch less than 3 feet. What is the surface area of the top of the car? What is the most accurate area Ernesto can use to buy his paint?

1. Read the statement carefully.

2. Reread the scenario and make a list of the known quantities.

   Length = 9 feet 11.25 inches

   Width = 35.5 inches (3 feet = 36 inches; $36 - \frac{1}{2}$ inch = 35.5 inches)

3. Read the statement again and look for the unknown or the variable.

   The scenario asks for the surface area of the car's top.

   Work with the accuracy component after calculating the surface area.

4. Create expressions and inequalities from the known quantities and variable(s).

   The surface area will require some assumptions. A soap box derby car is tapered, meaning it is wider at one end than it is at another. To be sure Ernesto has enough paint, he assumes the car is rectangular with the width being measured at the widest location.

   A = length × width = lw

   For step 2, we listed length and width, but they are not in units that can be multiplied.

   Convert the length to inches.

   Length = 9 feet 11.25 inches = 9(12) + 11.25 = 119.25 inches

   Width = 35.5 inches

5. Solve the problem.

   Substitute length and width into the formula $A = lw$.

   $A = lw$

   $A = 119.25 \bullet 35.5 = 4233.375$

   This gives a numerical result for the surface area, but the problem asks for the most accurate surface area measurement that can be calculated based on Ernesto's initial measurements. Since Ernesto only measured to the hundredths place, the answer can only be reported to the hundredths place.

   The surface area of the top of Ernesto's car is 4,233.38 square inches.

6. Convert to the appropriate units if necessary.

   When buying paint, the hardware store associate will ask how many square feet need to be covered. Ernesto has his answer in terms of square inches. Convert to square feet.

   There are 144 square inches in a square foot.

$$1\text{ ft}^2 = 144\text{ in}^2$$

$$4233.38\text{ in}^2 \bullet \frac{1\text{ ft}^2}{144\text{ in}^2}$$

$$4233.38\ \cancel{\text{in}^2} \bullet \frac{1\text{ ft}^2}{144\ \cancel{\text{in}^2}} = 29.398472\text{ ft}^2$$

7. Rounding must take place here again because Ernesto can only report to the hundredths place.

   Ernesto's surface area = 29.40 ft^2

Practice 1.2.1: Creating Linear Equations in One Variable

For the problem below, read each scenario and give the units you would use to work with each situation.

1. What units would you use to write equations for each scenario that follows?

   a. jogging

   b. speed of a giant tortoise on land

   c. speed of light

   d. cost of mailing a package

Read each scenario, write an equation, and then solve the problem. Remember to include the appropriate units.

2. The radius of a sphere is measured to be 3.12 cm. What is the most accurate volume of the sphere you can report?

3. The length of a dance floor to be replaced is 1 foot shorter than twice than width. You measured the width to be 12.25 feet. What is the area and what is the most accurate area you can report?

4. Leah's dog consumes four times as many calories a day as her cat. Her cat consumes 240 calories per day. How many calories per day does her dog consume?

continued

5. It costs Marcus an access fee for each visit to his gym, plus it costs him $3 in gas for each trip to the gym and back. This month it cost Marcus $108 for 6 trips to his gym. How much is Marcus's access fee per visit?

6. Rebecca bought x pairs of socks and received a 20% discount. Each pair of socks cost her $4.99. Her total cost without tax was $29.94. How many pairs of socks did Rebecca buy?

7. Amelia and 2 of her friends went out to lunch. Each girl ordered exactly the same meal. The total cost was $55.08, which included an 8% tax. What was the price of each meal, not including tax?

8. Alan mowed the lawn and trimmed the hedges in his yard. The amount of time he spent trimming the hedges was $\frac{1}{3}$ the amount of time it took him to mow the lawn. If it took him 1 hour and 15 minutes to mow the lawn, how long did it take him to trim the hedges?

9. The area of a football field is about $\frac{3}{4}$ the size of an international soccer field. The area of a football field, including the end zones, is 57,600 square feet. What is an approximate area of an international soccer field?

10. Alex and Brian park their bikes side-by-side. Alex leaves to visit friends, and Brian leaves 30 minutes later, headed for the same destination. Alex pedals 5 miles per hour slower than Brian. After 1 hour, Brian passes Alex. At what speed are they each pedaling?

Lesson 1.2.2: Creating Linear Inequalities in One Variable

Introduction

Inequalities are similar to equations in that they are mathematical sentences. They are different in that they are not equal all the time. An inequality has infinite solutions, instead of only having one solution like a linear equation. Setting up the inequalities will follow the same process as setting up the equations did. Solving them will be similar, with two exceptions, which will be described later.

Key Concepts

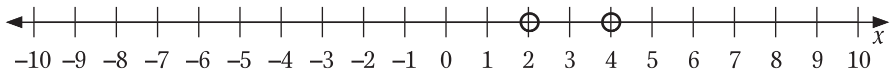
- The prefix *in-* in the word *inequality* means "not." Inequalities are sentences stating that two things are not equal. Remember earlier inequalities such as $12 > 2$ and $1 < 7$.
- Remember that the symbols >, <, ≥, ≤, and ≠ are used with inequalities.
- Use the table below to review the meanings of the inequality symbols and the provided examples with their **solution sets,** or the value or values that make a sentence or statement true.

Symbol	Description	Example	Solution set
>	greater than, more than	$x > 3$	all numbers greater than 3; does not include 3
≥	greater than or equal to, at least	$x \geq 3$	all numbers greater than or equal to 3; includes 3
<	less than	$x < 3$	all numbers less than 3; does not include 3
≤	less than or equal to, no more than	$x \leq 3$	all numbers less than or equal to 3; includes 3
≠	not equal to	$x \neq 3$	includes all numbers except 3

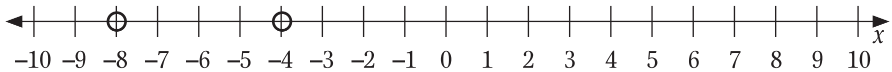
- Solving a linear inequality is similar to solving a linear equation. The processes used to solve inequalities are the same processes that are used to solve equations.
- Multiplying or dividing both sides of an inequality by a negative number requires reversing the inequality symbol. On the next page, there is a number line to show the process.

- First, look at the example of the inequality $2 < 4$.

$2 < 4$

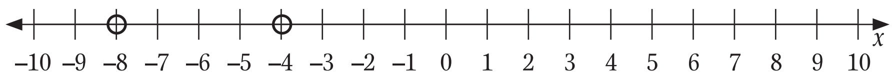


- Multiply both sides by –2 and the inequality becomes $2(-2) < 4(-2)$ or $-4 < -8$.



- Is –4 really less than –8?
- To make the statement true, you must reverse the inequality symbol: $-4 > -8$

$-4 > -8$



Creating Inequalities from Context

1. Read the problem statement first.
2. Reread the scenario and make a list or a table of the known quantities.
3. Read the statement again, identifying the unknown quantity or variable.
4. Create expressions and inequalities from the known quantities and variable(s).
5. Solve the problem.
6. Interpret the solution of the inequality in terms of the context of the problem.

Guided Practice 1.2.2

Example 1

Juan has no more than \$50 to spend at the mall. He wants to buy a pair of jeans and some juice. If the sales tax on the jeans is 4% and the juice with tax costs \$2, what is the maximum price of jeans Juan can afford?

1. Read the problem statement first.

2. Reread the scenario and make a list or a table of the known quantities.

   Sales tax is 4%.

   Juice costs \$2.

   Juan has no more than \$50.

3. Read the statement again, identifying the unknown quantity or variable.

   The unknown quantity is the cost of the jeans.

4. Create expressions and inequalities from the known quantities and variable(s).

   The price of the jeans + the tax on the jeans + the price of the juice must be less than or equal to \$50.

   $x + 0.04x + 2 \le 50$

5. Solve the problem.

$x + 0.04x + 2 \le 50$	Add like terms.
$1.04x + 2 \le 50$	Subtract 2 from both sides.
$1.04x \le 48$	Divide both sides by 1.04.
$x \le 46.153846$	

Normally, the answer would be rounded down to 46.15. However, when dealing with money, round up to the nearest whole cent as a retailer would.

$x \le 46.16$

6. Interpret the solution of the inequality in terms of the context of the problem.

Juan should look for jeans that are priced at or below $46.16.

Example 2

Alexis is saving to buy a laptop that costs $1,100. So far she has saved $400. She makes $12 an hour babysitting. What's the least number of hours she needs to work in order to reach her goal?

1. Read the problem statement first.

2. Reread the scenario and make a list or a table of the known quantities.

Alexis has saved $400.

She makes $12 an hour.

She needs at least $1,100.

3. Read the statement again, identifying the unknown quantity or variable.

You need to know the least number of hours Alexis must work to make enough money. Solve for hours.

4. Create expressions and inequalities from the known quantities and variable(s).

   Alexis's saved money + her earned money must be greater than or equal to the cost of the laptop.

   $400 + 12h \geq 1100$

5. Solve the problem.

$400 + 12h \geq 1100$	Subtract 400 from both sides.
$12h \geq 700$	Divide both sides by 12.
$h \geq 58.3\overline{3}$	

6. Interpret the solution of the inequality in terms of the context of the problem.

   In this situation, it makes sense to round up to the nearest half hour since babysitters usually get paid by the hour or half hour. Therefore, Alexis needs to work at least 58.5 hours to make enough money to save for her laptop.

Example 3

A radio station is giving away concert tickets. There are 40 tickets to start. They give away 1 pair of tickets every hour for a number of hours until they have at most 4 tickets left for a grand prize. If the contest runs from 11:00 A.M. to 1:00 P.M. each day, for how many days will the contest last?

1. Read the problem statement first.

2. Reread the scenario and make a list or a table of the known quantities.

   The contest starts with 40 tickets.

   The station gives away 2 tickets every hour.

   The contest ends with at most 4 tickets left.

3. Read the statement again, identifying the unknown quantity or variable(s).

   For how many days will the contest last?

   This is tricky because the tickets are given away in terms of hours.

   First, solve for hours.

4. Create expressions and inequalities from the known quantities and variable(s).

   40 tickets – 2 tickets given away each hour must be less than or equal to 4 tickets.

   $40 - 2h \leq 4$

5. Solve the problem.

$40 - 2h \leq 4$	Subtract 40 from both sides.
$-2h \leq -36$	Divide both sides by –2 and switch the inequality symbol.
$h \geq 18$	

6. Interpret the solution of the inequality in terms of the context of the problem.

   The inequality is solved for the number of hours the contest will last. The contest will last at least 18 hours, or 18 hours or more.

   The problem asks for the number of days the contest will last. If the contest lasts from 11:00 A.M. to 1:00 P.M. each day, that is 3 hours per day. Convert the units.

   1 day = 3 hours

   $$18 \text{ hours} \bullet \frac{1 \text{ day}}{3 \text{ hours}}$$

   $$18 \cancel{\text{ hours}} \bullet \frac{1 \text{ day}}{3 \cancel{\text{ hours}}} = 6 \text{ days}$$

   The contest will run for 6 days or more.

Practice 1.2.2: Creating Linear Inequalities in One Variable

Translate each phrase into an algebraic inequality.

1. An amusement park ride can hold 8 passengers.

2. An auditorium can seat 250 people or fewer.

3. The maximum weight an elevator can hold is 2,400 pounds.

Read each scenario, write an inequality to model the scenario, and then use the inequality to solve the problem.

4. Jeff is saving to purchase a new basketball that will cost at least $88. He has already saved $32. At least how much more does he need to save for the basketball?

5. Suppose you earn $15 per hour working part time as a carpenter. This month, you want to earn at least $950. How many hours must you work?

6. Mackenzie earned a score of 79 on her semester biology test. She needs to have a total of at least 160 points from her semester and final tests to receive a B for her grade. What score must Mackenzie earn on her final test to ensure her B?

continued

7. Arianna buys computer games from an online store. Each game she orders costs \$22, and shipping for her total order is \$9. Arianna can spend no more than \$75. How many computer games can Arianna buy?

8. A recreation center holds a lacrosse game every Saturday morning for young adults. The group agreed that at least 6 players are needed on each team. One team started out with 16 players. After an hour of playing, 2 players from that team started leaving every 7 minutes. For at least how long can they remain playing?

9. A radio station has no more than \$25,000 to give away. They have decided to give away \$1,000 three times a day every day until they have at least \$4,000 left to award as a grand prize. How many days will the contest run?

For problem 10, create your own context for the given inequality, and then solve the inequality. Be sure to express your solution in terms of the context of the problem.

10. $3x - 3 > 6$

Lesson 1.2.3: Creating Exponential Equations

Introduction

Exponential equations are equations that have the variable in the exponent. Exponential equations are found in science, finance, sports, and many other areas of daily living. Some equations are complicated, but some are not.

Key Concepts

- The general form of an exponential equation is $y = a \bullet b^x$, where a is the initial value, b is the base, and x is the time. The final output value will be y.
- Since the equation has an exponent, the value increases or decreases rapidly.
- The base, b, must always be greater than 0 ($b > 0$).
- If the base is greater than 1 ($b > 1$), then the exponential equation represents **exponential growth.**
- If the base is between 0 and 1 ($0 < b < 1$), then the exponential equation represents **exponential decay.**
- If the time is given in units other than 1 (e.g., 1 month, 1 hour, 1 minute, 1 second), use the equation $y = ab^{\frac{x}{t}}$, where t is the time it takes for the base to repeat.
- Another form of the exponential equation is $y = a(1 \pm r)^t$, where a is the initial value, r is the rate of growth or decay, and t is the time.
- Use $y = a(1 + r)^t$ for exponential growth (notice the plus sign). For example, if a population grows by 2% then r is 0.02, but this is less than 1 and by itself does not indicate growth.
- Substituting 0.02 for b into the formula $y = a \bullet b^x$ requires the expression $(1 + r)$ to arrive at the full growth rate of 102%, or 1.02.
- Use $y = a(1 - r)^t$ for exponential decay (notice the minus sign). For example, if a population decreases by 3%, then 97% is the factor being multiplied over and over again. The population from year to year is always 97% of the population from the year before (a 3% decrease). Think of this as 100% minus the rate, or in decimal form $(1 - r)$.
- Look for words such as *double*, *triple*, *half*, *quarter*—such words give the number of the base. For example, if an experiment begins with 1 bacterium that doubles (splits itself in two) every hour, determining how many bacteria

will be present after x hours is solved with the following equation: $y = (1)2^x$, where 1 is the starting value, 2 is the rate, x is the number of hours, and y is the final value.

- Look for the words *initial* or *starting* to substitute in for a.
- Look for the words *ended with* and *after*—these words will be near the final value given.
- Follow the same procedure as with setting up linear equations and inequalities in one variable:

Creating Exponential Equations from Context
1. Read the problem statement first. 2. Reread the scenario and make a list or a table of the known quantities. 3. Read the statement again, identifying the unknown quantity or variable. 4. Create expressions and inequalities from the known quantities and variable(s). 5. Solve the problem. 6. Interpret the solution of the exponential equation in terms of the context of the problem.

Guided Practice 1.2.3

Example 1

A population of mice quadruples every 6 months. If a mouse nest started out with 2 mice, how many mice would there be after 2 years? Write an equation and then use it to solve the problem.

1. Read the scenario and then reread it again, this time identifying the known quantities.

   The initial number of mice = 2.

   The base = quadruples, so that means 4.

   The amount of time = every 6 months for 2 years.

2. Read the statement again, identifying the unknown quantity or variable.

   The unknown quantity is the number of mice after 2 years. Solve for the final amount of mice after 2 years.

3. Create expressions and equations from the known quantities and variable(s).

   The general form of the exponential equation is $y = a \bullet b^x$, where y is the final value, a is the initial value, b is the base, and x is the time.

   $a = 2$

   $b = 4$

   x = every 6 months for 2 years

   Since the problem is given in months, you need to convert 2 years into 6-month time periods. How many 6-month time periods are there in 2 years?

