MOMENTUM!

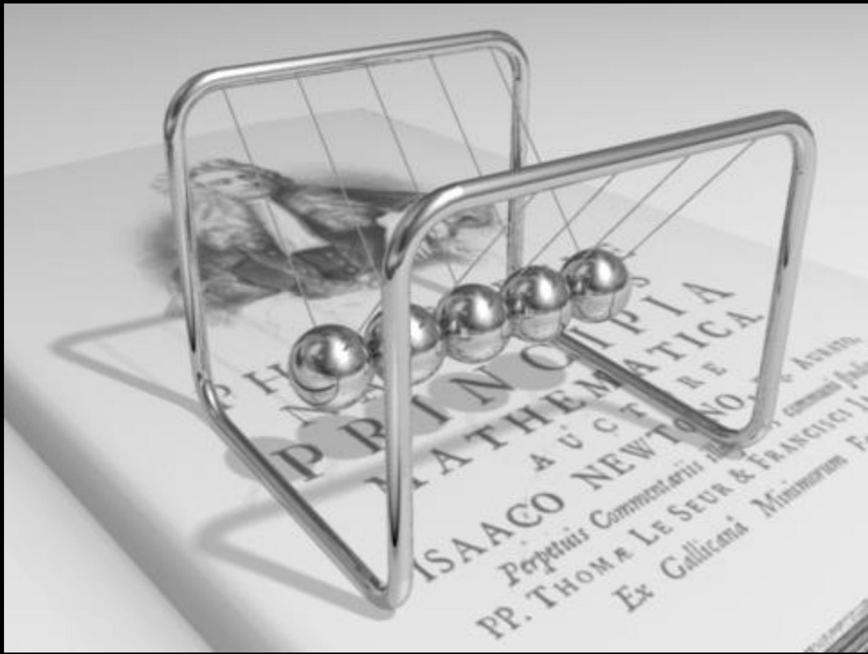
Momentum

- A measure of how hard it is to stop an object which is moving.
- Related to both mass and velocity.

Momentum = mass x velocity

$$p = m \times v$$

(in kg· m/s)



Why is momentum important?

The same momentum exists before and after a collision. Momentum lets us predict collisions…

….and explosions!

Elastic Collisions

(objects bounce and don't stick)

- ***Linear momentum*** is conserved
- Total ***energy*** is conserved
- Total ***kinetic energy*** is conserved

$TKE_{in} = TKE_{out}$

Inelastic Collisions

(objects collide and stick)

- ***Linear momentum*** is conserved
- Total ***energy*** is conserved
- ***Kinetic energy*** is **not** conserved

$TKE_{in} \neq TKE_{out}$

Momentum vs. Inertia

- **Inertia** is a property of mass that resists changes in velocity; however, inertia depends ***only on mass***.
- Inertia is a **scalar** quantity (no direction).
- **Momentum** is a property of moving mass that resists changes in a moving object's velocity.
- Momentum is a **vector** quantity that depends on ***both mass and velocity*** (has direction).

EXAMPLE: Momentum practice problem.

If a football player's momentum is 400 kg m/s, and he has a weight of 1200 N, what is his velocity (in m/s)?

$F_w = mg$

$m = \frac{F_W}{g}$

$m = \frac{1200\ N}{10\frac{m}{s^2}}$

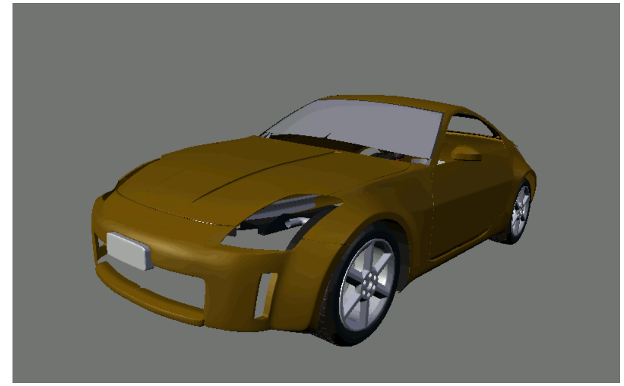
Ans. $p = m \cdot v$

$$\frac{p}{m} = v$$

$$\frac{400\ kg\frac{m}{s}}{120\ kg} = v$$

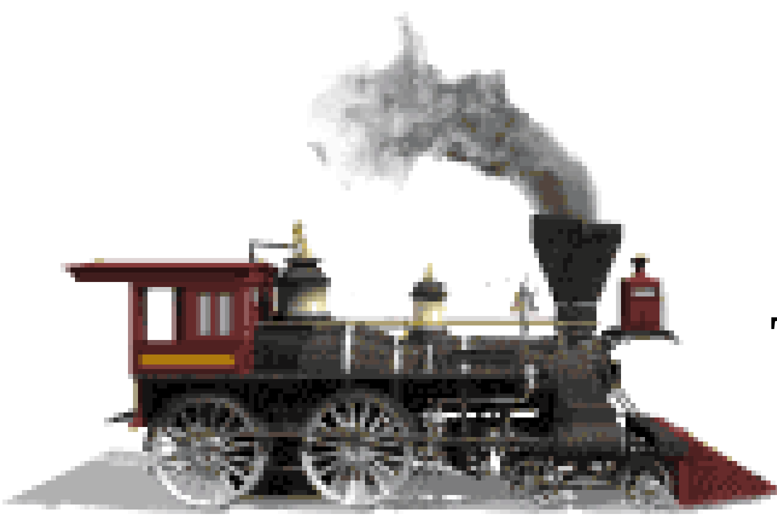
Ans. **3.33 m/s = v**

P.O.D. 1: Find the momentum of the following.



