CENTRIFUGAL PUMPS PERFORMANCE ESTIMATION WITH NON-NEWTONIAN FLUIDS: REVIEW AND CRITICAL ANALYSIS

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ABSTRACT

Centrifugal pumps are used in many applications in which non-Newtonian fluids are involved, such as food industry and oil&gas applications, producing the pump performance derating.

In order to give an overview of pros, cons of the different analytical approaches for pump performance derating a literature review on the most significant advances in this topic will be carried out. Moreover to deepen the knowledge about the internal flow and rheological behavior inside the centrifugal pumps working with non-Newtonian fluids, a detailed CFD analysis of two different pumps will be carried out.

The analysis will be focus on the apparent viscosity correction involved in the performance derating with analytical methods and the effects of different types of fluid. Moreover the comparison of the results with two pumps with very different typology, field of application, and dimensions will help to generalize the meaning of the analysis.

KEYWORDS
centrifugal pump, non-Newtonian fluid, analytical method, computational fluid dynamics

NOMENCLATURE

D_{imp}	Impeller diameter [m]
D_h	Equivalent hydraulic diameter [m]
H	Total head [m]
η	Efficiency [%]
k	Consistency index [Pas^n]
n	Viscosity index
$\dot{\gamma}$	Shear rate [s^{-1}]
ρ	Density [kg/m^3]
Q	Volume flow rate [m^3/s]
v	Velocity of the fluid through pipe[m/s]
N	Pump rotation speed [rpm]
z	Number of blades
w	Pump characteristic dimension [m]
ω	Pump rotation speed [rad/s]
μ	Dynamic viscosity [Pas]
μ_a	Apparent viscosity [Pas]
μ_{pl}	Plastic viscosity [Pas]
Re	$= \rho \cdot D_{imp}^2 \cdot \omega / \mu$ Pump Reynolds number
n'	$= \frac{d \ln(\tau)}{d \ln\left(\frac{8 \cdot v}{D_h}\right)}$
τ	Shear stress [Pa]
n_s	$= N \cdot Q^{1/2}/H^{3/4}$ Specific speed [m$^{3/4}$/s$^{5/2}$]
P	Shaft power [W]
BEP	Best efficiency point
HI	Hydraulic Institute

INTRODUCTION

Non-Newtonian fluids are characterized by a non-linear relation between shear stress and shear rate and for this reason, their viscosity depends on the local shear rate value. In this work the rheology of the non-Newtonian fluid will be addressed by means of two quantities: (i) the apparent viscosity that represents the ratio between the shear stress and shear rate values (Chhabra and

Richardson, 1999) and (ii) the plastic viscosity defined as the derivative of the shear stress/shear rate curve (Walker and Goulas, 1984). Pump efficiency, head and shaft power are affected by the non-Newtonian behavior of the fluids as well as in case of Newtonian fluids with high viscosity values. Therefore, the knowledge of non-Newtonian fluids effect on the pump performance is fundamental in the design process as well as in the pump selection phase (manufacturers provide pump performance obtained with water as operating fluid).

In literature, there is lack of knowledge about the behavior of centrifugal pumps handling high viscosity Newtonian fluids and even less regarding non-Newtonian fluids, whose rheology follows a non-Newtonian law. Among the pump performance prediction methods for high viscous Newtonian fluids, HI method is the most widespread in literature. For the non-Newtonian fluids there is not a specific method for the performance predictions, most of the authors apply the HI method relying on a representative value of viscosity like: the plastic viscosity in agreement with Walker and Goulas (1984), or the apparent viscosity, in agreement with Pullum et al. (2007), and Sery et al. (2006). Moreover Walker and Goulas (1984) and Graham et al. (2009), in order to predict the performance of the pump with non-Newtonian, proposed to correlate the Pump Reynolds number with the head and efficiency, using two different way to determine the viscosity to calculate the Pump Reynolds number.

In this paper several CFD analyses of two open-impeller centrifugal pumps operating with non-Newtonian pseudo-plastic fluids are reported. These fluids are considered isotropic with time-independent rheological characteristics. The present work is in the frame of the analysis of pump performance operating in particular conditions, such as food and oil&gas industry. The two pumps considered here are characterized by different typology, dimension and field of application. Three-dimensional numerical analysis gives the possibility to evaluate the local shear rate and consequently the performance without assumption of a representative viscosity. The comparison of the results coming from the CFD and the Walker and Goulas (1984) and Pullum et al. (2007) approaches highlights the capability and the criticisms involved in this specific application.

LITERATURE REVIEW AND METHODS

In this paper an overview of the state of the art in term of non-Newtonian fluid processing using centrifugal pumps is carried out. Different approaches and their capabilities to predict pump performance related to the HI method are reported. The HI is a well-established method to predict pump performance for Newtonian fluids characterized by constant viscosity values higher than water and uses a single viscosity value to predict the pump performance variation. Applying this method to the case of non-Newtonian fluids, it should define a reference viscosity value able to represent the characteristics of the non-Newtonian fluids in to the pump. Three different approaches can be used in order to obtain this value.

The first one is proposed by Walker and Goulas (1984) where the dynamic viscosity used in the HI method is replaced by the plastic viscosity obtained at the highest value of the shear rate tested during the rheological characterization of the fluid (usually within 100 s^{-1} and 1500 s^{-1}). The authors have demonstrated that the pump performance is well predicted near the *BEP* with a confidence band of 5 % with respect to the experimental results using a shear rate of 1500 s^{-1}. Other authors (Sery and Slatter, 2002 and Kalombo et al., 2014) have used this method obtaining wider confidence band respect to the experimental data.

The second approach is proposed by Pullum et al. (2007). The viscosity value used in the HI method is equal to the apparent viscosity calculated through the following steps. The flow inside the pump is modeled with the assumption to be equivalent to the flow inside the circular pipe of appropriate diameter D_h evaluated as follows:

$$D_h = \frac{4 \cdot w \cdot \pi \cdot D_{imp}}{2(\pi \cdot D_{imp} + w)} \tag{1}$$

The correspondent average fluid velocity is defined as

$$v = \frac{4 \cdot Q}{\pi \cdot D_h^2} \tag{2}$$

The fluid dynamic assumption models the fluid regime as laminar and, the shear rate is calculated by means of Rabinowitsch-Mooney law

$$\dot{\gamma} = \left(\frac{3 \cdot n' + 1}{4 \cdot n'}\right) \cdot \frac{8 \cdot v}{D_h} \tag{3}$$

With this procedure it is possible to calculate the apparent viscosity used in the HI method. In this approach the correct estimation of the pump characteristic dimension is the main issue. In order to perform this, it is necessary to know in advance the pump head curve when operating with a non-Newtonian fluid of known rheological characterization. In the Pullum et al. (2007) approach firstly an initial value of the pump characteristic dimension w is evaluated, then a global non linear minimization is applied to correct w in order to minimize the error between the calculated head and the experimental head. The value of w can now be used to predict the performance with other fluids pumped by the same pump. In according to Pullum et al. (2007), if the flow is turbulent the apparent viscosity is evaluated at a high shear rate. Pullum et al. (2007), reported that a 10% confidence bar can be obtained applying their method. The Pullum et al. (2007) approach has been later applied by Graham et al. (2009), Pullum et al. (2011), Kalombo et al. (2014) and Furlan et al. (2014). According to Graham et al. (2009) a value of approximately $w = 0{,}25 \cdot D_{imp}$ can be suitable for the prediction of the performance of any pump, but Kalombo et al. (2014) and Furlan et al. (2014) experimental data are in contrast with this statement.

