

CENTRIFUGAL FANS, BLOWERS AND COMPRESSORS

4.1 Introduction

Power absorbing turbomachines used to handle compressible fluids like air, gases etc, can be broadly classified into: (i) Fans (ii) Blowers and (iii) Compressors. These machines produce the head (pressure) in the expense of mechanical energy input. The pressure rise in centrifugal type machines are purely due to the centrifugal effects.

A fan usually consists of a single rotor with or without a stator. It causes only a small pressure rise as low as a few centimeters of water column. Generally it rises the pressure upto a maximum of 0.07 bar (70 cm WG). In the analysis of the fan, the fluid will be treated as incompressible as the density change is very small due to small pressure rise. Fans are used for air circulation in buildings, for ventilation, in automobiles in front of engine for cooling purposes etc.

Blower may consists of one or more stages of compression with its rotors mounted on a common shaft. The air is compressed in a series of successive stages and is passed through a diffuser located near the exit to recover the pressure energy from the large kinetic energy. The overall pressure rise may be in the range of 1.5 to 2.5 bars. Blowers are used in ventilation, power station, workshops etc.

Compressor is used to produce large pressure rise ranging from 2.5 to 10 bar or more. A single stage compressor can generally produce a pressure rise up to 4 bar. Since the velocities of air flow are quite high, the Mach number and compressibility effects may have to be taken into account in evaluating the stage performance of a compressor.

In general the centrifugal compressor may be known as a fan, blower, supercharger etc, depending on the need to be served. Broadly speaking, fans are the low-pressure compressors; blowers are the medium pressure compressors. It is therefore the analysis of one, say centrifugal compressor, will also holds good to the other machines like blower, fans.

4.2 Important Elements of a Centrifugal Compressor

Fig.4.1 shows the essential parts of a typical centrifugal compressor. It mainly consists of (i) inlet casing with the converging nozzle (ii) the impeller (iii) the diffuser and (iv) the outlet casing.

The function of the inlet casing with the converging nozzle is to accelerate the entering fluid to the impeller inlet. The inlet nozzle accelerate the fluid from the initial condition (state 0) to the entry of the Inlet Guide Vanes (IGV) which direct the flow in the desired direction at the inlet of the impeller (state 1).

The impeller convert the supplied mechanical energy into fluid energy whereby the fluid kinetic energy and the static pressure rises. An impeller is made of radial blades which are brazed to the shroud. It can be made from a single piece consisting of both the inducer and a largely radial portion. The inducer receives the flow between the hub and tip diameters (d_h and d_t) of the impeller eye and passes on to the radial portion of the impeller blades. The flow approaching the impeller may be with or without swirl. The inlet diameter

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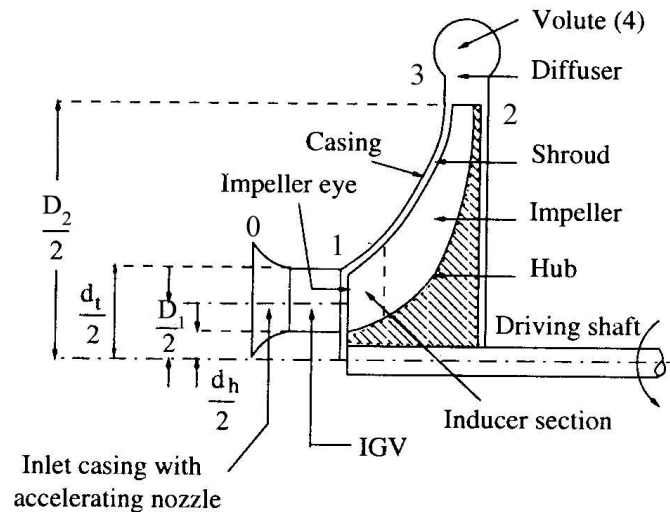


Fig. 4.1 Important elements of a centrifugal compressor

Fig.4.2 shows the general schematic diagram of a centrifugal compressor. The impeller may be of single-sided or double-sided as shown in Fig.4.3 (a) and (b). Double-sided impeller may be used where for the given size the compressor has to handle more flow.

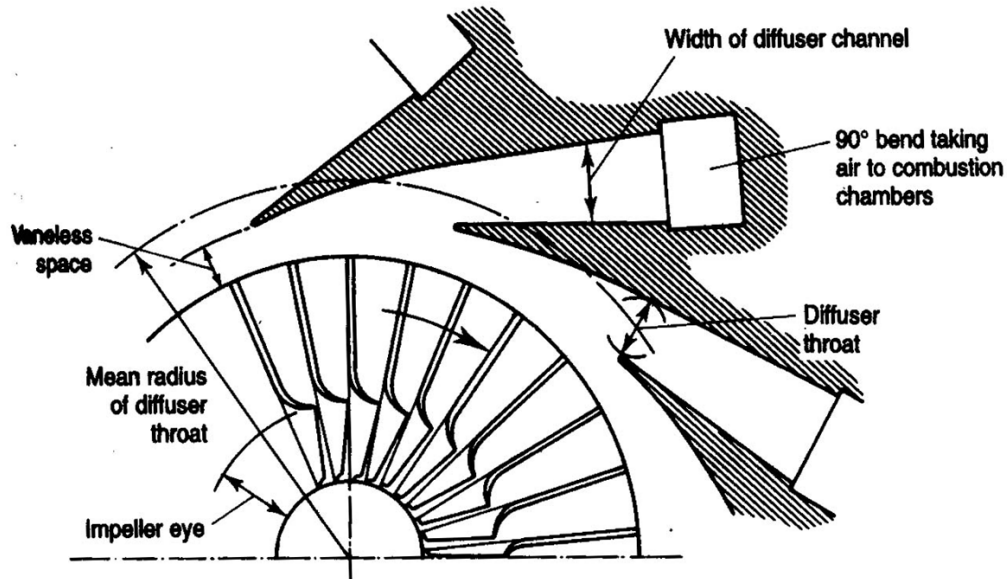


Fig. 4.2 Schematic diagram of a centrifugal compressor

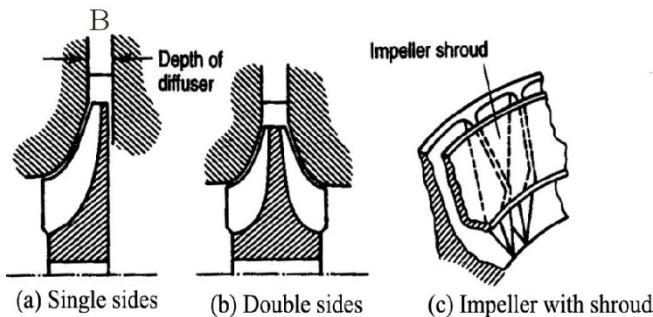


Fig.4.3 Types of impeller construction

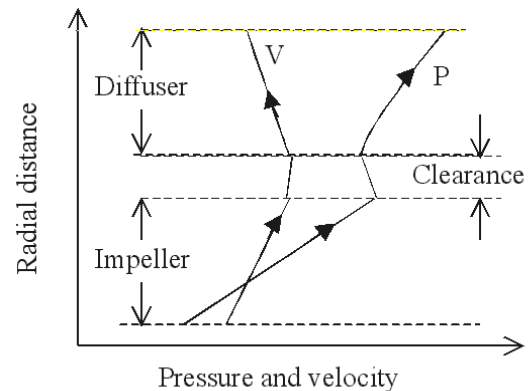


Fig.4.4 Variation of pressure and velocity in a centrifugal compressor

4.3 Variations of Pressure and Velocity

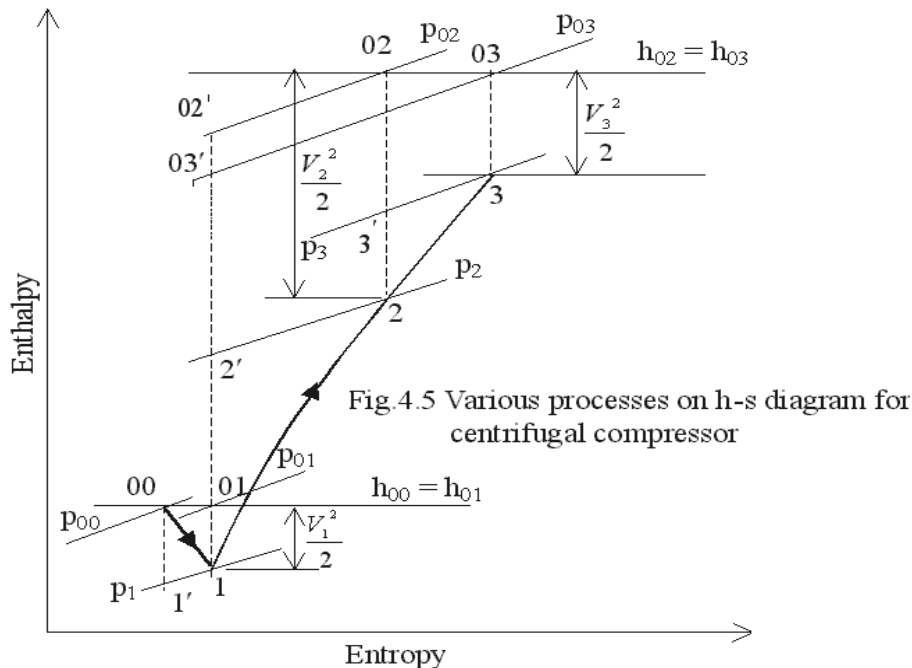
Fig.4.4 shows the variation of the pressure and the velocity across the impeller and the diffuser. As the fluid approaches the impeller, it is subjected to centrifugal effect thereby the kinetic energy (velocity) and the pressure of the fluid both increases along the radial direction. When the impeller discharges the fluid into the diffuser, the static pressure of the fluid rises due to the deceleration of the flow. Therefore the velocity reduces and the pressure still increases as shown in Fig.4.4. This is mainly due to the conversion of kinetic energy into pressure energy of the fluid.