   To determine this, think about how many 6-month time periods there are in 1 year. There are 2. Multiply that by 2 for each year. Therefore, there are four 6-month time periods in 2 years.

(*continued*)

Another way to determine the period is to set up ratios.

$$2\ \cancel{\text{years}} \bullet \frac{12 \text{ months}}{1\ \cancel{\text{year}}} = 24 \text{ months}$$

$$24\ \cancel{\text{months}} \bullet \frac{1 \text{ time period}}{6\ \cancel{\text{months}}} = 4 \text{ time periods}$$

Therefore, $x = 4$.

4. Substitute the values into the general form of the equation $y = a \bullet b^x$.

$y = a \bullet b^x$ OR $y = ab^{\frac{x}{t}}$

$y = (2) \bullet (4)^4$ OR $y = (2)(4)^{\frac{24}{6}}$

5. Follow the order of operations to solve the problem.

$y = (2) \bullet (4)^4$	Raise 4 to the 4th power.
$y = (2) \bullet 256$	Multiply 2 and 256.
$y = 512$	

6. Interpret the solution in terms of the context of the problem.

There will be 512 mice after 2 years if the population quadruples every 6 months.

Example 2

In sporting tournaments, teams are eliminated after they lose. The number of teams in the tournament then decreases by half with each round. If there are 16 teams left after 3 rounds, how many teams started out in the tournament?

1. Read the scenario and then reread it again, this time identifying the known quantities.

   The final number of teams = 16.

   The reduction = $\frac{1}{2}$.

   The amount of time = 3 rounds.

2. Read the statement again, identifying the unknown quantity or variable.

   The unknown quantity is the number of teams with which the tournament began. Solve for the initial or starting value, a.

3. Create expressions and equations from the known quantities and variable(s).

   The general form of the exponential equation is $y = a \bullet b^x$, where y is the final value, a is the initial value, b is the rate of decay or growth, and x is the time.

   $y = 16$

   $b = \frac{1}{2}$

   $x = 3$ rounds

4. Substitute the values into the general form of the equation $y = a \bullet b^x$.

   $y = a \bullet b^x$

   $16 = a \bullet \left(\frac{1}{2}\right)^3$

5. Follow the order of operations to solve the problem.

$16 = a \bullet \left(\frac{1}{2}\right)^3$ Raise the base to the power of 3.

$16 = a \bullet \frac{1}{8}$ Multiply by the reciprocal.

$a = 128$

6. Interpret the solution in terms of the context of the problem.

The tournament started with 128 teams.

Example 3

The population of a small town is increasing at a rate of 4% per year. If there are currently about 6,000 residents, about how many residents will there be in 5 years at this growth rate?

1. Read the scenario and then reread it again, this time identifying the known quantities.

The initial number of residents = 6,000.

The growth = 4%.

The amount of time = 5 years.

2. Read the statement again, identifying the unknown quantity or variable.

The unknown quantity is the number of residents after 5 years. Solve for the final value after 5 years.

3. Create expressions and equations from the known quantities and variable(s).

   The general form of the exponential growth equation with a percent increase is $y = a(1 + r)^t$, where y is the final value, a is the initial value, r is the rate of growth, and t is the amount of time.

   $a = 6000$

   $r = 4\% = 0.04$

   $t = 5$ years

4. Substitute the values into the general form of the equation $y = a(1 + r)^t$.

   $y = a(1 + r)^t$

   $y = 6000(1 + 0.04)^5$

5. Follow the order of operations to solve the problem.

$y = 6000(1 + 0.04)^5$	Add inside the parentheses first.
$y = 6000(1.04)^5$	Raise the base to the power of 5.
$y = 6000(1.21665)$	Multiply.
$y \approx 7300$	

6. Interpret the solution in terms of the context of the problem.

   If this growth rate continues for 5 years, the population will increase by more than 1,000 residents to about 7,300 people.

Example 4

You want to reduce the size of a picture to place in a small frame. You aren't sure what size to choose on the photocopier, so you decide to reduce the picture by 15% each time you scan it until you get it to the size you want. If the picture was 10 inches long at the start, how long is it after 3 scans?

1. Read the scenario and then reread it again, this time identifying the known quantities.

   The initial length = 10 inches.

   The reduction = 15% = 0.15.

   The amount of time = 3 scans.

2. Read the statement again, identifying the unknown quantity or variable.

   The unknown quantity is the length of the picture after 3 scans. Solve for the final value after 3 scans.

3. Create expressions and equations from the known quantities and variable(s).

   The general form of the exponential growth equation with a percent decrease is $y = a(1 - r)^t$, where y is the final value, a is the initial value, r is the rate of decay, and t is the amount of time.

   $a = 10$

   $r = 15\% = 0.15$

   $t = 3$ scans

4. Substitute the values into the general form of the equation $y = a(1 - r)^t$.

   $y = a(1 - r)^t$

   $y = 10(1 - 0.15)^3$

5. Follow the order of operations to solve the problem.

$y = 10(1 - 0.15)^3$	Subtract inside the parentheses first.
$y = 10(0.85)^3$	Raise the base to the power of 3.
$y = 10(0.614125)$	Multiply.
$y \approx 6.14$	

6. Interpret the solution in terms of the context of the problem.

After 3 scans, the length of the picture is about 6 inches.

Practice 1.2.3: Creating Exponential Equations

Use what you know about linear and exponential equations to complete problems 1–3.

1. Determine whether each scenario can be modeled by a linear or an exponential equation.
   a. The price of a gallon of gas increases by $0.75 every 2 months.

   b. Every 2 months, a gallon of gas costs three times as much as it did before.

2. Determine whether each scenario can be modeled by a linear or an exponential equation.
   a. A piece of jewelry appreciates (increases in value) so that after 20 years it's worth twice what you paid for it.

   b. A piece of jewelry appreciates so that its value doubles every 20 years.

3. Determine whether each scenario can be modeled by a linear or an exponential equation.
   a. A town's population declines by 3% each year.

   b. About 200 residents leave town each year.

For problems 4–10, write an equation to model each scenario. Then use the equation to solve the problem.

4. If you end with 1,920 bacteria in a Petri dish and the population doubled every hour, how many bacteria did you start with 6 hours ago?

continued

5. An investment doubles in value every 9 years. What was the starting value of the investment if it is worth $4,800 after 27 years?

6. An insect population triples every 4 months. If the population started out with 24 insects, how many insects would there be in 16 months?

7. The half-life of a radioactive substance is the time it takes for half of the substance to decay. The half-life of one form of rhodium, Rh-106, is about 30 seconds. If you start with 100 grams of Rh-106, how much will be left after 4 minutes?

8. The NCAA Division I Basketball tournament begins each year with a certain number of teams. After each round of games, the losing teams are cut from the tournament, so that each round has half as many teams playing as the previous round. After 3 rounds 8 teams are left. How many teams started out in the tournament?

9. A city's population grows by about 1% each year. If the city's population is 63,000 people now, what will the population be in 4 years?

10. A town's population decreases each year by about 1%. If the town's population is 3,000 now, what will the population be in 5 years? In 10 years?

Lesson 3: Creating and Graphing Equations in Two Variables

Common Core State Standards

A–CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

N–Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★

Essential Questions

1. What do the graphs of equations in two variables represent?
2. How do you determine the scales to use for the *x*- and *y*-axes on any given graph?
3. How do the graphs of linear equations and exponential equations differ? How are they similar?
4. How can graphing equations help you to make decisions?

WORDS TO KNOW

coordinate plane a set of two number lines, called the axes, that intersect at right angles

dependent variable labeled on the *y*-axis; the quantity that is based on the input values of the independent variable

exponential decay an exponential equation with a base, *b*, that is between 0 and 1 ($0 < b < 1$); can be represented by the formula $y = a(1 - r)^t$, where *a* is the initial value, $(1 - r)$ is the decay rate, *t* is time, and *y* is the final value

exponential equation an equation that has a variable in the exponent; the general form is $y = a \bullet b^x$, where *a* is the initial value, *b* is the base, *x* is the time, and *y* is the final output value.

Another form is $y = ab^{\frac{x}{t}}$, where *t* is the time it takes for the base to repeat.

exponential growth	an exponential equation with a base, b, greater than 1 ($b > 1$); can be represented by the formula $y = a(1 + r)^t$, where a is the initial value, $(1 + r)$ is the growth rate, t is time, and y is the final value
independent variable	labeled on the x-axis; the quantity that changes based on values chosen
linear equation	an equation that can be written in the form $ax + by = c$, where a, b, and c are rational numbers; can also be written as $y = mx + b$, in which m is the slope, b is the y-intercept, and the graph is a straight line
slope	the measure of the rate of change of one variable with respect to another variable; $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$
***x*-intercept**	the point at which the line intersects the x-axis at $(x, 0)$
***y*-intercept**	the point at which the line intersects the y-axis at $(0, y)$

Recommended Resources

- Math-Play.com. "Hoop Shoot."

  http://walch.com/rr/CAU1L3SlopeandIntercept

  This one- or two-player game includes 10 multiple-choice questions about slope and y-intercept. Correct answers result in a chance to make a 3-point shot in a game of basketball.

- Oswego City School District Regents Exam Prep Center. "Equations and Graphing."

  http://walch.com/rr/CAU1L3GraphLinear

  This site contains a thorough summary of the methods used to graph linear equations.

- Ron Blond Mathematics Applets. "The Exponential Function $y = ab^x$."

  http://walch.com/rr/CAU1L3ExponentialFunction

  This applet provides sliders for the variables a and b, and shows how changing the values of these variables results in changes in the graph.

Lesson 1.3.1: Creating and Graphing Linear Equations in Two Variables

Introduction

Many relationships can be represented by linear equations. Linear equations in two variables can be written in the form $y = mx + b$, where m is the slope and b is the y-intercept. The slope of a linear graph is a measure of the rate of change of one variable with respect to another variable. The y-intercept of the equation is the point at which the graph crosses the y-axis and the value of x is zero.

Creating a linear equation in two variables from context follows the same procedure at first for creating an equation in one variable. Start by reading the problem carefully. Once you have created the equation, the equation can be graphed on the coordinate plane. The **coordinate plane** is a set of two number lines, called the axes, that intersect at right angles.

Key Concepts

Reviewing Linear Equations:

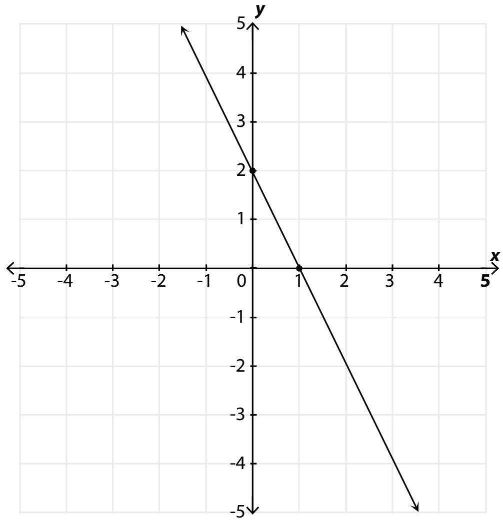
- The slope of a linear equation is also defined by the ratio of the rise of the graph compared to the run. Given two points on a line, (x_1, y_1) and (x_2, y_2), the slope is the ratio of the change in the y-values of the points (rise) to the change in the corresponding x-values of the points (run).

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

- The slope-intercept form of an equation of a line is often used to easily identify the slope and ***y*-intercept**, which then can be used to graph the line. The slope-intercept form of an equation is shown below, where m represents the slope of the line and b represents the y-value of the point where the line intersects the y-axis at point $(0, y)$.

$$y = mx + b$$

- Horizontal lines have a slope of 0. They have a run but no rise. Vertical lines have no slope.
- The ***x*-intercept** of a line is the point where the line intersects the x-axis at $(x, 0)$.
- If a point lies on a line, its coordinates make the equation true.
- The graph of a line is the collection of all points that satisfy the equation. The graph of the linear equation $y = -2x + 2$ is shown on the following page, with its x- and y-intercepts plotted.



Creating Equations
1. Read the problem statement carefully before doing anything.
2. Look for the information given and make a list of the known quantities.
3. Determine which information tells you the rate of change, or the slope, m. Look for words such as *each*, *every*, *per*, or *rate*.
4. Determine which information tells you the y-intercept, or b. This could be an initial value or a starting value, a flat fee, and so forth.
5. Substitute the slope and y-intercept into the linear equation formula, $y = mx + b$.

Determining the Scale and Labels When Graphing:

- If the slope has a rise and run between –10 and 10 and the y-intercept is 10 or less, use a grid that has squares equal to 1 unit.
- Adjust the units according to what you need. For example, if the y-intercept is 10,000, each square might represent 2,000 units on the y-axis. Be careful when plotting the slope to take into account the value each grid square represents.
- Sometimes you need to skip values on the y-axis. It makes sense to do this if the y-intercept is very large (positive) or very small (negative). For example, if your y-intercept is 10,000, you could start your y-axis numbering at 0 and "skip" to 10,000 at the next y-axis number. Use a short, zigzag line starting at 0 to about the first grid line to show that you've skipped values. Then continue with the correct numbering for the rest of the axis. For an illustration, see Guided Practice Example 3, step 4.

- Only use x- and y-values that make sense for the context of the problem. Ask yourself if negative values make sense for the x-axis and y-axis labels in terms of the context. If negative values don't make sense (for example, time and distance can't have negative values), only use positive values.
- Determine the independent and dependent variables.
- The independent variable will be labeled on the x-axis. The **independent variable** is the quantity that changes based on values you choose.
- The dependent variable will be labeled on the y-axis. The **dependent variable** is the quantity that is based on the input values of the independent variable.

Graphing Equations Using a Table of Values

Using a table of values works for any equation when graphing. For an example, see Guided Practice Example 1, step 7.

1. Choose inputs or values of x.
2. Substitute those values in for x and solve for y.
3. The result is an ordered pair (x, y) that can be plotted on the coordinate plane.
4. Plot at least 3 ordered pairs on the line.
5. Connect the points, making sure that they lie in a straight line.
6. Add arrows to the end(s) of the line to show when the line continues infinitely (if continuing infinitely makes sense in terms of the context of the problem).
7. Label the line with the equation.

Graphing Equations Using the Slope and y-intercept

For an example, see Guided Practice Example 2, step 6.

1. Plot the y-intercept first. The y-intercept will be on the y-axis.
2. Recall that slope is $\frac{\text{rise}}{\text{run}}$. Change the slope into a fraction if you need to.
3. To find the rise when the slope is positive, count up the number of units on your coordinate plane the same number of units in your rise. (So, if your slope is $\frac{3}{5}$, you count up 3 on the y-axis.)

(continued)

4. For the run, count over to the right the same number of units on your coordinate plane in your run, and plot the second point. (For the slope $\frac{3}{5}$, count 5 to the right and plot your point.)

5. To find the rise when the slope is negative, count down the number of units on your coordinate plane the same number of units in your rise. For the run, you still count over to the right the same number of units on your coordinate plane in your run and plot the second point. (For a slope of $-\frac{4}{7}$, count down 4, right 7, and plot your point.)

6. Connect the points and place arrows at one or both ends of the line when it makes sense to have arrows within the context of the problem.

7. Label the line with the equation.

Graphing Equations Using a TI-83/84:

Step 1: Press [Y=] and key in the equation using [X, T, θ, *n*] for *x*.

Step 2: Press [WINDOW] to change the viewing window, if necessary.

Step 3: Enter in appropriate values for Xmin, Xmax, Xscl, Ymin, Ymax, and Yscl, using the arrow keys to navigate.

Step 4: Press [GRAPH].