Kalombo et al. (2014) carried out a direct comparison showing that the method proposed by Pullum et al. (2007) is able to predict the pump head with greater precision respect to the Walker and Goulas (1984) method which, by contrast, performs better the estimation of the pump efficiency.

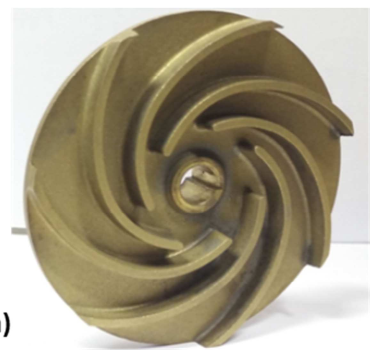
The third, and last, method refers to the strategy proposed by Sery et al. (2006). The authors proposed the viscosity calculation in correspondence of the pump impeller average shear rate determined using the method described in Metzner and Otto (1957). With this method, confidence band of 5 % for the pump head and 20 % for the pump efficiency have fixed.

In literature, there is lack of knowledge about numerical simulations of centrifugal pumps which process non-Newtonian fluids. Beyond the analysis of an open impeller centrifugal pump operating with tomato paste (Buratto et al., 2015) which is the basis of the present paper, two recent relevant works in this field are Allali et al (2015), in which the internal flow with different form of volutes in an open impeller pump is analyzed and Ye et al. (2015) in which the laminar/turbulent transition inside an open impeller pump is studied. In this case, the authors put the attention on the effects of shear stress, pump speed and fluid rheology.

NUMERICAL MODEL

In this work two centrifugal pumps are considered. Both machines have an open impeller but several differences can be highlighted. Figure 1 reports the geometric characteristics and the performance with water of the two pumps. Pump 1 was designed for operating with dirty water, Pump 2 was specifically designed for operating with tomato paste and was analyzed in Buratto et al. (2015).

The numerical simulations were carried out by means of the commercial CFD code ANSYS


a)


b)

	Pump 1	Pump 2
n_s	18	103
D_{imp} [m]	0.0955	0.61
z	7	3
H at BEP [m]	9.2	20
Q at BEP [m^3/h]	4.45	5000
N [rpm]	2900	903
P at BEP [kW]	0.19	317

Figure 1: a) the impeller of Pump1, b) the impeller of Pump 2 and pump characteristics

CFX 15.0. A second-order high-resolution advection scheme was adopted to calculate the advection terms in the discrete finite volume equations. The simulations were performed in a steady multiple frame of reference, taking into account the contemporary presence of moving and stationary domains. In particular, a mixing plane approach was imposed at the rotor/stator interface between the impeller and the volute and between the impeller and the inlet duct. A rotating frame of reference approach was used for the impeller domain with a rotation speed of 2900 rpm and 903 rpm for Pump 1 and 2 respectively. For numerical modeling purposes, the fluids were treated as incompressible and isothermal. As the inlet boundary condition, a constant velocity value with normal direction and a turbulent intensity equal to 5 % was imposed. The no-slip wall boundary condition was used for all the solid surfaces. At the outlet, a static pressure condition was used. As turbulence model, standard k-ω is chosen. This model has shown successful applications in the simulation of centrifugal pump as reported, for instance, by Zubanov et al. (2015) and Song et al. (2003). Figure 2 reports the numerical domains used in the present work. Both pumps are provided by a single impeller coupled with a volute. The numerical model comprises also the inlet and outlet ducts modeled as a stationary domain.

The grid used in the calculations is a hybrid grid composed of tetrahedral elements on the core and prismatic elements on walls. Pump 1 is discretized by using 11,238,529 elements while, Pump 2 is discretized by using 9,769,415 elements. Figures 3a and 3b report the comparison between numerical and experimental data, obtained using water as process fluid, for Pump 1 and 2 respectively. The numerical results obtained for Pump 1 are in good agreement with the experimental data. For Pump 2, the performance curve trend and shape are well reproduced by the numerical simulation but a high discrepancy between numerical and experimental values can be noticed (in Fig. 3b, error bar are equal to 20 %). This can explained by considering that the experimental data for Pump 2 were obtained during pump operation, directly connected with the industrial plant, equipped with portable instruments, which were installed in accessible sections