4.4 Principle of Operation

Fig. 4.5 shows the enthalpy-entropy diagram for a centrifugal compressor. Air enters the impeller eye through an accelerating nozzle. As the fluid velocity increases in the nozzle, there will be a pressures drop between the nozzle exit and the impeller inlet, and is represented by the process 00-1. The air is then enters the impeller with a static pressure and temperature p_1 and T_1

respectively. Even though there is increase of entropy due losses and the pressure drop in the accelerating nozzle and the IGv, the stagnation enthalpy at the inlet of the nozzle and the impeller inlet remains same ($h_{00}=h_{01}$) because of no work transfer during this process. The energy transfer occurs only in the impeller blades. The process 1-2 shows the actual compression process in the impeller where the pressure of air increases from p_1 to p_2 due to the centrifugal effect. The process 1-2' is the isentropic compression. The stagnation pressure corresponding to the exit state of the impeller is p_{02} .

The process 2-3 is the actual diffusion process in the diffuser where the large kinetic energy of the fluid is converted into pressure energy, thereby the static pressure rises further from p_2 to p_3 . The diffusion process would have been taken place isentropically then the process becomes 2-3'. The stagnation pressure at the exit of the diffuser is p_{03} . The stagnation enthalpy remains constant from state 2 to 3 (i.e., $h_{02}=h_{03}$) even though the stagnation pressure decreases progressively (i.e., $p_{02} > p_{03}$). This mainly due to the diffusion process is incomplete and as well as irreversible. If the isentropic compression would have been taken place from pressure p_1 to delivery pressure of the stage p_3 , the process would be 1-3' and in terms of stagnation states the process is 01-03'.



4.5 Entrance Velocity Triangle

Fig.4.6 shows the velocity triangle at the inlet of the impeller. It can be seen that the fluid enters the inducer section axially with no whirl velocity when there is no IGv, i.e., $V_1=V_{f1}$, $V_{u1}=0$, $\alpha_1=90^\circ$. This is the general case at the inlet for the maximum energy transfer condition.

Fig.4.7 shows the flow through axially straight inducer section in the presence of IGv's. Due to the presence the inlet guide vanes the fluid enters the inducer with α_1 so that it has some swirl velocity V_{u1} but the straight inducer blades made the relative velocity axial, i.e., $\beta_1=90^\circ$.

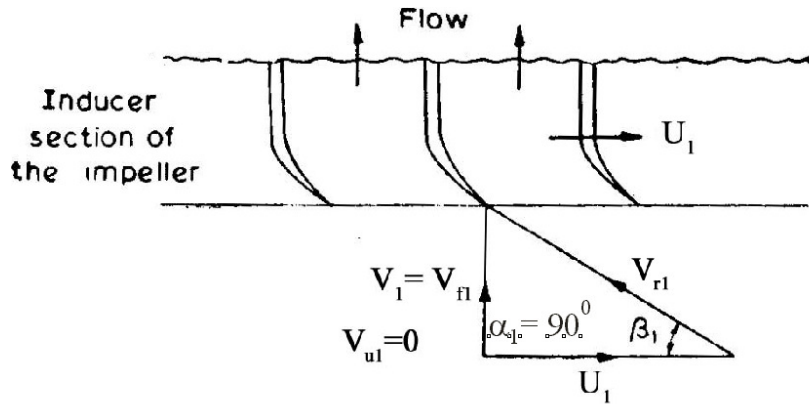


Fig.4.6 Flow at the inlet through the inducer without IGV

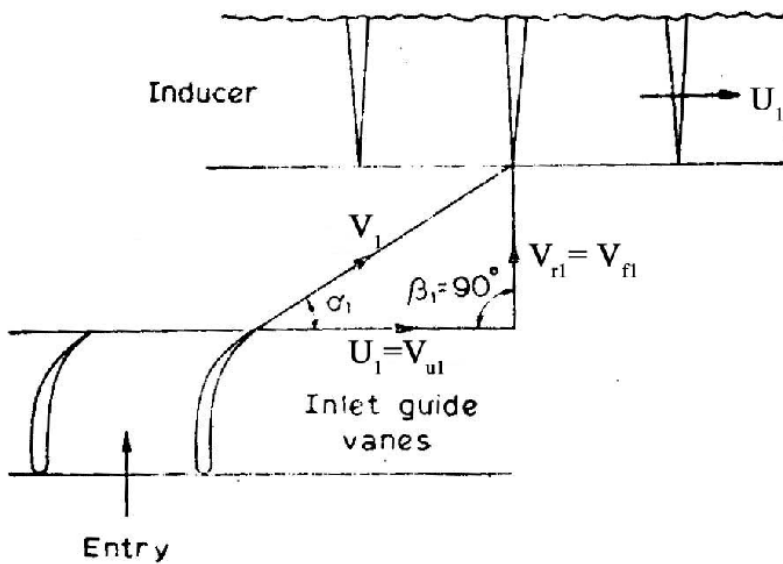


Fig.4.7 Flow at the inlet through the inducer with IGV

4.6 Optimum Inlet Velocity at the Impeller Eye

The magnitude of relative velocity at the inlet of the impeller eye is very important as the relative Mach number at the inlet is mainly depend on this velocity only. We know that more the Mach number value more will be the compressibility effect and hence it reduces the compressor efficiency. It is therefore necessary to keep the relative velocity value as low as possible. There is a value of eye tip speed which will give minimum relative velocity as can be seen from Fig.4.8.

The eye root diameter can be as small as possible which will be decided by the size of shaft and bearing arrangement. Then for the given flow Q the area of the eye flow may be large, giving a low inlet velocity V_1 and a high eye tip speed U_1 or it may be small, giving a large V_1 and small U_1 . In these two extreme cases the relative velocity V_{r1} is high and hence the minimum value is exist in between these. An expression for V_{r1} can be obtained in terms of eye tip diameter d_t using inlet velocity triangle as follows.

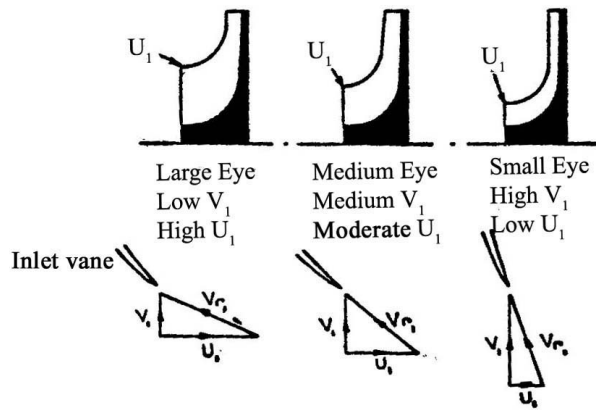


Fig.4.8 Eye conditions at the impeller inlet

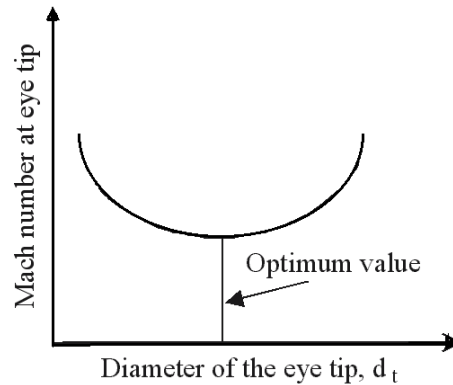


Fig. 4.9 Variation of Mach number with the eye tip diameter

$$V_{r1}^2 = U_1^2 + V_1^2$$

$$V_{r1} = \left[\left(\frac{\pi d_t N}{60} \right)^2 + \left(\frac{4Q/\pi}{(d_t^2 - d_i^2)} \right)^2 \right]^{1/2}$$

4.1

With the d_i , Q and N being fixed, differentiate the eqn.(4.1) with respect to d_t and equate to zero for getting the value of d_t for minimum value of V_{r1} . Fig. 4.9 shows the variation of relative velocity, hence the relative mach number, with the eye tip diameter. If there is no possibility to reduce the relative velocity further for a given machine, the relative Mach number can be reduced further using the pre-whirl at the inlet with the use of inlet guide vanes but the penalty will be the reduction in energy transfer in the impeller. Thus this technique is usually used only in high pressure ratio compressor, where the inlet relative Mach number exceeds unity and shock waves reduce the impeller efficiency.

4.7 Velocity Triangles at the Eye Hub and Tip

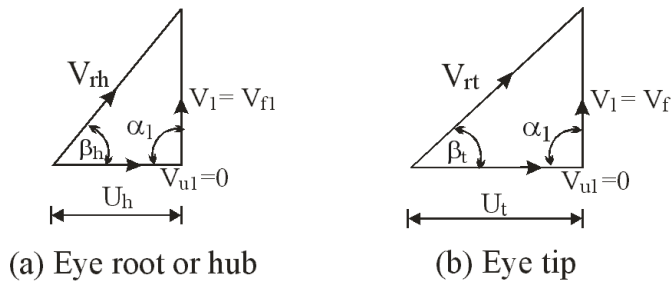


Fig. 4.10 Inlet velocity triangles eye

In an ideal condition the fluid enters the eye section radially with no whirl component. The velocity of flow remains constant from hub to tip of the eye. The tangential velocities of the impeller at the hub (root) and the tip of the eye are calculated based on the corresponding hub and tip diameters of the eye respectively. Fig.4.10 shows the inlet velocity triangles at the hub and tip of the eye. The relative blade angle at the hub β_h is slightly larger than that at the tip β_t as shown in Figure.

$$U_h = \frac{\pi d_h N}{60} \quad (4.2)$$

Tangential velocity of the impeller at the eye tip is

$$U_t = \frac{\pi d_t N}{60} \quad (4.3)$$

The mass flow rate of the air entering the impeller eye can be calculated as

$$\dot{m} = \pi/4 (d_t^2 - d_h^2) V_{f1} \rho_1 \quad (4.4)$$

4.8 Different Vane Shape

The impellers may be classified depending on the exit angle β_2 into (i) Backward curved vanes, (ii) Radial blades. and (iii) Forward curved blades. The velocity triangles are as shown in fig below:

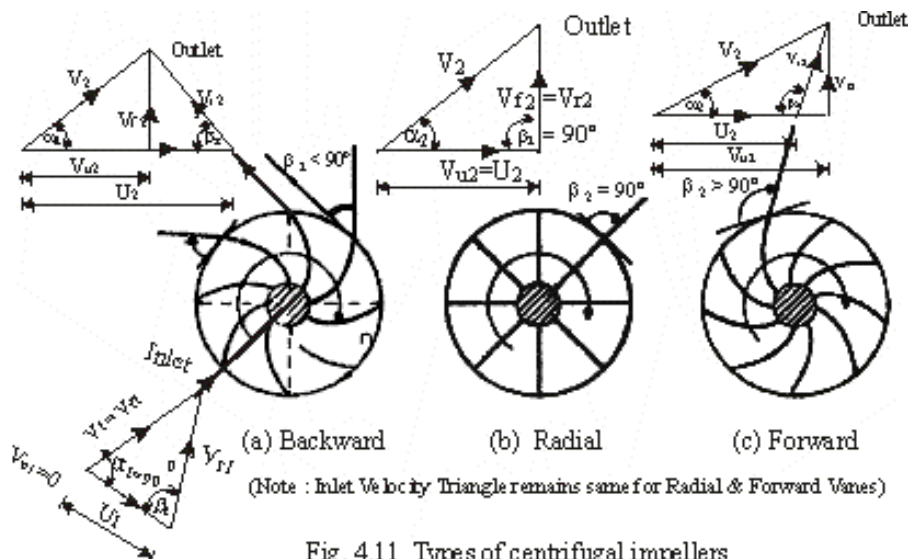


Fig. 4.11 Types of centrifugal impellers

Backward curved blades are those which make an angle less than 90° ($\beta_2 < 90^\circ$) at the exit of the impeller with respect to the tangential direction, radial blades will make an angle of 90° ($\beta_2 = 90^\circ$) and the forward curved blades will make an angle more than 90° ($\beta_2 > 90^\circ$) as shown in Fig.4.11.