Car: m = 1800 kg; v = 288 km /hr

Bus: m = 9000 kg; v = 16 m /s

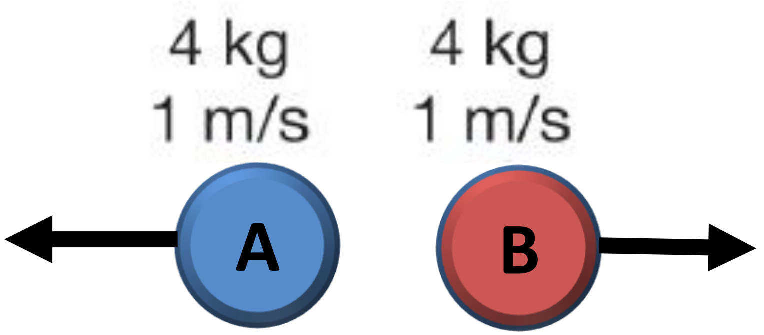


Train: $m = 3.6 \times 10^4$ kg; v = 4 m /s

Momentum vs. Kinetic Energy

- **Kinetic energy** and **momentum** are different quantities, even though both depend on **mass** and **velocity**.
- Kinetic energy (**TKE = $\frac{1}{2}mv^2$**) is a **scalar** quantity. **Mass** is always **+** and when you square the **velocity** you always get a **+** answer. Kinetic Energy doesn't depend on direction.

	Kinetic Energy	Momentum
A	$= \frac{1}{2}(4\text{ kg})(-1\text{ m/s})^2 =$ **2 J**	
B	$= \frac{1}{2}(4\text{ kg})(1\text{ m/s})^2 =$ **2 J**	



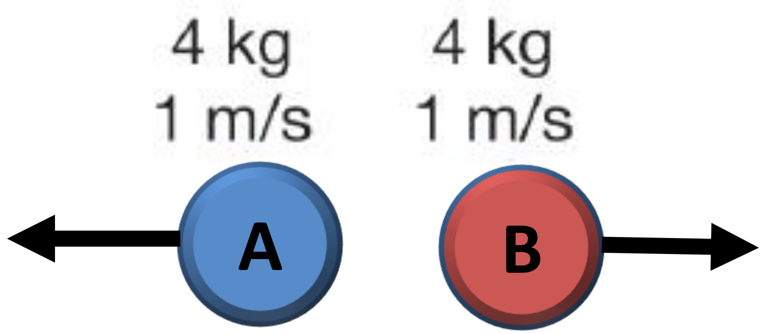
Momentum vs. Kinetic Energy

Momentum (**$p = mv$**)is a **vector,** so it always depends on **direction**.

Sometimes momentum is **+** if velocity is in the **+** direction and

sometimes momentum is **—** if the velocity is in the **—** direction.

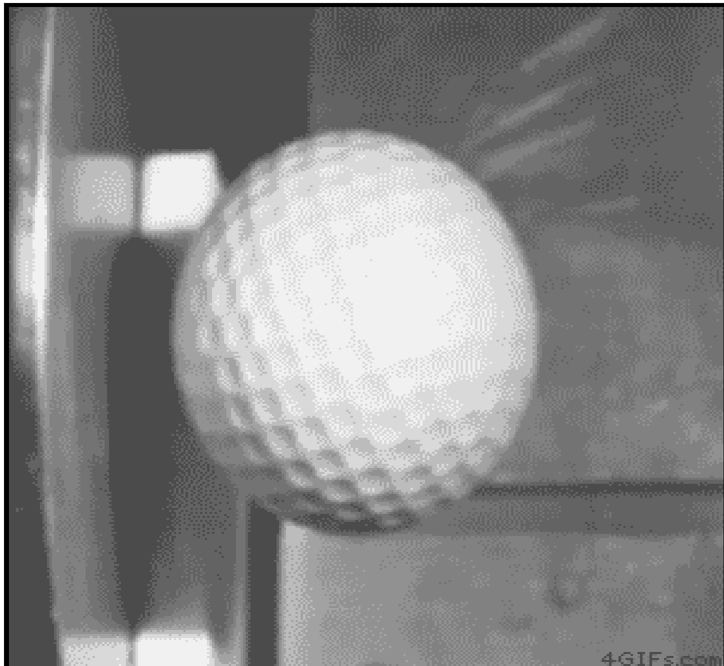
	Kinetic Energy	Momentum
A	$= \frac{1}{2}(4\text{ kg})(-1\text{ m/s})^2$ = **2 J**	= 4 kg ·–1 m/s = –**4 kg·m/s**
B	$= \frac{1}{2}(4\text{ kg})(1\text{ m/s})^2$ = **2 J**	= 4 kg ·1 m/s = **4 kg ·m/s**



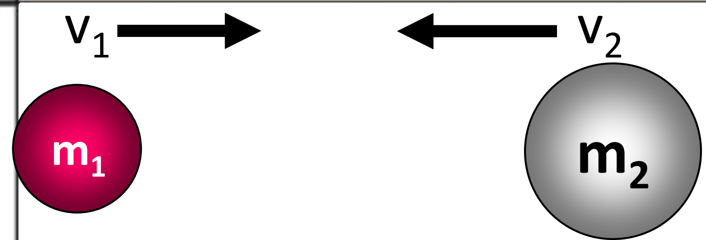
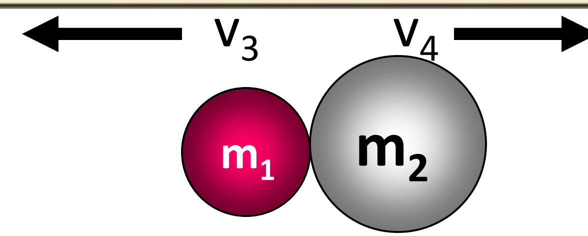
Two balls with the same mass and speed have the same kinetic energy but opposite momentum.