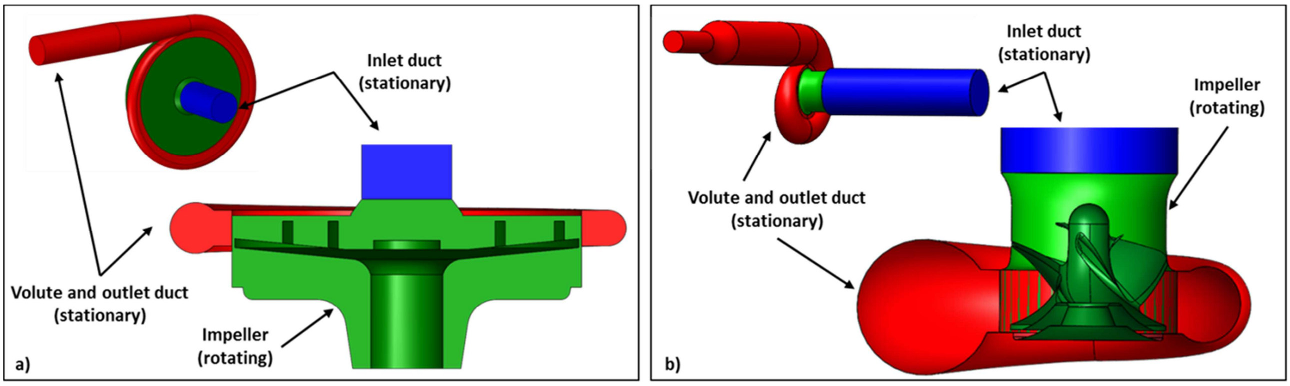


Figure 2: Numerical domains: a) Pump 1 and b) Pump 2

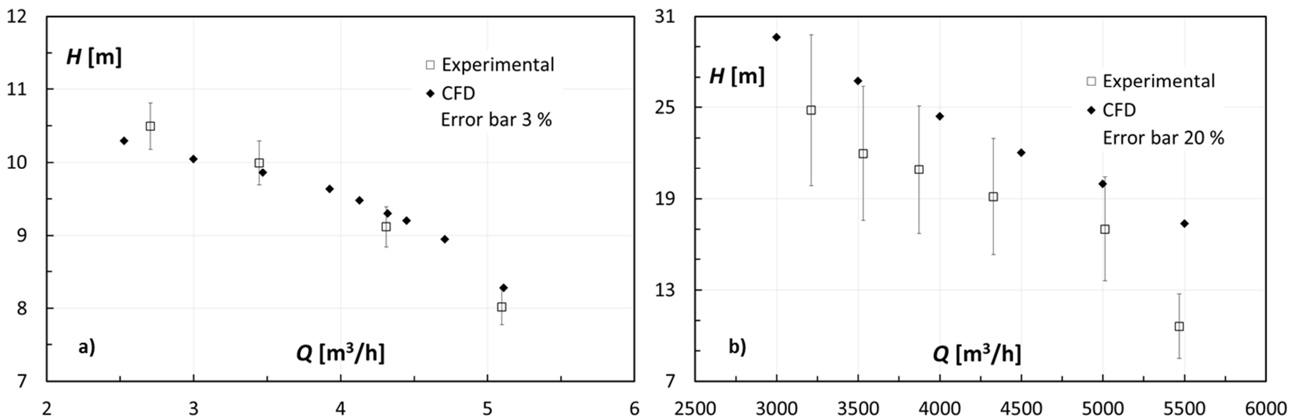


Figure 3: Comparison between CFD and experimental results with water: a) Pump 1 with 3 % error bar and b) Pump 2 with 20 % error bar

along the circuit rather than in the most appropriate positions. To better asses the simulation reliability, a grid independence analysis was been carried out to achieve the highest confidence as possible in the numerical results. The grid used is the result of this independence study, which is not reported in detail here, but more information can be found in Buratto et al. (2015).

Table 1 reports the characteristics of the non-Newtonian fluids used in this analysis. Each fluid is labelled by an alphanumeric code that contains the values of the consistency index and the viscosity index. In this work, pseudo-plastic fluids are chosen with time-independent characteristics. The numerical modelisation is implemented using a power law (Ostwald de Waele model)

$$\tau = k \cdot \dot{\gamma}^n \quad (4)$$

in the shear rate range between 10^{-4} s^{-1} and 10^{7} s^{-1}. As reported by Chabra and Richardson (1999), power law is the simplest representation of pseudo-plastic behavior and is the most widely used model in the literature dealing with process engineering applications. The model constants k and n are varied in order to perform a wider sensitivity analysis. All fluids are characterized by a density value of 1100 kg/m^3.

RESULTS

Performance analysis

Figure 4 reports the pump performance obtained with CFD numerical simulations in the case of water and non-Newtonian fluids. Processing non-Newtonian fluids, the pump shaft power increases while the efficiency and pump head decrease. Increasing the apparent viscosity at high shear rate (higher values of k and n), pump head and efficiency decrease while the shaft power increases. Comparing Pump 1 and 2 it is clear how the same fluid implies different performance modification (fluids K10N0.25, K10N0.5 and K10N0.75). Pump 1 appears more sensible to the fluid viscosity. In particular, the non-Newtonian behavior implies greater performance deration for the Pump 1 characterized by lower values of specific speed. For Pump 1 in the case of fluid K2N0.25, the pump head increases respect to water showing a phenomenon similar to the sudden rising head effect (Li 2011) obtained with newtonian fluid slightly more viscous than water and due to the low specific speed value as reported by Lazarkiewicz and Troskolanski (1965).

Shear Rate Analysis

The analytical methods presented before rely on the estimation of a representative value of shear rate to calculate the viscosity. CFD analysis represents an useful tool able to estimate this quantity, strongly related to the internal flow field and not measurable by means of experimental methods. Centrifugal pumps are characterized by a three-dimensional fluid structures that imply local variation of the shear rate values and in turn, local variation of the apparent viscosity with non-Newtonian fluids. As reported in Buratto et al. (2015) internal flow structure and local three-

Table 1: Fluid characteristics divided according to the type of pump

Pump 1			Pump 2		
Fluid	***k* [Pas^n]**	***n***	**Fluid**	***k* [Pas^n]**	***n***
K2N0.25	2	0.25	K10N0.25	10	0.25
K2N0.5	2	0.50	K10N0.5	10	0.50
K2N0.75	2	0.75	K10N0.75	10	0.75
K6N0.25	6	0.25	K100N0.25	100	0.25
K6N0.5	6	0.50	K100N0.5	100	0.50
K6N0.75	6	0.75	K100N0.75	100	0.75
K10N0.25	10	0.25	K200N0.25	200	0.25
K10N0.5	10	0.50	K200N0.5	200	0.50
K10N0.75	10	0.75	K200N0.75	200	0.75