Graphing Equations Using a TI-Nspire:

Step 1: Press the home key.
Step 2: Arrow over to the graphing icon (the picture of the parabola or the U-shaped curve) and press [enter].
Step 3: At the blinking cursor at the bottom of the screen, enter in the equation and press [enter].
Step 4: To change the viewing window: press [menu], arrow down to number 4: Window/Zoom, and click the center button of the navigation pad.
Step 5: Choose 1: Window settings by pressing the center button.
Step 6: Enter in the appropriate XMin, XMax, YMin, and YMax fields.
Step 7: Leave the XScale and YScale set to auto.
Step 8: Use [tab] to navigate among the fields.
Step 9: Press [tab] to “OK” when done and press [enter].

Guided Practice 1.3.1

Example 1

A local convenience store owner spent \$10 on pencils to resell at the store. What is the equation of the store's revenue if each pencil sells for \$0.50? Graph the equation.

1. Read the problem and then reread the problem, determining the known quantities.

   Initial cost of pencils: \$10

   Charge per pencil: \$0.50

2. Identify the slope and the y-intercept.

   The slope is a rate. Notice the word "each."

   Slope = 0.50

   The y-intercept is a starting value. The store *paid* \$10. The starting revenue then is –\$10.

   y-intercept = –10

3. Substitute the slope and y-intercept into the equation $y = mx + b$, where m is the slope and b is the y-intercept.

   $m = 0.50$

   $b = -10$

   $y = 0.50x - 10$

4. Change the slope into a fraction in preparation for graphing.

   $$0.50 = \frac{50}{100} = \frac{1}{2}$$

5. Rewrite the equation using the fraction.

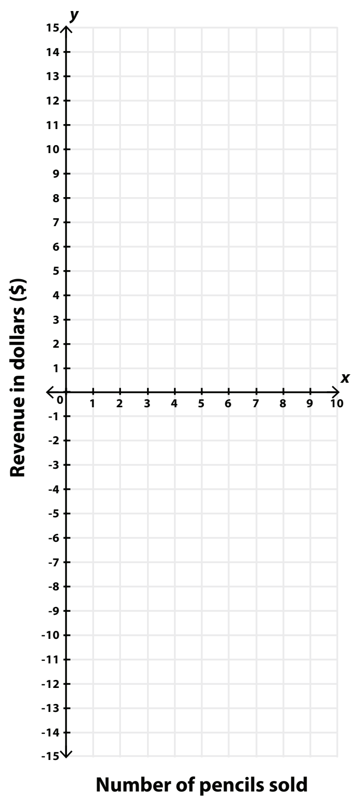
$$y=\frac{1}{2}x-10$$

6. Set up the coordinate plane and identify the independent and dependent variables.

In this scenario, x represents the number of pencils sold and is the independent variable. The x-axis label is "Number of pencils sold."

The dependent variable, y, represents the revenue the store will make based on the number of pencils sold. The y-axis label is "Revenue in dollars ($)."

Determine the scales to be used. Since the slope's rise and run are within 10 units and the y-intercept is –10 units, a scale of 1 on each axis is appropriate. Label the x-axis from 0 to 10, since you will not sell a negative amount of pencils. Label the y-axis from –15 to 15, to allow space to plot the $10 the store owner paid for the pencils (–10).

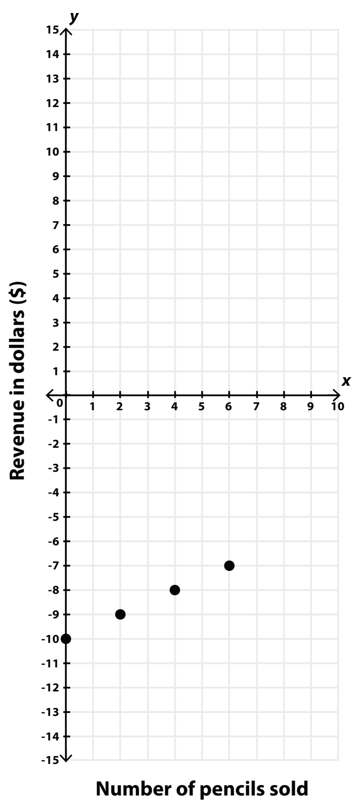


7. Plot points using a table of values.

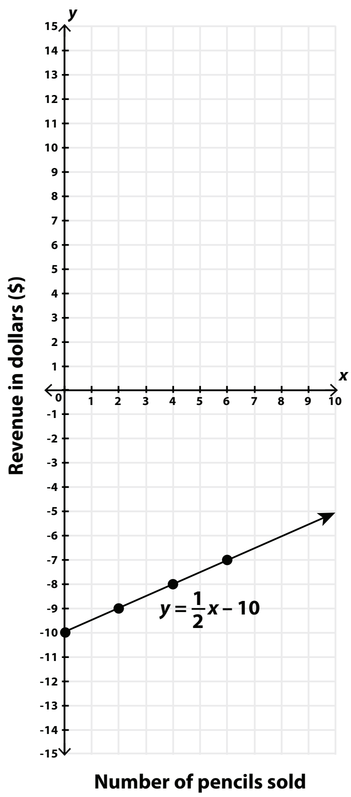
Substitute x values into the equation $y=\frac{1}{2}x-10$ and solve for y.

Choose any values of x to substitute. Here, it's easiest to use values of x that are even since after substituting you will be multiplying by $\frac{1}{2}$. Using even-numbered x values will keep the numbers whole after you multiply.

x	y
0	$\frac{1}{2}(0)-10=-10$
2	–9
4	–8
6	–7



8. Connect the points with a line and add an arrow to the right end of the line to show that the line of the equation goes on infinitely in that direction. Be sure to write the equation of the line next to the line on the graph.



Example 2

A taxi company in Kansas City charges \$2.50 per ride plus \$2 for every mile driven. Write and graph the equation that models this scenario.

1. Read the problem statement and then reread the problem, determining the known quantities.

   Initial cost of taking a taxi: \$2.50

   Charge per mile: \$2

2. Identify the slope and the y-intercept.

   The slope is a rate. Notice the word "every."

   Slope = 2

   The y-intercept is a starting value. It costs \$2.50 initially to hire a cab driver.

   y-intercept = 2.50

3. Substitute the slope and y-intercept into the equation $y = mx + b$, where m is the slope and b is the y-intercept.

   $m = 2$

   $b = 2.50$

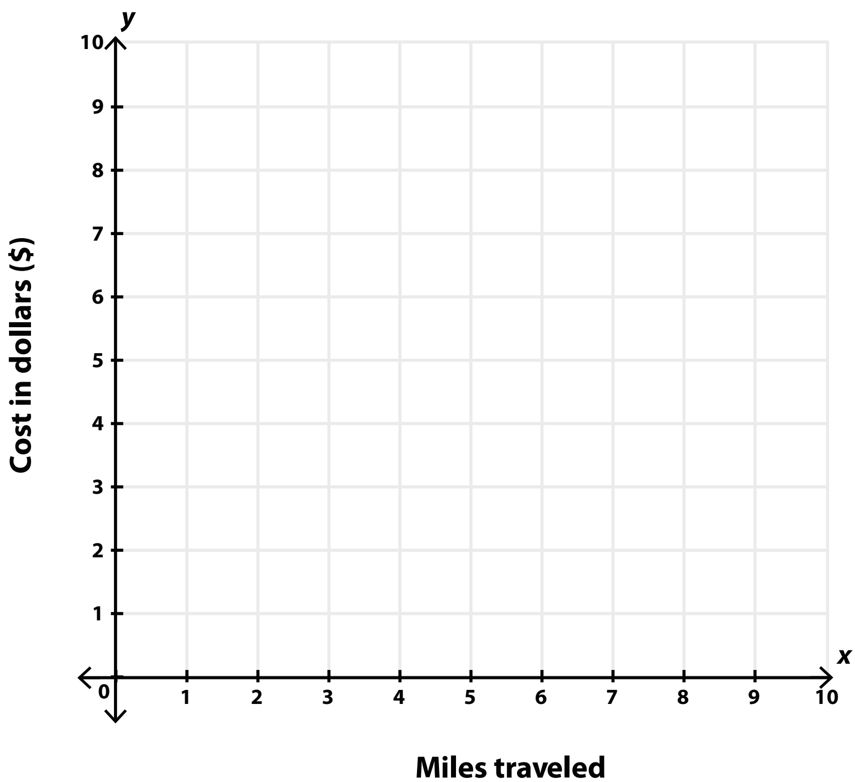
   $y = 2x + 2.50$

4. Set up the coordinate plane.

   In this scenario, x represents the number of miles traveled in the cab and is the independent variable. The x-axis label is "Miles traveled."

   The dependent variable, y, represents the cost of taking a cab based on the number of miles traveled. The y-axis label is "Cost in dollars ($)."

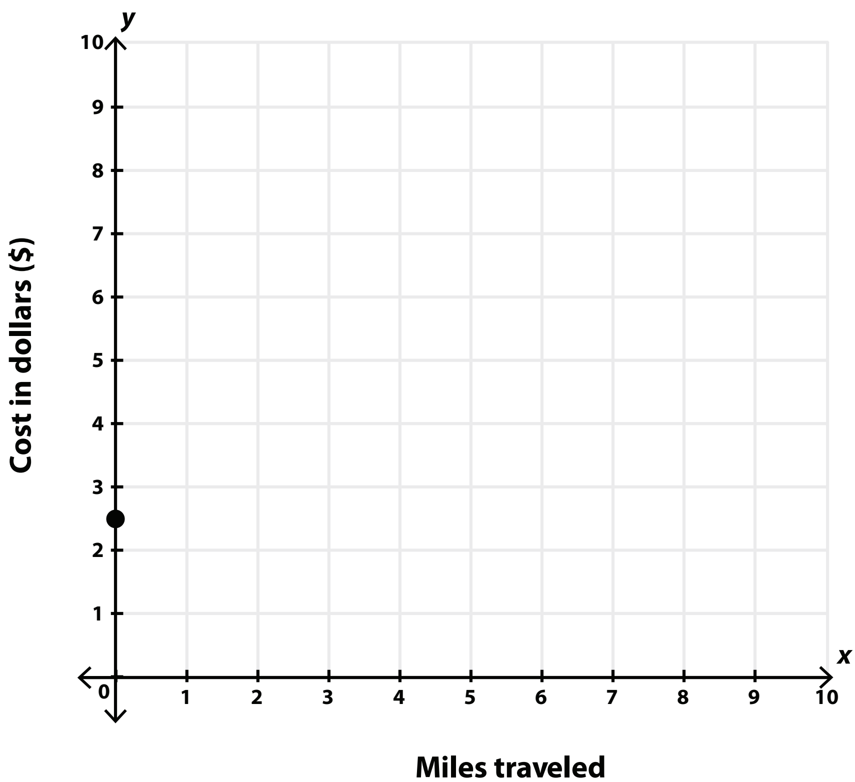
   Determine the scales to be used. Since the slope's rise and run are within 10 units and the y-intercept is within 10 units of 0, a scale of 1 on each axis is appropriate. Label the x-axis from 0 to 10, since miles traveled will only be positive. Label the y-axis from 0 to 10, since cost will only be positive.



5. Graph the equation using the slope and y-intercept. Plot the y-intercept first.

The y-intercept is 2.5. Remember that the y-intercept is where the graph crosses the y-axis and the value of x is 0. Therefore, the coordinate of the y-intercept will always have 0 for x. In this case, the coordinate of the y-intercept is (0, 2.5).

To plot points that lie in between grid lines, use estimation. Since 2.5 is halfway between 2 and 3, plot the point halfway between 2 and 3 on the y-axis. Estimate the halfway point.



6. Graph the equation using the slope and y-intercept. Use the slope to find the second point.

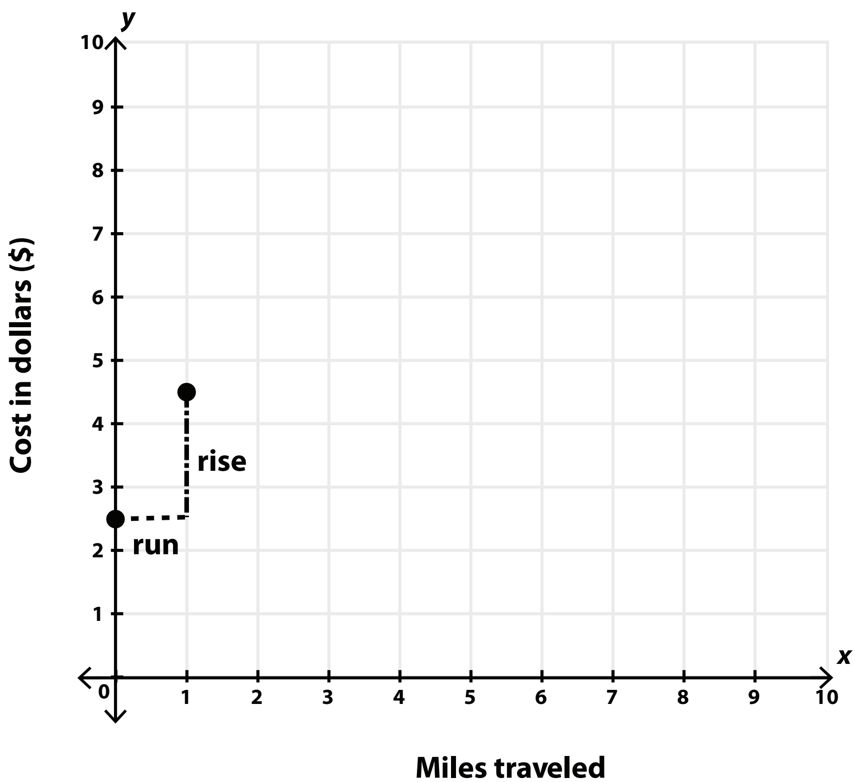
   Remember that the slope is $\frac{\text{rise}}{\text{run}}$. In this case, the slope is 2. Write 2 as a fraction.

   $$2=\frac{2}{1}=\frac{\text{rise}}{\text{run}}$$

   The rise is 2 and the run is 1.

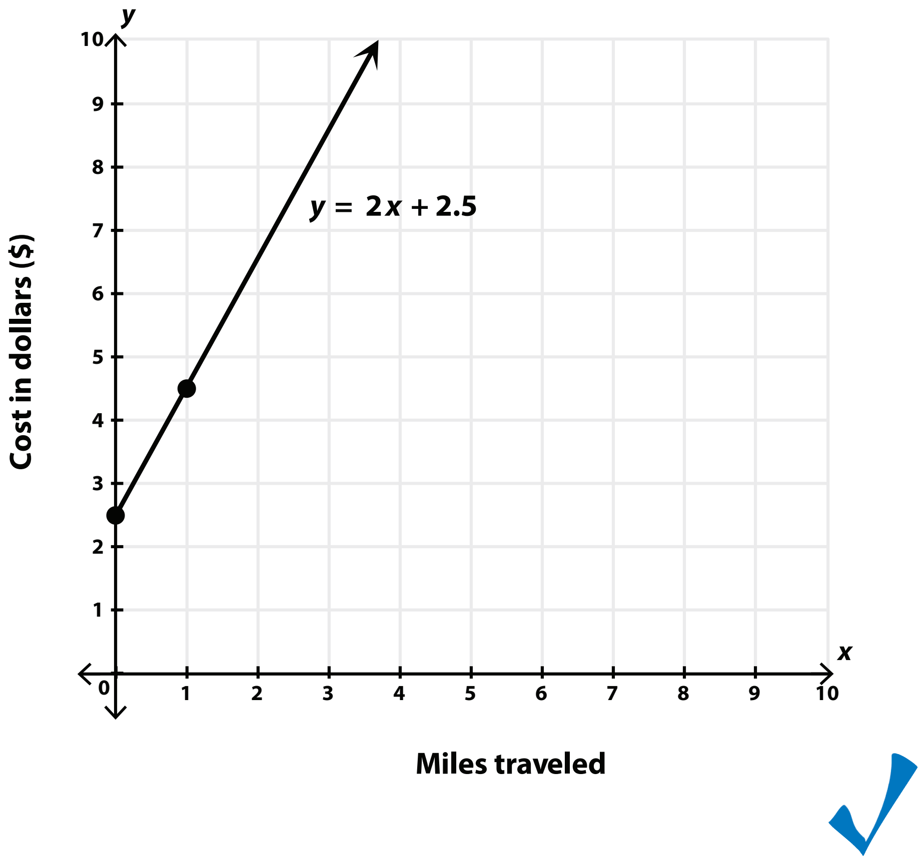
   Point your pencil at the y-intercept. Move the pencil up 2 units, since the slope is positive. Remember that the y-intercept was halfway between grid lines. Be sure that you move your pencil up 2 complete units by first going to halfway between 3 and 4 (3.5) and then halfway between 4 and 5 (4.5) on the y-axis.