Conservation of Momentum

The ***law of conservation of momentum*** states when a system of interacting objects ***is not influenced by outside forces (like friction***), the total momentum of the system cannot change.

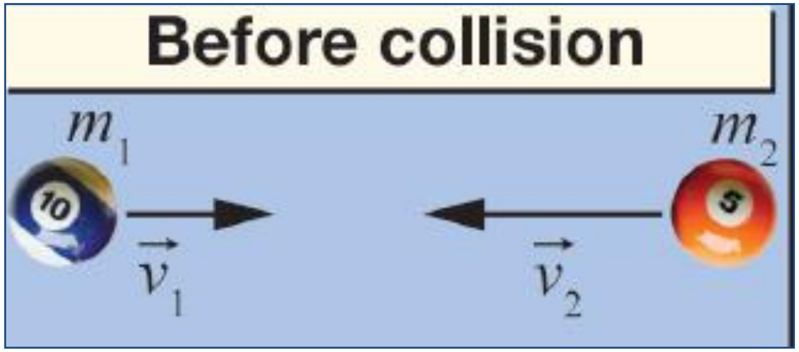


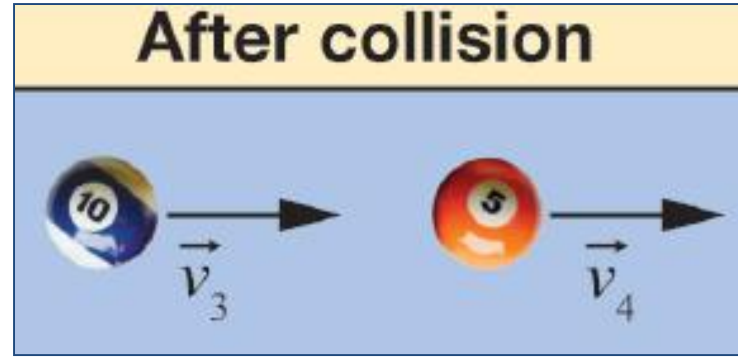
Collisions

Collision type	Before collision	After collision
Elastic: Objects bounce off each other in the collision.		