In centrifugal compressors generally radial blades are used but the backward curved vanes are also used more often in practice for special purposes. Forward curved vanes are rarely used.

4.9 Degree of Reaction (R)

The degree of reaction (R) for the radial flow power absorbing machines is already discussed in chapter-2 and hence from eqn.(2.55) it is given by

$$R = \frac{1}{2} \left(1 + \frac{V_{f2} \cot \beta_2}{U_2} \right) \quad (4.5)$$

The above equation can also be expressed using velocity triangles as

The above equation can also be expressed using velocity triangles as

$$\begin{aligned} R &= [1 - (V_2^2 - V_1^2) / 2U_2 V_{u2}] \\ &= 1 - (V_{f2}^2 + V_{u2}^2 - V_{f1}^2) / 2U_2 V_{u2} \end{aligned}$$

Since it is assumed that the flow velocity is constant through out, i.e., $V_{f1} = V_{f2}$, the degree of reaction can be expressed as

$$\begin{aligned} R &= 1 - (V_{f1}^2 + V_{u2}^2 - V_{f1}^2) / 2U_2 V_{u2} \\ &= 1 - 0.5 V_{u2} / U_2 \end{aligned} \quad (4.6)$$

4.10 Effects of Exit Blade Angle

The effect of vane shape on its efficiency can be studied by considering the constant value of tangential tip speed (U_2) and constant radial velocity of flow (V_{f2})

When $\beta_2 < 90^\circ$ as in Fig 4.11(a), the value of V_2 will be low and its tangential component (V_{u2}) also low. Hence the absolute velocity of air at exit of the impeller is less and therefore the losses at exit are reduced to get high pressure rise. Thus, the efficiency of the backward curved vanes are considerably high and the losses are less.

When $\beta_2 = 90^\circ$, Fig 4.11(b), the value of V_2 will be more or moderate and V_2 is also slightly high. Correspondingly, exit velocity of air V_2 will have a slightly higher value and hence losses at exit is slightly high for this type machine. Therefore, in the radial vanes impeller, the efficiency is moderate and the losses are also moderate.

When $\beta_2 > 90^\circ$, Fig.4.11(c), the value of V_2 will be very more and correspondingly the loss at the exit is much more. Consequently, an extremely well efficient diffuser has to be used to recover the pressure energy from the large kinetic energy at the exit. But due to high turbulence and thick boundary layers a complete conversion of kinetic energy for the corresponding pressure rise is impossible in the diffuser. Hence the forward curved vanes are unstable and less efficient. They need more input energy to operate.

Therefore from the above discussion it can be concluded that the impeller with large exit blade angles are less efficient than that of the impeller with smaller exit angles. Thus the backward vanes are used where high efficiency is desired and the radial blades are used when the high pressure rise is needed though the efficiency is not high. Forward curved blades are used very rarely. Generally the centrifugal compressor impellers are of radial type because of their easy manufacture and suitable for high speed.

4.11 Different Vane Shape and their Characteristics

The Euler's equation for energy transfer in a radial flow power absorbing machines with radial entry ($V_{u1}=0$) for one dimensional ideal flow conditions is given by:

$$H_e = U_2 V_{u2} / g = U_2 (U_2 - V_{f2} \cot \beta_2) / g$$

$$H_e = \frac{U_2^2}{g} - \frac{U_2 \cot \beta_2 Q}{\pi D_2 B_2 g} \quad (4.7)$$

$$\text{Hence } H_e = K_1 - K_2 Q \quad (4.8)$$

$$\text{Where Constants, } K_1 = \frac{U_2^2}{g} \text{ \& } K_2 = \frac{U_2 \cot \beta_2}{\pi D_2 B_2 g}$$

Eqn.(4.8) is known as the H-Q characteristic curve for the centrifugal fan, blower and compressor. The value of constant K_1 represents the kinetic energy of the fluid moving at the tangential tip speed of the impeller and the constant K_2 represents the slope of the H-Q curve which may be positive, zero or negative for fixed value of b_2 . Using eqn.(4.8) the theoretical H-Q relationship can be obtained as shown in Fig.4.12(a).

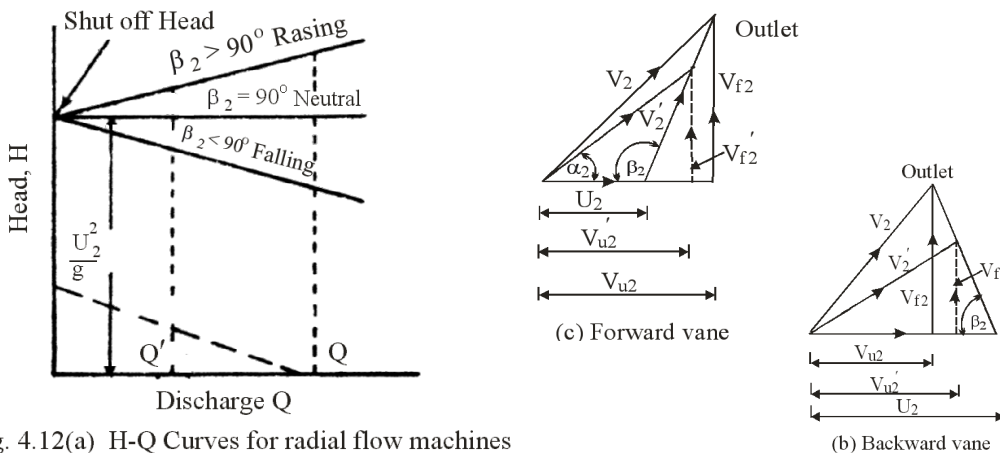


Fig. 4.12(a) H-Q Curves for radial flow machines

In backward curved blades, i.e., $\beta_2 < 90^\circ$, the value of $\cot \beta_2$ is positive, hence such type machine has a negative slope (i.e., K_2 is positive) & therefore H-Q curves is falling type as shown in Fig.4.12(a). In backward curved blades as the discharge increases, the head or the total enthalpy rise, Δh_0 , reduces as V_{u2} decreases for a given value of β_2 as can be seen in Fig.(b). The dashed line shows the initial value of flow, and the solid line represents the velocity triangle for a increased flow.

In radial blades i.e., $\beta_2 = 90^\circ$, the value of $\cot \beta_2$ is Zero. For such type of machine for any value of flow rates, the head remains constant as shown in Fig.4.12(a).

In forward curved blade, i.e., $\beta_2 > 90^\circ$, the value of $\cot \beta_2$ is negative, and H-Q curve has a positive slope as shown in Fig.4.12(a). Hence for increased discharge, head also increases as V_{u2} increases for a given β_2 as shown in Fig.4.12(c) and it has rising characteristics.

In eqn. (4.8), if $Q=0$, $H_e=H_s = U_2^2/g$. This head which is independent of vane shape is called "Shut-off head". The actual measured head at shut-off is much less than the value of (U_2^2/g) due to high turbulence and shock when pre-whirl exist as shown in inclined dash line as in Fig.4.12(a).

From Fig.4.12(b), it seen that for large value of β_2 , the value of V_2 also more. For backward curved vanes, the value of V_{u2} is less and hence energy transfer is less, but losses at exit is also less for forward curved vanes, V_{u2} is large, hence it transfer more energy but as the value of V_2 is more, the losses cannot be diffused in a fixed casing.

Hence backward curved vanes are generally used. The radial vanes are used for high pressure rise and are a reasonable compromise between high exit K.E. and high energy transfer, and also easy to design.

4.12 Actual Characteristics of Centrifugal Compressor

The actual performance characteristics show trends other than ideal due to the various losses in the flow passage. The types of losses that are commonly occurring in the compressor are: (i) Frictional losses due the flow over the blade surface (also called skin friction) which is proportional to the V_{f2} and hence proportional to m^2 (ii) Incidence losses due the improper incidence of fluid at the entry which is also called turning losses. The actual performance characteristic trends will be different than the ideal due to these losses in the flow passage. An account of these losses, the expected pressure rise reduces at any given flow rate. Fig.4.13 shows the actual characteristic of a radial bladed centrifugal machine.

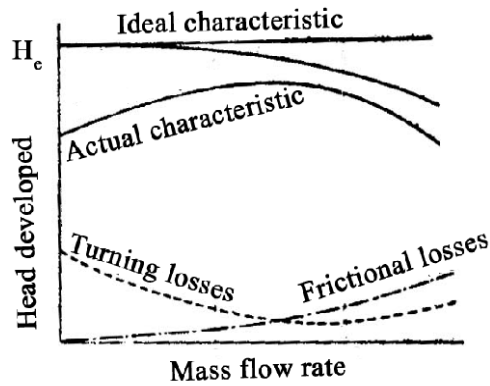


Fig.4.13 Actual characteristic of a centrifugal compressor

Actual head produced can therefore be obtained by deducting these losses from ideal (Euler's)

The total frictional losses that occur due to the flow in the blades passage and in the diffuser is:

$$h_{f1} = K Q^2 \quad (4.9)$$

Where, K = constant, Q = discharge of fluid.