   Now, move your pencil to the right 1 unit for the run and plot a point. This is your second point.



7. Connect the points and extend the line. Then, label your line.

Draw a line through the two points and add arrows to the right end of the line to show that the line continues infinitely in that direction. Label the line with the equation, $y = 2x + 2.5$.



Example 3

Miranda gets paid \$300 a week to deliver groceries. She also earns 5% commission on any orders she collects while out on her delivery run. Write an equation that represents her weekly pay and then graph the equation.

1. Read the problem statement and then reread the problem, determining the known quantities.

   Weekly payment: \$300

   Commission: 5% = 0.05

2. Identify the slope and the y-intercept.

   The slope is a rate. Notice the symbol "%," which means *percent*, or *per 100*.

   Slope = 0.05

   The y-intercept is a starting value. She gets paid \$300 a week to start with before taking any orders.

   y-intercept = 300

3. Substitute the slope and y-intercept into the equation $y = mx + b$, where m is the slope and b is the y-intercept.

   $m = 0.05$

   $b = 300$

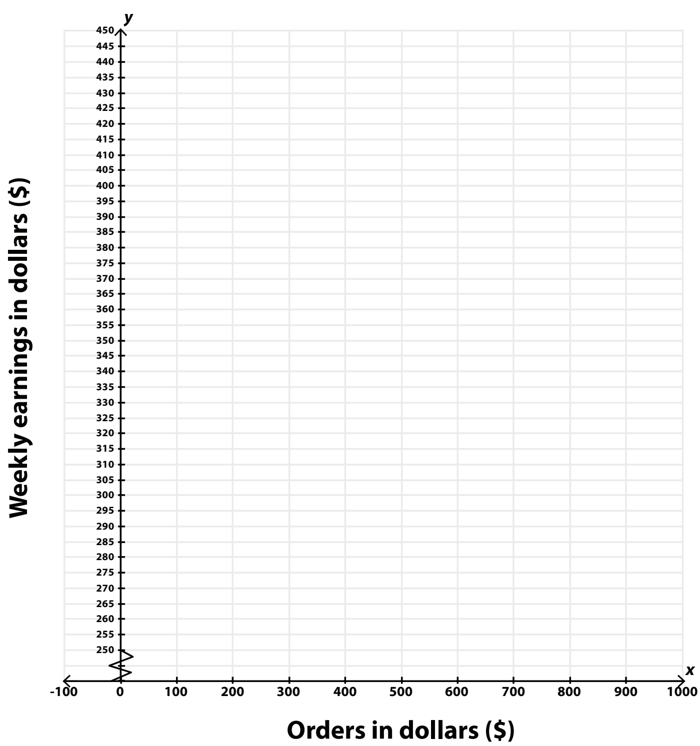
   $y = 0.05x + 300$

4. Set up the coordinate plane.

In this scenario, x represents the amount of money in orders Miranda gets. The x-axis label is "Orders in dollars ($)."

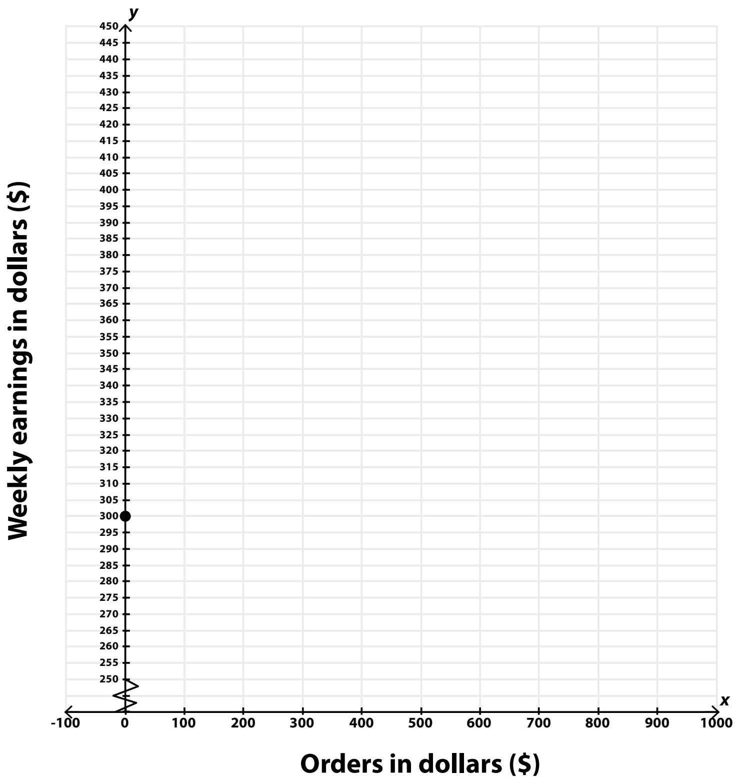
The dependent variable, y, represents her total earnings in a week. The y-axis label is "Weekly earnings in dollars ($)."

Determine the scales to be used. The y-intercept is in the hundreds and the slope is in decimals. Work with the slope first. The slope is 0.05 or $\frac{5}{100}$. The rise is a small number, but the run is big. The run is shown on the x-axis, so that will need to be in increments of 100. Start at –100 or 0, since the order amounts will be positive, and continue to 1,000. The rise is shown on the y-axis and is small, but remember that the y-intercept is $300. Since there's such a large gap before the y-intercept, the y-axis will need to skip values so the graph doesn't become too large. Start the y-axis at 0, then skip to 250 and label the rest of the axis in increments of 5 until you reach 450. Use the zigzag line to show you skipped values between 0 and 250.



5. Graph the equation using the slope and y-intercept. Plot the y-intercept first.

   The y-intercept is 300. Remember that the y-intercept is where the graph crosses the y-axis and the value of x is 0. Therefore, the coordinate of the y-intercept will always have 0 for x. In this case, the coordinate of the y-intercept is (0, 300).



6. Graph the equation using the slope and *y*-intercept. Use the slope to find the second point.

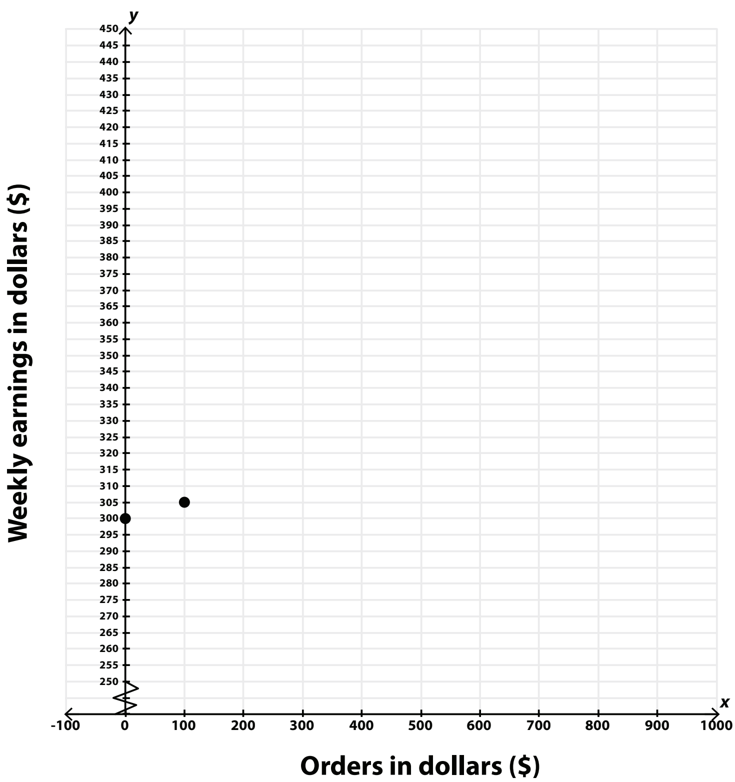
   Remember that the slope is $\frac{\text{rise}}{\text{run}}$. In this case the slope is 0.05. Rewrite 0.05 as a fraction.

   $$0.05 = \frac{5}{100} = \frac{\text{rise}}{\text{run}}$$

   The rise is 5 and the run is 100.

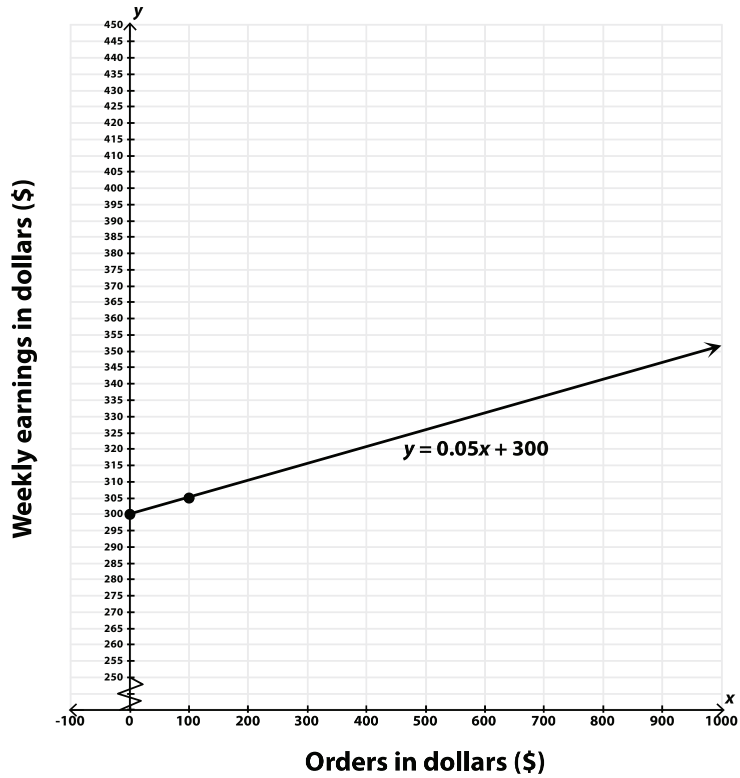
   Point your pencil at the *y*-intercept. Move the pencil up 5 units, since the slope is positive. On this grid, 5 units is one tick mark.

   Now, move your pencil to the right 100 units for the run and plot a point. On this grid, 100 units to the right is one tick mark. This is your second point.



7. Connect the points and extend the line. Then, label your line.

Draw a line through the two points and add an arrow to the right end of the line to show that the line continues infinitely in that direction. Label your line with the equation, $y = 0.05x + 300$.



Example 4

The velocity (or speed) of a ball thrown directly upward can be modeled with the following equation: $v = -gt + v_0$, where v is the speed, g is the force of gravity, t is the elapsed time, and v_0 is the initial velocity at time 0. If the force of gravity is equal to 32 feet per second, and the initial velocity of the ball is 96 feet per second, what is the equation that represents the velocity of the ball? Graph the equation.

1. Read the problem statement and then reread the problem, determining the known quantities.

   Initial velocity: 96 ft/s

   Force of gravity: 32 ft/s

   Notice that in the given equation, the force of gravity is negative. This is due to gravity acting on the ball, pulling it back to Earth and slowing the ball down from its initial velocity.

2. Identify the slope and the y-intercept.

   Notice the form of the given equation for velocity is the same form as $y = mx + b$, where $y = v$, $m = -g$, $x = t$, and $b = v_0$. Therefore, the slope = –32 and the y-intercept = 96.

3. Substitute the slope and y-intercept into the equation $y = mx + b$, where m is the slope and b is the y-intercept.

   $m = -g = -32$

   $b = v_0 = 96$

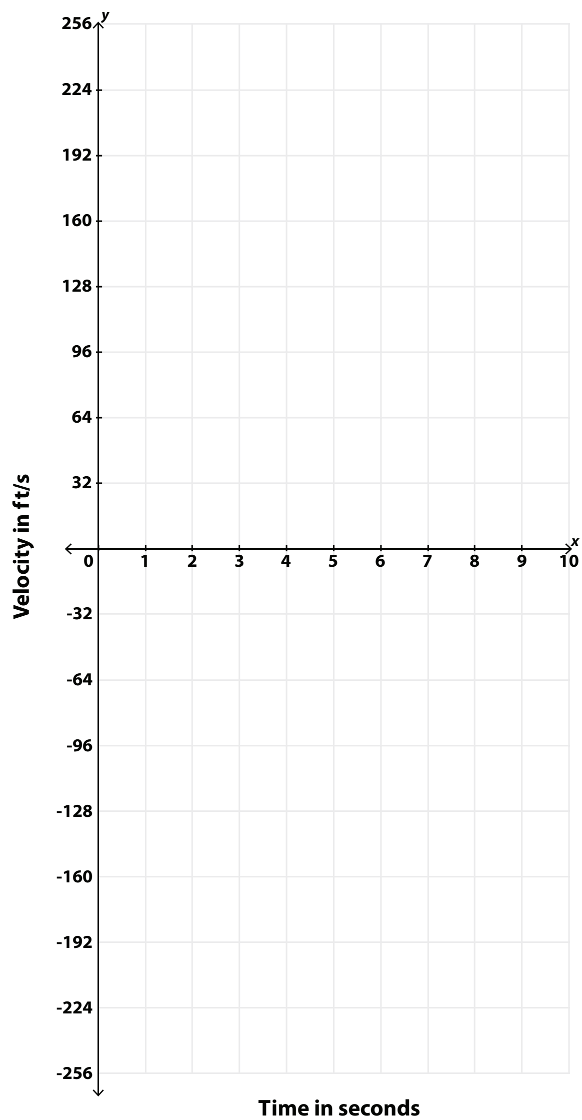
   $y = -32x + 96$

4. Set up the coordinate plane.

   In this scenario, x represents the time passing after the ball was dropped. The x-axis label is "Time in seconds."

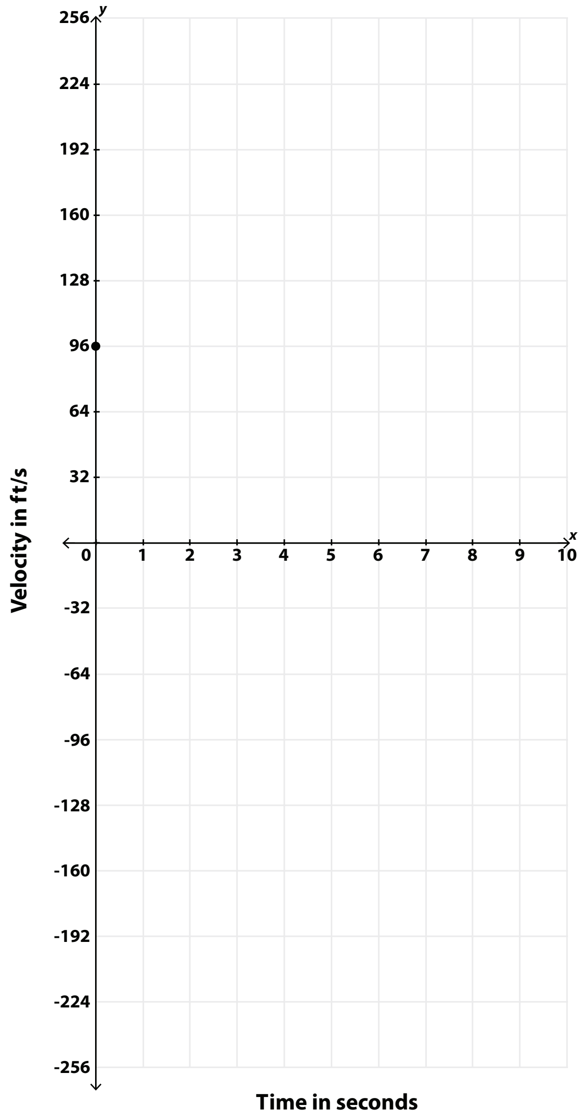
   The dependent variable, y, represents the velocity, or speed, of the ball. The y-axis label is "Velocity in ft/s."