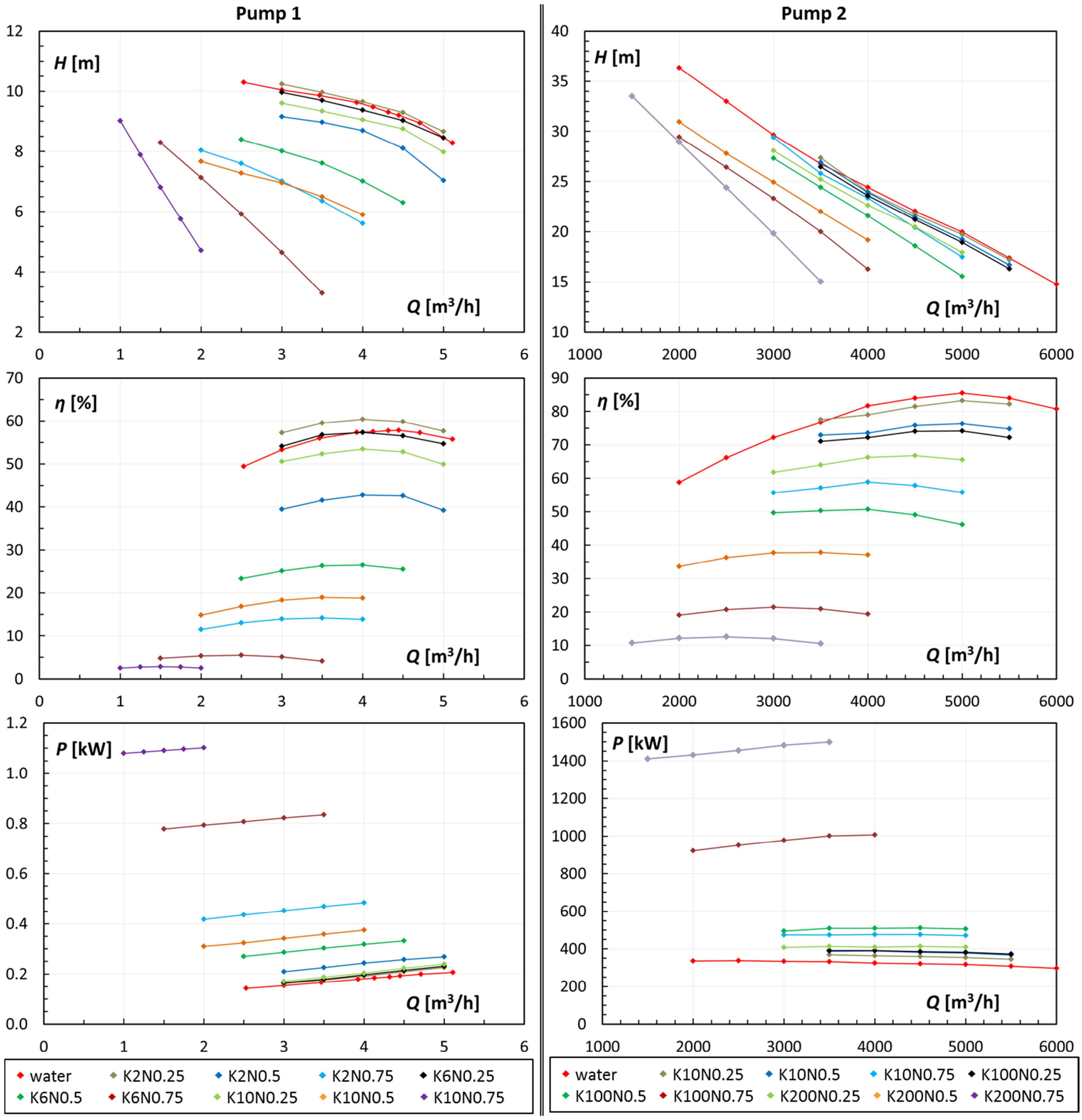


Figure 4: CFD Pump performance according to fluid characteristics

dimensional phenomena like tip leakage are related to the fluid rheology. Figure 5 reports the apparent viscosity on the impeller surface for both pumps. Table 2 reports the average shear rate values according to the pump regions (indicated in Fig. 6) and for five values of flow rate for each pump. The average shear rate value assumes higher value close to the walls, and as consequence, the average apparent viscosity assumes the lowest values in these regions. By contrast, higher values of average apparent viscosity is reached in the core of the volume (far from the walls) especially in the volute region. Finally, average shear rate values increase as a function of the flow rate provided by the pump in both cases but for both pumps the average shear rate on the blade tip and impeller back plate is not varying with respect to the flow rate. The independency of the back plate shear rate values from the pump flow rate are in line with those reported in literature (Lazarkiewicz and Troskolanski, 1965) highlighting that the CFD solution are reliable and representative of the actual pump operation.

For Pump 1 the average shear rate is also constant on the trailing edge. Comparing the same flow

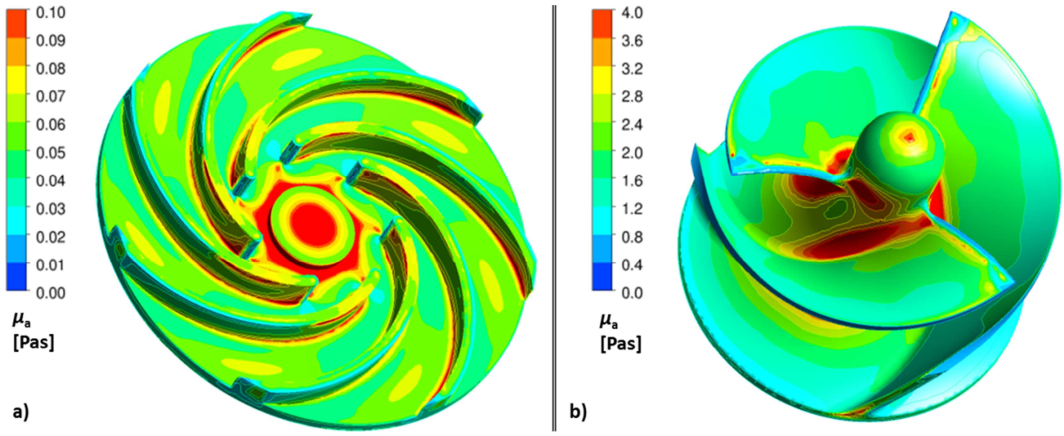


Figure 5: Apparent viscosity on the impeller: a) Pump 1 processing the fluid K6N0.5 at 4 m^3/h, b) Pump 2 processing the fluid K100N0.5 at 4000 m^3/h

Table 2: Average shear rate [s^{-1}] calculated through CFD simulations on the volumes and surfaces of the two pumps varying the flow rate

Pump	Fluid	Q [m^3/h]	volumes: Impeller volume	volumes: Volute volume	surfaces: Blade	surfaces: Hub	surfaces: Impeller back plate	surfaces: Leading edge	surfaces: Trailing edge	surfaces: Blade tip
1	K6N0.75	1.5	2207	685	4646	1477	5073	4306	10224	14876
1	K6N0.75	2.0	2231	760	4830	1677	5073	5491	10259	14906
1	K6N0.75	2.5	2258	831	5023	1885	5073	6639	10286	14941
1	K6N0.75	3.0	2287	902	5229	2100	5073	7767	10412	14988
1	K6N0.75	3.5	2318	974	5427	2317	5073	8870	10344	15022
2	K200N0.5	2000	248	44	2635	835	1660	8212	3768	25273
2	K200N0.5	2500	242	50	2818	1067	1633	8510	3709	25274
2	K200N0.5	3000	243	54	3021	1348	1633	9505	3888	25200
2	K200N0.5	3500	251	56	3208	1681	1646	10260	4322	25096
2	K200N0.5	4000	255	59	3396	1955	1664	11101	4248	24868