Losses due to improper fluid incidence on the blades at the inlet can be calculated as:

$$h_{f2} = C_d Q^2 \quad (4.10)$$

Where C_d is the total drag coefficient

$$\text{Total loss in the system, } h_f = h_{f1} + h_{f2} \quad (4.11)$$

developed by the machine, i.e.,

$$H = H_e - h_f \quad (4.12)$$

4.13 Slip and Slip Co-efficient (μ)

In deriving the Euler's equation, it was assumed that the velocities are constant (uniform) over the cross sectional area. But in actual practice this assumption is not correct because the velocities are not constant over a cross sectional area as shown in Fig.4.15.

Due to uneven pressure distribution and hence the velocity distribution, head developed by the machine is always less than that developed at the ideal condition. The energy transfer (work) to the shaft is therefore simply reduced by a certain amount due to slip.

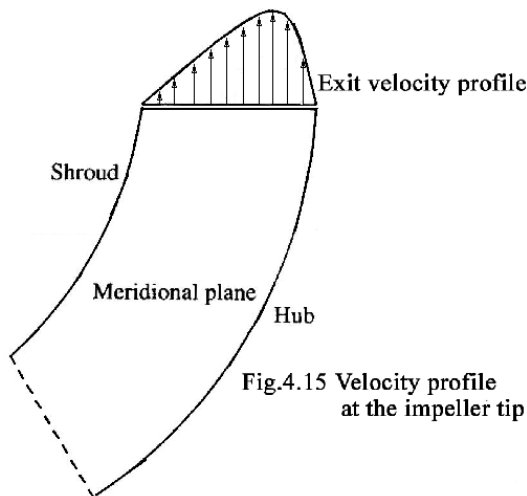


Fig.4.15 Velocity profile at the impeller tip

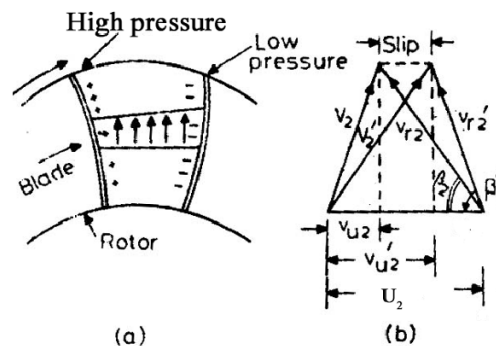


Fig.4.14 Effect of slip

In Fig.4.14, the components drawn with dash referred to the ideal conditions without slip and without dash refers to a ideal condition after considering the slip and non-uniform velocity distribution at the tip. The head based on the ideal velocity diagrams without slip is called Euler's head (H_e) and the head obtained corresponding to the slip is called ideal head (H_i) as the fluid is ideal. The actual head developed by the machine is related to the ideal head through the adiabatic or diagram efficiency (η) of a machine.

The difference between the Euler's head (H_e) and the ideal head (H_i) is called the slip. It can also be defined as the difference between tangential component of the velocity V_{u2} and V_{u2}' . Therefore

$$\text{Slip, } S = V_{u2}' - V_{u2} \quad (4.13)$$

The ratio of ideal head with the slip to the Euler's head without slip is called the slip coefficient. Therefore the slip coefficient is given by

$$\mu = H_i / H_e = V_{u2} / V_{u2}' \quad (4.14)$$

The slip factor is a parameter which limits the work capacity of the compressor even under ideal conditions and this quantity should be as high as possible. More the number of vanes, greater will be the value of slip factor, but increases the solidity of the impeller eye, i.e., a decrease in the effective flow area. This gives an additional frictional loss at the eye which is not recommended. It is therefore necessary to select the number of vanes in the impeller so as to give minimum losses. Generally, in practice, 19-20 vanes will be selected so that the slip factor value is around 0.9.

4.14 Energy Transfer

The general Euler's energy transfer without slip is given by

$$E_e = (U_2 V_{u2} - U_1 V_{u1}) / g_c$$

For ideal condition at inlet the fluid enters radially with no whirl component, the energy equation becomes

$$E_e = U_2 V_{u2} / g_c$$

For the maximum energy transfer the blades are assumed to be radial, i.e., $U_2 = V_{u2}$, hence the energy becomes

$$E_{e,\max} = U_2^2 / g_c$$

Now the ideal energy transfer with slip, using eqn.(4.14), is

$$E = \mu U_2^2 / g_c \quad (4.15)$$

Eqn.(4.15) represents the **theoretical** or maximum work done on the air.

In real fluid, some part of the power supplied by the impeller on the air is used to overcome the losses like windage, disc friction and casing losses. Therefore the power required is greater than the actual power to be supplied on the air and hence the actual power to be supplied is taken care by the term power input factor. The power input factor is defined as the ratio of actual work supplied to the theoretical work supplied. The power input factor or work done factor (Ψ) is

Actual energy transfer or
work done on the air per unit mass is

$$E = h_{02} - h_{01} = \Psi \mu U_2^2 / g_c \quad (4.18)$$

4.16 Overall

As there is no work transfer in the diffuser,
by energy balance we can write,

$$h_{03} - h_{01} = h_{02} - h_{01}$$

Therefore eqn (4.18) for the ideal gas can be written as

$$C_p (T_{03} - T_{01}) = \Psi \mu U_2^2 / g_c$$

$$\Rightarrow (T_{03} - T_{01}) = \Psi \mu U_2^2 / C_{p1} \quad (4.19)$$

4.15
Power
Input
Factor
or
Work
done
Factor
(Ψ)

Pressure ratio (p_{r0})

The overall total-to-total efficiency of the compressor is defined as

$$\begin{aligned}\eta_{t-t} &= \frac{\text{Total isentropic enthalpy rise between the inlet and exit}}{\text{Actual enthalpy rise between the same inlet and exit}} \\ \eta_{t-t} &= \frac{C_p (T'_{03} - T_{01})}{C_p (T_{03} - T_{01})} \\ (T'_{03} - T_{01}) &= \eta_{t-t} (T_{03} - T_{01}) \\ \frac{T'_{03}}{T_{01}} - 1 &= \frac{\eta_{t-t} (T_{03} - T_{01})}{T_{01}} \quad (4.20)\end{aligned}$$

Using isentropic relation and with the use of eqn.(4.19), the eqn.(4.20) is written for the overall pressure ratio as

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_{t-t} \Psi \mu U_2^2}{C_p T_{01}} \right]^{\frac{\gamma}{(\gamma-1)}} \quad (4.21)$$

The stagnation pressure rise across the impeller can also be calculated using the eqn.(4.21).

4.16 Loading Coefficient or Pressure Coefficient (Φ_p)

It is the ratio of isentropic work input across the impeller to the Euler's work input.

$$\begin{aligned}\Phi_p &= \frac{\text{Isentropic enthalpy rise across the impeller}}{\text{Euler's work input}} \\ \Phi_p &= \frac{C_p (T'_{02} - T_{01})}{U_2^2}\end{aligned}$$

Using the isentropic efficiency of the compressor

$$\Phi_p = \frac{\eta_c C_p (T_{02} - T_{01})}{U_2^2}$$

But the actual work input, $\Delta h_0 = C_p (T_{02} - T_{01}) = E = \Psi \mu U_2^2$

$$\therefore \Phi_p = \frac{\eta_c \Psi \mu U_2^2}{U_2^2}$$

$$\text{Pressure coefficient, } \Phi_p = \eta_c \Psi \mu \quad (4.22)$$

The loading or pressure coefficient can also be derived in terms of β_2 and the exit flow coefficient ϕ_2 as follows:

$$\Phi_p = \frac{\text{Isentropic enthalpy rise across the impeller}}{\text{Euler's work input}}$$

$$\begin{aligned} \Phi_p &= \frac{U_2 V_{u2}}{U_2^2} = \frac{V_{u2}}{U_2} \\ &= \frac{U_2 - V_{f2} \cot \beta_2}{U_2} \\ &= 1 - \frac{V_{f2} \cot \beta_2}{U_2} \end{aligned}$$

$$\Phi_p = 1 - \phi_2 \cot \beta_2 \quad (4.23)$$

Where $\phi_2 = \text{Flow coefficient at the exit} = V_{f2}/U_2$

4.17 Centrifugal Compressor Characteristics

Using group of various variables the compressible machines characteristics can be explained. Generally the characteristic of a centrifugal compressor is obtained by plotting overall pressure ratio, p_{03}/p_{01} , against the mass flow parameter, $m\sqrt{T_{01}/p_{01}}$ for a particular constant speed parameter, $N/\sqrt{T_{01}}$. Fig.4.16 shows the typical characteristic of a centrifugal compressor. If the control valve provided at the end of the delivery pipe after the diffuser is closed, then the air present in the impeller is simply subjected to the churning action. The head developed corresponding to this condition is called '**shut-off**' head as indicated by the state 1. As the control valve is opened the air start exiting from the system, the head developed by the machine now increases due to the diffuser's contribution as indicated by the raising portion of the curve 1-A. At A pressure ratio becomes maximum but still efficiency will not be maximum. The state where the efficiency is maximum, is called the design point, say B. Further increase in mass flow rate, the pressure ratio start decreasing due to ineffectiveness of the diffuser now to convert large velocities. Thus the portion of the curve A-D shows the falling nature, i.e, negative slope of the curve.

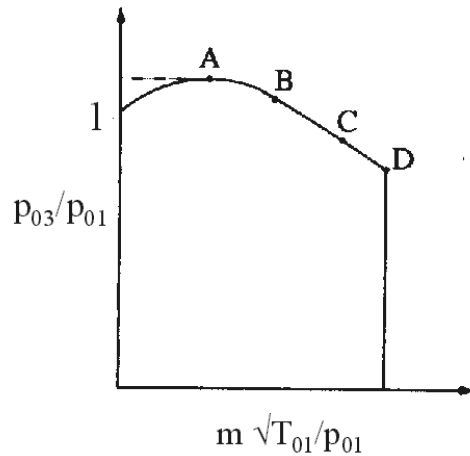


Fig.4.16 Characteristic of a centrifugal compressor for a constant speed

4.17.1 Surging

The phenomenon of momentary fluctuations in head and discharge due to unsteady flow, flow reversal and vibration at low flow rates is called “Surging”.