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_3 + m_2\vec{v}_4$$

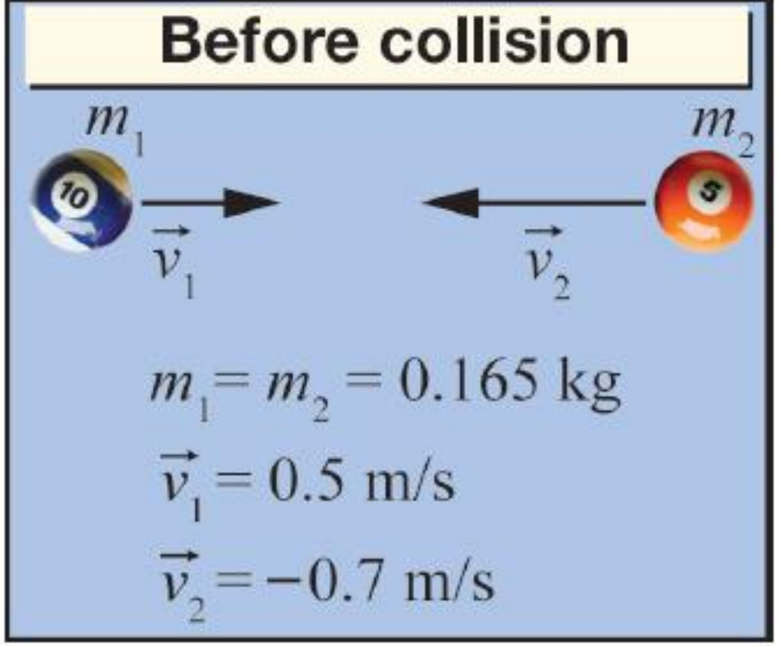
EXAMPLE: Elastic collisions

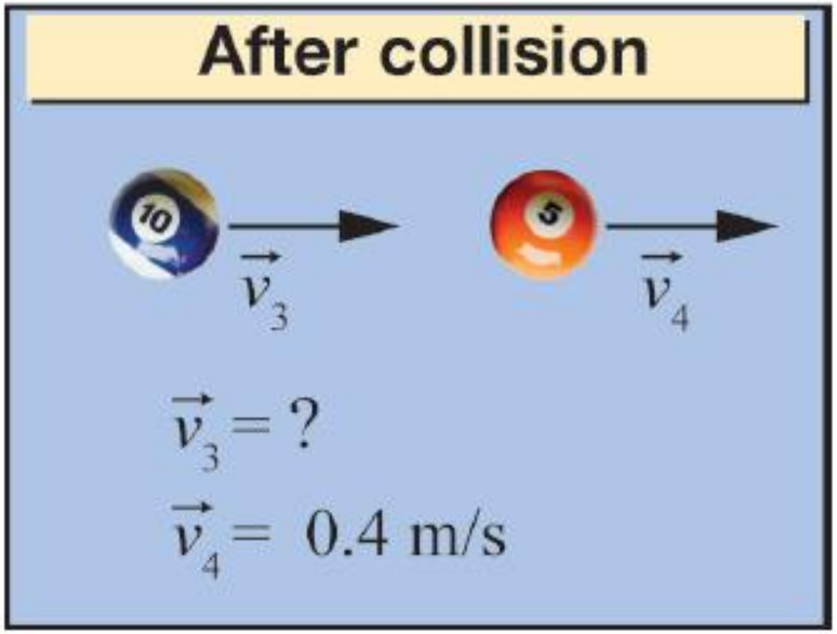




Two 0.165 kg billiard balls roll toward each other and collide head-on. Initially, the 5-ball has a velocity of 0.5 m/s. The 10-ball has an initial velocity of -0.7 m/s. The collision is elastic and the 10-ball rebounds with a velocity of 0.4 m/s, reversing its direction.

What is the velocity of the 5-ball after the collision?





Ans. G.U.E.S.S.

G. ivens: $m_1 = m_2 = 0.165$ kg, $v_1 = 0.5$ m/s, $v_2 = -0.7$ m/s, $v_4 = 0.4$ m/s

U. known: v_3

E. quation: $m_1v_1 + m_2v_2 = m_1v_3 + m_2v_4$

S. olve:

$$\begin{array}{rcl} m_1v_1 + m_2v_2 & = & m_1v_3 + m_2v_4 \\ -m_2v_4 & & -m_2v_4 \\ \hline \end{array}$$

$$m_1v_1 + m_2v_2 - m_2v_4 = m_1v_3$$

$$\frac{m_1v_1 + m_2v_2 - m_2v_4}{m_1} = v_3$$

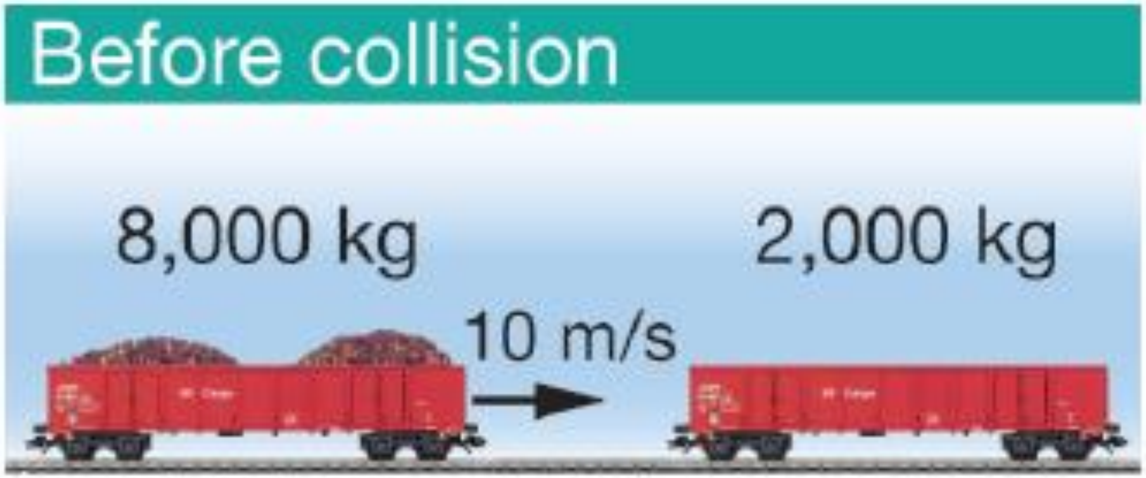
S. ubstitute:

$$\frac{(0.165\ kg)(0.5\frac{m}{s}) + (0.165\ kg)(-0.7\frac{m}{s}) - (0.165\ kg)(0.4\frac{m}{s})}{0.165\ \text{kg}} = v_3$$

$$-0.6\ \text{m/s} = v_3$$

P.O.D. 2: A 200 kg football player moves at 5 m/s towards a 150 kg player moving at –7 m/s. They collide and bounce off each other in opposite directions elastically. If the 200 kg player is moving at 3 m/s after the impact, how fast (in m/s) is the 150 kg player moving?

EXAMPLE: Inelastic collisions



A train car moving to the right at 10 m/s collides with a parked train car. They stick together and roll along the track. If the moving car has a mass of 8,000 kg and the parked car has a mass of 2,000 kg, **what is their combined velocity after the collision?**

Ans. G.U.E.S.S.