   Determine the scales to be used. The y-intercept is close to 100 and the slope is 32. Notice that 96 (the y-intercept) is a multiple of 32. The y-axis can be labeled in units of 32. Since the x-axis is in seconds, it makes sense that these units are in increments of 1. Since time cannot be negative, use only a positive scale for the x-axis.



5. Graph the equation using the slope and *y*-intercept. Plot the *y*-intercept first.

   The *y*-intercept is 96. Remember that the *y*-intercept is where the graph crosses the *y*-axis and the value of *x* is 0. Therefore, the coordinate of the *y*-intercept will always have 0 for *x*. In this case, the coordinate of the *y*-intercept is (0, 96).



6. Graph the equation using the slope and y-intercept. Use the slope to find the second point.

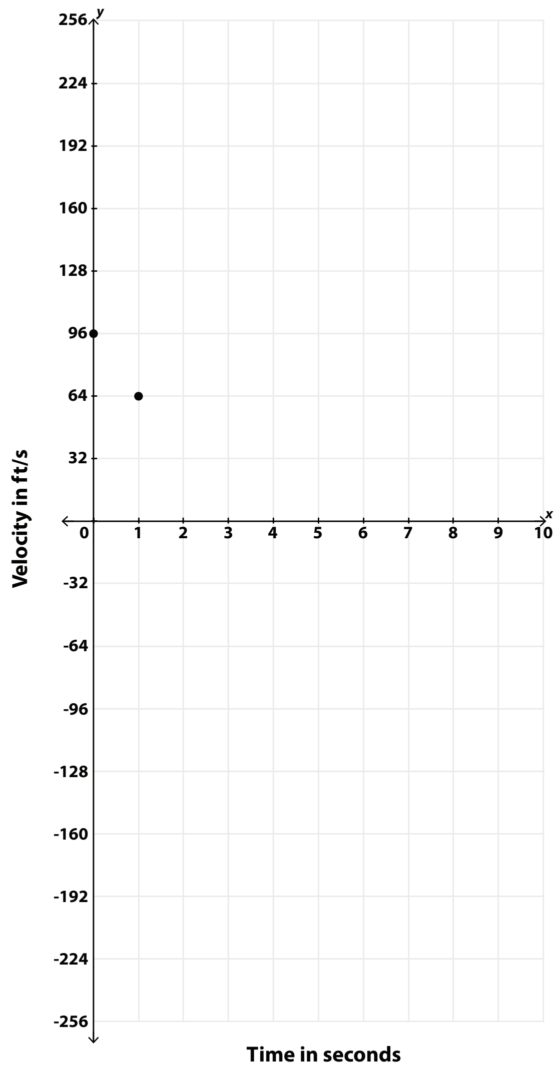
Remember that the slope is $\frac{\text{rise}}{\text{run}}$. In this case, the slope is –32. Rewrite –32 as a fraction.

$$-32=\frac{-32}{1}=\frac{\text{rise}}{\text{run}}$$

The rise is –32 and the run is 1.

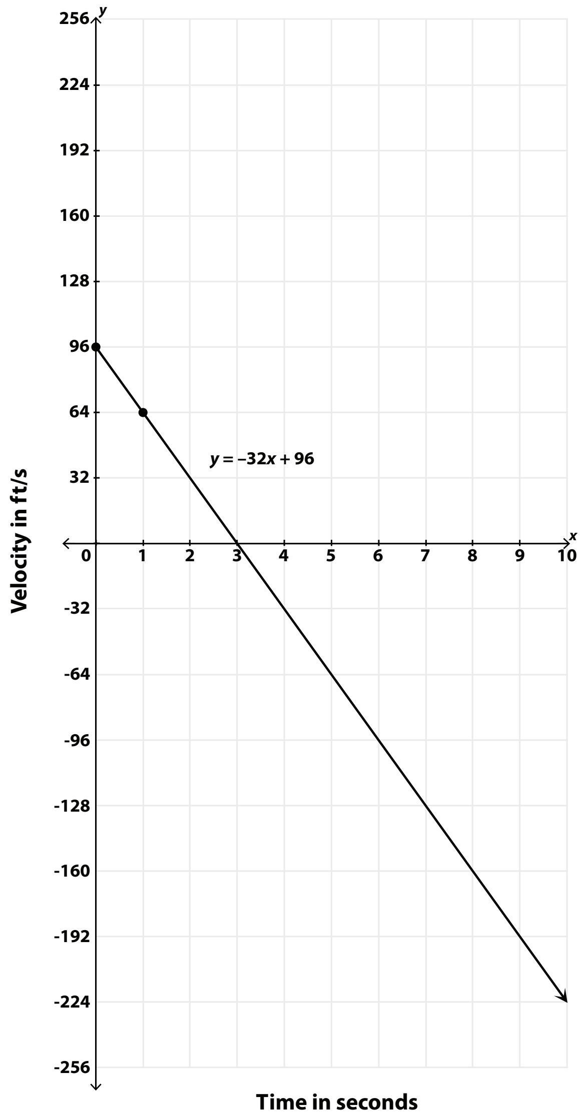
Point your pencil at the y-intercept. Move the pencil down 32 units, since the slope is negative. On this grid, 32 units is one tick mark.

Now, move your pencil to the right 1 unit for the run and plot a point. This is your second point.



7. Connect the points and extend the line. Then, label your line.

   Draw a line through the two points and add an arrow to the right end of the line to show that the line continues infinitely in that direction. Label your line with the equation, $y = -32x + 96$.



Example 5

A Boeing 747 starts out a long flight with about 57,260 gallons of fuel in its tank. The airplane uses an average of 5 gallons of fuel per mile. Write an equation that models the amount of fuel in the tank and then graph the equation using a graphing calculator.

1. Read the problem statement and then reread the problem, determining the known quantities.

   Starting fuel tank amount: 57,260 gallons

   Rate of fuel consumption: 5 gallons per mile

2. Identify the slope and the y-intercept.

   The slope is a rate. Notice the word "per" in the phrase "5 gallons of fuel per mile." Since the total number of gallons left in the fuel tank is decreasing at this rate, the slope is negative.

   Slope = –5

   The y-intercept is a starting value. The airplane starts out with 57,260 gallons of fuel.

   y-intercept = 57,260

   Substitute the slope and y-intercept into the equation $y = mx + b$, where m is the slope and b is the y-intercept.

   $m = 5$

   $b = 57{,}260$

   $y = -5x + 57{,}260$

3. Graph the equation on your calculator.

On a TI-83/84:

Step 1: Press [Y=].

Step 2: At Y_1, type in [(–)][5][X, T, θ, *n*][+][57260].

Step 3: Press [WINDOW] to change the viewing window.

Step 4: At Xmin, enter [0] and arrow down 1 level to Xmax.

Step 5: At Xmax, enter [3000] and arrow down 1 level to Xscl.

Step 6: At Xscl, enter [100] and arrow down 1 level to Ymin.

Step 7: At Ymin, enter [40000] and arrow down 1 level to Ymax.

Step 8: At Ymax, enter [58000] and arrow down 1 level to Yscl.

Step 9: At Yscl, enter [1000].

Step 10: Press [GRAPH].

On a TI-Nspire:

Step 1: Press the [home] key.

Step 2: Arrow over to the graphing icon and press [enter].

Step 3: At the blinking cursor at the bottom of the screen, enter in the equation [(–)][5][x][+][57260] and press [enter].

Step 4: Change the viewing window by pressing [menu], arrowing down to number 4: Window/Zoom, and clicking the center button of the navigation pad.

Step 5: Choose 1: Window settings by pressing the center button.

Step 6: Enter in the appropriate XMin value, [0], then press [tab].

Step 7: Enter in the appropriate XMax value, [3000], then press [tab].

Step 8: Leave the XScale set to "Auto." Press [tab] twice to navigate to YMin and enter [40000].

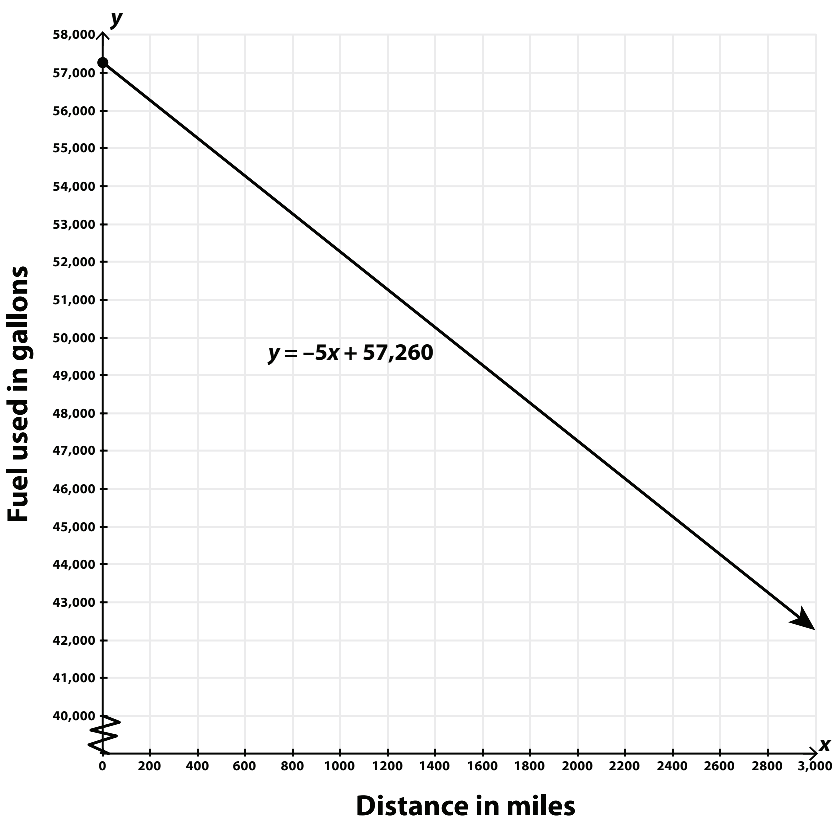
Step 9: Press [tab] to navigate to YMax. Enter [58000]. Press [tab] twice to leave YScale set to "auto" and to navigate to "OK."

Step 10: Press [enter].

Step 11: Press [menu] and select 2: View and 5: Show Grid.

4. Redraw the graph on graph paper.

   On the TI-83/84, the scale was entered in [WINDOW] settings. The X scale was 100 and the Y scale was 1000. Set up the graph paper using these scales. Label the *y*-axis "Fuel used in gallons." Show a break in the graph from 0 to 40,000 using a zigzag line. Label the *x*-axis "Distance in miles." To show the table on the calculator so you can plot points, press [2nd][GRAPH]. The table shows two columns with values; the first column holds the *x*-values, and the second column holds the *y*-values. Pick a pair to plot, and then connect the line. To return to the graph, press [GRAPH]. Remember to label the line with the equation. (*Note:* It may take you a few tries to get the window settings the way you want. The graph that follows shows an X scale of 200 so that you can easily see the full extent of the graphed line.)



(*continued*)

If you used a TI-Nspire, determine the scale that was used by counting the dots on the grid from your minimum *y*-value to your maximum *y*-value. In this case, there are 18 dots vertically between 40,000 and 58,000. The difference between the YMax and YMin values is 18,000. Divide that by the number of dots (18). The result (1,000) is the scale.

$$\frac{\text{Y Max} - \text{Y Min}}{\text{Number of dots}} = \frac{58{,}000 - 40{,}000}{18} = \frac{18{,}000}{18} = 1000$$

This means each dot is worth 1,000 units vertically. Label the *y*-axis "Fuel used in gallons." Use a zigzag line to show a break in the graph from 0 to 40,000.

Repeat the same process for determining the *x*-axis scale. The XMin = 0 and XMax = 3000. The number of dots = 30.

$$\frac{\text{X Max} - \text{X Min}}{\text{Number of dots}} = \frac{3000 - 0}{30} = \frac{3000}{30} = 100$$

This means each dot is worth 100 units horizontally.

Set up your graph paper accordingly. Label the *x*-axis "Distance in miles."

On your calculator, you need to show the table in order to plot points. To show the table, press [tab][T]. To navigate within the table, use the navigation pad. The table shows two columns with values; the first column holds the *x*-values, and the second column holds the *y*-values. Pick a pair to plot and then connect the line. Remember to label the line with the equation. To hide the table, navigate back to the graph by pressing [ctrl][tab]. Then press [ctrl][T].

Practice 1.3.1: Creating and Graphing Linear Equations in Two Variables

Graph each equation on graph paper.

1. $y = -x - 2$

2. $y = -x + 2$

3. $y = \frac{1}{2}x + 4$

4. A gear on a machine turns at a rate of $\frac{1}{2}$ revolution per second. Let x = time in seconds and y = number of revolutions. What is the equation that models the number of revolutions over time? Graph this equation.

5. The formula for converting temperature from degrees Fahrenheit to degrees Celsius is linear. To convert from Fahrenheit to Celsius, subtract 32 from the Fahrenheit temperature and then multiply by a rate of five-ninths. What is the equation that models the conversion of degrees Fahrenheit to degrees Celsius? Graph this equation.

6. A limousine company charges an initial rate of $50.00 and $75.00 for each hour. What is the equation that models the fee for hiring this limousine company? Graph this equation.

7. Angela receives a base weekly salary of $100 plus a commission of $65 for each computer she installs. What is the equation that models her weekly pay? Graph this equation.

8. A cable company charges a monthly fee of $59.00 plus $8 for each on-demand movie watched. What is the equation that models the company's total fees? Graph this equation.

9. Garrett borrowed $500 from his aunt. She doesn't charge any interest, and he makes $15 payments each month. What is the equation that models the amount Garrett owes? Graph this equation.

10. A small newspaper company is downsizing and has lost employees at a steady rate. Twelve months ago they had 65 employees, and now they have 29. What is the equation that models the loss of employees over time? Graph this equation.

Lesson 1.3.2: Creating and Graphing Exponential Equations

Introduction

Exponential equations in two variables are similar to linear equations in two variables in that there is an infinite number of solutions. The two variables and the equations that they are in describe a relationship between those two variables. Exponential equations are equations that have the variable in the exponent. This means the final values of the equation are going to grow or decay very quickly.

Key Concepts

Reviewing Exponential Equations:

- The general form of an exponential equation is $y = a \bullet b^x$, where a is the initial value, b is the rate of decay or growth, and x is the time. The final output value will be y.
- Since the equation has an exponent, the value increases or decreases rapidly.
- The base, b, must always be greater than 0 ($b > 0$).
- If the base is greater than 1 ($b > 1$), then the exponential equation represents exponential growth.
- If the base is between 0 and 1 ($0 < b < 1$), then the exponential equation represents exponential decay.
- If the base repeats after anything other than 1 unit (e.g., 1 month, 1 week, 1 day, 1 hour, 1 minute, 1 second), use the equation $y = ab^{\frac{x}{t}}$, where t is the time when the base repeats. For example, if a quantity doubles every 3 months, the equation would be $y = 2^{\frac{x}{3}}$.
- Another formula for exponential growth is $y = a(1 + r)^t$, where a is the initial value, $(1 + r)$ is the growth rate, t is time, and y is the final value.
- Another formula for exponential decay is $y = a(1 - r)^t$, where a is the initial value, $(1 - r)$ is the decay rate, t is time, and y is the final value.