Let us consider that the compressor is operating at the state C as shown in Fig.4.16. If the flow is reduced by gradual closing of the delivery valve, the operating point now shifted to stable equilibrium point B. On further decrease in flow the operating point shifts to the left side of the curve, eventually reaches the maximum pressure ratio point A. Any further decrease in flow will not increase the pressure ratio and hence starts reducing. At this condition there is a large pressure in the downstream of the system near exit than at compressor delivery and the flow stops momentarily, and may even flow in the reverse direction. This reduces the downstream pressure. After short interval of time, the compressor again starts to deliver the air and the operating point quickly shifts to C again. Again the pressure starts increasing and the operating point moves from right to left. If the downstream conditions are remain unchanged then once again the flow will breakdown after point A and the cycle will be repeated with a high frequency. This phenomenon is called ‘surging’ or ‘pumping’.

If the surging is severe enough then the compressor may be ultimately subjected to impact loads and high frequency vibration leads to the physical damage due to the producing of high pressures repeatedly. Because of this phenomenon at the low flow rates, the compressor can not operate on the positive slope of the curve, i.e., to the left portion of the point A.

4.17.2 Choking

At higher mass flow rates the behavior of the compressor will be different. If the mass flow rates are higher the characteristic curve will be along ABCD as shown in Fig.4.16. It can be seen from Fig.4.16 that for the increased mass flow rate the pressure ratio start decreasing and hence the density also. This effect cause the increase of absolute velocity and angle of incidence at the diffuser vane top. This leads to the rapid steepening in the slope of the curve and finally reaches a point D, beyond which there will be no further increase in mass flow rate for any value of pressure ratio. Therefore the characteristic curve at this point becomes vertical and the point D on the curve is called Choking point.

Choking is therefore defined as the phenomenon in which the mass flow rate reaches to a fixed value irrespective of any of pressure ratios. Choking means the velocity of fluid in the passage reaches the velocity of sound at that point within the compressor. Choking may occur any where with in the machine such as at the inlet, in the impeller or in the diffuser section.

4.17.3 Actual Performance Characteristic of Centrifugal Compressor

The actual characteristics and the total head efficiency of a centrifugal compressor are shown in Fig.4.17 and 4.18 respectively.

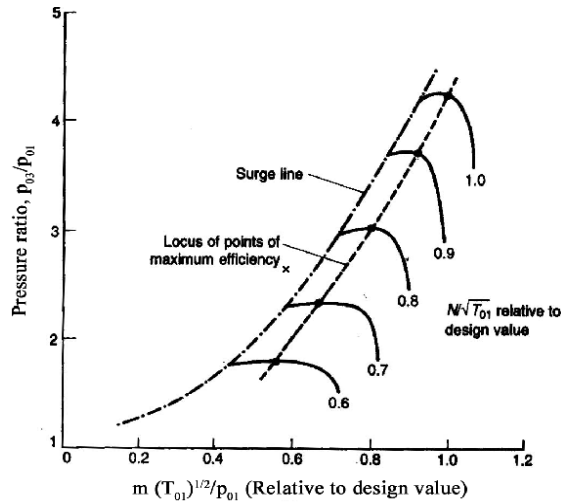


Fig.4.17 Actual characteristics of a centrifugal compressor

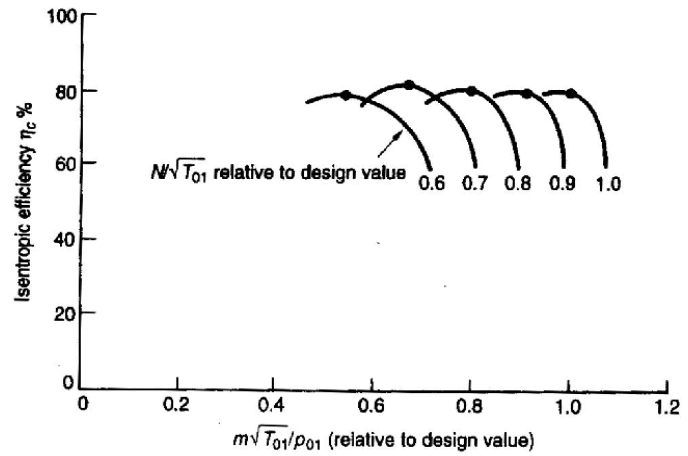


Fig.4.18 Total head efficiency of a centrifugal compressor

In Fig.4.17, the portion of the curve left to the maximum pressure ratio point is not operable due to the surging problem and the line joining these points is called the surge line. On the higher mass flow, the portion of the curve towards right is also limited because of the choking. The maximum efficiencies of the curve for the given speed are quite close to the surge line. Since the operating range for the best efficiency of the compressor is limited, the characteristics of the compressor and the turbine are to be matched properly in the gas turbine power plant, otherwise the problems of surging or low efficiency are to be experienced.

4.18 Pre-rotation or Pre-whirl

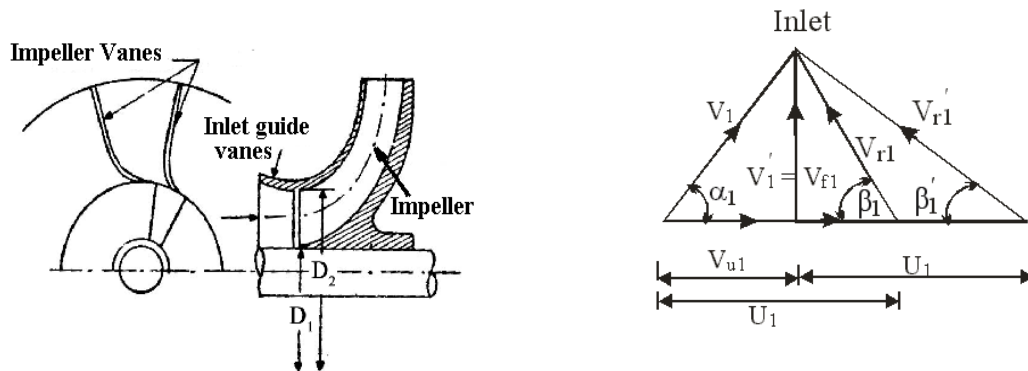


Fig.4.19 Pre-whirl and the resulting velocity triangle at the inlet

As discussed in Sec.4.6 and refers to the Fig.4.9 that the velocity at the inlet will have more effect on the Mach number at the inlet. It is seen that the relative velocity at the inlet should be minimum, which reduces the Mach number, for a given eye tip diameter. When the diameter of eye tip is fixed then an alternative method to reduce the Mach number is to provide pre-whirl at the inlet using the guide vanes. With the use of guide vanes some amount of whirl velocity will

be created so that the relative velocity is reduced. With the pre-whirl at the inlet the energy transfer to the air by the impeller is reduced by the amount $U_1 V_{u1}$. Compare to the energy transfer on ideal condition using $U_2 V_{u2}$, the amount of $U_1 V_{u1}$ is negligible, but the advantage of using pre-whirl that even smaller eye tip diameter can be used which gives smaller value of U_1 . Limiting value of Mach number is usually in the range of 0.7 - 0.8 for neglecting the compressible effects.

4.19 Diffuser

Diffuser is used in the centrifugal compressors to convert large kinetic energy of fluid exiting from the impeller into useful fluid or pressure energy. Therefore it plays an important role in static pressure rise. For a radial bladed impeller, the diffuser will compress and increase the pressure nearly equal to 50% of the overall static pressure rise. Diffuser may be (i) Vaneless type or (ii) Vaned type.

4.19.1 Vaneless Diffuser

In this type of diffuser, the diffusion process will take place in the vaneless space around the impeller before the fluid leaves the compressor stage through a volute casing. A vaneless diffuser is shown in Fig.4.20(a).

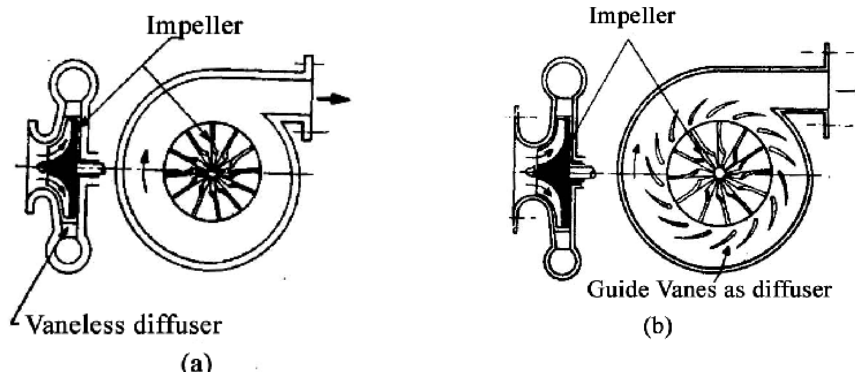


Fig.4.20 Diffuser of a centrifugal compressor or blower

The continuity equation at any radius for the uniform width (B) of the diffuser is written as

$$\text{Mass flow rate, } m = \rho A V_f = \rho (2\pi R B) V_f \quad (4.24)$$

The flow in a vaneless space is a free-vortex flow in which the angular momentum remains constant.

For constant width of the impeller, the ratio of tangential velocity at the exit to that at the inlet is given by

$$\frac{V_{u3}}{V_{u2}} = \frac{V_{f3}}{V_{f2}} = \frac{V_3}{V_2} = \frac{R_2}{R_3} \quad (4.25)$$

and the inlet angle of the diffuser, if used, is

$$\alpha_2 = \alpha_3 = \tan^{-1}(V_{f2}/V_{u2}) = \tan^{-1}(V_{f3}/V_{u3}) \quad (4.26)$$

Eqn.(4.25) implies that the diffusion is directly proportional to the diameter ratio (D_3/D_2) and hence it requires relatively large-sized diffuser which is major disadvantage of the vaneless diffuser. Because of the long path of flow in this type of diffuser the frictional effects are important and the efficiency is low. However in industrial applications the large size compressors are required, the vaneless diffuser is economical and has wide range of mass flow

rates. The most advantage of the vaneless diffuser is that it will not suffer from stalling and shock waves.

4.19.2 Vaned Diffuser

In vaned diffuser as shown in Fig.4.20(b) the vanes either straight type or aerofoil type are used to diffuse the large kinetic energy with shorter length and high efficiency compared to the vaneless diffuser. In this case the length of flow travel and the diameter are reduced. It consists of a ring of diffuser vanes around the impeller and the fluid enters the diffuser through the short vaneless space. The diffuser blades are such that the area towards the exit is kept on increasing so that more diffusion can be achieved with less travel length. The rate of diffusion mainly controlled by the diffuser blade angle which is usually kept less than 12° to avoid the boundary layer separation. More number of vanes also can not be employed since they increase the frictional losses. To avoid possibilities of rotating stall, boundary layer separation and frictional losses, less number of diffuser vanes are used than that of the impeller. In some cases the number of diffuser blades is kept one-third of the number of the impeller blades.