G. ivens: $m_1 = 8000$ kg $m_2 = 2000$ kg, $v_1 = 10$ m/s, $v_2 = 0$ m/s

U. known: v_3

E. quation: $m_1v_1 + m_2v_2 = (m_1 + m_2)v_3$

S. olve: $\frac{m_1v_1 + m_2v_2}{m_1 + m_2} = v_3$

S. ubstitute:

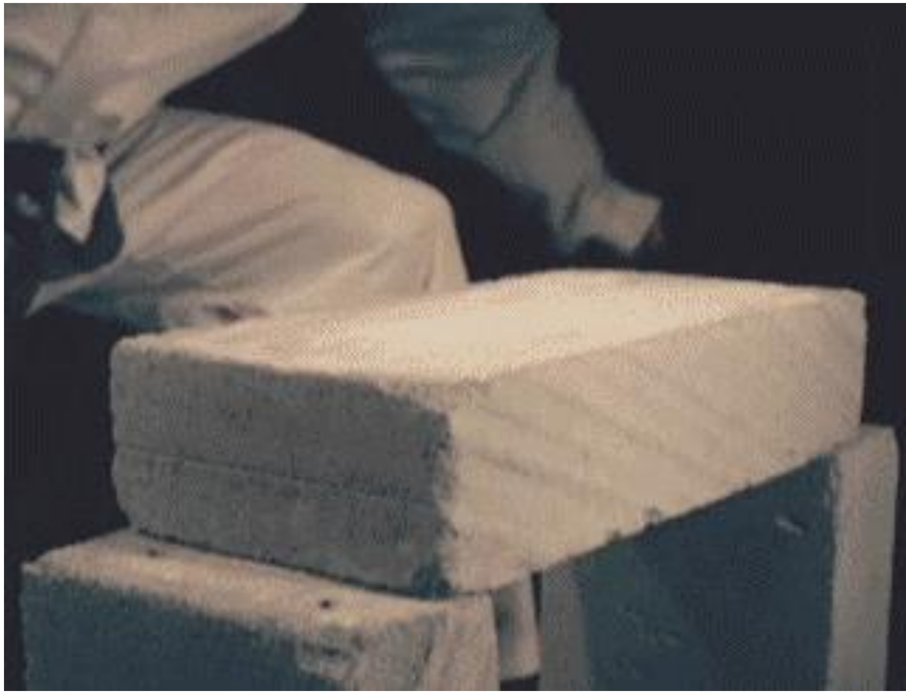
$$\frac{(8000\ kg)(10\frac{m}{s}) + (2000\ kg)(0\ \frac{m}{s})}{8000\ \text{kg} + 2000\ \text{kg}} = v_3$$

$$8\ \text{m/s} = v_3$$

P.O.D. 3: A 2000 kg bus rear ends a 2500 kg bus which is moving at 5 m/s. If the 2000 kg bus was moving at 30 m/s initially, *how fast (in m/s)* would the two buses move forward together after the collision?

Force is the Rate of Change of Momentum

- **Momentum changes when a net force is applied.**
- **The inverse is also true:**
  - **If momentum changes, forces are created.**
- **If momentum changes quickly, <u>large</u> forces are involved.**



This means that force and momentum are

directly proportional

Force and Momentum Change

The relationship between force and motion follows directly from Newton's Second Law.

$$F = m \cdot a \rightarrow F = m \cdot \frac{\Delta v}{\Delta t} \rightarrow F \cdot \Delta t = m \cdot \Delta v$$

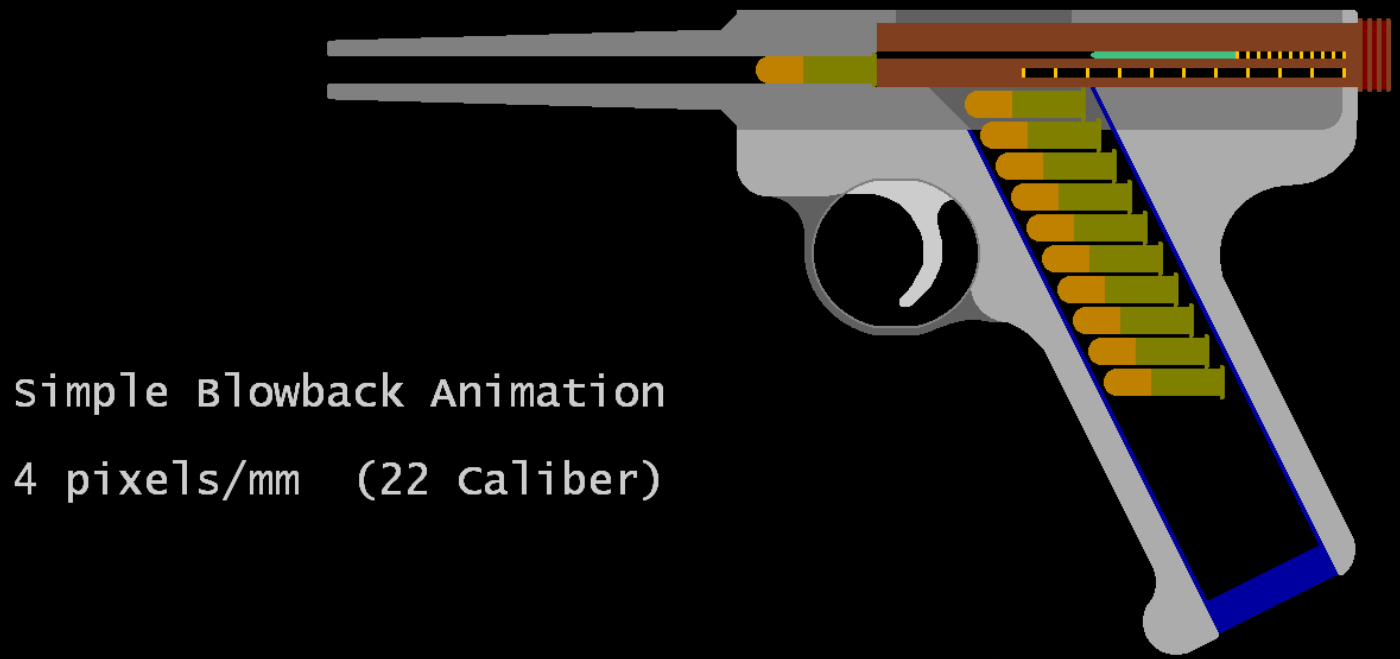
$$F \cdot \Delta t = \Delta \vec{p}$$

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

Force (N)

Change in time (sec)

Change in momentum (kg m/sec)



Impulse Defined

The product of $F \cdot \Delta t$ from the last slide is called **Impulse.**

The symbol for impulse is J. So, by definition:

$$J = F \cdot \Delta t$$

Example : A 2000-kg car traveling at 90 km/h crashes into a concrete wall that does not give at all.