Introducing the Compound Interest Formula:

- The general form of the compounding interest formula is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A is the initial value, r is the interest rate, n is the number of times the investment is compounded in a year, and t is the number of years the investment is left in the account to grow.
- Use this chart for reference:

Compounded…	***n* (number of times per year)**
Yearly/annually	1
Semi-annually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

- Remember to change the percentage rate into a decimal by dividing the percentage by 100.
- Apply the order of operations and divide r by n, then add 1. Raise that value to the power of the product of nt. Multiply that value by the principal, P.

Graphing Exponential Equations Using a Table of Values

1. Create a table of values by choosing x-values and substituting them in and solving for y.
2. Determine the labels by reading the context. The x-axis will most likely be time and the y-axis will be the units of the final value.
3. Determine the scales. The scale on the y-axis will need to be large since the values will grow or decline quickly. The value on the x-axis needs to be large enough to show the growth rate or the decay rate.

Graphing Equations Using a TI-83/84:

Step 1: Press [Y=] and key in the equation using [^] for the exponent and [X, T, θ, *n*] for *x*.

Step 2: Press [WINDOW] to change the viewing window, if necessary.

Step 3: Enter in appropriate values for Xmin, Xmax, Xscl, Ymin, Ymax, and Yscl, using the arrow keys to navigate.

Step 4: Press [GRAPH].

Graphing Equations Using a TI-Nspire:

Step 1: Press the home key.

Step 2: Arrow over to the graphing icon (the picture of the parabola or the U-shaped curve) and press [enter].

Step 3: At the blinking cursor at the bottom of the screen, enter in the equation using [^] before entering the exponents, and press [enter].

Step 4: To change the viewing window: press [menu], arrow down to number 4: Window/Zoom and click the center button of the navigation pad.

Step 5: Choose 1: Window settings by pressing the center button.

Step 6: Enter in the appropriate XMin, Xmax, YMin, and YMax fields.

Step 7: Leave the XScale and YScale set to auto.

Step 8: Use [tab] to navigate among the fields.

Step 9: Press [tab] to "OK" when done and press [enter].

Guided Practice 1.3.2

Example 1

If a pendulum swings to 90% of its height on each swing and starts out at a height of 60 cm, what is the equation that models this scenario? What is its graph?

1. Read the problem statement and then reread the scenario, identifying the known quantities.

   Initial height = 60 cm

   Decay rate = 90% or 0.90

2. Substitute the known quantities into the general form of the exponential equation $y = ab^x$, where a is the initial value, b is the rate of decay, x is time (in this case swings), and y is the final value.

   $a = 60$

   $b = 0.90$

   $y = ab^x$

   $y = 60(0.90)^x$

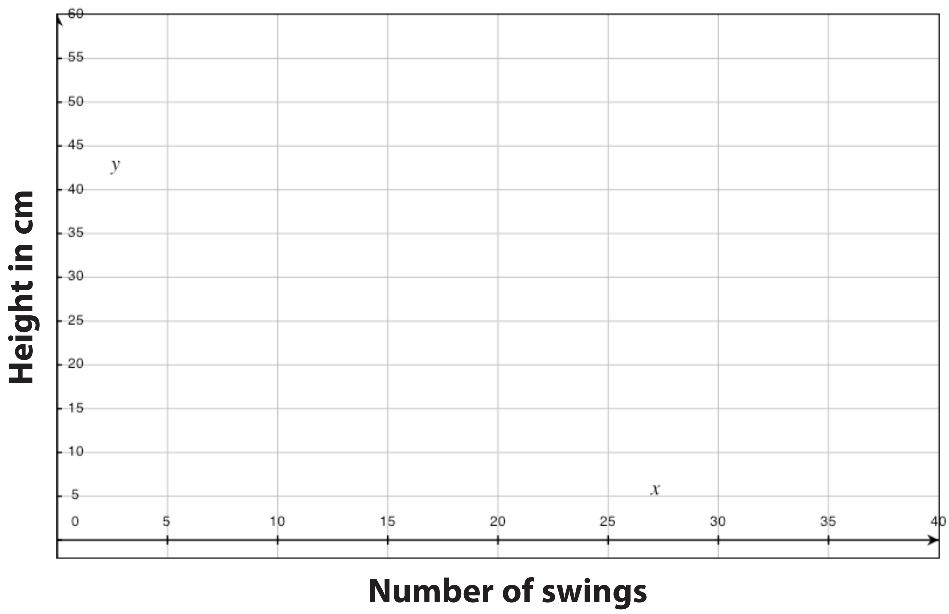
3. Create a table of values.

x	y
0	60
1	54
2	48.6
3	43.74
5	35.43
10	20.92
20	7.29
40	0.89

4. Set up the coordinate plane.

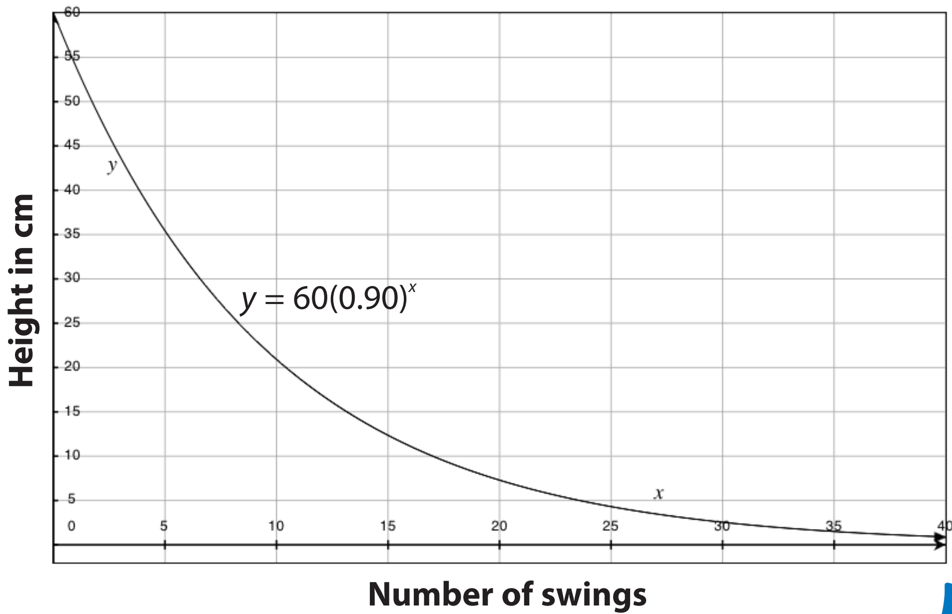
Determine the labels by reading the problem again. The independent variable is the number of swings. That will be the label of the x-axis. The y-axis label will be the height. The height is the dependent variable because it depends on the number of swings.

To determine the scales, examine the table of values. The x-axis needs a scale that goes from 0 to 40. Counting to 40 in increments of 1 would cause the axis to be very long. Use increments of 5. For the y-axis, start with 0 and go to 60 in increments of 5. This will make plotting numbers like 43.74 a little easier than if you chose increments of 10.



5. Plot the points on the coordinate plane and connect the points with a line (curve).

   When the points do not lie on a grid line, use estimation to approximate where the point should be plotted. Add an arrow to the right end of the line to show that the curve continues in that direction toward infinity.



Example 2

The bacteria *Streptococcus lactis* doubles every 26 minutes in milk. If a container of milk contains 4 bacteria, write an equation that models this scenario and then graph the equation.

1. Read the problem statement and then reread the scenario, identifying the known quantities.

   Initial bacteria count = 4

   Base = 2

   Time period = 26 minutes

2. Substitute the known quantities into the general form of the exponential equation $y = ab^x$, for which a is the initial value, b is the base, x is time (in this case 1 time period is 26 minutes), and y is the final value. Since the base is repeating in units other than 1, use the equation $y = ab^{\frac{x}{t}}$, where $t = 26$.

   $a = 4$

   $b = 2$

   $y = ab^{\frac{x}{26}}$

   $y = 4(2)^{\frac{x}{26}}$

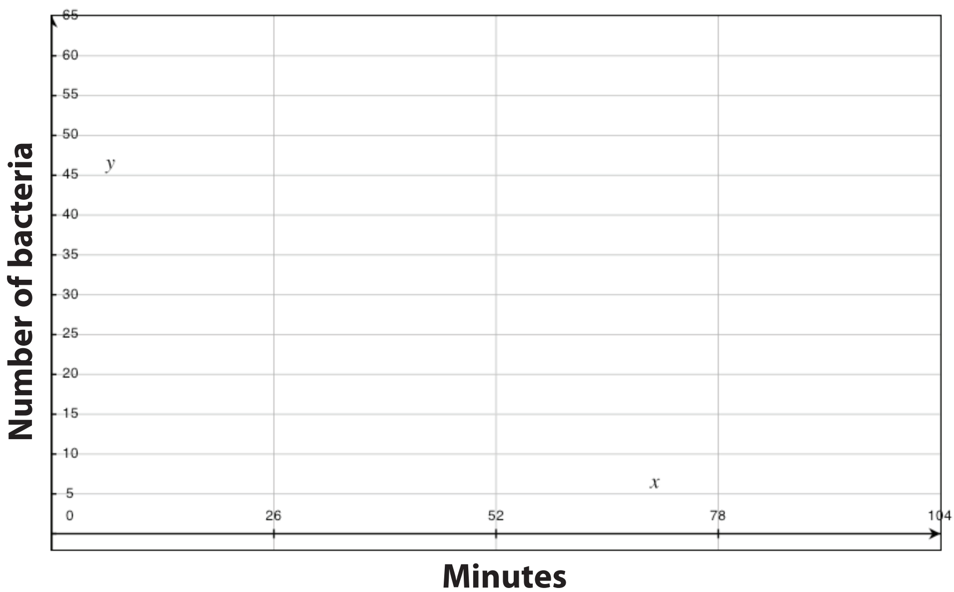
3. Create a table of values.

x	y
0	4
26	8
52	16
78	32
104	64

4. Set up the coordinate plane.

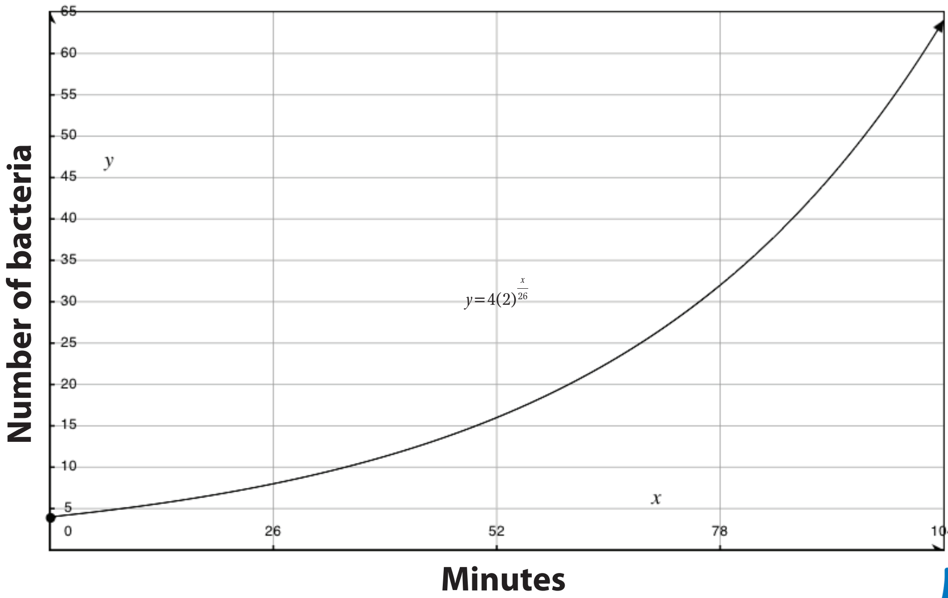
   Determine the labels by reading the problem again. The independent variable is the number of time periods. The time periods are in number of minutes. Therefore, "Minutes" will be the x-axis label. The y-axis label will be the "Number of bacteria." The number of bacteria is the dependent variable because it depends on the number of minutes that have passed.

   The x-axis needs a scale that reflects the time period of 26 minutes and the table of values. The table of values showed 4 time periods. One time period = 26 minutes and so 4 time periods = 4(26) = 104 minutes. This means the x-axis scale needs to go from 0 to 104. Use increments of 26 for easy plotting of the points. For the y-axis, start with 0 and go to 65 in increments of 5. This will make plotting numbers like 32 a little easier than if you chose increments of 10.



5. Plot the points on the coordinate plane and connect the points with a line (curve).

   When the points do not lie on a grid line, use estimation to approximate where the point should be plotted. Add an arrow to the right end of the line to show that the curve continues in that direction toward infinity.



Example 3

An investment of \$500 is compounded monthly at a rate of 3%. What is the equation that models this situation? Graph the equation.

1. Read the problem statement and then reread the scenario, identifying the known quantities.

   Initial investment = \$500

   $r = 3\%$

   Compounded monthly = 12 times a year

2. Substitute the known quantities into the general form of the compound interest formula, $A = P\left(1+\frac{r}{n}\right)^{nt}$, for which P is the initial value, r is the interest rate, n is the number of times the investment is compounded in a year, and t is the number of years the investment is left in the account to grow.

   $P = 500$

   $r = 3\% = 0.03$

   $n = 12$

   $$A = P\left(1+\frac{r}{n}\right)^{nt}$$

   $$A = 500\left(1+\frac{0.03}{12}\right)^{12t}$$

   $$A = 500(1.0025)^{12t}$$

   Notice that, after simplifying, this form is similar to $y = ab^x$. To graph on the x- and y-axes, put the compounded interest formula into this form, where $A = y$, $P = a$, $\left(1+\frac{r}{n}\right) = b$, and $t = x$.

   $A = 500(1.0025)^{12t}$ becomes $y = 500(1.0025)^{12x}$.

3. Graph the equation using a graphing calculator.

On a TI-83/84:

Step 1: Press [Y=].

Step 2: Type in the equation as follows: [500][×][1.0025][^][12][X, T, θ, *n*]

Step 3: Press [WINDOW] to change the viewing window.

Step 4: At Xmin, enter [0] and arrow down 1 level to Xmax.

Step 5: At Xmax, enter [10] and arrow down 1 level to Xscl.

Step 6: At Xscl, enter [1] and arrow down 1 level to Ymin.

Step 7: At Ymin, enter [500] and arrow down 1 level to Ymax.

Step 8: At Ymax, enter [700] and arrow down 1 level to Yscl.

Step 9: At Yscl, enter [15].

Step 10: Press [GRAPH].

On a TI-Nspire:

Step 1: Press the [home] key.

Step 2: Arrow over to the graphing icon and press [enter].

Step 3: At the blinking cursor at the bottom of the screen, enter in the equation [500][×][1.0025][^][12x] and press [enter].

Step 4: To change the viewing window: press [menu], arrow down to number 4: Window/Zoom, and click the center button of the navigation pad.

Step 5: Choose 1: Window settings by pressing the center button.

Step 6: Enter in the appropriate XMin value, [0], and press [tab].

Step 7: Enter in the appropriate XMax value, [10], and press [tab].

Step 8: Leave the XScale set to "Auto." Press [tab] twice to navigate to YMin and enter [500].

Step 9: Press [tab] to navigate to YMax. Enter [700]. Press [tab] twice to leave YScale set to "Auto" and to navigate to "OK."

Step 10: Press [enter].

Step 11: Press [menu] and select 2: View and 5: Show Grid.