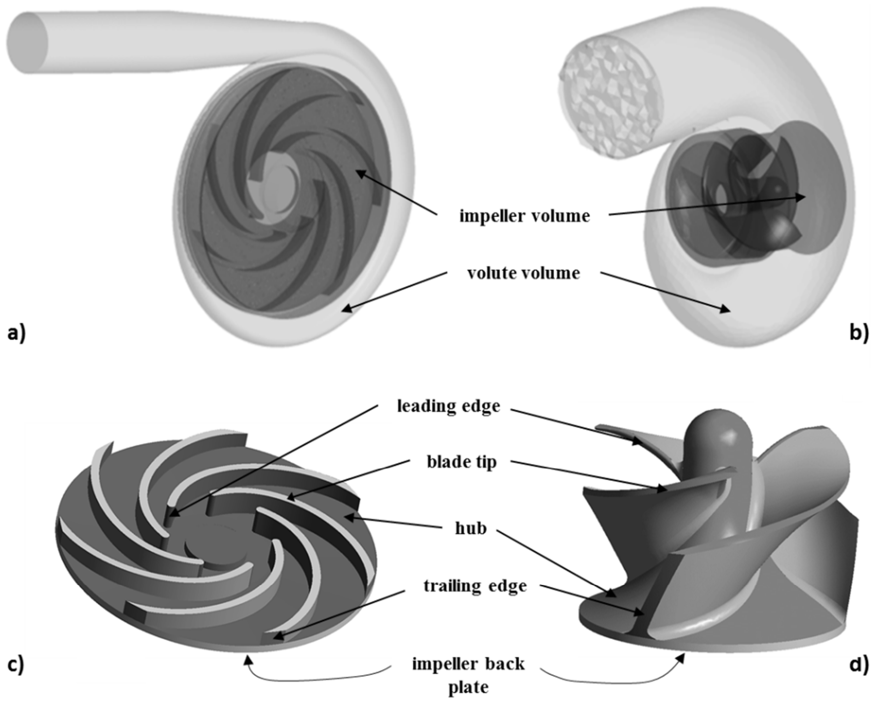


Figure 6: Reference names for parts of the pumps: a) names of volumes of Pump 1, b) names of volumes of Pump 2, c) names of surfaces of Pump 1, d) names of surfaces of Pump 2

rate varying the fluid characteristics (variation of the values of indexes *k* and *n*) other considerations can be done. Table 3 and 4 reports this analysis highlighting that by increasing the values of *k* and *n* the average shear rate decreases for both pumps. The last analysis refers to the comparison between Pump 1 and 2 operating with the same fluid (K10N0.75) at the BEP. Table 5 shows that in Pump 1 greater values of average shear rate are calculated for the volumes (impeller and volute) while lower values can be found closer to the walls respect to Pump 2. This means that the shear rate pattern, and in turn, the viscosity, depends on the pump.

PUMP PERFORMANCE ESTIMATION USING THE HYDRAULIC INSTITUTE METHOD

In the last part of this work, the analytical models proposed by Walker and Goulas (1984) and Pullum et al. (2007) are applied to both pumps. The viscosity estimated by these models was used in the Hydraulic Institute method (ANSI/HI 9.6.7, 2004) for predicting the pump performance. The outcome of this methods is compared to the CFD numerical results above presented.

Walker and Goulas Method

In this paper the plastic viscosity was evaluated at the shear rate of 1500 s^{-1}, as suggested by Walker and Goulas (1984). In Fig. 7 the comparison between the CFD results and the model

Table 3: Average shear rate [s^{-1}] calculated through CFD simulations on the volumes and surfaces of the two pumps at constant value of *n* and flow rate (3.5 m^3/h for Pump 1 and 4000 m^3/h for Pump 2) while varying *k*

Pump	Fluid	volumes		surfaces					
		Impeller volume	Volute volume	Blade	Hub	Impeller back plate	Leading edge	Trailing edge	Blade tip
1	K2N0.5	3657	1496	35099	22620	47332	124094	72175	32193
1	K6N0.5	3320	1343	14689	11291	23125	58549	31364	16901
1	K10N0.5	3015	1168	9736	7970	16696	39481	20466	15432
2	K10N0.5	329	78	18167	10250	12789	37423	21762	60803
2	K100N0.5	265	68	4792	2778	2689	15263	4876	31192
2	K200N0.5	255	59	3396	1955	1664	11101	4248	24868

Table 4: Average shear rate [s^{-1}] calculated through CFD simulations on the volumes and surfaces of the two pumps at constant value of *n* and flow rate (3.5 m^3/h for Pump 1 and 4000 m^3/h for Pump 2) while varying *n*

Pump	Fluid	volumes		surfaces					
		Impeller volume	Volute volume	Blade	Hub	Impeller back plate	Leading edge	Trailing edge	Blade tip
1	K6N0.25	3193	1389	141425	83188	168933	280564	235138	136651
1	K6N0.5	3320	1343	14689	11291	23125	58549	31364	16901
1	K6N0.75	2318	974	5427	2317	5073	8870	10344	15022
2	K100N0.25	311	75	22012	13003	16901	45963	25669	66689
2	K100N0.5	265	68	4792	2778	2689	15263	4876	31192
2	K100N0.75	247	56	1927	858	957	4390	2094	20000

Table 5: Comparison average shear rate [s^{-1}] with K10N0.75 at *BEP* on the volumes and surfaces of every pump

Pump	volumes		surfaces					
	Impeller volume	Volute volume	Blade	Hub	Impeller back plate	Leading edge	Trailing edge	Blade tip
1	2072	516	4015	960	3729	4243	10107	13483
2	282	80	4591	2626	4713	12018	3788	25396