(*a*) Estimate the time of collision, assuming that the car decelerates 30 m/s per second.

(*b*) Estimate the average force exerted by the wall on the car.

Ans. **90 km/hr** is 25 m/s

(a) The time of **deceleration** is given by $\mathbf{v_f = v_i - at}$

So $\mathbf{t = \frac{v_f - v_i}{a} \rightarrow t = \frac{0\frac{m}{s} - 25\frac{m}{s}_i}{-30\frac{m}{s^2}} \rightarrow t = \frac{-25\frac{m}{s}}{-30\frac{m}{s^2}} \rightarrow t = \frac{-25\frac{m}{s}}{-30\frac{m}{s^2}} = 0.83\ s}$

(b) $\mathbf{F_{net} \cdot t = m \cdot \Delta v} \rightarrow \mathbf{F_{net}} = \frac{m\Delta v}{t} \rightarrow \mathbf{F_{net}} = \frac{(2000\ kg)(-25\frac{m}{s})}{0.83\ \ s} \rightarrow \mathbf{-60{,}240.96\ N}$

P.O.D. 4

A sucker of mass 80 kg feels an impulse of 3000 kg ·m/s for 0.5 seconds when he gets kicked by a buddy.

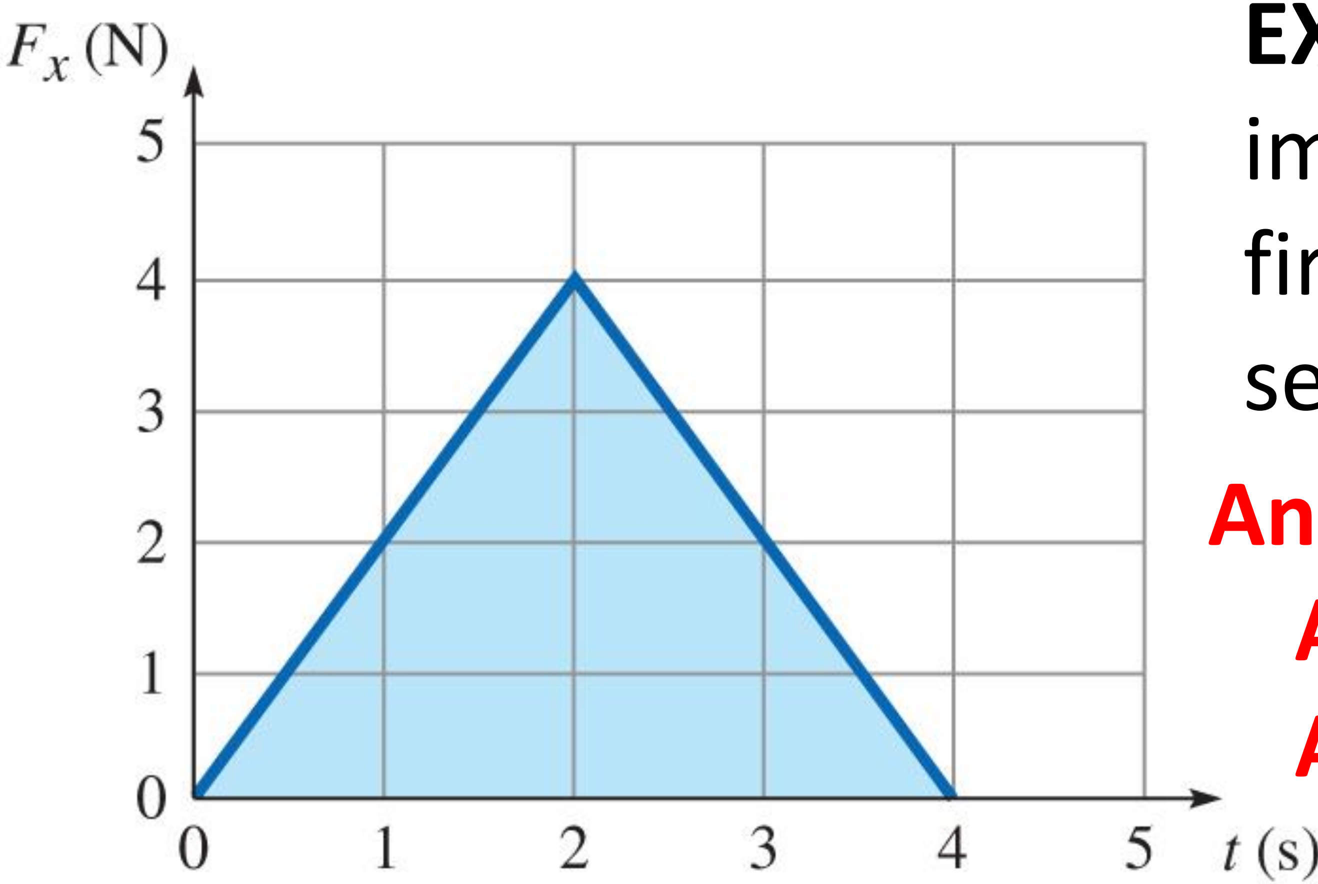
How much force (in N) does he feel?