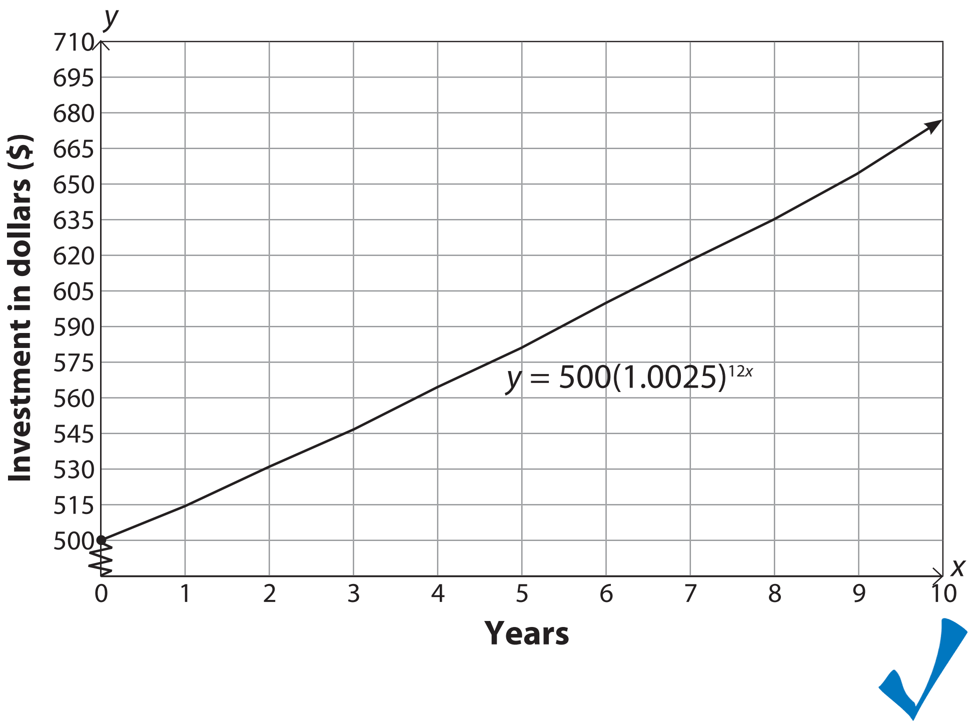
Note: To determine the y-axis scale, show the table to get an idea of the values for y. To show the table, press [ctrl] and then [T]. To turn the table off, press [ctrl][tab] to navigate back to the graphing window and then press [ctrl][T] to turn off the table.

4. Transfer your graph from the screen to graph paper.

   Use the same scales that you set for your viewing window.

   The x-axis scale goes from 0 to 10 years in increments of 1 year.

   The y-axis scale goes from \$500 to \$700 in increments of \$15. You'll need to show a break in the graph from 0 to 500 with a zigzag line.



Practice 1.3.2: Creating and Graphing Exponential Equations

Use a table of values to graph the following exponential equations.

1. $y = 3(2)^x$

2. $y = 30(0.95)^x$

3. $y = 800(1.00267)^{12x}$

Write an equation to model each scenario, and then graph the equation.

4. The NCAA Division I Basketball Tournament begins each year with 64 teams. After each round a team is eliminated, reducing the number of teams by half.

5. A type of bacteria doubles every 36 hours. A Petri dish starts out with 16 of these bacteria.

6. The population of a town is increasing by 1.7% per year. The current population is 9,000 people.

7. The population of a town is decreasing by 2.2% per year. The current population is 15,000 people.

8. An investment of $2,500 earns 2.3% interest and is compounded monthly.

9. An investment of $300 earns 3.1% interest and is compounded weekly.

10. An investment of $500 earns 1.9% interest and is compounded daily.

Lesson 4: Representing Constraints

Common Core State Standard

A–CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★

Essential Questions

1. How can you model real-world applications using equations?
2. How can you solve real-world applications by graphing systems of equations?
3. How can you model real-world applications using inequalities?
4. Why are there constraints when solving and graphing real-world applications?

WORDS TO KNOW

algebraic inequality	an inequality that has one or more variables and contains at least one of the following symbols: $<$, $>$, $\leq$, $\geq$, or $\neq$
constraint	a restriction or limitation on either the input or output values
inequality	a mathematical sentence that shows the relationship between quantities that are not equivalent
solution set	the value or values that make a sentence or statement true
system of equations	a set of equations with the same unknowns
system of inequalities	a set of inequalities with the same unknowns

Recommended Resources

- NCTM Illuminations. "Dirt Bike Dilemma."

  http://walch.com/rr/CAU1L4SysEquations

  Students use a system of equations to maximize profits for a dirt bike manufacturer.

- Purplemath.com. "Linear Programming: Word Problems."

  http://walch.com/rr/CAU1L4SysInequalities

  This site offers a review of systems of inequalities and constraints associated with real-world situations. It also includes graphs of feasible regions.

Lesson 1.4.1: Representing Constraints

Introduction

Situations in the real world often determine the types of values we would expect as answers to equations and inequalities. When an inequality has one or more variables and contains at least one inequality symbol ($<$, $>$, $\leq$, $\geq$, or $\neq$), it is called an **algebraic inequality**.

Sometimes there are limits or restrictions on the values that can be substituted into an equation or inequality; other times, limits or restrictions are placed on answers to problems involving equations or inequalities. These limits or restrictions are called **constraints**.

Key Concepts

- Many real-world situations can be modeled using an equation, an inequality, or a **system of equations** or **inequalities**. A system is a set of equations or inequalities with the same unknowns.
- When creating a system of equations or inequalities, it is important to understand that the solution set is the value or values that make each sentence in the system a true statement.
- Being able to translate real-world situations into algebraic sentences will help with the understanding of constraints.

Guided Practice 1.4.1

Example 1

Determine whether the coordinate (–2, 9) is a solution to the inequality $y \le 5x + 6$.

1. Substitute the values for x and y into the original inequality.

   $y \le 5x + 6$

   $9 \le 5(-2) + 6$

2. Simplify the sentence.

$9 \le 5(-2) + 6$	Multiply 5 and –2.
$9 \le -10 + 6$	Add –10 and 6.
$9 \le -4$	

3. Interpret the results.

   9 is NOT less than or equal to –4; therefore, (–2, 9) is not a solution to the inequality $y \le 5x + 6$.

Example 2

A taxi company charges \$2.50 plus \$1.10 for each mile driven. Write an equation to represent this situation. Use this equation to determine how far you can travel if you have \$10.00. What is the minimum amount of money you will spend?

1. Translate the verbal description into an algebraic equation. Let m represent the number of miles driven and let C represent the total cost of the trip.

   $2.50 + 1.10m = C$

2. The total cost of the trip can't be more than \$10.00 because that is all you have to spend. Substitute this amount in for C.

   $2.50 + 1.10m = 10.00$

3. Although you have \$10.00 to spend, you could also spend less than that. Change the equal sign to a less than or equal to sign (≤).

   $2.50 + 1.10m \le 10.00$

4. Solve the inequality by isolating the variable.

$2.50 + 1.10m \le 10.00$	Subtract 2.50 from both sides.
$1.10m \le 7.50$	Divide both sides by 1.10.
$m \le 6.82$	

   You can travel up to 6.82 miles and not pay more than \$10.00. Because the company charges by the mile, you can travel no more than 6 miles.

5. The minimum the taxi driver charges is \$2.50, but it is unlikely that he or she will charge you if you get in the cab and get right back out without going anywhere. You will pay \$1.10 if you travel 1 mile or less; add this to the minimum charge of \$2.50 to arrive at \$3.60.

6. You will spend a minimum of \$3.60, but no more than \$10.00. ✓

Example 3

A school supply company produces wooden rulers and plastic rulers. The rulers must first be made, and then painted.

- It takes 20 minutes to make a wooden ruler. It takes 15 minutes to make a plastic ruler. There is a maximum amount of 480 minutes per day set aside for making rulers.
- It takes 5 minutes to paint a wooden ruler. It takes 2 minutes to paint a plastic ruler. There is a maximum amount of 180 minutes per day set aside for painting rulers.

Write a system of inequalities that models the making and then painting of wooden and plastic rulers.

1. Identify the information you know.

   There is a maximum of 480 minutes for making rulers.

   - It takes 20 minutes to make a wooden ruler.
   - It takes 15 minutes to make a plastic ruler.

   There is a maximum of 180 minutes for painting rulers.

   - It takes 5 minutes to paint a wooden ruler.
   - It takes 2 minutes to paint a plastic ruler.

2. Write an inequality to represent the amount of time needed to make the rulers. Let w represent the wooden rulers and p represent the plastic rulers.

   $20w + 15p \le 480$

3. Write an inequality to represent the amount of time needed to paint the rulers. Use the same variables to represent wooden and plastic rulers.

   $5w + 2p \le 180$

4. Now consider the constraints on this situation. It is not possible to produce a negative amount of either wooden rulers or plastic rulers; therefore, you need to limit the values of w and p to values that are greater than or equal to 0.

   $w \ge 0$

   $p \ge 0$

5. Combine all the inequalities related to the situation and list them in a brace, {. These are the constraints of your scenario.

$$\begin{cases} 20w + 15p \le 480 \\ 5w + 2p \le 180 \\ w \ge 0 \\ p \ge 0 \end{cases}$$

Example 4

Use the system of inequalities created in Example 3 to give a possible solution to the system.

1. We know from this situation that you cannot produce a negative amount of rulers, so none of our solutions can be negative.

2. In future lessons, we discuss more precise ways of determining the solution set to a system. For now, we can use our knowledge of numbers and ability to solve algebraic sentences to find possible solutions.

3. Choose a value for w.

   Let $w = 0$. Substitute 0 for each occurrence of w in the system and solve for p.

$20w + 15p \leq 480$	
$20(0) + 15p \leq 480$	Substitute 0 for w.
$15p \leq 480$	Divide both sides by 15.

   For the first inequality, $p \leq 32$.

$5w + 2p \leq 180$	
$5(0) + 2p \leq 180$	Substitute 0 for w.
$2p \leq 180$	Divide both sides by 2.

   For the second inequality, $p \leq 90$.

4. Interpret the results.

   In 480 minutes, the company can make no more than 32 plastic rulers if 0 wooden rulers are produced.

   In 180 minutes, the company can paint no more than 90 plastic rulers if there are no wooden rulers to paint.

Practice 1.4.1: Representing Constraints

Determine whether each coordinate listed below is a solution to the given algebraic sentence.

1. Is the coordinate (3, 1) a solution to the equation $y = -15x + 14$?

2. Is the coordinate (2, 5) a solution to the inequality $y \le 11x - 7$?

3. Is the coordinate (1, 3) a solution to the inequality $y < 6x - 3$?

Read each scenario and use it to complete the parts that follow.

4. Given the inequalities $y < -2x + 8$ and $y \ge 4x + 1$, find a point that
   a. satisfies both inequalities.

   b. satisfies neither inequality.

   c. satisfies one inequality, but not the other.

5. A water company charges a monthly fee of \$7.90 plus a usage fee of \$2.60 per 1,000 gallons used.
   a. Write an equation to find the total monthly charges, including the number of gallons used.

   b. Use your equation to determine the maximum number of gallons you can use each month if you have budgeted \$30 a month for water.

continued

Use the information in each scenario to complete problems 6–10.

6. A clothing store is selling graphic tees for $7.50 and solid tees for $5.00. You have $30. Write an inequality to represent this situation. What can you buy?

7. A store offers two rental options for a television. Plan A charges an up-front fee of $15 plus another $7 each month. Plan B charges $20 up front plus $5 each month. Write a system of equations that represents the cost of renting a television under both plans. Be sure to include any necessary constraints.

8. You have 200 feet of fence to create a rectangular garden. The width of the garden can't be more than 20 feet. What constraints represent this situation?

9. The local bakery never has more than a combined total of 32 strawberry cakes and carrot cakes and never more than 6 carrot cakes. Write a system of inequalities that represents this situation. Be sure to include all constraints.

10. Emilee wants to restock her fish tank. She can add no more than 15 new fish, and she wants to include Tiger Barbs and catfish. She would like to have at least 4 catfish. Write a system of inequalities that represents this situation. Be sure to include all constraints.

Lesson 5: Rearranging Formulas

Common Core State Standard

A–CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*★

Essential Questions

1. How is solving a literal equation or formula for a specific variable similar to solving an equation with one variable?
2. How could solving a literal equation or formula for a specific variable be helpful?
3. How do you determine for which variable a literal equation or formula should be solved?

WORDS TO KNOW

formula a literal equation that states a specific rule or relationship among quantities

inverse a number that when multiplied by the original number has a product of 1

literal equation an equation that involves two or more variables

reciprocal a number that when multiplied by the original number has a product of 1

Recommended Resources

- CRCTLessons.com. "Solving Equations Game."

  http://walch.com/rr/CAU1L5SolvingEquations

  Practice solving equations for a given variable with this online basketball game.

- Purplemath.com. "Solving Literal Equations."

  http://walch.com/rr/CAU1L5LitEquations

  This site has an overview of literal equations, with worked examples on how to solve equations and formulas for a given variable.

Lesson 1.5.1: Rearranging Formulas

Introduction

Literal equations are equations that involve two or more variables. Sometimes it is useful to rearrange or solve literal equations for a specific variable in order to find a solution to a given problem. In this lesson, literal equations and **formulas**, or literal equations that state specific rules or relationships among quantities, will be examined.

Key Concepts

- It is important to remember that both literal equations and formulas contain an equal sign indicating that both sides of the equation must remain equal.
- Literal equations and formulas can be solved for a specific variable by isolating the specified variable.
- To isolate the specified variable, use inverse operations. When coefficients are fractions, multiply both sides of the equation by the **reciprocal**. The reciprocal of a number, also known as the **inverse** of a number, can be found by flipping a number. Think of an integer as a fraction with a denominator of 1. To find the reciprocal of the number, flip the fraction. The number 2 can be thought of as the fraction $\frac{2}{1}$. To find the reciprocal, flip the fraction: $\frac{2}{1}$ becomes $\frac{1}{2}$. You can check if you have the correct reciprocal because the product of a number and its reciprocal is always 1.

Guided Practice 1.5.1

Example 1

Solve $6y - 12x = 18$ for y.

1. Begin isolating y by adding $12x$ to both sides.

$$\begin{array}{r} 6y - 12x = 18 \\ \underline{\quad + 12x \qquad + 12x} \\ 6y = 18 + 12x \end{array}$$

2. Divide each term by 6.

$$\frac{6y}{6} = \frac{18}{6} + \frac{12x}{6}$$

$$y = 3 + 2x$$

Example 2

Solve $15x - 5y = 25$ for y.

1. Begin isolating y by subtracting $15x$ from both sides of the equation.

$$\begin{array}{r} 15x - 5y = 25 \\ \underline{-15x \qquad\quad -15x} \\ -5y = 25 - 15x \end{array}$$

2. To further isolate y, divide both sides of the equation by the coefficient of y. The coefficient of y is –5. Be sure that each term of the equation is divided by –5.

$$\frac{-5y}{-5} = \frac{25 - 15x}{-5}$$

$$\frac{-5y}{-5} = \frac{25}{-5} - \frac{15x}{-5}$$

$$y = -5 + 3x$$

Example 3

Solve $4y + 3x = 16$ for y.

1. Begin isolating y by subtracting $3x$ from both sides of the equation.

$$\begin{array}{r} 4y+3x=16 \\ \underline{\quad -3x \quad -3x} \\ 4y=16-3x \end{array}$$

2. To further isolate y, divide both sides of the equation by the coefficient of y. The coefficient of y is 4. Be sure that each term of the equation is divided by 4.

$$\frac{4y}{4}=\frac{16-3x}{4}$$

$$y=\frac{16}{4}-\frac{3x}{4}$$

$$y=4-\frac{3}{4}x$$

Example 4

The formula for finding the area of a triangle is $A=\frac{1}{2}bh$, where b is the length of the base and h is the height of the triangle. Suppose you know the area and height of the triangle, but need to find the length of the base. In this case, solving the formula for b would be helpful.

1. Begin isolating b by multiplying both sides of the equation by the reciprocal of $\frac{1}{2}$, or 2.

   $$A=\frac{1}{2}bh$$
   $$2\bullet A=2\bullet\left(\frac{1}{2}bh\right)$$
   $$2A=bh$$

   Multiplying both sides of the equation by the reciprocal is the same as dividing both sides of the equation by $\frac{1}{2}$. The result will be the same.