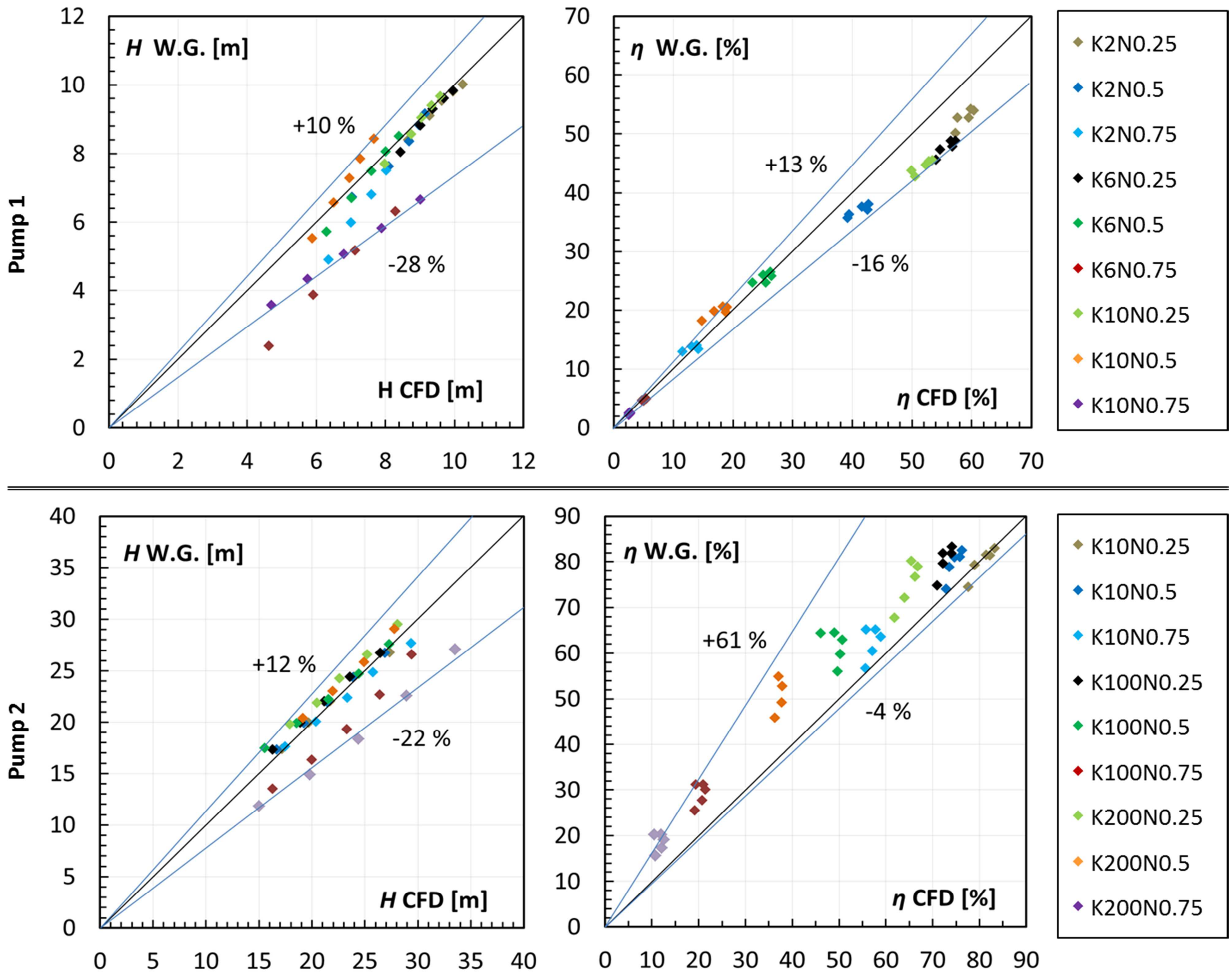


Figure 7: Comparison between CFD and Walker and Goulas (W.G.) method results

prediction with Walker and Goulas method is reported. For Pump 1, the 95 % of the Walker and Goulas points for pump head is comprises in the range [-28 %, 10 %] respect to the CFD results while for the Pump 2 this range assumes the values of [-22 %, 12 %]. The pump efficiency has different values for this range, in particular, Pump 1 has a range of [-16 %, 13 %] while Pump 2 has a range equal to [-4 %, 61 %]. If the pump works with a fluid which is responsible for a great decrease of performance respect to the water, the difference between CFD and Walker and Goulas results is greater, especially for the fluids K2N0.75, K6N0.75 and K10N0.75 for Pump 1 and K100N0.75 and K200N0.75 for Pump 2 (see Fig. 4 for more details about performance characteristics).

Pullum et al. Method

The Pullum et al. (2007) method is based on the estimation of *w*. In this work *w* was obtained with an iterative method based on guessing the *w* value used in the head prediction by the method of Pullum et al (2007) in order to minimize the difference with respect to the pump head calculated via CFD simulations. This procedure was carried out for a chosen fluid. The minimization of the difference was based on the least squares error calculation and the *w* obtained at the end of the procedure was used to predict the performance with the other fluids. Figure 8 reports the comparison between CFD results and the model predictions. The Pullum points of Pump 1 are obtained using a characteristic dimension equal to 0.00652 m (w/D_{imp} = 6.8 %) based on the head CFD obtained with the fluid K6N0.5. The Pullum points of Pump 2 are obtained using a characteristic dimension equal to 0.0915 m (w/D_{imp} = 15 %) based on the head CFD obtained with the fluid K100N0.5. For Pump 1, the 95 % of the Pullum points for pump head is comprises in the