What is his acceleration (in m/s^2)?

With what velocity (in m/s) does he move away from the impact?

Variable Impulse

Find the **area** under the F(t) graph for a ***variable*** force to find the impulse, or **integrate** (∫)



(a)

EX: What is the impulse in the first four seconds?

Ans. A = ½bh

A = ½(4 s)(4 N)

A = 8 N·s

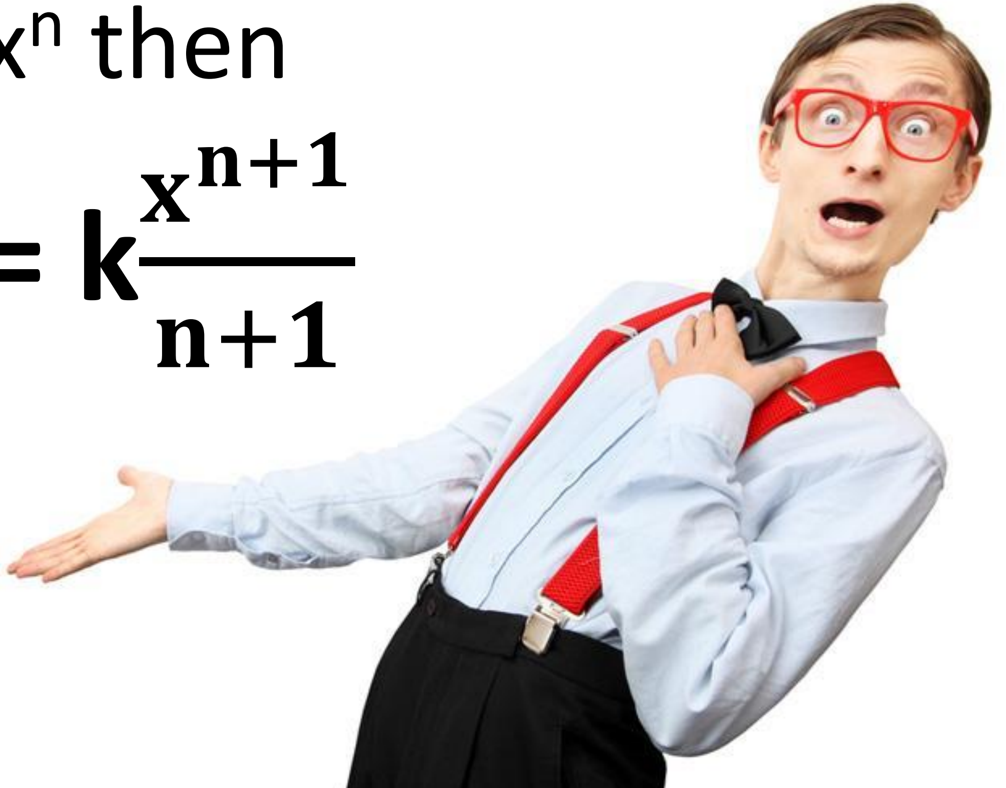
INTEGRATION REFRESHER:

Finding the "area under the curve" is the same thing as integrating.

The trick for integrating is:

If $f(x) = kx^n$ then

$$\int f(x)\,dx = k\frac{x^{n+1}}{n+1}$$

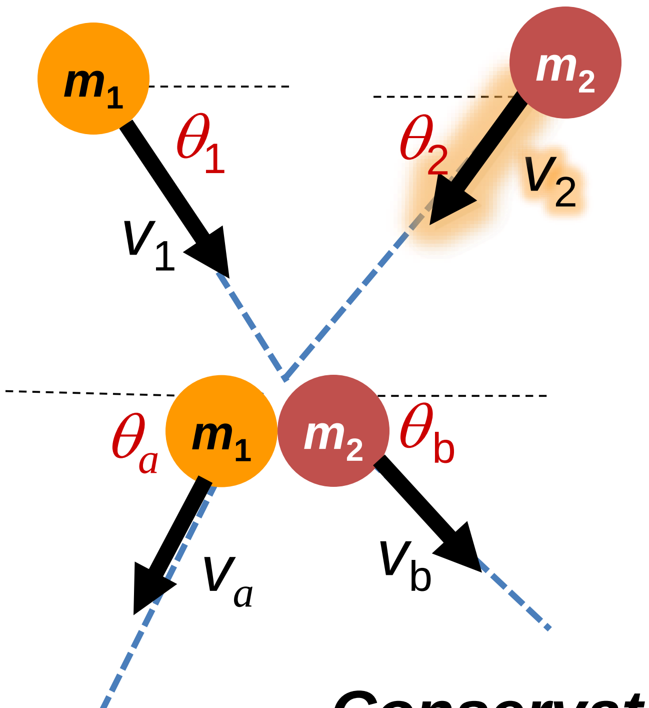


P.O.D. 5

The force acting on a farm hand by a horse's kick is given by $F(t) = 9t^2 - 4t + 6$. The farm hand is initially at rest at $t = 0$ s. Find the **impulse** on the farm hand over the first 40 milliseconds.