2. To further isolate b, divide both sides of the equation by h.

   $$\frac{2A}{h}=\frac{bh}{h} \text{ or } b=\frac{2A}{h}$$
   $$\frac{2A}{h}=b$$

3. The formula for finding the length of the base of a triangle can be found by doubling the area and dividing the result by the height of the triangle.

Example 5

The distance, d, that a train can travel is found by multiplying the rate of speed, r, by the amount of time that it is travelling, t, or $d = rt$. Solve this formula for t to find the amount of time the train will travel given a specific distance and rate of speed.

1. Isolate t by dividing both sides of the equation by r.

$$\frac{d}{r}=\frac{rt}{r}$$

$$t=\frac{d}{r}$$

2. The formula for finding the amount of time it will take a train to travel a given distance at a given speed is $t=\frac{d}{r}$.

Practice 1.5.1: Rearranging Equations and Formulas

For problems 1–4, solve each equation for y.

1. $4y + 24 = 40x$

2. $7y + 14x = 63$

3. $24x - 72 = 8y$

4. $39 - 3y = 15x$

Read each scenario and solve for the given variable.

5. The formula $C = \pi d$ is used to calculate the circumference of a circle. Solve this formula for d.

6. The formula for calculating distance given rate of speed and time is $d = rt$. Solve this formula for r.

7. The formula for calculating simple interest is $I = prt$. Solve this formula for t.

8. The formula for calculating the surface area of a right square pyramid is $A = s^2 + 2sl$. Solve this formula for l.

9. The formula for converting degrees Fahrenheit to degrees Celsius is $C = \frac{5}{9}(F - 32)$. Solve this formula for F.

10. The formula for calculating the volume of a square pyramid is $V = \frac{1}{3}b^2h$. Solve this formula for h.

Answer Key

Unit 1: Relationships Between Quantities

Lesson 1: Interpreting Structure in Expressions

Practice 1.1.1: Identifying Terms, Factors, and Coefficients, pp. 7–8

1. terms: $14x^2$, $2x$, -9
   factors: 14 and x^2, 2 and x
   coefficients: 14, 2
   constant: -9
3. terms: $(4x^3)/5$, $9x$
   factors: $4/5$ and x^3, 9 and x
   coefficient: $4/5$
   constants: there are none
5. expression: $x^6 + 3x$
   terms: x^6, $3x$
   factors: 3 and x
   coefficient: 3
   constants: there are none
7. expression: $2x + 0.05(x) = 2.05x$
   terms: $2.05x$
   factors: 2.05 and x
   coefficient: 2.05
   constants: there are none
9. expression: $x + x + (x - 4) + (x - 4) = 2(x) + 2(x - 4) = 4x - 8$
   terms: $4x$, -8
   factors: 4 and x
   coefficient: 4
   constant: -8

Practice 1.1.2: Interpreting Complicated Expressions, pp. 12–13

1. The order of operations indicates that exponents must be applied before multiplying.
3. The number of books does not affect the value of m; the number of books is a constant and remains unchanged by the number of magazines.
5. The value of the expression will be greater than 9.
7. The amount will be increased.
9. The values of $(1 + r)$ would be less than 1.

Lesson 2: Creating Equations and Inequalities in One Variable

Practice 1.2.1: Creating Linear Equations in One Variable, pp. 28–29

1. Answers may vary. Possible answers:
   a. miles per hour
   b. inches per minute or miles per hour
   c. meters per second or feet per second
   d. dollars per pound

3. 287.88 ft^2
5. \$15
7. \$17 per lunch
9. 76,800 square feet

Practice 1.2.2: Creating Linear Inequalities in One Variable, pp. 36–37

1. $x \leq 8$
3. $w \leq 2400$
5. $15x \geq 950$; $x \geq 63\ 1/3$; You must work at least 63 1/3 hours.
7. $22x + 9 \leq 75$; $x \leq 3$; Arianna can buy up to 3 computer games.
9. $25{,}000 - 3000x \geq 4000$; $x \leq 7$ days. The giveaway should end in 7 days or less in order to have \$4,000 or more to give away for the grand prize.

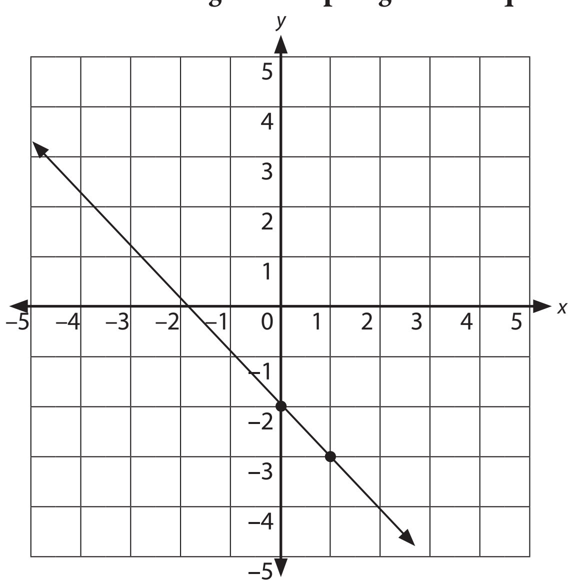
Practice 1.2.3: Creating Exponential Equations, pp. 47–48

1. a. linear; b. exponential
3. a. exponential; b. linear
5. $4800 = a(2)^3$; \$600
7. $y = 100(0.5)^8$; 0.39 grams
9. $y = 63{,}000(1.01)^4$; about 65,559 people

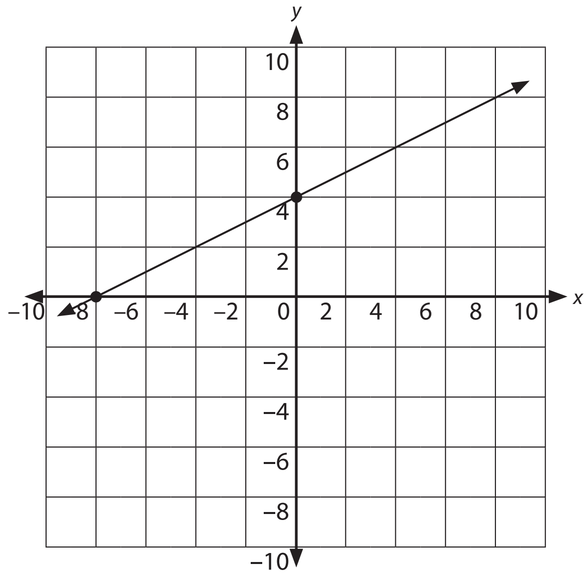
Lesson 3: Creating and Graphing Equations in Two Variables

Practice 1.3.1: Creating and Graphing Linear Equations in Two Variables, p. 79

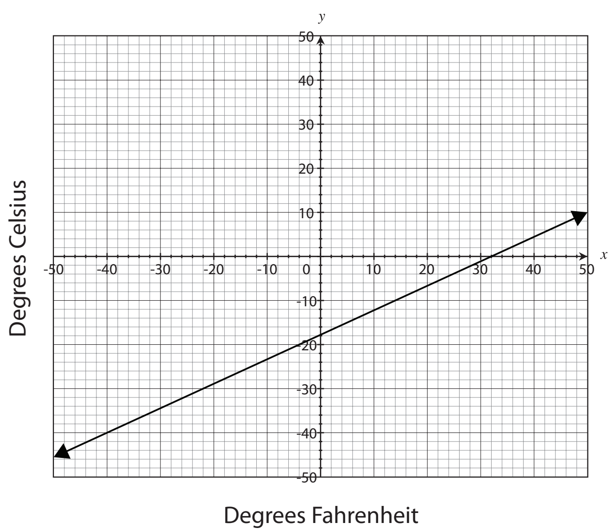
1.



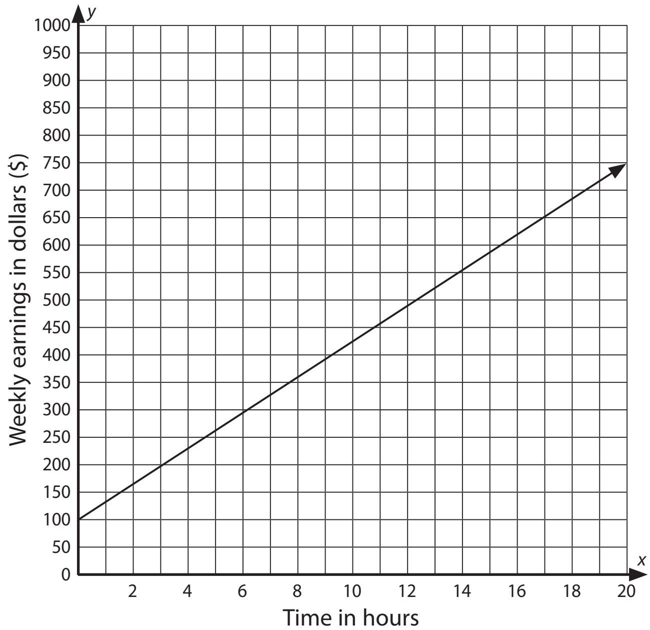
3.



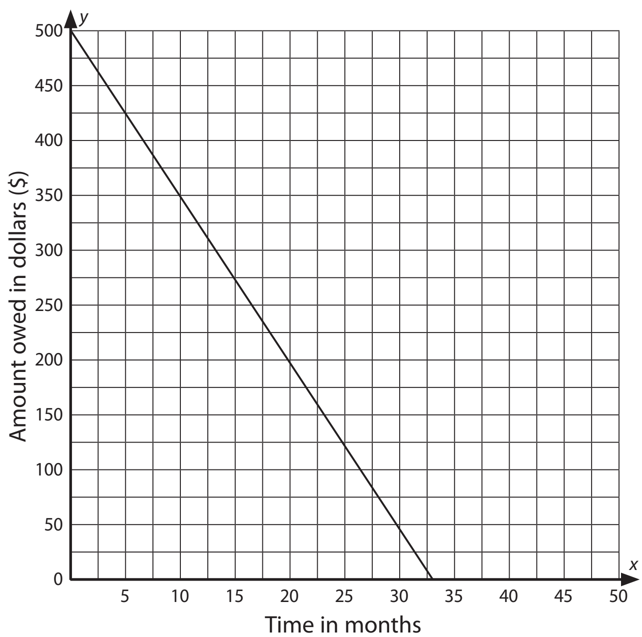
5. $y = 5/9(x - 32)$; slope = 5/9; y-intercept: (0, –17 7/9)



7. $y = 65x + 100$; slope $= 65$; y-intercept: $(0, 100)$

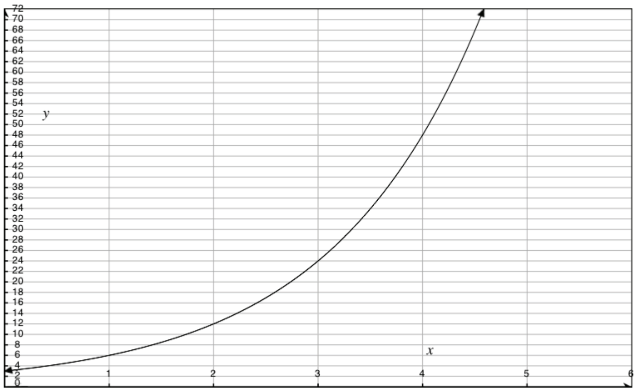


9. $y = -15x + 500$; slope $= -15$; y-intercept: $(0, 500)$

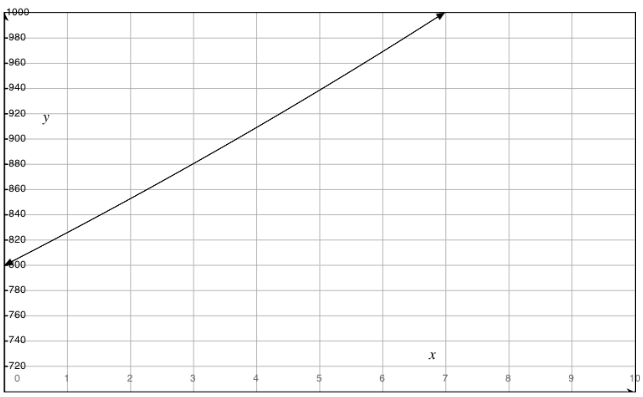


Practice 1.3.2: Creating and Graphing Exponential Equations, p. 92

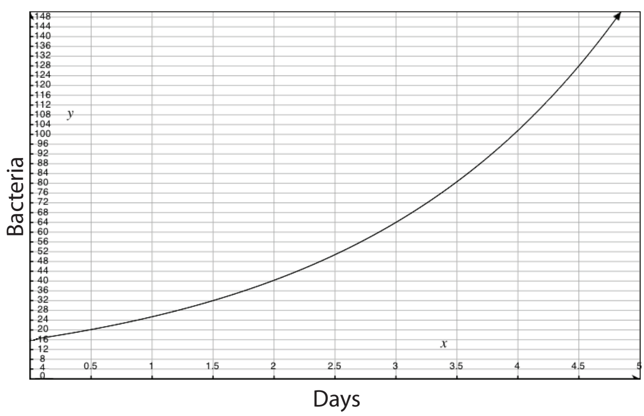
1.



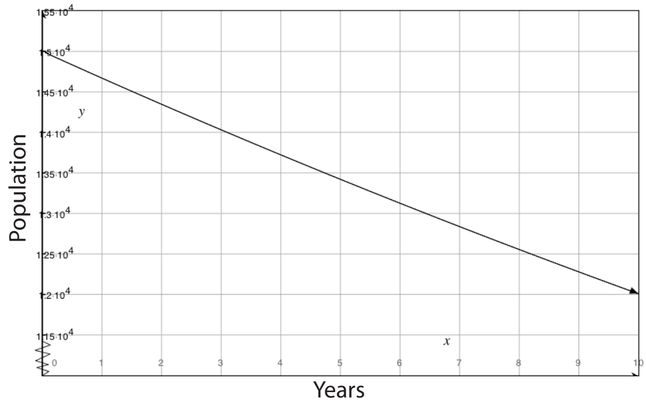
3.



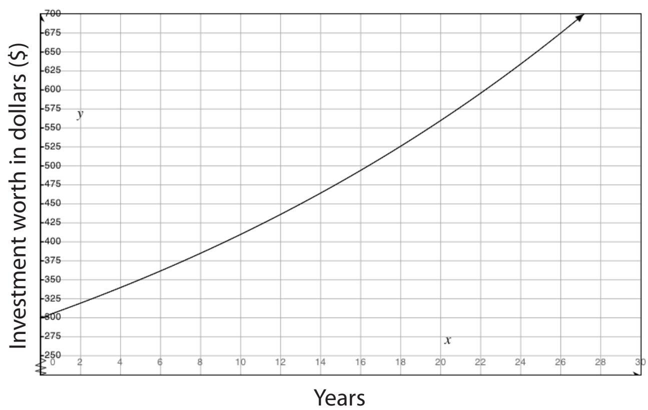
5. $y = 16(2)^{24x/36} = 16(2)^{2x/3}$, for which x is in days



7. $y = 15{,}000(0.978)^x$



9. $y = 300(1.0006)^{52x}$



Lesson 4: Representing Constraints

Practice 1.4.1: Representing Constraints, pp. 99–100

1. no
3. no
5. a. $y = 2.60x + 7.90$; b. $x = 8.5$; maximum = 8,500 gallons
7. $\begin{cases} y = 7x + 15 \\ y = 5x + 20 \\ x \geq 1 \end{cases}$
9. $\begin{cases} x + y \leq 32 \\ y \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$

Lesson 5: Rearranging Formulas

Practice 1.5.1: Rearranging Equations and Formulas, p. 107

1. $y = 10x - 6$
3. $y = 3x - 9$
5. $d = C/\pi$
7. $t = I/pr$
9. $F = (9/5)C + 32$