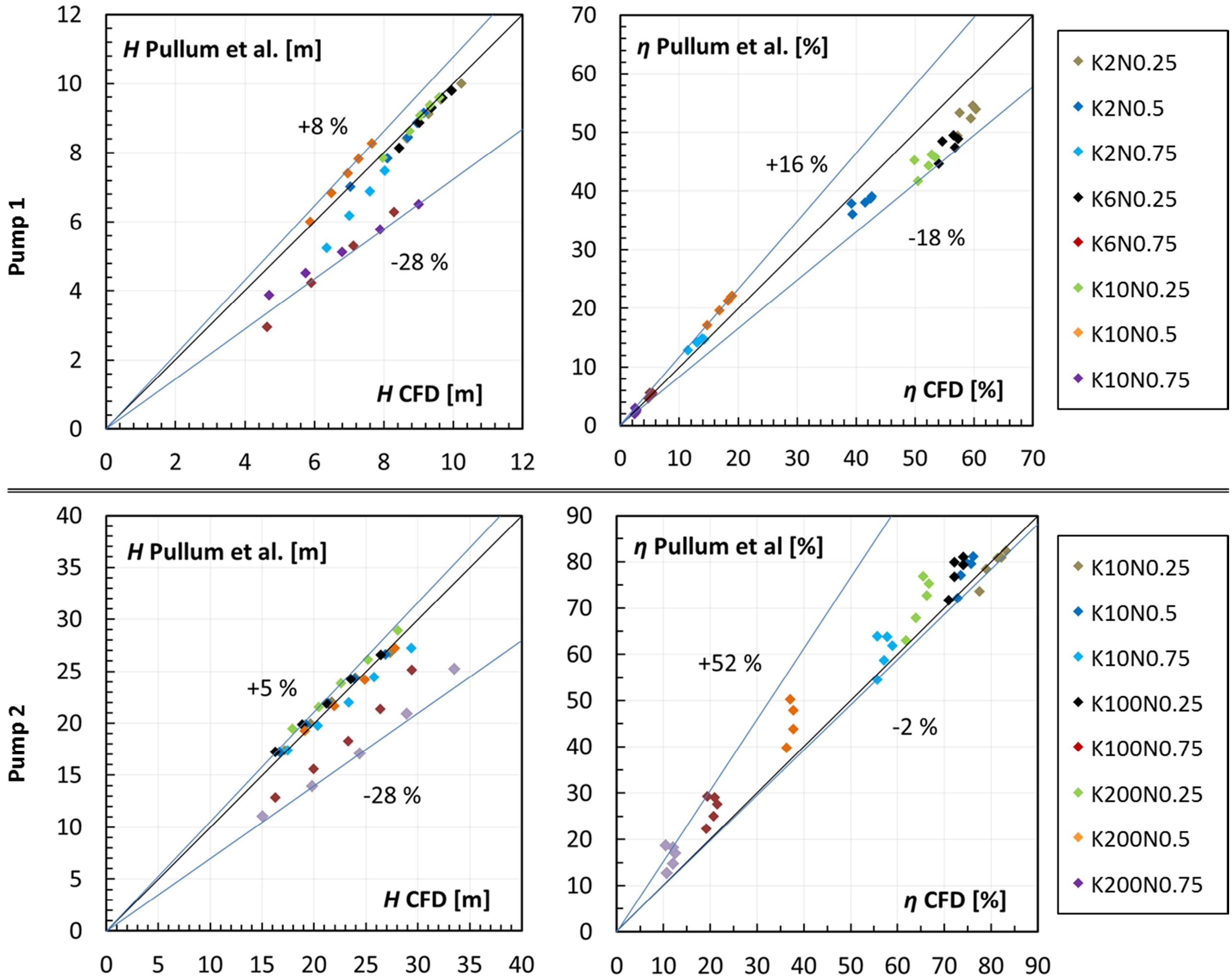


Figure 8: Comparison between CFD and Pullum et al. method results

range of [-28 %, 8 %], while for the Pump 2 this range assumes the values of [-28 %, 5 %]. The pump efficiency has different values for this range, in particular, Pump 1 has a range of [-18 %, 16 %] while Pump 2 has a range equal to [-2 %, 52 %]. If the pump works with a fluid which is responsible for a great decrease of performance respect to the water, the difference is greater, especially for the fluids K2N0.75, K6N0.75 and K10N0.75 for Pump 1 and K100N0.75 and K200N0.75 for Pump 2 (see Fig. 4 for more details about performance characteristics).

Remarks and Criticisms

Table 6 summarizes the comparison between CFD and analytical methods. Both methods provide almost same variations with respect to the CFD results even if the two pumps are very different in terms of size and power. Walker and Goulas (1984) and Pullum et al. (2007) methods are in agreement with CFD in the case of fluid in which the pump derating is low.

Walker and Goulas (1984) approach uses the value of the plastic viscosity, obtained at the highest value of the shear rate tested during the rheological characterization of the fluid, into the HI method, without providing a clear motivation for this choice. In this approach the fluid is modeled as a Bingham non-Newtonian fluid where the estimation of the representative viscosity of the fluid does not take into account the yield stress but only the plastic viscosity. Walker and Goulas (1984) assuming 1500 s^{-1} as a representative value of shear rate, obtained a confidence band of 5 % of analytical predictions respect to experimental results, but other authors (Sery and Slatter, 2002 and Kalombo et al., 2014) found an even wider confidence band. In the present work the variation obtained between the CFD results and the model prediction is higher than 5 %. This is probably due to the fact that the flow field inside the centrifugal pump is three-dimensional and the shear rate

Table 6: Summary of the error margin respect CFD obtained by Walker and Goulas and Pullum et al. approach

	Walker and Goulas	Pullum et al.
Head – Pump 1	[-28 %, 10 %]	[-28 %, 8 %]
Head - Pump 2	[-22 %, 12 %]	[-28 %, 5 %]
Efficiency - Pump 1	[-16 %, 13 %]	[-18 %, 16 %]
Efficiency - Pump 2	[-4 %, 61 %]	[-2 %, 52 %]

changes according to the pump geometry, flow rate, rotational velocity and fluid characteristics. For these reasons, the value of shear rate at which refers the calculation of the plastic viscosity is to be intended as a representative value, hardly generalizable and specific for each studied case.

Pullum et al. (2007) approach uses the value of the apparent viscosity, into the HI method. The apparent viscosity seems to be more suitable with respect to the plastic viscosity that has no fundamental rheological meaning as reported by Graham et al. (2009). The shear rate value that corresponds to the apparent viscosity is estimated using the Rabinowitsch-Mooney correlation able to take into account the characteristic dimension of the pump. This correlation takes into account the shear rate variation related to the variation of the pump flow rate and the fluid characteristics (through the exponent n). This method appears more suitable for taking into account different aspect of pump and fluids, but at the same time does not consider the influence of the coefficient k that, in the same way of the exponent n, modifies the pump performance as reported in the present work.

The analysis reported in the present paper shows also that the pump characteristic dimension, and then the hydraulic diameter of the Rabinowitsch-Mooney equation, depends on the fluid characteristics (see Tables 7 and 8). This evidence is in contrast with the method reported by Pullum et al. (2007) which use the same values of w for all type of fluid. For example, in the case of Pump 1, with the fluid K10N0.5, w is three times greater than the case with the fluid K6N0.75 and for the Pump 2, with the fluid K10N0.25, w is eleven times greater than the case with the fluid K200N0.75. The present analysis demonstrates that even in the case of same fluids (K10N0.25, K10N0.5 e K10N0.75) the ratio w/D_{imp} assumes different values for the two pumps (as reported in the Tables 7 and 8), in agreement with Kalombo et al. (2014) and Furlan et al. (2014) and contrarily with those reported by Graham et al. (2009) which assumes that the relation w/D_{imp} = 25 % represents an universal relation that could be used for any combination of pump and fluid. Therefore, as a function of the hydraulic diameter, the value of shear rate and in turn, the value of the apparent viscosity changes according to the fluid characteristics.

In the present work, the variation between the prediction obtained by the Pullum method and the CFD results is greater with respect to those reported in literature (Pullum et al., 2007, Graham et al., 2009 and Kalombo et al., 2014) and it is probably due to the sensible difference between the rheological behavior of the fluids simulated which needs different values of characteristic dimension of the pump to be properly represented.