Conservation of Momentum in 2-D

To handle a collision in 2-D, we conserve momentum in each dimension separately.



Choosing down & right as positive:

before:

$$p_x = m_1 v_1 \cos\theta_1 - m_2 v_2 \cos\theta_2$$

$$p_y = m_1 v_1 \sin\theta_1 + m_2 v_2 \sin\theta_2$$

after:

$$p_x = -m_1 v_a \cos\theta_a + m_2 v_b \cos\theta_b$$

$$p_y = m_1 v_a \sin\theta_a + m_2 v_b \sin\theta_b$$

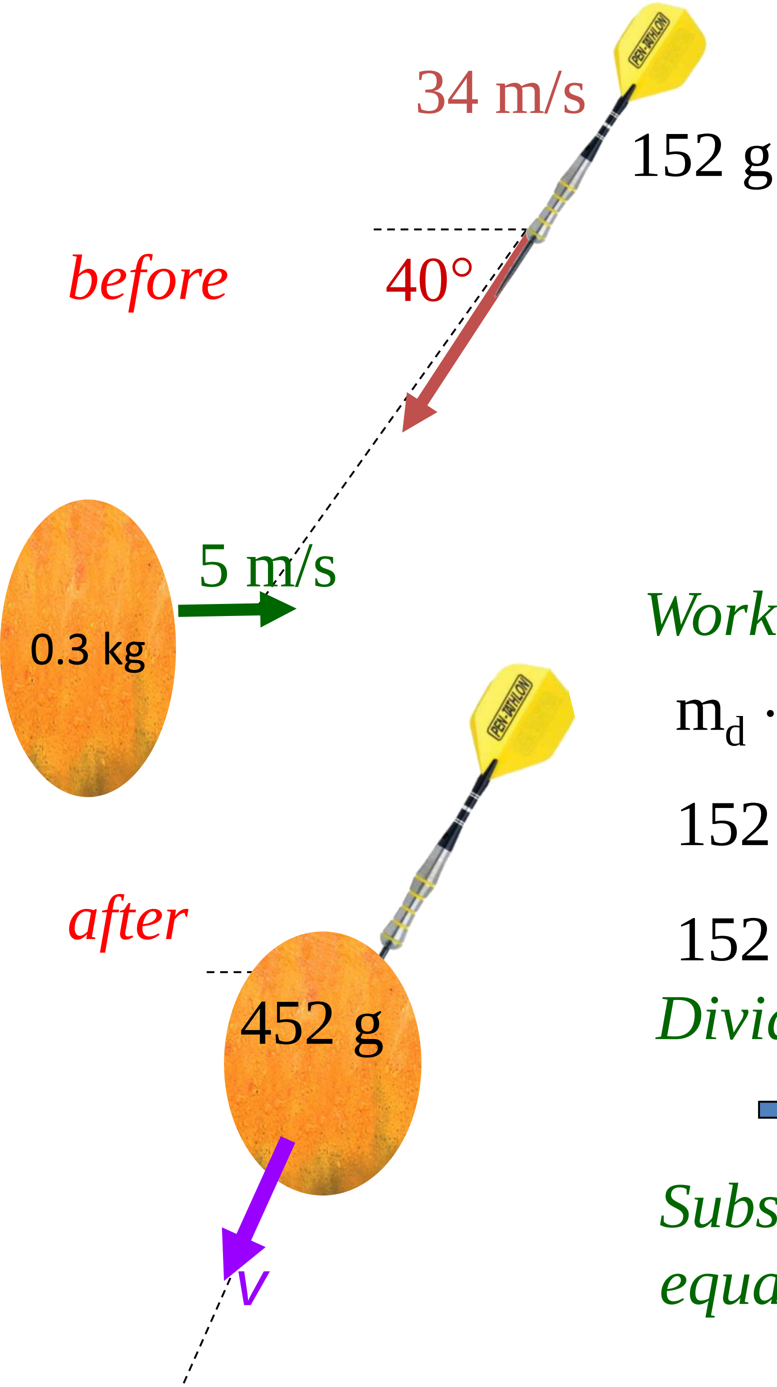
Conservation of momentum equations:

$$\boldsymbol{p_{before} = p_{after}}$$

$$m_1 v_1 \cos\theta_1 - m_2 v_2 \cos\theta_2 = -m_1 v_a \cos\theta_a + m_2 v_b \cos\theta_b$$

$$m_1 v_1 \sin\theta_1 + m_2 v_2 \sin\theta_2 = m_1 v_a \sin\theta_a + m_2 v_b \sin\theta_b$$

Example



A mean, old dart strikes an innocent mango that was just passing by minding its own business. Which way and how fast do they move off together?

Working in grams and taking left & down as + :

$$m_d \cdot v_d = (m_d + m_m)\, v_f$$

$$152\,(34)\sin 40° \;=\; 452\,v\sin\theta$$

$$152\,(34)\cos 40° - 300\,(5) \;=\; 452\,v\cos\theta$$

Dividing equations : $1.35097 \;=\; \tan\theta$

$\Rightarrow$ $\theta = 53.4908°$

Substituting into either of the first two equations : $v = 9.14$ m/s

P.O.D. 6:

A pool player hits a cue ball in the x-direction at 0.80 m/s. The cue ball knocks into the 8-ball, which moves at a speed of 0.30 m/s at an angle of 35^{o} angle above the x-axis. Determine the angle of deflection of the cue ball. Assume the masses of the balls are the same.