The diffuser efficiency is defined as

The diffuser efficiency is defined as

$$\eta_D = \frac{\text{Ideal enthalpy drop}}{\text{Actual enthalpy drop}} \quad (4.27)$$

From h-s diagram of Fig.4.5,

$$\eta_D = \frac{h'_3 - h_2}{h_3 - h_2} = \frac{T_2 (T'_3/T_2 - 1)}{(T_3 - T_2)}$$

Using isentropic relation,

$$\eta_D = \frac{T_2 [(p_3/p_2)^{(\gamma-1/\gamma)} - 1]}{(T_3 - T_2)} \quad (4.28)$$

4.20 Volute Casing

A simplest form of volute or scroll casing is as shown in Fig.4.21. The volute casing collects and guide the flow from diffuser or from the impeller when there is no diffuser. It possesses the circular passage of increasing cross sectional area along the direction of flow towards the discharge end. As there is gradual increase in area it provides a constant uniform velocity around the impeller which results in equal pressures around the compressor casing, and hence no radial thrust on the shaft. Of course if any deviation in the flow rate from the design condition the radial thrust exists which will try to bend the shaft. Normally 20-30% of the exit kinetic energy from the impeller is recovered in the simple volute casing.

For the fans and low pressure blowers the simple volute casing is employed since it is very economical as they handle the air at very low pressures.

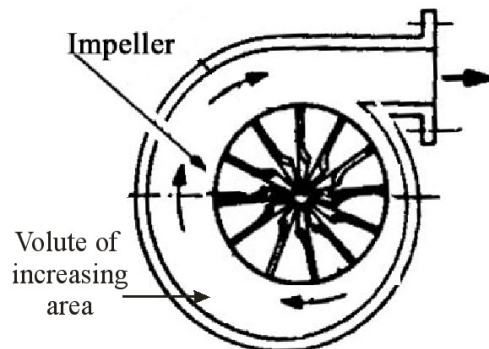


Fig.4.21 Volute or Scroll casing

Solved Examples

Example 4.1: The following data refers to a centrifugal compressor:-

(i) Free air delivered = 1200 kg/hr. (ii) Suction conditions are 1 bar and 290⁰ K (iii) Velocity of air at entry = 60 m/s (iv) Isentropic efficiency of compressor = 70%. (v) Total head pressure ratio = 3, (vi) Mechanical efficiency = 95%

Find the total head, temperature of air at exit and power required for compressor. [Sep. 1989 MYS]

Solution: Given: $\dot{m} = 1200 \text{ kg/hr} = 0.33 \text{ kg/s}$, $p_1 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$, $T_1 = 290 \text{ K}$, $V_1 = 60 \text{ m/s}$, $\eta_{tt} = 0.7$, $p_{02}/p_{01} = 3$, $\eta_m = 0.95$. To find: H , T_{02} , P .

The stagnation temperature at inlet is,

$$T_{01} = T_1 + V_1^2 / (2g_c C_p) = 290 + 60^2 / (2 \times 1 \times 1005)$$

$$T_{01} = 291.8 \text{ K}$$

Isentropic process for a perfect gas is given by

$$\frac{T_{02}'}{T_{01}} = \left(\frac{p_{02}}{p_{01}} \right)^{(\gamma-1)/\gamma} = (3)^{(1.4-1)/1.4} = 1.269$$

$$T_{02}' = 1.269 \times T_{01} = 1.269 \times 291.8 = 369.47 \text{ K}$$

Isentropic enthalpy change,

$$\Delta h_0' = C_p (T_{02}' - T_{01}) = 1005 (369.47 - 291.8)$$

$$\Delta h_0' = 108.2 \text{ kJ/kg}$$

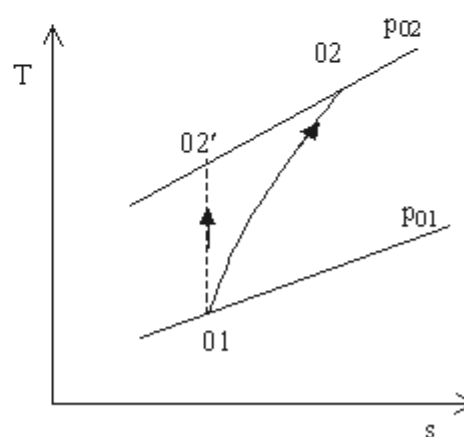


Fig. T-s diagram

(a) **Total Head Developed (H_e) :-**

We know that, $g H_e = \Delta h_0'$

$$H_e = \Delta h_0' / g = 108.2 \times 10^3 / 9.81 = 11030.4 \text{ m of air} \quad (\text{Ans})$$

$$\text{or } H_e = 13.24 \text{ m of water}$$

(b) **Temperature at exit (T_{02}) :**

$$\text{Isentropic efficiency, } \eta_{tt} = \frac{\text{Isentropic enthalpy change}}{\text{Actual enthalpy change}} = \frac{\Delta h_0'}{\Delta h_0}$$

$$\Delta h_0 = \Delta h_0' / \eta_{tt} = 108.2 / 0.7 = 154.57 \text{ kJ/kg}$$

$$\text{Also } \Delta h_0 = C_p(T_{02} - T_{01}) \Rightarrow 154.57 = 1.005 (T_{02} - 291.8)$$

$$\therefore T_{02} = 445.6 \text{ K}$$

(c) **Power required (P)**

$$\text{Power required, } P = \dot{m} \Delta h_0 / \eta_m = 0.33 \times 154.57 / 0.95$$

$$P = 53.69 \text{ kW}$$

Example 4.2 :- A centrifugal compressor rotor has inlet radius of 30 cm and exit radius of 60 cm. Entry is radial with a component of 60 m/s which is constant throughout. The compressor requires 700 kW of power to handle 20 kg of air per second. Find the blade angles at inlet and outlet if the compressor runs at 5100 RPM. Calculate the width at inlet and outlet, if specific volumes at inlet and outlet are respectively $0.85 \text{ m}^3/\text{kg}$ and $0.71 \text{ m}^3/\text{kg}$. What is the degree of reaction ? [Mar 89. MYS]

Solution: Given : $R_1 = 0.3 \text{ m}$, $\therefore D_1 = 0.6 \text{ m}$, $R_2 = 0.6 \text{ m}$, $\therefore D_2 = 1.2 \text{ m}$, Entry is radial, $V_{u1} = 0$ & $V_1 = V_{f1} = 60 \text{ m/s} = V_{f2} = V_f$, $P = 700 \text{ kW}$, $\dot{m} = 20 \text{ kg/s}$, $N = 5100 \text{ RPM}$.

$$v_1 = 0.85 \text{ m}^3/\text{kg}, \therefore \rho_1 = 1/v_1 = 1/0.85 = 1.176 \text{ kg/m}^3$$

$$v_2 = 0.71 \text{ m}^3/\text{kg}, \therefore \rho_2 = 1/v_2 = 1/0.71 = 1.408 \text{ kg/m}^3$$

To find :- β_1 & β_2 , B_1 and B_2 , R ,

Tangential velocity of rotor at inlet:

$$U_1 = \pi D_1 N / 60 = \pi \times 0.6 \times 5100 / 60$$

$$U_1 = 160.2 \text{ m/s}$$

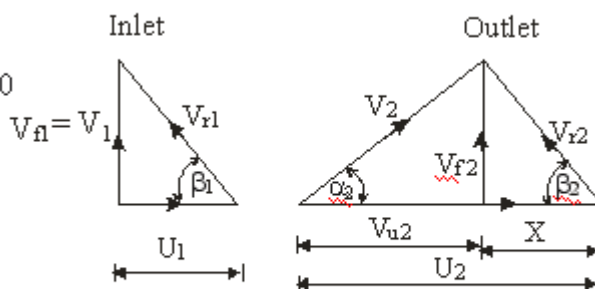
Similarly, $U_2 = 320.4 \text{ m/s}$

(a) **Blade Angles β_1 & β_2 :**

From inlet velocity triangle :-

$$\tan(\beta_1) = V_f / U_1 = 60 / 160.2 = 0.375$$

$$\beta_1 = \tan^{-1}(0.375) = 20.5^\circ$$



Note: Here the overall isentropic efficiency is assumed as unity as it is not mentioned.

$$\text{i.e., } \eta_0 = 1 = \Delta h_0' / \Delta h_0$$

$$\Delta h_0' = P / \dot{m} = 700 \times 10^3 / 20 = 35000 \text{ J/kg}$$

$$\text{Also } \Delta h_0' = U_2 V_{u2} / g_c \quad (\text{Since } V_{u1} = 0)$$

$$V_{u2} = \Delta h_0' \times g_c / U_2 = 35000 / 320.4 = 109.2 \text{ m/s}$$

From outlet velocity triangle :

$$X = U_2 - V_{u2} = 320.4 - 109.2 = 211.2 \text{ m/s}$$

$$\tan \beta_2 = V_f / X = 60 / 211.2 = 0.284.$$

$$\beta_2 = \tan^{-1} (0.284) = \underline{15.86^\circ}.$$

(b) Width at inlet and outlet :

$$\begin{aligned} \text{Mass flow rate, } \dot{m} &= \rho_1 A_{f1} V_{f1} = \rho_1 \pi D_1 B_1 V_f \\ 20 &= 1.176 \times \pi \times 0.6 \times B_1 \times 60. \quad \Rightarrow B_1 = \underline{0.15 \text{ m}} \end{aligned}$$

$$\begin{aligned} \text{Also, } \dot{m} &= \rho_2 A_{f2} V_{f2} = \rho_2 \pi D_2 B_2 V_f \\ 20 &= 1.408 \times \pi \times 1.2 \times B_2 \times 60. \quad \Rightarrow B_2 = \underline{0.063 \text{ m}} \end{aligned}$$

(c) Degree of Reaction (R) : For Radial flow machines,

$$R = \frac{1}{2} \left[1 + \frac{V_f}{U_2} \cot \beta_2 \right] = \frac{1}{2} \left[1 + \frac{60}{320.4} \cot (15.86) \right]$$

$$R = 0.83.$$

OR

$$R = \frac{E - [(V_2^2 - V_1^2) / 2g_c]}{E}$$

$$\text{Where } V_2 = \sqrt{U_2^2 + V_{u2}^2} = \sqrt{60^2 + 109.2^2} = 124.6 \text{ m/s}$$

$$R = \frac{35000 - [(124.6^2 - 60^2) / 2]}{35000} = 0.83$$

Example 4.3 :- A centrifugal compressor running at 6000 RPM having an impeller tip diameter of 101 cm has the following test data : (i) mass flow rate = 25 kg/s, (ii) static pressure ratio = 2.12 (iii) pressure at inlet 100 kPa, temperature = 28°C. (iv) mechanical efficiency = 0.97. Find (i) slip coefficient (ii) temperature of air at exit (iii) power input (iv) power coefficient [Mar 90, MYS]

Solution : Given : $N = 6000 \text{ RPM}$, $D_2 = 1.01 \text{ m}$, $\dot{m} = 25 \text{ kg/s}$, $p_2/p_1 = 2.12$, $p_1 = 100 \text{ kPa}$,

$T_1 = 273 + 28 = 301 \text{ K}$, $\eta_m = 0.97$, To find: η_0, T_2, P , Power coefficient (ψ_P)

Note :- As the static pressure ratio is given, all values are referred to static state only.