Table 7: Pump 1 - Characteristic dimension and equivalent duct diameter

	K2N0.25	K2N0.5	K2N0.75	K6N0.25	K6N0.5	K6N0.75	K10N0.25	K10N0.5	K10N0.75
w [m]	0.00300	0.00590	0.00352	0.00484	0.00652	0.00285	0.00623	0.00750	0.00293
D_h [m]	0.00594	0.01160	0.00696	0.00953	0.01280	0.00565	0.01220	0.01460	0.00580
w/D_{imp} [%]	3.1	6.1	3.7	5.1	6.8	3.0	6.5	7.9	3.1

Table 8: Pump 2 - Characteristic dimension and equivalent duct diameter

	K10N0.25	K10N0.5	K10N0.75	K100N0.25	K100N0.5	K100N0.75	K200N0.25	K200N0.5	K200N0.75
w [m]	0.2290	0.1460	0.0397	0.1460	0.0915	0.0267	0.1380	0.0842	0.0215
D_h [m]	0.4090	0.2710	0.0778	0.2710	0.1750	0.0527	0.2580	0.1610	0.0425
w/D_{imp} [%]	37.5	23.9	6.5	23.9	15.0	4.3	22.6	13.8	3.5

CONCLUSIONS

In this paper a detailed analysis of the analytical methods for predicting the pump performance operating with non-Newtonian fluids is reported. These models allow for the calculation of the viscosity value used in the Hydraulic Institute methods. These methods has been developed to provide the modification of the pump head and efficiency according to the fluids characteristics. The model evaluation is based on CFD numerical analysis related to two types of centrifugal pump that differ from size and specific speed. The numerical results, have shown that centrifugal pumps with greater value of specific speed are much less sensitive to the fluid viscosity.

The model prediction for the pump performance is based on the shear rate values. CFD analysis has shown that the shear rate varies from region to region inside the pump, and it is very difficult to define an unique value that represents the pump operating condition. In addition, these values are subjected by pump flow rate and fluid characteristics. For these reasons, in the last part of this work a detailed comparison is proposed in order to establish pros and cons of the analytic methods. Both methods shown results comparable to CFD ones, but at the same time, some model parameters are considered constant and not dependent to the pump specific speed and fluids characteristics as state instead by CFD results.

In the future, based on the CFD tools, analytical models could be improved becoming an even more useful tool during the pump design process or, from the user point of view, a ready-to-use calculation able to link the pump performance obtained for water into pump performance representative to the pump operation with non-Newtonian fluid. Further development, using CFD transient calculation, will be devoted to discover the modification of the radial equilibrium related to the different rheology of the processed fluid in order to enhance the reliability of the non-Newtonian pump design.

REFERENCES

Chabra R. P., Richardson J.F., (1999), *Non-Newtonian Flow In The Process Industries*, First Edition, Butterworth-Heinemann, Great Britain

Walker C. I., Goulas A., (1984), *Performance Characteristics of Centrifugal Pumps When Handling Non-Newtonian Homogeneous Slurries*, Proc. Inst. Mech Eng., Part A, **198**(A), pp. 41–49

Pullum L., Graham L. J. W., Rudman M., (2007), *Centrifugal Pump Performance Calculation for Homogeneous and Complex Heterogeneous Suspensions*, Journal of the Southern African Institute of Mining and Metallurgy, **107**, June 2007, pp. 373–379

Sery G., Kabamba B., Slatter P., (2006), *Paste Pumping with Centrifugal Pumps: Evaluation of the Hydraulic Institute Chart De-Rating Procedures*, Proceedings of the Ninth International Seminar on Past and Thickened Tailings 3-7 April 2006, Limerick Ireland. Australian Centre for Geomechanics, Perth, 2006. pp. 403-412

Graham L. J. W., Pullum L., Slatter P., Sery G., Rudman M., (2009), *Centrifugal Pump Performance Calculation for Homogeneous Suspensions*, Can. J. Chem. Eng., **87**, pp. 526–533

Buratto C., Pinelli M., Spina P. R., Vaccari A., Verga C., (2015), *CFD Study on Special Duty Centrifugal Pumps Operating with Viscous and Non-Newtonian Fluids*, Proceedings of 11thEuropean Conference on Turbomachinery Fluid dynamics & Thermodynamics ETC11, March 23-27, 2015, Madrid, Spain

Sery G.A., Slatter P. T., (2002), *Centrifugal pump derating for non-Newtonian slurries*, Proc. Hydrotransport *15*, BHR Group Limited, 2002, vol. **II**, pp.679–692

Kalombo J.J. N., Haldenwang R., Chhabra R.P., Fester V.G., (2014), *Centrifugal Pump Derating for Non-Newtonian Slurries*, Journal of Fluids Engineering, **136**, p. 031302

Pullum L., Graham L., Wu J., (2011), *Centrifugal pump performance with non-newtonian slurries*, T&S 15th International Conference on transport and sedimentation of solid particles 6-9 September 2011, Wroclaw, Poland

Furlan J., Visintainer R., Sellgren A., (2014), *Centrifugal pump performance when handling highly non-newtonian clays and tailings slurries* , BHR Group's 19th International Conference on Hydrotransport, Golden, Colorado, USA 24-26 September, 2014

Metzner A. B., Otto R.E., (1957), *Agitation of Non-Newtonian Fluids*, AlChE Journal, **3**(1), 1957, pp. 3-10

Allali A., Belbachir S., Lousdad A., Merahi L., (2015), *Numerical approach based design of centrifugal pump volute*, Mechanika, **21**, pp 301-306

Ye D. X., Li H., Zou C. H., Jiang B., (2015), *Characterization of the fluidization of medium consistency pulp suspensions in a centrifugal pump*, WIT Transactions on Engineering Sciences 2015, **89**, pp. 67-76

Zubanov, V. M., Shabliy, L. S., Krivcov, A. V., (2015), *Rotational Technique for Multistage Centrifugal Pump CFD-Modeling*, ASME Paper GT20115-42070

Song, X., Wood, H. G:, Day, S. W:, Olsen, D. B., (2003), *Studies of Turbulence Models in a Computational Fluid Dynamics Model of a Blood Pump*, Proceedings of 10th Congress of the International Society of Rotary Blood Pumps, September 11-14, 2002, Osaka, Japan

Li W.G., (2011), *Effect of exit blade angle, viscosity and roughness in centrifugal pumps investigated by CFD computation*, Task Quaterly (2011) 15 No1 pp. 21-41

Lazarkiewicz S., Troskolanski A., (1965), *Impeller Pumps*, Translated from Polish by D. K. Rutter, Pergamon Press

ANSI/HI 9.6.7, (2004), *Effects of Liquid Viscosity on Rotodynamic (Centrifugal and Vertical) Pump Performance*, Hydraulic Institute, Parsippany