Tangential speed of impeller, $U_2 = \pi D_2 N / 60 = \frac{\pi \times 1.01 \times 6000}{60} = 317.3 \text{ m/s}$

(i) Isentropic process for perfect gas is given by,

$$T_2' / T_1 = (p_2 / p_1)^{(\gamma-1)/\gamma} = (2.12)^{0.286} = 1.24$$

Ideal temperature at the exit, $T_2' = 1.24 \times 301 = 373.24 \text{ K}$

Isentropic enthalpy change, $\Delta h_0' = C_p (T_2' - T_1) = 1.005 (373.24 - 301)$

$$\Delta h_0' = 72.6 \text{ kJ/kg.}$$

(ii) **Theoretical work input**,

$$E_{\max} = U_2^2 / g_c = 317.3^2 / 1000 = 100.68 \text{ kJ/kg}$$

$$\text{Slip coefficient, } \mu = \Delta h_0' / E_{\max} = 72.6 / 100.68 = 0.726$$

(iii) **Power input (P)** :-

$$\text{Power required, } P = \dot{m} \Delta h_0' / \eta_m = 25 \times 72.6 / 0.97 = \underline{1871.1 \text{ kW.}}$$

(iv) **Power Coefficient (ψ_P)** :-

$$\psi_P = \frac{P}{\rho N^3 D_2^5} = \frac{P}{\dot{m} U_2^2 / g_c} = \frac{1871.1}{25 \times 100.68} = 0.744$$

Example 4.4 :- A centrifugal compressor runs at 15,000 RPM and produces stagnation pressure ratio of 4 between the impeller inlet and outlet. The stagnation conditions of air at the compressor intake are 1 bar and 25°C respectively. The absolute velocity at the compressor intake is axial. If the compressor has radial blades at the exit such that the relative velocity at the exit = 135 m/s, and the total-to-total efficiency of the compressor is 0.78, Drawn the velocity triangles at the exit of the rotor and compute the slip as well as the slip coefficient. Rotor diameter at the outlet is 58 cm. [Aug 93 MYS]

Solution: Given :- $N = 15,000 \text{ RPM}$, $p_{o2}/p_{o1} = 4$, $p_{o1} = 1 \text{ bar}$, $T_{o1} = 25 + 273 = 298 \text{ K}$,

V_1 is axial, $\beta_2 = 90^\circ$, $V_{r2} = V_{f2} = 135 \text{ m/s}$, $\eta_t = 0.78$, $D_2 = 0.58 \text{ m}$. To find :- slip, μ

For ideal case when no slip occurs, the outlet velocity triangle is shown by ADG and hence $V_{u2}' = U_2 = \pi D_2 N / 60 = \pi \times 0.58 \times 15000 / 60 = \underline{455.6 \text{ m/s}}$ and $V_{r2}' = V_{f2} = 135 \text{ m/s}$ and $\beta_2' = 90^\circ$

For ideal case when the slip occurs, the outlet velocity triangle is shown by the triangle ABG. The value of V_{u2} (i.e., with slip) is calculated as follows :-

$$T_{02}' = T_{01} (p_{02}/p_{01})^{(\gamma-1)/\gamma} = 298(4)^{0.286}$$

$$T_{02}' = 443 \text{ K}$$

Isentropic enthalpy rise ,

$$\Delta h_0' = C_p (T_{02}' - T_{01}) = 1.005(443 - 298)$$

$$\Delta h_0' = 145.7 \text{ kJ/kg}$$

$$\eta_{\text{tt}} = \frac{\text{Isentropic enthalpy rise}}{\text{Actual enthalpy rise}} = \frac{\Delta h_0'}{\Delta h_0}$$

$$\text{Actual enthalpy drop } \Delta h_0 = \Delta h_0' / \eta_{\text{tt}} = 145.7 / 0.78 = 186.8 \text{ kJ/kg}$$

$$\text{Also } \Delta h_0 = U_2 V_{u2} / g_c$$

$$186.8 \times 10^3 = 455.6 \times V_{u2}$$

$$V_{u2} = 410 \text{ m/s}$$

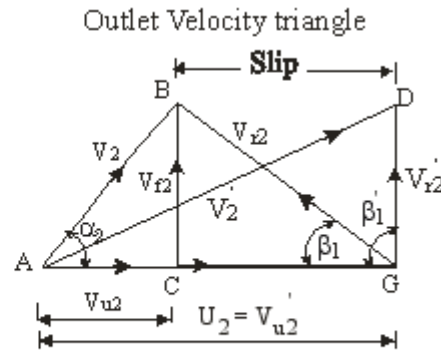
$$(i) \text{ Slip, } S = V_{u2}' - V_{u2} = 455.6 - 410 = 45.6$$

$$(ii) \text{ Slip coefficient, } \mu = V_{u2} / V_{u2}' = 410 / 455.6 = 0.9$$

Example 4.5 :- A centrifugal compressor running at 5950 RPM having an impeller tip diameter = 100 cm. Mass flow rate of air is 30 kg/s, total pressure ratio = 2.125, pressure at inlet is 1 bar and temperature is 25°C, slip coefficient, $\mu = 0.9$ and $\eta_{\text{mech}} = 0.97$. Find (i) Total efficiency (ii) Temperature of air at exit (iii) Power input needed and (iv) pressure coefficient. [Mar 89 MYS]

Solution : Given : $N = 5950 \text{ RPM}$, $D_2 = 1.0 \text{ m}$, $\dot{m} = 30 \text{ kg/s}$, $p_3/p_1 = 2.125$, $p_1 = 1 \text{ bar}$,

$T_1 = 298 \text{ K}$, $\mu = 0.9$, $\eta_{\text{mech}} = 0.97$, To Find :- η_{tt} , T_3 , P , Φ_p



Tangential velocity of the impeller,

$$U_2 = \pi D_2 N / 60 = \pi \times 1.0 \times 5950 / 60 = 312 \text{ m/s},$$

Actual enthalpy rise with slip,

$$\Delta h_0 = \mu U_2 V_{u2} / g_c = 0.9 \times 312^2 / 1000$$

$$\Delta h_0 = 87.61 \text{ kJ/kg} = C_p (T_{03} - T_{01})$$

$$T_{03} - T_{01} = \Delta h_0 / C_p = 87.61 / 1.005 = 87.11 \text{ K}$$

$$\therefore T_{03} = T_{01} + \Delta h_0 / C_p = 298 + 87.11$$

Total temperature of the air at the exit,

$$T_{03} = 385.17 \text{ K}$$

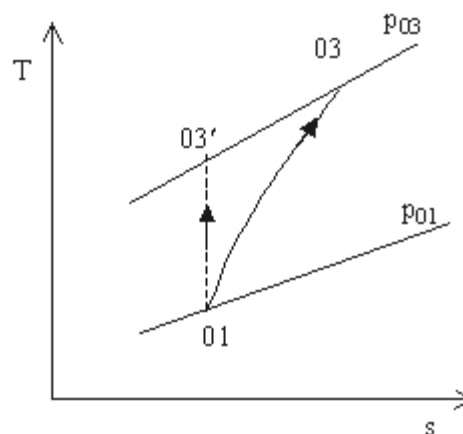


Fig. T-s diagram

Ideal enthalpy change, $\Delta h_0' = C_p (T_{03}' - T_{01})$

$$\text{Where } T_{03}' = T_{01} (P_{03}/P_{01})^{(\gamma-1)/\gamma} = 298 (2.125)^{0.286} = 369.6 \text{ K}$$

$$\therefore \Delta h_0' = 1.005 \times (369.6 - 298) = 71.97 \text{ kJ/kg}$$

$$\text{Total efficiency, } \eta_{tt} = \Delta h_0' / \Delta h_0 = 71.97 / 87.17 = 82.56\%$$

$$\text{Power input, } P = \dot{m} \Delta h_0 / \eta_m = 30 \times 87.61 / 0.97 = 2709.6 \text{ kW}$$

Pressure Coefficient: (Φ_p)

$$\Phi_p = \Delta h_0' / U_2^2 = 71.97 \times 1000 / 312^2 = 0.74$$

OR

$$\Phi_p = \mu \times \eta_{tt} = 0.9 \times 0.8256 = 0.74$$

Example 4.6: A centrifugal compressor has a an inlet eye 15 cm diameter. The impeller revolves at 20,000 rpm and the inlet air has an axial velocity of 107 m/s, inlet stagnation temperature and pressure are 294 K and 1.03 bar respectively. Determine (i) the theoretical angle of the blade at this point and (ii) Mach number of the flow at the tip of the eye.

Solution: Given: Inlet eye diameter, $D_2 = 0.15$ m, $N = 20000$ RPM, $V_{f1} = V_1 = 107$ m/s, $T_{01} = 294$ K, $p_{01} = 1.03$ bar

Tangential speed of the eye tip,

$$U_1 = \pi D_1 N / 60 = \pi \times 0.15 \times 20000 / 60 = 157.08 \text{ m/s}$$

(i) In let blade angle,

$$\beta_1 = \tan^{-1} (V_{f1} / U_1) = \tan^{-1} (107 / 157.08) = 34.26^\circ$$

Static temperature at the eye inlet,

$$T_1 = T_{01} - (V_1^2 / 2 C_p) = 294 - (107^2 / 2 \times 1005) = 288.3 \text{ K}$$

Velocity of sound at the eye tip,

$$C = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 288.3} = 340.35 \text{ m/s}$$

From velocity triangle, $V_{r1} = \sqrt{U_1^2 + V_1^2} = \sqrt{157.09^2 + 107^2} = 190.1 \text{ m/s}$

(ii) Relative Mach number at the eye tip

Pressure Coefficient: (Φ_p)

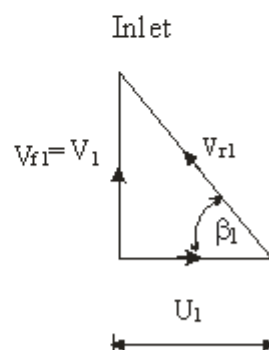
$$\Phi_p = \Delta h_0' / U_2^2 = 71.97 \times 1000 / 312^2 = 0.74$$

OR

$$\Phi_p = \mu \times \eta_{tt} = 0.9 \times 0.8256 = 0.74$$

Example 4.6: A centrifugal compressor has a an inlet eye 15 cm diameter. The impeller revolves at 20,000 rpm and the inlet air has an axial velocity of 107 m/s, inlet stagnation temperature and pressure are 294 K and 1.03 bar respectively. Determine (i) the theoretical angle of the blade at this point and (ii) Mach number of the flow at the tip of the eye.

Solution: Given: Inlet eye diameter, $D_2 = 0.15$ m, $N = 20000$ RPM, $V_{f1} = V_1 = 107$ m/s, $T_{01} = 294$ K, $p_{01} = 1.03$ bar



Tangential speed of the eye tip,

$$U_1 = \pi D_1 N / 60 = \pi \times 0.15 \times 20000 / 60 = 157.08 \text{ m/s}$$

(i) Inlet blade angle,

$$\beta_1 = \tan^{-1}(V_{f1}/U_1) = \tan^{-1}(107/157.08) = 34.26^\circ$$

Static temperature at the eye inlet,

$$T_1 = T_{01} - (V_1^2 / 2 C_p) = 294 - (107^2 / 2 \times 1005) = 288.3 \text{ K}$$

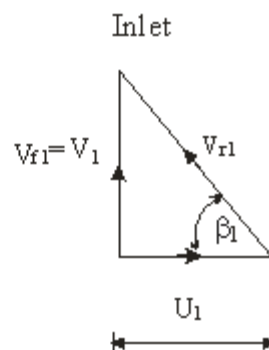
Velocity of sound at the eye tip,

$$C = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 288.3} = 340.35 \text{ m/s}$$

From velocity triangle, $V_{r1} = \sqrt{U_1^2 + V_1^2} = \sqrt{157.08^2 + 107^2} = 190.1 \text{ m/s}$

(ii) Relative Mach number at the eye tip

$$M = V_{r1} / C = 190.1 / 340.35 = 0.558$$



Example 4.7: A centrifugal compressor takes in gas at 5°C and 0.7 bar, and delivers at 1.05 bar. The total-to-total efficiency of the compressor is 83%. Calculate the final temperature of the gas and the work done per kg of gas.

If the gas were further compressed by passing through a second compressor having the same pressure ratio and efficiency, and with no cooling between the two compressors, what would be the overall efficiency of the complete process? Take $C_p = 1.005 \text{ kJ/kg-K}$ and $C_v = 0.718 \text{ kJ/kg-K}$ for gas.

Solution: Given: $T_{01} = 278 \text{ K}$, $p_{01} = 0.7 \text{ bar}$, $p_{02} = 1.05 \text{ bar}$, $\eta_{tt} = 0.83$, $C_p = 1.005 \text{ kJ/kg-K}$, $C_v = 0.718 \text{ kJ/kg-K}$, $\gamma = C_p / C_v = 1.4$, To find: T_{02} , W_a , η_o ?

(a) For single stage compression:

Using isentropic relation,

$$T_{02} = T_{01} (p_{02}/p_{01})^{(\gamma-1)/\gamma} = 278 (1.05/0.7)^{0.286} = 312.2 \text{ K}$$

Exit temperature, $T_{02} = T_{01} + (T_{02}' - T_{01})/\eta_{tt}$

$$T_{02} = 278 + (312.2 - 278)/0.83 = 319.2 \text{ K}$$

Actual work input, $\dot{W}_a = \dot{m} h_0 = C_p (T_{02} - T_{01}) = 1.005 \times (319.2 - 278)$

$$\dot{h}_0 = 41.41 \text{ kJ/kg}$$

(b) When compression taken place in two stages:

Since the pressure ratio is same in both the stages,

$$T_{03}' = T_{02} (p_{03}/p_{02})^{(\gamma-1)/\gamma} = 319.2 (1.05/0.7)^{0.286} \\ = 358.45 \text{ K}$$

Exit temperature from the last stage,

$$T_{03} = T_{02} + (T_{03}' - T_{02})/\eta_{tt}$$

$$T_{03} = 319.2 + (358.45 - 319.2)/0.83 = 366.5 \text{ K}$$

Overall efficiency:

For the process 01-03'',

$$T_{03}'' = T_{01} (p_{03}/p_{01})^{(\gamma-1)/\gamma} = T_{01} (p_{02}/p_{01})^{2(\gamma-1)/\gamma} \\ = 278 (1.05/0.7)^{2 \times 0.286} = 350.6 \text{ K}$$

$$\eta_0 = (T_{03}'' - T_{01}) / (T_{03} - T_{01})$$

$$= (350.6 - 278) / (366.5 - 278) = \mathbf{0.82 \text{ or } 82 \%}$$

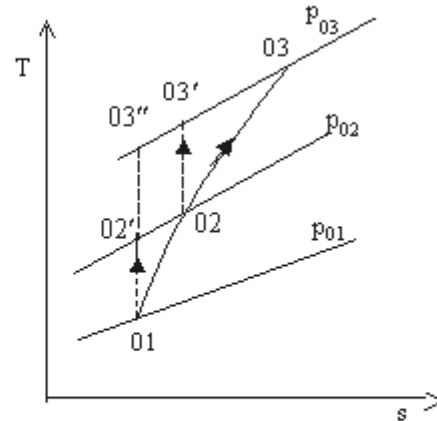


Fig. T-s diagram

AXIAL FLOW COMPRESSOR

5.1 Introduction:

An axial flow compressor is essentially an axial flow turbine driven in the reverse direction except that in order to achieve a sufficiently high efficiency, it is necessary to design the blades by taking extreme care. As already mentioned in chapter 2, the turning angle is very small preferably lower than 30° .

A schematic diagram of an axial flow compressor is as shown in Fig 5.1. It consists of a number of fixed blades which are attached to the casing and alternative rows of moving blades on to the shaft which is mounted on bearings. Air progresses from one blade row to the next blade rows guided through the fixed blades. Fixed blades serve the function of diffuser and hence the pressure of air increases when it comes out of it. As the air passes parallel to the axis of rotation of the shaft, the flow is axial and hence the name is axial flow compressor.

The usual type of compressor is of 50% degree of reaction in which static enthalpy change in rotor (static head) is half the stage static enthalpy change (total head). In this case, the velocity triangles are symmetric at inlet and outlet. In 50% Reaction type of compressor, it is necessary to active prewhirl at inlet so as to maintain the Mach number below 0.9 as it required for high efficiency. Because of relatively small turning angle (up to 30°), the pressure rise per stage will be small. However, the large axial velocity of flow makes more

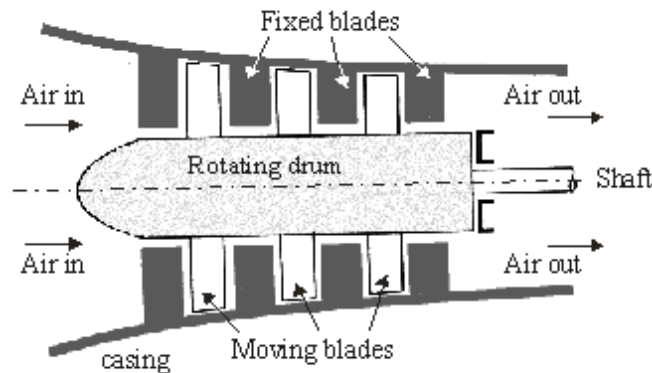


Fig. 5.1 Principal components of the axial flow compressor.

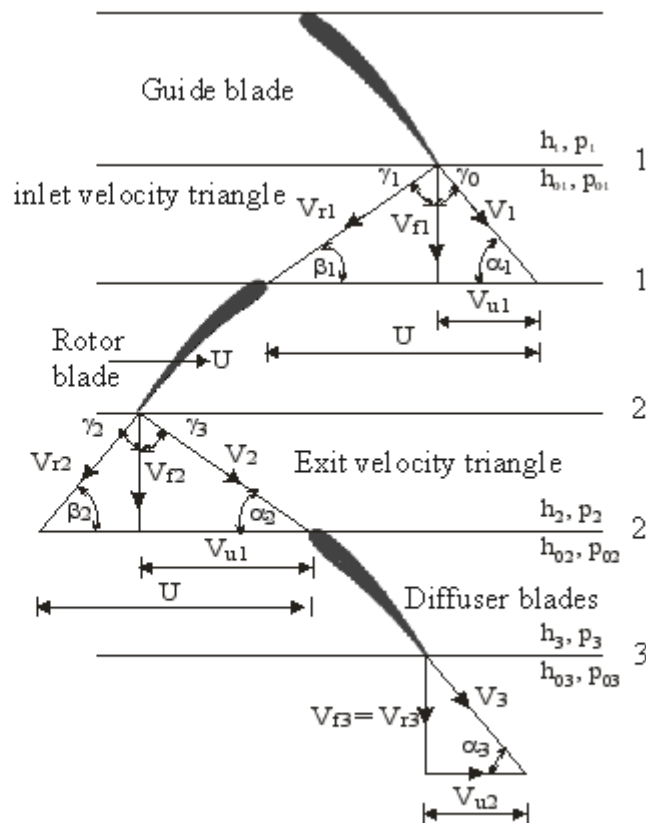


Fig 5.2 General velocity triangles for axial flow compressor